Edit Distance

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Programming, Data Structures and Algorithms using Python

Week 9

Document similarity

- "The students were able to appreciate the concept optimal substructure property and its use in designing algorithms"
- "The lecture taught the students to appreciate how the concept of optimal substructures can be used in designing algorithms"

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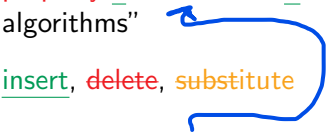

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Edit distance

- Minimum number of edit operations needed

converting "its" to "be" above can be done in several ways.
1. deleting "its" then adding "be"
2. Substitute "ts" with "be" and delete "i" (as done above)

We need to find minimum number of edit operations to achieve the same result

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- Minimum number of edit operations needed
- In our example, 24 characters inserted, 18 ~~deleted~~, 2 ~~substituted~~

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Edit distance

- Minimum number of edit operations needed
- In our example, 24 characters inserted, 18 ~~deleted~~, 2 ~~substituted~~
- Edit distance is at most 44

Edit distance

- Minimum number of editing operations needed to transform one document to the other
  - Insert a character
  - Delete a character
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  - Vladimir Levenshtein, 1965

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- Applications
  - Suggestions for spelling correction
  - Genetic similarity of species

What is Edit Distance?

Measure of similarity between two strings based on the minimum number of operations (insertions, deletions, and substitutions) required to transform one string into the other.

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Edit distance and LCS

- Longest common subsequence of u, v

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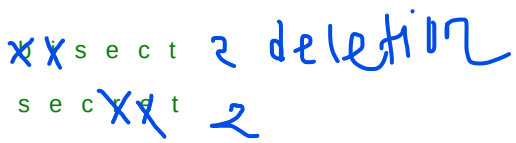
Edit distance and LCS

- Longest common subsequence of u (m), v (n)
  - Minimum number of deletes needed to make them equal

Suppose LCS(u, v) = k

then deletes required to make both equal will be (m-k) for u and (n-k) for v



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- Deleting a letter from u is equivalent to inserting it in v

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  - Delete `b`, `i` in `bisect` and `r`, `e` in `secret`

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Edit distance and LCS

- Longest common subsequence of u, v
  - Minimum number of deletes needed to make them equal
- Deleting a letter from u is equivalent to inserting it in v in terms of edit distance
  - `bisect`, `secret` — LCS is `sect`

method 1
- Delete `b`, `i` in `bisect` and `r`, `e` in `secret` — 4 operations

method 2
- Delete `b`, `i` and then insert `r`, `e` in ~~`b`~~`sect` — 4 operations

As focusing and changing one string (or document) is easier, we focus on method 2

Edit distance

sti ual to the edit distance
between those strings when only
ins s are
cons stitutions.

In other words, if you exclude
is
an orm
one into another will be the
gth of the LCS of the
two strings

Edit distance and LCS

- Longest common subsequence of u, v
  - Minimum number of deletes needed to make them equal
- Deleting a letter from u is equivalent to inserting it in v
  - `bisect`, `secret` — LCS is `sect`
  - Delete `b`, `i` in `bisect` and `r`, `e` in `secret`
  - Delete `b`, `i` and then insert `r`, `e` in `bisect`
- LCS equivalent to edit distance without substitution

Inductive structure for edit distance

- $u = a_0 a_1 \ldots a_{m-1}$
- $v = b_0 b_1 \ldots b_{n-1}$

Inductive structure for edit distance

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- Recall LCS

Inductive structure for edit distance

- $u = a_0 a_1 \ldots a_{m-1}$
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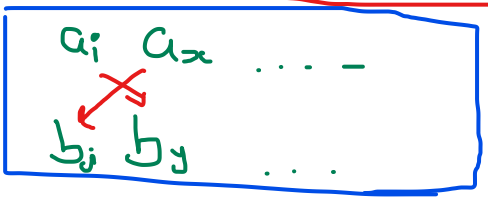
Remember you cannot have both because if you have both a_i and b_j then the LCS looks like this:

u = a_i...a_x...

v = b_j...b_y...

But we know a_x = b_j this means LCS is a_i....a_x... b_j...b_y...
But we also know a_i = b_y this means LCS is a_i...a_x... and b_j...b_y...

Thus both cannot occur toghther



- Recall LCS
- If $a_i = b_j$,
  $LCS(i,j) = 1 + LCS(i{+}1, j{+}1)$
- If $a_i \neq b_j$,
  $LCS(i,j) = \max[\ LCS(i, j{+}1),\ LCS(i{+}1, j)\ ]$

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- Edit distance — aim is to transform u to v

So instead of editing both u and v to make them equal, we will do uni-direction editing. i.e, we will transform u to v to find the EDIT DISTANCE

Inductive structure for edit distance

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  - Substitute a_i by b_j

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- If $a_i = b_j$, $ED(i, j) = ED(i+1, j+1)$

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- Base cases
  - $ED(m,n) = 0$

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- Base cases
  - $ED(m,n) = 0$
  - $ED(i,n) = m - i$ for all $0 \leq i \leq m$
    Delete $a_i a_{i+1} \ldots a_{m-1}$ from u

Inductive structure for edit distance

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SEE NOTEBOOK FOR NOTES OF THIS SLIDE

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  - $ED(i,n) = m - i$ for all $0 \leq i \leq m$
    Delete $a_i a_{i+1} \ldots a_{m-1}$ from u
  - $ED(m,j) = n - j$ for all $0 \leq j \leq n$
    Insert $b_j b_{j+1} \ldots b_{n-1}$ into u

Subproblem dependency

- Subproblems are $ED(i, j)$, for $0 \leq i \leq m$, $0 \leq j \leq n$

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- Table of $(m+1) \cdot (n+1)$ values

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b							
1	i							
2	s							
3	e							
4	c							
5	t							
6	•							

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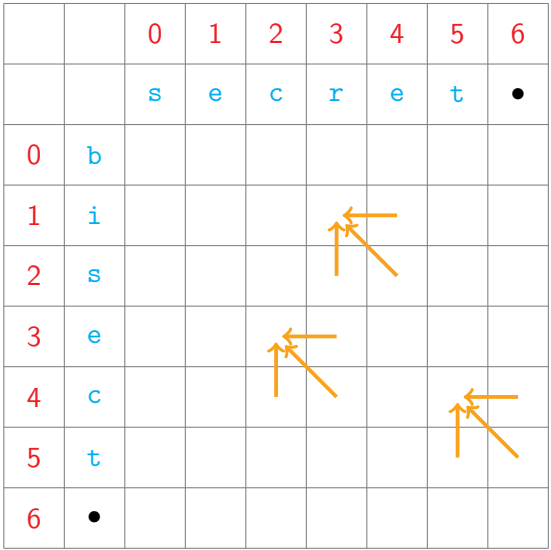

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We are going from BISECT to SECRET

		0	1	2	3	4	5	6
		v						
	u	s	e	c	r	e	t	•
0	b	u = "bisect" v= "", delete " bisect" ED=6						6
1	i							5
2	s							4
3	e							3
4	c	u = "ct" v= "", delete "ct" ED=2						2
5	t	u = "t" v= "", delete "t" ED=1						1
6	•	u = "", v = "" => ED() = 0						0

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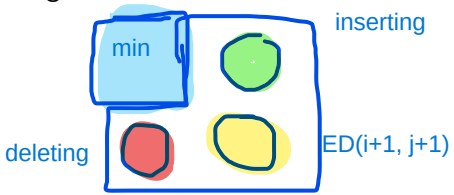

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b						5	6
1	i						4	5
2	s						3	4
3	e						2	3
4	c	u="ct", v="t" delete "c" OR insert "t", u = "tct"					1	2
5	t	ED("t", "t") = ED("", "")					0	1
6	•	u = "", v = "t" insert "t"					1	0

min

Subproblem dependency

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u = "t" v = "et"
inserting "e", u = "et" takes 1 operation
deleting we need to make u="", v= "" takes 3 operations in total (remember all operations are done on u)

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b					**4**	**5**	**6**
1	i					**3**	**4**	**5**
2	s					**2**	**3**	**4**
3	e					**1**	**2**	**3**
4	c					**1**	**1**	**2**
5	t					**1**	**0**	**1**
6	•					**2**	**1**	**0**

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		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b				**4**	**4**	**5**	**6**
1	i				**3**	**3**	**4**	**5**
2	s				**2**	**2**	**3**	**4**
3	e				**2**	**1**	**2**	**3**
4	c				**2**	**1**	**1**	**2**
5	t				**2**	**1**	**0**	**1**
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- No dependency for $ED(m, n)$ — start at bottom right and fill by row, column or diagonal

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b			**4**	**4**	**4**	**5**	**6**
1	i			**3**	**3**	**3**	**4**	**5**
2	s			**3**	**2**	**2**	**3**	**4**
3	e			**3**	**2**	**1**	**2**	**3**
4	c			**2**	**2**	**1**	**1**	**2**
5	t			**3**	**2**	**1**	**0**	**1**
6	•			**4**	**3**	**2**	**1**	**0**

Subproblem dependency

- Subproblems are $ED(i, j)$, for $0 \leq i \leq m$, $0 \leq j \leq n$
- Table of $(m + 1) \cdot (n + 1)$ values
- Like LCS, $ED(i, j)$ depends on $ED(i+1, j+1)$, $ED(i, j+1)$, $ED(i+1, j)$
- No dependency for $ED(m, n)$ — start at bottom right and fill by row, column or diagonal

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b		4	4	4	4	5	6
1	i		4	3	3	3	4	5
2	s		3	3	2	2	3	4
3	e		2	3	2	1	2	3
4	c		3	2	2	1	1	2
5	t		4	3	2	1	0	1
6	•		5	4	3	2	1	0

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		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b	4	4	4	4	4	5	6
1	i	3	4	3	3	3	4	5
2	s	2	3	3	2	2	3	4
3	e	3	2	3	2	1	2	3
4	c	4	3	2	2	1	1	2
5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

For entire string bisect and secret we need to make minimum of 4 operations to make them equal

Subproblem dependency

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- Like LCS, $ED(i,j)$ depends on $ED(i{+}1,j{+}1)$, $ED(i,j{+}1)$, $ED(i{+}1,j)$
- No dependency for $ED(m,n)$ — start at bottom right and fill by row, column or diagonal

Reading off the solution

- Transform `bisect` to `secret`

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b	4	4	4	4	4	5	6
1	i	3	4	3	3	3	4	5
2	s	2	3	3	2	2	3	4
3	e	3	2	3	2	1	2	3
4	c	4	3	2	2	1	1	2
5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

Subproblem dependency

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- No dependency for $ED(m, n)$ — start at bottom right and fill by row, column or diagonal

Reading off the solution

- Transform `bisect` to `secret`
- Delete `b`

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b	4	4	4	4	4	5	6
1	i	3	4	3	3	3	4	5
2	s	2	3	3	2	2	3	4
3	e	3	2	3	2	1	2	3
4	c	4	3	2	2	1	1	2
5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

Subproblem dependency

- Subproblems are $ED(i,j)$, for $0 \leq i \leq m$, $0 \leq j \leq n$
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- No dependency for $ED(m,n)$ — start at bottom right and fill by row, column or diagonal

Reading off the solution

- Transform `bisect` to `secret`
- Delete `b` , Delete `i`

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b	4	4	4	4	4	5	6
1	i	3	4	3	3	3	4	5
2	s	2	3	3	2	2	3	4
3	e	3	2	3	2	1	2	3
4	c	4	3	2	2	1	1	2
5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

Subproblem dependency

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Reading off the solution

- Transform `bisect` to `secret`
- Delete `b` , Delete `i` , Insert `r`

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b	4	4	4	4	4	5	6
1	i	3	4	3	3	3	4	5
2	s	2	3	3	2	2	3	4
3	e	3	2	3	2	1	2	3
4	c	4	3	2	2	1	1	2
5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

Subproblem dependency

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Reading off the solution

diagonal line substitution or when a[i] = b[j]

- Transform `bisect` to `secret`
- Delete `b` , Delete `i` , Insert `r` , Insert `e`

		0	1	2	3	4	5	6
		s	e	c	r	e	t	•
0	b	4	4	4	4	4	5	6
1	i	3	4	3	3	3	4	5
2	s	2	3	3	2	2	3	4
3	e	3	2	3	2	1	2	3
4	c	4	3	2	2	1	1	2
5	t	5	4	3	2	1	0	1
6	•	6	5	4	3	2	1	0

Implementation

```
def ED(u,v):
  import numpy as np
  (m,n) = (len(u),len(v))
  ed = np.zeros((m+1,n+1))

  for i in range(m-1,-1,-1):
    ed[i,n] = m-i
  for j in range(n-1,-1,-1):
    ed[m,j] = n-j

  for j in range(n-1,-1,-1):
    for i in range(m-1,-1,-1):
      if u[i] == v[j]:
        ed[i,j] = ed[i+1,j+1]
      else:
        ed[i,j] = 1 + min(ed[i+1,j+1],
                          ed[i,j+1],
                          ed[i+1,j])
  return(ed[0,0])
```

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  for j in range(n-1,-1,-1):
    ed[m,j] = n-j

  for j in range(n-1,-1,-1):
    for i in range(m-1,-1,-1):
      if u[i] == v[j]:
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Complexity

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    ed[m,j] = n-j

  for j in range(n-1,-1,-1):
    for i in range(m-1,-1,-1):
      if u[i] == v[j]:
        ed[i,j] = ed[i+1,j+1]
      else:
        ed[i,j] = 1 + min(ed[i+1,j+1],
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```

Complexity

- Again $O(mn)$, using dynamic programming or memoization

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    ed[m,j] = n-j

  for j in range(n-1,-1,-1):
    for i in range(m-1,-1,-1):
      if u[i] == v[j]:
        ed[i,j] = ed[i+1,j+1]
      else:
        ed[i,j] = 1 + min(ed[i+1,j+1],
                          ed[i,j+1],
                          ed[i+1,j])
  return(ed[0,0])
```

Complexity

- Again $O(mn)$, using dynamic programming or memoization
  - Fill a table of size $O(mn)$
  - Each table entry takes constant time to compute