Loss Minimizing Gear Shifting Algorithm based on Optimal Current Sets for IPMSM

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***Abstract*—Two speed gear is preferable in the commercial EVs such as EV trucks and EV bus, since they require a high starting torque and high speed driving, too. In this work, two speed gear with 12.5:1 and 5:1 is considered. The wheel torque is mapped to different points in the torque-speed plane of the motor depending on the reduction ratio. To determine the efficiency, it is necessary to calculate the losses for the two cases. To be fair, optimal current set (i_d, i_q) should be found for each case before calculating the loss, since the copper and inverter losses depend heavily on the current magnitude. The operation points are found by the MTPA method, or at the intersection between the torque and voltage limit curve. Ferrari's method is utilized to solve the fourth order equation resulted from the intersection. After comparing the losses for different gear ratios, the wheel torque-speed plane is separated, and from which an optimal gear shifting algorithm is developed. Simulation was performed by using NEDC. The results show that the proposed gear shift algorithm is superior to a fixed gear in minimizing the loss.**

I. Introduction

Motor has high efficiency in wide region of torque-speed plane. Thus, electric vehicles commonly utilize traction machine with a single speed gearbox [1]. However, electric vehicle with different gear ratio has better road load resistance performance than single speed gearbox [2]. Furthermore, high torque performance at low speed and high speed operation can be achieved by using the two-speed gearbox [3][4]. Optimal design procedure of the motor and gear ratio was proposed considering wide operating range and acceleration performance [5]. For a given wheel torque and speed, the change of gear ratio makes a motor operates at different conditions. Therefore, we can improve the efficiency of EV powertrain by selecting the suitable gear ratio.

In this work, the two-speed gear is utilized: First gear ratio is determined considering high torque for uphill starting and second gear ratio is used to meet maximum vehicle speed with high efficiency. In general, decision of gear shift is determined by vehicle speed [6]. Instead, we proposed a novel gear shift algorithm considering torque and speed to improve the efficiency. The algorithm can be obtained by optimal gear shifting line which is induced by comparing the losses for each gear ratios. Furthermore, hysteresis band technique is applied to solve the practical implementation issue. Simulations were carried out for NEDC drive cycle to show the usefulness of the algorithm.

II. Vehicle Dynamics and Driving Cycle

It is considered here a gear shifting algorithm for a commercial pick up truck that optimizes the fuel efficiency over a given driving cycle. Trucks and buses require high torque for uphill starting especially when the load is full. For this purpose, the gear ratio is as high as 12:1∼16:1. With such a high gear ratio, the motor speed will be extremely high at a high speed. Since the motor loss is large in the high speed region, it is better to switch to a low gear ratio such as 5:1∼6:1.

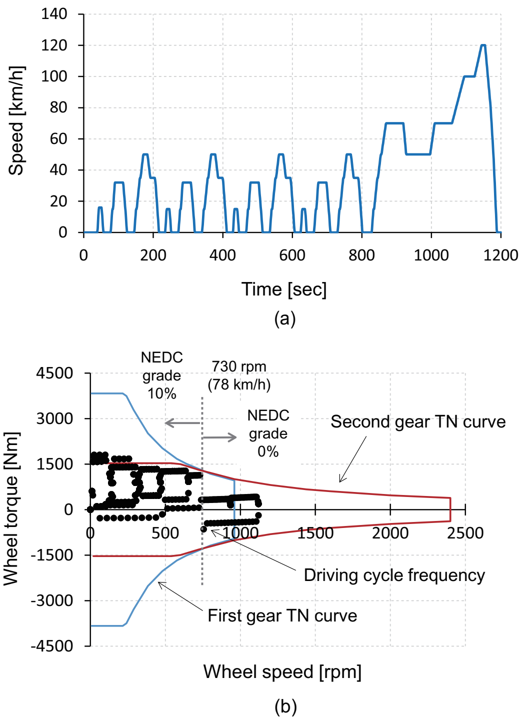

Fig. 1. Driving Cycle : (a) New European Driving Cycle(NEDC) (b) Wheel torque speed curve for two gear ratios with driving cycle frequency.

Fig. 1 shows NEDC mode driving cycles. Note that NEDC is a low speed urban driving cycle with low acceleration and deceleration. To calculate the required wheel torque we consider a simple vehicle dynamics:

$$\frac{dV_x}{dt} = \frac{1}{m_v}\left[F_x - \frac{\rho C_d A_F}{2}V_x^2 - f_r m_v g\cos\alpha - m_v g\sin\alpha\right] \quad (1)$$

where F_x: traction force, m_v: vehicle gross weight, V_x: vehicle speed, ρ: air density, C_d: drag coefficient, A_F: vehicle frontal area, f_r: rolling coefficient, g: gravity constant, and α: slope angle. The motor is connected to wheel shaft via a reduction gear with reduction ratio, g_{dr}. We also assume that the drive train efficiency from motor to wheel is η_{dr}. Then the motor shaft torque, T_e is related to the wheel torque such that

$T_w = T_e \eta_{dr} g_{dr}$. Therefore,

$$F_x = \frac{T_w}{r_w} = \frac{T_e \eta_{dr} g_{dr}}{r_w} \tag{2}$$

where r_w is an effective wheel radius. There is a wheel slip, $s_x = \frac{r_w \omega_w - V_x}{r_w \omega_w}$. Then the wheel speed is equal to

$$\omega_w = \frac{V_x}{r_w(1 - s_x)} \tag{3}$$

The vehicle parameters that we used for simulation are listed in Table I. Driving cycle frequency, which consists of wheel torque and speed, is defined by using vehicle dynamics and driving cycle. Aggressive mode is utilized in process of obtaining driving cycle frequency. A ten percent slope grade is applied from 0 to 730rpm and zero percent is applied above 730rpm. The driving cycle frequency obtained via this process is shown in Fig. 1 (b). It also shows a wheel torque-speed curves based on Table II and gear ratios. The blue curve and red curve are obtained by applying first gear ratio and second gear ratio to motor TN curve.

TABLE I
VEHICLE DYNAMICS.

Parameters	Values	Parameters	Values
C_d	0.3	α	0, 5.71
f_r	0.0077	ρ	1.225 kg/m^3
A_F	1.767 m^2	m_v	2700 kg
r_w	0.32 m	η_{dr}	0.9
s_x	0.08	First gear ratio, g_{dr1}	12.5
g	9.81 m/s^2	Second gear ratio, g_{dr2}	5

TABLE II
MOTOR SPECIFICATIONS.

Parameters	Values	Parameters	Values
Pole Numbers	8	Number of slots	48
DC-link Voltage	360V	Maximum Speed	12000rpm
Maximum Power	100kW	Rated Speed	2750 rpm
Maximum Current	320 Arms	Back EMF	0.0984V/ra
Maximum Torque	320Nm	Rated Power	60kW

III. OPTIMAL CURRENT COMMAND SELECTION

Maximum torque per ampere (MTPA) offers the minimum current magnitude for a given torque under the base speed. However in the field weakening range, it is required to find the intersection between the voltage limit and torque curves:

$$\omega_e^2(L_d i_d + \psi_m)^2 + \omega_e^2(L_q i_q)^2 \le \frac{V_{dc}^2}{3} \tag{4}$$

$$T = \frac{3P}{4}\Big[\psi_m i_q + (L_d - L_q) i_d i_q\Big] \tag{5}$$

where L_d, L_q, and ψ_m are d, q inductances, and PM flux linkage, respectively. i_d, i_q, ω_e, V_{dc}, T, and P are d, q axis current, inverter dc link voltage, electrical angular frequency, motor shaft torque, and pole number, respectively. Combining voltage limit (4) and torque equation (5), we obtain a quartic equation in i_d :

$$x^4 + Ax^3 + Bx^2 + Cx + D = 0 \tag{6}$$

where $A = \frac{2\psi_m}{L_d - L_q}\Big(2 - \frac{L_q}{L_d}\Big)$, $B = \frac{\psi_m^2}{(L_d-L_q)^2} + \frac{4\psi_m^2}{L_d(L_d-L_q)} + \frac{\psi_m^2}{L_d^2} - \frac{V_s^2}{(\omega_e L_d)^2}$, $C = \frac{2\psi_m}{L_d}\Big(\frac{\psi_m^2}{(L_d-L_q)^2} + \frac{\psi_m^2}{L_d(L_d-L_q)} - \frac{V_s^2}{\omega_e^2 L_d(L_d-L_q)}\Big)$, and $D = \frac{1}{(L_d-L_q)^2}\Big(\frac{\psi_m^4}{L_d^2} + \frac{L_q^2}{L_d^2}\cdot\frac{16T^2}{9P^2} - \frac{V_s^2}{\omega_e^2}\cdot\frac{\psi_m^2}{L_d^2}\Big)$. A general solution of fourth order equation (6) can be found by utilizing Ferrari's method [7].

$$x(= i_d) = -\frac{A}{4} \pm_s \frac{\eta}{2} \pm_t \frac{\mu}{2} \tag{7}$$

where

$$\mu = \sqrt{\frac{3}{4}A^2 - \eta^2 - 2B \pm_s \frac{1}{4\eta}(4AB - 8C - A^3)}\,,$$
$$\nu = \sqrt{\frac{\epsilon^2}{4} + \frac{\alpha^3}{27}},\ \gamma = \frac{B}{3} + \sqrt[3]{-\frac{\epsilon}{2} + \nu} + \sqrt[3]{-\frac{\epsilon}{2} - \nu},$$
$$\eta = \sqrt{\frac{A^2}{4} - B + \gamma},$$

$\alpha = \frac{1}{3}(3AC - 12D - B^2)$, and $\epsilon = \frac{1}{27}(-2B^3 + 9ABC + 72BD - 27C^2 - 27A^2D)$. In other words, if operating point of motor torque and electrical angular speed was determined, optimal current is calculated by Ferrari's Method in field weakening region.

IV. EFFICIENCY COMPARISON BETWEEN TWO GEAR RATIOS

To find a better gear ratio for a given torque and speed the losses are calculated for each gear set. Fig. 2 (a) shows a set of NEDC operation points in the wheel torque-speed plane. The wheel torque and speed are converted differently to the motor torque and speed depending on the gear ratio. Two reduction gear ratios are considered here: 12.5:1 and 5:1. Two sample points, A=(300rpm, 1150Nm) and B=(600rpm, 330Nm) are chosen for the purpose of illustration. They are mapped into A_1=(3750rpm, 102Nm) and B_1=(7500rpm, 29Nm) in the motor torque-speed plane via 12.5:1 gear shown as Fig. 2 (b). On the other hand, A_2=(1500rpm, 256Nm) and B_2=(3000rpm, 73Nm) via 5:1 gear shown as Fig. 2 (c).

A. Copper Loss

The motor losses are calculated and compared for A_1 and A_2. For each point best possible current set is chosen for loss calculation. In other words, the copper loss minimizing points are selected. The speed of A_2 is less than the rated speed, thereby the point of MTPA yields the loss minimizing solution. But the speed of A_1 is higher than the rated one. Thus the optimal solution is selected at the intersection between speed limit and torque curves. The current magnitude for A_1 is dramatically smaller than that of A_2 as shown in Fig. 3 (a). The same thing is repeated for B_1 and B_2. That result shows magnitude of B_2 is smaller than B_1 shown as Fig. 3 (b).

B. Iron Loss

Iron loss consists of hysteresis loss and eddy current loss. In this paper both losses are integrated into one formula [8]:

$$P_{iron} = C_{fe}(\lambda_d^2 + \lambda_q^2)\omega_e^\gamma \tag{8}$$

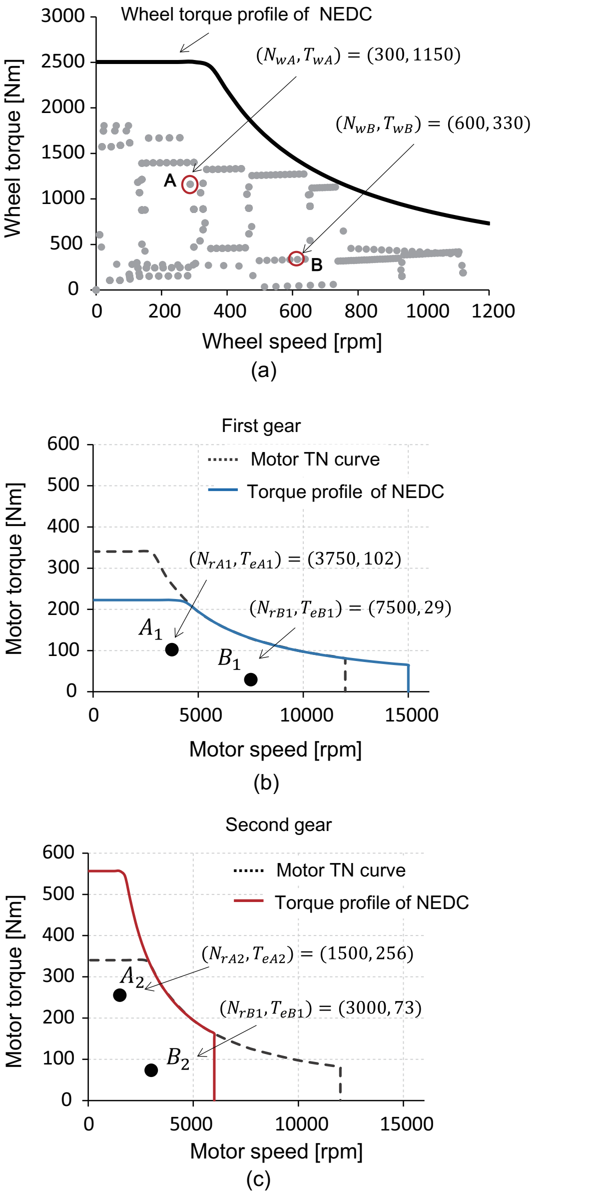

Fig. 2. Torque and speed curve: (a) Driving cycle frequency and wheel torque profile of NEDC (b) Motor torque profile based on first gear (c) Motor torque profile based on second gear.

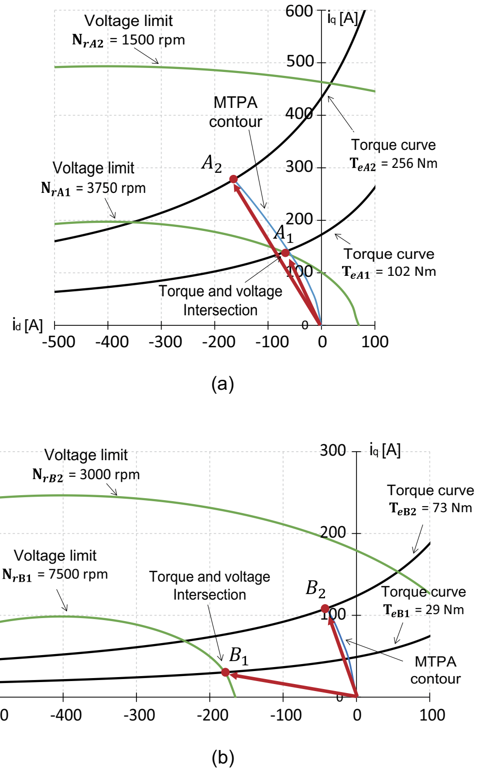

Fig. 3. Current optimization by using MTPA and Ferrari's method : (a) Comparison of point A_1 with A_2 (b) Comparison of point B_1 with B_2.

where λ_d and λ_q are d, q flux linkage and $C_{fe} = \frac{3}{2}$ and $\gamma = 1.6$. Note that the flux is proportional to the voltage magnitude. The iron losses for A_1 and B_1 are larger than those for A_2 and B_2, respectively, since the operation voltage and speed are higher at A_1 and B_1.

C. Inverter Loss

The total inverter loss are simply calculated as follows:

$$P_{inv} = \Big((\frac{2V_{dc}}{\pi}) I_s (t_r + t_f) + (\frac{2V_{on}}{\pi}) I_s t_{on} + (\frac{V_{dc}}{\pi}) I_{rr} t_{rr} \Big) f_{sw} \quad (9)$$

where I_s, t_r, V_{on}, I_{rr}, t_{rr} and f_{sw} are current magnitude, rise time, fall time, collector-emitter on-state voltage, reverse recovery current, reverse recovery time and switching frequency, respectively. Inverter loss consists of switching loss, conduction loss and reverse recovery loss [9]. The first term represents the IGBT switching loss, the second conduction loss, and the third loss caused by diode reverse recovery current. In the calculation we let $V_{dc} = 360$ and assume that the switching frequency is f_{sw} = 8kHz. We also utilized a typical automotive IGBT data; $t_r = 0.09\mu$s, $t_f = 0.16\mu$s, $I_{rr} = \frac{1}{3} I_s$, and $t_{rr} = 0.06\mu$s. The inverter loss depends basically on the current magnitude. Therefore, the loss for A_2 is larger than A_1 and B_1 is also lager than B_2.

D. Total Loss and Comparison

Note again that A_1 and A_2 yield the same wheel torque, but the gear ratios are different. The losses are compared in Fig. 4 (a). As expected, A_1, offering a shorter current magnitude, creates lower copper and inverter losses than A_2. However the iron loss is larger with A_1 since it utilizes a higher frequency. The total loss is lower for A_1, i.e, 12.5:1 gear ratio is better for such wheel speed and torque, (300rpm, 1150Nm). The situation is different for (600rpm, 330Nm). B_2 of 5:1 gear offers lower total loss. That is, a low gear ratio is more preferable for low torque high speed operation.

E. Efficiency Comparison between Two Gear

The total loss calculation is repeated for each point in the (wheel) torque-speed plane under different gear ratios. Each operation point is mapped into the motor torque-speed plane. Then, the optimal current (i_d, i_q) is found according to MTPA, or at an intersection with the voltage limit curve. To find the intersection, we utilized the Ferrari's method. Once the current magnitude is determined, the copper, iron, and inverter losses are calculated according to (8) and (9).

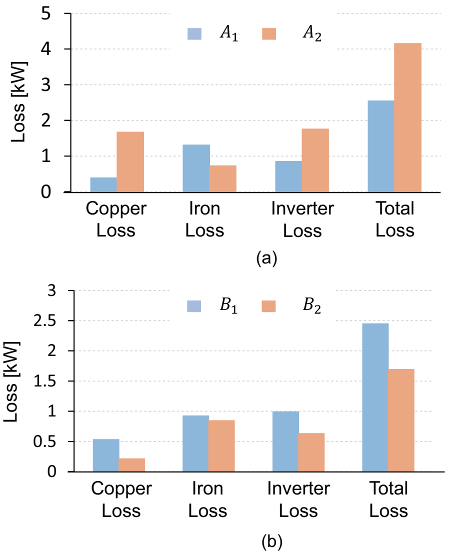

Fig. 4. Motor and inverter loss comparison : (a) Loss comparison between point A_1 with A_2 (b) Loss comparison between point B_1 with B_2.

The results are summarized as bar plots. Fig. 5 (a) and (b) show the total losses when the gear ratios are 12.5:1 and 5:1, respectively. The two cases are compared for each point, and loss difference is shown in Fig. 5 (c) . Positive value of loss difference is preferable to use first gear ratio, 12.5:1. On the other hand, negative value is preferable to use second gear ratio, 5:1.

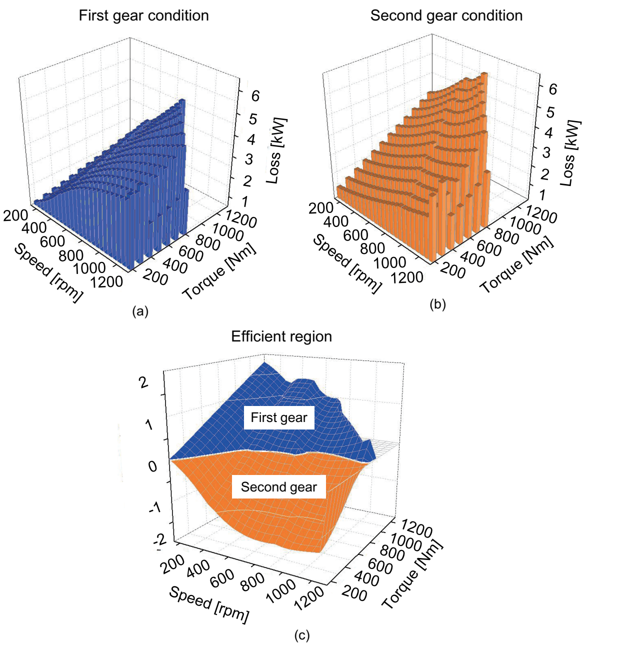

Fig. 5. loss calculation according to wheel torque and speed at set intervals: (a) 12.5:1 gear , (b) 5:1 gear, (c) Efficient region determined by using difference value of first and second gear condition.

Just looking at the choices of lower loss, we obtain Fig. 6 (a). 2-dimension of Fig. 5 (c) also indicates Fig. 6 (a). However, due to the speed limit of the motor, the first gear (12.5:1) cannot be used in the high speed region. Reflecting it, the separation regions are modified slightly as shown in Fig. 6 (b). We have to provide the hysteresis band between the region to prevent unnecessarily high frequent gear shifting. The bandwidth will be determined by trading the shifting frequency with the efficiency. Fig. 6 (c) shows an optimal gear shifting region with a hysteresis band. It is claimed here that an optimal gear shifting algorithm can be developed with the region separation diagram shown in Fig. 6 (c).

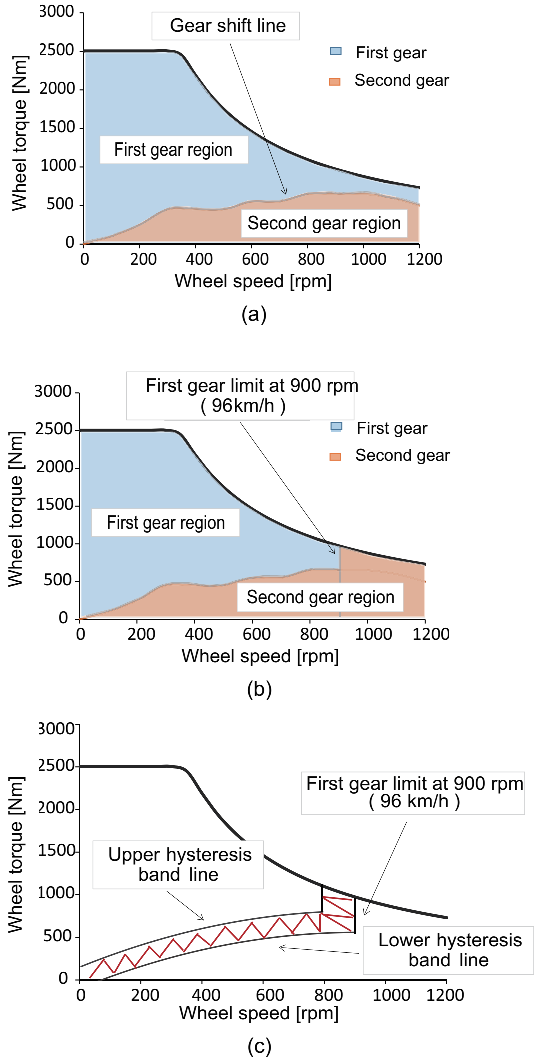

Fig. 6. Separation of regions for 12.5:1 and 5:1 gears based on loss minimization: (a) separation based on lower losses, (b) modification reflecting the motor speed limit, and (c) hysteresis band preventing frequent gear shift.

V. Gear Shift Algorithm

Note that point of wheel speed and torque, (N_w, T_W), can be defined by analyzing driving cycle with vehicle dynamics. Operating points based on NEDC are depicted in Fig. 2 (a). In Fig. 6 (a), wheel torque profile and gear shift line based on NEDC was shown. The gear shift line determines whether operating point is located in first or second gear region. Therefore, gear shift algorithm is generated from gear shift line. Note that use of first gear is limited over 900rpm shown in Fig. 6 (b). Therefore, second gear is only used over 900rpm in this algorithm. Except for speed limit, all case is determined by gear shift line. It is defined as quadratic function in wheel speed.

$$T_w = AN_w^2 + BN_w + C, \tag{10}$$

In this algorithm, one more thing is also considered. To prevent continuous gear shifting near the gear shift line, idea of hysteresis band is added to this algorithm, since frequent gear shift is not good and stressful to system durability.

$$T_{upper} = AN_w^2 + BN_w + D, \tag{11}$$

$$T_{lower} = AN_w^2 + BN_w + E, \tag{12}$$

where (11) is upper hysteresis band line. This line determines gear shift from second gear to first gear. Similarly, (12) is lower hysteresis band line which determines gear shift from first gear to second. If data of required torque and speed, (N_w, T_W), is received while driving a vehicle, algorithm determines whether gear shift to be executed or not. Gear shift algorithm related with (10) proceed in the following:

If $N_w < 800$ rpm,

1) Requirement of (N_{w0}, T_{w0}), F.G

2) Requirement of (N_{w1}, T_{w1}) at first gear,
$T_{w1} \geq T_{lower} = AN_{w1}^2 + BN_{w1} + E$, M.C.G

3) Requirement of (N_{w2}, T_{w2}) at first gear,
$T_{w2} < T_{lower} = AN_{w2}^2 + BN_{w2} + E$, S.G

4) Requirement of (N_{w3}, T_{w3}) at second gear,
$T_{w3} \leq T_{upper} = AN_{w3}^2 + BN_{w3} + D$, M.C.G

5) Requirement of (N_{w4}, T_{w4}) at second gear,
$T_{w4} > T_{upper} = AN_{w4}^2 + BN_{w4} + D$, F.G

If $N_w \geq 900$rpm use only second gear and 800rpm $\leq N_w < 900$rpm maintain current gear.

Where F.G, S.G and M.C.G indicate using first gear, using second gear and maintaining current gear. Note that first gear is determined as a starting gear due to $C < 0$ in case of initial condition, (N_{w0}, T_{w0}).

VI. Simulation Results

Gear shift algorithm was applied to the two speed gear system in the simulation. Fig. 7 (a) and (b) show phase current and total loss based on NEDC for single speed gear and two speed gear, respectively. Fig. 7 (c) shows loss comparison of copper loss, iron loss, inverter loss and total loss for single speed gear and two speed gear. Compared with Fig. 7 (a), Fig. 7 (b) shows lower current magnitude during NEDC mode. Consequently, copper loss and inverter loss are reduced in the two speed gear system, since these losses depend on current magnitude. Two speed gear with algorithm also shows lower iron loss than first gear in Fig. 7 (c). Total loss for single speed gear is 1220kJ. On the other hand, total loss for two-speed gear with algorithm shows 992kJ. Accordingly, result shows that two speed gear with algorithm reduce about 18.7 percent of total loss in comparison with single speed gear. In detail copper loss, iron loss and inverter loss reduce about 37.72, 13.38 and 17.56 percent of loss, respectively. Consumption power of single and two speed gear is 2.265kWh and 2.202kWh. Accordingly, two speed gear system with algorithm consumes lower power during one cycle NEDC mode. In case of 25kWh battery, single gear system drives 121.4km. On the other hand, two speed gear system drives 124.9km. Therefore, results shows that two speed gear can increase about 3.5km driving distance.

TABLE III
Comparison between Single Speed Gear and Two Speed Gear.

	Single Speed Gear	Two Speed Gear
Copper Loss	182.23kJ	113.58kJ
Iron Loss	592.82kJ	513.48kJ
Inverter Loss	444.72kJ	364.62kJ
Total Loss	1219.77kJ	991.68kJ
NEDC Distance	11km	11km
Consumption Power	2.265kWh	2.202kWh
Distance/1kWh	4.857km	4.996km
Distance/25kWh	121.4km	124.9km

VII. Conclusion

This paper has presented a two speed gear system for commercial EVs and novel algorithm related to loss minimization. In the case of two speed gear system, high torque and high speed operation at start-up are possible. Furthermore, The efficiency can be improved through two speed gear with gear shift algorithm. It shows the novel gear shift algorithm to minimize the loss and increase the mileage. In this paper, the overall operating point is shown on wheel torque-speed curve using New European Driving Cycle and vehicle dynamics. Thereafter, operating points are mapped in motor torque-speed plane according to reduction gear. To determine the efficiency, it is necessary to calculate the losses for first and second gear ratio. The losses consist of motor loss for copper loss and iron loss and inverter loss. To minimize the losses, optimal current set (i_d, i_q) should be found for each case, since copper loss and inverter loss depend on the current magnitude. Optimal current set is found by MTPA method, or at the intersection between torque and voltage limit curve. Ferrari's method is utilized to solve quartic equation resulted from the intersection. After calculating the losses with optimal current set, the wheel torque-speed plane is separated through comparison of first and second gear loss map. Thereafter, an optimal gear shifting algorithm is developed from the boundary between first and second gear. Furthermore, hysteresis band is applied to solve frequent gear shift related to practical implementation issue. Simulation was carried out based on NEDC, and the phase current of two speed gear system showed lower current than single speed gear during one cycle. Consequently, two speed gear with algorithm reduce about 18.7 percent of total loss and increase the driving distance about 3.5km in comparison with single speed gear.

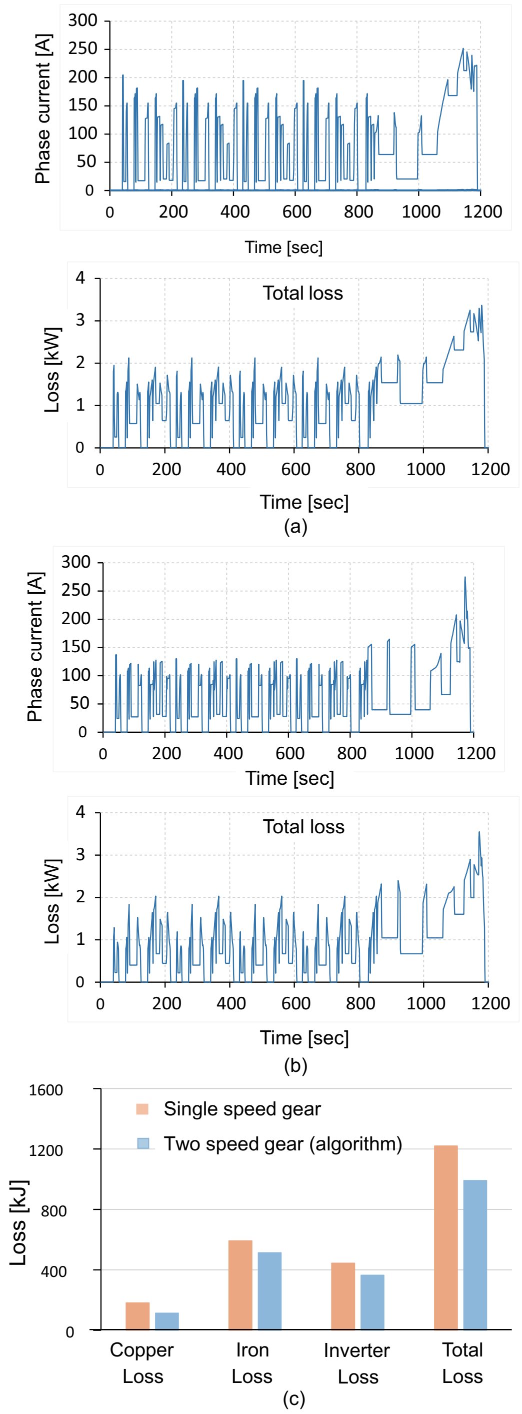

Fig. 7. Phase current and total loss based on NEDC mode: (a) phase current and total loss when a single speed gear (7.767:1) is used, (b) phase current and total loss when the two speed gear is used, and (c) total loss comparison.

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