

1. We want to show that $x > \log_2 x$ for all $x \in \mathbb{R}^+$. First, we note that

$$\int_1^\infty \tan(\alpha) d\alpha = \sum_{\beta=0}^\infty \left(\beta \cdot \arg \min_{\gamma \in \mathbb{C}} (1938\gamma^3 + \gamma) \right)$$

by the Borsuk–Ulam theorem. Furthermore, $1 = 1 + 1$ by the Banach–Tarski paradox, and thus, we are done.¹ \square

2. We want to compute the derivative of $\sin(e^x)$. We have the following.

$$\begin{aligned} \frac{d}{dx} \sin(e^x) &= \cos(e^x) \cdot \frac{d}{dx} e^x && \text{(chain rule)} \\ &= \cos(e^x) \cdot e^x \end{aligned}$$

3. See margin.

4. Let A be an orthogonal matrix. We want to show that A^2 is also orthogonal. We have the following.

$$\begin{aligned} (A^2)^\top (A^2) &= A^\top A^\top A A \\ &= A^\top I A && \text{(orthogonality of } A) \\ &= A^\top A \\ &= I && \text{(orthogonality of } A) \end{aligned}$$

I have discovered a truly beautiful solution to this problem, but unfortunately, it simply will not fit in this tiny margin.

Thus, A^2 is orthogonal. \square

5. I was unable to find the exact answer to this problem, but I was able to narrow it down to two possibilities (see Table 1).

Table 1: Possible answers to Problem 5.

True	False
This one seems likely because a lot of mathematical statements are true. For example, $1 + 1 = 2$ and $1 \times 0 = 0$ are both true statements.	This one also makes a lot of sense to me because sometimes things are false. An example of a false statement would be “I am confident in my solution to Problem 1”.

6. We simplify the expression as follows.

$$\begin{aligned} &\sqrt{1938^2 + 62^2 + 1938 \times 124 + \sin^2(1938) + \cos^2(1938) - 1} \\ &= \sqrt{1938^2 + 62^2 + 1938 \times 124 + 1 - 1} \\ &= \sqrt{1938^2 + 2 \times 1938 \times 62 + 62^2} \\ &= \sqrt{(1938 + 62)^2} \\ &= 1938 + 62 \\ &= 2000 \end{aligned}$$

¹This same technique can be easily adapted to prove $P=NP$.

7. Below is a demonstration of the execution of the merge sort algorithm:

	1	5	2	4	7	8	6	3
\hookrightarrow	5		2		8		6	
\hookrightarrow		1				8		
\hookrightarrow				1				
\hookrightarrow	1	2	3	4	5	6	7	8

8. *This proof is left as an exercise to the grader.*

9. We want to find the limit of $f(x) = \frac{\sin x}{\ln x}$ as x tends to infinity. Let $g(x) = -\frac{1}{\ln x}$ and $h(x) = \frac{1}{\ln x}$. Note that f is bounded below by g and above by h . Since the logarithm function increases without bound, we have $\lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} h(x) = 0$. Thus, by the squeeze theorem, we have $\lim_{x \rightarrow \infty} f(x) = 0$.

10. *Sorry, Professor — my ~~dog~~ cyber-dog ate my solution to this problem.*