PHYS4070

Week 12 Worksheet — Exact Diagonalization in Quantum Optical Systems (Part I)

A two-level atom driven by a classical oscillating electric field is described by the following Hamiltonian (in a frame rotating at the field frequency and after making the rotating-wave approximation):

$$\hat{\mathcal{H}} = -\Delta\hat{\sigma}_z + \hbar\Omega\hat{\sigma}_x\,. \tag{1}$$

Here, $\Omega \in \mathbb{R}$ is proportional to the amplitude of the driving field, $\Delta \in \mathbb{R}$ is the detuning between drive and atom, and

$$\hat{\sigma}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \hat{\sigma}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{2}$$

(a) Use the LAPACK routine DSYEV to obtain the eigendecomposition of Eq. (1). Plot the eigenvalues as functions of Ω/Δ. How do the eigenvectors vary with Ω/Δ?

A two-level atom evolves according to the Schrödinger equation,

$$i\,\frac{d\psi}{dt} = \hat{\mathcal{H}}\psi\,, \tag{3}$$

where $\psi \equiv \psi(t)$ is a two-component vector.

(b) Solve for the time dynamics of an atom that is initially in the state $\psi(0) = (1,\,0)^T$ by using Runge–Kutta integration. What happens as Ω increases from $\Omega = 0$ to $\Omega \gg \Delta$? Why is this?

Now, consider two adjacent two-level atoms. When the atoms are sufficiently close together (less than one wavelength apart), the oscillating dipole moment $\mathbf{d}(t)$ of one atom produces an electric field that drives the neighbouring atom. Ignoring decay, the Hamiltonian governing this interaction and an on-resonance drive can be written as follows:

$$\hat{\mathcal{H}} = \hbar\Omega\left[\hat{\sigma}_x^{(0)} + \hat{\sigma}_x^{(1)}\right] + \hbar g\hat{\sigma}_z^{(0)}\hat{\sigma}_z^{(1)}\,, \tag{4}$$

where $g \in \mathbb{R}$ is the dipole-dipole interaction strength.

(c) Solve for the dynamics of this system, starting with both atoms in the state $\psi(0) = (1,\,0)^T$. How do the dynamics change as g is increased from $g = 0$ to $g \gg \Omega$? Provide a general explanation for this.

Note, you may refer to Part 2 of Project 3 for additional helpful information.