Mathematical Formulation

Sets:

- **G**: Let G be a graph with nodes set N and 0 be the depot.
- **C** = {1,2,3,.....,N} : Set of customers
- **N** = {0,1,2,3,.....,N} : A collection of nodes including a depot.
- **V** = {bolero_1, bolero_2,.......,bolero_10, tata_1, tata_2,........., tata_20,ape_1,ape_2,.....,ape_15,bike_1,bike_2}

Indices:

- $i, j \in$ **N**: Nodes
- 0: Depots
- $v \in$ **V**: Vehicles

Parameters:

- q_j : **demand associated with each customer (i.e. the weight to be delivered).**
- S_j : **service time associated with each customer j.**
- f_v : **free km for vehicle v**
- **A time window [E_j, L_j] where E_j is the earliest time the service can begin and L_j is the latest time for every customer.**
- Q_v : **maximum capacity of vehicle v.**
- FC_v : **Fixed cost of vehicle v.**
- D_{ij} : **Distance between two nodes.**
- max_kms_v : **maximum km vehicle v can travel in a day.**
- max_time_v : **maximum time a vehicle v can travel in a day.**
- $speed_v$: **uniform speed of different vehicles v.**
- T_{ij}^{v}: **Travel time between two nodes =** D_{ij} / $speed_v$.
- VC_v : **This is the variable cost for vehicle type v.**
- $maxcustomer_v$: **maximum number of customers served by v vehicles.**
- $maxdemand_v$: **maximum demand of customers served by v vehicles.**
- $mindemand_v$: **minimum demand of customers served by v vehicles.**

Decision Variables:

- **A decision variable that decides if vehicle v has taken route $i -> j$ or not.**

$$x_{ij}^{v} = \begin{Bmatrix} 1, if \ \ vehicle\ v\ has\ taken\ the\ route\ i->j \\ 0, otherwise \end{Bmatrix}$$

- $extrakms_v$: **A decision variable that calculates extra km that vehicle v will travel in its route.**

($extrakms_v \in R^+$**)** ***(low_bound = 0)***

- d_v : **total distance traveled by vehicle v** = $\sum_{i \in N} \sum_{j \in N} (D_{ij} * x_{ij}^{v}) \ \ \forall v \in V$

($d_v \in R^+$**)**

- **A decision variable that indicates whether the vehicle v is used or not.**

$$u_v = \begin{Bmatrix} 1, if\ vehicle\ v\ is\ in\ use \\ 0, otherwise \end{Bmatrix}$$

- **A decision variable that indicates whether vehicle v visited customer i or not.**

$$y_{i}^{v} = \begin{Bmatrix} 1, if\ vehicle\ v\ visits\ node\ \ i \\ 0, otherwise \end{Bmatrix}$$

- t_i^v: **time of vehicle v to visit node i** **(**$t_i^v \in R^+$**)**
- f_{ij}^{v}**: number of units of product taken by vehicle v from node i to j**

($f_{ij}^{v} \in R^+$**)**

Objective Function:

Our objective is to minimize the sum of the vehicle acquisition costs and routing costs.

Minimize (z) : $\sum_{v \in V} VC_v * \ extra\ kms_v \ + \ \sum_{v \in V} FC_v \ * \ u_v$

Constraints:

(1) **This constraint says that when the value of cumulative distance traveled by vehicle v is negative, the value of $extrakms_v$ becomes 0, otherwise the value of $extrakms_v$ is equal to the cumulative distance traveled by vehicle v.**

$$extrakms_v \geq \ d_v - f_v \qquad \forall\ v \in V$$

(2) **This constraint is a big M constraint which tells that if any vehicle v is in use then it has at least visited one node and if it is not in use, then the value of LHS becomes 0 because x_{ij}^v is a binary variable and has its lower bound zero.**

$$\sum_{i \in N} \sum_{j \in N} x_{ij}^v \leq M(u_v) \qquad \forall\ v \in V$$

(3) **This constraint says that each customer in C is visited exactly once by any vehicle v.**

$$\sum_{v \in V} y_j^v = 1 \qquad \forall j \ \in \ C$$

(4) **This constraint is a flow balancing constraint which tells that the number of vehicles coming in and out of a customer's location is the same and it can only be by only one vehicle v.**

$$\sum_{i \in N} \ x_{ij}^v = \sum_{i \in N} \ x_{ji}^v = y_j^v \qquad \forall\ j \in N, \forall\ v \in V$$

(5) **This constraint says that every customer j demand must be bounded between the minimum and maximum demand, any vehicle v can serve.**

$$y_j^v mindemand_v \leq q_j y_j^v \leq y_j^v maxdemand_v \qquad \forall\, j \in C, \forall\, v \in V$$

(6) **This constraint says that the number of customers per vehicle v must be less than the maximum number of customers it can serve.**

$$\sum_{j \in C} y_j^v \leq maxcustomer_v + 1 \qquad \forall v \in V$$

7) **This constraint says that the total number of units taken by vehicle v between two nodes = demand of the node i.**

$$\sum_{v \in V} \sum_{j \in N} f_{ij}^v - \sum_{v \in V} \sum_{j \in N} f_{ji}^v = q_i \qquad \forall i \in C$$

8) **This constraint says that the flow of units between two nodes must be less than the maximum weight a vehicle v can take between two nodes.**

$$0 <= f_{ij}^v <= Q_v x_{ij}^v \qquad \forall\, i,\, j \in N, \forall\, v \in V$$

(9) **Time Window Constraints.**

$$t_i^v + S_i + T_{ij}^v \leq t_j^v + M(1 - x_{ij}^v) \qquad \forall i \in N,\, j \in C\,,\, v \in V$$

(10) **This constraint says that the start time of any vehicle must lie between the time windows of each customer j.**

$$E_j y_j^v \leq t_j^v \leq L_j y_j^v \qquad \forall\, j \in C\,,\, v \in V$$

(11) **Upper bounds of variables.**

$$d_v \leq max\_kms_v \qquad \forall v \in V$$