Linear Programming: Bandwidth Allocation

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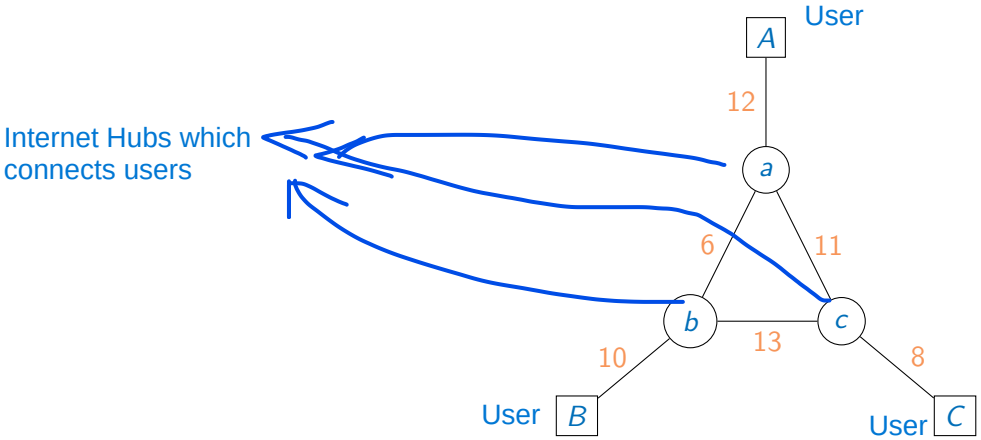
Programming, Data Structures and Algorithms using Python

Week 11

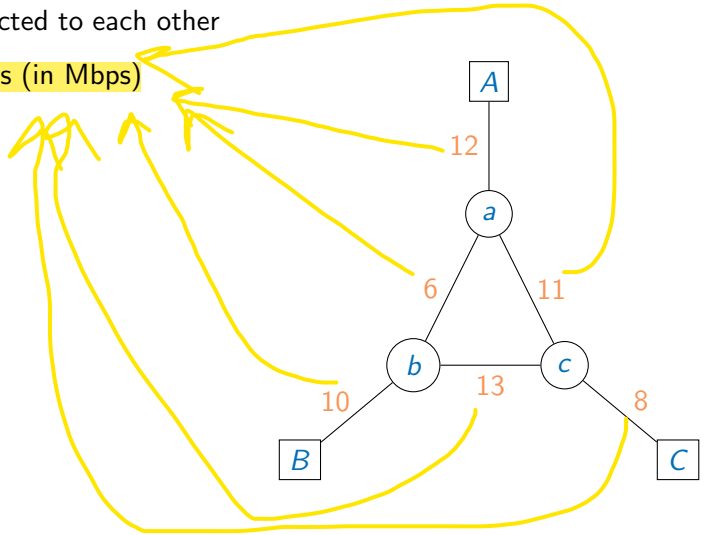
Network bandwidth

- 3 users, A, B, C to be connected to each other



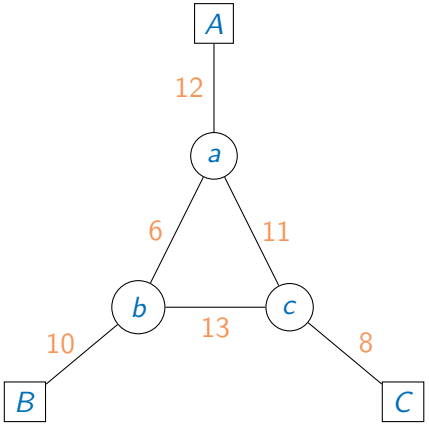
Network bandwidth

- 3 users, A, B, C to be connected to each other
- Link have capacity constraints (in Mbps)



Network bandwidth

- 3 users, A, B, C to be connected to each other
- Link have capacity constraints (in Mbps)
- Each connection A–B, B–C, A–C should have at least 2 Mbps of bandwidth



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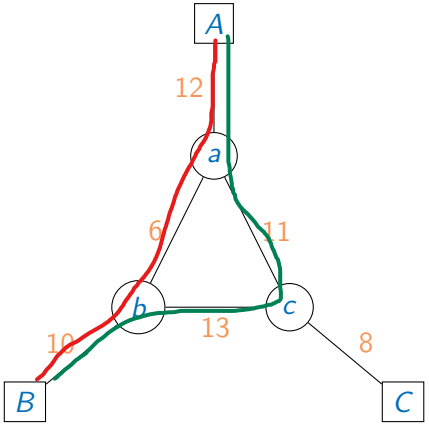

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  - A–a–b–B
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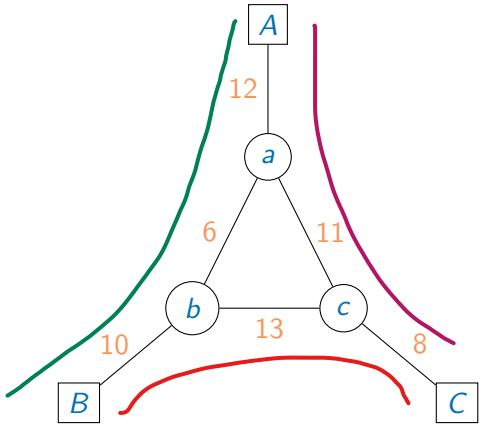
- Each connection earns revenue, per Mbps
  - A–B, Rs 300/Mbps
  - B–C, Rs 200/Mbps
  - A–C, Rs 400/Mbps



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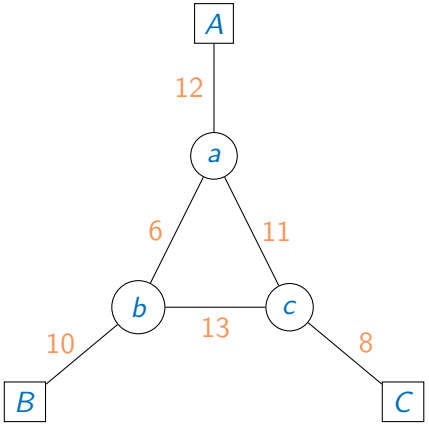
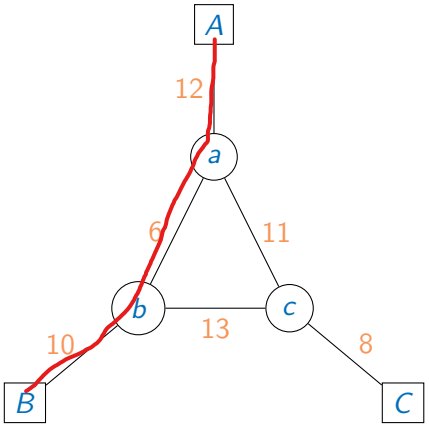
- Allocate bandwidth to maximize revenue



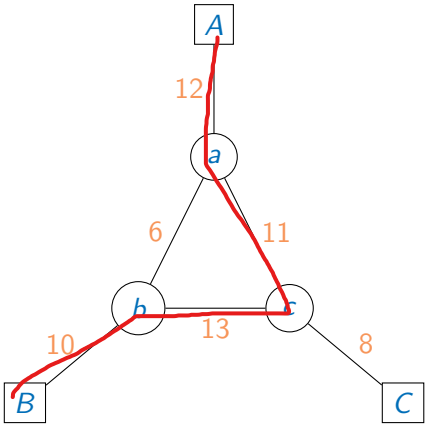
Linear program

- x_{AB} – bandwidth via short connection A–a–b–B,



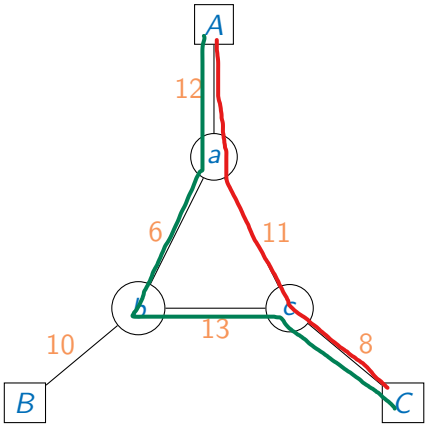
Linear program

- x_{AB} – bandwidth via short connection A–a–b–B,
- y_{AB} – bandwidth via long connection A–a–c–b–B



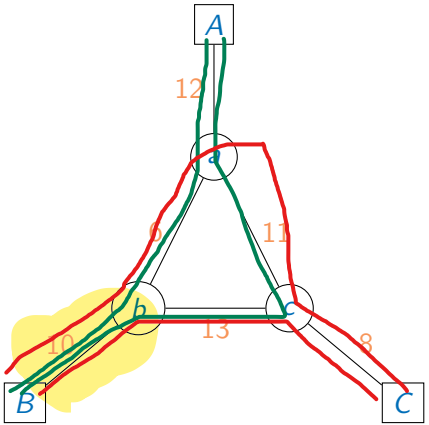
Linear program

- x_{AB} – bandwidth via short connection A–a–b–B,
- y_{AB} – bandwidth via long connection A–a–c–b–B
- Likewise, x_{AC}, y_{AC}, x_{BC}, y_{BC}



Linear program

- x_{AB} – bandwidth via short connection A–a–b–B,
- y_{AB} – bandwidth via long connection A–a–c–b–B
- Likewise, x_{AC}, y_{AC}, x_{BC}, y_{BC}
- x_{AB}, y_{AB} both flow via edge b–B, as do x_{BC}, y_{BC}
  - $x_{AB} + y_{AB} + x_{BC} + y_{BC} \leq 10$



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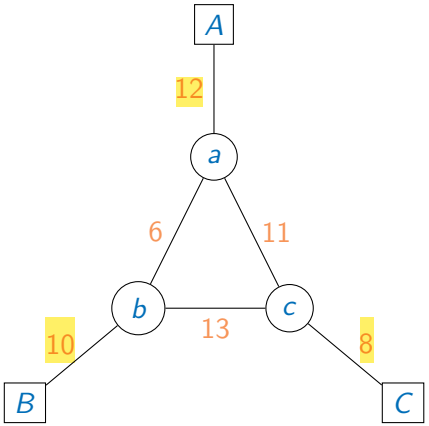
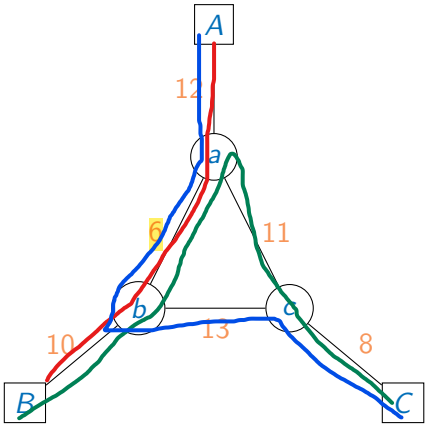
- y_{AB} – bandwidth via long connection A–a–c–b–B
- Likewise, x_{AC}, y_{AC}, x_{BC}, y_{BC}
- x_{AB}, y_{AB} both flow via edge b–B, as do x_{BC}, y_{BC}
  - $x_{AB} + y_{AB} + x_{BC} + y_{BC} \leq 10$
- Likewise
  - $x_{AB} + y_{AB} + x_{AC} + y_{AC} \leq 12$
  - $x_{AC} + y_{AC} + x_{BC} + y_{BC} \leq 8$



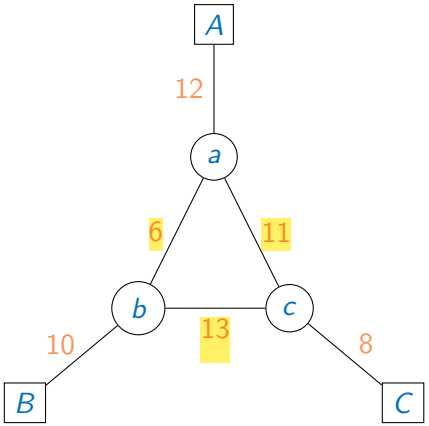
Linear program

- x_{AB}, y_{AC}, y_{BC} all flow via edge a–b
  - $x_{AB} + y_{AC} + y_{BC} \leq 6$



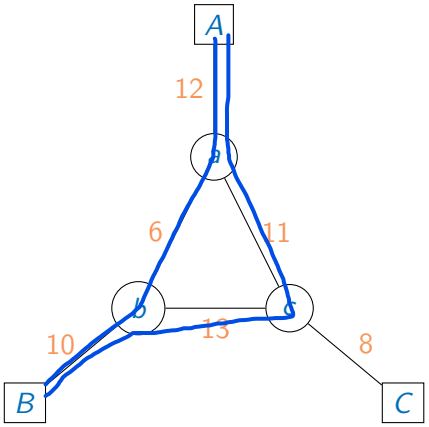
Linear program

- x_{AB}, y_{AC}, y_{BC} all flow via edge a–b
  - $x_{AB} + y_{AC} + y_{BC} \leq 6$
- Likewise
  - $y_{AB} + x_{BC} + y_{AC} \leq 13$
  - $y_{AB} + y_{BC} + x_{AC} \leq 11$



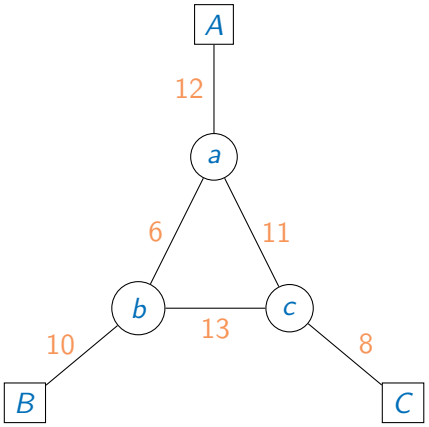
Linear program

- x_{AB}, y_{AC}, y_{BC} all flow via edge a–b
  - $x_{AB} + y_{AC} + y_{BC} \leq 6$
- Likewise
  - $y_{AB} + x_{BC} + y_{AC} \leq 13$
  - $y_{AB} + y_{BC} + x_{AC} \leq 11$
- Pairwise bandwidth at least 2 Mbps
  - $x_{AB} + y_{AB} \geq 2$
  - $x_{BC} + y_{BC} \geq 2$
  - $x_{AC} + y_{AC} \geq 2$



Linear program

- x_{AB}, y_{AC}, y_{BC} all flow via edge a–b
  - $x_{AB} + y_{AC} + y_{BC} \leq 6$
- Likewise
  - $y_{AB} + x_{BC} + y_{AC} \leq 13$
  - $y_{AB} + y_{BC} + x_{AC} \leq 11$
- Pairwise bandwidth at least 2 Mbps
  - $x_{AB} + y_{AB} \geq 2$
  - $x_{BC} + y_{BC} \geq 2$
  - $x_{AC} + y_{AC} \geq 2$
- Traffic on all routes is nonnegative
  - $x_{AB}, y_{AB}, x_{AC}, y_{AC}, x_{BC}, y_{AC} \geq 0$



Objective

- Revenue
  - A–B, Rs 300/Mbps
  - B–C, Rs 200/Mbps
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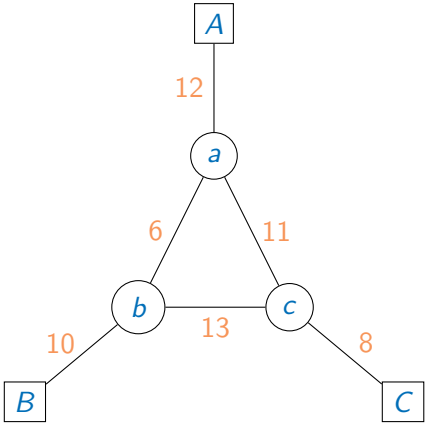

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- Revenue
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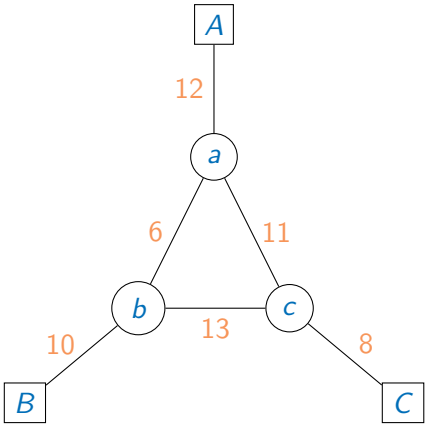
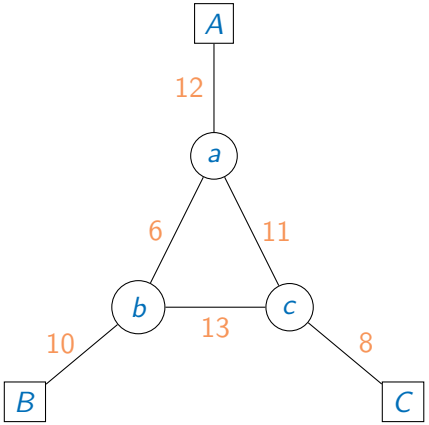
- Maximize
  - $300(x_{AB} + y_{AB}) +$
    $200(x_{BC} + y_{BC})$
    $400(x_{AC} + y_{AC}) +$



Solution

- Simplex yields
  - $x_{AB} = 0$, $y_{AB} = 7$ — 7 A-B
    $x_{BC} = 1.5$, $y_{BC} = 1.5$ — 3 B-C
    $x_{AC} = 0.5$, $y_{AC} = 4.5$ — 5 A-C



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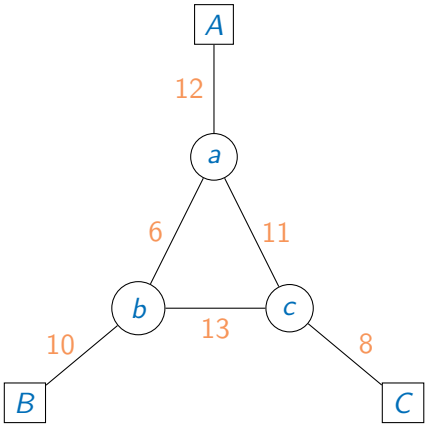

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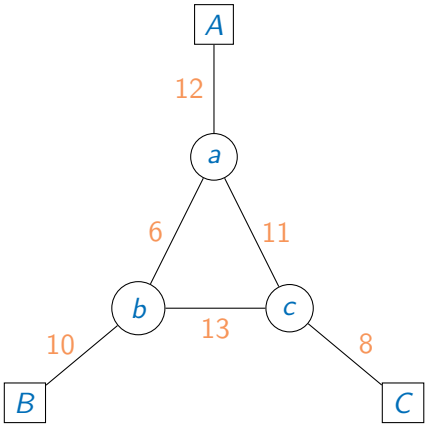

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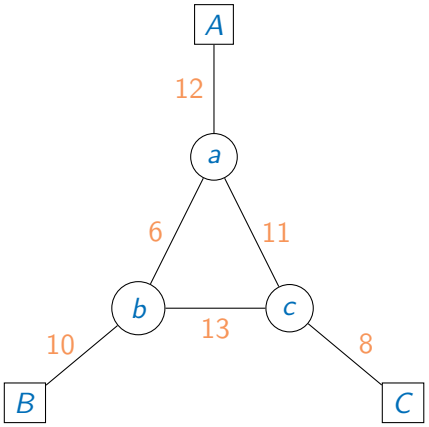

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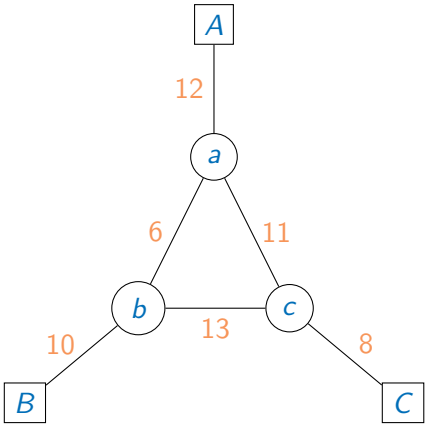

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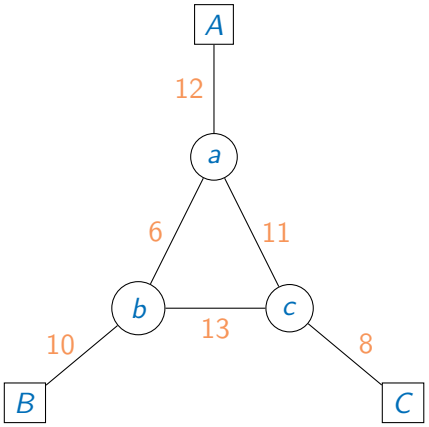
- One variable per path — number of paths is exponential, in general



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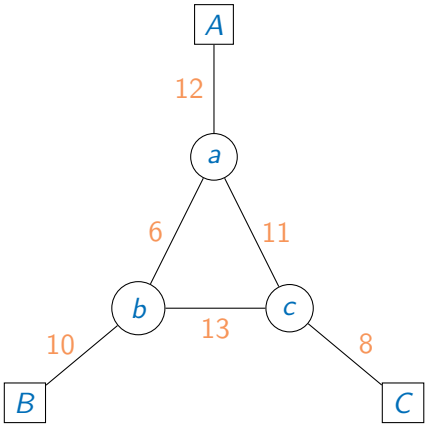

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- Will look at a better approach to analyze such network flows