

$$1b) u(x, y) = a_1x + b_1y + c_1$$

$$v(x, y) = a_2x + b_2y + c_2$$

These contains 6 unknowns

$$P = [a_1, b_1, c_1, a_2, b_2, c_2]^T$$

A pixel at (x, y) in frame t will move to

$$x' = x + u(x, y)$$

$$y' = y + v(x, y)$$

$$x' = x + a_1x + b_1y + c_1$$

$$y' = y + a_2x + b_2y + c_2$$

Applying Brightness Constancy

$$I(x, y, t) \approx I(x', y', t+1)$$

$$I(x', y', t+1) \approx I(x, y, t) + I_x u + I_y v + I_t$$

$$(I_x x) a_1 + (I_x y) b_1 + (I_x) c_1 + (I_y x) a_2 + (I_y y) b_2 + (I_y) c_2 = -I_t$$

for each pixel,

$$I = [I_x x, I_x y, I_x, I_y x, I_y y, I_y]$$

Now we can write it as

$$I \cdot \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{bmatrix} = P \cdot I_t$$

$$J_p = -I_t$$

Symbols for every pixel.

$$J_{1P} = -I_{t_1}$$

$$J_{2P} = -I_{t_2}$$

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$$J_{NP} = -I_{t_N}$$

$$A = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_N \end{bmatrix} \begin{bmatrix} I_x, x_1, I_x, y_1, I_n, I_y, x_1, I_y, y_1, I_y, \\ I_x, x_2, I_x, y_2, I_n, I_y, x_2, I_y, y_2, I_y, \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ I_x, x_N, I_x, y_N, I_n, I_y, x_N, I_y, y_N, I_y, \end{bmatrix}$$

$$b = \begin{bmatrix} -I_{t_1} \\ -I_{t_2} \\ \vdots \\ -I_{t_N} \end{bmatrix}$$

Now, we will use

$$(A^T A) p = A^T b$$