EE 5111: Estimation

Jan - May 2019

Mini Project Question

February 15, 2019

1 Problem

Consider the following OFDM system model:

$$Y = \mathbf{XFh} + N, \tag{1}$$

where $Y \in \mathbb{C}^{1024}$ is the set of observations, $\mathbf{X}$ is a 1024 dimensional diagonal matrix with known symbols, $\mathbf{h}$ is the L tap time domain channel vector, $\mathbf{F}$ is the $1024 \times L$ matrix performing IDFT[1] and N is complex Gaussian noise with variance σ^2.

For the following set of experiments, generate a set of random bits and modulate them as QPSK symbols to generate[2] $\mathbf{X}$. $\mathbf{h}$ is a multipath Rayleigh fading channel vector with an exponentially decaying power-delay profile $\mathbf{p}$ where $p[k] = e^{-\lambda(k-1)}, k = 1, 2...L$. That is, each component of $\mathbf{h}$ will be $h[k] = \dfrac{1}{\|\mathbf{p}\|_2}(a[k] + ib[k])p[k]$, where $a[k], b[k] \sim \mathcal{N}(0, \frac{1}{2})$; $k = 1, 2...L$. Here, λ is the decay factor (and choose $\lambda = 0.2$ for your simulations). Now, perform the following experiments on the described problem set up.

1. Estimate $\mathbf{h}$ using least squares method of estimation with $L = 16$.[3]
2. Now, suppose that $\mathbf{h}$ is sparse with just 6 non zero taps. Assuming that you know the non zero locations, estimate $\mathbf{h}$ using Least squares with the sparsity information.
3. Next, introduce guard band of 200 symbols on either side[4], i.e. now we have reduced number of observations. For this case:
   a Repeat (1),(2) for the above set up.
   b Apply regularization and redo least squares. Use various values of α for regularization with $\alpha\mathbf{I}$ and compare the estimation results.

[1] $\mathbf{F}(i,j) = e^{\frac{j2\pi(i-1)(j-1)}{1024}}; i = 1, \cdots, 1024, j = 1, \cdots, L$

[2] $X_{i,i} \in \{1 + 1j, -1 + 1j, 1 - 1j, -1 - 1j\}$

[3] Note that you are dealing with complex data now and hence the least squares estimate for the model $y = Xb$ shall now be $\hat{b} = (X^H X)^{-1} X^H y$

[4] Suppress to zero the first and last 200 symbols in X

4. Next perform least squares estimation on $\mathbf{h}$ with the following linear constraints :

$$h[1] = h[2]$$
$$h[3] = h[4]$$
$$h[5] = h[6]$$

For each of the above experiments, you have to compare $\mathbb{E}[\hat{\mathbf{h}}]$ and $\mathbf{h}$, theoretical and simulated MSE of estimation, all averaged over 10000 random trials. (Generate different instances of X and N for each trial.) Repeat the experiments for $\sigma^2 = \{0.1, 0.01\}$ for each case. Plot $\hat{\mathbf{h}}$ and $\mathbf{h}$ for one trial in each of the above cases.