Magnetostatics

Basic Concepts in Magnetism:

Magnetic field Intensity or Intensity of magnetization or Magnetic field Strength (H): Magnetic field Intensity at any point is the force experienced by a unit north pole at that point.

Magnetic moment (m): Magnetic moment m of a magnet is the product of magnetic pole strength and the distance between the two poles.

Magnetization (M): The magnetic moment/unit volume of a magnet is called magnetization. $M = \frac{m}{V}$

Magnetic Susceptibility (χ): Magnetic Susceptibility is the ratio of magnetization to the strength of the field or magnetic field intensity $\chi = \frac{M}{H}$

Magnetic Flux (ϕ): It is the measure of the strength of a magnetic field over a given area. The flux lines will be from North Pole to South Pole.

Magnetic Induction or Magnetic flux density (B):

The magnetic flux density or the magnetic induction is the magnetic flux over a unit area of a surface held normally to the flux. $B = \frac{\phi}{A} Tesla$

Permeability (μ):

The permeability of medium is defined as the ability of the medium to permit magnetic flux to permeate through itself, it given by the product of relative permeability and the permeability of free space. $\mu = \mu_o \mu_r H / m$ .For vaccum $\mu_r = 1$

The permeability of vacuum or free space is taken as the standard reference with respect to which permeabilities of other materials are expressed. The permeability of vaccum is given by $\mu_o = 4\pi \times 10_{-7} H / m$

Relation between B and H: H is related to B through the equation

$$B = \mu H$$
$$B = \mu_o \mu_r H$$
$$for vaccum \mu_r = 1$$
$$B = \mu_o H -----(1)$$

Relation between B, M and H:

Eqn (1) is the magnetic flux density for the flux set up by a magnetic field in a region where there is vaccum. Into this region if we bring a material medium, then it develops a magnetic moment in the influence of magnetic filed. Due to this magnetic moment extra flux lines will be set up inside the material medium.

The magnetic flux density due to extra flux in the medium is given by the product $\mu_o M$

$$\therefore B = \mu_o (H + M)$$

Where H is the magnetic field intensity, M is the Magnetization and μ_o is the permeability of free space.

Magnetostatics is the study of magnetic fields in systems where the currents are steady (not changing with time). It is the magnetic analogue of electrostatics, where the charges are stationary.

Magnetic Field: A region around a magnetic material or a moving electric charge within which the magnetic force acts. Its denoted by symbol B. SI unit is Tesla.

Magnetic Flux Density: it is the measure of the number of magnetic lines of force per unit of cross-sectional area.

Biot-Savarts Law

Two French physicists, Jean Baptiste Biot and Felix Savart derived the mathematical expression for magnetic flux density at a point due a nearby current carrying conductor, in 1820. Viewing the deflection of a magnetic compass needle, two scientists concluded that any current element projects a magnetic field in the space.

Biot Savarts law states that the magnetic field produced due to current carrying conductor is directly proportional to the length of the element dl, the current I, the sine of the angle and θ between direction of the current and the vector joining a given point of the field and the current element and is inversely proportional to the square of the distance of the given point from the current element, r.

$$Hence,\ dB \propto \frac{Idl sin\theta}{r^2} \quad or \quad dB = k\frac{Idl sin\theta}{r^2}$$

Where, k is a constant, depends upon the magnetic properties of the medium. Which is given by

$$k = \frac{\mu_o \mu_r}{4\pi}$$

The final Biot savarts law equation is given by

$$dB = \frac{\mu_o \mu_r}{4\pi} \times \frac{Idl sin\theta}{r^2}$$

Scalar Product and Vector Product

Vectors can be multiplied in two different ways: the scalar and vector product. As the name says, a scalar product of two vectors results in a scalar quantity, and a vector product in a vector quantity.

Scalar Product: The result of this product is a scalar quantity. The scalar product between two vector is denoted by a thick dot:

$\vec{A} \cdot \vec{B} = |\vec{A}|\ |\vec{B}| \cos\theta$ and $\vec{B} \cdot \vec{A} = \vec{A} \cdot \vec{B}$

Vector Product: The result of this product is a vector quantity. The vector product between two vector is denoted by a cross (the product is sometimes also called "cross-product"):

$\vec{A} \times \vec{B} = \vec{C}$ and $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$

The vector $\vec{C}$ is perpendicular to the plane of $\vec{A}$ and $\vec{B}$, and its magnitude is given by:

$|\vec{C}| = |\vec{A}|\ |\vec{B}| \sin\theta$

Operators

Del: it is commonly symbolized by the inverted capital Greek letter delta, is considered as an *operator* with the following definition:

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

Gradient: when **del** is combined with the scalar function *f*, we get Gradient.

$$grad \quad f = \nabla f \qquad \nabla f = \mathbf{i}\frac{\partial f}{\partial x} + \mathbf{j}\frac{\partial f}{\partial y} + \mathbf{k}\frac{\partial f}{\partial z}$$

another way of writing gradient is $< \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} >$

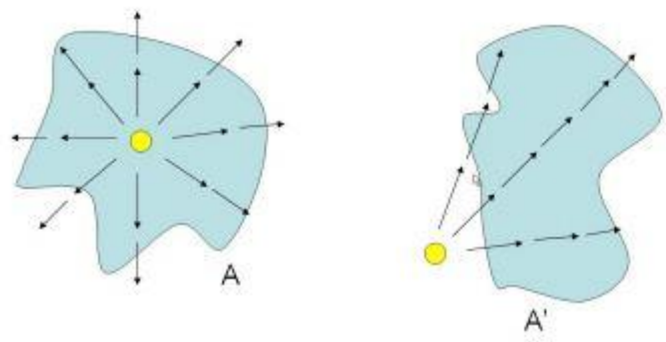
Divergence: it is also known as del dot, a dot product of del with vector. Which is given by

$$\nabla \bullet \mathbf{F} = (\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}) \bullet (\mathbf{i}F_x + \mathbf{j}F_y + \mathbf{k}F_z)$$

Curl: it is also known as del cross, a cross product of del with vector. Which is given by

$$\operatorname{curl}\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

Fundamental theorem for divergences: Gauss theorem.



Left: particle source inside closed surface A. Flux is nonzero.   Right: source outside closed surface. Flux through $A^{|}$ is zero.

Mathematically the divergence of ~v is just $\partial_i v_i = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z$. Consider the volumes inside A and $A^{|}$, $A = \partial V$ and $A^{|} = \partial V^{|}$ (the symbol "∂" here means "the boundary of"). Remembering that we're thinking of v as a current density for now, let's ask ourselves how much flux comes out, passing through the surface. We can divide up the whole volume into infinitesimal cubes dxdydz and consider each cube independently. First imagine that v is constant. Then the surface integral over the cube will give zero, because the flux in one face will be exactly cancelled by the flux out the other face. If v is not a constant, the vectors on opposite faces will not be quite equal, e.g. at x we'll have v_x but at x + dx we'll have $v_x + dv_x$. So the flow rate out of the cube in this direction will be equal to $(\partial v_x/\partial x)dx$. Integrating over the whole cube will give the whole flow rate out:

$$\int_{\text{cube}} \vec{\nabla} \cdot \vec{v}\, dxdydz = \int_{\partial\ \text{cube}} \vec{v} \cdot d\vec{a}.$$

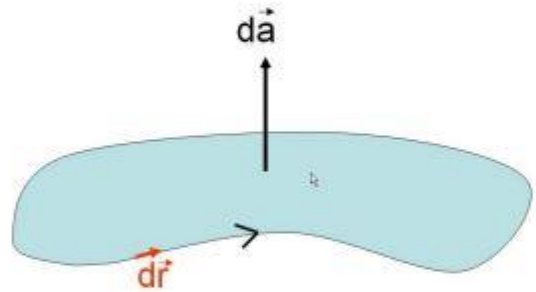
But if we consider any arbitrary volume τ to be a sum of such cubes, the surface integrals of the interior infinitesimal cubes will cancel, because their normals ˆa are oppositely directed. Therefore the total volume integral over τ is similarly related to the total surface integral of ∂τ :

$$\int_{\tau} \vec{\nabla} \cdot \vec{v}\, d\tau = \int_{\partial\tau} \vec{v} \cdot d\vec{a}.$$

This is called the divergence theorem or Gauss's theorem (not Gauss's law!).

Fundamental theorem for curls: Stokes theorem

Stokes theorem states that the surface integral of the curl of a function over any surface bounded by a closed path is equal to the line integral of a particular vector function round that path.



Directed area measure is perpendicular to loop according to right hand rule.

In this case we consider a directed closed path in space, the boundary of a 2D area A.

$$\int_A (\vec{\nabla} \times \vec{v}) \cdot d\vec{a} = \oint_{\partial A} \vec{v} \cdot d\vec{r}.$$

Remarks (Stokes and divergence thm):

Define the normal to the area according to the right hand rule using the sense of the loop, as in the above Fig.

If $\vec{v} = \vec{\nabla} \times \vec{B}$, then

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0 \Rightarrow \oint (\vec{\nabla} \times \vec{A}) \cdot d\vec{a} = 0$$

since

$$\int_V \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) d\tau = \int_{\partial V} (\vec{\nabla} \times \vec{A}) \cdot d\vec{a}$$

If $\vec{v} = \vec{\nabla}\phi$, then $\vec{\nabla} \times \vec{v} = 0$, so $\oint \vec{v} \cdot d\vec{r} = 0$. $\vec{v}$ is conservative.

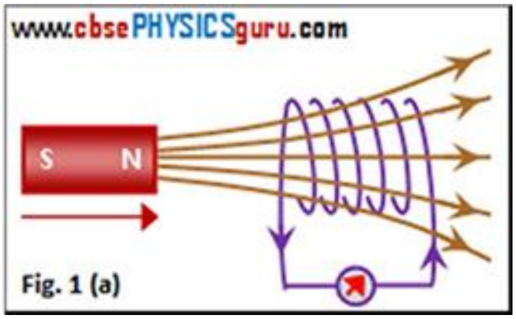
Faradays law: when the magnetic flux linking a circuit changes, an electromotive force is induced in the circuit proportional to the rate of change of the flux linkage.
the rate of change of flux linkage is equal to induced emf.

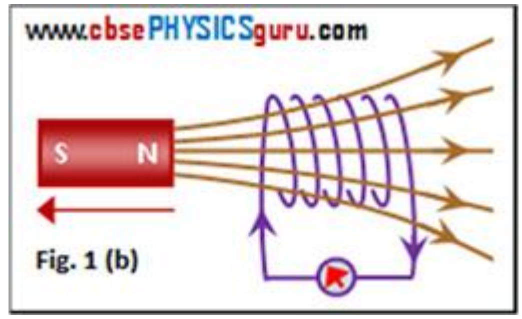
$$E = N\frac{d\phi}{dt}$$

Considering Lenz's Law. $E = -N\frac{d\phi}{dt}$ where Φ in Wb = B.A
B = magnetic field strength A = area of the coil

The Experiments of Faraday and Henry

Magnet and coil Experiment





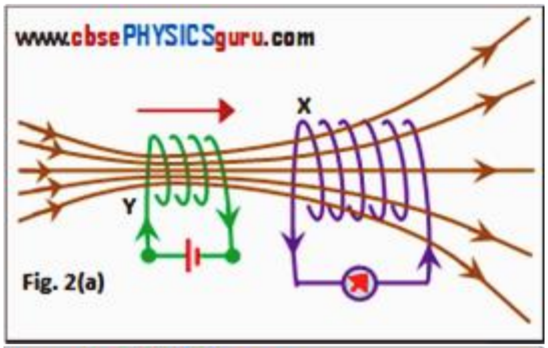
When the North-pole of a bar magnet is moved towards the coil or loop, the galvanometer connected to the coil shows deflection. It confirms the flow of electric current in the coil. It is also noted that the deflection lasts as long as the bar magnet is in motion. No deflection is shown by the galvanometer when the magnet does not move. See figure 1(a). When the North-pole of the bar magnet is moved away from the coil, the galvanometer shows deflection in the opposite direction. It indicates that the current flows in the reverse direction. See figure 1(b).

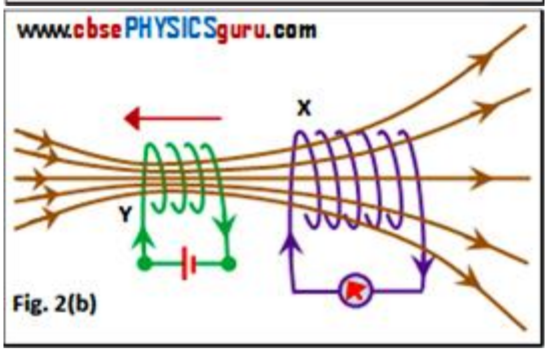
If the same experiment is repeated with the South-pole of the bar magnet, the deflections shown by the galvanometer are just opposite to that observed with the North-pole for similar movements (towards the coil or away from the coil). In other words, if the South-pole is moved towards the coil, we will get the result shown in figure 1(b). If the South-pole is moved away from the coil, we will get the result shown in figure 1(a)

The deflection, and consequently current becomes larger if the magnet is moved towards or away from the coil at a quicker rate.

Now, if the coil is moved towards or away from the stationary magnet, the same results are produced. It establishes that electric current in the coil is induced due to the relative motion between the magnet and the coil. It does not matter which is moving.

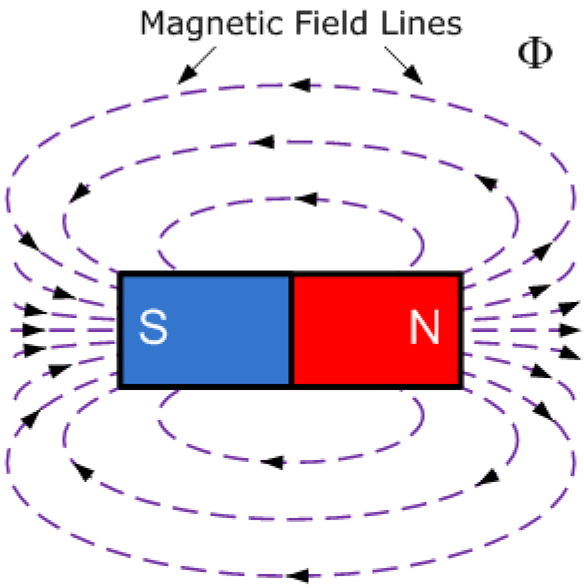
coil and coil Experiment





If a second coil Y carrying a steady current drawn from a battery is moved towards the coil X connected to a galvanometer, then the galvanometer shows a deflection. This confirms that an electric 2 current is induced in coil X. See figure 2(a). The deflection in the galvanometer lasts as long as the coil Y is in motion.
When coil Y is moved away from the coil X, the galvanometer shows a deflection in the opposite direction. See figure 2(b). It is also noted that the deflection lasts as long as the coil Y is in motion. No deflection is shown by the galvanometer when the coil Y does not move.
Now, if the coil X is moved towards or away from the stationary coil Y, the same results are produced. It establishes that electric current in the coil X is induced due to the relative motion between the two coils. It does not matter which is moving.
In both the cases, the deflection, and consequently current becomes larger if a coil is moved towards or away from the other coil at a quicker rate.

Magnetic Field due to Bar magnet

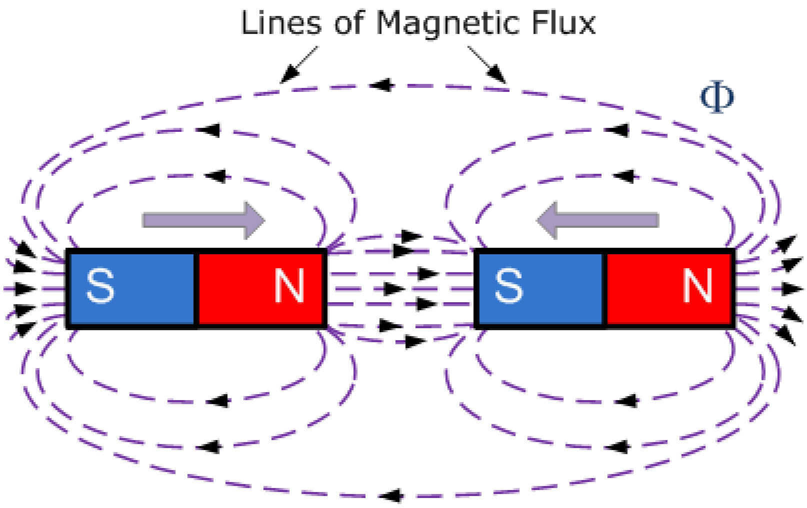


As shown above, the magnetic field is strongest near to the poles of the magnet were the lines of flux are more closely spaced. The general direction for the magnetic flux flow is from the North (N) to the South (S) pole. In addition, these magnetic lines form closed loops that leave at the north pole of the magnet and enter at the south pole. Magnetic poles are always in pairs.
However, magnetic flux does not actually flow from the north to the south pole or flow anywhere for that matter as magnetic flux is a static region around a magnet in which the magnetic force exists. In other words magnetic flux does not flow or move it is just there and is not influenced by gravity. Some important facts emerge when plotting lines of force:

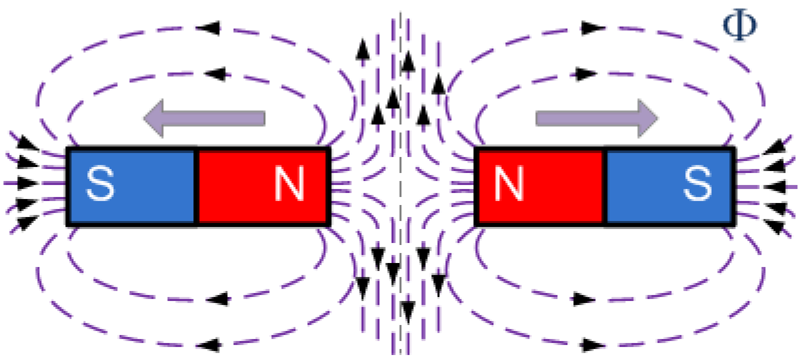
Properties of Field lines.

- Lines of force NEVER cross.
- Lines of force are CONTINUOUS.
- Lines of force always form individual CLOSED LOOPS around the magnet.
- Lines of force have a definite DIRECTION from North to South.
- Lines of force that are close together indicate a STRONG magnetic field.
- Lines of force that are farther apart indicate a WEAK magnetic field.

Magnetic Field of Like and Unlike Poles



Unlike Poles – "Attract"



Like Poles – "Repel"

When plotting magnetic field lines with a compass it will be seen that the lines of force are produced in such a way as to give a definite pole at each end of the magnet where the lines of force leave the North pole and re-enter at the South pole. Magnetism can be destroyed by heating or hammering the magnetic material, but cannot be destroyed or isolated by simply breaking the magnet into two pieces.

So if you take a normal bar magnet and break it into two pieces, you do not have two halves of a magnet but instead each broken piece will somehow have its own North pole and a South pole. If you take one of those pieces and break it into two again, each of the smaller pieces will have a North pole and a South pole and so on. No matter how small the pieces of the magnet become, each piece will still have a North pole and a South pole.

Classification of Diamagnetic, Paramagnetic and ferromagnetic materials

Depending upon the magnitude and sign of response to the applied magnetic field and also on the basis of effect of temperature all magnetic materials are classified into following three categories

1. Diamagnetic Materials

Diamagnetic Materials are those materials which experience a repelling force when brought near the poles of a strong magnet. They acquire a feeble magnetization in a direction opposite to that of the applied field.

Examples: Antimony, bismuth, copper, gold, lead, mercury, silver etc…

Properties:

1. The relative permeability is less than unity
2. The susceptibility is negative
3. The susceptibility does not vary with temperature.
4. When this substance is subjected to non uniform magnetic field they move from stronger region to weaker region of the magnetic field
5. The net magnetic dipole moment is zero.
6. B (magnetic flux density) is a linear function of H (Applied field strength)

2. Paramagnetic Materials:

Paramagnetic Materials are those which experience a feeble attractive force when brought near the poles of a magnet. It possesses a feeble magnetisation along the direction of the applied field

Examples: Aluminium, chromium, manganese chloride, platinum, sodium etc…

Properties

1. Susceptibility is small, positive
2. Permeability is greater than one
3. Paramagnetic susceptibility is dependent on temperature and it is given by $\chi = \frac{C}{T}$ Where C is a constant called curie point.
4. When this substance is subjected to non uniform magnetic field they move from weaker region to stronger region of the magnetic field
5. It acquires the net magnetic dipole moment (of small magnitude)

3. Ferromagnetic materials

The substances which are strongly attracted by a magnet are called ferromagnetic substances. These are the permanent magnets which exhibit hysteresis. They are strongly magnetized along the direction of the applied field and field lines crowd into the material.

Examples: iron, cobalt, nickel and their alloys.

Properties

1. The relative permeability is very high (>>1)
2. The susceptibility is very high
3. The magnetic susceptibility depends on the temperature which is given by, $\chi = \frac{C}{(T - T_c)}$ Where C is the Curie point and T_c is the Curie temperature and $T > T_c$

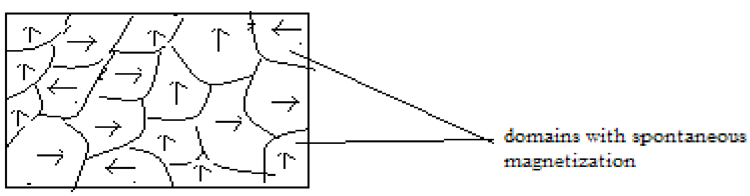
4. M (in turn B) doesn't vary linearly with that of H. If the magnetic induction is plotted against the applied field H, it forms a hysteresis curve

5. It posses a permanent net magnetic dipole moment

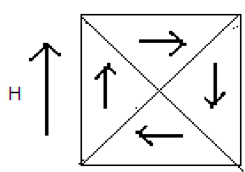
Concept of domains:

The volume of the ferromagnetic specimen is divided into number of regions (as shown in the diagram) which has spontaneous magnetization. These regions are called domains.

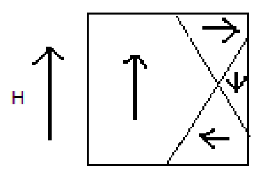
In the absence of the external magnetic field, the relative orientation of magnetic moments of various domains will be completely random. So the net magnetic moment is the vectorial sum of the magnetic moments of the constituent domains, turns out to be zero.



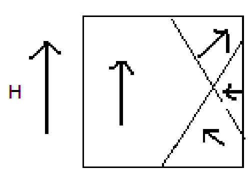
Effect of external magnetic field on the domain:



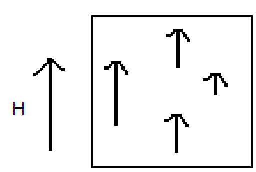
(a) domain orientationin the absence of magnetic field



(b) domain enlargement/shrinkage due to weak field



(c) domain rotation due to strong field



(d) Saturation due to very high field

When an external field is applied, the magnetization effect takes place either by domain wall movement or by rotation of domains. This occurs broadly in four stages depending on the strength of the applied magnetic field.

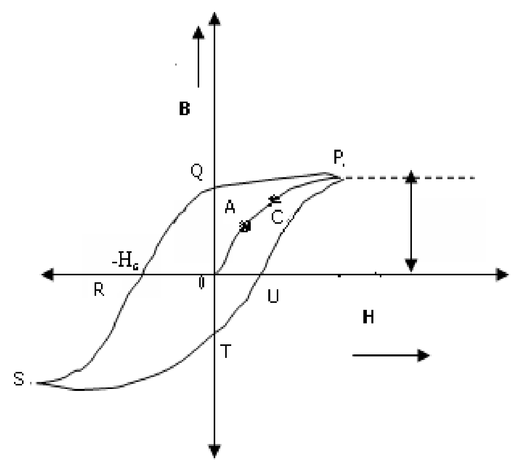
- In the absence of the external magnetic field, the domain orientation is random. So the net magnetic moment of the material is zero(fig a)

- When the field is weak, the domains which have their resultant magnetic moments in direction parallel to the direction of the applied field expand their size and the domains which are not oriented along the direction of magnetic field will shrink as shown in fig(b).
- When the field becomes strong, the domains magnetic moment rotate partially, and tend to align in the direction of the magnetic field (fig c), which results in further increase in the magnetization of the material.

For a very strong field, the magnetic moments of each and every domain align precisely along the direction of the field (fig d), which indicates the stage of magnetization. This state of magnetization is called saturation beyond which further increase in magnetization is impossible.

B H Curve (Hysteresis):

Hysteresis is the phase lag of the magnetic induction B in ferromagnetic materials w.r.t to the cyclic variation of an applied magnetic field H when the temperature of the specimen is below Curie temperature



The hysteresis can be explained as follows.

1. The magnetic field H is increased from zero value in the positive direction. The value of B also increases and takes the path along OP.
2. As H is further increased, B remains constant. This value of B at P is called saturation value and state of specimen is called saturation magnetization.
3. The value of H is then decreased B also starts decreasing but the curve takes the path PQ instead PO.
4. When H becomes zero, there remains certain amount of flux in the material represented by OQ in the diagram called the remanent flux density or remanent induction. The material remains magnetized even in the absence of an external field.

   Remanent induction is the residual induction left in the specimen when the applied magnetic field is zero and this effect is called retentivity.
5. Now the direction of H is reversed and increased. When it attains the value of OR, B becomes zero. This value of H=OR is called Coercive field and the effect is called coercivity. So **Coercive field is the field whose strength is just enough to demagnetize a specimen that has been magnetized to its saturation.**
6. With further increase in H, B increases in the opposite direction and reaches saturation value at S. The field is then decreased gradually to zero, for which the curve traces the path ST.
7. The direction of H is now reversed and increased, the specimen get completely demagnetized once again for H=OU. As H is increased further the curve traces the path UP

The closed path PQRSTUP is called Hysteresis curve and the area under the curve gives the energy loss/unit volume/cycle.

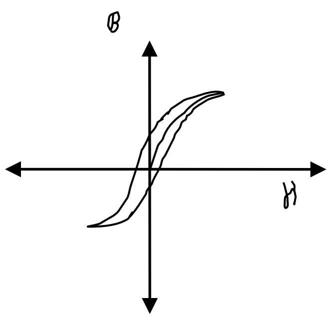
Hysteresis curve is the closed magnetization curve obtained by plotting the magnetic induction B in ferromagnetic materials w.r.t applied magnetic field H which is varied in cycles, when the specimen is at a temperature below its curie temperature.

Hysteresis loss: The energy loss that occurs during magnetization and demagnetization processes which are carried out in cycles on a ferromagnetic specimen is called hysteresis loss.

Soft magnetic materials:

Magnetic materials that are characterized by smaller area of hysteresis loop and lower energy loss are called soft magnetic materials. They cannot be permanently magnetized. It can be magnetized and demagnetized easily as hysteresis loop is small

Ex: silicon-iron alloy, mild Steel, perm alloy, pure nickel, amorphous ferrous alloy



Properties:

1. Low remnant magnetism
2. High permeability
3. High susceptibility
4. Low coercive field
5. Low hysteresis energy loss
6. Thin hysteresis loop

Uses:

1. They are used in the construction of cores of transformers, electric motors, generators and in electromagnets.
2. The magnetic mild steel is used for relays, reed switches and pole pieces for electromagnets.
3. They are used for audio frequency applications. (used in those instruments which work under a.c conditions)

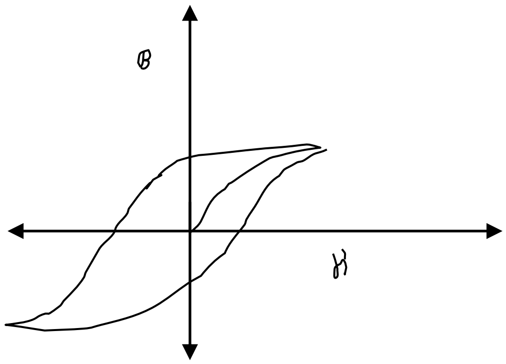
5. Perm alloy and certain ferrites are used as reading or recording heads in tape readers/players

Hard magnetic materials:

Hard magnetic materials are those which are characterized by large hysteresis loop due to which they retain some amount of magnetic energy when the external field is switched off.

They can be neither magnetized nor demagnetized easily

Ex: Alnico alloy, Platinum cobalt alloy, Samarium cobalt alloy, tungsten-steel alloy, alloys of aluminium



Properties:

1. Large hysteresis loop
2. High coercive field
3. High remanent induction
4. High permeability
5. High hysteresis energy loss
6. High saturation magnetization
7. Eddy current loss is more in metallic type and less in ceramic type

Uses:

1. They are used in magnetic detectors.
2. These magnets are used in measuring meters like galvanometers, ammeters, voltmeters, speedometers and recorders.
3. Tungsten steel is used in chucks and latches, tool holders, magnetic bearings and mixers.
4. They are used in audio systems like in speakers and microphones.
5. They can be used as flexible magnets embedded in the plastics like in gaskets of refrigerator doors

- These magnets are used in measuring meters like galvanometers, ammeters, voltmeters, speedometers and recorders.

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