1. We want to show that  $x > \log_2 x$  for all  $x \in \mathbb{R}^+$ . First, we note that

$$\int_{1}^{\infty} \tan(\alpha) \, d\alpha = \sum_{\beta=0}^{\infty} \left( \beta \cdot \arg\min_{\gamma \in \mathbb{C}} \left( 1938 \gamma^{3} + \gamma \right) \right)$$

by the Borsuk–Ulam theorem. Furthermore, 1 = 1 + 1 by the Banach–Tarski paradox, and thus, we are done.<sup>1</sup>

2. We want to compute the derivative of  $sin(e^x)$ . We have the following.

$$\frac{d}{dx}\sin(e^x) = \cos(e^x) \cdot \frac{d}{dx}e^x \qquad \text{(chain rule)}$$
$$= \cos(e^x) \cdot e^x$$

- 3. See margin.
- 4. Let A be an orthogonal matrix. We want to show that  $A^2$  is also orthogonal. We have solution to this the following.

ollowing.  

$$(A^{2})^{\mathsf{T}}(A^{2}) = A^{\mathsf{T}}A^{\mathsf{T}}AA$$

$$= A^{\mathsf{T}}IA \qquad (\text{orthogonality of } A)$$

 $= A^{\mathsf{T}}IA \qquad \text{(orthogonality of } A)$   $= A^{\mathsf{T}}A$   $= I \qquad \text{(orthogonality of } A)$ 

Thus,  $A^2$  is orthogonal.

5. I was unable to find the exact answer to this problem, but I was able to narrow it down to two possibilities (see Table 1).

Table 1: Possible answers to Problem 5.

True	False
lot of mathematical statements are	This one also makes a lot of sense to me because sometimes things are false. An example of a false state- ment would be "I am confident in my solution to Problem 1".

6. We simplify the expression as follows.

$$\sqrt{1938^2 + 62^2 + 1938 \times 124 + \sin^2(1938) + \cos^2(1938) - 1}$$

$$= \sqrt{1938^2 + 62^2 + 1938 \times 124 + 1 - 1}$$

$$= \sqrt{1938^2 + 2 \times 1938 \times 62 + 62^2}$$

$$= \sqrt{(1938 + 62)^2}$$

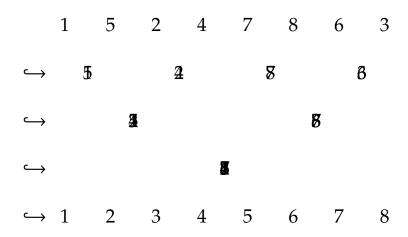
$$= 1938 + 62$$

$$= 2000$$

I have discovered a truly beautiful solution to this problem, but unfortunately, it simply will not fit in this tiny margin.

<sup>&</sup>lt;sup>1</sup>This same technique can be easily adapted to prove P=NP.

7. Below is a demonstration of the execution of the merge sort algorithm:



- 8. This proof is left as an exercise to the grader.
- 9. We want to find the limit of  $f(x) = \frac{\sin x}{\ln x}$  as x tends to infinity. Let  $g(x) = -\frac{1}{\ln x}$  and  $h(x) = \frac{1}{\ln x}$ . Note that f is bounded below by g and above by h. Since the logarithm function increases without bound, we have  $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} h(x) = 0$ . Thus, by the squeeze theorem, we have  $\lim_{x\to\infty} f(x) = 0$ .
- 10. Sorry, Professor my dog cyber-dog ate my solution to this problem.