ESO-208A
Assignment 2
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Q1) Test data

Test data: $4x_1 + 2x_2 = 10$

$2x_1 + 4x_2 + x_3 = 11.5$

$x_2 + 5x_3 = 4.5$

Input
3
4 2 0 10
2 4 1 11.5
0 1 5 4.5

Output
Gauss Elimination (without pivoting)
Gauss Elimination without Pivoting :
1.500000
2.000000
0.500000

Gauss Elimination (with partial pivoting)
Gauss Elimination with Partial Pivoting :
1.500000
2.000000
0.500000

Dolittle method (without pivoting)
Dolittle Method :

1.500000
2.000000
0.500000

L:

1.000000	0.000000	0.000000
0.500000	1.000000	0.000000
0.000000	0.333333	1.000000

U:

4.000000	2.000000	0.000000
0.000000	3.000000	1.000000
0.000000	0.000000	4.666667

Crout Method (without pivoting)

Crout Method :

1.500000
2.000000
0.500000

L:

1.000000	0.000000	0.000000
0.500000	3.000000	0.000000
0.000000	1.000000	4.666667

U:

4.000000	2.000000	0.000000
0.000000	1.000000	0.333333
0.000000	0.000000	1.000000

Cholesky Decomposition (without pivoting)

Cholesky Method :

1.500000
2.000000
0.500000

L:

2.000000	0.000000	0.000000
1.000000	1.732051	0.000000
0.000000	0.577350	2.160247

U:

2.000000	1.000000	0.000000
0.000000	1.732051	0.577350
0.000000	0.000000	2.160247

Q2) Test data

Test Case:

$$A = \begin{bmatrix} 8 & -1 & -1 \\ -1 & 4 & -2 \\ -1 & -2 & 10 \end{bmatrix}$$

Maximum iterations: 50

Maximum relative approximate error: 0.001%

Find Eigenvalue closest to: 8

Input

```
3
8.0 -1.0    -1.0
-1.0         4.0    -2.0
-1.0        -2.0    10.0
100
0.001
8.0
```

Output

Power Method

```
Power Method:

Eigenvalue:
10.778672

Eigenvector:
-0.267485  -0.255625      1.000000

Iterations:
30.000000

Iteration 1.000000 : 7.000000
Iteration 2.000000 : 8.857143
Iteration 3.000000 : 9.870968
Iteration 4.000000 : 10.297386
```

Iteration 5.000000 : 10.477944
Iteration 6.000000 : 10.568391
Iteration 7.000000 : 10.623473
Iteration 8.000000 : 10.661701
Iteration 9.000000 : 10.689930
Iteration 10.000000 : 10.711272
Iteration 11.000000 : 10.727521
Iteration 12.000000 : 10.739905
Iteration 13.000000 : 10.749333
Iteration 14.000000 : 10.756501
Iteration 15.000000 : 10.761944
Iteration 16.000000 : 10.766072
Iteration 17.000000 : 10.769201
Iteration 18.000000 : 10.771570
Iteration 19.000000 : 10.773364
Iteration 20.000000 : 10.774721
Iteration 21.000000 : 10.775747
Iteration 22.000000 : 10.776523
Iteration 23.000000 : 10.777110
Iteration 24.000000 : 10.777554
Iteration 25.000000 : 10.777890
Iteration 26.000000 : 10.778143
Iteration 27.000000 : 10.778335
Iteration 28.000000 : 10.778480
Iteration 29.000000 : 10.778589
Iteration 30.000000 : 10.778672

*The method is slow to convergence

Inverse Power Method

Inverse Power Method:

Eigenvalue:
3.074933

Eigenvector:
0.269593   1.000000   0.327737

Iterations:
12.000000

Iteration 1.000000 : 2.500000
Iteration 2.000000 : 2.887701
Iteration 3.000000 : 3.010374
Iteration 4.000000 : 3.052061
Iteration 5.000000 : 3.066700
Iteration 6.000000 : 3.071940
Iteration 7.000000 : 3.073839
Iteration 8.000000 : 3.074534
Iteration 9.000000 : 3.074790
Iteration 10.000000 : 3.074885
Iteration 11.000000 : 3.074920
Iteration 12.000000 : 3.074933

*The method is faster because the eigenvector corresponds close to $[\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}]$, the starting vector.

Inverse Power Method with Shift
Inverse Power Method:

Eigenvalue:
8.146131

Eigenvector:
0.935570 -0.308527 0.171816

Iterations:
4.000000

Iteration 1.000000 : 8.182574
Iteration 2.000000 : 8.146411
Iteration 3.000000 : 8.146131
Iteration 4.000000 : 8.146131

This method is again efficient since it targets the eigenvector to be found and is also fast because the eigenvector happens to be close to the starting vector.

QR Decomposition
QR Decomposition Method:

Eigen Values:

10.778678
8.146319
3.074941

Iterations: 23

As can be seen, the method is extremely tedious and computationally heavy. But it gives the eigenvalues and eigenvectors of a matrix directly. So due to less assumptions in its methodology, it becomes complex but much more useful for high dimensional vector spaces where taking assumptions is not humanely possible.