MAT257 RSG 3

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1. (Kunal) Show that $\mathrm{Bd}(\mathrm{Bd}(\mathrm{Bd}(A))) = \mathrm{Bd}(\mathrm{Bd}(A))$ for all $A \subset \mathbb{R}^n$ (*Hint*: Think about what definition of Bd to use.)

2. (Kunal) Given $A \subset \mathbb{R}^n$ what are the necessary and sufficient conditions to get $\mathrm{Bd}(\mathrm{Bd}(A)) = \mathrm{Bd}(A)$?*

3. Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable everywhere with $Df(a) = 0$ for all $a \in \mathbb{R}^n$. Is f necessarily a constant function?

4. Suppose $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable. Show that $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$.

5. Show that all 'tiny' functions are continuous at 0. Give an example of a function that is continuous at 0 but not tiny.

6. Show that $\min : \mathbb{R}^2 \to \mathbb{R}$ is continuous. Use this to conclude that $\min : \mathbb{R}^n \to \mathbb{R}$ is continuous. Is it differentiable?

7. Consider the following proof for Q2b in HW2:
   "Let $A \subset \mathbb{R}^n$ be closed and $B \subset \mathbb{R}^n$ be compact such that $A \cap B = \phi$. We see that $B \subset A^c$ where A^c is open. Hence for every $x \in B$, there exists a $\delta_x > 0$ such that $B_{\delta_x}(x) \subset A^c$. This forms an open cover of B, hence by compactness there exists a finite subcover for B, say $B_{\delta_1}(x_1), \ldots, B_{\delta_m}(x_m)$. We can take our δ to be $\min\{\delta_1, \ldots, \delta_m\}$. Then we must have that for all $x \in B, B_\delta \subset A^c$."
   Why does it not work?

8. Let X be a set. Let $\mathrm{cl} : \mathcal{P}(X) \to \mathcal{P}(X)$ be a map that satisfies the following properties for any $Y, Z \in \mathcal{P}(X)$:

   (i) $\mathrm{cl}(\phi) = \phi$

   (ii) $Y \subset \mathrm{cl}(Y)$

   (iii) $\mathrm{cl}(\mathrm{cl}(Y)) = \mathrm{cl}(Y)$

   (iv) $\mathrm{cl}(Y \cup Z) = \mathrm{cl}(Y) \cup \mathrm{cl}(Z)$

   Let $\mathcal{C} = \{F \in \mathcal{P}(X) : \mathrm{cl}(F) = F\}$. Show that $X - \mathcal{C} = \{X - F : F \in \mathcal{C}\}$ is a topology on X. Show that cl agrees with our usual definition of closure.