Quantum Algo Track: Summary Report (Problems 0–4)

Bob's Quantum Journey: From Diffusion to Ballistics

November 2025

Abstract

This report summarizes the foundational analysis and implementation strategies for the initial phase of the Quantum Algo Track, detailing the progression from classical diffusive random walks to ballistic quantum dynamics on lines, graphs, and energy lattices.

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1 Classical and Deterministic Walks (Problems 0 and 1)

These problems established the non-quantum baseline for measuring speedup.

1.1 Problem 0: The Classical Random Walk

- **Model:** Unbiased classical random walk on the integer line $\mathbb{Z}$.
- **Result:** The walk spreads **diffusively**. The Root-Mean-Squared (RMS) displacement scales as the square root of time:

$$\text{RMS}(t) = \sqrt{\langle x^2 \rangle} = \sqrt{\mathbf{t}}$$

1.2 Problem 1: A Quantum Coin Flip (Pauli-X Gate)

- **Model:** Discrete walk using the Pauli-X gate (X) as the coin, starting in $|x=0\rangle \otimes |0\rangle$.
- **Result:** The walk is **deterministic and periodic** ($0 \leftrightarrow 1 \leftrightarrow 0 \ldots$).
- **Conclusion:** Superposition is the necessary ingredient for quantum spreading; a simple flip between basis states yields classical-like, non-spreading behavior.

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2 Quantum Spreading: Superposition and Interference (Problem 2)

This section analyzes the fundamental mechanism for the quantum speedup using the standard Discrete-Time Quantum Walk (DTQW).

- **Model:** DTQW using the **Hadamard Coin** (H) in a superposition $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.
- **Dynamics:** The total evolution is given by the unitary operator $U = S \cdot C$ (Shift $\cdot$ Coin). The position becomes a quantum superposition of possibilities.
- **Mechanism: Quantum Interference** drives the spread.
  - Destructive interference suppresses amplitude near the origin.
  - Constructive interference reinforces amplitude away from the origin.
- **Key Result:** The RMS displacement scales **ballistically** (linearly with time):

$$\text{RMS}(t) \propto \mathbf{t}$$

- **Comparison:** This is a **quadratic speedup** over the classical diffusive walk ($\propto t$ vs. $\propto \sqrt{t}$).

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3 Quantum Search on Graphs (Problem 3)

The DTQW model is generalized to search a known graph structure $G = (V, E)$.

- **Model:** DTQW on an arbitrary graph (e.g., C_4 cycle graph) with a local Coin operator. The total Hilbert space dimension is $D_{total} = \sum_{v \in V} \text{degree}(v)$.
- **Operators:** The unitary step $U = S \cdot C$ is constructed from large permutation matrices:
  - **C**: Block-diagonal matrix applying the local coin to the local edge subspace.
  - **S**: Permutation matrix that maps an outgoing edge state to the incoming edge state at the neighboring vertex.
- **Search Metric:** The success probability of finding a target vertex t is calculated after T steps by summing the probabilities of all basis states associated with t.
- **Key Finding:** The quantum walk exhibits high probability concentration at short times (e.g., $P_{success} = 0.5000$ on C_4 at $T = 2$) compared to the classical steady-state ($P_{classical} = 0.2500$), followed by periodic collapse.

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4 Quantum Oscillator Dynamics (Problem 4)

The walk is adapted to the energy levels of the Quantum Harmonic Oscillator (QHO), represented by number states $\{|n\rangle\}$.

- **Model:** Movement is conditional on the coin state ($|q\rangle$):
  - Coin $|0\rangle \to$ Creation Operator $a^\dagger$ (move up, $n \to n+1$).
  - Coin $|1\rangle \to$ Annihilation Operator a (move down, $n \to n-1$).
- **Joint Map (for $n > 0$):** The time step evolution acts as:

$$U|n\rangle|0\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{n+1}|n+1\rangle|0\rangle + \sqrt{n}|n-1\rangle|1\rangle\right)$$

- **RMS Metric:** The root-mean-square energy level is computed as $\text{RMS}(t) = \sqrt{\sum_n n^2 P(n,t)}$.
- **Scaling Comparison:** The walk maintains the same fundamental **ballistic scaling** as the 1D quantum walk: $\text{RMS}(\mathbf{t}) \propto \mathbf{t}$.