Mathematics of Uncertainty

Project 5 - Decision making under uncertainty
Due: 12:00 pm IST on Friday, June 17, 2022

Name: ______________________ **Email:** ______________________

Submission: This assignment must be submitted online as a single PDF file on Gradescope. Please use this page with your name and email as the cover page for your submission.

Problem Description:

Assume that you are interested in purchasing a 16" Apple Macbook Pro with a specified configuration. A new Macbook Pro with that configuration is available on www.apple.com for $3,000. There are also several used Macbook Pros available for purchase at your local computer store. All of them have identical specifications, but different purchase prices. The computer store has set the prices based on the amount of time the previous owner has used it for. Assume that your purchase decision is dependent on only two attributes: cost (x_1), and the life (x_2) of the MacBook. Your multi-attribute utility function for this decision is:

$$U(x_1, x_2) = k_1 u_1(x_1) + k_2 u_2(x_2)$$

where, $u_1(x_1)$ and $u_2(x_2)$ are the single-attribute utility functions in cost and life, respectively.

The utility function for cost is:

$$u_1(x_1) = \begin{cases} 1, & \text{if } 0 \le x_1 < 1000 \\ -6 \times 10^{-5} \left(\frac{x_1}{100}\right)^3 + 1.3 \times 10^{-3} \left(\frac{x_1}{100}\right)^2 - 2.64 \times 10^{-2} \left(\frac{x_1}{100}\right) + 1.1907, & \text{if } 1000 \le x_1 < 2950 \\ 0, , & \text{if } 2950 \le x_1 \end{cases}$$

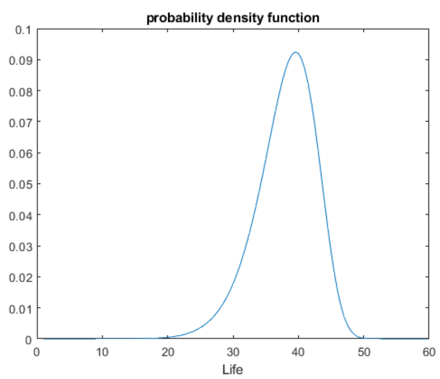
The utility function for the (remaining) life in months is:

$$u_2(x_2) = \begin{cases} 5 \times 10^{-6} x_2^3 + 7 \times 10^{-5} x_2^2 + 0.002 x_2 + 0.0011, & \text{if } 0 \le x_2 < 54 \\ 1, & \text{if } 54 \le x_2 \end{cases}$$

k_1 and k_2 are scaling factors, which will be provided to you during the lab.

Assume that the life of a Macbook from the date of the first use is given by the following (Weibull) probability density function where x is the life in months, $k = 10$ and $\lambda = 40$ (see plot).

$$f(x; \lambda, k) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$$



Tasks:

(a) Plot the single-attribute utility functions $u_1(x_1)$ and $u_2(x_2)$.

(b) Determine whether the decision maker is risk averse, risk prone, or risk neutral for the life attribute, x_2. Justify.

(c) Calculate the decision maker's local risk aversion for $x_2 = 30$ months.

(d) Using the multi-attribute utility function, $U(x_1, x_2)$, we would like to the best alternative among four MacBook Pro laptops (one brand new, and three used ones). The details of the four alternatives, including the price, and the number of months that it has already been used for, are shown in the table below.

Alternative	Price	Used for (in months)
A_0: New Macbook Pro	\$ 3,000	0
A_1: Used	\$ 1,500	24
A_2: Used	\$ 2,000	18
A_3: Used	\$ 2,500	12

For this task, write a Matlab code to calculate the expected utility $\mathbb{E}[U(x_1, x_2)]$ of each alternative. The alternative that maximizes the expected utility is the best option.

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Mini Project 5 - Decision Making Under Uncertainty

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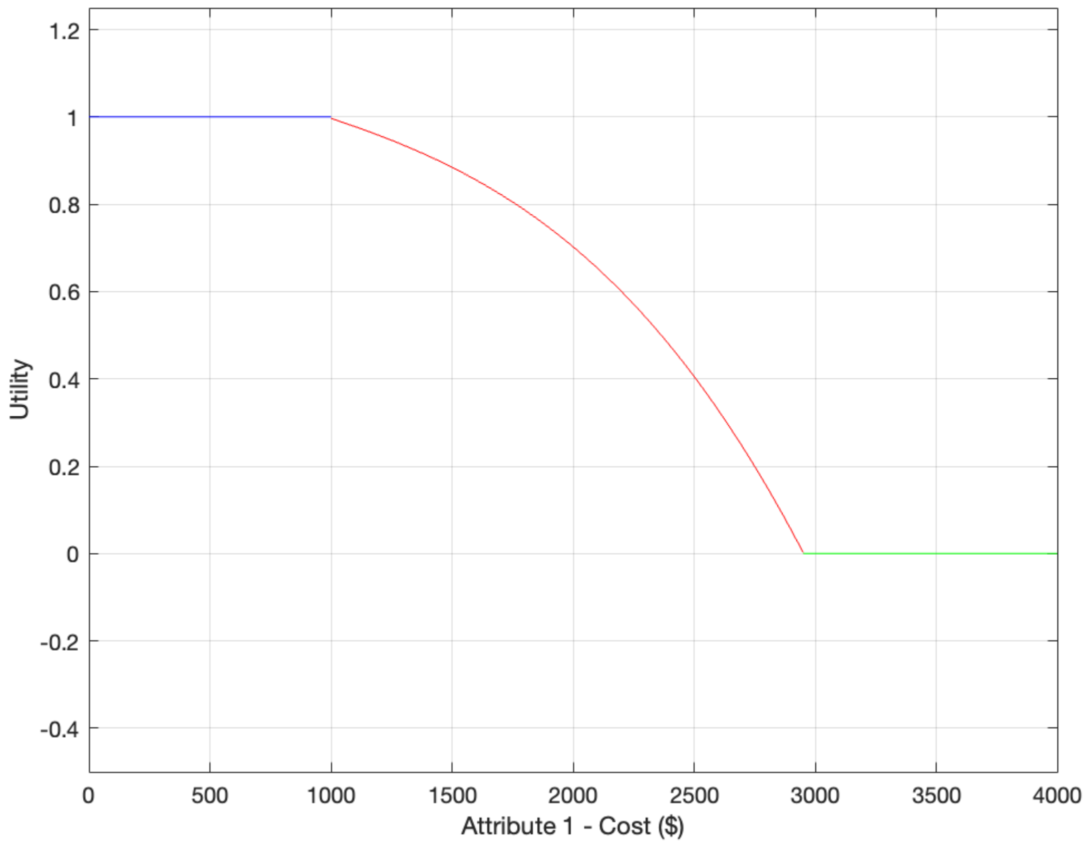
Section 2

Question 1 - Plot the single attribute utility functions $u_1(x_1)$ and $u_2(x_2)$.

Answer -

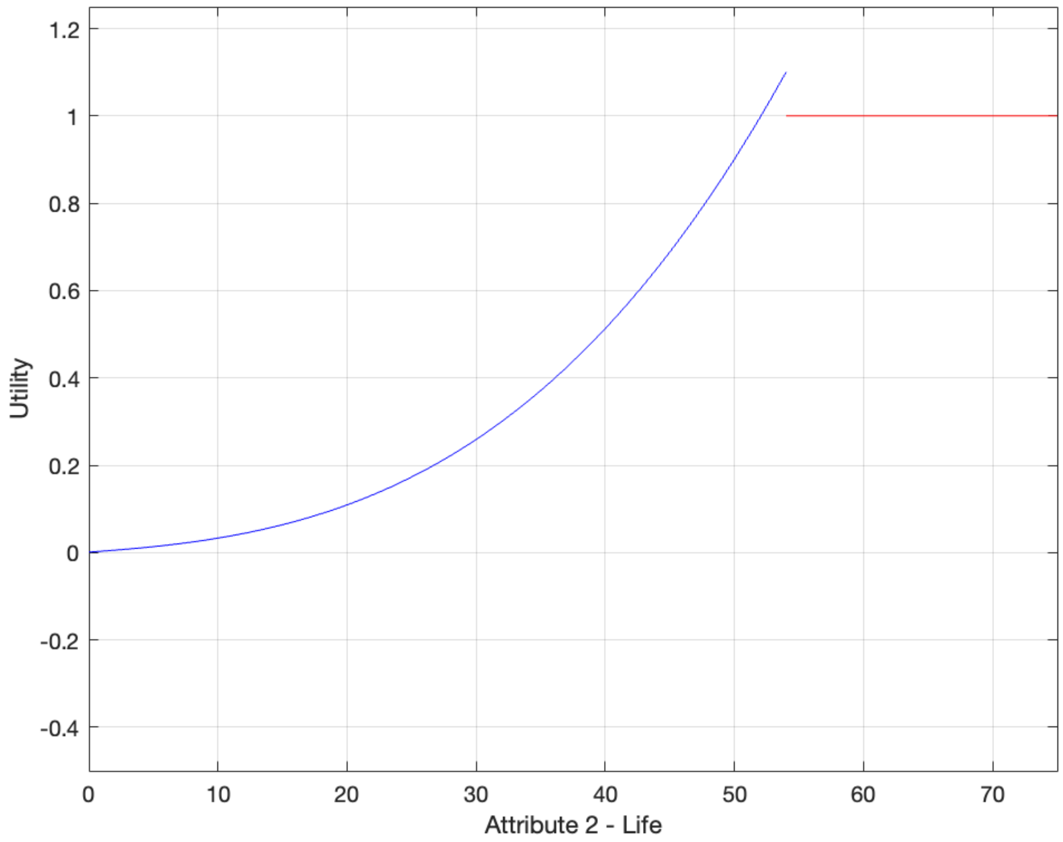
1. $u_1(x_1)$

```
fplot (@(x) 1, [0,1000],'b')
ylim ([-0.5 1.25])
grid on
xlabel('Attribute 1 - Cost ($)')
ylabel('Utility')
hold on
fplot (@(x) -6*10^-5*(x/100)^3+1.3*10^-3*(x/100)^2-2.64*10^-2*(x/100)+1.1907, [1000,2950],'r')
hold on
fplot (@(x) 0, [2950, 4000], 'g')
hold off
```



2. $u_2(x_2)$

```
fplot (@(x) 5*10^-6*x^3+7*10^-5*x^2+0.002*x+0.0011, [0,54],'b')
ylim ([-0.5 1.25])
grid on
xlabel('Attribute 2 - Life')
ylabel('Utility')
hold on
fplot (@(x) 1, [54,75],'r')
hold off
```



Question 2 - Determine whether the decision maker is risk averse, risk prone or risk neutral for the life attribute x_2. Justify.

Answer - Because the plot of $u_2(x_2)$ is convex, the decision maker is risk prone.

Question 3 - Calculate the decision maker's local risk aversion for $x_2 = 30$ months.

Answer -

$$-0.05279187817258883165436613638357 6$$

Code -

```
syms x
f = 5*10^-6*x^3+7*10^-5*x^2+0.002*x+0.0011;
risk_aversion = -(diff(f,2)/diff(f));
vpa(subs(risk_aversion,x,30))
```

Question 4 - Using the multi-attribute utility function, $U(x_1, x_2)$, we would like to the best alternative among four MacBook Pro laptops (one brand new, and three used ones). The details of the four alternatives, including the price, and the number of months that it has already been used for, are shown
in the table below.

For this task, write a Matlab code to calculate the expected utility $E[U(x1, x2)]$ of each alternative.
The alternative that maximizes the expected utility is the best option.

Answer - Alternative 1 (A_1) is the best option.

Code -

```
clear all;

k1 = 0.5;
k2 = 0.5;
% Uncomment for each alternative
% Alternative A0:
%x1 = 3000;
%x2 = wblrnd(40,10,10000,1) ;
% Alternative A1:
%x1=1500;
%x2 = wblrnd(40,10,10000,1) - 24 ;
% Alternative A2:
%x1=2000;
%x2 = wblrnd(40,10,10000,1) - 18 ;
% Alternative A3:
% x1=2500;
%x2 = wblrnd(40,10,10000,1) -12 ;

if (x1 >= 0 && x1 < 1000)
    u1 = 1;
elseif (x1 >= 1000 && x1 <= 2950)
    u1 = -6*10^(-5)*((x1/100)^3) + 1.3*10^(-3)*((x1/100)^2) - 2.64*10^(-2)*(x1/100) + 1.1907;
else
    u1 = 0;
end

for i = 1 : 10000
if (x2(i) >= 0 && x2(i) < 54)
    u2(i) = 5*10^(-6)*(x2(i)^3) + 7*10^(-5)*(x2(i)^2) + 0.002*x2(i) + 0.0011;
else
    u2(i) = 1;
end
end

for i = 1:10000
    U(i) = k1*u1 + k2*u2(i);
end
disp(mean(U));
```