ECE406 **Algorithm Design & Analysis** **Winter 2025**

Assignment #6
due Fri Apr 04 11:59pm
See Appendix for submission instructions

Exercise 1 (NP-completeness) Consider the following two problems

> Bin-Packing
>
> **Input:** A set of n items where item i has size s_i; m bins each with capacity $C > 0$.
>
> **Output:** An assignment of each item to one bin such that the total size of items in each bin is $\leq C$, or determine that no such assignment exists.

For example, suppose we have five items $(s_1, s_2, s_3, s_4, s_5) = (2, 1, 3, 2, 3)$ and $m = 3$ bins each with capacity $C = 4$. A solution is bin $1 = \{\text{item } 1, \text{item } 4\}$, bin $2 = \{\text{item } 2, \text{item } 3\}$, and bin $3 = \{\text{item } 5\}$. Bins 1 and 2 are filled to capacity, while bin 3 still has one unit of space.

> Partition
>
> **Input:** A set of integers $A = \{a_1, a_2, \ldots, a_n\}$.
>
> **Output:** Split A into two subsets A_1 and A_2 with the same sum, or the determine that no such subset exists.

Using the example of $A = \{1, 4, -1, 2\}$, a solution is $A_1 = \{1, 2\}$ and $A_2 = \{4, -1\}$, both of which sum to 3. Notice that the union of A_1 and A_2 is A.

(i) Verify that Bin-Packing is in NP.

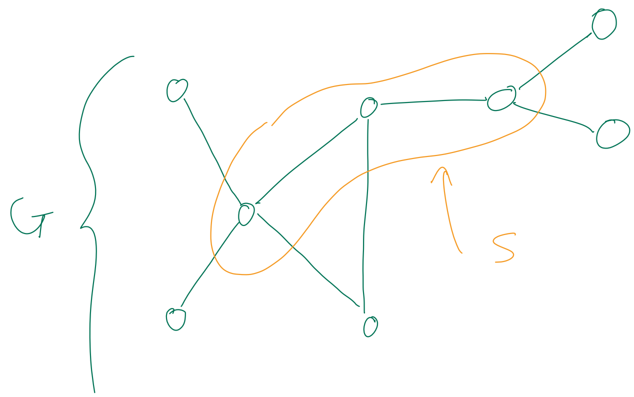
(ii) Verify that Partition is in NP.

(iii) Give a reduction from Partition to Bin-Packing. This reduction shows that one problem is "at least as hard" as the other. Which one is at least as hard as the other?

Exercise 2 (Vertex Cover and Set Cover) Consider the following two problems:

VERTEX COVER
Input: An undirected graph $G = (V, E)$
Output: A subset of vertices $S \subseteq V$ such that every edge has an endpoint in S.
Goal: Minimize $|S|$.

Example of Vertex Cover:



SET COVER
Input: A set of n elements B; sets $S_1, \ldots, S_m \subseteq B$
Output: A selection of the S_i whose union is B
Goal: Minimize the number of sets picked.

Example of Set Cover: see lecture notes (Sec. 5.6).

(i) What are the search versions of Vertex Cover and Set Cover?

(ii) Describe a reduction from the search version of Vertex Cover to the search version of Set Cover.

Note: earlier in class we derived a greedy algorithm for the optimization version of Set Cover. However, this algorithm only provided an *approximate* solution. Finding an exact solution to Set Cover is hard, and in fact the search version of Set Cover is NP-Complete.

Exercise 3 (Set cover to ILP) Describe a reduction from the search version of set cover to the search version of integer linear programming.

Recall from lecture, the search version of ILP is to find solution to $Ax \leq b$, where x is a vector of non-negative integers, or determine no such solution exists. Note that this formulation has *only* constraints and no objective function. This is because if we want to reach some "goal" g that represents the minimum objective value for any solution, i.e. $c^T x \geq g$, then this can be incorporated as an extra constraint of the LP: $-c^T x \leq -g$.

Appendix

How do I submit this assignment? To submit the assignment, please complete the following steps.

(i) Upload your typed or handwritten work for *each* question using the ECE406 **Crowdmark** site.

(ii) That's it! There's no programming question this time, so no Dropbox upload is needed.