# LAGRANGE COHERENT STRUCTURES

Course Project - AE 308

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## Problem Formulation:

The given transfer is

$$G(S) = \frac{S + 500}{S(S + 0.0325)(S^2 + 2.57S + 6667)} \tag{1}$$

Given conditions:

$$K_V = \text{Ramp Error Constant} = 100$$
$$\text{Phase Margin} > 35°$$

Now, using these above conditions, we need to design a controller using Root Locus Method and we need to determine Resonant Frequency, Resonant Peak and Bandwidth of the Compensated System. Also determine the Transient Response of the System.

## Theory:

Lets look at some of the basics on what controllers are what are types of controllers one can use.

## Theory:

Lets look how different types of controllers impact the closed loop behaviour of a given system.

a) **P Controller:**

   (i) Modifies overall loop gain and hence steady state behaviour.

   (ii) Change dominant pole location and thereby relative and transient response.

b) **PD Controller:**

   (i) It can reduce the overshoot of a proportional controller because PD controller takes into account the rate of change in error.

   (ii) It can improve system tolerances to external disturbances.

c) **PI Controller:**

(i) It is mainly used for tracking of step and ramp inputs.

(ii) Eliminates steady state error.

d) **PID Controller:**

(i) It can reduce overshoot of system.

(ii) Improve system tolerances to external disturbances.

(iii) Steady state error decreases.

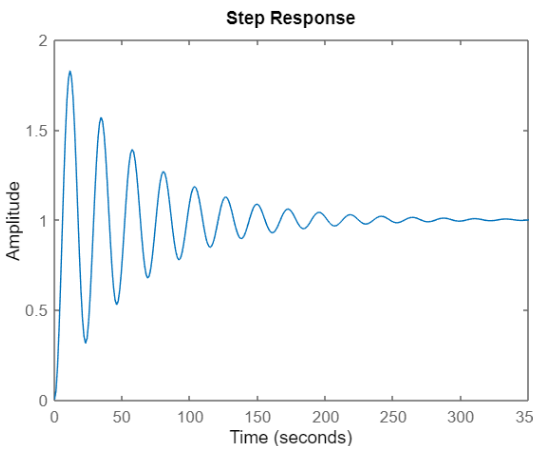
e) **Lead and Lag Compensator:**

(i) A Lead Compensator can increase the stability or speed of response of a system.

(ii) A Lag Compensator can reduce the steady state error.

## Design Strategy:

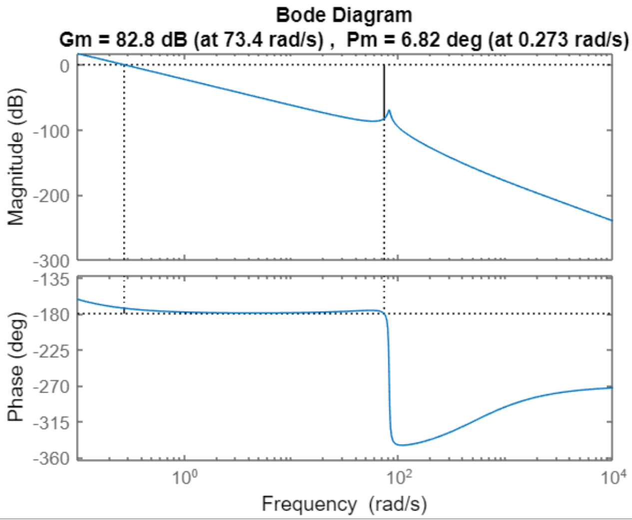
Let us first understand the system given to us observing the step response of the uncompensated system.




```
    RiseTime: 3.9851
SettlingTime: 231.8658
 SettlingMin: 0.3127
 SettlingMax: 1.8291
   Overshoot: 82.9073
  Undershoot: 0
        Peak: 1.8291
    PeakTime: 11.4717
```

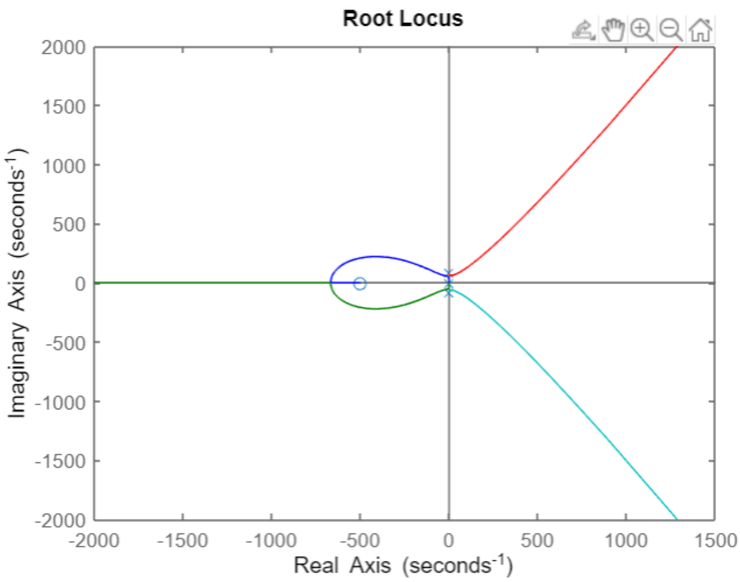
The uncompensated closed loop response has a settling time of 231.689 seconds and O.S of 82.7% which are very large and also has a $k_v$ of 2.3 which is very little compared to our requirement and hence we need to add quite a bit of gain to the system.

Also let us look at the bode plot of the given plant system.




You observe that the phase margin of the uncompensated system is way too less than what is required.

Let us observe the root locus of the given plant.




From the root locus we can see that we need to shift the root locus to the left side and make our poles lie on the right. This is a classic example of Lead compensator.

Lets assume the controller is of the form,

$$K_c \cdot \frac{S + Z_c}{S + P_c} \tag{2}$$

The given uncompensated system,

$$G(S) = \frac{S+500}{S(S+0.0325)(S^2+2.57S+6667)} \tag{3}$$

From the condition, $PM = 35°$.

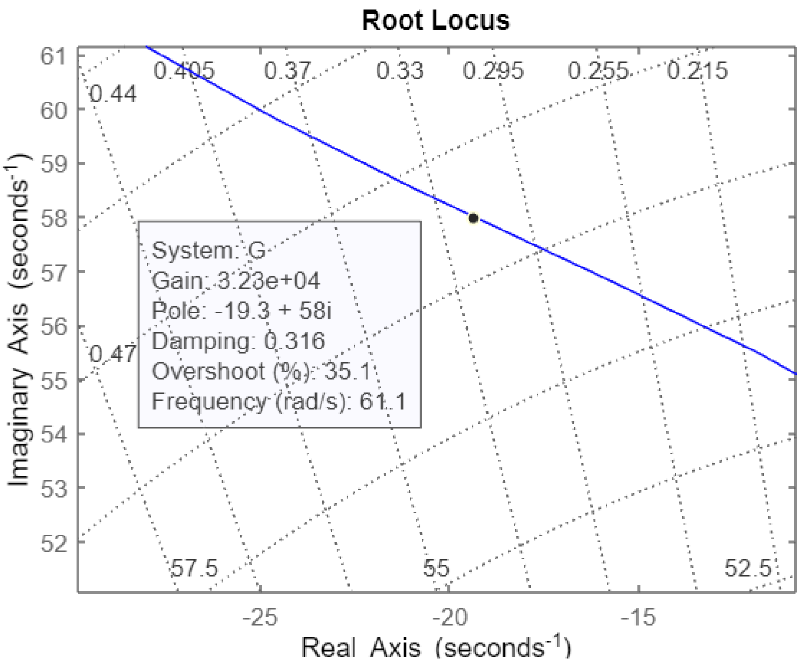
Lets find the damping ratio,

$$\begin{aligned} PM &= \tan^{-1}\left(\frac{2\tau}{\sqrt{-2\tau^2+\sqrt{1+4\tau^4}}}\right) \\ 35° &= \tan^{-1}\left(\frac{2\tau}{\sqrt{-2\tau^2+\sqrt{1+4\tau^4}}}\right) \\ 0.7 &= \frac{2\tau}{\sqrt{-2\tau^2+\sqrt{1+4\tau^4}}} \\ \tau &= 0.316 \end{aligned} \tag{4}$$

## Uncompensated:

Search along the line $\tau = 0.316$ and

find the operating point $= -19.3 + 58i$ and Gain $= 3.23 \times 10^4$




$$G(S) = \frac{3.23 \times 10^4 (S+500)}{S(S+0.0325)(S^2+2.57S+6667)} \tag{5}$$

$$\begin{aligned} K_v &= \lim_{S \to 0} G(S) \\ &= \frac{32300 \times 500}{0.0325 \times 6667} = 74500 \end{aligned} \tag{6}$$

But our required value = 100.

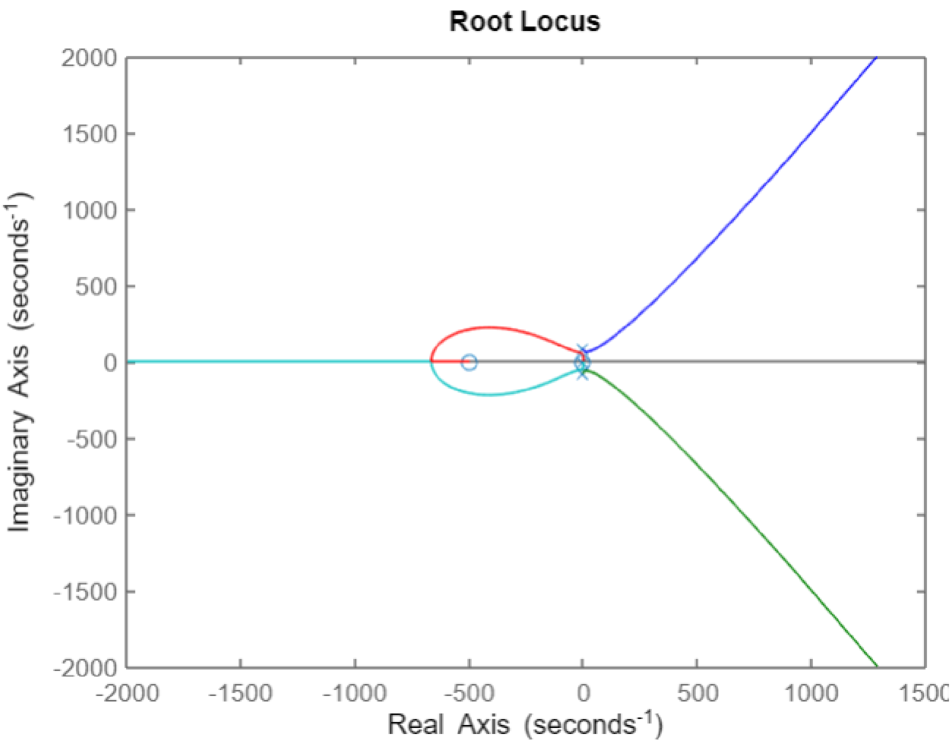
$\therefore$ The improvement of $K_v$ from uncompensated system to compensated system,

$$\frac{100}{74500} = 0.00134 \tag{7}$$

Our Controller $T.f. = \frac{K(S+Z_c)}{S+P_c}$.

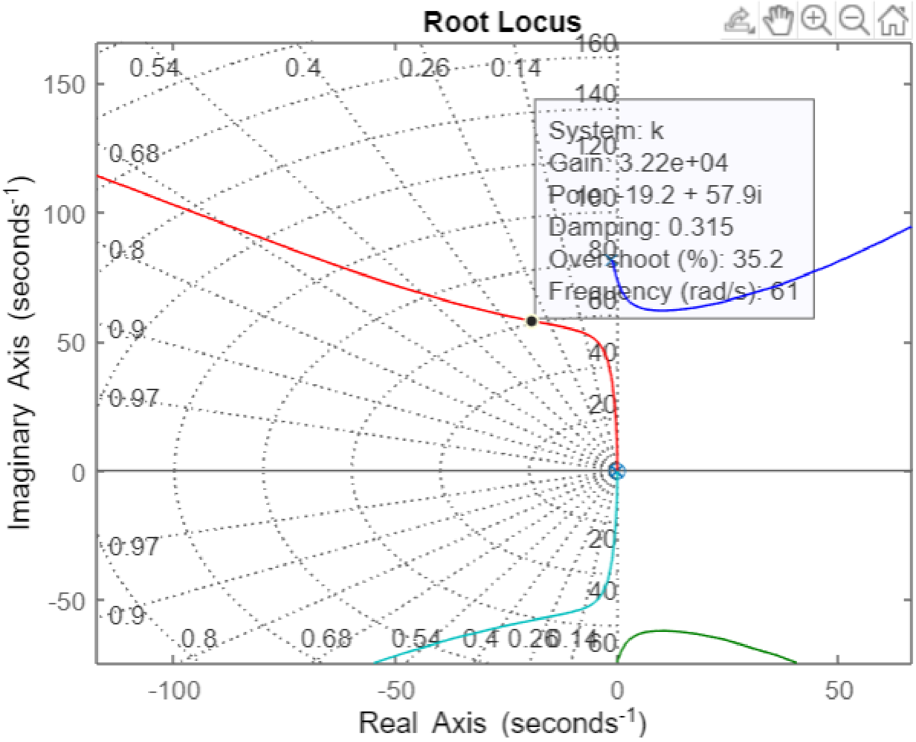
Let us arbitrarily select $Z_c = 0.000134$, $P_c = 0.1$

Now, our $G_c(S) = \frac{K(S+0.000134)}{(S+0.1)}$.




## Compensated:

We added a pole at -0.1 and zero at -0.000134. Search along $\tau = 0.316$ line.




We find operating point as $-19.2 + 57.9i$ and Gain $= 3.22 \times 10^4$.

$\therefore$ Our Final Transfer Function becomes,

$$G(S) = 3.23 \times 10^4 \cdot \frac{(S + 0.000134)}{(S + 0.1)} \cdot \frac{(S + 500)}{S(S + 0.0325)(S^2 + 2.57S + 6667)} \tag{8}$$

$$\begin{aligned} K_v &= \lim_{S \to 0} G(S) \\ &= \frac{3.22 \times 10^4 \times 0.000134 \times 500}{0.1 \times 0.0325 \times 6667} \\ &= 99.56 \end{aligned} \tag{9}$$

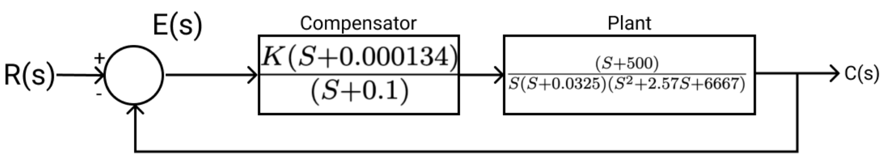
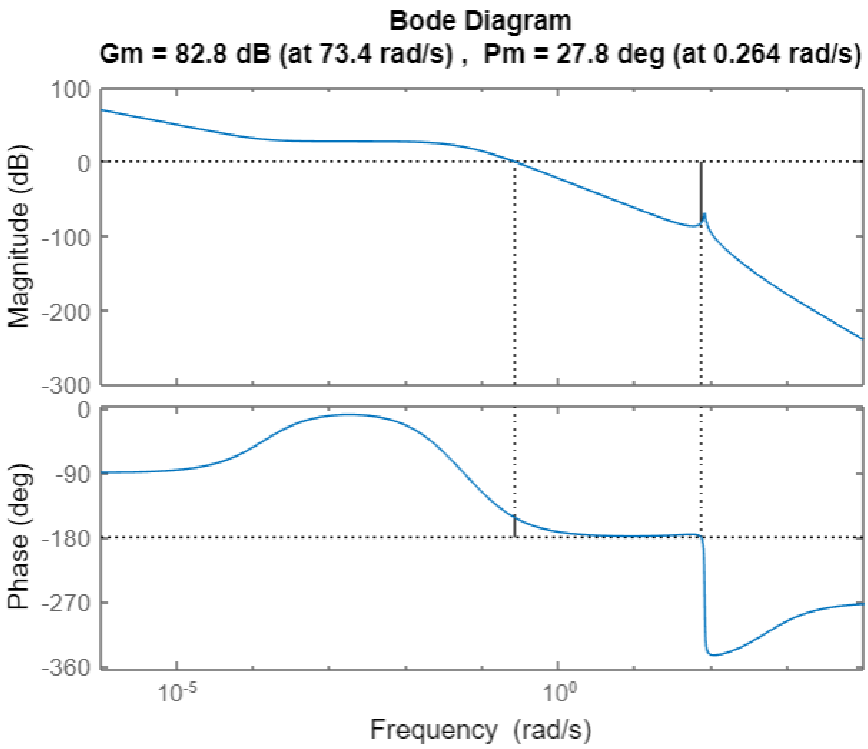



Figure 1: Closed-loop Block Diagram

Let us look at the Bode diagram of the Compensated system.




| Parameter | Uncompensated | Compensated |
|---|---|---|
| Plant and Compensator | $\frac{K(S+500)}{S(S+0.0325)(S^2+2.57S+6667)}$ | $\frac{K(S+0.000134)}{(S+0.1)}\frac{(S+500)}{S(S+0.0325)(S^2+2.57S+6667)}$ |
| K | 32300 | 32200 |
| $K_v$ | 74500 | 99.56 |
| Dominant Second Order Pole | $-19.3 \pm 58i$ | $-19.2 \pm 57.9i$ |

$\therefore$ Our Controller $= G(S) = 3.23 \times 10^4 \cdot \frac{(S+0.000134)}{(S+0.1)}$

with Poles $= -19.2 + 57.9i$

From above Dominant Poles,

$$19.2 = \sigma = \tau\omega_n \tag{10}$$

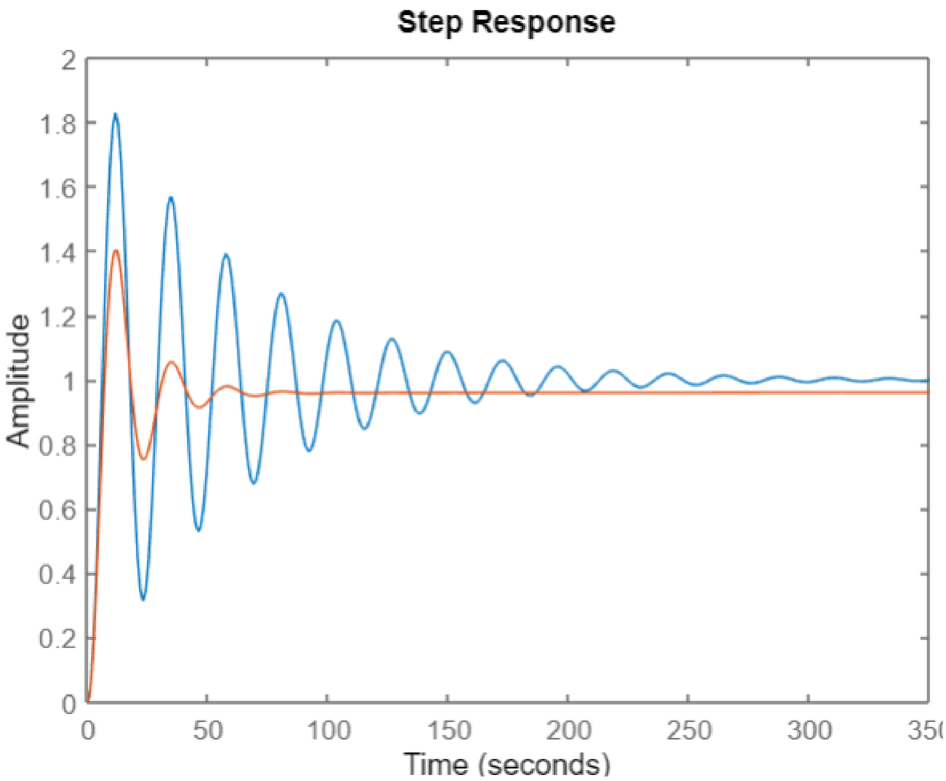
$$\omega_n = 60.79 \text{ rad/s} \tag{11}$$

$$\begin{aligned}\text{Resonant Frequency} = \omega_r &= \omega_n\sqrt{1-2\tau^2} \\ &= 60.79\sqrt{1-2(0.316)^2} \\ &= 54.3 \text{ rad/s}\end{aligned} \tag{12}$$

$$\begin{aligned}\text{Resonant Peak} &= \frac{1}{2\tau\sqrt{1-\tau^2}} \\ &= \frac{1}{2\times 0.316\sqrt{1-(0.316)^2}} \\ &= 1.667\end{aligned} \tag{13}$$

$$\begin{aligned}\text{Bandwidth} &= \omega_n(1-2\tau^2+\sqrt{4\tau^4-4\tau^2+2})^{\frac{1}{2}} \\ &= 87.69\ \text{rad/s}\end{aligned} \tag{14}$$

Let's see both Compensated and Uncompensated transient responses.



```
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    SettlingMin: 0.3127
    SettlingMax: 1.8291
      Overshoot: 82.9073
     Undershoot: 0
           Peak: 1.8291
       PeakTime: 11.4717
```

```
       RiseTime: 4.6472
   SettlingTime: NaN
    SettlingMin: 0.7518
    SettlingMax: 1.4033
      Overshoot: 40.3287
     Undershoot: 0
           Peak: 1.4033
       PeakTime: 11.8176
```

```
%code for plot1
clear all
clc
s = zpk(0,[],1);
Gu =  (s+500)/(s*(s+0.0325)*(s^2+ 2.57*s+ 6667));
stepinfo(Gu/(1+Gu))


%code for plot2
clear all
clc
s = zpk(0,[],1);
k =  (s+500)/(s*(s+0.0325)*(s^2+ 2.57*s+ 6667));
bode(k)


%code for plot3
clear all
clc
s = zpk(0,[],1);
k =  (s+500)/(s*(s+0.0325)*(s^2+ 2.57*s+ 6667));
rlocus(k)


%Code for plot4
clear all
clc
s = zpk(0,[],1);
Gu =  (s+0.000134)(s+500)/((s+0.1)*s(s+0.0325)*(s^2+ 2.57*s+ 6667));
```

```
rlocus(Gu)


%code for plot5
clear all
clc
s = zpk(0,[],1);
k =  (s+0.000134)(s+500)/((s+0.1)*s(s+0.0325)*(s^2+ 2.57*s+ 6667));
margin(k)


%code for plot6
clear all
clc
s = zpk(0,[],1);
Gu =  (s+500)/(s*(s+0.0325)*(s^2+ 2.57*s+ 6667));

Gc = (s+0.000134)/(s+0.1);

Gce=Gu*Gc;
Tu=feedback(Gu,1);
Tc=feedback(Gce,1);
step(Tu)
hold
step(Tc)
```