1)

```
Message:  2148321
Public Key: (N): 12982131232583, (e): 5
Encoded Message:  66285311511014
Secret Key:  7789272102893
Decoded Message:  2148321
Encoder Working?:  True


part iii:
p: 1334737, q: 9726359
part iv:
d:  7789272102893
part v:
Encoded message:  66285311511014
part vi:
Decoded message:  2148321
```

2)

i) $T(n) = 3\,T(n/2) + O(1)$

$a=3,\ b=2,\ d=0$ $\Big\}$ $d < \log_b a$ $\quad\therefore T(n) = O(n^{\log_2 3})$

ii) $T(n) = 4\,T(n/5) + O(n)$

$a=4,\ b=5,\ c=1$ $\Big\}$ $d > \log_b a$ $\quad\therefore T(n) = O(n)$

iii) $T(n) = 4\,T(n/8) + O(n^2)$

$a=4,\ b=8,\ d=2$ $\Big\}$ $d > \log_b a$ $\quad\therefore T(n) = O(n^2)$

iv) $T(n) = T(n-1) + n$, $T(0) = 0$

$$\begin{aligned} T(n) &= T(n-1) + n \\ &= T(n-2) + n + n \\ &\vdots \\ &= n(n) \\ &= n^2 \end{aligned}$$

$T(n) = O(n^2)$

3)

```python
# Assume its sorted from smallest to largest
def logn(A):
    l = 0
    r = len(A)-1

    while (l <= r):
        mid = (l+r) // 2
        if (A[mid] > mid):
            r = mid - 1
        elif (A[mid] < mid):
            l = mid + 1
        else:
            return True
    return False
```

Checks center of the array and eliminates half of the array for analysis if A[i] != i. The center of the remaining half is checked to recursively remove remaining halves until an answer is found.

$$T(n) = T(n/2) + O(1)$$

$$\left.\begin{array}{l} a=1 \\ b=2 \\ d=0 \end{array}\right\} \quad d = \log_b a \quad \Rightarrow T(n) = O(n^0 \log n)$$

$$\therefore T(n) = O(\log n)$$

4) $T(n) = 3T(n/3) + O(n)$

$a=3$, $b=3$, $d=1$ $\Big\}$ $d = \log_b a$ $\therefore$ $T(n) = O(n \log n)$