CS 2340 - Computer Architecture

3 Data Representations
Dr. Alice Wang

BINARY

It's as easy
as 01,10,11

Housekeeping

- Released on eLearning are practice questions similar to those that will be on the exam
- After each lecture you can go and try the questions for the specific lecture
- I will be going over these practice questions in the exam review

Review

Last time

- MIPS Assembly Coding Basics
  - Assembler Directives
  - Registers
  - Instructions: Load / Store / Add / Sub / Add immediate
  - Pseudo-instructions: li, la, move
  - System Calls
- MARS demo

Mars demo

- How to startup MARS
  - Look at syllabus if you have a MAC
- How to edit your code
- How to save your code
- The Register Table
- The Data memory

Step-by-step demo in the reference section

- How to Assemble your code
- How to Run your code
- Run I/O vs Mars Messages windows
- **How to Debug your code using breakpoints**
- **How to view data in decimal, ASCII & hex**

Number Systems

Humans: Decimal

Computers: Binary, Hexadecimal

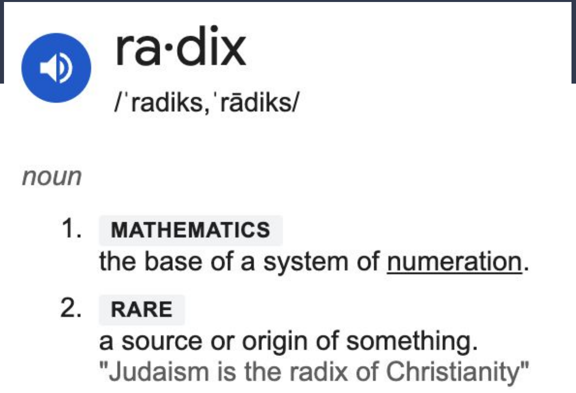
Computers and Humans have different number systems

What is decimal vs binary?

Decimal counts using numbers 0~9:
0,1, 2, 3, 4, 5, … 9, 10, 11, 12, …

Binary counts using number 0~1:
0, 1, 10, 11, 100, 101, 110, …

Decimal numbers



ra·dix

/ˈradiks,ˈrādiks/

noun

1. **MATHEMATICS**
   the base of a system of numeration.
2. **RARE**
   a source or origin of something.
   "Judaism is the radix of Christianity"

- Base 10 (Radix 10)

1000's column

100's column

10's column

1's column

$$5374_{10} = \underbrace{5 \times 10^3}_{\text{five thousands}} + \underbrace{3 \times 10^2}_{\text{three hundreds}} + \underbrace{7 \times 10^1}_{\text{seven tens}} + \underbrace{4 \times 10^0}_{\text{four ones}}$$

Indicates Base 10

Binary numbers

- Base 2 (Radix 2)

8's column
4's column
2's column
1's column

$$1101_2 = \underset{\text{one eight}}{1 \times 2^3} + \underset{\text{one four}}{1 \times 2^2} + \underset{\text{no two}}{0 \times 2^1} + \underset{\text{one one}}{1 \times 2^0} = 13_{10}$$

Indicates Base 2

Nomenclature

	Prefix	Examples	Subscript	Examples
Binary	0b	0b0010_1101	2	0010_1101_2
Decimal	0d blank	0d45 45	10	45_{10}

**If there is no prefix or subscript,
the default is decimal**

Base 2 or Radix 2

- It’s useful to know base 2 binary numbers from 0 to 15
    - 0 = 0b0000
    - 1 = 0b0001
    - 2 = 0b0010
    - 3 = 0b0011
    - 4 = 0b0100
    - 5 = 0b0101
    - 6 = 0b0110
    - 7 = 0b0111
    - 8 = 0b1000
    - 9 = 0b1001
    - 10 = 0b1010
    - 11 = 0b1011
    - 12 = 0b1100
    - 13 = 0b1101
    - 14 = 0b1110
    - 15 = 0b1111

Common base 2 numbers

- It’s useful to memorize power of 2 up to 2^{15}
  - $2^0 = 1$
  - $2^1 = 2$
  - $2^2 = 4$
  - $2^3 = 8$
  - $2^4 = 16$
  - $2^5 = 32$
  - $2^6 = 64$
  - $2^7 = 128$
  - $2^8 = 256$
  - $2^9 = 512$
  - $2^{10} = 1024$
  - $2^{11} = 2048$
  - $2^{12} = 4096$
  - $2^{13} = 8192$
  - $2^{14} = 16384$
  - $2^{15} = 32768$

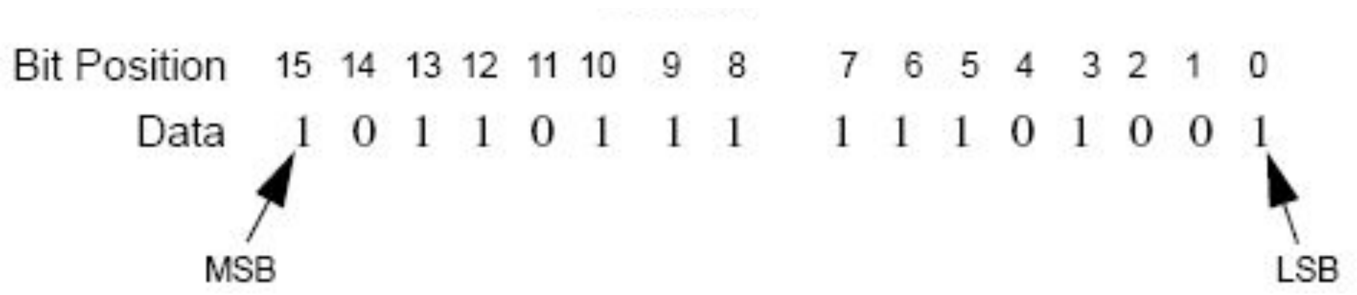
Large Powers of Two

Larger Powers of Two vs Ten

- 2^{10} = 1 kilo ≈ 10^3 = 1000 (1024)
- 2^{20} = 1 mega ≈ 10^6 = 1 million (1,048,576)
- 2^{30} = 1 giga ≈ 10^9 = 1 billion (1,073,741,824)

MSB and LSB

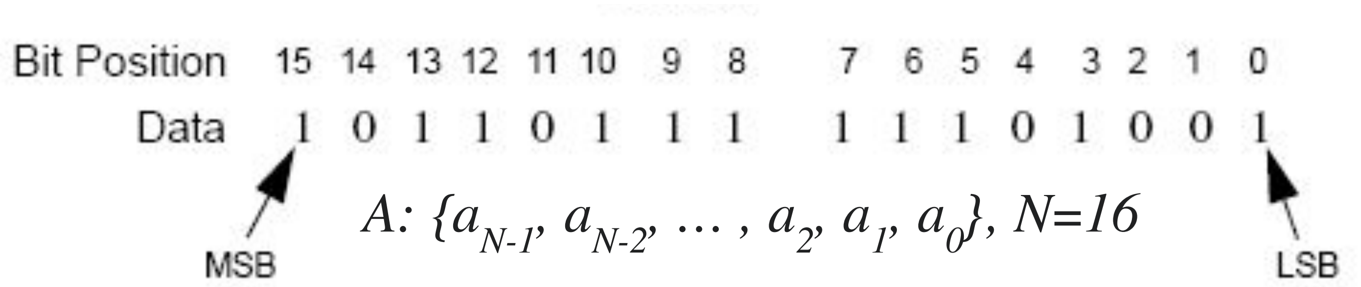
Most Significant Bit (MSB)	This bit has the highest value (greatest weight) and is located at the far left of the bit string
Least Significant Bit (LSB)	This bit has the lowest value (bit position zero) and is located at the far right of the bit string.



Bit Position	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
Data	1	0	1	1	0	1	1	1	1	1	1	0	1	0	0	1

Bit position

Most Significant Bit (MSB)	This bit has the highest value (greatest weight) and is located at the far left of the bit string
Least Significant Bit (LSB)	This bit has the lowest value (bit position zero) and is located at the far right of the bit string.



Bit Position	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
Data	1	0	1	1	0	1	1	1	1	1	1	0	1	0	0	1

$$A: \{a_{N-1}, a_{N-2}, \ldots, a_2, a_1, a_0\}, N=16$$

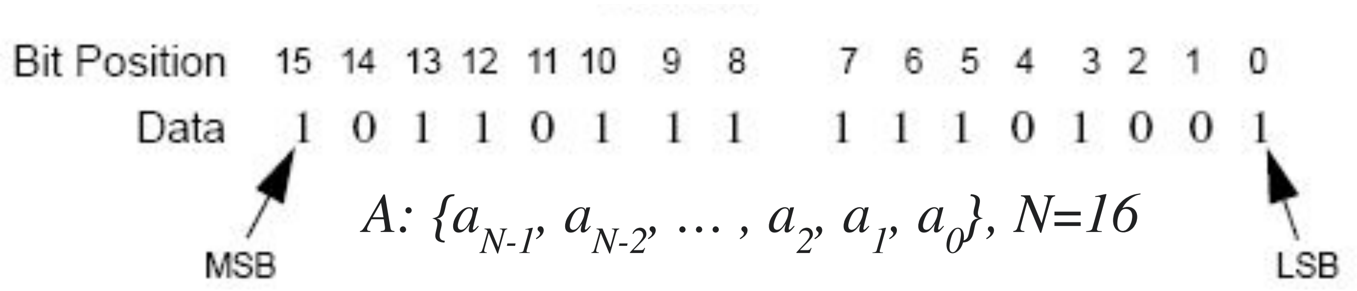
A[15] = a_{15} = 0b1 (MSB) → 15th element of Array A

A[0] = a_0 = 0b1 (LSB) → 0th element of Array A

A[11] = a_{11} = ___________

Bit position

Most Significant Bit (MSB)	This bit has the highest value (greatest weight) and is located at the far left of the bit string
Least Significant Bit (LSB)	This bit has the lowest value (bit position zero) and is located at the far right of the bit string.



Bit Position	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
Data	1	0	1	1	0	1	1	1	1	1	1	0	1	0	0	1

$$A: \{a_{N-1}, a_{N-2}, \ldots , a_2, a_1, a_0\}, N=16$$

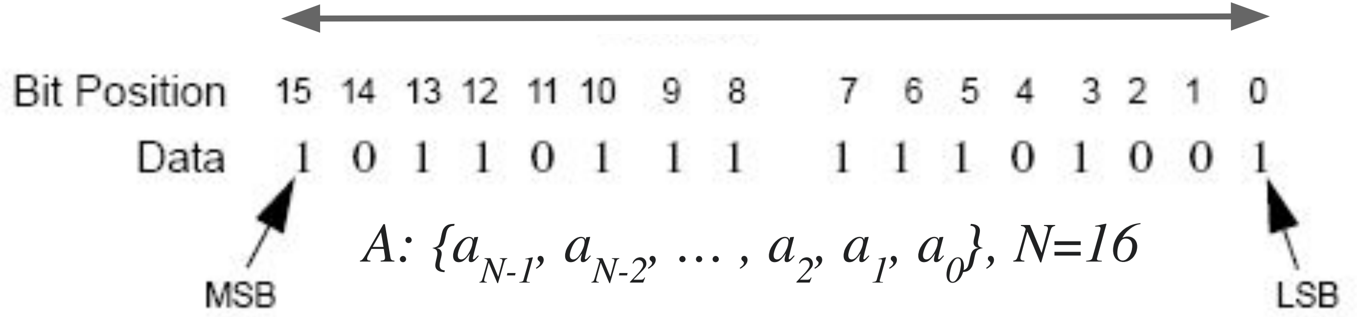
A[15:10] = 0b10_1101

A[2:0] = ________

Bit width

Bit width	Number of binary digits (bits) used to represent a value in a computer system

Bit width = ______ bits



Bit Position	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
Data	1	0	1	1	0	1	1	1	1	1	1	0	1	0	0	1

$A: \{a_{N-1}, a_{N-2}, \ldots , a_2, a_1, a_0\}, N=16$

Convert Decimal to Binary – My Turn

Use the Successive Division approach

- Convert 6_{10} to binary

Ans: ________$_2$ or 0b__________

Successively divide by radix (2 for binary)
Results are the remainder

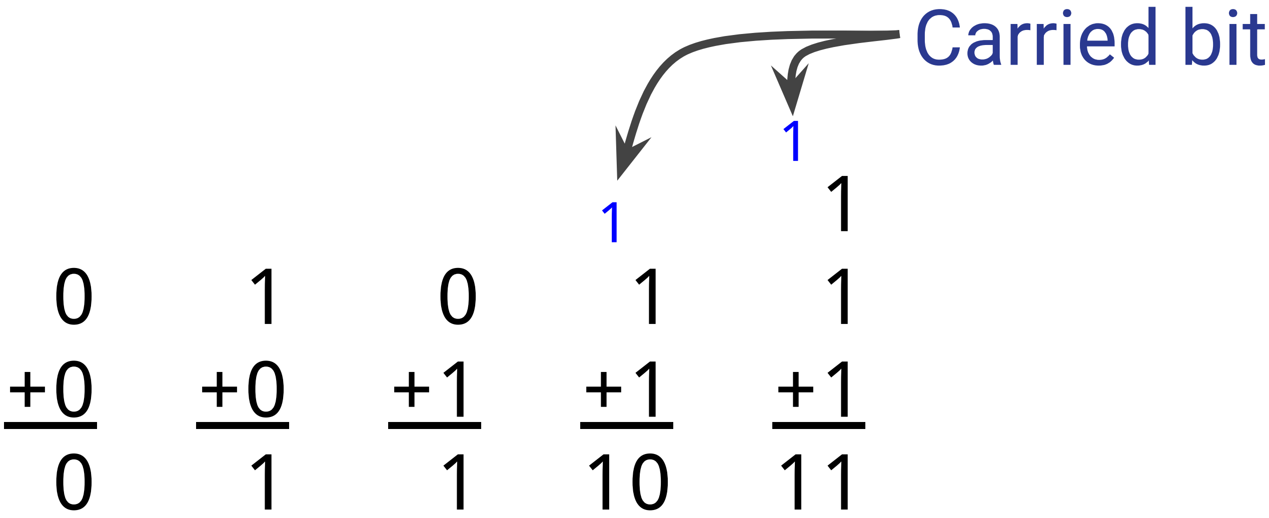
Convert Decimal to Binary – Your Turn

Use the Successive Division approach

- Convert 40_{10} to binary

Ans: _______ $_2$ or 0b________

Simple Binary Addition



$$\begin{array}{rrrrr}
 & & & & 1 \\
 & & & 1 & 1 \\
0 & 1 & 0 & 1 & 1 \\
+0 & +0 & +1 & +1 & +1 \\
\hline
0 & 1 & 1 & 10 & 11
\end{array}$$

Carried bit

Binary Addition

- Decimal

$$\begin{array}{rccccl} & & 1 & 1 & & \leftarrow \text{carries} \\ & 3 & 7 & 3 & 4 & \\ + & 5 & 1 & 6 & 8 & \\ \hline & 8 & 9 & 0 & 2 & \end{array}$$

- Binary

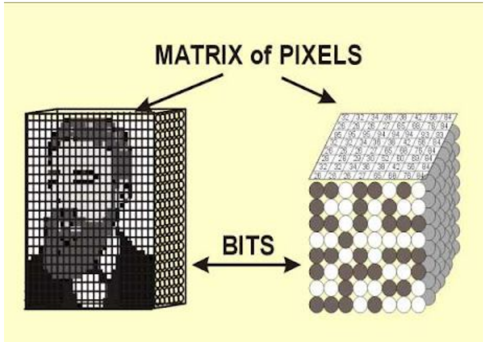
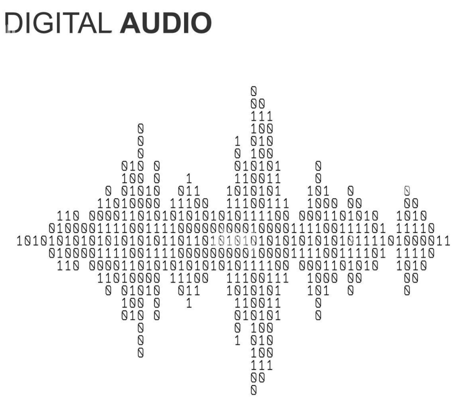
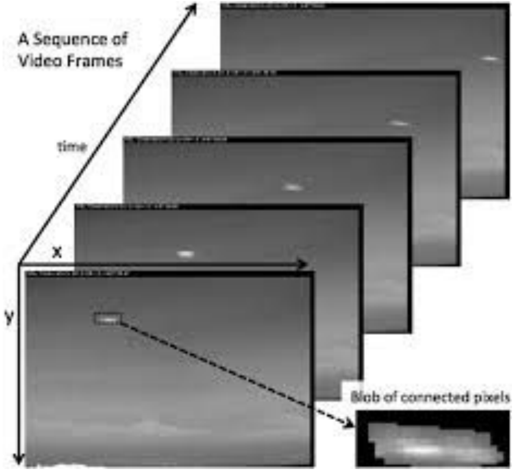
$$\begin{array}{rccccl} & & 1 & 1 & & \leftarrow \text{carries} \\ & 1 & 0 & 1 & 1 & \\ + & 0 & 0 & 1 & 1 & \\ \hline & 1 & 1 & 1 & 0 & \end{array}$$

Binary Addition Example

Binary	Decimal
0101	
+1001	+_____

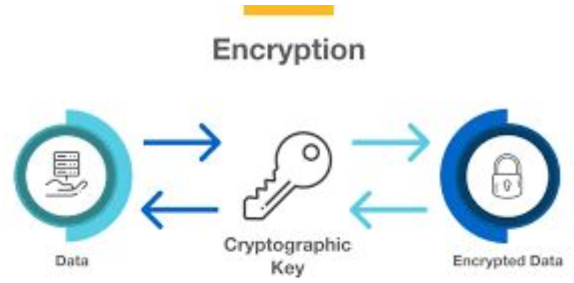
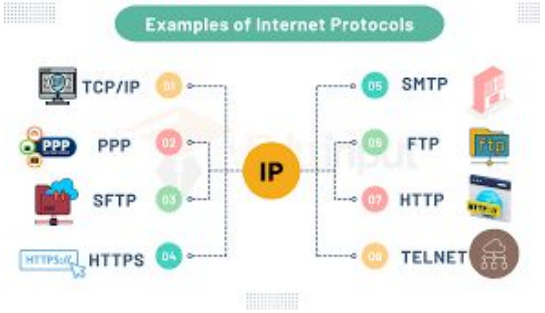
Other than computers where is binary used?

- Binary is used in many multimedia applications

		 A Sequence of Video Frames time x y Blob of connected pixels
Digital images are stored as binary files where each pixel is represented by a binary value.	Audio files are stored as sequences of binary numbers that represent sound waveforms.	Video files are composed of binary data that encodes a sequence of frames (images) together with audio tracks.

Other than computers where is binary used?

- Binary is also used in communications and security

		 Examples of Internet Protocols TCP/IP 01 PPP 02 SFTP 03 HTTPS 04 IP 05 SMTP 06 FTP 07 HTTP 08 TELNET
Binary codes are used to encode and decode data for transmission over digital communication channels.	Binary is essential in encryption algorithms, which secure data by converting it into a binary format that can only be decrypted with a key.	Binary is used in networking protocols and data transmission, ensuring accurate communication between devices.

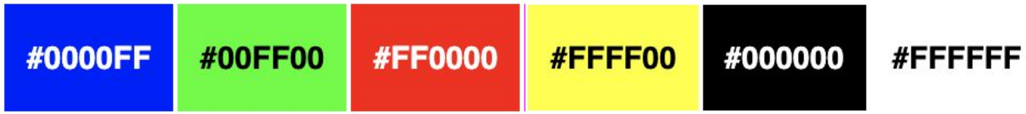
Binary can get kind of cumbersome...

- Once a number gets > 8 digits it becomes hard to read, write and remember
- This is why we commonly use higher base numbers like Hexadecimal instead of Binary

- Examples: 0010_1010 vs 2A

Where is Hexadecimal used?

- Used widely
- HTML Colour Codes



#0000FF #00FF00 #FF0000 #FFFF00 #000000 #FFFFFF

- MAC Addresses
  - Physical address is given as 6 address pairs

    E.g. “A0-1D-48-FE-5E-F5”

What is decimal vs binary vs hex?

Decimal counts using numbers 0~9:
0, 1, 2, 3, 4, 5, … 9, 10, 11, 12, …

Binary counts using numbers 0~1:
0, 1, 10, 11, 100, 101, 110, …

Hexadecimal counts using numbers 0~9, A~F
0, 1, 2, 3, …, 8, 9, A, B, …, F, 10, 11, 12, …, 19, 1A, 1B, ….

Hex/Dec/Bin table

Hex	Dec	Bin
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111

Hex	Dec	Bin
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

- Hex: Base 16
- Common notation “0x” prefix
- Or Subscript$_{16}$

Nomenclature

	Prefix	Examples	Subscript	Examples
Binary	0b	0b0010_1101	2	0010_1101_2
Decimal	0d blank	0d45 45	10	45_{10}
Hexadecimal	0x	0x2D	16	$2D_{16}$

If there is no prefix or subscript, the default is decimal

Convert Decimal to Hex – My Turn

Use the Successive Division approach

- Convert 18_{10} to hex

Ans: $\_\_\_\_\_\_\_\__{16}$ or 0x_________

Successively divide by radix (16 for hex)
Results are the remainder

Convert Decimal to Hex – Your Turn

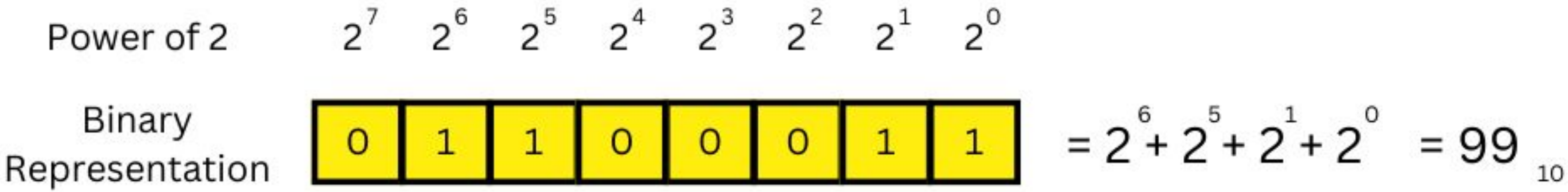
Use the Successive Division approach

- Convert 40_{10} to hex

Ans: _______ $_{16}$ or 0x________

Convert Hex or Bin to Decimal

Given a sequence of bin or hex numbers: $a_{N-1}, \ldots, a_1, a_0$
converting to decimal:



Power of 2	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Binary Representation	0	1	1	0	0	0	1	1

$= 2^6 + 2^5 + 2^1 + 2^0 = 99_{10}$

$$\Sigma\ a_i * r^i \text{ from } i = 0 \text{ to } N-1$$

where r = radix 2 (bin) or 16 (hex)

Multiplying the individual numbers by powers of 2 (bin) or 16 (hex) starting with 0 for the right most digit

Hex or Bin to Decimal conversion

My Turn: Convert 1100_2 to decimal

1	1	0	0
2^3	2^2	2^1	2^0

=

=

Your Turn: Convert $4AF_{16}$ to decimal

16^2	16^1	16^0

=

=

Multiplying the individual numbers by powers of 2 (bin) or 16 (hex) starting with 0 for the right most digit

Binary to Hex, Hex to Binary – Example

Convert every 4 bits to hex or each hex number to 4 bits
Use Look-up Tables, Convert to Decimal or Memorize!

My Turn: Convert 0b1010_0011 to hex

=

=

Your Turn: Convert $4AF_{16}$ to binary

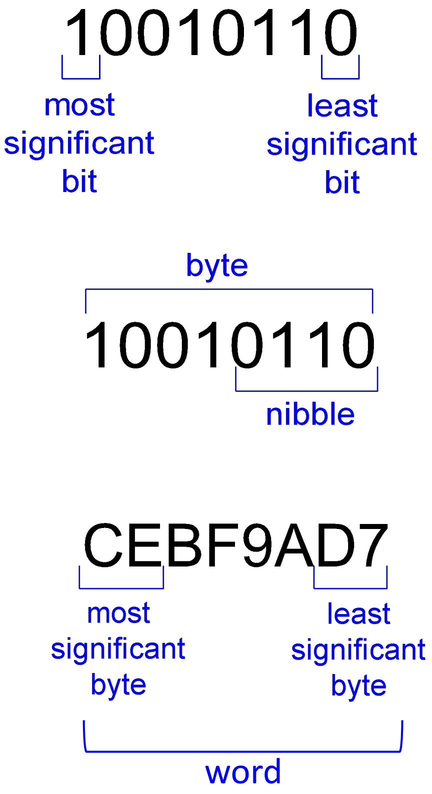
=

=

"_" underscore used for readability

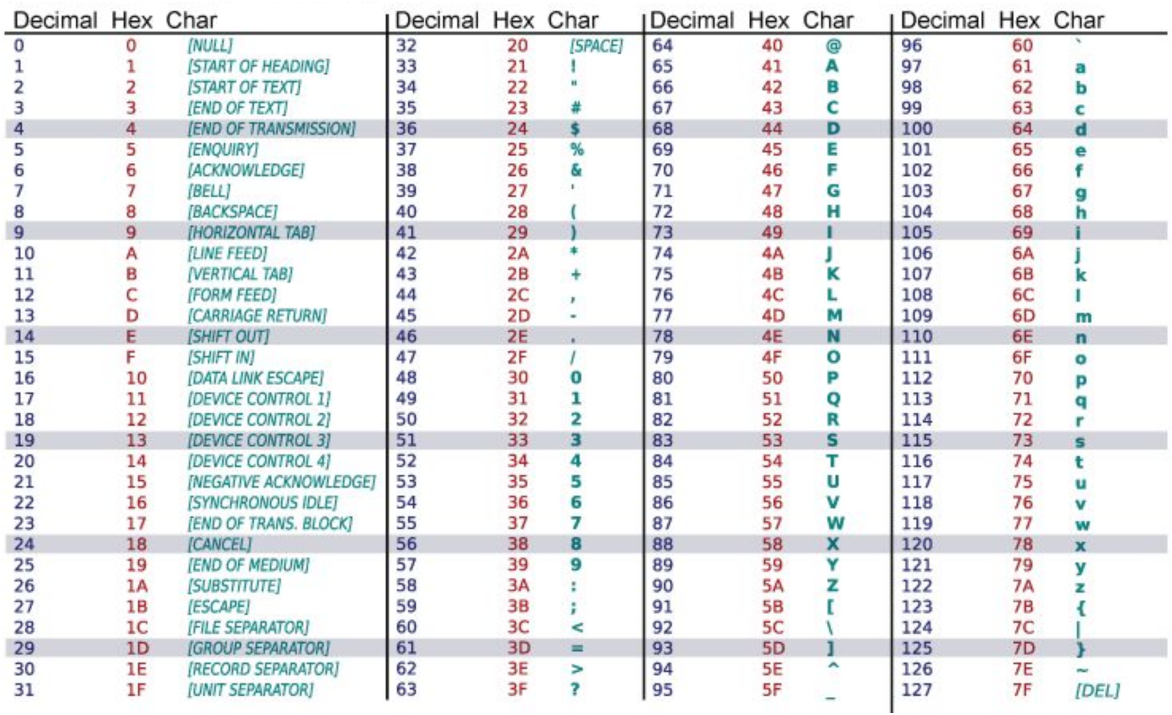
Important: Bits, Bytes and Words

- Bits
- Hex digits = Nibbles = 4 bits
- Bytes = 8 bits
- Words = 32 bits = 4 bytes



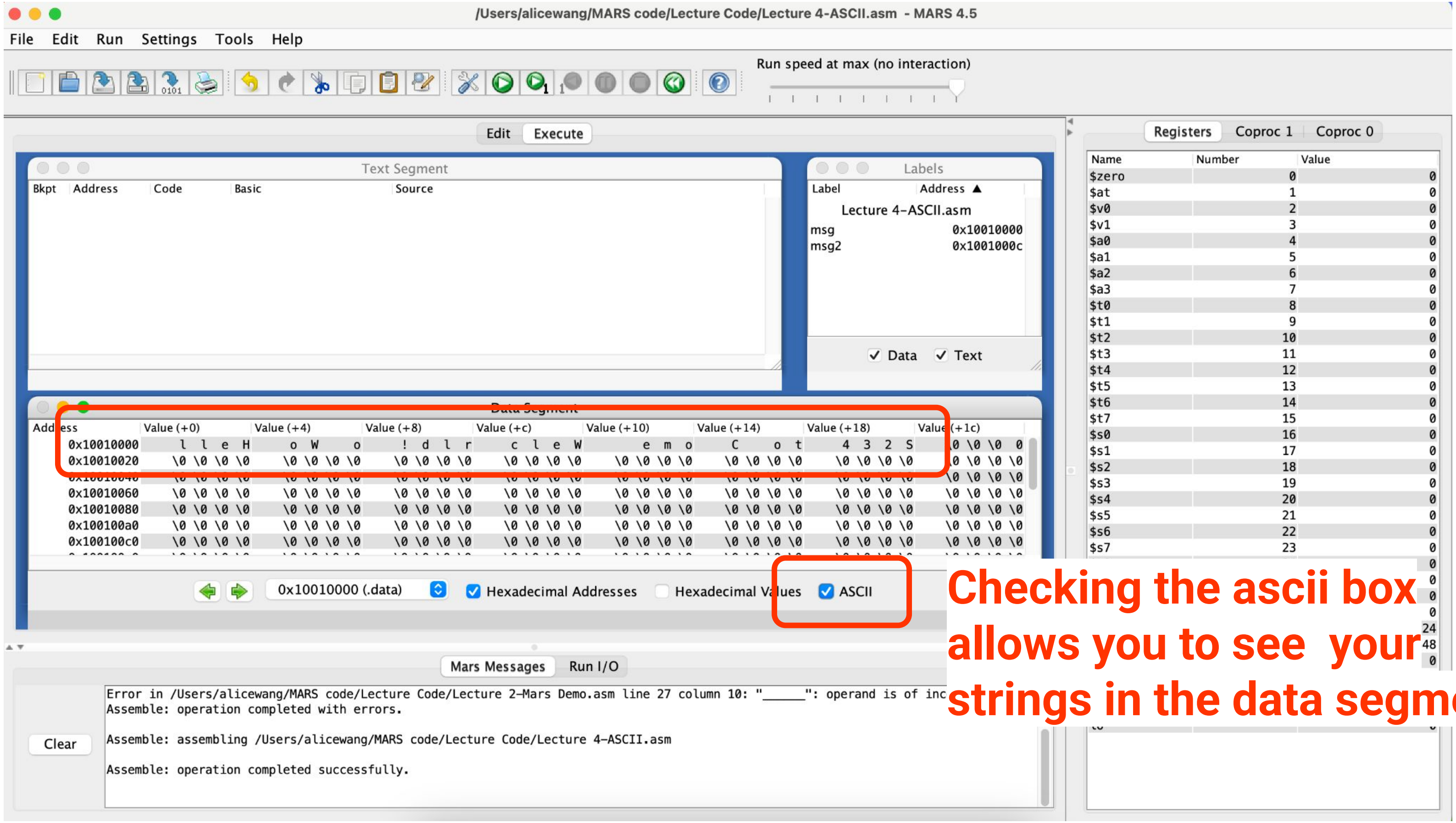
Review: ASCII

- Most commonly used format for text files
- Each character is represented by 7-bit binary (or 2 hex digits)



Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char
0	0	[NULL]	32	20	[SPACE]	64	40	@	96	60	`
1	1	[START OF HEADING]	33	21	!	65	41	A	97	61	a
2	2	[START OF TEXT]	34	22	"	66	42	B	98	62	b
3	3	[END OF TEXT]	35	23	#	67	43	C	99	63	c
4	4	[END OF TRANSMISSION]	36	24	$	68	44	D	100	64	d
5	5	[ENQUIRY]	37	25	%	69	45	E	101	65	e
6	6	[ACKNOWLEDGE]	38	26	&	70	46	F	102	66	f
7	7	[BELL]	39	27	'	71	47	G	103	67	g
8	8	[BACKSPACE]	40	28	(	72	48	H	104	68	h
9	9	[HORIZONTAL TAB]	41	29	)	73	49	I	105	69	i
10	A	[LINE FEED]	42	2A	*	74	4A	J	106	6A	j
11	B	[VERTICAL TAB]	43	2B	+	75	4B	K	107	6B	k
12	C	[FORM FEED]	44	2C	,	76	4C	L	108	6C	l
13	D	[CARRIAGE RETURN]	45	2D	-	77	4D	M	109	6D	m
14	E	[SHIFT OUT]	46	2E	.	78	4E	N	110	6E	n
15	F	[SHIFT IN]	47	2F	/	79	4F	O	111	6F	o
16	10	[DATA LINK ESCAPE]	48	30	0	80	50	P	112	70	p
17	11	[DEVICE CONTROL 1]	49	31	1	81	51	Q	113	71	q
18	12	[DEVICE CONTROL 2]	50	32	2	82	52	R	114	72	r
19	13	[DEVICE CONTROL 3]	51	33	3	83	53	S	115	73	s
20	14	[DEVICE CONTROL 4]	52	34	4	84	54	T	116	74	t
21	15	[NEGATIVE ACKNOWLEDGE]	53	35	5	85	55	U	117	75	u
22	16	[SYNCHRONOUS IDLE]	54	36	6	86	56	V	118	76	v
23	17	[END OF TRANS. BLOCK]	55	37	7	87	57	W	119	77	w
24	18	[CANCEL]	56	38	8	88	58	X	120	78	x
25	19	[END OF MEDIUM]	57	39	9	89	59	Y	121	79	y
26	1A	[SUBSTITUTE]	58	3A	:	90	5A	Z	122	7A	z
27	1B	[ESCAPE]	59	3B	;	91	5B	[	123	7B	{
28	1C	[FILE SEPARATOR]	60	3C	<	92	5C	\	124	7C	|
29	1D	[GROUP SEPARATOR]	61	3D	=	93	5D	]	125	7D	}
30	1E	[RECORD SEPARATOR]	62	3E	>	94	5E	^	126	7E	~
31	1F	[UNIT SEPARATOR]	63	3F	?	95	5F	_	127	7F	[DEL]

Viewing ASCII in MARS



Label	Address
msg	0x10010000
msg2	0x1001000c

```
Address     Value (+0)  Value (+4)  Value (+8)  Value (+c)  Value (+10) Value (+14) Value (+18) Value (+1c)
0x10010000     l  l  e  H     o  W     o     !  d  l  r     c  l  e  W        e  m  o     C     o  t     4  3  2  S  \0 \0 \0  0
0x10010020  \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0
0x10010040  \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0
0x10010060  \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0
0x10010080  \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0
0x100100a0  \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0
0x100100c0  \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0 \0
```

0x10010000 (.data) ☑ Hexadecimal Addresses ☐ Hexadecimal Values ☑ ASCII

Name	Number	Value
$zero	0	0
$at	1	0
$v0	2	0
$v1	3	0
$a0	4	0
$a1	5	0
$a2	6	0
$a3	7	0
$t0	8	0
$t1	9	0
$t2	10	0
$t3	11	0
$t4	12	0
$t5	13	0
$t6	14	0
$t7	15	0
$s0	16	0
$s1	17	0
$s2	18	0
$s3	19	0
$s4	20	0
$s5	21	0
$s6	22	0
$s7	23	0

```
Error in /Users/alicewang/MARS code/Lecture Code/Lecture 2-Mars Demo.asm line 27 column 10: "______": operand is of inc
Assemble: operation completed with errors.

Assemble: assembling /Users/alicewang/MARS code/Lecture Code/Lecture 4-ASCII.asm

Assemble: operation completed successfully.
```

Checking the ascii box allows you to see your strings in the data segment

Reference: Conversion Rules

Given a number: $a_{N-1}, a_{N-2}, \ldots, a_1, a_0$

From	To		Example	
Decimal	Binary	Divide by 2, Remainder is binary	9	9/2 = 4R**1**, 4/2 = 2R**0**, 2/2 = 1R**0,** 1/2 = 0R**1** → 0b1001
Decimal	Hexadecimal	Divide by 16, Remainder is binary	93	93/16 = 5R**13** “D”, 5/16 = 0R**5** → 0x5D
Binary	Decimal	$\Sigma\ a_i * 2^i$ from i = 0 to N-1	0b0110	$0*2^3 + 1*2^2 + 1*2^1 + 0* 2^0 = 6$
Hexadecimal	Decimal	$\Sigma\ a_i * 16^i$ from i = 0 to N-1	0xB8	$11* 16^1 + 8 * 16^0 = 184$
Binary	Hexadecimal	Convert every 4 bits to hex	0b0011_1110	0b0011 → 0x**3**, 0b1110 → **E** 0x3E
Hexadecimal	Binary	Convert every hex digit to bits	0x59	5 → 0101, 9 → 1001: 0b0101_1001

Next lecture

Signed number representation