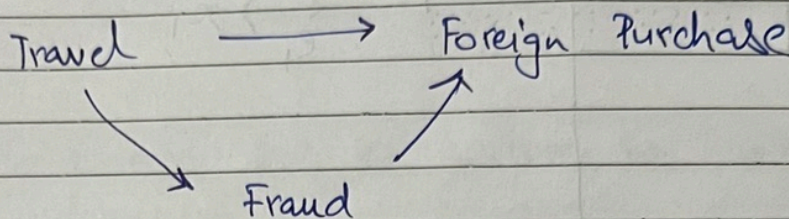


Bayesian Network

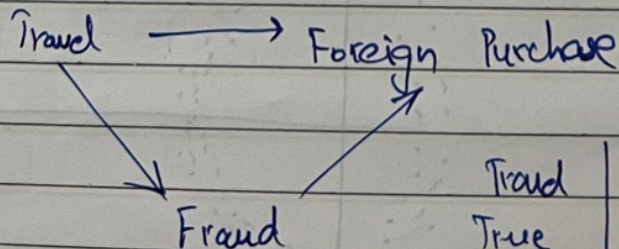


+ Increased probability of travel makes fraud more likely. Travel can cause fraud.

+ Increased probability of foreign purchase makes fraud more likely. Foreign purchase is evidence for fraud.

+ Travel and fraud can each cause foreign purchase. Travel explains foreign purchase & so is evidence against fraud.

True False
0.05 0.95



Travel	True	False
True	0.01	0.99
False	0.004	0.998

Travel	Fraud	True	False
True	True	0.90	0.10 0.10
False	True	0.10	0.90
True	False	0.90	0.10
False	False	0.01	0.99

i) Classify the hidden variables

travel = ?, foreign purchase = true, fraud = ?

$$P(\text{fraud} = \text{true} \mid \text{Foreign purchase} = \text{true})$$

$$= \alpha * [P(\text{fraud} = \text{true} \mid \text{travel} = \text{true}) * P(\text{foreign purchase} = \text{true})]$$

$$= P(\text{fraud} = \text{true} \mid \text{travel} = \text{false}) * P(\text{foreign purchase} = \text{true})$$

$$\text{travel} = \text{false}, \text{fraud} = \text{true}) * P(\text{travel} = \text{false})$$

$$= \alpha * [0.01 * 0.90 * 0.05 + 0.002 * 0.10 * 0.95]$$

$$= \alpha * [0.00045 + 0.00019]$$

$$= 0.00064\alpha$$

$$P(\text{fraud} = \text{False} \mid \text{Foreign purchase} = \text{true})$$

$$= \alpha * [P(\text{fraud} = \text{False} \mid \text{travel} = \text{true}) * P(\text{foreign purchase} = \text{true})$$

$$\text{travel} = \text{true}, \text{fraud} = \text{false}) * P(\text{travel} = \text{true})$$

+

$$P(\text{fraud} = \text{false} \mid \text{travel} = \text{false}) * P(\text{foreign purchase} = \text{true})$$

$$\text{travel} = \text{false}, \text{fraud} = \text{false}) * P(\text{travel} = \text{false})]$$

$$= \alpha * [0.99 * 0.90 * 0.05 + 0.998 * 0.01 * 0.95]$$

$$= \alpha * [0.04455 + 0.009481] = 0.054031\alpha$$

$$\alpha = \frac{1}{0.00064 + 0.05405} = 18.291$$

$$\begin{aligned} P(\text{fraud} = \text{true} | \text{foreign-purchase} = \text{true}) \\ &= 0.00064 \alpha \\ &= 0.00064 \times 18.291 \\ &= 0.01170 \end{aligned}$$

$$\therefore P(\text{fraud} = \text{true} | \text{foreign-purchase} = \text{true}) = 1.17\%$$

$$\begin{aligned} 2.) \quad P(\text{fraud} = \text{true} | \text{foreign-purchase} = \text{true}, \text{travel} = \text{true}) \\ &= \alpha \times 0.00045 \end{aligned}$$

$$P(\text{fraud} = \text{false} | \text{foreign-purchase} = \text{true}, \text{travel} = \text{true})$$

$$= \alpha \times 0.04455$$

$$\therefore \alpha = \frac{1}{0.00045 + 0.04455} = 22.222$$

$$P(\text{fraud} = \text{true} | \text{foreign-purchase} = \text{true}, \text{travel} = \text{true})$$

$$= \alpha \times 0.00045$$

$$= 22.222 \times 0.00045$$

$$= 0.01$$

$$\therefore P(\text{fraud} = \text{travel}, \text{foreign-purchase} = \text{true}, \text{travel} = \text{true}) = 1.0\%$$