7) 3D point : $X = (x, y, z, 1)^T$

Two unknown Cameras : $P = R_{3\times4}$, $P' = R_{3\times4}$

Image points : $x = (u, v, 1)^T \sim PX$, $x' = (u', v', 1)^T \sim P'X$

For any corresponding pair (x, x') we have epipolar constraint

$$x'^T F x = 0$$

We seek Homographies H and H'

$$\tilde{x} = Hx = (\tilde{u}, \tilde{v}, 1)^T, \quad \tilde{x}' = H'x' = (\tilde{u}', \tilde{v}', 1)^T$$

$$\tilde{v} = \tilde{v}' \text{ (same row)}$$

$$d = \tilde{u} - \tilde{u}'$$

We have focal length 'f' and Base line B

$$d = \frac{fB}{Z} \Rightarrow Z = \frac{fB}{d}$$

In uncalibrated case

$$Z = \frac{\alpha}{d}, \quad \alpha = fB \text{ (unknown Constant)}$$

$$\hat{Z} = \frac{1}{d}$$

$$\underline{\underline{Z = \alpha \hat{Z}.}}$$

In the rectified pair the first camera can be choosen as

$$P_0 = [I|0]$$

$$P_0' = [I|b]$$

where,

$$b = (B, 0, 0)^T \text{ (unknown baseline)}$$

Using pinhole model

$$\hat{X} = \frac{\tilde{u}}{d}, \quad \hat{Y} = \frac{\tilde{v}}{d}, \quad \hat{Z} = \frac{1}{d} \quad (d = \tilde{u} - \tilde{u}')$$

$$\hat{P} = \begin{pmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{pmatrix} = \frac{1}{d} \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ 1 \end{pmatrix}$$

True distance between A and B is L_{AB}^{true}

$$d_A = \tilde{u}_A - \tilde{u}'_A, \qquad d_B = \tilde{u}_B - \tilde{u}'_B$$

$$\hat{P}_A = \frac{1}{d_A} \begin{pmatrix} \tilde{u}_A \\ \tilde{v}_A \\ 1 \end{pmatrix} \qquad \hat{P}_B = \frac{1}{d_B} \begin{pmatrix} \tilde{u}_B \\ \tilde{v}_B \\ 1 \end{pmatrix}$$

$$\hat{L}_{AB} = |\hat{P}_A - \hat{P}_B|$$

$$\text{Scale Factor } s = \frac{L_{AB}^{true}}{\hat{L}_{AB}}$$

- Compute $\hat{P}_i, \hat{P}_j$
- Projective Distance $\hat{L}_{ij} = |\hat{P}_i - \hat{P}_j|$

$$L_{ij} = s\hat{L}_{ij} = \frac{L_{AB}^{true}}{\hat{L}_{AB}} \hat{L}_{ij}$$

$$\boxed{L_{ij} = s \, \|\hat{P}_i - \hat{P}_j\|}$$