String Matching: Boyer-Moore algorithm

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Programming, Data Structures and Algorithms using Python

Week 10

Speeding up the brute force algorithm

- Text `t`, pattern `p` of of lengths n, m
- For each starting position `i` in `t`, compare `t[i:i+m]` with `p`
  - Scan `t[i:i+m]` right to left

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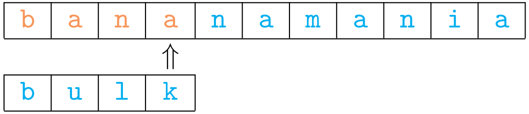
- While matching, we find a letter in `t` that does not appear in `p`
    - `t = bananamania`, `p = bulk`



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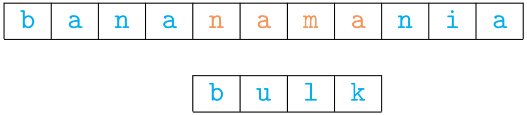
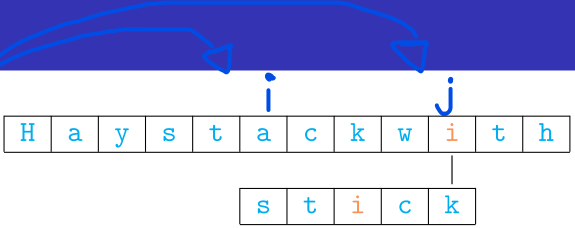
- While matching, we find a letter in `t` that does not appear in `p`
  - `t = bananamania`, `p = bulk`
- Shift the next scan to position after mismatched letter
- What if the mismatched letter does appear in `p`?



Sliding the search

- Suppose `c = t[i+j] != p[j]`, but `c` does occur somewhere in `p[j]`

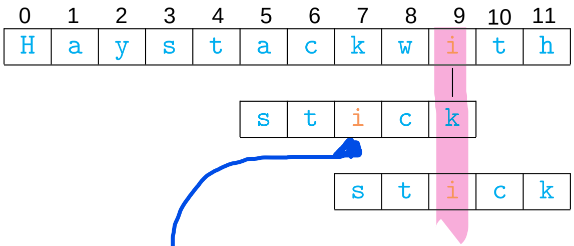
i - index of text
j - index of pattern



1. We find "i" != "k"
2. But "i" is present in pattern (p)
3. So, shifting slice by one does not make sense as we want to get "i" (in p) aligned with "i" (in t)

Sliding the search

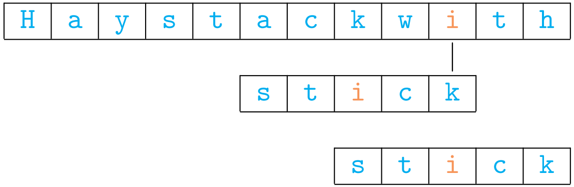
- Suppose c = t[i+j] != p[j], but c does occur somewhere in p[j]
- Align rightmost occurrence of c in p with t[i+j]



1. Here t[i+j] = "i"
2. Rightmost occurrence of "i" in p
3. i = 5, j = 4

Sliding the search

- Suppose `c = t[i+j] != p[j]`, but `c` does occur somewhere in `p[j]`
- Align rightmost occurrence of `c` in `p` with `t[i+j]`
- Scan this substring of `t` next



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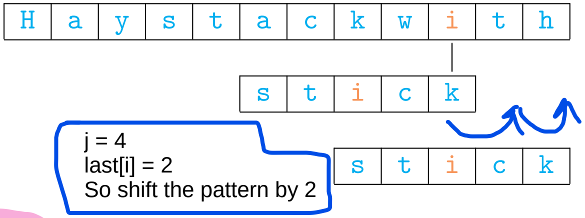
  - Shift pattern by `j - last[c]`



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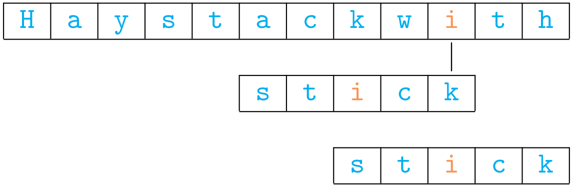
- If `c` not in `p`, shift pattern by `j+1`



We saw this case in last lecture where "a" was not present in p "bulk" so we shifted the pattern by 4

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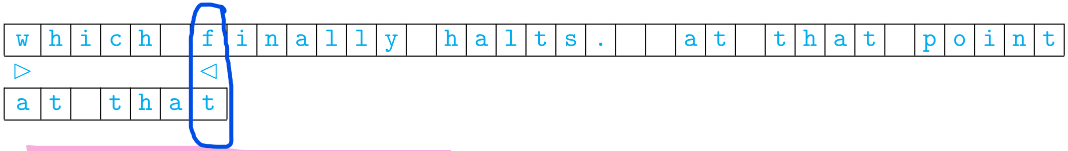

w h i c h   f i n a l l y   h a l t s .   a t   t h a t   p o i n t
▷ ◁
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We start comparing from RIGHT to LEFT

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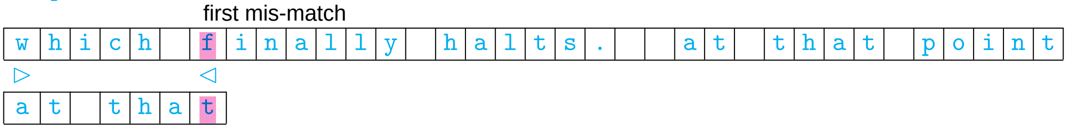
first mis-match
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▷ ◁
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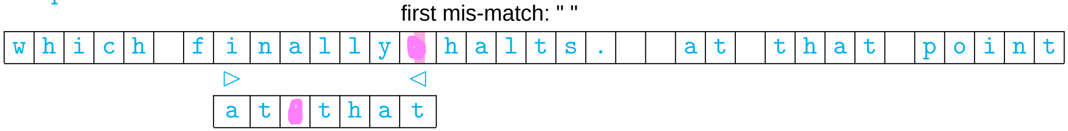

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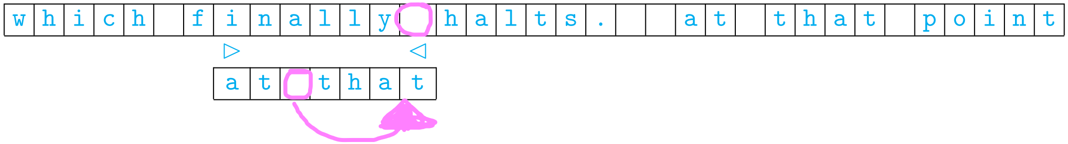

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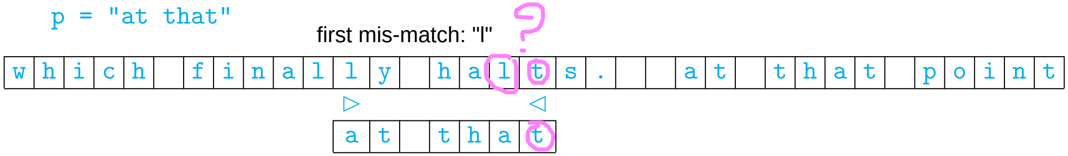

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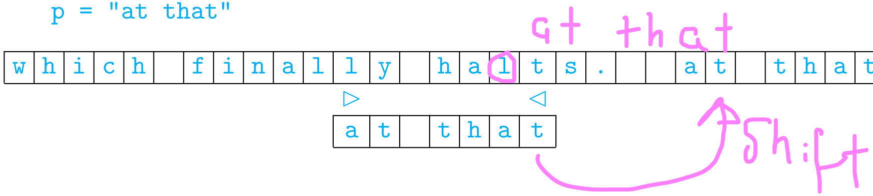

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at that
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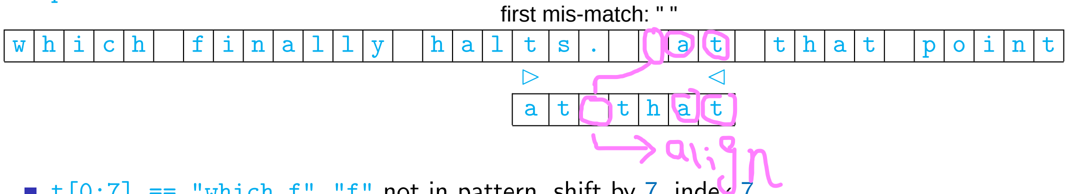

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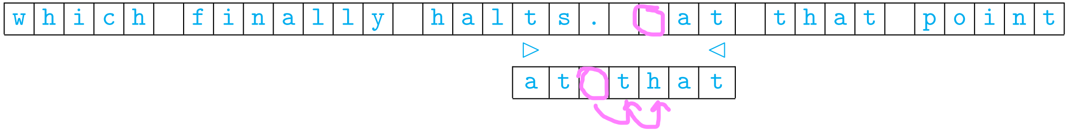

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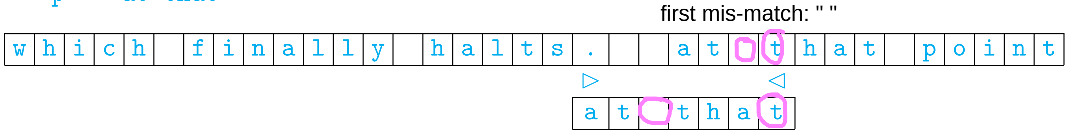

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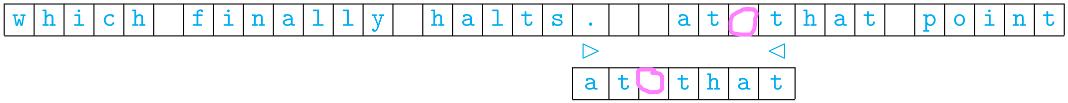

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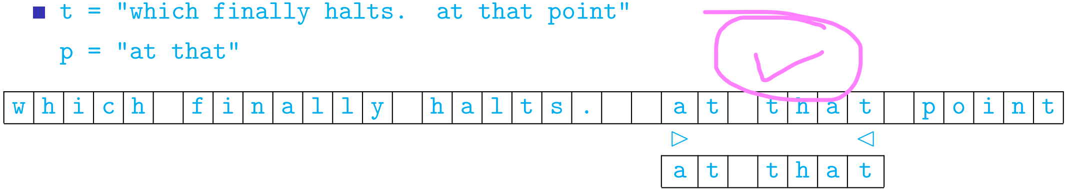

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NOTE: We cannot stop checking from here, as there could be more matches in the text.
NOTE: We shift by one as there can be OVERLAPPING pattern. Example t = "aaa" p = "aa"

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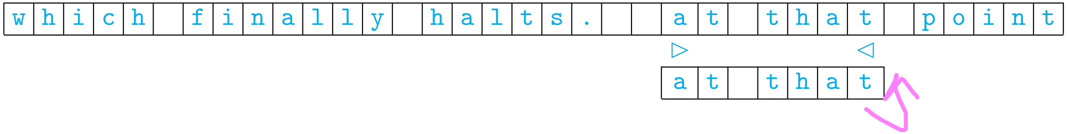
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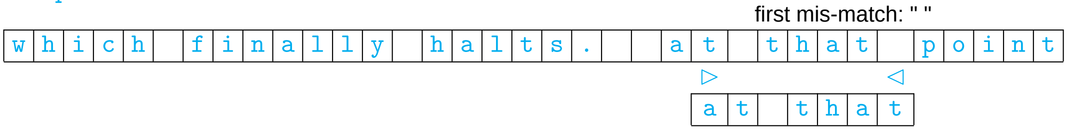

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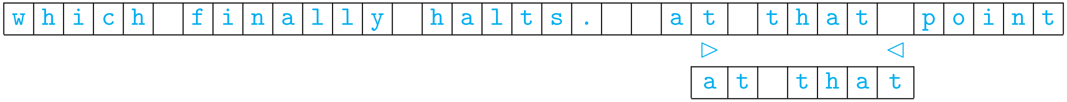

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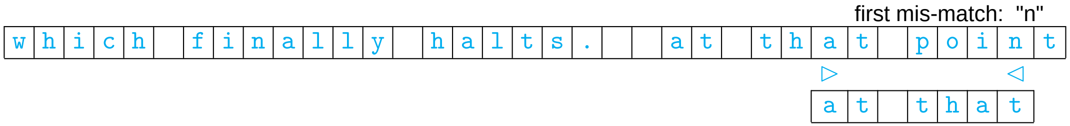

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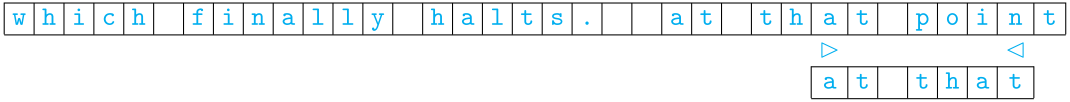

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- `t[27:34] == "at poin"`, `"n"` not in pattern, shift by 7, index 34, stop

Implementation

- Initialize `last[c]` for each `c` in `p`
    - Single scan, rightmost value is recorded

```
def boyermoore(t,p):
  last = {}                    # Preprocess
  for i in range(len(p)):
    last[p[i]] = i

  poslist,i = [],0             # Loop
  while i <= (len(t)-len(p)):
    matched,j = True,len(p)-1
    while j >= 0 and matched:
      if t[i+j] != p[j]:
        matched = False
      j = j - 1
    if matched:
      poslist.append(i)
      i = i + 1
    else:
      j = j + 1
      if t[i+j] in last.keys():
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      else:
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  return(poslist)
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- Nested loop, compare each segment `t[i:i+len(p)]` with `p`
- If `p` matches, record and shift by `1`

As we decremented j after mismatch was found. So, we need to restore it, so increment j again by 1

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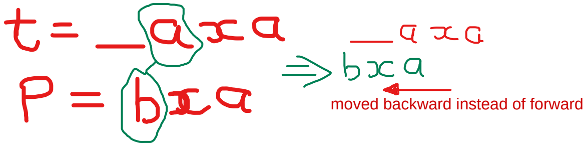

1. The first mis-match occurs at "a", not matching with "b"
2."a" is there in p but it is in front
3. In this case aligning p[j] and t[i+j] would move p backwards
4. We don't want this because "a" in "bxa" was already aligned in the past

- We find a mismatch at `t[i+j]`
  - If `j > last[t[i+j]]`, shift by `j - last[t[i+j]]`
  - If `last[t[i+j]] > j`, shift by `1`
    - Should not shift `p` to left!
  - If `t[i+j]` not in `p`, shift by `j+1`

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  - "Sublinear"
  - Experimentally — English text, 5 character pattern, average number of comparisons is 0.24 per character
  - Performance improves as pattern length grows — more characters skipped

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- Boyer-Moore works well, in practice
  - "Sublinear"
  - Experimentally — English text, 5 character pattern, average number of comparisons is 0.24 per character
  - Performance improves as pattern length grows — more characters skipped
- Often used in practice — `grep` in Unix