EE 5111: Estimation

Jan - May 2019

Mini Project 2

March 11, 2019

1 Performance of MLE

The aim of this exercise is to study the variation in the performance of MLE with increase in the number of samples. Consider the following equation:

$$x_i = A + n_i \qquad\qquad i = 1, ..., N \tag{1}$$

where A is scalar and n_i are the noise samples. Compute the maximum likelihood estimate of A for the following cases:

1. $n_i \sim \mathcal{N}(0, 1)$. In this case, use the following expression derived in class:
$$\hat{A} = \frac{1}{N}\sum_{i=1}^{N} x_i.$$
2. $n_i \sim Lap(0, 1/\sqrt{2})$, i.e., Laplace distribution with zero mean and unit variance. In this case, the MLE is derived as:
$$\hat{A} = median(x_i).$$
3. $n_i \sim Cauchy(0, \gamma)$. Use $\gamma = 1/\sqrt{2C_g}$ where $C_g = 1.78$. Refer Q3 of Tutorial 3 for MLE in case of Cauchy noise. The closed form solution for MLE is not available and hence, MLE should be computed through numerical evaluation. Use Newton Raphson or any other appropriate numerical method.

Repeat the above experiments for $N = 10, 100, 1000, 10000$ where N is the number of samples considered for estimation and for $A = 1$ and $A = 10$.
Present the following for each noise distribution:

1. Tabulate the values of $\mathbb{E}[\hat{A}]$ against the number of samples for both values of A. What do you infer?
2. Tabulate the values of $Var(\hat{A})$ against the number of samples for both values of A. What do you infer?
3. Plot the PDF of the estimate for different number of samples for $A = 1$. What can you say about the PDF? Justify. Do you observe the following relation?
$$\sqrt{N}(\hat{A} - A_0) \sim \mathcal{N}(0, I(A)^{-1})$$
Here, I denotes the Fisher information.

Consider appropriate number of iterations for computing mean, variance and the PDF of $\hat{A}$.