Data Science 2022 Tutorial 6

Machine Learning

1. a) $P(H) = \lambda$

$P(T) = 1 - \lambda$

$P(\text{first head at } k+1) = (1-\lambda)^{k} \lambda$

b) Let M be no. of tosses required to get the first head & let $s = E[M]$

As tosses are independent & expression is additive

$s = \lambda \times 1 + (1-\lambda)(s+1)$

$s = \lambda + s + 1 - \lambda s - \lambda$

$\therefore s\lambda = 1$

$\therefore s = \frac{1}{\lambda}$

Q2. $X \rightarrow$ random variable

a. Variance of X : $Var(X) = E[(X - E[X])^2]$

To Prove: $Var(X) = E[X^2] - E[X]^2$

$Var(X) = E[(X - E[X])^2]$

$= E[X^2 - 2X E[X] + E[X]^2]$

$$= E[x^2] - 2E[xE[x]] + E[x]^2$$

$$= E[x^2] - 2E[x]^2 + E[x]^2$$

$$= E[x^2] - E[x]^2$$

b. $E[x] = 0$ & $E[x^2] = 1$

To find : ① Variance of x

② If $Y = a + bX$, $Var(Y) = ?$

① $Var(x) = E[x^2] - E[x]^2$

$$= 1 - 0^2 = 1$$

② $Y = a + bX$

$$E[Y^2] = E[(a+bX)^2]$$

$$= E[a^2 + 2abX + b^2X^2]$$

$$= a^2 + 2ab\,E[X] + b^2E[X^2]$$

$$= a^2 + 2ab(0) + b^2$$

$$\therefore E[Y^2] = a^2 + b^2$$

$$E[y] = E[a+bx] = a + bE[x]$$

$$= a + b(0)$$

$$E[y] = a$$

$$Var(x) = E[y^2] - E[r]^2 = a^2 + b^2 - a^2$$

$$\therefore Var(Y) = b^2$$

8. $A \Rightarrow$ Aku predicts that a given horse is a winning horse

$\sim A \Rightarrow$ " is not "

Similarly, let B be the event that the given horse wins & $\sim B$ be the event that the given horse does not win.

a. Given a horse, the probability that it wins

$$P(B) = P(B,A) + P(B \cap A)$$
$$= P(B|A) \cdot P(A) + P(B|\sim A) \cdot P(\sim A)$$

$$= 0.99 \times 10^{-5} + (1 - 0.99999) \times (1 - 10^{-5})$$

$$P(B) \approx 1.99 \times 10^{-5}$$

b. Probability that Aku predicts a black beauty is winning

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(A|B) . P(A)}{P(B)} = \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$$

$$\therefore P(A|B) = 0.497$$