PROJECT REPORT

On

" TO FIND SOLUTIONS OF LAST MILE ROUTING PROBLEM (CAPACITATED VEHICLE ROUTING PROBLEM WITH TIME WINDOWS) IN REASONABLE AMOUNT OF TIME"

Submitted in partial fulfilment of the requirements for the award of the degree of

MASTER OF OPERATIONAL RESEARCH

Undertaken at

Cogneau Systems Private Limited.

Regd. Office: C-30, SF, Mayfield Garden, Sector-50,

Gurugram- 122018, Haryana

UNIVERSITY OF DELHI

DEPARTMENT OF OPERATIONAL RESEARCH

SUBMITTED BY

Rishi Mehdiratta

Master of Operational Research

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Title of the Project	**:**	**To find solutions of Last Mile Routing Problem (Capacitated Vehicle Routing Problem with Time Windows) in reasonable amount of time**
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CERTIFICATE

This is to certify that the project entitled "To find solutions of Last Mile Routing Problem (Capacitated Vehicle Routing Problem with Time Windows) in reasonable amount of time" submitted in partial fulfilment of the Degree of Master of Operational Research, done by Mr. Rishi Mehdiratta is an authentic work carried out by him at Cogneau Systems Private Limited., Gurugram, Haryana under my guidance. The matter embodied in this project work has not been submitted earlier for award of any degree or diploma to the best of my knowledge and belief.

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Lastly I would like to thanks all my batchmates and seniors for their cooperation and help without which I would not have been able to complete this project successfully.

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Definition of Operational Research

"The high tech field of OR offers numerous excellent opportunities to boost performance immediately. Yet OR Practitioner time remains to skim the cream before everyone wakes up to the projects. When most do wake up, USA companies that are not taking full advantage of OR will leave serious money on the table and be outflanked competitors."

--**Randy Robinson**

"Operations Research is the systematic application of quantitative methods, techniques and tools to the analysis of problems involving the operations of systems."

--Daellanbach and George 1978

"OR is a scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilizations of limited resources.

--H A Taha

"OR is the application of scientific methods, techniques and tools to problems involving the operations of a system so as to provide those in control of the system with *optimum solutions* to the problem."

--C W Churchman, R L Ackoff & E L Arnoff

Operations Research may be described as a scientific approach to decision-making that involves the operations of organizational system."

--F S Hiller and G J Lieberman, 1980

OPERATIONAL RESEARCH – AN OVERVIEW

Operational Research (OR) is an interdisciplinary branch of applied mathematics and formal science that uses methods like mathematical modeling, statistics, queuing theory, game theory, decision analysis, and simulation to arrive at optimal or near optimal solutions to complex problems. Because of the computational nature of these fields, OR also has ties to computer science, and operations researchers regularly use custom-written or off-the-shelf software.

The term operational research was first coined in 1940 by McClosky and Trefthen in a small town Bowdsey, of United Kingdom. The new science came into existence in military context during World War II. The procedures of operations research gave effective assistance during World War II in missions such as deploying radar, searching for enemy submarines, and getting supplies where they were most needed. Following the war, new analytical methods were developed and numerous peacetime applications emerged, leading to the use of operations research in many industries and occupations.

The prevalence of operations research in the Nation's economy reflects the growing complexity of managing large organizations that require the effective use of money, materials, equipment, and people. Operations researches analysts help determine better ways to coordinate these elements by applying analytical methods from mathematics, science, and engineering. They solve problems in different ways and propose alternative solutions to management, which then chooses the course of action that best meets the organization's goals. In general, operations research analysts may be concerned with diverse issues such as top-level strategy, planning, forecasting, resource allocation, performance measurement, scheduling, design of production facilities and systems, supply chain management, pricing, transportation and distribution, and analysis of data in large databases.

In a nutshell, operations research (O.R.) is the discipline of applying advanced analytical methods to help make better decisions.

By using techniques such as mathematical modelling to analyze complex situations, operations research gives executives the power to make more effective decisions and build more productive systems based on:

- More complete data
- Consideration of all available options

HISTORY OF OPERATIONAL RESEARCH

O.R. has its beginning in World War II. The term Operational Research was coined by McClosky and Trefthen in 1940 in U.K. British scientists set up the first field installations of radar's during the battle and observed air operations. Their analysis of these led to suggestions that greatly improved and increased the effectiveness of British fighters, and contributed to successful British defense. Operations Research was then extended to antisubmarine warfare and to all phases of military, naval and air operations both in Britain and the United States, and was incorporated in the postwar military establishments of both the countries. In India, Operational Research came into existence in 1949 with the opening of an O.R unit at the Regional Research Laboratory at Hyderabad. At the same time, another group was set up I Defense Science Laboratory which devoted itself to problem of stores, purchase and planning. In 1953, an O.R unit was established in the Indian Statistical Institute, Calcutta, for the application of O.R methods in national planning and survey. The O.R society of India was formed in 1955. It became a member of the International Federation of O.R Societies in 1959.

MODELS IN OPERATIONAL RESEARCH

Classification of Models

A model in O.R. is a simplified representation of an operation or a process in which only the basic aspects or the most important features of a typical problem under investigation are considered. Constructing a model aids to input the complexities and possible uncertainties attending a decision making problem, into a logical framework amenable to comprehensive analysis. Such a model clarifies the decision alternatives, their anticipated effects, indicate the relevant data for analyzing the alternatives, and leads to informative conclusions. The objective a model is to provide a means for analyzing the behavior of the system for the purpose of improving its performance.

Although the classification of models is a subjective problem, they may be distinguished as follows:

MODELS BY DEGREE OF ABSTRACTION

These models are based on the past data/information of the problems under consideration and can be categorized into:

(a) **Language Models**

(b) **Case Studies**.

MODELS BY FUNCTION

These models consist of:

(a) **Descriptive Models**- These models describe explain and predict facts and relationships among the various activities of the problem. These are used to describe mathematically some particular aspects of the system being modeled. These models do not have an objective function as apart of the model to evaluate decision alternatives. Thus, in a descriptive model it is possible to get information as to how one or more factors charge as a result of changes in other factors.

(b) **Predictive Models**- These model indicates that “if this occurs then that will follow” They relate independent and dependent variables and permit trying out, ‘what if’ questions. In other words, these models are used to predict the outcomes due to a given set of the alternatives for the problem. These models do not have an objective function as a part of the model to evaluate decision alternatives.

(c) **Normative Models**- These models provide the "best" or "optimal" solution to the problems subject to limitation on the use of resources. These models provide recommended courses of action. For example, in mathematical programming, models are formulated for optimizing the given objective function, subject to certain restrictions and non-negativity of the decision variables.

MODELS BY STRUCTURE

These models are represented by:

(a) **Iconic Models**- Iconic model retains some of the physical properties and characteristics of the system they represent. An iconic model is either in an idealized form of or a scaled scale version of the system. In the other words, such models represent the system as it is by scaling it up or down.

(b) **Analogue Models**- Analogue models are more abstract then iconic ones for there is no 'look- alike' correspondence between these models and real life items. They are built by utilizing one set of properties to represent another set of properties.

(c) **Symbolic Models**-These models are more abstract in nature. They employ asset of mathematical symbols to represent the components of the real system. These models are more general and precise.

MODELS BASED ON DEGREE OF CERTAINITY

These models are represented by:

(a) **Deterministic Models**- If all the parameters, constants and functional relationships are assumed to be known with certainty when decision is made, then the model is said to be deterministic. Thus in such a case, the outcome associated with particular course of action is known. That is for a specific set of input values there is uniquely determined output which represents the solution of the model under conditions of certainty. Linear programming models are example of deterministic models.

(b) **Probabilistic (Stochastic) Models**- Models in which at least one parameters or decision variables is a random variable is called probabilistic models. These models reflect, to some extent the complexity of the real world and the uncertainty surrounding it.

MODELS BY EXTENT OF GENERALITY

These models are represented by:

- **Specific Models**- When a model presents a system at some specific time, it is known as a specific model. In these models if time factor is not considered, then they are termed as static model and dynamic model otherwise.

- **General Models**- Simulation and heuristic models fall under general models. These models are mainly used to explore alternative strategies which have been overlooked previously. These models do not yield any optimum solution to the problem, but give a solution to the problem depending on the assumptions based on the past experience.

METHODOLOGY OF OPERATIONAL RESEARCH

The systematic methodology developed for operations research study deals with problems involving conflicting multiple objectives, policies and alternatives. O.R. in the final analysis is a scientific methodology which is applied to the study of operations of large complex organization and activities with a view to assessing the overall implications of various alternative courses of action, thus providing an improved basis for managerial decision.

Then O.R. approach to problem solving consists of the following six steps:

- **Formulation of the problem:** It involves analysis of the physical system, setting up of objectives, determination of restriction constraints against which decision should be adopted, alternative courses of action and measurement of effectiveness.

- **Construction of a mathematical model:** after formulation of the problem, the next step is to express all the relevant variables of the problem into a mathematical model. a generalized mathematical model might take the form: $\mathbf{E = f(x_i, y_j)}$

- **Deriving the solution from the model:** once the mathematical model is formulated, the next step is to determine the values of the decision variables that optimize the given objective function. This deals with the mathematical calculations for obtaining the solution to the model.

- **Validity of the model:** the model should be validated to measure its accuracy. That is in the order for a model to be useful, the degree to which it actually represents the system or problem being modeled must be established. A model is valid or accurate if (a) it contains all the objectives, constraints, and the decision variables relevant to the problem, (b) the objectives, constraints, and the decision variables are all relevant to, or the actually part of the problem, and (c) the functional relationships are valid.

- **Establishing control over the solution*:*** After testing the model and its solution, the next step of the study is to establish control over the solution, by proper feedback of the information on the variables which deviated significantly. As soon as one or more of the control variables

change significantly, the solution goes out of the control in such as situation the model may accordingly be modified.

- **Implementation of the final results*:*** Finally, the tested results of the model are implemented to work. This would basically involve a careful explanation of the solution to be adopted and its relationship with the operating realities. This stage of O.R. investigation is executed primarily through the cooperation of both the O.R. experts and those who are responsible for managing and operating the system.

SCIENTIFIC METHODS IN OPERATIONAL RESEARCH

The scientific method in operations research consists of the following three phases:

***Judgement Phase*:** This phase includes:

- Identification of the real life problem.
- Selection of an appropriate goal and the values of the various variables related to the goals.
- Appropriate scale of measurement
- Formulation of an appropriate model of the problem, abstracting the essential formulation so that the solution at the decision makers can be sought.

***Research phase*:** this phase is the largest and longest among the other two. However other two also equally important as they provide basis for a scientific method. This phase utilizes

- (i) Observations and data collection for better understanding of what the problem is.
- (ii) Formulation of hypothesis and models.
- (iii) Observation and experiment to test the hypothesis on the basis of additional data.
- (iv) Analysis of the available information and verification of the hypothesis using pre-established measures of effectiveness.
- (v) Predictions of the various results from the hypothesis, and
- (vi) Generalization of the results and consideration of alternative methods.

***Action phase*:** This phase consists of making recommendations for decision process by those who first posed the problem for consideration, or by anyone in a position to make a decision influencing the operation in which the problem occurred.

APPLICATIONS OF OPERATIONAL RESEARCH

O.R is mainly concerned with the techniques of applying scientific knowledge, besides the development of science. In recent years, O.R. has successfully entered many different areas of research in Defense, Govt. service organizations and Industry. Some applications of O.R. in the functional areas of management are:

FINANCE, BUDGETING AND INVESTMENT:

- Cash flow analysis, long-range capital requirement, investment portfolios, dividend policies, etc.
- Credit policies, credit risks etc.

PHYSICAL DISTRIBUTION:

- Location, size of warehouses, distribution centers, retail outlet etc.
- Distribution policy.

PURCHASE, PROCUREMENT AND EXPLORATION:

- Determining the quantity and timing of purchase of raw materials, machinery etc.
- Bidding policies.
- Equipment replacement policies.

MANUFACTURING AND FACILITY PLANNING:

- Production scheduling.
- Project scheduling and allocation of resources
- Number and location of factories, warehouse and their size.
- Determining the optimum production mix.
- Maintaining crew sizes.
- Maintenance policies and preventive maintenance.

MARKETING:

- Product selection, timing competitive actions.
- Advertising strategy and choice of different media of advertising.
- Number of salesman, frequency of calling of accounts, etc.
- Effectiveness of market research.

- Size of stock to meet the future demand.

PERSONAL:

- Recruitment policies and assignment of jobs.
- Selection of suitable personal on minimum salary.
- Mixer of age and skills.
- Establishing equitable bonus systems.

RESEARCH AND DEVELOPMENT:

- Determination of areas of concentration of research and development.
- Reliability and evaluation of alternative designs.
- Control of development projects.
- Co-ordination of multiple research projects.

USES OF OPERATIONAL RESEARCH

Formulation of industrial problems may be generalized into different groups of classical problems, the package program for which is available for mechanization and manual solutions.

Various problem of optimization can be brought to the model of linear program for which solution is available. While formulating the problem, the class of the problem is to be decided and the parameters are to be defined accordingly.

Inventory control, production planning, product mix, transportation problem, etc. are very common to the industries. The cost reduction with the help of these tools is very much powerful in comparison to any other conventional method. We can enumerate the advantages of these techniques as:

- Optimum use of production factors: linear programming techniques indicate how a manager can most effectively employ his production factors by more efficiently selecting and distributing these elements.

- Improved quality of decision: the computation table gives a clear picture of happenings within the basic restriction and the possibilities of compound behavior of the elements involved in the problems. The effect on the profitability due to changes in the production pattern will be clearly indicated in the table, e.g. simplex tables.

- Modification of mathematical solution: O.R. presents a possible practical solution when one exists, but it is always a responsibility of the manager to accept or modify the solution before its use. The effects of these modifications may be evaluated from the computational steps and tables.

- Alternative solution: O.R. techniques will suggest all the alternative solution available for the same profit so that the management may decide on the basis of its strategies.

LIMITATIONS OF OPERATIONAL RESEARCH

O.R. has certain limitations. These limitations are as follows:

- **Magnitude of computation:** O.R. tries to find out the optimal solution taking all the factors into account. In the modern society, these factors are numerous and expressing them in quantity and establishing relationship among these, require huge calculations. All these calculations cannot be handled manually and require electronic computers which bear a very heavy cost. Thus the use of O.R. is limited to only very large organizations.

- **Absence of qualification:** O.R. provides solution only when all the elements related to a problem can be quantified. The tangible factors such as product, price, etc., can be expressed in terms of quantity, but intangible factors such as human relations etc. cannot be quantified. Thus these intangible elements of the problem are excluded from the study, though these might be equal or more important than quantifiable intangible factors as far as possible.

- **Distance between managers and operations research:** O.R. being specialist job requires a mathematician or a statistician, who might not be aware of business problems. Similarly, a manager may fail to understand the complex working of O.R. Thus, there is a gap between one who provides the solution and one who uses a solution. Thus, the manager who becomes suspicious about the optimal solution. This problem is mainly of training. Both the persons should have a working knowledge of each other's job to have better understanding of insights of the problem and its optimal solution.

CHARACTERISTICS OF OPERATIONAL RESEARCH

- Such a team, when confronted with a problem, determines the solution of the problem by following inter-disciplinary approach. Every expert of the team tries to abstract the essence of the problem and determines if the same type of problem has been previously undertaken or not. If a similar problem has been dealt previously then it becomes quite easy to apply the same technique to determine the solution of current problem as well. In this way, each member of the team, by utilizing his experience and expertise may be in a position to suggest an approach that otherwise may not be thought of. Thus, Operational Research makes use of experience Interdisciplinary Team Approach. It is an important characteristic of operational research. According to this characteristic, no single individual can be an expert on all aspects of a problem under consideration. Thus, Operational Research utilizes the inter-disciplinary approach i.e., an Operational Research team comprises of experts from different disciplines and expertise of people from different disciplines for developing new methods and procedures.

- Operational Research is a continuing process. It cannot stop on the application of the model to one problem, for this may create new problems in other sectors and in the implementation of the decision taken. Operational Research must also specify the organizational changes required to implement decisions and control the result thereof. Without this, the work of Operational Research practitioner is incomplete.

- Objective. Operational Research attempts to find the best or optimal solution to the problem under consideration. To do this, it is necessary to define a measure of effectiveness that takes into account the goals (objective) of the organization. In other words, “Operational Research is the scientific study of large systems with a view to identify problem areas and provide the managers with a quantitative basis for decisions which will enhance their effectiveness in achieving the specified objectives.”

- Operational Research gives only bad answers to the problem where only worse could be given i.e.; it cannot give perfect answers to the problems. Thus, Operational Research improves only the quality of the solution.

- Methodological Approach. Operational Research utilizes the scientific method. Specifically, the process begins with the careful observation and formulation of the problem. The next step is to construct a scientific (typically mathematical or simulation) model that attempts to abstract the essence of the real problem. From this model, conclusions or solutions are obtained which are also valid for the real problem. In an iterative fashion, the model is then verified through appropriate experimentation.

TECHNIQUES OF OPERATIONAL RESEARCH

The techniques discussed below can be freely used by a progressive manager in the decision making process.

- **Probability:** It is rarely possible to predict the future of the business world with complete certainty. There is always an element of uncertainty as far as the future courses of events are concerned. The probability concepts try to analyze the uncertainties and bring out necessary data with reasonable accuracy for the purpose of decision making. Probabilities are of two types: Objective Probability and Subjective Probability. The objective probability is a probability for which there is a definite historical evidence and common experience. On the other hand, the subjective probability is a probability where historical evidence is not available and the businessman has to rely on own estimation of a situation and the likelihood of various outcomes.

- **Decision theory:** The basic elements in a decision theory are:
    1. Alternative course of action(strategies)
    2. Various states of nature.
    3. Knowledge about the likelihood of occurrence of each state of nature.
    4. Net value (pay-off) to decision-maker for each outcome.
    5. Decision maker's objectives.

The basic premise of decision theory is that the behavior of the future is probabilistic and not deterministic. Various probabilities are assigned to the state of nature on the basis of available information or subjective judgment and the likely outcomes of the alternative courses of action are evaluated accordingly before a particular alternative is selected.

This technique of decision making is based on expected monetary value and utility. By analyzing the monetary value of utility from a given set of circumstances, a model is constructed and accordingly decisions are taken. It is a well-known fact that the environment within which decisions are taken can be logically divided into three parts: certainty, risk and uncertainty. Certainty exists when one can specify exactly what will happen when during the period for which the decision is being made. Risk refers to a situation where one can specify a probability distribution for the possible outcomes. Uncertainty refers to the condition when one cannot specify the relative likelihood of the outcomes.

Although some business decision can be made under condition nearing certainty, elements of risk and uncertainty underline most of the decisions which mangers make.

- **Linear Programming.** It is a method for selecting an optimum combination of Factors from a series of inter-related alternatives, each subjective to limitation. It involves the development of linear equations to obtain the best solution for the allocation problem. An allocation problem "...arise whenever there are a number of activities to perform but limitations on either the amount of resources or the way they can be spent prevent us from performing each activity in the most effective way conceivable. In such situations we wish to allot the available resources to the activities in a way that will optimize the effectiveness".

  Linear programming consists of:

  1. *The Simplex Method.* This aims at maximizing or minimizing a given function, subject to constraints in respect of each variable.

  2. *The Transportation Problem.* This deals with problems of matching the origins (stores, warehouse, and factories) with the outlets (process center, market etc.) at a minimum cost of distribution and transportation.

  3. *The Assignment Problem.* It can handle the problems of assigning a given number of agents each one to the same number of tasks so as to result in maximum efficiency or minimum cost.

- **Dynamic Programming:** This technique deals with the problems that arise in connection with multi period analysis and decisions. In contrast to Linear Programming there does not exists a standard mathematical formulation of Dynamic Programming, rather it is a general type of approach to problem solving and particular equation must be developed to fit each individual situation. However, the basic approach used in Dynamic programming is to break down a problem into series of problems in such a way that answer to the first sub problem can be used in deriving the solution to the next sub-problem and so forth finally giving solution to the whole problem.

- **Sequencing:** This method solves problem where effectiveness measure (in terms of cost, time, mileage, etc.), depends upon the sequence of performing given jobs. This helps to determine a sequence in which given jobs should be performed if the objective is to minimize the total efforts.

- **Game Theory**: Developed by Jon von Neumann and Morgenson, this is a mathematical theory applicable to competitive business problems. This technique deals with situations where two or more (finite) individuals are making decisions involving conflicting interests. However, the final decision depends upon the decisions of the parties concerned.

- **Inventory Control and Management:** Inventory problems(models) are mainly concerned with inventory decisions, the basic inventory decisions are:

    a. How much to order at one time, and
    b. When to order this quantity.

    The first decision namely – how much to order at one time, is to be balanced between two pressures – the first pressure is to order huge lots so as to minimize ordering costs and the other pressure is order small lots so as to minimize carrying costs. The optimum course of action is a compromise between the two extremes. Arriving at a model for deriving the economic order quantity (E.O.Q) can do this.
    The second decision namely – when to order this quantity, is decided by ascertaining the re-ordering level. The re-ordering level is the point lying between the maximum and the minimum levels at which time it is essential to intimate the purchase department for fresh supplies of the material. This point will usually be slightly higher than the minimum stock, to cover such emergencies as abnormal usage of material or unexpected delay in delivery of fresh supplies. Re-ordering level depends upon lead-time, rate of consumption and economic ordering quantity.

- **Queuing Theory:** Waiting lines at any service center are common phenomena and Queuing theory is devoted to mathematical study of waiting lines. Various alternative models have been used to describe such situations, but they basically share the following common features: (i)

Units requiring services are generated from an input source from different kinds of queues for service, (ii) a service discipline by which the queue members are selected for receiving service, (iii) a service mechanism which defines the type of service, after the completion of which unit leaves the system or rejoins it for further processing. Many alternative assumptions can be made about these common elements, and these give rise to the different queuing problems.

- **Network Analysis (PERT/CPM)**: PERT or Program Evaluation and Review technique and CPM or Critical Path Method are powerful management tools for planning and control of complex jobs involving a large number of activities. A project consists of well-defined collection of jobs or activities, some of which can be started independently of others and all the jobs, have to be carried out in technological sequence. The objective of PERT/CPM technique is to establish time duration for each activity and to shorten the total duration acquired for the completion of the project incurring the optimum cost. CPM introduces the concept of critical path, i.e., the longest time required to complete a project and emphasizes the reduction of the duration of the activities by the application of more resources by obtaining a trade-off between cost and time completion.

- **Simulation:** It is highly versatile technique of operational research. It has a wide-ranging application in business situations. Simulation is particularly appropriate where it is difficult to build a model for the real life situation mathematically or if at all it is modeled, it is difficult to solve the model analytically. It may be noted that simulation is a manipulation of a model constructed from the formal statements of mathematical representation in respect of logical relations between the elements in a structure or a system expressed in measurable terms. Thus, simulation is a process of designing an experiment, which will duplicate or present as nearly as possible the real situation and then watching what does happen. In every walk of life, the test of adequacy of our decisions is the test of our reality. However, in practical life, modern business cannot afford the luxury of testing the consequences of the major decisions in the real-life world. Instead of finding out the characteristics of an aircraft or a skyscraper, by actually building or constructing it, we can simulate its performance on the basis of its characteristics. In this way we can experiment without incurring the cost of failure. The simulation techniques allow the modern managers to examine the probable

consequences of his decisions without the risk of real life experimentation.

- **Reliability:** Reliability theory is concerned with quantifying the frequency of failures and developing an indicator of quality and dependability of a product. It is closely associated with probability theory and therefore facilitates statistical analysis and measurement.

The assessment of reliability of equipment is most useful to the designer in improving the quality of critical parts as well as in deciding how much to provide by way of standby.

- **Some Advance O.R. Techniques:**

1. *Non-linear programming* is that form of programming in which some or all of the variables are curvilinear. In other words, this means that either the objective functions or the constraints or both are not in liner form. In most of the practical situations, we encounter with non-linear programming problems.

2. *Integer programming*. Integer programming applies when the values of decision variables are restricted to integers. Applications include Financial Management and Plant Location.

3. *Goal programming*. Goal programming deals with the problems having multiple objectives. It is a technique quite similar to linear programming. Applications include production scheduling, transportation problems, portfolio analysis and crop selection in agriculture.

4. *Heuristic programming* also known as discovery method refers to step by step search towards an optimal when a problem cannot be expressed in mathematical programming form. The search procedure examines successively a series of combinations that lead to

stepwise improvement in the solution and the search stops when near optimal has been found.

5. *Algorithmic programming* is just the opposite of Heuristic programming. It may also be termed as mathematical programming. This programming refers to a thorough and exhaustive mathematical approach to investigate all aspects of given variables in order to obtain optimal solution.

6. *Quadratic programming* refers to a modification of linear programming, in which the objective function and constraint equations appear in quadratic form, i.e., they contain squared terms.

7. *Probabilistic programming* also known as stochastic programming refers to linear programming that includes an evaluation of relative risks and uncertainties in various alternatives of choices.

COMPANY PROFILE

Cogneau Systems Private Limited

Cogneau Systems Private Limited is a Private incorporated on 03 December 2015. It is classified as Non-govt company and is registered at Registrar of Companies, Delhi. Its authorized share capital is Rs. 3,000,000 and its paid up capital is Rs. 191,030. It is involved in Other computer related activities [for example maintenance of websites of other firms/ creation of multimedia presentations for other firms etc.

Cogneau Systems Private Limited's Annual General Meeting (AGM) was last held on 31 December 2020 and as per records from Ministry of Corporate Affairs (MCA), its balance sheet was last filed on 31 March 2020.

Directors of Cogneau Systems Private Limited are Rahul Dogar, Sunil Mehta, Ramesh Kumar Mehta and Hitesh Batra.

Cogneau Systems Private Limited's Corporate Identification Number is (CIN) U72900HR2015PTC057452 and its registration number is 57452.Its Email address is sm@cogneau.com and its registered address is C-30, SF MAYFIELD GARDEN, SECTOR-50 GURGAON Gurgaon HR 122018 IN , - , .

Current status of Cogneau Systems Private Limited is – Active.

WHAT IS OPTIMIZATION?

Mathematical optimization or mathematical programming is the selection of a best element, with regard to some criterion, from some set of available alternatives. Optimization problems of sorts arise in all quantitative disciplines from computer science and engineering to operations research and economics, and the development of solution methods has been of interest in mathematics for centuries.

In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. The generalization of optimization theory and techniques to other formulations constitutes a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective function given a defined domain (or input), including a variety of different types of objective functions and different types of domains.

Fermat and Lagrange found calculus-based formulae for identifying optima, while Newton and Gauss proposed iterative methods for moving towards an optimum.

The term "linear programming" for certain optimization cases was due to George B. Dantzig, although much of the theory had been introduced by Leonid Kantorovich in 1939. (Programming in this context does not refer to computer programming, but comes from the use of program by the United States military to refer to proposed training and logistics schedules, which were the problems Dantzig studied at that time.) Dantzig published the Simplex algorithm in 1947, and John von Neumann developed the theory of duality in the same year.

DIFFERENT TYPES OF OPTIMIZATION PROBLEMS

- **Convex programming** studies the case when the objective function is convex (minimization) or concave (maximization) and the constraint set is convex. This can be viewed as a particular case of nonlinear programming or as generalization of linear or convex quadratic programming.
- **Linear programming (LP)**, a type of convex programming, studies the case in which the objective function f is linear and the constraints are specified using only linear equalities and inequalities. Such a constraint set is called a polyhedron or a polytope if it is bounded.
- **Second-order cone programming (SOCP)** is a convex program, and includes certain types of quadratic programs.
- **Semidefinite programming (SDP)** is a subfield of convex optimization where the underlying variables are semidefinite matrices. It is a generalization of linear and convex quadratic programming.
- **Conic programming** is a general form of convex programming. LP, SOCP and SDP can all be viewed as conic programs with the appropriate type of cone.
- **Geometric programming** is a technique whereby objective and inequality constraints expressed as polynomials and equality constraints as monomials can be transformed into a convex program.
- **Integer programming** studies linear programs in which some or all variables are constrained to take on integer values. This is not convex, and in general much more difficult than regular linear programming.
- **Quadratic programming** allows the objective function to have quadratic terms, while the feasible set must be specified with linear equalities and inequalities. For specific forms of the quadratic term, this is a type of convex programming.
- **Fractional programming** studies optimization of ratios of two nonlinear functions. The special class of concave fractional programs can be transformed to a convex optimization problem.
- **Nonlinear programming** studies the general case in which the objective function or the constraints or both contain nonlinear parts. This may or may not be a convex program. In general, whether the program is convex affects the difficulty of solving it.

- **Stochastic programming** studies the case in which some of the constraints or parameters depend on random variables.
- **Robust optimization** is, like stochastic programming, an attempt to capture uncertainty in the data underlying the optimization problem. Robust optimization aims to find solutions that are valid under all possible realizations of the uncertainties defined by an uncertainty set.
- **Combinatorial optimization** is concerned with problems where the set of feasible solutions is discrete or can be reduced to a discrete one.
- **Stochastic optimization** is used with random (noisy) function measurements or random inputs in the search process.
- **Infinite-dimensional optimization** studies the case when the set of feasible solutions is a subset of an infinite-dimensional space, such as a space of functions.
- Heuristics and metaheuristics make few or no assumptions about the problem being optimized. Usually, heuristics do not guarantee that any optimal solution need be found. On the other hand, heuristics are used to find approximate solutions for many complicated optimization problems.
- Constraint satisfaction studies the case in which the objective function f is constant (this is used in artificial intelligence, particularly in automated reasoning).
- **Constraint programming** is a programming paradigm wherein relations between variables are stated in the form of constraints.
- **Disjunctive programming** is used where at least one constraint must be satisfied but not all. It is of particular use in scheduling.
- **Space mapping** is a concept for modeling and optimization of an engineering system to high-fidelity (fine) model accuracy exploiting a suitable physically meaningful coarse or surrogate model.
- In a number of subfields, the techniques are designed primarily for optimization in dynamic contexts (that is, decision making over time):
- Calculus of variations seeks to optimize an action integral over some space to an extremum by varying a function of the coordinates.
- **Optimal control theory** is a generalization of the calculus of variations which introduces control policies.
- **Dynamic programming** is the approach to solve the stochastic optimization problem with stochastic, randomness, and unknown model parameters. It studies the case in which the optimization strategy is based on splitting the problem into smaller subproblems. The equation that describes the relationship between these subproblems is called the Bellman equation.

WHAT IS NP HARD PROBLEM?

A decision problem *H* is NP-hard when for every problem *L* in NP, there is a polynomial-time many-one reduction from *L* to *H*. An equivalent definition is to require that every problem *L* in NP can be solved in polynomial time by an oracle machine with an oracle for *H*. Informally, an algorithm can be thought of that calls such an oracle machine as a subroutine for solving *H* and solves *L* in polynomial time if the subroutine call takes only one step to compute.

Another definition is to require that there be a polynomial-time reduction from an NP-complete problem *G* to *H*. As any problem *L* in NP reduces in polynomial time to *G*, *L* reduces in turn to *H* in polynomial time so this new definition implies the previous one. Awkwardly, it does not restrict the class NP-hard to decision problems, and it also includes search problems or optimization problems.

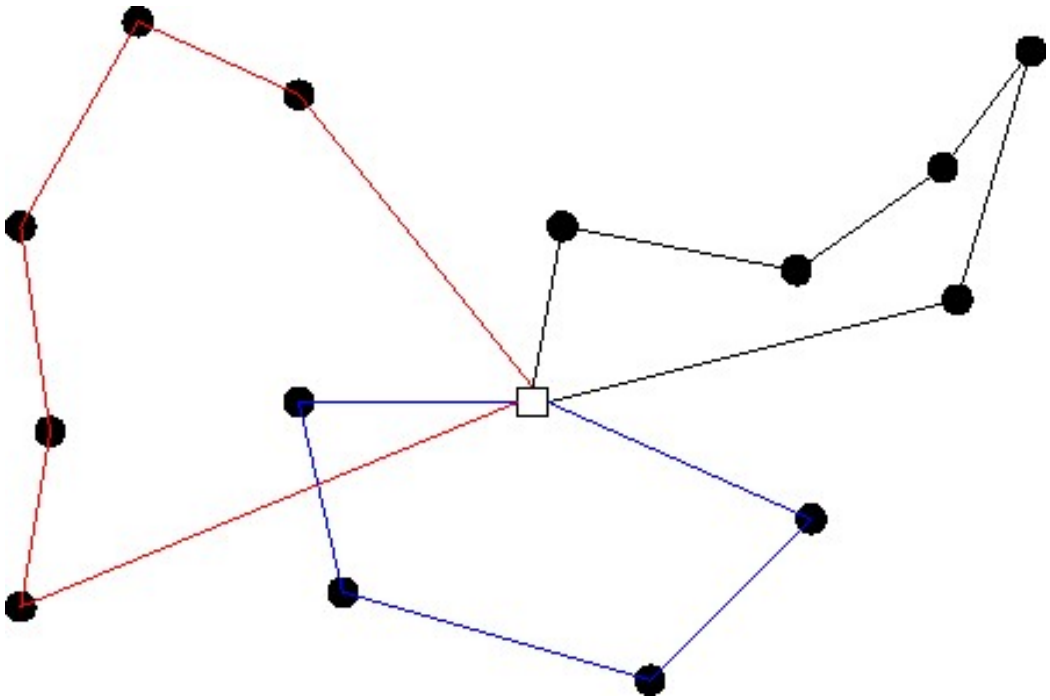
In mathematical terms, the VRP is classified as an *NP-hard* problem, meaning that the required solution time increases exorbitantly with size. The number of possible solutions to the VRP is of the order of *n!*, where *n* is the number of *nodes* (locations the vehicle must reach) in the network.

WHAT IS VEHICLE ROUTING PROBLEM?

The **vehicle routing problem** (**VRP**) is a combinatorial optimization and integer programming problem which asks "What is the optimal set of routes for a fleet of vehicles to traverse in order to deliver to a given set of customers?". It generalizes the well-known travelling salesman problem (TSP).

It first appeared in a paper by George Dantzig and John Ramser in 1959, in which the first algorithmic approach was written and was applied to petrol deliveries. Often, the context is that of delivering goods located at a central depot to customers who have placed orders for such goods. The objective of the VRP is to minimize the total route cost. In 1964, Clarke and Wright improved on Dantzig and Ramser's approach using an effective greedy algorithm called the savings algorithm.

Determining the optimal solution to VRP is NP-hard, so the size of problems that can be solved, optimally, using mathematical programming or combinatorial optimization may be limited. Therefore, commercial solvers tend to use heuristics due to the size and frequency of real world VRPs they need to solve.



One of the most important applications of optimization is *vehicle routing*, in which the goal is to find the best routes for a fleet of vehicles visiting a set of locations. Usually, "best" means routes with the least total distance or cost. Here are a few examples of routing problems:

- A package delivery company wants to assign routes for drivers to make deliveries.
- A cable TV company wants to assign routes for technicians to make residential service calls.
- A ride-sharing company wants to assign routes for drivers to pick up and drop off passengers.

The most famous routing problem is the *Traveling Salesperson Problem* (TSP): find the shortest route for a salesperson who needs to visit customers at different locations and return to the starting point. A TSP can be represented by a graph, in which the nodes correspond to the locations, and the edges (or arcs) denote direct travel between locations.

In the *Vehicle Routing Problem (VRP)*, the goal is to find optimal routes for multiple vehicles visiting a set of locations. (When there's only one vehicle, it reduces to the Traveling Salesperson Problem.)

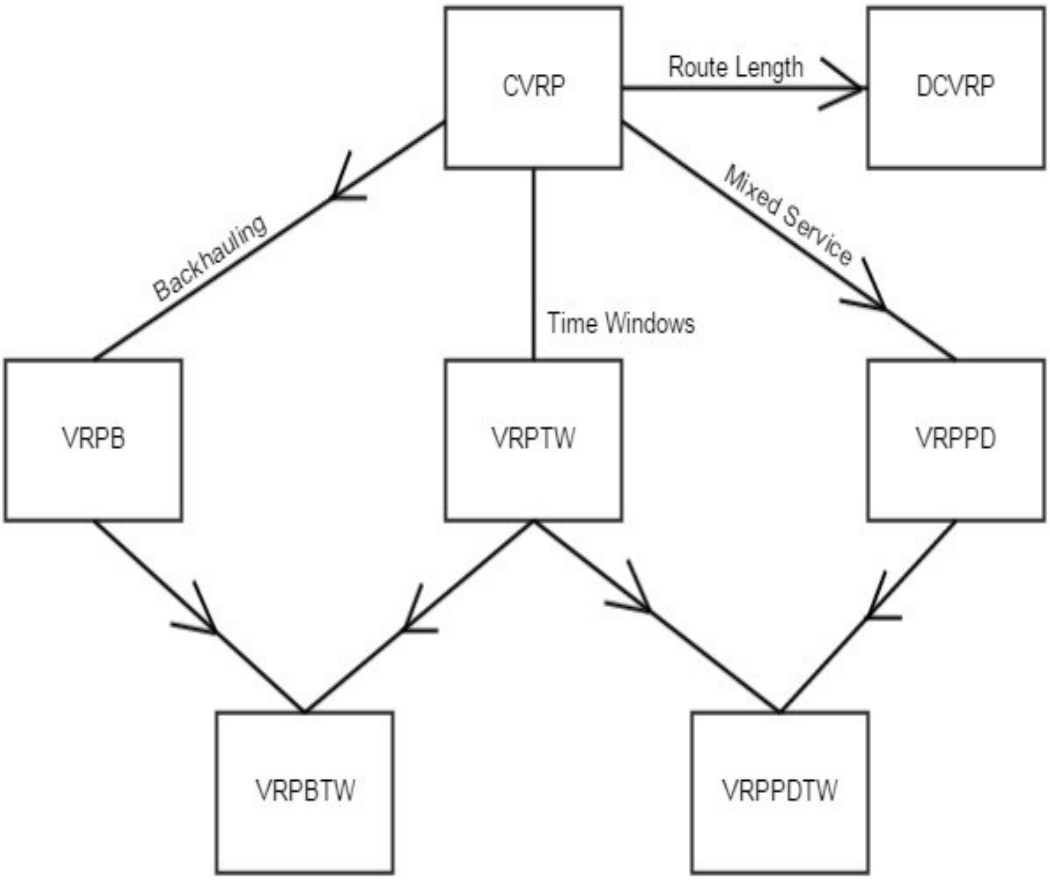
But what do we mean by "optimal routes" for a VRP? One answer is the routes with the least total distance. However, if there are no other constraints, the optimal solution is to assign just one vehicle to visit all locations, and find the shortest route for that vehicle. This is essentially the same problem as the TSP.

A better way to define optimal routes is to minimize the length of the longest single route among all vehicles. This is the right definition if the goal is to complete all deliveries as soon as possible. The VRP example below finds optimal routes defined this way.

In later sections, we'll describe other ways of generalizing the TSP by adding constraints on the vehicles, including:

- **Capacity constraints:** the vehicles need to pick up items at each location they visit, but have a maximum carrying capacity.
- **Time windows:** each location must be visited within a specific time window.

WHAT ARE VRP INVARIANTS?



Several variations and specializations of the vehicle routing problem exist:

- **Vehicle Routing Problem with Profits (VRPP):** A maximization problem where it is not mandatory to visit all customers. The aim is to visit once customers maximizing the sum of collected profits while respecting a vehicle time limit. Vehicles are required to start and end at the depot. Among the most known and studied VRPP, we cite:
  - The Team Orienteering Problem (TOP) which is the most studied variant of the VRPP
  - The Capacitated Team Orienteering Problem (CTOP),
  - The TOP with Time Windows (TOPTW).

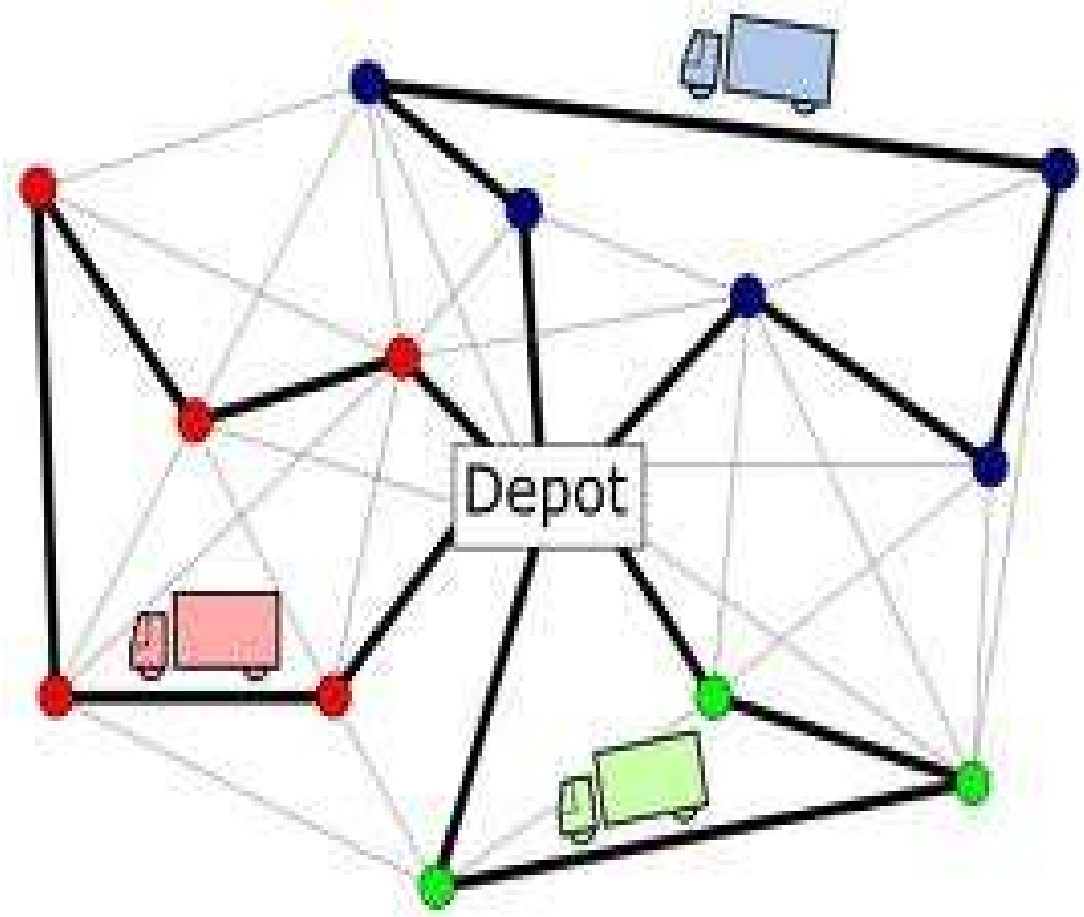
- **Vehicle Routing Problem with Pickup and Delivery (VRPPD):** A number of goods need to be moved from certain pickup locations to other delivery locations. The goal is to find optimal routes for a fleet of vehicles to visit the pickup and drop-off locations.
- **Vehicle Routing Problem with LIFO:** Similar to the VRPPD, except an additional restriction is placed on the loading of the vehicles: at any delivery location, the item being delivered must be the item most recently picked up. This scheme reduces the loading and unloading times at delivery locations because there is no need to temporarily unload items other than the ones that should be dropped off.
- **Vehicle Routing Problem with Time Windows (VRPTW**): The delivery locations have time windows within which the deliveries (or visits) must be made.
- **Capacitated Vehicle Routing Problem:** CVRP or CVRPTW. The vehicles have a limited carrying capacity of the goods that must be delivered.
- **Vehicle Routing Problem with Multiple Trips (VRPMT):** The vehicles can do more than one route.
- **Open Vehicle Routing Problem (OVRP):** Vehicles are not required to return to the depot.
- Inventory Routing Problem (IRP): Vehicles are responsible for satisfying the demands in each delivery point.
- **Multi-Depot Vehicle Routing Problem (MDVRP):** Multiple depots exist from which vehicles can start and end.

Several software vendors have built software products to solve various VRP problems. Numerous articles are available for more detail on their research and results.

Although VRP is related to the Job Shop Scheduling Problem, the two problems are typically solved using different techniques.

WHAT IS CAPACITATED VEHICLE ROUTING PROBLEM?

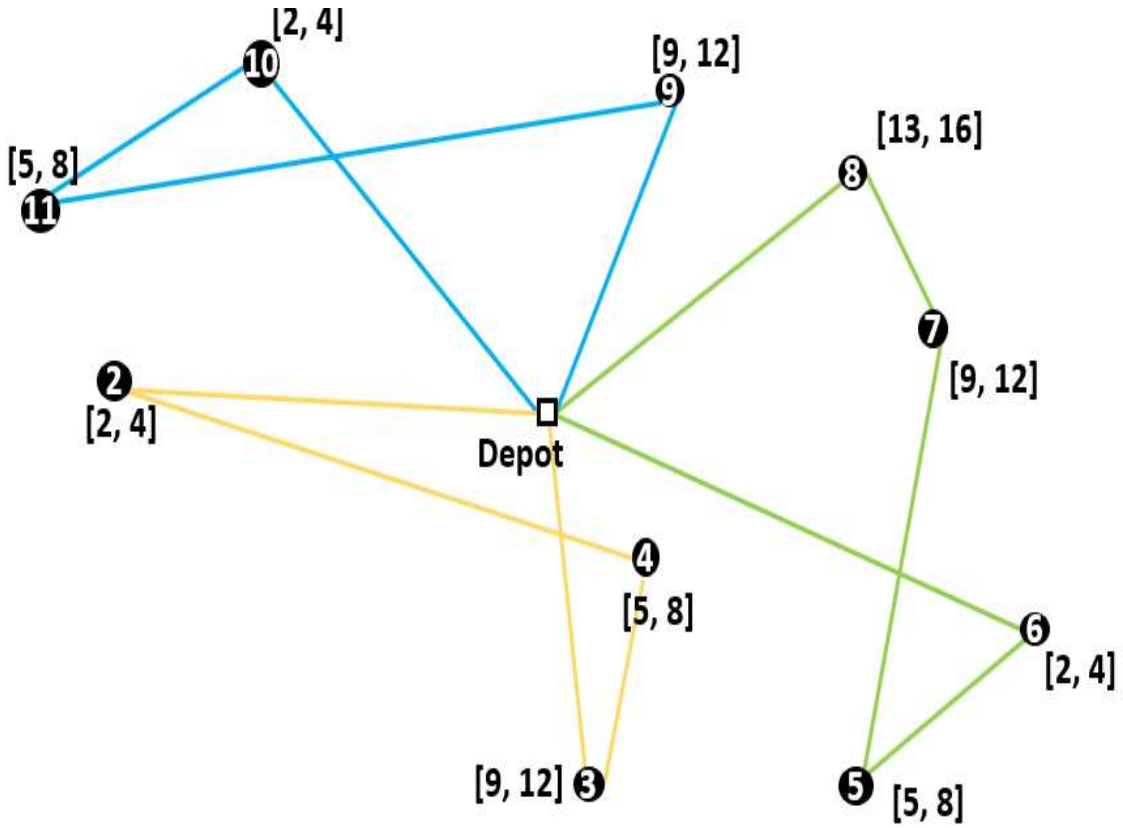
The *capacitated vehicle routing problem* (CVRP) is a VRP in which vehicles with limited carrying capacity need to pick up or deliver items at various locations. The items have a quantity, such as weight or volume, and the vehicles have a maximum *capacity* that they can carry. The problem is to pick up or deliver the items for the least cost, while never exceeding the capacity of the vehicles.



WHAT IS VEHICLE ROUTING PROBLEM WITH TIME WINDOWS?

Many vehicle routing problems involve scheduling visits to customers who are only available during specific time windows. These problems are known as *vehicle routing problems with time windows* (VRPTWs).

The *Vehicle Routing Problem with Time Windows* (VRPTW) is the extension of the *Capacitated Vehicle Routing Problem* (CVRP) where the service at each customer must start within an associated time interval, called a time window. Time windows may be hard or soft. In case of hard time windows, a vehicle that arrives too early at a customer must wait until the customer is ready to begin service. In general, waiting before the start of a time window incurs no cost. In the case of soft time windows, every time window can be violated barring a penalty cost. The time windows may be one-sided, e.g., stated as the latest time for delivery.



WHAT ARE DIFFERENT HUERISTICS APPROACHES TO SOLVE ROUTING PROBLEM?

1. **Genetic Algorithm:** The term Genetic Algorithm was first used by John Holland They are designed to mimic the Darwinian theory of evolution, which states that populations of species evolve to produce more complex organisms and fitter for survival on Earth. Genetic algorithms operate on string structures, like biological structures, which are evolving in time according to the rule of survival of the fittest by using a randomized yet structured information exchange. Thus, in every generation, a new set of strings is created, using parts of the fittest members of the old set The algorithm terminates when the satisfactory fitness level has been reached for the population or the maximum generations have been reached.
2. **Tabu Search Algorithm**: Tabu search (TS) is a heuristic algorithm created by Fred Glover. Using a gradient-descent search with memory techniques to avoid cycling for determining an optimal solution. It does so by forbidding or penalizing moves that take the solution, in the next iteration, to points in the solution space previously visited. The algorithm spends some memory to keep a Tabu list of forbidden moves, which are the moves of the previous iterations or moves that might be considered unwanted.
3. **Simulated Annealing Algorithm:** The Simulated Annealing Algorithm was developed by Kirkpatrick et. al. in 1983, and is based on the analogy of ideal crystals in thermodynamics. The annealing process in metallurgy can make particles arrange themselves in the position with minima potential as the temperature is slowly decreased. The Simulation Annealing algorithm mimics this mechanism and uses the objective function of an optimization problem instead of the energy of a material to arrive at a solution
4. **Path Cheapest Arc:** Starting from a route "start" node, connect it to the node which produces the cheapest route segment, then extend the route by iterating on the last node added to the route.

WHAT ARE HUERISTICS APPROACHES I USED TO SOLVE CVRPTW PROBLEM?

- **Path Cheapest Arc:** Starting from a route "start" node, connect it to the node which produces the cheapest route segment, then extend the route by iterating on the last node added to the route.

WHAT IS GUROBI SOLVER?

The Gurobi Optimizer is a state-of-the-art solver for mathematical programming. The solvers in the Gurobi Optimizer were designed from the ground up to exploit modern architectures and multicore processors, using the most advanced implementations of the latest algorithms.

The Gurobi interface for MATLAB® allows users to build an optimization model, pass the model to Gurobi, and obtain the optimization result, all from within the MATLAB environment. It can be used to solve optimization problems using any of the following forms: linear constraints, bound constraints, integrality constraints, cone constraints, and quadratic constraints.

Gurobi includes a linear programming solver (LP), quadratic programming solver (QP), quadratically constrained programming solver (QCP), mixed-integer linear programming solver (MILP), mixed-integer quadratic programming solver (MIQP), and a mixed-integer quadratically constrained programming solver (MIQCP).

The Gurobi Optimizer:

- Is more than 50% faster than the solver of our closest competitor.
- Has achieved an average 60% improvement in speed every year since its first release in 2009 – and is now almost 60 times faster than the first version.
- Is used by our customers in a wide variety of mission-critical applications in their organizations to achieve numerous business objectives including maximizing resource utilization, maximizing profits, and reducing delays.

WHAT IS GOOGLE OR TOOLS?

OR-Tools is open source software for *combinatorial optimization*, which seeks to find the best solution to a problem out of a very large set of possible solutions. Here are some examples of problems that OR-Tools solves:

- Vehicle routing: Find optimal routes for vehicle fleets that pick up and deliver packages given constraints (e.g., "this truck can't hold more than 20,000 pounds" or "all deliveries must be made within a two-hour window").
- Scheduling: Find the optimal schedule for a complex set of tasks, some of which need to be performed before others, on a fixed set of machines, or other resources.
- Bin packing: Pack as many objects of various sizes as possible into a fixed number of bins with maximum capacities.

In most cases, problems like these have a vast number of possible solutions—too many for a computer to search them all. To overcome this, OR-Tools uses state-of-the-art algorithms to narrow down the search set, in order to find an optimal (or close to optimal) solution.

OR-Tools includes solvers for:

- **Constraint Programming**
  - A set of techniques for finding feasible solutions to a problem expressed as *constraints* (e.g., a room can't be used for two events simultaneously, or the distance to the crops must be less than the length of the hose, or no more than five TV shows can be recorded at once).
- **Linear and Mixed-Integer Programming**
  - The Glop linear optimizer finds the optimal value of a linear objective function, given a set of linear inequalities as constraints (e.g., assigning people to jobs, or finding the best allocation of a set of resources while minimizing cost). Glop and the mixed-

integer programming software SCIP are also available via the Google Apps Script Optimization Service.

- **Vehicle Routing**
  - A specialized library for identifying best vehicle routes given constraints.
- **Graph Algorithms**
  - Code for finding shortest paths in graphs, min-cost flows, max flows, and linear sum assignments.

HOW GOOGLE OR TOOLS IS BETTER THAN GUROBI?

1. Google OR Tool is time efficient than Gurobi.
2. Google OR Tools is free open source tools rather Gurobi is paid Software tool.
3. Google OR Tools solve NP Hard Problems like VRP Problem in reasonable amount of time by using different heuristics techniques whereas here GUROBI fails as if the problem gets larger and larger, GUROBI takes a lot of time and resources to solve the problem.

Project Statement: **Given a set of n (250 approx) nodes and m (47) vehicles of k (4) types, to find a solution that can solve the CVRPTW Problem within 15 minutes. Our objective is to minimize the total cost (variable cost + fixed cost).**

Requirements:

Input Data:

A. Customer data

1. **customer_id:** Unique ID of the customer.
2. **buyer_lat:** Latitude of the buyer
3. **buyer_long:** Longitude of the buyer
4. **weight(kg):** maximum demand of the customer
5. **transaction time(min):** time taken by a vehicle at each customer node
6. **delivery_slot_start:** starting time of the customer at which it can be visited.
7. **delivery_slot_end:** ending time of the customer at which it can be visited.
8. **hub_lat:** Latitude of Hub
9. **hub_long:** Longitude of Hub
10. **E:** conversion of **delivery_slot_start** in minutes (considering the start time to be 0 minutes)
11. **L:** conversion of **delivery_slot_end** in minutes
12. **customer_no:** unique name to each customer (for simplification)

B. Vehicle Data

1. **vehicle_type:** different types of vehicles
2. **Max Buyers/Customers (per day):** Maximum number of buyers that a vehicle can visit in a day in its journey.
3. **Fixed Cost:** Fixed cost of the vehicle, applied only when a vehicle is in use that is when the vehicle leaves the hub.
4. **Free KMs:** free kilometers of the vehicle.
5. **Variable Cost:** Variable cost of the vehicle, applied only when the distance traveled by a vehicle in its journey is more than free km.
6. **Max KMs (per day):** Maximum kilometers of the vehicle that it can travel in a day.
7. **Min weight per buyer/customer:** Minimum weight a buyer can demand.
8. **Max weight per buyer/customer:** Maximum weight a buyer can demand.
9. **Max Time (Minutes):** Maximum time of a vehicle in its journey.

C. Constraints

1. **Capacity constraints**
2. **No of buyers**
3. **Distance**
4. **Time**
5. **Max load per customer**
6. **Min load per Customer**
7. **Max Load per Vehicle Type**
8. **Min load per Vehicle Type**

Mathematical Formulation

Sets:

- **G: Let G be a graph with nodes set N and 0 be the depot.**
- **C = {1,2,3,….,N} : Set of customers**
- **N = {0,1,2,3,….,N} : A collection of nodes including a depot.**
- **V = {bolero_1, bolero_2,……,bolero_10, tata_1, tata_2,……., tata_20,ape_1,ape_2,….,ape_15,bike_1,bike_2}**

Indices:

- $i, j \in$ **N**: **Nodes**
- 0: **Depots**
- $v \in$ **V**: **Vehicles**

Parameters:

- q_j : **demand associated with each customer (i.e. the weight to be delivered).**
- S_j : **service time associated with each customer** j.
- f_v : **free km for vehicle v**
- **A time window [E_j, L_j] where E_j is the earliest time the service can begin and L_j is the latest time for every customer.**
- Q_v : **maximum capacity of vehicle v.**
- FC_v : **Fixed cost of vehicle v.**
- D_{ij} : **Distance between two nodes.**
- max_kms_v : **maximum km vehicle v can travel in a day.**
- max_time_v : **maximum time a vehicle v can travel in a day.**
- $speed_v$: **uniform speed of different vehicles v.**
- T_{ij}^v: **Travel time between two nodes =** $D_{ij} / speed_v$.
- VC_v : **This is the variable cost for vehicle type v.**
- $maxcustomer_v$: **maximum number of customers served by v vehicles.**
- $maxdemand_v$: **maximum demand of customers served by v vehicles.**
- $mindemand_v$: **minimum demand of customers served by v vehicles.**

Decision Variables:

- **A decision variable that decides if vehicle v has taken route $i -> j$ or not.**

$$x_{ij}^{v} = \left\{ \begin{matrix} 1, if\ \ vehicle\ v\ has\ taken\ the\ route\ i{->}j \\ 0, otherwise \end{matrix} \right\}$$

- $extrakms_{v}$: **A decision variable that calculates extra km that vehicle v will travel in its route.**

($extrakms_{v} \in R^{+}$) *(low_bound = 0)*

- d_{v} : **total distance traveled by vehicle v** = $\sum_{i \in N} \sum_{j \in N} (D_{ij} * x_{ij}^{v}) \quad \forall v \in V$

($d_{v} \in R^{+}$)

- **A decision variable that indicates whether the vehicle v is used or not.**

$$u_{v} = \left\{ \begin{matrix} 1, if\ vehicle\ v\ is\ in\ use \\ 0, otherwise \end{matrix} \right\}$$

- **A decision variable that indicates whether vehicle v visited customer i or not.**

$$y_{i}^{v} = \left\{ \begin{matrix} 1, if\ vehicle\ v\ visits\ node\ \ i \\ 0, otherwise \end{matrix} \right\}$$

- t_{i}^{v}: **time of vehicle v to visit node i** **($t_{i}^{v} \in R^{+}$)**
- f_{ij}^{v}: **number of units of product taken by vehicle v from node i to j**

($f_{ij}^{v} \in R^{+}$)

Objective Function:

Our objective is to minimize the sum of the vehicle acquisition costs and routing costs.

Minimize (z) : $\sum_{v \in V} VC_v * extra\ kms_v + \sum_{v \in V} FC_v * u_v$

Constraints:

(1) **This constraint says that when the value of cumulative distance traveled by vehicle** v **is negative, the value of** $extrakms_v$ **becomes 0, otherwise the value of** $extrakms_v$ **is equal to the cumulative distance traveled by vehicle** v **.**

$$extrakms_v \geq d_v - f_v \qquad \forall v \in V$$

(2) **This constraint is a big M constraint which tells that if any vehicle** v **is in use then it has at least visited one node and if it is not in use, then the value of LHS becomes 0 because** x_{ij}^v **is a binary variable and has its lower bound zero.**

$$\sum_{i \in N} \sum_{j \in N} x_{ij}^v \leq M(u_v) \qquad \forall v \in V$$

(3) **This constraint says that each customer in C is visited exactly once by any vehicle** v **.**

$$\sum_{v \in V} y_j^v = 1 \qquad \forall j \in C$$

(4) **This constraint is a flow balancing constraint which tells that the number of vehicles coming in and out of a customer's location is the same and it can only be by only one vehicle** v **.**

$$\sum_{i \in N} x_{ij}^v = \sum_{i \in N} x_{ji}^v = y_j^v \qquad \forall j \in N, \forall v \in V$$

(5) **This constraint says that every customer j demand must be bounded between the minimum and maximum demand, any vehicle v can serve.**

$$y_j^v mindemand_v \leq q_j y_j^v \leq y_j^v maxdemand_v \qquad \forall j \in C, \forall v \in V$$

(6) **This constraint says that the number of customers per vehicle v must be less than the maximum number of customers it can serve.**

$$\sum_{j \in C} y_j^v \leq maxcustomer_v + 1 \qquad \forall v \in V$$

7) **This constraint says that the total number of units taken by vehicle v between two nodes = demand of the node i.**

$$\sum_{v \in V} \sum_{j \in N} f_{ij}^v - \sum_{v \in V} \sum_{j \in N} f_{ji}^v = q_i \qquad \forall i \in C$$

8) **This constraint says that the flow of units between two nodes must be less than the maximum weight a vehicle v can take between two nodes.**

$$0 <= f_{ij}^v <= Q_v x_{ij}^v \qquad \forall i, j \in N, \forall v \in V$$

(9) **Time Window Constraints.**

$$t_i^v + S_i + T_{ij}^v \leq t_j^v + M(1 - x_{ij}^v) \qquad \forall i \in N, j \in C, v \in V$$

(10) **This constraint says that the start time of any vehicle must lie between the time windows of each customer j.**

$$E_j y_j^v \leq t_j^v \leq L_j y_j^v \qquad \forall j \in C, v \in V$$

(11) **Upper bounds of variables.**

$$d_v \leq max\_kms_v \qquad \forall v \in V$$

Vehicle Type	Max Buyers/Customers (in a trip)	Fixed Cost	Variable Cost	Capacity(Kg)	Free KMs (	KMs (per	Min Weight per Buyer/Customer	Max Weight per Buyer/Customer	Max Time(hrs)	uniform speed (km/min)
bolero_1	30	2000	8	2000	100	250	20	2000	720	2
bolero_2	30	2000	8	2000	100	250	20	2000	720	2
bolero_3	30	2000	8	2000	100	250	20	2000	720	2
bolero_4	30	2000	8	2000	100	250	20	2000	720	2
bolero_5	30	2000	8	2000	100	250	20	2000	720	1
bolero_6	30	2000	8	2000	100	250	20	2000	720	1
bolero_7	30	2000	8	2000	100	250	20	2000	720	1
bolero_8	30	2000	8	2000	100	250	20	2000	720	1
bolero_9	30	2000	8	2000	100	250	20	2000	720	1
bolero_10	30	2000	8	2000	100	250	20	2000	720	1
tata_1	20	1500	6	1000	70	150	10	100	720	0.5
tata_2	20	1500	6	1000	70	150	10	100	720	0.5
tata_3	20	1500	6	1000	70	150	10	100	720	0.5
tata_4	20	1500	6	1000	70	150	10	100	720	0.5
tata_5	20	1500	6	1000	70	150	10	100	720	0.5
tata_6	20	1500	6	1000	70	150	10	100	720	0.5
tata_7	20	1500	6	1000	70	150	10	100	720	0.5
tata_8	20	1500	6	1000	70	150	10	100	720	0.5
tata_9	20	1500	6	1000	70	150	10	100	720	0.5
tata_10	20	1500	6	1000	70	150	10	100	720	0.5
tata_11	20	1500	6	1000	70	150	10	100	720	0.5
tata_12	20	1500	6	1000	70	150	10	100	720	0.5
tata_13	20	1500	6	1000	70	150	10	100	720	0.5
tata_14	20	1500	6	1000	70	150	10	100	720	0.5
tata_15	20	1500	6	1000	70	150	10	100	720	0.5
tata_16	20	1500	6	1000	70	150	10	100	720	0.5
tata_17	20	1500	6	1000	70	150	10	100	720	0.5
tata_18	20	1500	6	1000	70	150	10	100	720	0.5
tata_19	20	1500	6	1000	70	150	10	100	720	0.5
tata_20	20	1500	6	1000	70	150	10	100	720	0.5
ape_1	18	1000	5	600	60	120	10	50	720	0.3
ape_2	18	1000	5	600	60	120	10	50	720	0.3
ape_3	18	1000	5	600	60	120	10	50	720	0.3
ape_4	18	1000	5	600	60	120	10	50	720	0.3
ape_5	18	1000	5	600	60	120	10	50	720	0.3
ape_6	18	1000	5	600	60	120	10	50	720	0.3
ape_7	18	1000	5	600	60	120	10	50	720	0.3
ape_8	18	1000	5	600	60	120	10	50	720	0.3
ape_9	18	1000	5	600	60	120	10	50	720	0.3
ape_10	18	1000	5	600	60	120	10	50	720	0.3
ape_11	18	1000	5	600	60	120	10	50	720	0.3
ape_12	18	1000	5	600	60	120	10	50	720	0.3
ape_13	18	1000	5	600	60	120	10	50	720	0.3
ape_14	18	1000	5	600	60	120	10	50	720	0.3
ape_15	18	1000	5	600	60	120	10	50	720	0.3
bike_1	15	600	3	40	10	50	0	10	720	0.1
bike_2	15	600	3	40	10	50	0	10	720	0.1

customer_id	buyer_lat	buyer_long	weight(Kg)	transaction_time(mins)	delivery_slot_start	delivery_slot_end	customer_no	E	L
c_110002_5	28.63793355	77.22481809	0.29	5	7:00 AM	1:00 PM	56	0	360
c_110004_48	28.61914613	77.18340505	0.42	5	4:00 PM	7:00 PM	99	540	720
c_110002_79	28.6179006	77.23758293	0.83	5	1:00 PM	4:00 PM	230	360	540
c_110005_95	28.64482289	77.1661627	0.9	5	7:00 AM	1:00 PM	246	0	360
c_110007_9	28.65612464	77.17849524	1.1	5	7:00 AM	1:00 PM	60	0	360
c_110005_21	28.63983231	77.16359098	1.66	5	7:00 AM	1:00 PM	72	0	360
c_110006_34	28.63805451	77.21270938	1.92	5	10:00 AM	1:00 PM	185	180	360
c_110005_65	28.6696162	77.17918711	2.07	5	4:00 PM	7:00 PM	116	540	720
c_110007_80	28.6871583	77.18344219	2.69	5	10:00 AM	1:00 PM	231	180	360
c_110004_38	28.58806927	77.17576419	2.8	5	7:00 AM	1:00 PM	39	0	360
c_110009_59	28.70238436	77.20186248	2.84	5	7:00 AM	1:00 PM	210	0	360
c_110001_97	28.63157109	77.21091197	2.87	5	7:00 AM	1:00 PM	148	0	360
c_110002_42	28.6592502	77.26869353	2.93	5	7:00 AM	1:00 PM	43	0	360
c_110005_71	28.650284	77.1674444	2.95	5	1:00 PM	4:00 PM	222	360	540
c_110007_89	28.67010225	77.19091388	3.02	5	1:00 PM	4:00 PM	140	360	540
c_110003_13	28.56836312	77.2100634	3.55	5	4:00 PM	7:00 PM	64	540	720
c_110007_94	28.69785194	77.21079358	3.9	5	7:00 AM	1:00 PM	145	0	360
c_110007_66	28.68523946	77.20212341	3.9	5	10:00 AM	1:00 PM	217	180	360
c_110003_31	28.56924436	77.20660465	4.18	5	7:00 AM	1:00 PM	32	0	360
c_110002_1	28.65950184	77.23631164	4.2	5	7:00 AM	1:00 PM	152	0	360
c_110007_28	28.69185592	77.17291608	4.24	5	1:00 PM	4:00 PM	179	360	540
c_110009_47	28.73670323	77.19775186	4.38	5	1:00 PM	4:00 PM	198	360	540
c_110002_49	28.63414924	77.22955465	4.64	5	1:00 PM	4:00 PM	50	360	540
c_110006_19	28.64388351	77.24457793	4.7	5	4:00 PM	7:00 PM	20	540	720
c_110006_92	28.66618942	77.22905961	4.74	5	10:00 AM	1:00 PM	243	180	360
c_110007_1	28.67607912	77.20847836	4.81	5	7:00 AM	1:00 PM	2	0	360
c_110008_72	28.62938199	77.14963032	6.78	7	7:00 AM	1:00 PM	223	0	360
c_110002_50	28.62366059	77.24514423	8.72	7	4:00 PM	7:00 PM	201	540	720
c_110002_35	28.65774353	77.22712613	8.87	7	7:00 AM	1:00 PM	36	0	360
c_110002_57	28.62535745	77.22412521	9.21	7	4:00 PM	7:00 PM	208	540	720
c_110009_41	28.72274686	77.17056804	9.3	7	10:00 AM	1:00 PM	42	180	360
c_110008_37	28.63121454	77.16245053	9.54	7	4:00 PM	7:00 PM	188	540	720

c_110008_1	28.63488869	77.15112967	10	7	10:00 AM	1:00 PM	52	180	360
c_110002_84	28.6274357	77.26005325	13.27	7	10:00 AM	1:00 PM	235	180	360
c_110005_58	28.64696794	77.18582968	13.55	7	4:00 PM	7:00 PM	209	540	720
c_110009_64	28.71175652	77.18759836	15.65	7	10:00 AM	1:00 PM	115	180	360
c_110004_97	28.59574103	77.17850077	15.82	7	1:00 PM	4:00 PM	248	360	540
c_110007_33	28.67915246	77.17370501	16.25	7	7:00 AM	1:00 PM	34	0	360
c_110006_0	28.65714197	77.22083115	17.34	7	4:00 PM	7:00 PM	51	540	720
c_110001_33	28.6235263	77.20878894	18.72	7	7:00 AM	1:00 PM	184	0	360
c_110002_70	28.64360096	77.25678639	21.85	7	7:00 AM	1:00 PM	121	0	360
c_110001_29	28.63091278	77.19092065	22.96	7	4:00 PM	7:00 PM	30	540	720
c_110006_32	28.67203727	77.22044042	23.9	7	4:00 PM	7:00 PM	33	540	720
c_110004_4	28.59642879	77.18076835	24.54	7	7:00 AM	1:00 PM	55	0	360
c_110006_8	28.65122524	77.22343717	24.62	7	4:00 PM	7:00 PM	9	540	720
c_110006_10	28.63832346	77.23225975	25.69	7	7:00 AM	1:00 PM	161	0	360
c_110009_87	28.69934864	77.20528521	26.32	7	7:00 AM	1:00 PM	138	0	360
c_110004_84	28.59636467	77.19118774	26.38	7	4:00 PM	7:00 PM	135	540	720
c_110007_22	28.66671501	77.20113847	26.69	7	4:00 PM	7:00 PM	173	540	720
c_110001_44	28.60184229	77.20164015	29.51	7	4:00 PM	7:00 PM	95	540	720
c_110002_62	28.61719188	77.23148049	30.27	7	4:00 PM	7:00 PM	113	540	720
c_110006_98	28.66663814	77.20370172	34.62	7	4:00 PM	7:00 PM	249	540	720
c_110008_40	28.67014007	77.1555373	38.66	7	4:00 PM	7:00 PM	91	540	720
c_110003_30	28.60471833	77.23181966	42.88	7	4:00 PM	7:00 PM	181	540	720
c_110001_23	28.62599599	77.22960466	45.36	7	4:00 PM	7:00 PM	24	540	720
c_110008_50	28.64766331	77.17532384	46.66	7	7:00 AM	1:00 PM	101	0	360
c_110001_39	28.6166146	77.19716136	48.09	7	10:00 AM	1:00 PM	40	180	360
c_110001_54	28.60447247	77.23528627	49.68	7	1:00 PM	4:00 PM	205	360	540
c_110007_15	28.66994016	77.2021008	50.49	7	1:00 PM	4:00 PM	166	360	540
c_110006_91	28.63871985	77.20484533	52.22	7	4:00 PM	7:00 PM	142	540	720
c_110008_5	28.63547271	77.18426281	53.34	7	10:00 AM	1:00 PM	6	180	360
c_110005_25	28.66757278	77.20783211	54.04	7	4:00 PM	7:00 PM	176	540	720
c_110003_47	28.56589374	77.21762095	54.63	7	4:00 PM	7:00 PM	98	540	720
c_110004_37	28.60153166	77.20305918	55.39	7	7:00 AM	1:00 PM	88	0	360
c_110008_63	28.63357281	77.18815208	56.01	7	4:00 PM	7:00 PM	114	540	720

c_110001_54	28.61138114	77.21410135	56.95	7	4:00 PM	7:00 PM	105	540	720
c_110004_11	28.58806992	77.20147458	58.3	7	7:00 AM	1:00 PM	62	0	360
c_110003_44	28.56778499	77.20537501	58.52	7	4:00 PM	7:00 PM	45	540	720
c_110001_98	28.60309871	77.19228288	60.54	7	4:00 PM	7:00 PM	149	540	720
c_110009_4	28.70341682	77.17533717	63.66	7	4:00 PM	7:00 PM	5	540	720
c_110002_19	28.61702087	77.22755459	65.1	7	4:00 PM	7:00 PM	170	540	720
c_110005_69	28.67378695	77.17195029	65.7	7	10:00 AM	1:00 PM	120	180	360
c_110008_17	28.63856007	77.18879565	65.96	7	7:00 AM	1:00 PM	68	0	360
c_110009_6	28.73007515	77.17067082	66.16	7	7:00 AM	1:00 PM	157	0	360
c_110006_17	28.66181684	77.20906791	66.69	7	4:00 PM	7:00 PM	18	540	720
c_110003_42	28.61389886	77.22108375	67.26	7	4:00 PM	7:00 PM	193	540	720
c_110001_9	28.60496634	77.22606828	67.75	7	1:00 PM	4:00 PM	10	360	540
c_110008_26	28.64739889	77.16906566	69.74	7	7:00 AM	1:00 PM	177	0	360
c_110009_48	28.73613941	77.19606421	73.2	7	7:00 AM	1:00 PM	199	0	360
c_110002_10	28.63587318	77.22406707	75.32	7	10:00 AM	1:00 PM	61	180	360
c_110003_93	28.60030065	77.22491567	76.26	7	1:00 PM	4:00 PM	144	360	540
c_110002_25	28.64563045	77.26901991	78.3	7	10:00 AM	1:00 PM	76	180	360
c_110002_41	28.64021447	77.25547338	80.04	7	7:00 AM	1:00 PM	92	0	360
c_110001_30	28.61016272	77.1995874	80.6	7	7:00 AM	1:00 PM	31	0	360
c_110003_25	28.60872902	77.250002	81.19	7	7:00 AM	1:00 PM	26	0	360
c_110001_36	28.64996499	77.2061677	82.3	7	4:00 PM	7:00 PM	37	540	720
c_110004_12	28.62710343	77.17464996	83.21	7	4:00 PM	7:00 PM	13	540	720
c_110002_6	28.63703619	77.26164569	84.98	7	1:00 PM	4:00 PM	57	360	540
c_110008_80	28.66433606	77.16461557	86.07	7	10:00 AM	1:00 PM	131	180	360
c_110007_36	28.69058661	77.18384121	86.29	7	1:00 PM	4:00 PM	187	360	540
c_110007_86	28.68605773	77.20848302	87.39	7	1:00 PM	4:00 PM	237	360	540
c_110008_14	28.67228208	77.19099244	87.77	7	7:00 AM	1:00 PM	65	0	360
c_110002_43	28.61940316	77.25294481	89.23	7	4:00 PM	7:00 PM	94	540	720
c_110002_14	28.63351134	77.26785218	90.66	7	4:00 PM	7:00 PM	165	540	720
c_110008_69	28.66105071	77.14399042	91.19	7	1:00 PM	4:00 PM	220	360	540
c_110007_2	28.65262826	77.18031131	91.88	7	7:00 AM	1:00 PM	153	0	360
c_110001_83	28.63490157	77.23191827	92.41	7	4:00 PM	7:00 PM	234	540	720
c_110009_3	28.72205645	77.17342065	93.14	7	4:00 PM	7:00 PM	54	540	720

c_110003_76	28.59967415	77.21988067	96.21	7	1:00 PM	4:00 PM	127	360	540
c_110003_91	28.57444133	77.20810836	96.28	7	4:00 PM	7:00 PM	242	540	720
c_110002_43	28.62636812	77.24638249	97.15	7	7:00 AM	1:00 PM	194	0	360
c_110003_73	28.60808442	77.22192358	97.81	7	7:00 AM	1:00 PM	224	0	360
c_110009_74	28.73209102	77.21548374	98.02	7	1:00 PM	4:00 PM	225	360	540
c_110002_46	28.63941715	77.22535898	98.44	7	7:00 AM	1:00 PM	197	0	360
c_110001_49	28.63824754	77.19675601	98.73	7	7:00 AM	1:00 PM	100	0	360
c_110001_55	28.60867252	77.22795933	99.08	7	1:00 PM	4:00 PM	106	360	540
c_110005_68	28.64986897	77.19008029	102	10	1:00 PM	4:00 PM	219	360	540
c_110002_85	28.614045	77.22733688	103.45	10	4:00 PM	7:00 PM	236	540	720
c_110005_31	28.66505737	77.18806962	103.92	10	7:00 AM	1:00 PM	182	0	360
c_110001_14	28.60491671	77.21436647	104.96	10	10:00 AM	1:00 PM	15	180	360
c_110005_64	28.65908262	77.19902814	105.93	10	4:00 PM	7:00 PM	215	540	720
c_110004_81	28.60529059	77.18383712	106.43	10	7:00 AM	1:00 PM	232	0	360
c_110004_6	28.60616761	77.20403457	111.41	10	7:00 AM	1:00 PM	7	0	360
c_110002_9	28.63216644	77.24788405	115.88	10	7:00 AM	1:00 PM	160	0	360
c_110001_78	28.59832819	77.2259096	118.82	10	10:00 AM	1:00 PM	129	180	360
c_110009_27	28.69903388	77.2087808	119.74	10	7:00 AM	1:00 PM	28	0	360
c_110002_88	28.62462998	77.26687729	120.59	10	7:00 AM	1:00 PM	139	0	360
c_110002_11	28.62918723	77.22468663	122.96	10	1:00 PM	4:00 PM	12	360	540
c_110001_49	28.64597071	77.19875764	124.26	10	10:00 AM	1:00 PM	200	180	360
c_110004_41	28.60691859	77.1803425	125.02	10	1:00 PM	4:00 PM	192	360	540
c_110007_72	28.68921198	77.19952659	125.95	10	4:00 PM	7:00 PM	123	540	720
c_110007_16	28.68635745	77.18349257	126.24	10	7:00 AM	1:00 PM	67	0	360
c_110007_34	28.67004119	77.18547253	136.78	10	7:00 AM	1:00 PM	35	0	360
c_110004_90	28.62582016	77.19505571	141	10	7:00 AM	1:00 PM	241	0	360
c_110002_96	28.66439245	77.26200609	144.05	10	7:00 AM	1:00 PM	147	0	360
c_110007_5	28.65535086	77.19931806	145.04	10	4:00 PM	7:00 PM	156	540	720
c_110009_55	28.70213891	77.17716229	145.12	10	1:00 PM	4:00 PM	206	360	540
c_110005_62	28.63664752	77.16716737	149.52	10	7:00 AM	1:00 PM	213	0	360
c_110007_63	28.6897511	77.21685462	159.58	10	10:00 AM	1:00 PM	214	180	360
c_110004_99	28.61989839	77.17225828	161.02	10	7:00 AM	1:00 PM	150	0	360
c_110008_7	28.63370158	77.18812156	161.85	10	7:00 AM	1:00 PM	8	0	360

c_110001_61	28.61855042	77.20648322	162.62	10	10:00 AM	1:00 PM	112	180	360
c_110001_47	28.63738711	77.20181407	163.84	10	7:00 AM	1:00 PM	48	0	360
c_110001_61	28.64963159	77.20972707	168.5	10	4:00 PM	7:00 PM	212	540	720
c_110005_77	28.66506581	77.2041685	168.6	10	7:00 AM	1:00 PM	128	0	360
c_110004_58	28.62490705	77.18018731	169.78	10	1:00 PM	4:00 PM	109	360	540
c_110006_28	28.66023127	77.23863082	172.76	10	7:00 AM	1:00 PM	79	0	360
c_110007_60	28.67275937	77.1926204	184.05	10	10:00 AM	1:00 PM	111	180	360
c_110004_8	28.62731748	77.21683638	184.25	10	4:00 PM	7:00 PM	59	540	720
c_110003_20	28.57676229	77.2291858	185.28	10	4:00 PM	7:00 PM	71	540	720
c_110002_27	28.65938095	77.24397809	191.14	10	7:00 AM	1:00 PM	78	0	360
c_110005_79	28.63148423	77.17209643	191.31	10	1:00 PM	4:00 PM	130	360	540
c_110008_31	28.64743075	77.15884838	194.19	10	4:00 PM	7:00 PM	82	540	720
c_110008_3	28.67308808	77.15599975	196.53	10	10:00 AM	1:00 PM	4	180	360
c_110008_29	28.65694866	77.14872492	197.01	10	7:00 AM	1:00 PM	180	0	360
c_110003_34	28.57373798	77.20977498	197.1	10	1:00 PM	4:00 PM	85	360	540
c_110003_99	28.61798582	77.21913797	201.62	10	1:00 PM	4:00 PM	250	360	540
c_110005_18	28.64333108	77.16657609	207.68	10	4:00 PM	7:00 PM	19	540	720
c_110007_19	28.68442112	77.1855336	207.88	10	10:00 AM	1:00 PM	70	180	360
c_110005_28	28.6376213	77.17528314	208.02	10	7:00 AM	1:00 PM	29	0	360
c_110008_67	28.62962682	77.14178611	214.36	10	4:00 PM	7:00 PM	218	540	720
c_110007_30	28.68124063	77.20303962	219.36	10	4:00 PM	7:00 PM	81	540	720
c_110005_35	28.6551064	77.19271639	220.62	10	7:00 AM	1:00 PM	86	0	360
c_110002_45	28.65445616	77.25851107	230.09	10	1:00 PM	4:00 PM	196	360	540
c_110008_20	28.67621229	77.15742127	231.7	10	4:00 PM	7:00 PM	171	540	720
c_110006_76	28.64791688	77.22024075	231.96	10	7:00 AM	1:00 PM	227	0	360
c_110009_67	28.68716655	77.18593595	234.42	10	7:00 AM	1:00 PM	118	0	360
c_110001_57	28.64920245	77.21103834	234.79	10	4:00 PM	7:00 PM	108	540	720
c_110008_15	28.64175992	77.16021936	236.4	10	4:00 PM	7:00 PM	16	540	720
c_110008_46	28.67059573	77.16867957	238.8	10	7:00 AM	1:00 PM	97	0	360
c_110003_56	28.58417578	77.24479692	240.27	10	4:00 PM	7:00 PM	207	540	720
c_110002_85	28.66626405	77.2184204	247.21	10	7:00 AM	1:00 PM	136	0	360
c_110005_3	28.66301914	77.19820441	247.93	10	1:00 PM	4:00 PM	154	360	540
c_110007_18	28.67028556	77.1993291	248.25	10	7:00 AM	1:00 PM	169	0	360

c_110002_2	28.64977128	77.2617096	248.46	10	4:00 PM	7:00 PM	3	540	720
c_110008_74	28.64871855	77.14577982	249.37	10	7:00 AM	1:00 PM	125	0	360
c_110001_12	28.60449127	77.22639419	253.32	10	7:00 AM	1:00 PM	163	0	360
c_110009_46	28.72700147	77.18231411	253.51	10	4:00 PM	7:00 PM	47	540	720
c_110003_22	28.56848222	77.20183224	257.57	10	1:00 PM	4:00 PM	23	360	540
c_110005_86	28.63722209	77.20030167	259.04	10	7:00 AM	1:00 PM	137	0	360
c_110001_90	28.60144666	77.19403985	265.5	10	4:00 PM	7:00 PM	141	540	720
c_110006_82	28.63565502	77.2208022	267.06	10	4:00 PM	7:00 PM	233	540	720
c_110009_65	28.73331249	77.19771965	267.96	10	7:00 AM	1:00 PM	216	0	360
c_110009_15	28.73223858	77.21609761	268.17	10	4:00 PM	7:00 PM	66	540	720
c_110004_10	28.60314302	77.1887138	270.79	10	7:00 AM	1:00 PM	11	0	360
c_110002_88	28.64814799	77.24832697	270.87	10	10:00 AM	1:00 PM	239	180	360
c_110007_8	28.68461064	77.19300678	272.81	10	4:00 PM	7:00 PM	159	540	720
c_110007_92	28.68671059	77.19613221	275.55	10	1:00 PM	4:00 PM	143	360	540
c_110003_95	28.56898705	77.25340137	277.29	10	4:00 PM	7:00 PM	146	540	720
c_110001_16	28.6350677	77.23986446	284.83	10	7:00 AM	1:00 PM	167	0	360
c_110006_51	28.66839745	77.22946332	288.89	10	1:00 PM	4:00 PM	202	360	540
c_110006_53	28.66100042	77.20850375	292.31	10	10:00 AM	1:00 PM	204	180	360
c_110005_23	28.64241642	77.21228073	293.14	10	10:00 AM	1:00 PM	74	180	360
c_110009_35	28.69719497	77.18882766	296.19	10	1:00 PM	4:00 PM	186	360	540
c_110003_77	28.5905024	77.21549412	304.55	10	7:00 AM	1:00 PM	228	0	360
c_110003_75	28.57167176	77.23304461	306.1	10	4:00 PM	7:00 PM	226	540	720
c_110003_39	28.60911081	77.20442203	310.57	10	4:00 PM	7:00 PM	90	540	720
c_110004_22	28.60838759	77.22243518	310.92	10	4:00 PM	7:00 PM	73	540	720
c_110005_40	28.67887835	77.17772184	315.64	10	4:00 PM	7:00 PM	41	540	720
c_110003_33	28.60834685	77.24276633	320.08	10	10:00 AM	1:00 PM	84	180	360
c_110003_68	28.57010699	77.24168489	320.38	10	7:00 AM	1:00 PM	119	0	360
c_110005_12	28.64522636	77.18199662	331.79	10	7:00 AM	1:00 PM	63	0	360
c_110005_44	28.67229896	77.15934148	339.56	10	4:00 PM	7:00 PM	195	540	720
c_110003_38	28.58509834	77.20790251	342.66	10	7:00 AM	1:00 PM	189	0	360
c_110004_89	28.61134189	77.2134912	346.46	10	10:00 AM	1:00 PM	240	180	360
c_110001_43	28.61687066	77.2076316	350.9	10	4:00 PM	7:00 PM	44	540	720
c_110002_51	28.63606195	77.22919851	352.85	10	10:00 AM	1:00 PM	102	180	360

c_110001_32	28.6306516	77.2374989	358.46	10	1:00 PM	4:00 PM	183	360	540
c_110005_0	28.65434206	77.20699608	375.26	10	4:00 PM	7:00 PM	1	540	720
c_110006_24	28.66303421	77.25349332	375.61	10	4:00 PM	7:00 PM	25	540	720
c_110005_66	28.63619352	77.21095348	375.73	10	7:00 AM	1:00 PM	117	0	360
c_110003_7	28.57780248	77.20973506	376.45	10	7:00 AM	1:00 PM	158	0	360
c_110002_39	28.62073986	77.22063632	380.93	10	7:00 AM	1:00 PM	190	0	360
c_110004_73	28.60077176	77.17875518	381.42	10	7:00 AM	1:00 PM	124	0	360
c_110005_26	28.66834713	77.20302584	385.06	10	1:00 PM	4:00 PM	77	360	540
c_110001_32	28.62336619	77.23991419	386.19	10	1:00 PM	4:00 PM	83	360	540
c_110009_21	28.70716872	77.19035693	390	10	7:00 AM	1:00 PM	172	0	360
c_110002_59	28.63190041	77.2199066	390.42	10	4:00 PM	7:00 PM	110	540	720
c_110005_78	28.67165364	77.15893281	390.67	10	4:00 PM	7:00 PM	229	540	720
c_110009_52	28.69522544	77.22115456	391.8	10	4:00 PM	7:00 PM	203	540	720
c_110004_38	28.5874253	77.18834968	393.43	10	7:00 AM	1:00 PM	89	0	360
c_110005_93	28.65643662	77.17213748	393.55	10	4:00 PM	7:00 PM	244	540	720
c_110005_96	28.64329325	77.1946524	398.63	10	4:00 PM	7:00 PM	247	540	720
c_110003_52	28.59410184	77.22006523	398.88	10	4:00 PM	7:00 PM	103	540	720
c_110004_48	28.61598681	77.20061768	401.82	10	7:00 AM	1:00 PM	49	0	360
c_110001_81	28.60523879	77.21533127	401.94	10	10:00 AM	1:00 PM	132	180	360
c_110006_37	28.64082206	77.24427646	402.72	10	1:00 PM	4:00 PM	38	360	540
c_110002_13	28.64249706	77.26256846	404.86	10	1:00 PM	4:00 PM	164	360	540
c_110007_75	28.69936367	77.22408874	406.04	10	7:00 AM	1:00 PM	126	0	360
c_110008_71	28.6440203	77.18963771	407.29	10	10:00 AM	1:00 PM	122	180	360
c_110001_56	28.60250512	77.20265742	407.36	10	7:00 AM	1:00 PM	107	0	360
c_110002_60	28.64316473	77.23184355	417.74	10	7:00 AM	1:00 PM	211	0	360
c_110004_20	28.58888717	77.21606912	418.16	10	4:00 PM	7:00 PM	21	540	720
c_110005_16	28.64424873	77.21150479	423.78	10	7:00 AM	1:00 PM	17	0	360
c_110001_2	28.63321358	77.21350753	427.75	10	4:00 PM	7:00 PM	53	540	720
c_110006_23	28.6505692	77.19893716	428.67	10	7:00 AM	1:00 PM	174	0	360
c_110009_42	28.71977631	77.20656773	436.13	10	4:00 PM	7:00 PM	93	540	720
c_110006_24	28.67850827	77.23828887	438.16	10	7:00 AM	1:00 PM	75	0	360
c_110007_87	28.69759804	77.19417248	442.96	10	4:00 PM	7:00 PM	238	540	720
c_110007_17	28.67509623	77.21259761	444.61	10	4:00 PM	7:00 PM	168	540	720

c_110003_83	28.61595023	77.22874648	445.12	10	4:00 PM	7:00 PM	134	540	720
c_110004_40	28.63631868	77.18374819	445.19	10	10:00 AM	1:00 PM	191	180	360
c_110003_4	28.60751366	77.20475166	447.41	10	4:00 PM	7:00 PM	155	540	720
c_110008_36	28.6290085	77.18909688	447.54	10	1:00 PM	4:00 PM	87	360	540
c_110005_26	28.63044543	77.16788484	447.76	10	4:00 PM	7:00 PM	27	540	720
c_110003_18	28.60614279	77.23995697	451.83	10	4:00 PM	7:00 PM	69	540	720
c_110007_53	28.65825443	77.17326397	455.12	10	10:00 AM	1:00 PM	104	180	360
c_110002_7	28.65438128	77.23145597	457.97	10	4:00 PM	7:00 PM	58	540	720
c_110005_45	28.6699412	77.19537381	470.88	10	7:00 AM	1:00 PM	96	0	360
c_110008_21	28.63476806	77.13927834	475.54	10	10:00 AM	1:00 PM	22	180	360
c_110007_13	28.67826741	77.17345762	476.11	10	7:00 AM	1:00 PM	14	0	360
c_110004_0	28.6204898	77.17971733	477.25	10	4:00 PM	7:00 PM	151	540	720
c_110003_27	28.57492043	77.21457497	477.91	10	1:00 PM	4:00 PM	178	360	540
c_110003_11	28.61518896	77.22179971	479.04	10	10:00 AM	1:00 PM	162	180	360
c_110006_45	28.66463467	77.25313792	482.18	10	10:00 AM	1:00 PM	46	180	360
c_110009_29	28.72225456	77.19230257	485.35	10	7:00 AM	1:00 PM	80	0	360
c_110008_24	28.66375624	77.14809815	488.95	10	7:00 AM	1:00 PM	175	0	360
c_110008_82	28.67181412	77.17208513	489.08	10	4:00 PM	7:00 PM	133	540	720
c_110003_70	28.61839299	77.22486317	489.3	10	1:00 PM	4:00 PM	221	360	540
c_110002_94	28.61752737	77.25781429	497.9	10	7:00 AM	1:00 PM	245	0	360

CVRPTW Problem Solution containing 50 Nodes and 5 Vehicles using Gurobi.

```
"C:\Users\Rishi Mehdiratta\PycharmProjects\pythonProject7\venv\Scripts\python.exe" "C:/Users/Rishi Mehdiratta/PycharmProjects/pythonProject7/main.py"
C:\Users\Rishi Mehdiratta\PycharmProjects\pythonProject7\venv\lib\site-packages\pulp\pulp.py:1352: UserWarning: Spaces are not permitted in the name. Converted to '_'
  warnings.warn("Spaces are not permitted in the name. Converted to '_'")
Set parameter Username
Academic license - for non-commercial use only - expires 2022-06-13
Set parameter MIPFocus to value 1
Set parameter Heuristics to value 0.5
Set parameter MIPGap to value 0.2
Set parameter Symmetry to value 2
Set parameter SolFiles to value "sol_"
Set parameter TimeLimit to value 1800
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 31942 rows, 31224 columns and 214824 nonzeros
Model fingerprint: 0x4f0ff455
Variable types: 15612 continuous, 15612 integer (0 binary)
Coefficient statistics:
  Matrix range     [1e-01, 2e+04]
  Objective range  [8e+00, 2e+03]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 7e+02]
Presolve removed 612 rows and 0 columns
Presolve time: 0.62s
Presolved: 31330 rows, 31224 columns, 195342 nonzeros
Variable types: 15612 continuous, 15612 integer (15612 binary)

Deterministic concurrent LP optimizer: primal and dual simplex
```

```
Showing first log only...


Use crossover to convert LP symmetric solution to basic solution...
Concurrent spin time: 0.05s

Solved with dual simplex

Root relaxation: objective 3.225806e+03, 1631 iterations, 0.67 seconds (0.25 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 3225.80645    0  492          - 3225.80645      -     -   19s
     0     0 6076.92308    0  456          - 6076.92308      -     -   51s
H    0     0                    10000.000000 6076.92308  39.2%     -   54s
     0     0 6683.57308    0  477 10000.0000 6683.57308  33.2%     -   60s
     0     0 6683.57308    0  476 10000.0000 6683.57308  33.2%     -   61s
     0     0 8606.65000    0  526 10000.0000 8606.65000  13.9%     -   74s

Cutting planes:
  Gomory: 1
  Cover: 18
  Implied bound: 376
  Clique: 8
  MIR: 78
  Flow cover: 76
  Network: 2
  RLT: 4
  Relax-and-lift: 23
  BQP: 3
```

```
59   PSD: 3
60
61 Explored 1 nodes (53545 simplex iterations) in 74.56
   seconds (30.72 work units)
62 Thread count was 8 (of 8 available processors)
63
64 Solution count 1: 10000
65
66 Optimal solution found (tolerance 2.00e-01)
67 Best objective 1.000000000000e+04, best bound 8.
   606650000000e+03, gap 13.9335%
68 Gurobi status= 2
69 Optimal
70 10000.0
71  route 0 _ 46 _ bolero_1 :1.0
72  flow 0 _ 46 _ bolero_1  : 1065.3899999946373
73 ____________________________________________________
   ____________
74  route 14 _ 21 _ bolero_1 :1.0
75  flow 14 _ 21 _ bolero_1  : 513.6799999998684
76 ____________________________________________________
   ____________
77  route 16 _ 50 _ bolero_1 :1.0
78  flow 16 _ 50 _ bolero_1  : 208.0199999999279
79 ____________________________________________________
   ____________
80  route 21 _ 16 _ bolero_1 :1.0
81  flow 21 _ 16 _ bolero_1  : 256.1099999998684
82 ____________________________________________________
   ____________
83  route 31 _ 36 _ bolero_1 :1.0
84  flow 31 _ 36 _ bolero_1  : 663.9999999969143
85 ____________________________________________________
   ____________
86  route 36 _ 14 _ bolero_1 :1.0
87  flow 36 _ 14 _ bolero_1  : 618.6399999979017
88 ____________________________________________________
   ____________
89  route 46 _ 48 _ bolero_1 :1.0
90  flow 46 _ 48 _ bolero_1  : 811.8799999946373
91 ____________________________________________________
```

```
 91 ____________
 92  route 48 _ 31 _ bolero_1 :1.0
 93  flow 48 _ 31 _ bolero_1  : 730.689999994725
 94 ____________________________________________________
    _____________
 95  route 50 _ 0 _ bolero_1 :1.0
 96  flow 50 _ 0 _ bolero_1  : 0.0
 97 ____________________________________________________
    _____________
 98  route 0 _ 12 _ bolero_2 :1.0
 99  flow 0 _ 12 _ bolero_2  : 1949.3699999977853
100 ____________________________________________________
    _____________
101  route 4 _ 38 _ bolero_2 :1.0
102  flow 4 _ 38 _ bolero_2  : 986.9599999983275
103 ____________________________________________________
    _____________
104  route 10 _ 11 _ bolero_2 :1.0
105  flow 10 _ 11 _ bolero_2  : 1199.7399999982506
106 ____________________________________________________
    _____________
107  route 11 _ 4 _ bolero_2 :1.0
108  flow 11 _ 4 _ bolero_2  : 1003.2099999982506
109 ____________________________________________________
    _____________
110  route 12 _ 30 _ bolero_2 :1.0
111  flow 12 _ 30 _ bolero_2  : 1837.9599999977852
112 ____________________________________________________
    _____________
113  route 15 _ 0 _ bolero_2 :1.0
114  flow 15 _ 0 _ bolero_2  : 0.0
115 ____________________________________________________
    _____________
116  route 27 _ 15 _ bolero_2 :1.0
117  flow 27 _ 15 _ bolero_2  : 475.53999999990447
118 ____________________________________________________
    _____________
119  route 30 _ 10 _ bolero_2 :1.0
120  flow 30 _ 10 _ bolero_2  : 1601.5599999982505
121 ____________________________________________________
    ____________
```

```
route 38 _ 27 _ bolero_2 :1.0
flow 38 _ 27 _ bolero_2  : 539.1999999988457
_____________________________________________________
route 0 _ 25 _ bolero_3 :1.0
flow 0 _ 25 _ bolero_3  : 1782.4899999548684
_____________________________________________________
route 18 _ 28 _ bolero_3 :1.0
flow 18 _ 28 _ bolero_3  : 24.62
_____________________________________________________
route 19 _ 18 _ bolero_3 :1.0
flow 19 _ 18 _ bolero_3  : 506.79999996072684
_____________________________________________________
route 25 _ 39 _ bolero_3 :1.0
flow 25 _ 39 _ bolero_3  : 1407.2299999556908
_____________________________________________________
route 28 _ 0 _ bolero_3 :1.0
flow 28 _ 0 _ bolero_3  : 0.0
_____________________________________________________
route 29 _ 19 _ bolero_3 :1.0
flow 29 _ 19 _ bolero_3  : 574.5499999575482
_____________________________________________________
route 37 _ 43 _ bolero_3 :1.0
flow 37 _ 43 _ bolero_3  : 1008.6599999562403
_____________________________________________________
route 39 _ 37 _ bolero_3 :1.0
flow 39 _ 37 _ bolero_3  : 1384.269999955759
_____________________________________________________
route 43 _ 29 _ bolero_3 :1.0
flow 43 _ 29 _ bolero_3  : 657.7599999568934
_____________________________________________________
route 0 _ 8 _ bolero_4 :1.0
```

```
153  flow 0 _ 8 _ bolero_4  : 1907.11999998973
154 ____________________________________________________
    _____________
155  route 3 _ 47 _ bolero_4 :1.0
156  flow 3 _ 47 _ bolero_4  : 1115.1199999938842
157 ____________________________________________________
    _____________
158  route 7 _ 33 _ bolero_4 :1.0
159  flow 7 _ 33 _ bolero_4  : 440.0799999954177
160 ____________________________________________________
    _____________
161  route 8 _ 13 _ bolero_4 :1.0
162  flow 8 _ 13 _ bolero_4  : 1904.1899999902305
163 ____________________________________________________
    _____________
164  route 9 _ 17 _ bolero_4 :1.0
165  flow 9 _ 17 _ bolero_4  : 1687.009999992518
166 ____________________________________________________
    _____________
167  route 13 _ 9 _ bolero_4 :1.0
168  flow 13 _ 9 _ bolero_4  : 1850.8499999913627
169 ____________________________________________________
    _____________
170  route 17 _ 45 _ bolero_4 :1.0
171  flow 17 _ 45 _ bolero_4  : 1677.7099999937225
172 ____________________________________________________
    _____________
173  route 26 _ 7 _ bolero_4 :1.0
174  flow 26 _ 7 _ bolero_4  : 442.8799999947363
175 ____________________________________________________
    _____________
176  route 33 _ 49 _ bolero_4 :1.0
177  flow 33 _ 49 _ bolero_4  : 435.37999999757176
178 ____________________________________________________
    _____________
179  route 41 _ 3 _ bolero_4 :1.0
180  flow 41 _ 3 _ bolero_4  : 1119.2999999937224
181 ____________________________________________________
    _____________
182  route 42 _ 0 _ bolero_4 :1.0
183  flow 42 _ 0 _ bolero_4  : 0.0
```

```
184 ____________________________________________________
    ____________
185  route 45 _ 41 _ bolero_4 :1.0
186  flow 45 _ 41 _ bolero_4  : 1201.5999999937223
187 ____________________________________________________
    ____________
188  route 47 _ 26 _ bolero_4 :1.0
189  flow 47 _ 26 _ bolero_4  : 691.3399999941078
190 ____________________________________________________
    ____________
191  route 49 _ 42 _ bolero_4 :1.0
192  flow 49 _ 42 _ bolero_4  : 315.6399999987825
193 ____________________________________________________
    ____________
194  route 0 _ 2 _ bolero_6 :1.0
195  flow 0 _ 2 _ bolero_6  : 1902.2799999177246
196 ____________________________________________________
    ____________
197  route 1 _ 24 _ bolero_6 :1.0
198  flow 1 _ 24 _ bolero_6  : 1115.6199999191679
199 ____________________________________________________
    ____________
200  route 2 _ 5 _ bolero_6 :1.0
201  flow 2 _ 5 _ bolero_6  : 1821.6799999177247
202 ____________________________________________________
    ____________
203  route 5 _ 20 _ bolero_6 :1.0
204  flow 5 _ 20 _ bolero_6  : 1684.8999999190287
205 ____________________________________________________
    ____________
206  route 6 _ 1 _ bolero_6 :1.0
207  flow 6 _ 1 _ bolero_6  : 1120.4299999191678
208 ____________________________________________________
    ____________
209  route 20 _ 23 _ bolero_6 :1.0
210  flow 20 _ 23 _ bolero_6  : 1561.9399999191678
211 ____________________________________________________
    ____________
212  route 22 _ 0 _ bolero_6 :1.0
213  flow 22 _ 0 _ bolero_6  : -7.455917436102366e-08
214 ____________________________________________________
```

```
214 _____________
215  route 23 _ 34 _ bolero_6 :1.0
216  flow 23 _ 34 _ bolero_6  : 1400.089999919168
217 ____________________________________________________
    _____________
218  route 24 _ 35 _ bolero_6 :1.0
219  flow 24 _ 35 _ bolero_6  : 1110.9799999191678
220 ____________________________________________________
    _____________
221  route 32 _ 22 _ bolero_6 :1.0
222  flow 32 _ 22 _ bolero_6  : 402.7199999254242
223 ____________________________________________________
    _____________
224  route 34 _ 6 _ bolero_6 :1.0
225  flow 34 _ 6 _ bolero_6  : 1129.299999919168
226 ____________________________________________________
    _____________
227  route 35 _ 44 _ bolero_6 :1.0
228  flow 35 _ 44 _ bolero_6  : 692.8199999191678
229 ____________________________________________________
    _____________
230  route 40 _ 32 _ bolero_6 :1.0
231  flow 40 _ 32 _ bolero_6  : 610.3999999205682
232 ____________________________________________________
    _____________
233  route 44 _ 40 _ bolero_6 :1.0
234  flow 44 _ 40 _ bolero_6  : 634.2999999204242
235 ____________________________________________________
    _____________
236
237 Process finished with exit code 0
238
```

CVRP Problem Solution containing 250 Nodes and 45 Vehicles using Google OR Tools.

File - main

```
"C:\Users\Rishi Mehdiratta\PycharmProjects\pythonProject10\venv\Scripts\python.exe
" "C:/Users/Rishi Mehdiratta/PycharmProjects/pythonProject10/main.py"
Objective: 1218929
Route for vehicle 0:
 0 Load(0) ->  173 Load(267.96) ->  0 Load(267.96)
Distance of the route: 10402m
Load of the route: 267.96

Route for vehicle 1:
 0 Load(0) ->  136 Load(169.78) ->  232 Load(614.97) ->  99 Load(711.1800000000001
) ->  73 Load(777.1400000000001) ->  142 Load(968.45) ->  180 Load(1253.28) ->  185
Load(1557.83) ->  164 Load(1806.08) ->  67 Load(1864.3799999999999) ->  0 Load(1864.
3799999999999)
Distance of the route: 86238m
Load of the route: 1864.3799999999999

Route for vehicle 2:
 0 Load(0) ->  229 Load(442.96) ->  169 Load(700.53) ->  97 Load(792.9399999999999
) ->  163 Load(1040.87) ->  157 Load(1275.29) ->  168 Load(1528.8) ->  196 Load(1879
.6999999999998) ->  78 Load(1949.4399999999998) ->  0 Load(1949.4399999999998)
Distance of the route: 68728m
Load of the route: 1949.4399999999998

Route for vehicle 3:
 0 Load(0) ->  179 Load(277.29) ->  198 Load(635.75) ->  235 Load(1083.51) ->  214
Load(1482.3899999999999) ->  188 Load(1793.31) ->  147 Load(1994.9299999999998) ->
0 Load(1994.9299999999998)
```

```
20 Distance of the route: 46753m
21 Load of the route: 1994.9299999999998
22
23 Route for vehicle 4:
24  0 Load(0) ->  117 Load(120.59) ->  141 Load(311.73) ->  218 Load(716.59) ->  236
   Load(1168.42) ->  206 Load(1554.6100000000001) ->  161 Load(1794.88) ->  66 Load(
   1851.8300000000002) ->  101 Load(1948.9800000000002) ->  63 Load(2003.6100000000004
   ) ->  0 Load(2003.6100000000004)
25 Distance of the route: 68708m
26 Load of the route: 2003.6100000000004
27
28 Route for vehicle 5:
29  0 Load(0) ->  64 Load(55.39) ->  113 Load(166.8) ->  250 Load(664.7) ->  68 Load(
   723.22) ->  69 Load(783.76) ->  166 Load(1033.13) ->  249 Load(1522.43) ->  170 Load
   (1781.47) ->  110 Load(1886.43) ->  94 Load(1977.0900000000001) ->  0 Load(1977.
   0900000000001)
30 Distance of the route: 67703m
31 Load of the route: 1977.0900000000001
32
33 Route for vehicle 6:
34  0 Load(0) ->  202 Load(376.45) ->  200 Load(752.06) ->  127 Load(897.18) ->  212
   Load(1290.73) ->  80 Load(1366.05) ->  151 Load(1580.4099999999999) ->  124 Load(
   1721.4099999999999) ->  98 Load(1814.55) ->  140 Load(1999.83) ->  0 Load(1999.83)
35 Distance of the route: 57246m
36 Load of the route: 1999.83
37
38 Route for vehicle 7:
```

```
 0 Load(0) ->  162 Load(247.21) ->  246 Load(732.5600000000001) ->  225 Load(1160.31
) ->  139 Load(1344.56) ->  155 Load(1576.26) ->  187 Load(1886.83) ->  84 Load(1967
.4299999999998) ->  0 Load(1967.4299999999998)
Distance of the route: 42503m
Load of the route: 1967.4299999999998

Route for vehicle 8:
 0 Load(0) ->  223 Load(418.16) ->  239 Load(889.04) ->  207 Load(1279.04) ->  199
Load(1654.3) ->  184 Load(1950.49) ->  0 Load(1950.49)
Distance of the route: 29311m
Load of the route: 1950.49

Route for vehicle 9:
 0 Load(0) ->  181 Load(288.89) ->  248 Load(777.97) ->  231 Load(1223.0900000000001
) ->  244 Load(1702.13) ->  76 Load(1769.39) ->  108 Load(1872.8400000000001) ->
111 Load(1978.7700000000002) ->  0 Load(1978.7700000000002)
Distance of the route: 40552m
Load of the route: 1978.7700000000002

Route for vehicle 10:
 0 Load(0) ->  182 Load(292.31) ->  138 Load(476.36) ->  104 Load(574.8) ->  100
Load(671.0799999999999) ->  86 Load(753.3799999999999) ->  83 Load(833.4199999999998
) ->  125 Load(977.4699999999998) ->  0 Load(977.4699999999998)
Distance of the route: 38975m
Load of the route: 977.4699999999998

Route for vehicle 11:
```

```
59  0 Load(0) ->  189 Load(315.64) ->  178 Load(591.19) ->  93 Load(680.4200000000001
   ) ->  88 Load(765.4000000000001) ->  144 Load(961.9300000000001) ->  0 Load(961.
   9300000000001)
60 Distance of the route: 26832m
61 Load of the route: 961.9300000000001
62
63 Route for vehicle 12:
64  0 Load(0) ->  230 Load(444.61) ->  77 Load(512.36) ->  58 Load(562.04) ->  156 Load
   (794.0) ->  132 Load(956.62) ->  54 Load(999.5) ->  0 Load(999.5)
65 Distance of the route: 31919m
66 Load of the route: 999.5
67
68 Route for vehicle 13:
69  0 Load(0) ->  245 Load(482.18) ->  89 Load(568.25) ->  219 Load(974.29) ->  0 Load(
   974.29)
70 Distance of the route: 15110m
71 Load of the route: 974.29
72
73 Route for vehicle 14:
74  0 Load(0) ->  233 Load(447.41) ->  237 Load(902.53) ->  71 Load(967.63) ->  0 Load(
   967.63)
75 Distance of the route: 14675m
76 Load of the route: 967.63
77
78 Route for vehicle 15:
79  0 Load(0) ->  221 Load(407.36) ->  183 Load(700.5) ->  146 Load(897.6) ->  81 Load(
   973.86) ->  48 Load(1000.24) ->  0 Load(1000.24)
```

```
 80 Distance of the route: 26738m
 81 Load of the route: 1000.24
 82
 83 Route for vehicle 16:
 84  0 Load(0) ->  153 Load(220.62) ->  204 Load(602.04) ->  190 Load(922.1199999999999
    ) ->  75 Load(988.81) ->  0 Load(988.81)
 85 Distance of the route: 18917m
 86 Load of the route: 988.81
 87
 88 Route for vehicle 17:
 89  0 Load(0) ->  226 Load(428.67) ->  210 Load(820.47) ->  119 Load(944.73) ->  57
    Load(992.82) ->  0 Load(992.82)
 90 Distance of the route: 19054m
 91 Load of the route: 992.82
 92
 93 Route for vehicle 18:
 94  0 Load(0) ->  143 Load(194.19) ->  145 Load(391.2) ->  209 Load(781.87) ->  87
    Load(865.08) ->  114 Load(980.96) ->  0 Load(980.96)
 95 Distance of the route: 22785m
 96 Load of the route: 980.96
 97
 98 Route for vehicle 19:
 99  0 Load(0) ->  224 Load(423.78) ->  238 Load(881.75) ->  56 Load(928.41) ->  70
    Load(992.0699999999999) ->  31 Load(1001.3699999999999) ->  0 Load(1001.
    3699999999999)
100 Distance of the route: 26580m
101 Load of the route: 1001.3699999999999
```

```
102
103 Route for vehicle 20:
104  0 Load(0) ->  222 Load(417.74) ->  192 Load(749.53) ->  121 Load(875.48) ->  92
    Load(963.25) ->  53 Load(1001.91) ->  0 Load(1001.91)
105 Distance of the route: 23647m
106 Load of the route: 1001.91
107
108 Route for vehicle 21:
109  0 Load(0) ->  197 Load(352.85) ->  203 Load(733.78) ->  160 Load(972.5799999999999
    ) ->  50 Load(1002.0899999999999) ->  0 Load(1002.0899999999999)
110 Distance of the route: 18929m
111 Load of the route: 1002.0899999999999
112
113 Route for vehicle 22:
114  0 Load(0) ->  172 Load(267.06) ->  171 Load(532.56) ->  118 Load(655.52) ->  128
    Load(805.04) ->  122 Load(931.28) ->  61 Load(984.62) ->  37 Load(1000.44) ->  0
    Load(1000.44)
115 Distance of the route: 32509m
116 Load of the route: 1000.44
117
118 Route for vehicle 23:
119  0 Load(0) ->  126 Load(145.04) ->  72 Load(210.74) ->  150 Load(418.76) ->  95
    Load(509.95) ->  112 Load(616.38) ->  186 Load(922.48) ->  82 Load(1000.78) ->  0
    Load(1000.78)
120 Distance of the route: 26856m
121 Load of the route: 1000.78
122
```

```
123 Route for vehicle 24:
124  0 Load(0) ->  211 Load(393.43) ->  242 Load(870.6800000000001) ->  120 Load(995.7
    ) ->  0 Load(995.7)
125 Distance of the route: 10358m
126 Load of the route: 995.7
127
128 Route for vehicle 25:
129  0 Load(0) ->  174 Load(268.17) ->  234 Load(715.71) ->  90 Load(802.0) ->  135
    Load(970.6) ->  51 Load(1000.87) ->  0 Load(1000.87)
130 Distance of the route: 17904m
131 Load of the route: 1000.87
132
133 Route for vehicle 26:
134  0 Load(0) ->  208 Load(390.42) ->  148 Load(598.1) ->  134 Load(766.6) ->  116
    Load(886.34) ->  91 Load(973.73) ->  47 Load(1000.0500000000001) ->  0 Load(1000.
    0500000000001)
135 Distance of the route: 20562m
136 Load of the route: 1000.0500000000001
137
138 Route for vehicle 27:
139  0 Load(0) ->  194 Load(342.66) ->  205 Load(727.72) ->  165 Load(976.1800000000001
    ) ->  44 Load(1000.72) ->  0 Load(1000.72)
140 Distance of the route: 16674m
141 Load of the route: 1000.72
142
143 Route for vehicle 28:
144  0 Load(0) ->  216 Load(401.94) ->  130 Load(562.96) ->  201 Load(938.69) ->  65
```

```
144 Load(994.7) ->  0 Load(994.7)
145 Distance of the route: 12679m
146 Load of the route: 994.7
147
148 Route for vehicle 29:
149  0 Load(0) ->  191 Load(320.38) ->  149 Load(528.26) ->  228 Load(966.4200000000001
    ) ->  42 Load(989.3800000000001) ->  33 Load(999.3800000000001) ->  0 Load(999.
    3800000000001)
150 Distance of the route: 19312m
151 Load of the route: 999.3800000000001
152
153 Route for vehicle 30:
154  0 Load(0) ->  243 Load(477.91) ->  107 Load(579.9100000000001) ->  36 Load(595.
    5600000000001) ->  19 Load(599.74) ->  0 Load(599.74)
155 Distance of the route: 22478m
156 Load of the route: 599.74
157
158 Route for vehicle 31:
159  0 Load(0) ->  167 Load(253.32) ->  195 Load(599.78) ->  0 Load(599.78)
160 Distance of the route: 5024m
161 Load of the route: 599.78
162
163 Route for vehicle 32:
164  0 Load(0) ->  240 Load(475.54) ->  115 Load(594.36) ->  22 Load(598.74) ->  16
    Load(602.29) ->  0 Load(602.29)
165 Distance of the route: 24256m
166 Load of the route: 602.29
```

```
167
168 Route for vehicle 33:
169  0 Load(0) ->  217 Load(402.72) ->  96 Load(494.6) ->  105 Load(593.33) ->  32 Load
    (602.87) ->  0 Load(602.87)
170 Distance of the route: 14396m
171 Load of the route: 602.87
172
173 Route for vehicle 34:
174  0 Load(0) ->  158 Load(234.79) ->  159 Load(471.19) ->  60 Load(523.41) ->  55
    Load(568.77) ->  40 Load(587.49) ->  34 Load(600.76) ->  0 Load(600.76)
175 Distance of the route: 20022m
176 Load of the route: 600.76
177
178 Route for vehicle 35:
179  0 Load(0) ->  220 Load(407.29) ->  137 Load(580.05) ->  41 Load(601.9) ->  0 Load(
    601.9)
180 Distance of the route: 8325m
181 Load of the route: 601.9
182
183 Route for vehicle 36:
184  0 Load(0) ->  213 Load(398.63) ->  102 Load(496.44) ->  103 Load(594.46) ->  27
    Load(601.24) ->  0 Load(601.24)
185 Distance of the route: 14119m
186 Load of the route: 601.24
187
188 Route for vehicle 37:
189  0 Load(0) ->  177 Load(272.81) ->  123 Load(409.59000000000003) ->  109 Load(513.
```

```
189 51) ->  79 Load(586.71) ->  38 Load(602.96) ->  0 Load(602.96)
190 Distance of the route: 13519m
191 Load of the route: 602.96
192
193 Route for vehicle 38:
194  0 Load(0) ->  215 Load(401.82) ->  133 Load(565.66) ->  35 Load(579.2099999999999
    ) ->  30 Load(588.42) ->  28 Load(597.14) ->  21 Load(601.38) ->  10 Load(604.18
    ) ->  0 Load(604.18)
195 Distance of the route: 29618m
196 Load of the route: 604.18
197
198 Route for vehicle 39:
199  0 Load(0) ->  175 Load(270.79) ->  62 Load(324.83000000000004) ->  131 Load(486.
    68000000000006) ->  52 Load(521.3000000000001) ->  59 Load(571.7900000000001) ->
    46 Load(597.4800000000001) ->  14 Load(600.4300000000002) ->  11 Load(603.
    2700000000002) ->  6 Load(604.9300000000002) ->  0 Load(604.9300000000002)
200 Distance of the route: 27692m
201 Load of the route: 604.9300000000002
202
203 Route for vehicle 40:
204  0 Load(0) ->  154 Load(230.09) ->  193 Load(569.65) ->  49 Load(596.34) ->  17
    Load(600.24) ->  9 Load(602.9300000000001) ->  0 Load(602.9300000000001)
205 Distance of the route: 14073m
206 Load of the route: 602.9300000000001
207
208 Route for vehicle 41:
209  0 Load(0) ->  227 Load(436.13) ->  129 Load(595.71) ->  5 Load(596.8100000000001
```

```
209 ) ->  8 Load(598.8800000000001) ->  13 Load(601.8100000000001) ->  0 Load(601.
    8100000000001)
210 Distance of the route: 15531m
211 Load of the route: 601.8100000000001
212
213 Route for vehicle 42:
214  0 Load(0) ->  247 Load(488.95) ->  43 Load(512.85) ->  85 Load(594.04) ->  12 Load
    (596.91) ->  15 Load(599.93) ->  18 Load(603.8299999999999) ->  0 Load(603.
    8299999999999)
215 Distance of the route: 14072m
216 Load of the route: 603.8299999999999
217
218 Route for vehicle 43:
219  0 Load(0) ->  106 Load(99.08) ->  152 Load(318.44) ->  176 Load(589.31) ->  26
    Load(594.1199999999999) ->  23 Load(598.7599999999999) ->  24 Load(603.
    4599999999999) ->  0 Load(603.4599999999999)
220 Distance of the route: 11193m
221 Load of the route: 603.4599999999999
222
223 Route for vehicle 44:
224  0 Load(0) ->  241 Load(476.11) ->  39 Load(493.45) ->  29 Load(502.32) ->  45 Load
    (526.9399999999999) ->  74 Load(593.0999999999999) ->  25 Load(597.8399999999999
    ) ->  20 Load(602.04) ->  1 Load(602.3299999999999) ->  7 Load(604.2499999999999
    ) ->  3 Load(605.0799999999999) ->  2 Load(605.4999999999999) ->  4 Load(606.
    3999999999999) ->  0 Load(606.3999999999999)
225 Distance of the route: 25452m
226 Load of the route: 606.3999999999999
```

File - main

```
227
228 Total distance of all routes: 1218929m
229 Total load of all routes: 46834.670000000006
230
231 Process finished with exit code 0
232
```

VRPTW Problem Solution containing 50 Nodes and 17 Vehicles using Google OR Tools.

File - main

```
"C:\Users\Rishi Mehdiratta\PycharmProjects\pythonProject8\venv\Scripts\python.exe" "
C:/Users/Rishi Mehdiratta/PycharmProjects/pythonProject8/main.py"
Objective: 8161
Route for vehicle 0:
0 Time(14,14) -> 21 Time(360,360) -> 0 Time(360,360)
Time of the route: 360min

Route for vehicle 1:
0 Time(0,0) -> 3 Time(331,331) -> 0 Time(331,331)
Time of the route: 331min

Route for vehicle 2:
0 Time(0,0) -> 17 Time(289,289) -> 0 Time(289,289)
Time of the route: 289min

Route for vehicle 3:
0 Time(0,0) -> 15 Time(276,276) -> 44 Time(613,613) -> 0 Time(613,613)
Time of the route: 613min

Route for vehicle 4:
0 Time(0,0) -> 34 Time(226,226) -> 7 Time(521,521) -> 0 Time(521,521)
Time of the route: 521min

Route for vehicle 5:
0 Time(21,21) -> 11 Time(246,246) -> 46 Time(540,540) -> 0 Time(540,540)
Time of the route: 540min

```

```
27 Route for vehicle 6:
28 0 Time(144,144) -> 48 Time(360,360) -> 35 Time(615,615) -> 0 Time(615,615)
29 Time of the route: 615min
30
31 Route for vehicle 7:
32 0 Time(0,0) -> 14 Time(196,196) -> 12 Time(394,394) -> 27 Time(616,616) -> 0 Time(
   616,616)
33 Time of the route: 616min
34
35 Route for vehicle 8:
36 0 Time(154,154) -> 19 Time(360,360) -> 30 Time(563,563) -> 0 Time(563,563)
37 Time of the route: 563min
38
39 Route for vehicle 9:
40 0 Time(1,1) -> 16 Time(180,180) -> 2 Time(368,368) -> 38 Time(565,565) -> 0 Time(565
   ,565)
41 Time of the route: 565min
42
43 Route for vehicle 10:
44 0 Time(3,3) -> 10 Time(180,180) -> 45 Time(349,354) -> 29 Time(540,540) -> 0 Time(
   540,540)
45 Time of the route: 540min
46
47 Route for vehicle 11:
48 0 Time(0,0) -> 8 Time(158,158) -> 50 Time(321,321) -> 4 Time(490,490) -> 32 Time(671
   ,671) -> 0 Time(671,671)
49 Time of the route: 671min
```

```
50
51 Route for vehicle 12:
52 0 Time(55,55) -> 13 Time(207,207) -> 49 Time(374,374) -> 43 Time(540,540) -> 42 Time
   (698,698) -> 0 Time(698,698)
53 Time of the route: 698min
54
55 Route for vehicle 13:
56 0 Time(110,110) -> 18 Time(230,230) -> 5 Time(360,360) -> 23 Time(496,496) -> 26
   Time(635,635) -> 0 Time(635,635)
57 Time of the route: 635min
58
59 Route for vehicle 14:
60 0 Time(143,143) -> 9 Time(249,249) -> 22 Time(360,360) -> 20 Time(467,467) -> 39
   Time(604,604) -> 0 Time(604,604)
61 Time of the route: 604min
62
63 Route for vehicle 15:
64 0 Time(206,206) -> 47 Time(273,273) -> 1 Time(360,360) -> 24 Time(453,453) -> 37
   Time(563,563) -> 36 Time(685,685) -> 0 Time(685,685)
65 Time of the route: 685min
66
67 Route for vehicle 16:
68 0 Time(477,477) -> 41 Time(540,540) -> 33 Time(635,635) -> 0 Time(635,635)
69 Time of the route: 635min
70
71 Route for vehicle 17:
72 0 Time(471,471) -> 6 Time(504,504) -> 28 Time(540,540) -> 31 Time(578,578) -> 25
```

File - main

```
72 Time(622,622) -> 40 Time(674,674) -> 0 Time(674,674)
73 Time of the route: 674min
74
75 Total time of all routes: 10155min
76
77 Process finished with exit code 0
78
```

OUTPUT OF CVRP SOLVED FROM GOOGLE OR TOOLS

For each location on a route, the output shows:

- The index of the location.
- The total load carried by the vehicle when it departs the location.

OUTPUT OF VRPTW SOLVED FROM GOOGLE OR TOOLS

For each location on a route, Time(a,b) is the solution window: the vehicle that visits the location must do so in that time interval to stay on schedule.

REFERENCES

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3) https://www.gurobi.com/
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5) https://www.hindawi.com/journals/mpe/2012/104279/