Boolean Algebra and logic gates

→ **Introduction**:

In 1854, Irish logician and mathematician George Boole, developed a mathematical system for formulating logical statements with symbols, the problems could be written and solved in a manner similar to ordinary algebra. Boolean algebra, finds application in the analysis and design of digital systems.

→ **Logical operators**:

To represent and solve arithmetic expressions we use arithmetic operators such as +, −, × and ÷. Similarly we can use logical operators to represent and solve logical expressions.

There are three basic logical operators: NOT / INVERTER, AND and OR.

Inverter / NOT gate

The inverter (NOT circuit) performs a basic logic function called "inversion" or "complementation".

Symbol

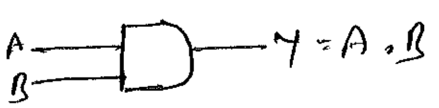
IC 7404

Truth table

I/p	O/p
0	1
1	0

AND gate

Symbol

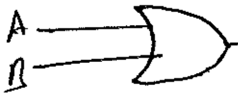


IC 7408

Truth table

I/p's		O/p
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR gate



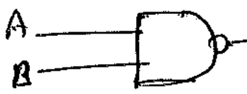
$Y = A + B$

IC 7432

Truth table

I/p's		O/p's
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

NAND gate



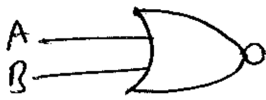
$Y = \overline{A \cdot B}$

IC 7400

Truth table

I/p's		O/p's
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR gate



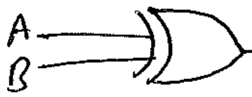
$Y = \overline{A + B}$

IC 7402

Truth table

I/p's		O/p's
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Ex-OR gate



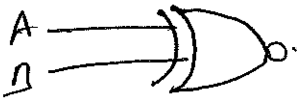
$Y = A \oplus B$

IC 7486

Truth table

I/p's		O/p's
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Ex-NOR gate



$Y = \overline{A \oplus B}$ or $A \odot B$

IC 4077

Truth table

I/p's		O/p's
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

1. MINIMIZATION TECHNIQUES AND LOGIC GATES

→ Boolean algebra :-

Boolean algebra is used to rearrange Boolean Equation to make simple logic circuit.

→ Laws of Boolean algebra :-

* Commutative laws

Truth tables

Law 1 : $A+B = B+A$

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

≡

B	A	B + A
0	0	0
0	1	1
1	0	1
1	1	1

Law 2 : $AB = BA$

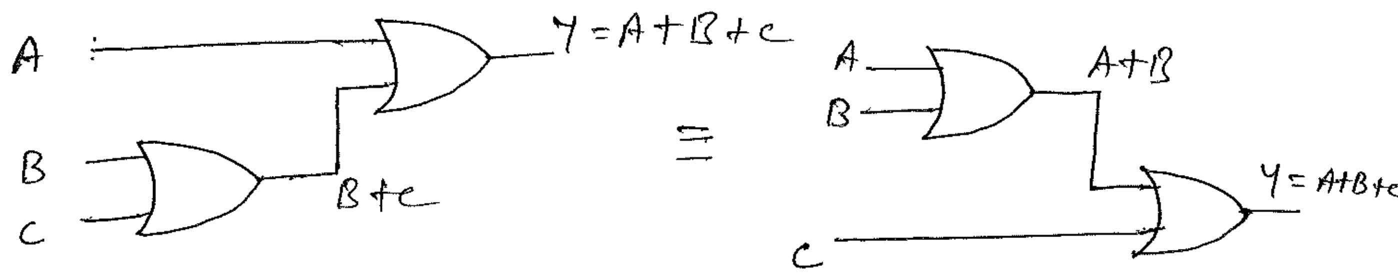
A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

≡

B	A	BA
0	0	0
0	1	0
1	0	0
1	1	1

* Associative Laws :-

Law 1 : $A+(B+C) = (A+B)+C$

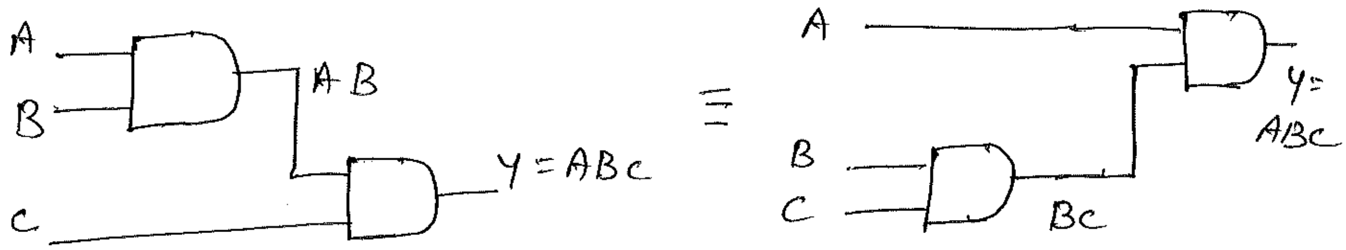


Truth tables

A	B	C	A+B	(A+B)+C
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

A	B	C	B+C	A+(B+C)
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Law 2 : $(AB)C = A(BC)$



Truth table

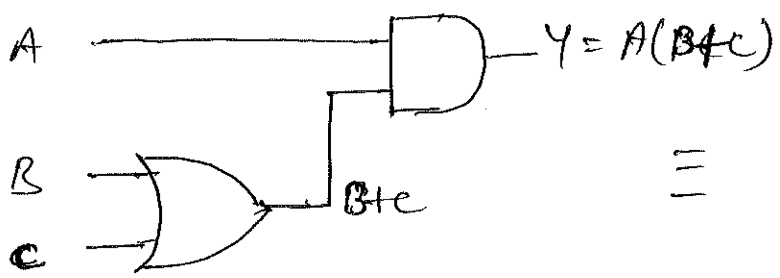
A	B	C	AB	(AB)C
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

≡

A	B	C	BC	A(BC)
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

→ Distributive law :-

LAW : $A(B+C) = AB + AC$



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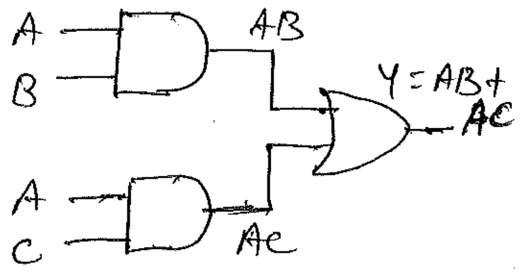


A	B	C	(B+C)	A(B+C)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

≡

A	B	C	AB	AC	AB+AC
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

→ Rules in Boolean algebra

* Use capital letters for representing variables and functions of variables. Any single variable or a function of several variables can have either a 1 or 0 value, using positive logic, a binary 1 will represent a HIGH level, and a binary 0 will represent a LOW level in Boolean Equations.
* The complement of a variable is represented by a "bar" over the letter.

Eg:- The complement of a variable A will be denoted by $\bar{A}$. If $A = 1$, $\bar{A} = 0$ and if $A = 0$, $\bar{A} = 1$.

a prime symbol (') is used to denote the complement.

Eg:- The complement of A can be written as A'.

* The logical AND function of two variables is represented either by writing a "dot" b/w the two variables, $A \cdot B$ this dot is omitted and written as AB.
* The logical OR function of two variables is represented by the sign "+" between the two variables such as $A+B$ (read as A OR B)

Rule	Proof	Dual	Proof
$A+0=A$	$\bar{A}+1=1$	$A\cdot 1=A$	$\bar{A}+AB=\bar{A}+B$
$A+1=1$	$\overline{\overline{\bar{A}+1}}$	$A\cdot 0=0$	$\overline{\overline{\bar{A}+AB}}$
$A+A=A$	$\overline{\bar{\bar{A}}\cdot\bar{1}}$	$A\cdot A=A$	$\overline{\bar{\bar{A}}\cdot\overline{AB}}$
$A+\bar{A}=1$	$\overline{A\cdot\bar{1}}$	$A\cdot\bar{A}=0$	$\overline{A(\bar{A}+\bar{B})}$
$A+AB=A$	$\overline{A\cdot 0}$	$A\cdot(A+B)=A$	$\overline{A\bar{A}+A\bar{B}}=\overline{A\bar{B}}$
	$\bar{0}=1$		$=\bar{A}+\bar{\bar{B}}$
			$=\bar{A}+B$

$A+\bar{A}B=A+B$, $\quad A\cdot(\bar{A}+B)=AB$

$(A+B)(A+C)=A+BC$, $\quad AB+AC=A(B+C)$

$AB+\bar{A}C+BC=AB+\bar{A}C$, $\quad (A+B)(\bar{A}+C)(B+C)=(A+B)(\bar{A}+C)$

$\bar{A}+1=1$, $\quad \bar{A}+AB=\bar{A}+B$

Rule of $A+AB=A$

LHS $A+AB$

$A(1+B)$

$A(1)=A$

Rule of $A+\bar{A}B=A+B$

LHS $=A+\bar{A}B$

$=A+AB+\bar{A}B$

$=A+B(A+\bar{A})$

$=A+B(1)$

Rule of $(A+B)(A+C)=A+BC$

LHS $=(A+B)(A+C)$

$=AA+AC+AB+BC$

$=A+AC+AB+BC$

$=A(1+C)+AB+BC$

$=A(1)+AB+BC=A(1+B)+BC=A+BC$

→ Principle of Duality :-

The duality theorem says that;

* changing each OR sign to an AND sign.
* changing each AND sign to an OR sign.
* complementing any 0 or 1 appearing in the expression.

Rule 1 says that $A + 0 = A$

The dual relation is $A \cdot 1 = A$

This dual property is obtained by changing the OR sign to an AND sign, and by complementing the 0 to get a 1.

The duality theorem is useful in a new Boolean relation.

Eg:- Distributive law states that

$$A(B+C) = AB + AC$$

By changing each OR and AND operation, we get the dual relation

$$A + BC = (A+B)(A+C)$$

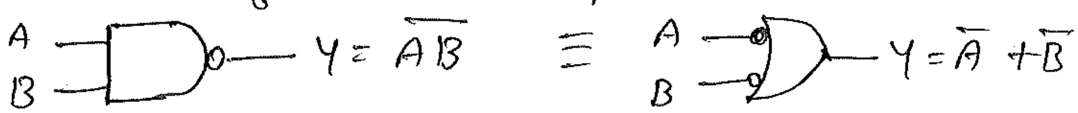
(Construct the truth table and check will be identical the Boolean relation is true)

→ DeMorgan's Theorems :-

DeMorgan suggested two theorems, in the Equation form, they are:

* $\overline{AB} = \overline{A} + \overline{B}$

The complement of a product is Equal to the sum of the complements.

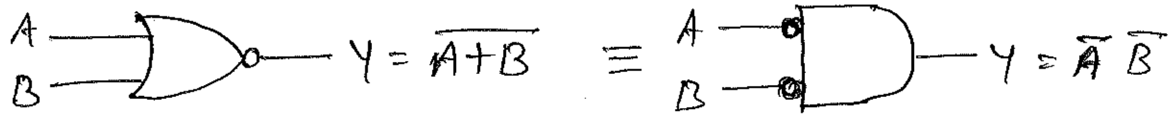


Truth table

A	B	$\overline{AB}$	$\bar{A}+\bar{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

* $\overline{A+B} = \bar{A}\,\bar{B}$

The complement of a sum is Equal to the product of the complements.



Truth table

A	B	$\overline{A+B}$	$\bar{A}\,\bar{B}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

→ Consensus theorem :-

In simplification of Boolean Expression, an Expression of the form $AB + \bar{A}C + BC$ the term BC is redundant and can be eliminated to form the Equivalent Expression $AB + \bar{A}C$. The theorem used for this simplification is known as consensus theorem.

$$AB + \bar{A}C + BC = AB + \bar{A}C$$

To find a pair of terms, one of which contains a variable and the other contains its complement.

Proof: $AB + \bar{A}C + BC = AB + \bar{A}C + (A + \bar{A})BC$

$= AB + \bar{A}C + AB + \bar{A}C$

$= AB + \bar{A}C.$

Eg: Solve the given Expression using consensus theorem.

$$\bar{A}\bar{B} + AC + B\bar{C} + BC + AB$$

Soln:- $\bar{A}\bar{B} + AC + B\bar{C} + BC + AB$

$= \bar{A}\bar{B} + AC + B\bar{C} + AB$

$= \bar{A}\bar{B} + AC + B\bar{C}$

Dual of consensus theorem:-

The dual form of consensus theorem is stated as,

$$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$$

Proof: $(A\bar{A} + AC + \bar{A}B + BC)(B+C) = A\bar{A} + AC + \bar{A}B + BC$

$(A\bar{A} + AC + \bar{A}B)(B+C) = A\bar{A} + AC + \bar{A}B$

$(AC + \bar{A}B)(B+C) = AC + \bar{A}B$

$ABC + AC + \bar{A}B + \bar{A}BC = AC + \bar{A}B$

$(A+\bar{A})BC + AC + \bar{A}B = AC + \bar{A}B$

$BC + AC + \bar{A}B = AC + \bar{A}B$

$AC + \bar{A}B = AC + \bar{A}B$

Ep: Solve the following Boolean Expression using dual of consensus theorem

$$(A+B)(\bar{A}+C)(B+C)(\bar{A}+D)(B+D)$$

Soln: $(A+B)(\bar{A}+C)(B+C)(\bar{A}+D)(B+D)$

$= (A+B)(\bar{A}+C)(\bar{A}+D)(B+D)$

$= (A+B)(\bar{A}+C)(\bar{A}+D)$

→ Applications of DeMorgan's Theorem:-

* The De Morgan theorems change + (plus) sign to • (dot) sign and vice-versa
  i.e. AND operation to OR operation and vice-versa.
* These theorems are used to convert Boolean Expression of SOP form to POS form and vice-versa.

Eg:- 1) Apply DeMorgans theorem to the following Expressions.

(a) $\overline{(A+B+C)D}$ (b) $\overline{ABC+DEF}$

Soln:- (a) $Y = \overline{(A+B+C)D}$

$= \overline{(A+B+C)} + \overline{D}$

$Y = \bar{A}\bar{B}\bar{C} + \bar{D}$

(b) $Y = \overline{ABC+DEF}$

$\overline{ABC}.\overline{DEF}$

$Y = (\bar{A}+\bar{B}+\bar{C})(\bar{D}+\bar{E}+\bar{F})$

2) Simplify the Expressions

(a) $\overline{\overline{A+B}+\bar{C}}$ (b) $\overline{\overline{A+B+\bar{C}}+D(\overline{E+F})}$

Soln. $Y = \overline{\overline{A+B}+\bar{C}}$

$= \overline{\overline{A+B}}.\bar{\bar{C}}$

$Y = (A+B).C$

$F = \overline{\overline{A+B+\bar{C}}+D(\overline{E+F})}$

$= (\overline{\overline{A+B+\bar{C}}}).[\overline{D(\overline{E+F})}]$

$= (A+B+\bar{C}).(\bar{D}+\overline{\overline{E+F}})$

$F = (A+B+\bar{C}).(\bar{D}+E+F)$

3) Simplify the Expression $\overline{\overline{AB}+\bar{A}+AB}$

Soln:

$F = \overline{\overline{AB}+\bar{A}+AB}$

$= \overline{\bar{A}+\bar{B}+\bar{A}+AB}$

$= \overline{\bar{A}+\bar{B}+AB}$

$= \bar{\bar{A}}.\bar{\bar{B}}.\overline{AB}$

$= A.B(\bar{A}+\bar{B})$

$= \cancel{A\bar{A}}B + AB\cancel{\bar{B}} = 0$

$\bar{A}A = 0$

$\bar{B}B = 0$

4) Simplify the Expression $F = (A+B)(\bar{A}+C)(B+C)$

Soln:

$$F = (A+B)(\bar{A}+C)(B+C)$$

$$= (A\bar{A} + AC + \bar{A}B + BC)(B+C)$$

$$= \cancel{A\bar{A}}B + \cancel{A\bar{A}}C + ABC + ACC + \bar{A}BB + \bar{A}BC + BBC + BCC$$

$$= ABC + AC + \bar{A}B + \bar{A}BC + BC + BC$$

$$= AC(B+1) + \bar{A}B(C+1) + BC$$

$$= AC.1 + \bar{A}B.1 + BC$$

$$= AC + \bar{A}B$$

5) Reduce the Expression $A[B+\bar{C}(\overline{AB+A\bar{C}})]$

Soln

$$Y = A[B+\bar{C}(\overline{AB+A\bar{C}})]$$

$$= A[B+\bar{C}(\bar{A}+\bar{B}).(\bar{A}+C)]$$

$$= A[B+\bar{C}(\bar{A}+\bar{B}).(\bar{A}+C)]$$

$$= A[B+\bar{C}(\bar{A}\bar{A}+\bar{A}C+\bar{A}\bar{B}+\bar{B}C)]$$

$$= A[B+\bar{C}(\bar{A}+\bar{A}C+\bar{A}\bar{B}+\bar{B}C)]$$

$$= A[B+\bar{A}\bar{C}+\bar{A}\bar{C}C+\bar{A}\bar{B}\bar{C}+\bar{B}\bar{C}C]$$

$$= A[B+\bar{A}\bar{C}+0+\bar{A}\bar{B}\bar{C}+0]$$

$$= AB+\bar{A}A\bar{C}+A\bar{A}\bar{B}\bar{C} = AB+0+0 = AB$$

6) Simplify the Expression $Y = AB + A\bar{B}.(\overline{\bar{A}\bar{C}})$

Soln:

$$Y = AB + A\bar{B}.(\overline{\bar{A}\bar{C}})$$

$$= AB + A\bar{B}.(\bar{\bar{A}}+\bar{\bar{C}})$$

$$= AB + A\bar{B}(A+C)$$

$$= AB + AA\bar{B} + A\bar{B}C$$

$$= AB + A\bar{B} + A\bar{B}C = AB + A\bar{B}(C+1)$$

$\cancel{(B+\bar{B}=1)}$

$$= \cancel{A(B+\bar{B})+A\bar{B}C}$$

$$= \cancel{AB+AB+A\bar{B}C}$$

$$= AB + A\bar{B}(1)$$

$$= A(\bar{B}+B) \qquad \bar{B}+B=1$$

$$Y = A$$

7) Simplify ~~the~~ following Expression.

$F = (A+B)(A+\overline{AB})C + \overline{A}(B+\overline{C}) + \overline{A}B + ABC$

Soln

$F = (A+B)(A+\overline{AB})C + \overline{A}(B+\overline{C}) + \overline{A}B + ABC$

$= (A+B)(A+\overline{A}+\overline{B})C + \overline{A}(B+\overline{C}) + \overline{A}B + ABC$

$= (A+B)(1+\overline{B})C + \overline{A}(B+\overline{C}) + \overline{A}B + ABC$

$= (A+B)(C+\overline{B}C) + \overline{A}B + \overline{A}\overline{C} + \overline{A}B + ABC$

$= AC + A\overline{B}C + BC + \cancel{B\overline{B}C}^{0} + \overline{A}B + \overline{A}\overline{C} + \overline{A}B + ABC$

$[\overline{B}+1=1]$ $= AC(\overline{B}+1) + BC + \overline{A}B + \overline{A}\overline{C} + \overline{A}B + ABC$

$= AC + BC + \overline{A}B + \overline{A}\overline{C} + ABC$

$= AC + \overline{A}B + \overline{A}\overline{C} + BC(1+A)$

$= AC + BC + \overline{A}B + \overline{A}\overline{C}$

$= C(A+B) + \overline{A}(B+\overline{C}) =$

$B(\cancel{C+\overline{A}})$

8) Simplify $XY + XYZ + XY\overline{Z} + \overline{X}YZ$

Soln: $F = XY + XYZ + XY\overline{Z} + \overline{X}YZ$

$= XY(1+Z) + XY\overline{Z} + \overline{X}YZ$ $\left[\begin{matrix} 1+Z=1 \\ 1+\overline{Z}=1 \end{matrix}\right]$

$= XY + XY\overline{Z} + \overline{X}YZ$

$= XY(1+\overline{Z}) + \overline{X}YZ$

$= XY + \overline{X}YZ$ $[X+\overline{X}Z = X+Z]$

$= Y(X+\overline{X}Z) = Y(X+Z)$

9) Simplify $AB + \overline{AC} + A\overline{B}C(AB+C)$

$Y = AB + \overline{AC} + A\overline{B}C(AB+C)$

$= AB + \overline{AC} + \cancel{A\overline{B}CAB}^{0} + A\overline{B}CC$

$= AB + \overline{AC} + A\overline{B}C$ $[\overline{A}+AB = \overline{A}+B]$

$= AB + \overline{A} + \overline{C} + A\overline{B}C$

$A+\overline{A}B = A+B$

$B+\overline{B}C = B+C$

~~$\overline{AB}+\overline{C}+A\overline{B}$~~$C$ $= AB + A\overline{B}C + \overline{A} + \overline{C}$

~~$\overline{A}+A\overline{B}C+\overline{B}$~~$+\overline{C}$ $= A(B+\overline{B}C) + \overline{A} + \overline{C}$

$= A(B+C) + \overline{A} + \overline{C}$

$= 1 + B + \overline{C}$ $= AB + AC + \overline{A} + \overline{C}$

$= 1 + \overline{C} = \underline{1}$ $= AB + \overline{A} + AC + \overline{C}$

$= \overline{A} + B + \overline{C} + A = \overline{A} + A + B + \overline{C}$

10) Simplify $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$$
$$= \bar{A}\bar{C}(\bar{B} + B) + \bar{A}BC$$
$$= \bar{A}\bar{C} + \bar{A}BC$$
$$= \bar{A}(\bar{C} + BC) = \underline{\bar{A}(\bar{C} + B)}$$

$[\bar{C} + BC = \bar{C} + B]$

11) Reduce the following Boolean Expressions:

$\overline{A\bar{B} + ABC} + A(B + A\bar{B})$

Soln:

$$Y = \overline{\overline{A\bar{B} + ABC} + A(B + A\bar{B})}$$
$$= \overline{\overline{A\bar{B} + ABC}} \cdot \overline{A(B + A\bar{B})}$$
$$= (A\bar{B} + ABC)(\overline{AB + AA\bar{B}})$$
$$= A(\bar{B} + BC)(\overline{AB + A\bar{B}})$$
$$= A(\bar{B} + C)(\overline{A(B + \bar{B})})$$
$$= (A\bar{B} + AC)(\overline{A(1)})$$
$$= (A\bar{B} + AC)(\bar{A} + \bar{1})$$
$$= (A\bar{B} + AC)(\bar{A} + 0)$$
$$= \cancelto{0}{A\bar{A}}\bar{B} + \cancelto{0}{A\bar{A}}C = \underline{0}$$

$(\bar{A} + AB = \bar{A} + B)$

12) Simplify the Expression $bc + \bar{a}\bar{b}c + ab\bar{c}$

Soln: Bring this to minterm canonical form

$$Y = (a + \bar{a})bc + \bar{a}\bar{b}c + ab\bar{c}$$
$$= abc + \bar{a}bc + \bar{a}\bar{b}c + ab\bar{c}$$

rearranging

$$= abc + ab\bar{c} + \bar{a}bc + \bar{a}\bar{b}c$$
$$= ab(c + \bar{c}) + \bar{a}c(b + \bar{b})$$
$$= \underline{ab + \bar{a}c}$$

13) Simplify the Expression.

$$\bar a\bar b\bar c + \bar a\bar b c + \bar a b\bar c + a\bar b\bar c + a\bar b c + ab\bar c$$

Soln: Rearranging

$$Y = \bar a\bar b\bar c + \bar a\bar b c + a\bar b\bar c + a\bar b c + \bar a b\bar c + ab\bar c$$

$$= \bar b(\bar a\bar c + \bar a c + a\bar c + ac) + b\bar c(a+\bar a)$$

$$= \bar b[\bar a(c+\bar c) + a(c+\bar c)] + b\bar c(1)$$

$$= \bar b[\bar a(1) + a(1)] + b\bar c = \bar b\bar a + a\bar b + b\bar c$$

$$= \bar b(a+\bar a) + b\bar c$$

$$= \bar b + b\bar c$$

$$= \bar b + \bar c$$

$$= \bar b + b\bar c$$

$$= \overline{\overline{\bar b + b\bar c}}$$

$$= \overline{\bar{\bar b}\cdot\overline{b\bar c}}$$

$$= \overline{b\cdot\overline{b\bar c}}$$

$$= \overline{b(\bar b + \bar{\bar c})} = \overline{b(\bar b + c)} = \overline{\cancel{b\bar b}^{\,0} + bc}$$

$$= \overline{0 + bc}$$

$$= \bar 0\cdot\overline{bc}$$

$$= 1\cdot\bar b + \bar c$$

14) Prove the following identities.

(i) $(a+b)(\bar a\bar c + c)(\overline{\bar b + ac}) = \bar a b$

Soln:

$$= (a+b)(\bar a\bar c + c)(\overline{\bar b + ac})$$

$$= (a+b)(c+\bar a)(\overline{\bar b + ac})$$

$$= (a+b)(c+\bar a)(\bar{\bar b}\cdot\overline{ac})$$

$$= (a+b)(c+\bar a)\,b(\bar a + \bar c)$$

$$= (a+b)(c+\bar a)(\bar a+\bar c)\,b$$

$$= (a+b)(\bar a c + \cancel{c\bar c}^{\,0} + \bar a\bar a + \bar a\bar c)b$$

$$= (a+b)(\bar a c + \bar a + \bar a\bar c)b$$

$$= (a+b)(\bar a(c+1) + \bar a\bar c)b$$

$$= (a+b)(\bar a + \bar a\bar c)b$$

$$= (a+b)[\bar a(\bar c+1)]b$$

$$= c + \bar a\bar c$$

$$= \overline{\overline{c + \bar a\bar c}}$$

$$= \overline{\bar c\cdot\overline{\bar a\bar c}}$$

$$= \overline{\bar c(\bar{\bar a} + \bar{\bar c})}$$

$$= \overline{\bar c(a+c)}$$

$$= \overline{\bar c a + \cancel{\bar c c}^{\,0}}$$

$$= \overline{\bar c a + 0}$$

$$= (\bar{\bar c} + \bar a)\bar 0$$

$$= (c+\bar a)1$$

$= (a+b)\,[\bar{a}b]$

$= \cancel{a\bar{a}b}^{\,0} + \bar{a}bb$

$= \bar{a}b$

(ii) $a\bar{b} + b\bar{c} + \bar{a}c = \bar{a}b + \bar{b}c + a\bar{c}$

LHS $= a\bar{b} + b\bar{c} + \bar{a}c = a\bar{b}.1 + b\bar{c}.1 + \bar{a}c.1$

$= a\bar{b}(c+\bar{c}) + b\bar{c}(a+\bar{a}) + \bar{a}c(b+\bar{b})$

$= a\bar{b}c + a\bar{b}\bar{c} + ab\bar{c} + \bar{a}b\bar{c} + \bar{a}bc + \bar{a}\bar{b}c$

rearranging

$= \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}c + \bar{a}\bar{b}c + a\bar{b}\bar{c} + ab\bar{c}$

$= \bar{a}b(c+\bar{c}) + \bar{b}c(a+\bar{a}) + a\bar{c}(b+\bar{b})$

$= \bar{a}b + \bar{b}c + a\bar{c}$

15) Using theorems and laws of Boolean algebra simplify the following

(a) $(a+b+\bar{c}+d)(a+b+\bar{c}+\bar{d})(a+\bar{b}+\bar{c}+d)(a+\bar{b}+\bar{c}+\bar{d})$
$(\bar{a}+\bar{b}+\bar{c}+d)(\bar{a}+\bar{b}+\bar{c}+\bar{d})$

Soln:- $(a+b+\bar{c}+d)(a+b+\bar{c}+\bar{d})(a+\bar{b}+\bar{c}+d)$
$(a+\bar{b}+\bar{c}+\bar{d})(\bar{a}+\bar{b}+\bar{c}+d)(\bar{a}+\bar{b}+\bar{c}+\bar{d})$

$= (a+b+\bar{c}+d\bar{d})(a+\bar{b}+\bar{c}+d\bar{d})(\bar{a}+\bar{b}+\bar{c}+d\bar{d})$

$= (a+b+\bar{c})(a+\bar{b}+\bar{c})(\bar{a}+\bar{b}+\bar{c})$

$= (a+b+\bar{c})(a\bar{a}+\bar{b}+\bar{c})$

$= (a+b+\bar{c})(\bar{b}+\bar{c})$

$= a\bar{b} + a\bar{c} + \cancel{b\bar{b}}^{\,0} + b\bar{c} + \bar{b}\bar{c} + \bar{c}\bar{c}$

$= a\bar{b} + a\bar{c} + b\bar{c} + \bar{b}\bar{c} + \bar{c}$

$$= a\bar{b} + a\bar{c} + \bar{c}(b+\bar{b}) + \bar{c}$$
$$= a\bar{b} + a\bar{c} + \bar{c} + \bar{c}$$
$$= a\bar{b} + a\bar{c} + \bar{c}$$
$$= a\bar{b} + \bar{c}(a+1) = \underline{a\bar{b} + \bar{c}}$$

(b) $\bar{a}\bar{b}\bar{c}d + a\bar{b}\bar{c}d + bd + bc\bar{d}$

Soln: $Y = \bar{a}\bar{b}\bar{c}d + a\bar{b}\bar{c}d + bd + bc\bar{d}$

$$= \bar{b}\bar{c}d(\bar{a}+a) + b(d+\bar{d}c)$$
$$= \bar{b}\bar{c}d + b(d+c)$$
$$= \bar{b}\bar{c}d + bd + bc$$
$$= d(\bar{b}\bar{c} + b) + bc$$
$$= d(b+\bar{c}) + bc$$
$$= bd + \bar{c}d + bc$$
$$= bd(c+\bar{c}) + \bar{c}d + bc$$
$$= bdc + bd\bar{c} + \bar{c}d + bc$$
$$= bc(1+d) + \bar{c}d(b+1)$$
$$Y = \underline{bc + \bar{c}d}$$

$$= \bar{b}\bar{c} + b$$
$$= \overline{\overline{\bar{b}\bar{c} + b}}$$
$$= \overline{(\bar{\bar{b}} + \bar{\bar{c}})\bar{b}}$$
$$= \overline{(b+c)\bar{b}}$$
$$= \overline{b\bar{b} + \bar{b}c}$$
$$= \overline{0 + \bar{b}c}$$
$$= \bar{0}.(\overline{\bar{b}c})$$
$$= 1(\bar{\bar{b}} + \bar{c})$$
$$= (b+\bar{c})$$

→ **Boolean Formulas and Functions** :-

The Boolean Expressions, which are constructed by connecting the Boolean constants and variables with the Boolean operations. Boolean Expressions are also known as Boolean formulas.

Eg:- If the Boolean Expression $(A+\bar{B})C$ is used to describe the function f, then Boolean function is written as,

$$f(A,B,C) = (A+\bar{B})C \quad \text{or} \quad f = (A+\bar{B})C$$

→ Normal Formulas

Let us consider the four-variable Boolean function.

$$f(A, B, C, D) = A + \bar{B}C + AC\bar{D} \quad - (1)$$

In this Boolean function the variables are appeared either in a complemented or an uncomplemented form each occurrence of a variable is called a literal.

Boolean function (1) consists of six literals. They appear in the product terms. A product term is defined as either a literal or a product of literals.

Function (1) contains three products terms, A, $\bar{B}C$ and $AC\bar{D}$

Let us consider another four variable Boolean function.

$$f(A, B, C, D) = (B+\bar{D})(A+\bar{B}+C)(\bar{A}+C) \rightarrow (2)$$

The above Boolean function consists of seven literals. They appear in the sum terms. A sum term is defined as either a literal or a sum of literal.

Function (2) contains three sum terms, $(B+\bar{D})$, $(A+\bar{B}+C)$ and $(\bar{A}+C)$.

The literals and terms are arranged in two forms:

* Sum of product form (SOP) and
* product of sum form (POS).

→ Sum of product form. (SOP)

The word sum and product are derived from the symbolic representations of the OR and AND functions by + and . (add and Multiply).

A sum of products (SOP) is a group of product terms ORed together.

Eg:- $ABC + A\bar{B}\bar{C}$

$XY + XY\bar{Z} + XZ$

→ Product of Sum form (POS)

A product of sums is any groups of sum terms ANDed together.

Eg:- $(A+\bar{B}+C)(B+\cancel{C}+\bar{D})$

$(P+Q)(Q+\bar{R}+S)$

Each of these product of sums Expressions consist of two or more sum terms (OR) that are ANDed together.

→ Canonical formulas

Canonical formulas are also known as standard SOP and POS forms.

Standard SOP form or Minterm Canonical formula.

In Expression $AB+AB\bar{C}$ the first product term do not contain literal C. If each term in SOP form contains all the literals then the SOP form is known as standard or canonical SOP form. Each individual term in the standard SOP form is called minterm.

∴ canonical ~~f~~ SOP form is also known as minterm canonical formula.

In Expression $AB\bar{C} + ABC + \bar{A}BC + A\bar{B}C$ all the literals are present in each product term.

Standard POS form or Maxterm canonical formula :-

In POS form contains all the literals then the POS form is known as standard or canonical POS form. Each individual term in the standard POS form is called maxterm.

∴ Canonical POS form is also known as maxterm canonical formula.

Eg: $Y = (A+B+C)(A+\bar{B}+C)$

Eg:- 1) Convert the given expression in standard SOP form, $Y = AC+AB+BC$

Soln: $Y = AC + AB + BC$

$$= AC(B+\bar{B}) + AB(C+\bar{C}) + BC(A+\bar{A})$$

$$= ABC + A\bar{B}C + ABC + AB\bar{C} + ABC + \bar{A}BC$$

$$= ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$$

2) Convert the given expression in standard SOP form. $Y = A+AB+ABC$

Soln: $Y = A + AB + ABC$

$$= A(B+\bar{B}).(C+\bar{C}) + AB.(C+\bar{C}) + ABC$$

$$= (AB+A\bar{B}).(C+\bar{C}) + ABC + AB\bar{C} + ABC$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC + AB\bar{C} + ABC$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$$

3) convert the given Expression in Standard POS form. $Y = (A+B)(B+C)(A+C)$

Soln:

$$Y = (A+B)(B+C)(A+C)$$

$$= (A+B+C.\bar{C})(B+C+A.\bar{A})(A+C+B.\bar{B})$$

$$= (A+B+C)(A+B+\bar{C})(A+B+C)(\bar{A}+B+C)(A+B+C)(A+\bar{B}+C)$$

$$= (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)(A+\bar{B}+C)$$

4) convert the given Expression in Standard POS form. $Y = A(A+B)(A+B+C)$

Soln:

$$Y = A(A+B)(A+B+C)$$

$$= (A+B.\bar{B}+C.\bar{C})(A+B+C.\bar{C})(A+B+C)$$

$$= (A+B.\bar{B}+C)(A+B.\bar{B}+\bar{C})(A+B+C)(A+B+\bar{C})(A+B+C)$$

$$= (A+B+C)(A+\bar{B}+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})(A+B+\bar{C})(A+B+C)$$

$$= (A+B+C)(A+\bar{B}+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})$$

→ M - Notations

Each individual term in standard SOP form is called minterm and each individual term in standard POS form is called maxterm. The minterms and maxterms for a three literal / variable logical function where the number of minterms as well as maxterms is $2^3 = 8$.

In general, for an n-variable logical function there are 2^n minterms and an equal number of maxterms.

Minterms and maxterms for three variables

Variables			Minterms	Maxterms
A	B	C	m_i	M_i
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A+B+\bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$A+\bar{B}+C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A+\bar{B}+\bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A}+B+C = M_4$
1	0	1	$A\bar{B}C = m_5$	$\bar{A}+B+\bar{C} = M_5$
1	1	0	$AB\bar{C} = m_6$	$\bar{A}+\bar{B}+C = M_6$
1	1	1	$ABC = m_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$

Logical function can be represented as follows:

* $Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C}$
$= m_0 + m_1 + m_3 + m_6$
$= \sum m(0, 1, 3, 6)$

* $Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$
$= M_1 + M_3 + M_6$
$= \pi M(1, 3, 6)$

where $\sum$ denotes Sum of product
while π denotes product of Sum.

Eg 1) Simplify the following three variable Expression using boolean algebra.
$Y = \sum m(1, 3, 5, 7)$

Soln. write Expression in Sum of products form
$Y = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

Common terms for factorization and apply boolean rules.

$$Y = \bar{A}C(\bar{B}+B) + AC(\bar{B}+B)$$

$$= \bar{A}C + AC \qquad [\bar{A}+A = 1]$$

$$= C(\bar{A}+A) = \underline{\underline{C}}$$

2) Simplify the following three variable Expression using boolean algebra.

$$Y = \pi M(3,5,7)$$

Soln: Expression in POS form and convert it into SOP form.

$$Y = (A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$= (A\bar{A} + AB + A\bar{C} + \bar{A}\bar{B} + B\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C} + B\bar{C} + C\bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$= (AB + A\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C} + B\bar{C} + \bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$= (AB + A\bar{C} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + B\bar{C} + \bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$= (AB + \bar{C}(A+\bar{A}) + \bar{A}\bar{B} + \bar{C}(B+\bar{B}) + \bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$= (AB + \bar{C}(1) + \bar{A}\bar{B} + \bar{C}(1) + \bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$= (AB + \bar{C} + \bar{A}\bar{B})(\bar{A}+\bar{B}+\bar{C})$$

$$= A\cancel{\bar{A}}B^{0} + A\cancel{B}\bar{B}^{0} + AB\bar{C} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{C}\bar{C} + \bar{A}\bar{A}\bar{B} + \bar{A}\bar{B}\bar{B} + \bar{A}\bar{B}\bar{C}$$

$$= AB\bar{C} + \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{C} + \bar{A}\bar{B} + \bar{A}\bar{B} + \bar{A}\bar{B}\bar{C}$$

$$= \bar{C}(\bar{A}+AB) + \bar{C}(1+\bar{B}) + \bar{A}\bar{B} + \bar{A}\bar{B}\bar{C}$$

$$= \bar{C}(\bar{A}+B) + \bar{C}(1) + \bar{A}\bar{B}(1+\bar{C})$$

$$= \bar{A}\bar{C} + B\bar{C} + \bar{C} + \bar{A}\bar{B}(1)$$

$$= \bar{A}\bar{C} + \bar{C}(1+B) + \bar{A}\bar{B}$$

$$= \bar{A}\bar{C} + \bar{C}(1) + \bar{A}\bar{B}$$

$$= \bar{C}(\bar{A}+1) + \bar{A}\bar{B}$$

$$Y = \underline{\underline{\bar{C} + \bar{A}\bar{B}}}$$

→ Complements of canonical formulas :-

A product of sums form derived from a truth table is logically equivalent to a sum of products form derived from the truth table.

A	B	C	Y	
0	0	0	1	← $\bar{A}\bar{B}\bar{C}$
0	0	1	1	← $\bar{A}\bar{B}C$
0	1	0	0	← $A+\bar{B}+C$ → POS form
0	1	1	1	← $\bar{A}BC$
1	0	0	1	← $A\bar{B}\bar{C}$
1	0	1	0	← $\bar{A}+B+\bar{C}$ → POS form
1	1	0	1	← $AB\bar{C}$
1	1	1	1	← ABC

The standard SOP and POS from the truth table.

SOP form : $Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$

POS form : $Y = (A+\bar{B}+C)(\bar{A}+B+\bar{C})$

POS form we have;

$$Y = \cancelto{0}{A\bar{A}} + AB + A\bar{C} + \bar{A}\bar{B} + \cancelto{0}{B\bar{B}} + \bar{B}\bar{C} + \bar{A}C + BC + \cancelto{0}{C\bar{C}}$$

$$= AB + A\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}C + BC$$

Converting to standard SOP form we have,

$$Y = AB(C+\bar{C}) + A\bar{C}(B+\bar{B}) + \bar{A}\bar{B}(C+\bar{C}) + \bar{B}\bar{C}(A+\bar{A}) + \bar{A}C(B+\bar{B}) + BC(A+\bar{A})$$

$$= ABC + AB\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}C + ABC + \bar{A}BC$$

$$Y = ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}BC$$

rearranging,

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$

$\therefore$ The POS and SOP derived from the truth table are logically Equivalent.

In terms of minterms and maxterms can write,

$Y = m_0 + m_1 + m_3 + m_4 + m_6 + m_7 = M_2 + M_5$

$Y = \sum m(0, 1, 3, 4, 6, 7)$

$Y = \pi M(2, 5)$

Eg 1) Find the complements of each of the following in SOP and POS form.

$f(a, b, c) = \sum m(0, 3, 5, 6, 7)$

a	b	c	y	
0	0	0	1	$\leftarrow \bar{A}\bar{B}\bar{C}$
0	0	1	0	$\leftarrow A+B+\bar{C} \rightarrow$
0	1	0	0	$\leftarrow A+\bar{B}+C \rightarrow$ POS form
0	1	1	1	$\leftarrow \bar{A}BC$
1	0	0	0	$\leftarrow \bar{A}+B+C \rightarrow$
1	0	1	1	$\leftarrow A\bar{B}C$
1	1	0	1	$\leftarrow AB\bar{C}$
1	1	1	1	$\leftarrow ABC$

The standard SOP and POS form from the truth table.

SOP form ; $Y = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

POS form ; $Y = (A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)$

we have

$Y = (AA + A\bar{B} + AC + AB + \cancel{B}\bar{B}^{0} + BC + A\bar{C} + \bar{B}\bar{C} + \cancel{C}\bar{C}^{0})$
$(\bar{A}+B+C)$

$= (A + A\bar{B} + AC + AB + BC + A\bar{C} + \bar{B}\bar{C})(\bar{A}+B+C)$

$= [A(\bar{B}+1) + \cancel{AC} + AB + BC + A(C+\bar{C}) + \bar{B}\bar{C}](\bar{A}+B+C)$

$$= (A + AB + BC + A + \overline{B}\overline{C})(\overline{A} + B + C)$$
$$= (A(B+1) + BC + A + \overline{B}\overline{C})(\overline{A} + B + C)$$
$$= (A + A + BC + \overline{B}\overline{C})(\overline{A} + B + C)$$
$$= (A + BC + \overline{B}\overline{C})(\overline{A} + B + C)$$
$$= \cancel{A\overline{A}}^{0} + AB + AC + \overline{A}BC + BBC + BCC + \overline{A}\overline{B}\overline{C} + \cancel{B\overline{B}}^{0}\overline{C} + \overline{B}\cancel{\overline{C}C}^{0}$$
$$= AB + AC + \overline{A}BC + BC + BC + \overline{A}\overline{B}\overline{C}$$
$$= AB + AC + BC + \overline{A}BC + \overline{A}\overline{B}\overline{C}$$
$$= AB + AC + BC(\overline{A} + 1) + \overline{A}\overline{B}\overline{C}$$
$$= AB + AC + BC + \overline{A}\overline{B}\overline{C}$$

converting to standard SOP form.

$$= AB(C + \overline{C}) + AC(B + \overline{B}) + BC(A + \overline{A}) + \overline{A}\overline{B}\overline{C}$$
$$= ABC + AB\overline{C} + ABC + A\overline{B}C + ABC + \overline{A}BC + \overline{A}\overline{B}\overline{C}$$
$$= \overline{A}\overline{B}\overline{C} + \overline{A}BC + AB\overline{C} + A\overline{B}C + ABC$$

$$Y = m_0 + m_3 + m_5 + m_6 + m_7 = M_1 + M_2 + M_4$$
$$Y = \sum m(0, 3, 5, 6, 7)$$
$$Y = \prod M(1, 2, 4)$$

→ Equation complementation :-

For every Boolean function f there is a associated a complementory function $\overline{f}$ in which,

$$\overline{f}(A_1, A_2, A_3 ---, A_n) = 1 \text{ if } f(A_1, A_2, A_3, ---, A_n) = 0$$

and $\overline{f}(A_1, A_2, A_3, ---, A_n) = 0$ if $f(A_1, A_2, A_3, ---, A_n) = 1$

for all combinations of values of $A_1, A_2, A_3, ---, A_n$.

A Boolean formula for $\overline{f}$ is obtained by complementing the Boolean Expression for f.

Eg:- $f = AB\overline{C} + B(C + \overline{D})$

The function $\overline{f}$ is described by

$$\overline{f} = [\overline{AB\overline{C} + B(C + \overline{D})}]$$

$= \overline{AB\bar{C}} \cdot \overline{B(C+\bar{D})}$

$= (\bar{A}+\bar{B}+\bar{\bar{C}}) \cdot (\bar{B} + (\overline{C+\bar{D}}))$

$= (\bar{A}+\bar{B}+C)(\bar{B}+\bar{C}\bar{\bar{D}})$

$= (\bar{A}+\bar{B}+C)(\bar{B}+\bar{C}D)$

Eg:- 1) Reduce the given Expression using Equation complementation $f = \overline{AB} + \bar{A} + AB$

Soln $f = \overline{AB} + \bar{A} + AB$

using equation complementation $\bar{f}$

$\bar{f} = \overline{\overline{AB} + \bar{A} + AB}$

$= \overline{\bar{A} + \bar{B} + \bar{A} + AB}$

$= \overline{\bar{A} + \bar{B} + \bar{A} + B}$

$= \overline{\bar{A}+1} = \bar{1} = \underline{\underline{0}}$

2) verify the following Boolean algebraic manipulation. Justify each step with a reference to a postulate or theorem:

$(AB+C+D)(\bar{C}+D)(\bar{C}+D+E) = AB\bar{C}+D$

Soln $= (AB+C+D)(\bar{C}+D)(\bar{C}+D+E)$

$= (AB\bar{C}+ABD+C\bar{C}+CD+\bar{C}D+DD)(\bar{C}+D+E)$

$= (AB\bar{C}+ABD+CD+\bar{C}D+D)(\bar{C}+D+E)$

$= [AB\bar{C}+ABD+CD+D(\bar{C}+1)](\bar{C}+D+E)$

$= [AB\bar{C}+ABD+CD+D](\bar{C}+D+E)$

$= [AB\bar{C}+ABD+D(C+1)](\bar{C}+D+E)$

$= (AB\bar{C}+ABD+D)(\bar{C}+D+E)$

$= AB\bar{C}\bar{C}+AB\bar{C}D+AB\bar{C}E+AB\bar{C}D+ABDD$
$+ABDE+\bar{C}D+DD+DE$

$= AB\bar{C}+AB\bar{C}D+AB\bar{C}E+AB\bar{C}D+ABD+ABDE$
$+\bar{C}D+D+DE$

$= AB\bar{C}(1+D)+AB\bar{C}E+ABD(\bar{C}+1)+ABDE+D(\bar{C}+1)$
$+DE$

$= AB\overline{C} + AB\overline{C}E + ABD + ABDE + D + DE$

$= AB\overline{C}(1+E) + ABD(1+E) + D(1+E)$

$= AB\overline{C} + ABD + D$

$= AB\overline{C} + D(AB+1)$

$= AB\overline{C} + D$

3) Simplify the following expression.

$a + a\overline{b} + a\overline{b}\,\overline{c} + a\overline{b}\,\overline{c}\,\overline{d} + -----$

Soln: $y = a + a\overline{b} + a\overline{b}\,\overline{c} + a\overline{b}\,\overline{c}\,\overline{d} + ----$

$= a[1 + \overline{b} + \overline{b}\,\overline{c} + \overline{b}\,\overline{c}\,\overline{d} + ----]$

$= a$

→ Expansion about a variable :-

To expand the Boolean function about the single variable is given by the theorem known as Shannon's Expansion theorem.

* Shannon's Expansion Theorem :-

Theorem 1 :

$$f(A_1, A_2, A_3, ----A_i, ---A_n) = A_i \cdot f(A_1, A_2, A_3 --, 1, --A_n) + \overline{A_i} \cdot f(A_1, A_2, A_3 --, 0, -- A_n)$$

Theorem 2 :

$$f(A_1, A_2, A_3, ---A_i, -- A_n) = [A_i + f(A_1, A_2, A_3, --, 0, --A_n)] \cdot [\overline{A_i} + f(A_1, A_2, A_3, --, 1, ---A_n)]$$

Eg:- 1) Expand the given Boolean function using Shannon's Expansion theorem.

$f(A, B, C, D) = A\overline{B} + (AC+B)D$

Soln: $f(A,B,C,D) = A\bar{B} + (AC+B)D$

using Theorem 1 → $= A[1.\bar{B} + (1.C+B)D] + \bar{A}[0.\bar{B} + (0.C+B)D]$

$= A[\bar{B} + (C+B)D] + \bar{A}(BD)$

$= A\bar{B} + A(C+B)D + \bar{A}BD$

2) Expand the given Boolean function using Shannon's Expansion theorem.

$f(A,B,C,D) = \bar{A}C + (B+AD)C$

Soln: $f(A,B,C,D) = \bar{A}C + (B+AD)C$

$= A[\bar{1}.C + (B+1.D)C] +$

using Theorem 1 → $\bar{A}[\bar{0}.C + (B+0.D)C]$

$= A[0.C + (B+D)C] + \bar{A}[1.C + BC]$

$= A(B+D)C + \bar{A}(C+BC)$

using Theorem 2

$f(A,B,C,D) = \bar{A}C + (B+AD)C$

$= [A + (\bar{0}C + (B+0D)C)] . [\bar{A} + (\bar{1}C + (B+1D)C]$

$= [A + (1C + BC)] . [\bar{A} + (0C + (B+D)C]$

$= (A + C + BC) . (\bar{A} + (B+D)C)$

using Theorem 2 for first Example

$f(A,B,C,D) = A\bar{B} + (AC+B)D$

$= [A + (0\bar{B}) + (0C+B)D] . [\bar{A} + (1.\bar{B} + (1.C+B)D]$

$= (A + BD) . (\bar{A} + \bar{B} + (C+B)D)$

3). Apply Shannon's theorem to Expand

$$f(a,b,c,d) = \bar{a}bc\bar{d} + d(b\bar{c} + a\bar{b})$$

Isolating variables b in both forms.

Soln: Apply Shannon's theorem, Theorem 1

$$f(a,b,c,d) = b\,f(a,1,c,d) + \bar{b}\,f(a,0,c,d)$$

$$f(a,b,c,d) = \bar{a}bc\bar{d} + d(b\bar{c} + a\bar{b})$$

$$= b[1\bar{a}c\bar{d} + d(1.\bar{c} + a\bar{1})] + \bar{b}[0\bar{a}c\bar{d} + d(0\bar{c} + a1)]$$

$$= b[\bar{a}c\bar{d} + d(\bar{c} + a0)] + \bar{b}[0 + 0 + ad]$$

$$= b(\bar{a}c\bar{d} + \bar{c}d) + \bar{b}(ad)$$

and using theorem 2

$$f(a,b,c,d) = [b + f(a,0,c,d)][\bar{b} + f(a,1,c,d)]$$

$$f(a,b,c,d) = \bar{a}bc\bar{d} + d(b\bar{c} + a\bar{b})$$

$$= [b + (\bar{a}0c\bar{d}) + d(0\bar{c} + a\bar{0})] \cdot [\bar{b} + (\bar{a}1c\bar{d}) + d(1\bar{c} + a\bar{1})]$$

$$= [b + d(a1)] \cdot [\bar{b} + \bar{a}c\bar{d} + d(\bar{c} + a0)]$$

$$= (b + ad)(\bar{b} + \bar{a}c\bar{d} + \bar{c}d)$$

4) Expand using Shannon's theorem to isolate variable a in the following expressions

a(i) $\bar{a}(\bar{b} + c) + \bar{c}$ (ii) $(b + \bar{c})(a\bar{b} + c)$

Soln: $f(a,b,c) = \bar{a}(\bar{b} + c) + \bar{c}$

Theorem 1

$$= a.\bar{1}(\bar{b} + c) + \bar{c} + \bar{a}.\bar{0}(\bar{b} + c) + \bar{c}$$

$$= a.0(\bar{b} + c) + \bar{c} + \bar{a}1(\bar{b} + c) + \bar{c}$$

$$f(a,b,c) = \bar{c} + \bar{a}(\bar{b} + \bar{c}) + \bar{c}$$

Theorem 2

$f(a,b,c) = \bar{a}(\bar{b}+\bar{c}) + \bar{c}$

$= (a + \bar{0}(\bar{b}+c)+\bar{c})(\bar{a}+\bar{1}(\bar{b}+c)+\bar{c})$

$= (a+1(\bar{b}+c)+\bar{c})(\bar{a}+0(\bar{b}+c)+\bar{c})$

$= (a+(\bar{b}+c)+\bar{c})(\bar{a}+\bar{c})$

(ii) $f(a,b,c) = (b+\bar{c})(a\bar{b}+c)$

Theorem 1 $= a.(b+\bar{c})(1\bar{b}+c) + \bar{a}.(b+\bar{c})(0\bar{b}+c)$

$= a(b+\bar{c})(\bar{b}+c) + \bar{a}(b+\bar{c})c$

or

$= a(b+\bar{c})(\bar{b}+c) + \bar{a}(0+bc+0+\cancel{c\bar{c}}^{0})$

$f = a(b+\bar{c})(\bar{b}+c) + \bar{a}bc$

Theorem 2

$f(a,b,c) = (b+\bar{c})(a\bar{b}+c)$

$= (a+(b+\bar{c})(0\bar{b}+c)).(\bar{a}+(b+\bar{c})(1\bar{b}+c))$

$= (a+(0+bc+0+\cancel{c\bar{c}}^{0})).(\bar{a}+(b+\bar{c})(\bar{b}+c))$

$f = (a+bc)(\bar{a}+(b+\bar{c})(\bar{b}+c))$

5) Apply the Expansion theorem to Express

$f(a,b,c) = ab+\bar{b}c+a\bar{c}$

as $f(a,b,c) = af_1(b,c) + \bar{a}f_2(b,c)$

and $f(a,b,c) = (b+g_1(a,c))(\bar{b}+g_2(a,c))$

Soln: The Expansion formula are

$f(a,b,c) = af(1,b,c) + \bar{a}f(0,b,c)$

and $f(a,b,c) = (b+f(a,0,c)).(\bar{b}+f(a,1,c))$

given $= ab+\bar{b}c+a\bar{c}$

Theorem 1 take 'a' out side $= a.[1.b+(\bar{b}c)+1.\bar{c}] + \bar{a}.[0.b+\bar{b}c+0\bar{c}]$

$= a.[b+\bar{b}c+\bar{c}] + \bar{a}\bar{b}c$

$= ab+a\bar{b}c+a\bar{c}+\bar{a}\bar{b}c$

$= ab+\bar{b}c(a+\bar{a})+a\bar{c} = ab+\bar{b}c+a\bar{c}$

$f(a,b,c) = ab + \bar{b}c + a\bar{c}$

Apply Theorem 2 take 'b' outside

$= (b + (0.a + \bar{0}c + a\bar{c})).(\bar{b} + (a1 + \bar{1}c + a\bar{c}))$

$= (b + (1.c + a\bar{c})).(\bar{b} + (a + 0 + a\bar{c}))$

$= (b + (a+c)).(\bar{b} + (a + a\bar{c}))$

$= (b + (a+c))(\bar{b} + (a+c))$

* Shannon's reduction theorems :-

Theorem 1

a) $A_i . f(A_1, A_2, A_3, ---, A_i, --- A_n) = A_i . f(A_1, A_2, A_3, ---, 1, ---, A_n)$

b) $A_i + f(A, A_2, A_3, ---, A_i, --- A_n) = A_i + f(A_1, A_2, A_3, ---, 0, ---, A_n)$

where $f(A_1, A_2, A_3, ---, K, --- A_n)$ for $K = 0, 1$ denotes the formula $f(A_1, A_2, A_3, ---, A_i, ---, A_n)$ upon the substitution of the constant K for all occurrences of the variable A_i.

Eg:- 1) Reduce the following Boolean function using Shannon's reduction theorem.

$f(A, B, C, D) = A[\bar{A}(B+C) + (A+D)]$

Soln:- $f(A, B, C, D) = A[\bar{A}(B+C) + (A+D)]$

$= A.[\bar{1}.(B+C) + (1+D)]$

$= A.[0(B+C) + (1+D)]$

$= A(1+D) = A$

$f(A, B, C, D) = A[\bar{A}(B+C) + (A+D)]$

$= A + (\bar{0}(B+C)) + (0+D)$

$= A + (1(B+C)) + D$

$= A + (B+C) + D$

27. Reduce the following Boolean function using Shannon's reduction theorem.

$f(A, B, C, D) = A + \bar{A}B + A\bar{C}(B+C)(B+D)$

Soln:- $f(A, B, C, D) = A + \bar{A}B + A\bar{C}(B+C)(B+D)$

$= A.[1 + \bar{1}B + 1.\bar{C}(B+C)(B+D)]$

$= A.[1 + 0B + \bar{C}(B+C)(B+D)]$

$= A[1 + \bar{C}(B+C)(B+D)]$

$f(A, B, C, D) = A + \bar{A}B + A\bar{C}(B+C)(B+D)$

$= A + [0 + \bar{0}B + 0\bar{C}(B+C)(B+D)]$

$= A + [1.B + 0]$

$= A + B$

Theorem 2:-

a) $\bar{A_i} \cdot f(A_1, A_2, A_3, ---, A_i, ---, A_n) = \bar{A_i} \cdot f(A_1, A_2, A_3, --, 0, --, A_n)$

b) $\bar{A_i} + f(A_1, A_2, A_3, --, A_i, --, A_n) = \bar{A_i} + f(A_1, A_2, A_3, ---, 1, --, A_n)$

where $f(A_1, A_2, A_3, ---, K, ---, A_n)$ for $K = 0, 1,$ denotes the formula $f(A_1, A_2, A_3, ---, A_i, ---, A_n)$ upon the substitution of the constant K for all occurrences of the variable A_i.

Eg:- 1) Reduce the following Boolean function using Shannon's reduction theorem.

$f(A, B, C, D) = \bar{A}[(A+C)(D+\bar{A}B)]$

Soln:- $f(A, B, C, D) = \bar{A}[(A+C)(D+\bar{A}B)]$

$= \bar{A}.[\bar{0}(0+C)(D+\bar{0}B)]$

$= \bar{A}.[1(0+C)(D+1B)]$

$= \bar{A}[C(D+B)]$

$$f(A,B,C,D) = \bar{A}\,[(A+C)(D+\bar{A}B)]$$
$$= \bar{A} + [\bar{1}(1+C)(D+\bar{1}B)]$$
$$= \bar{A} + [0(1+C)(D+0B)]$$
$$= \bar{A} + [0 \cdot D + 0 \cdot B]$$
$$= \bar{A}$$

2) Reduce the following Boolean function using Shannon's reduction theorem.

$$f(A,B,C,D) = \bar{A} + AB + \bar{A}(B+D)$$

Soln:- $f(A,B,C,D) = \bar{A} + AB + \bar{A}(B+D)$

$$= \bar{A} \cdot [\bar{0} + 0B + \bar{0}(B+D)]$$
$$= \bar{A} \cdot [1 + 0 + 1(B+D)]$$
$$= \bar{A}\,[1 + B + D]$$

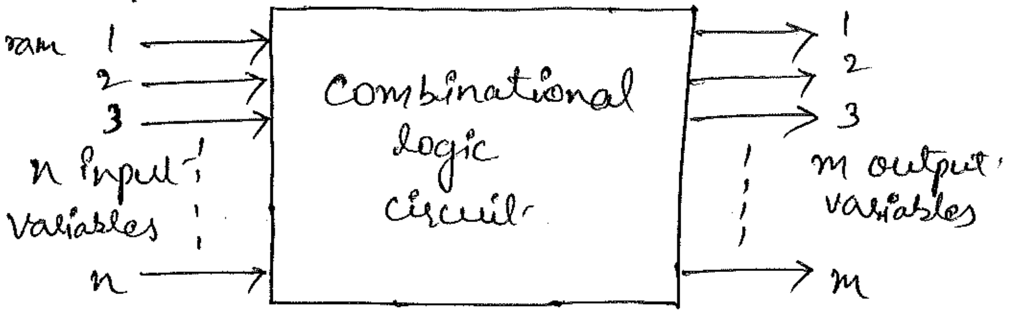
$$f(A,B,C,D) = \bar{A} + AB + \bar{A}(B+D)$$
$$= \bar{A} + [\bar{1} + 1B + \bar{1}(B+D)]$$
$$= \bar{A} + [0 + B + 0(B+D)]$$
$$= \bar{A} + B$$

→ combinational circuits :-

* If any logic circuit output depends only on the present input, with no storage involved, that type of logic circuit called as a combinational logic circuits

Eg:- Arithematic circuits, code converter, multiplexer/ demultiplexer, encoder/decorder etc

Fig:- Block diagram of a combinational logic circuit



* Combinational circuits are easy to design but require more hardware.
* More expensive circuit.
* Combinational circuits are faster in speed, because delay between inputs and outputs is due to propogation delay of gates.
* The behaviour is defined by the set of output functions only.
* Designer has less flexibility since the output depends only on the present inputs.
  Eg:- parallel Adder.

→ Incomplete Boolean functions and Don't care conditions :-

In some logic circuits, certain input conditions never occur, therefore the corresponding output never appears. In such cases the output level is not defined, it can be either HIGH or LOW.

These output levels are indicated by 'X' or 'd' in the truth table and are called don't care o/p's or don't care conditions or incompletely specified functions.

Here o/p's are defined for i/p conditions from 000 to 101. for remaining two conditions of i/p, o/p is not defined hence these are called don't care conditions.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	X
1	1	1	X

* Don't care conditions in logic design :-

consider the logic circuit for an even parity generator for 4-bit BCD number.

A	B	C	D	P
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	—
1	0	1	1	—
1	1	0	0	—
1	1	0	1	—
1	1	1	0	—
1	1	1	1	—

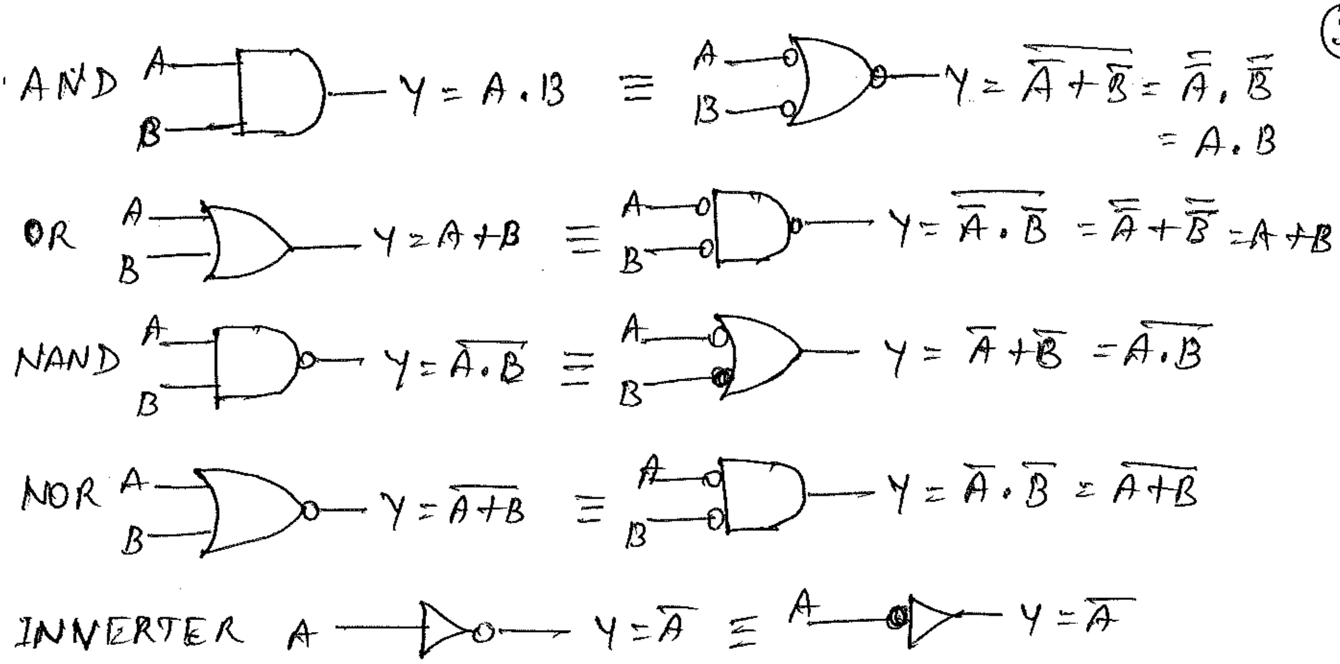
The Boolean function for even parity generator with 4-bit BCD input can be Expressed in minterm canonical formula as,

$$f(A,B,C,D) = \sum m\,(1,2,4,7,8) + d\,(10,11,12,13,14,15)$$

→ Boolean operations and gates :-

* Alternative logic-gate representation

The five basic logic gates are AND, OR, INVERTER, NAND and NOR and the standard symbols used to represent them on logic-network diagrams.



AND $Y = A \cdot B \equiv Y = \overline{\bar{A} + \bar{B}} = \bar{\bar{A}} \cdot \bar{\bar{B}} = A \cdot B$

OR $Y = A + B \equiv Y = \overline{\bar{A} \cdot \bar{B}} = \bar{\bar{A}} + \bar{\bar{B}} = A + B$

NAND $Y = \overline{A \cdot B} \equiv Y = \bar{A} + \bar{B} = \overline{A \cdot B}$

NOR $Y = \overline{A + B} \equiv Y = \bar{A} \cdot \bar{B} = \overline{A + B}$

INVERTER $Y = \bar{A} \equiv Y = \bar{A}$

→ Logic gates – Definitions

* Logic gate is an Electronic circuit that performs a Boolean algebraic function.
* Not gate or Inverter is A logic gate that changes its input logic level to the opposite logic level.
* Bubble is A small circle indicating logical inversion on a circuit symbol.
* AND gate is A logic circuit whose output is HIGH when all inputs are HIGH.
* OR gate is A logic circuit whose output is high when atleast one input is high.
* NAND gate is A logic circuit whose output is low when all inputs are HIGH.
* NOR gate is A logic circuit whose output is low when any one inputs are high.
* Ex (Exclusive) OR gate or XOR gate is A two input logic circuit whose output is HIGH when one input (but not both) is HIGH.
* Ex NOR gate or XNOR gate is A two input logic circuit whose output is the complement of ExOR gate.

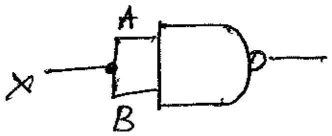
→ Universal Gates :-

The NAND and NOR gates are known as universal gates, since any logic function can be implemented using NAND or NOR gates.

* NAND gate :-

The NAND gate can be used to generate the NOT function, the AND function, the OR function, and the NOR function.

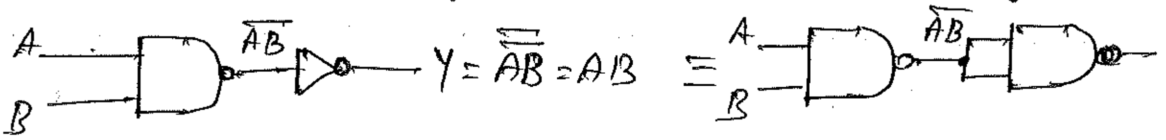
NOT function :- An inverter can be made from a NAND gate by connecting all of the inputs together and creating, in effect, a single common input, for a two-input gate.



$Y = \overline{AB} = \overline{XX} = \overline{X} + \overline{X} = \overline{X}$ when $X = 0$, $Y = 1$; $X = 1$, $Y = 0$

Fig:- NOT function using NAND gate.

AND function :- An AND function can be generated using only NAND gates. It is generated by simply inverting output of NAND gate.



$Y = \overline{\overline{AB}} = AB \equiv Y = \overline{\overline{AB}} = AB$

Fig:- AND function using NAND gates

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

≡

A	B	$\overline{AB}$	$\overline{\overline{AB}}$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

OR function :- OR function is generated using only NAND gates $Y = A + B = \overline{\overline{A}} + \overline{\overline{B}} = \overline{\overline{A} \cdot \overline{B}}$

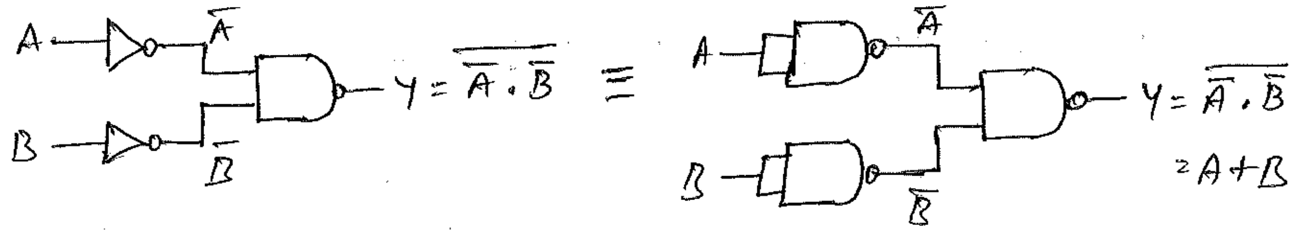


Fig: OR function using only NAND gates

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

≡

A	B	$\bar{A} \cdot \bar{B}$	$\overline{\bar{A} \cdot \bar{B}}$
0	0	1	0
0	1	0	1
1	0	0	1
1	1	0	1

NOR function: NOR function is generated using only NAND gates. $Y = \overline{A+B} = \bar{A} \cdot \bar{B} = \overline{\overline{\bar{A} \cdot \bar{B}}}$

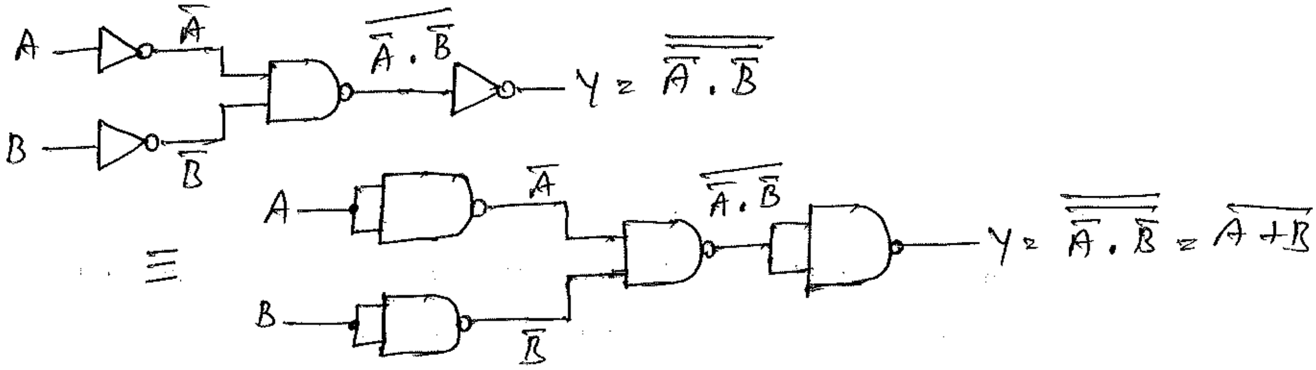


Fig: NOR function using only NAND gates

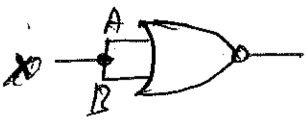
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

≡

A	B	$\bar{A} \cdot \bar{B}$	$\overline{\bar{A} \cdot \bar{B}}$	$\overline{\overline{\bar{A} \cdot \bar{B}}}$
0	0	1	0	1
0	1	0	1	0
1	0	0	1	0
1	1	0	1	0

* NOR Gate: The NOR gate is also a universal gate, since it can be used to generate the NOT, AND, OR and NAND functions.

NOT function: An inverter can be made from a NOR gate by connecting all of the inputs together and creating, in effect, a single common input.



$Y = \overline{A+B} = \overline{x+x} = \bar{x}$ when $x=0, Y=1$
$x=1, Y=0$

Fig: NOT function using NOR gate.

OR function: An OR function can be generated using only NOR gates.

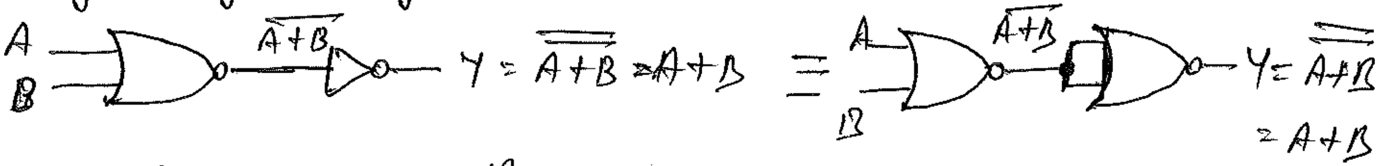


Fig: OR function using NOR gates.

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

≡

A	B	$\overline{A+B}$	$\overline{\overline{A+B}}$
0	0	1	0
0	1	0	1
1	0	0	1
1	1	0	1

AND function:

AND function is generated using only NOR gates.

$Y = A.B = \overline{\overline{A}.\overline{B}} = \overline{\overline{A}+\overline{B}}$

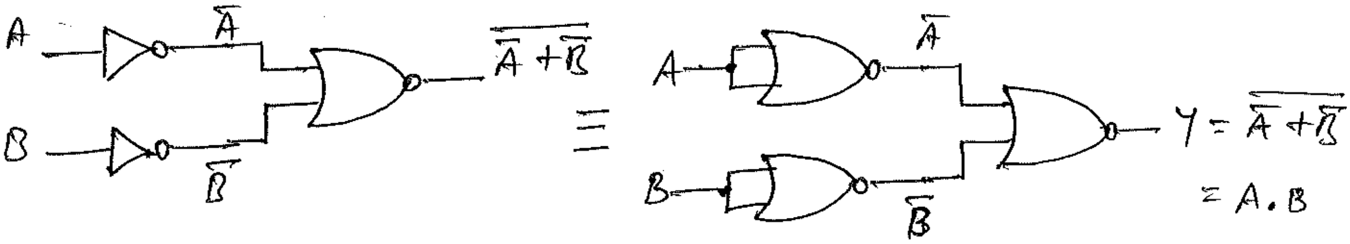


Fig: AND function using NOR gates

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

≡

A	B	$\overline{A}+\overline{B}$	$\overline{\overline{A}+\overline{B}}$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

NAND function :-

NAND function is generated using only NOR gates.

$Y = \overline{A \cdot B} = \overline{A} + \overline{B} = \overline{\overline{\overline{A} + \overline{B}}}$.

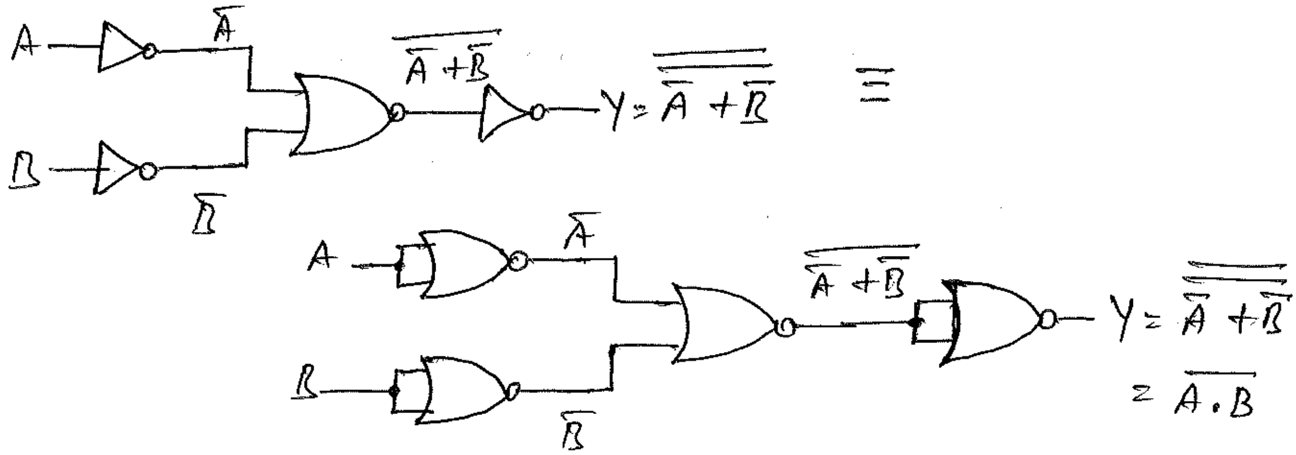


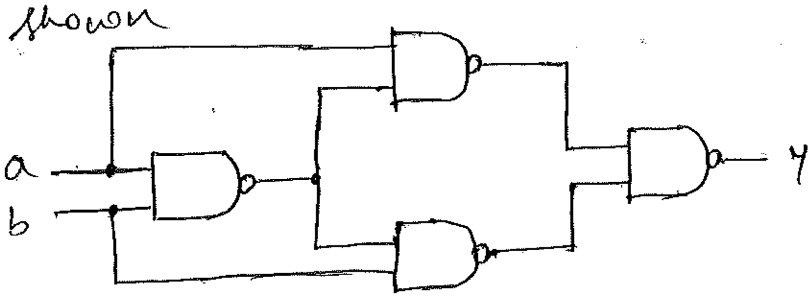
Fig: NAND function using only NOR gates

A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

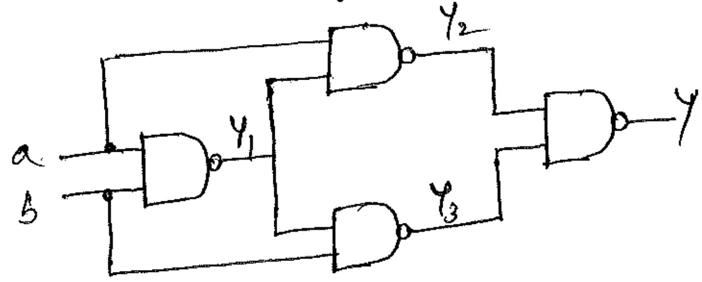
≡

A	B	$\overline{A}+\overline{B}$	$\overline{\overline{A}+\overline{B}}$	$\overline{\overline{\overline{A}+\overline{B}}}$
0	0	1	0	1
0	1	1	0	1
1	0	1	0	1
1	1	0	1	0

Eg: 1) Write the Boolean Expression for the schematic shown



Soln. Label the gate outputs as Y_1, Y_2 and Y_3



$Y_1 = \overline{ab}$, $Y_2 = \overline{aY_1} = \overline{a\overline{ab}}$,

$Y_3 = \overline{bY_1} = \overline{b\overline{ab}}$

$Y = \overline{Y_2 \cdot Y_3} = \overline{\overline{a\overline{ab}} \cdot \overline{b\overline{ab}}}$

$= \overline{\overline{a\overline{ab}}} + \overline{\overline{b\overline{ab}}}$

$= a\overline{ab} + b\overline{ab}$

$= \overline{ab}(\overline{a}+\overline{b})aa$

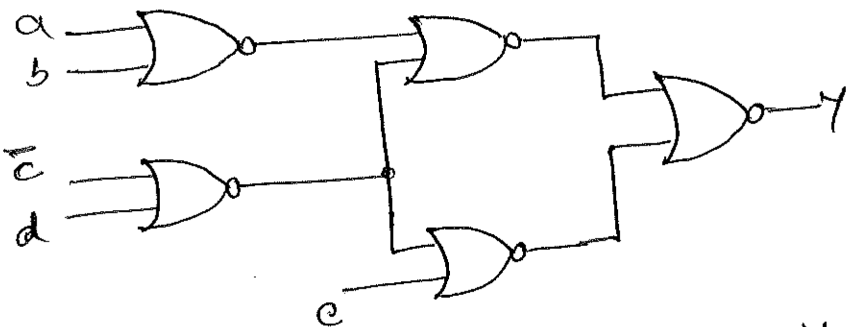
$= \overline{\overline{a}\overline{b}}\,(a+b)$

$= (\overline{\overline{a}}+\overline{\overline{b}})(a+b)$

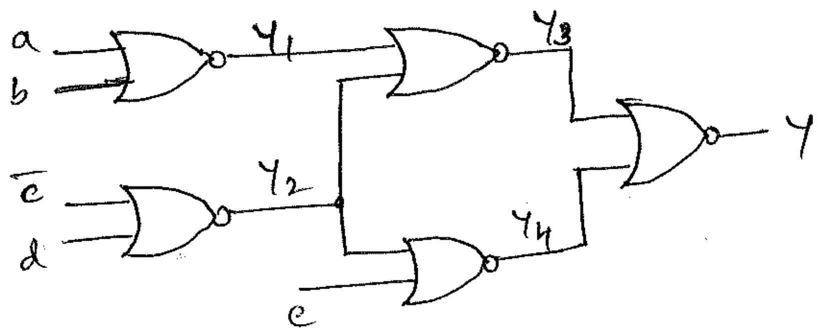
$= \cancel{a\overline{a}}^{0} + \overline{a}b + a\overline{b} + \cancel{b\overline{b}}^{0}$

$Y = \overline{a}b + a\overline{b}$

2) Write the Boolean Expression for the schematic shown below



Soln: Label the gate o/p's as Y_1, Y_2, Y_3 and Y_4



$Y_1 = \overline{a+b}$,

$Y_2 = \overline{\overline{c}+d}$

$Y_3 = \overline{Y_1+Y_2}$

$= \overline{\overline{a+b}+\overline{\overline{c}+d}}$

$Y_4 = \overline{Y_2+e}$

$= \overline{\overline{\overline{c}+d}+e}$

$Y = \overline{Y_3+Y_4}$

$= \overline{\overline{\overline{\overline{a+b}+\overline{\overline{c}+d}}} + \overline{\overline{\overline{c}+d}+e}}$

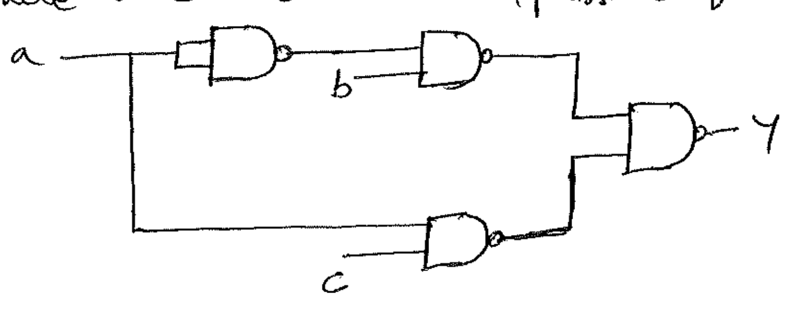
$= \left(\overline{\overline{\overline{a+b}+\overline{\overline{c}+d}}}\right)\left(\overline{\overline{\overline{\overline{c}+d}+e}}\right)$

$= \left(\overline{a+b}+\overline{\overline{c}+d}\right)\left(\overline{\overline{c}+d}+e\right)$

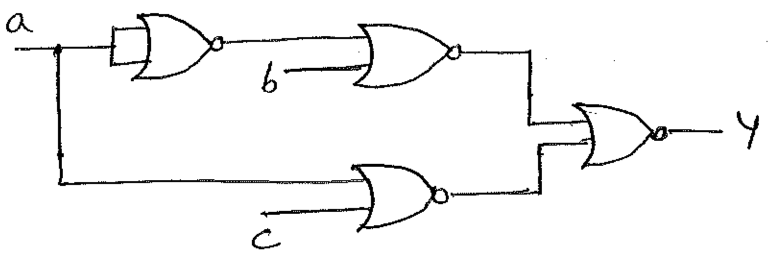
$= \cancel{a+b}\;(\overline{a}\,\overline{b}+c\overline{d})(c\overline{d}+e)$

3) Write the Boolean Expression for the following schematics.

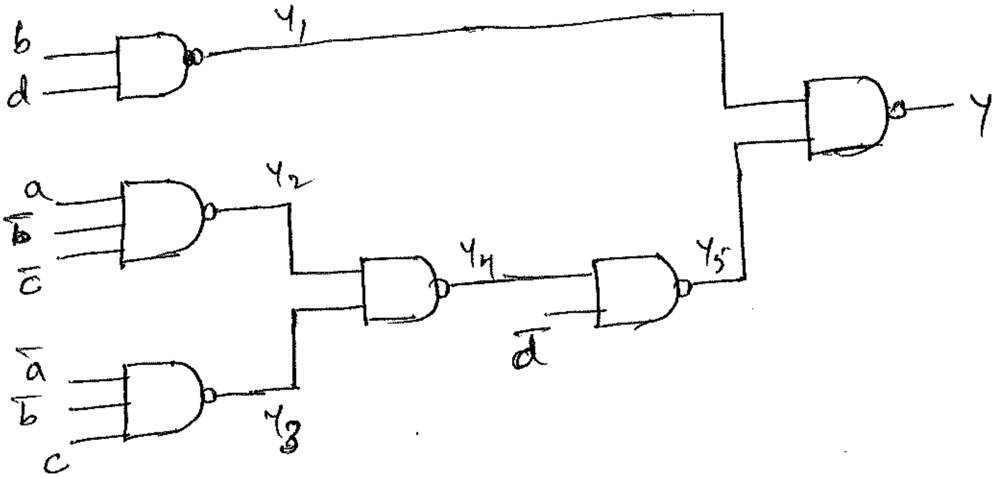
a)



Soln. $\overline{a}b + ac$

b) 

Soln: $(\overline{\bar{a}}+b)(a+c)$

c) 

$Y_1 = \overline{bd}$, $Y_2 = \overline{a\bar b\bar c}$, $Y_3 = \overline{\bar a\bar b c}$,

$Y_4 = \overline{Y_2\, Y_3} = \overline{\overline{a\bar b\bar c}\;\overline{\bar a\bar b c}} = \overline{\overline{a\bar b\bar c}} + \overline{\overline{\bar a\bar b c}}$

$= a\bar b\bar c + \bar a\bar b c$

$Y_5 = \overline{(a\bar b\bar c+\bar a\bar b c)\,\bar d}$ $= \overline{a\bar b\bar c\bar d + \bar a\bar b c\bar d}$

$= \overline{(a\bar b\bar c+\bar a\bar b c)} + \bar{\bar d}$ $= \overline{a\bar b\bar c\bar d}\cdot\overline{\bar a\bar b c\bar d}$

$= (\overline{a\bar b\bar c})(\overline{\bar a\bar b c}) + d$ $= (\bar a+b+c+d)(a+b+\bar c+d)$

$= (\bar a+\bar{\bar b}+\bar{\bar c})(\bar{\bar a}+\bar{\bar b}+\bar c)+d$

$= (\bar a+b+c)(a+b+\bar c)+d$

$= a\bar a + \bar a b + \bar a\bar c + ad + ab + bb + b\bar c + bd + ac + ab$
$+ c\bar c + cd + ab + bd + \bar c d + dd$

$= \bar a b + \bar a\bar c + ab + b + b\bar c + bd + ac + cd + d$

$= b(a+\bar a) + \bar a\bar c + \bar c(b+1) + bd + ac + d(c+1)$

$= b + \bar a\bar c + \bar c + bd + ac + d$

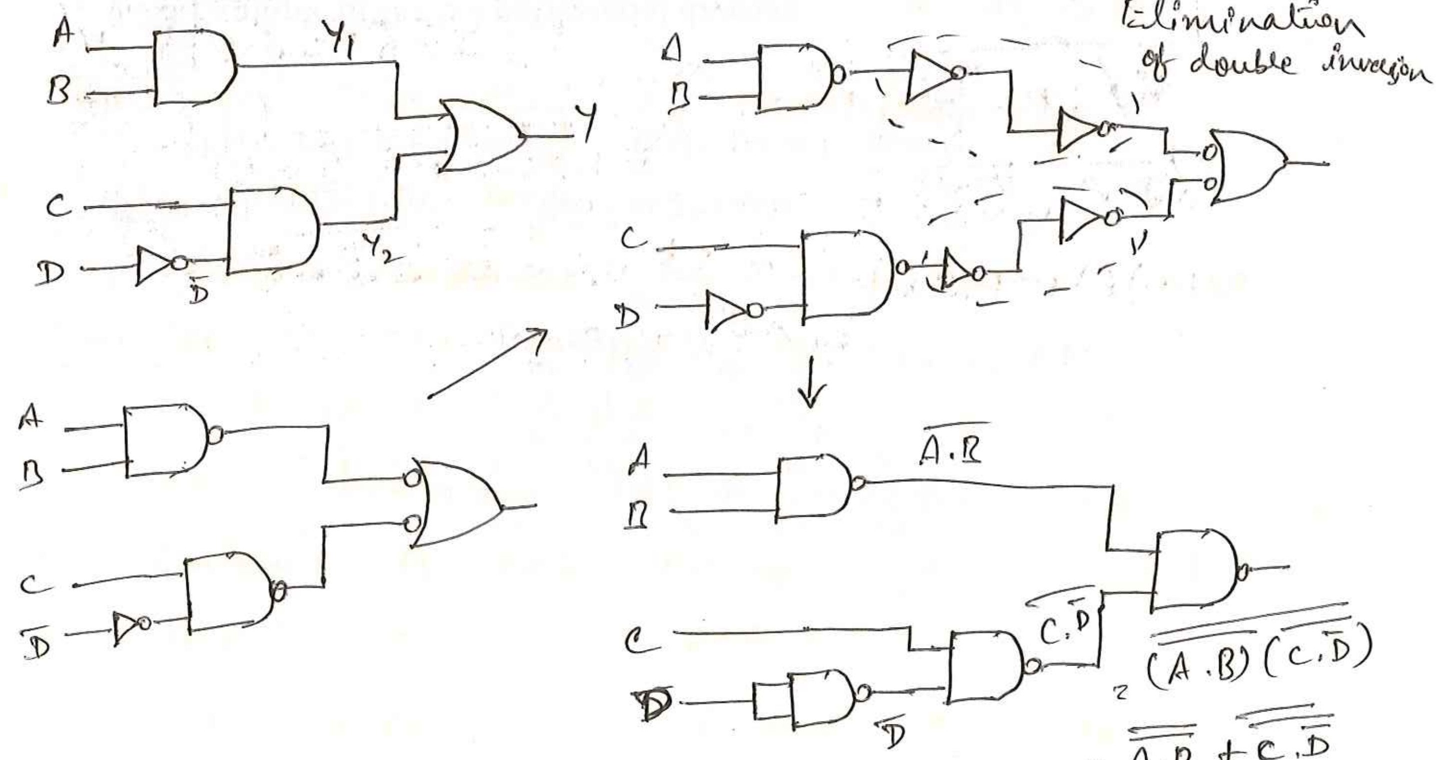
$= b(d+1) + \bar c(\bar a+1) + ac + d$

$Y = b + \bar c + ac + d$

Conversion of AND/OR/NOT Logic to NAND/NOR logic circuit

To implement Boolean Expression $AB+C\overline{D}$, we require two AND gates, one OR gate and one inverter. This requires three standard IC's. Two AND gates from AND IC and only OR gate and one inverted are utilized from OR and Inverter IC's. other gates from three ICs are not utilized. To improve utilization of ICs and to reduce number of ICs required, one can use only NAND/NOR gates to implement Boolean Expression. We have to convert given AND/OR/NOT Boolean Expression logic to NAND/NOR logic.

Eg: $AB+C\overline{D}$ Using NAND circuit.

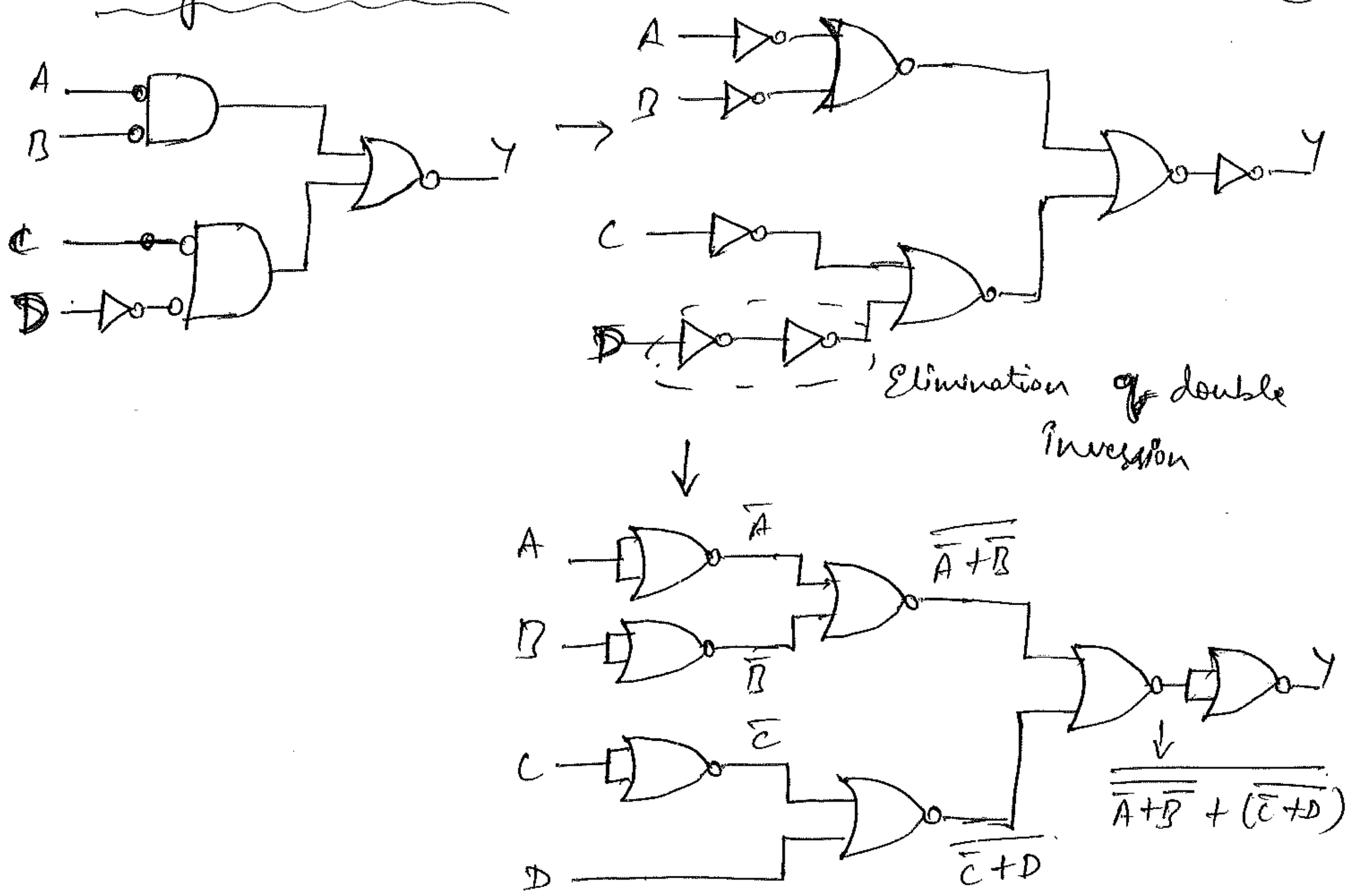


$= \overline{\overline{(A.B)}\,\overline{(C.\overline{D})}}$

$= \overline{\overline{A.B}} + \overline{\overline{C.\overline{D}}}$

$= A.B + C.\overline{D}$

using NOR circuit



$Y = AB + C\bar{D}$

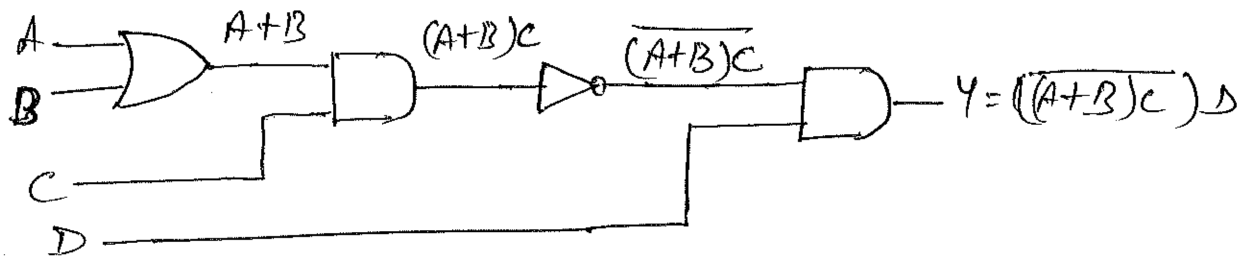
Steps for converting to NAND/NOR logic

* Draw AND/OR logic
* If NAND hardware has been chosen, add bubbles on the output of each AND gate; and bubbles on inputs side to all OR gates.
* If NOR hardware has been chosen add bubbles on output of each OR gate and bubble on input of each AND gate.
* Add or subtract an inverter on each line that received a bubble in step 2 or 3.
* Replace bubbled OR by NAND and bubbled AND by NOR.
* Eliminate double inversions.

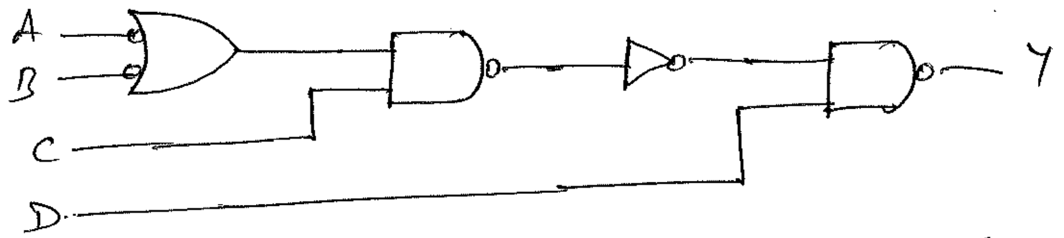
Eg: 1) Boolean Expression : $(\overline{(A+B)C})D$

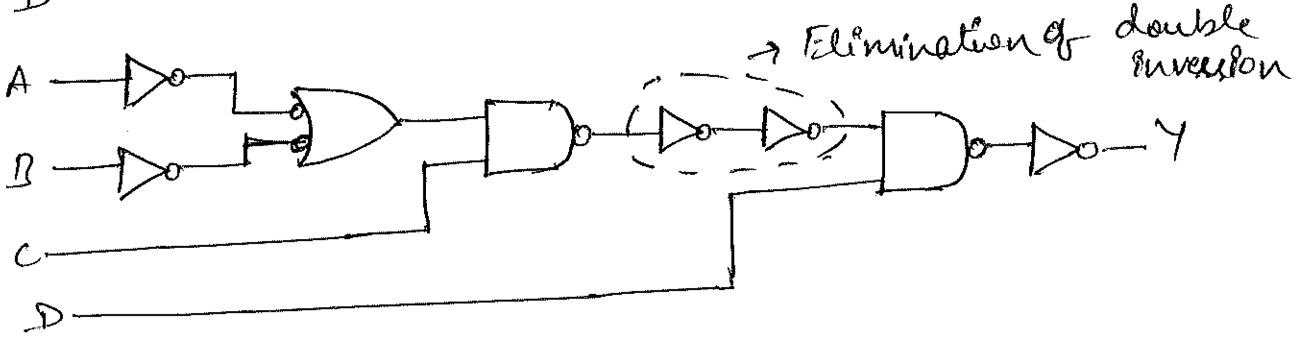
Design using NAND and NOR circuit.

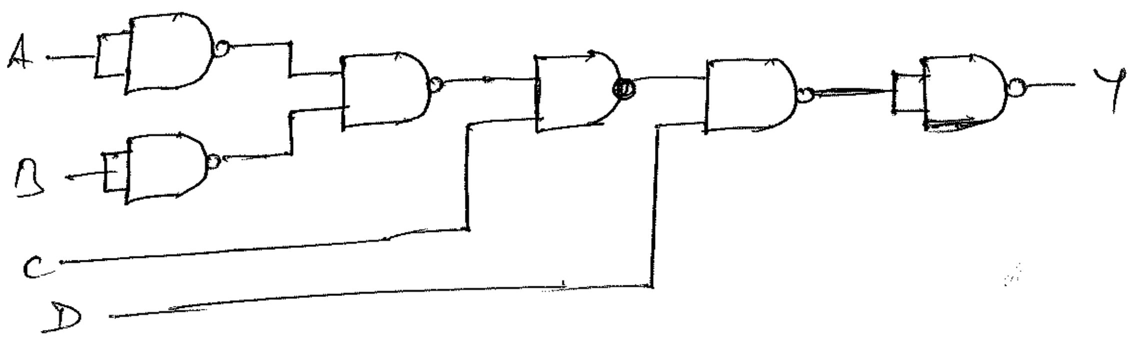
Soln:



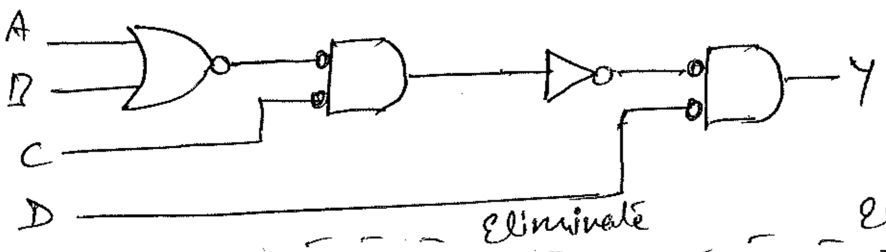
NAND circuit

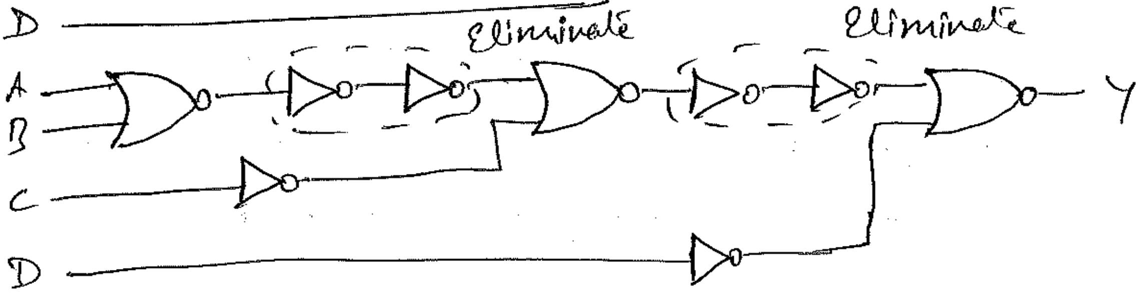


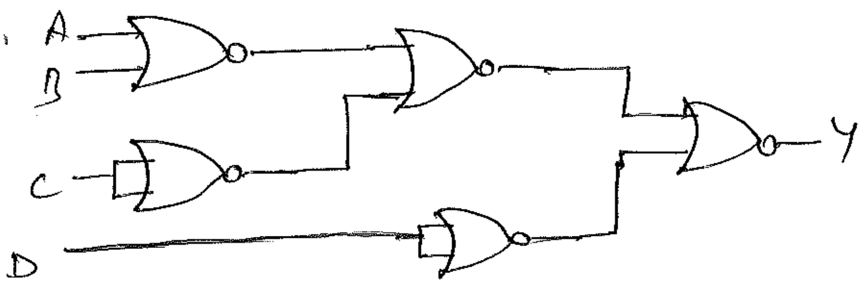




NOR circuit

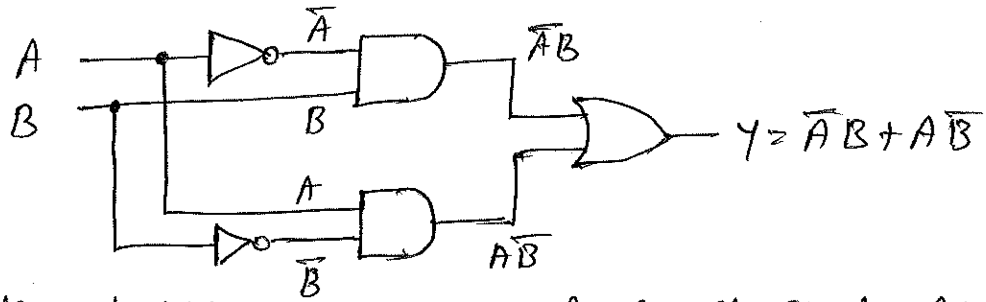




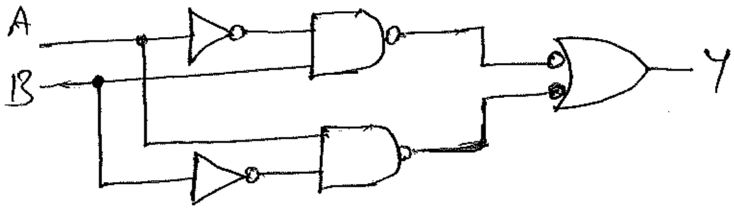


2) Implement the Boolean Expression for EX-OR gate using NAND gates.

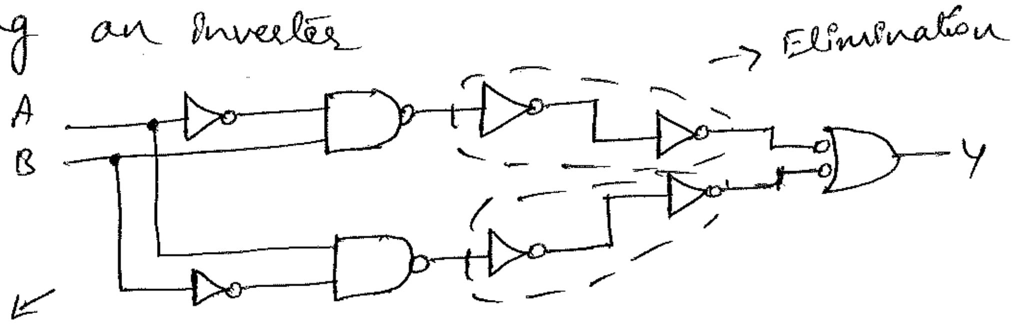
Soln: Boolean Expression for Ex-OR gate is $\bar{A}B + A\bar{B}$

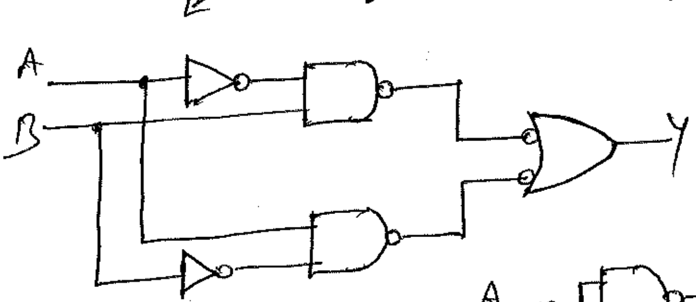


Adding bubbles on the output of each AND gates and on the inputs of each OR gate

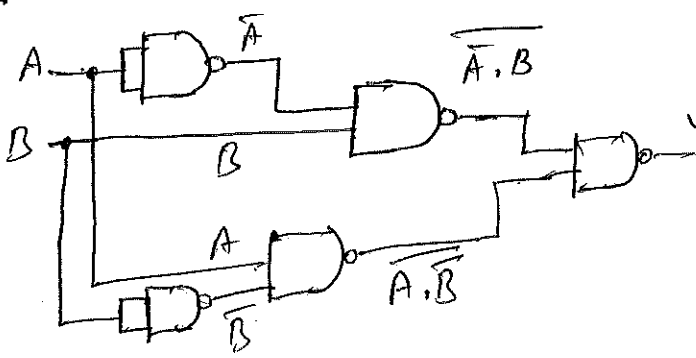


Adding an Inverter





Replacing Inverter and bubbled OR with NAND



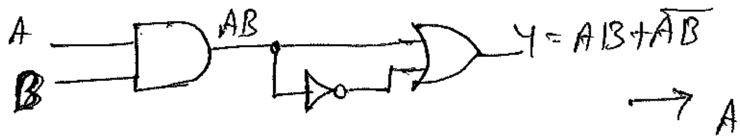
$Y = \overline{\overline{\bar{A}B} \cdot \overline{\bar{B}A}}$

$= \bar{A}B + A\bar{B}$

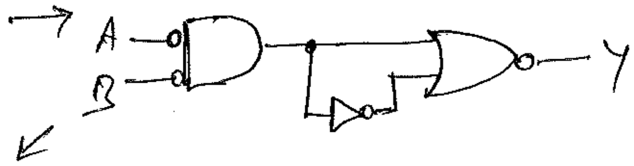
3) Implement Boolean Expression for Ex NOR gate using NOR gates.

Soln: Boolean Expression for Ex-NOR gate is

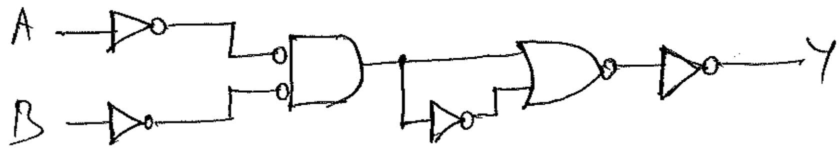
$AB + \overline{AB}$



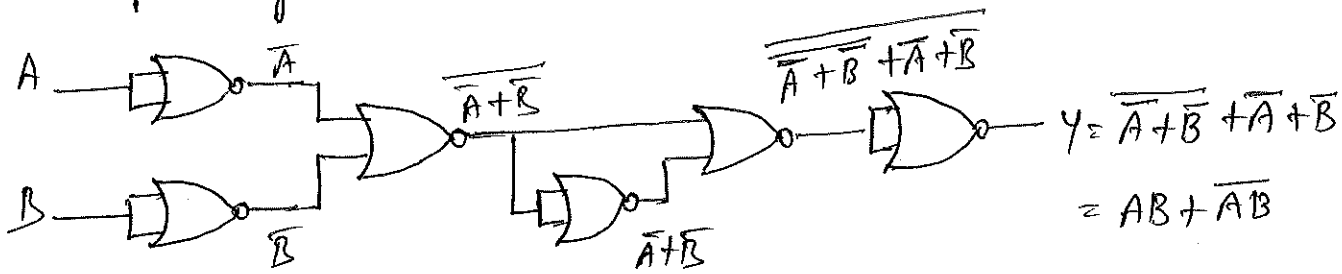
Adding bubbles



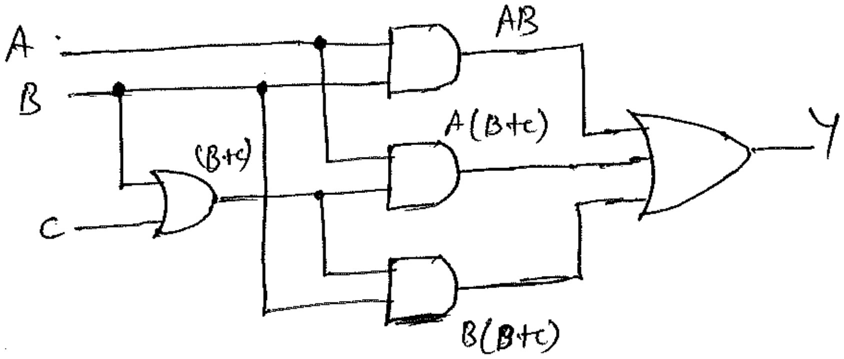
Adding an inverter



Replacing inverter and bubbled AND with NOR



4) Simplify the given logic circuit shown in below figure and implement simplified logic circuit using logic gates.



Solution

$Y = AB + A(B+c) + B(B+c)$

$= AB + AB + Ac + BB + Bc$

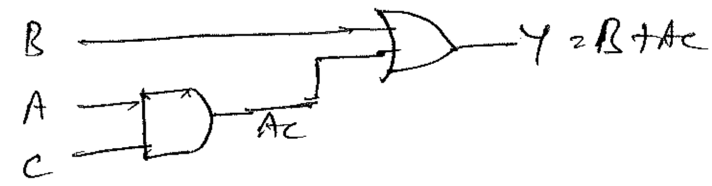
$= AB + Ac + B + Bc$

$= AB + Ac + B(1+c) = AB + Ac + B = B(A+1) + Ac$

$= \underline{B + Ac}$

Simplified logic circuit.

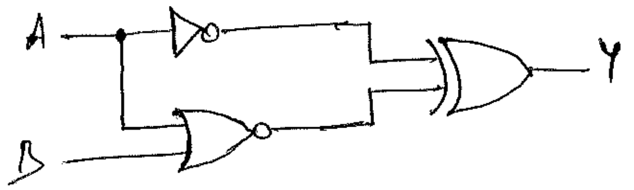
$Y = B + AC$



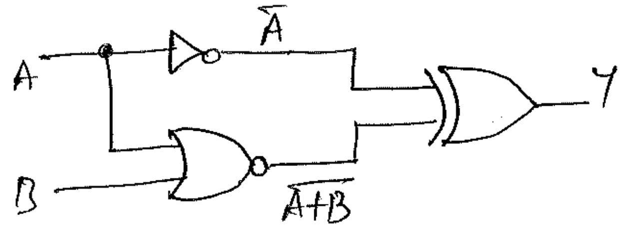
(OR)

5) Draw a logic circuit to implement the function $F = AB + A(B+C) + B(B+C)$ and simplify the function and draw logic circuit for the simplified function.

6) Determine the truth table for the circuit shown in below figure.



Soln



$Y = \overline{A} \oplus \overline{A+B}$

$= \overline{\overline{A}}\;\overline{A+B} + \overline{A}\;\overline{\overline{A+B}}$

$= \cancel{A\;\overline{A+B}}$

$= A(\overline{A} \cdot \overline{B}) + \overline{A}(A+B)$

$= A\overline{A}\,\overline{B} + \overline{A}A + \overline{A}B$

$= \overline{A}B$

Truth table

A	B	$\overline{A}$	$\overline{A}B$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	0	0

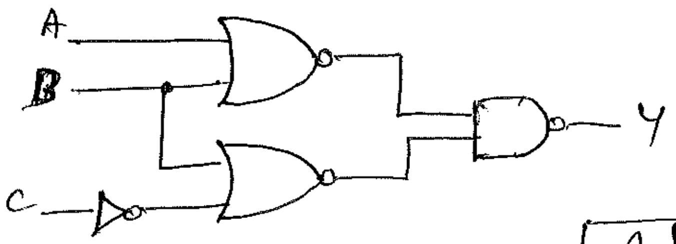
A	$\overline{A}$	B	$\overline{B}$	$\overline{A}B$	$A\overline{B}$	$\overline{A}B + A\overline{B}$
0	1	0	1	0	0	0
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	0	1	0	0	0	0

$Y = \overline{A}B + A\overline{B} = A \oplus B$

$A \oplus B = \overline{A}B + A\overline{B}$

$\overline{A} \oplus \overline{A+B} = \overline{\overline{A}}\;\overline{A+B} + \overline{A}\;\overline{\overline{A+B}}$

Eg:- Determine the truth table for the circuit shown in below figure.



Soln.

$$Y = \overline{\overline{A+B} \cdot \overline{B+\bar{C}}}$$

$$= \overline{\overline{A+B}} + \overline{\overline{B+\bar{C}}}$$

$$= A+B+B+\bar{C}$$

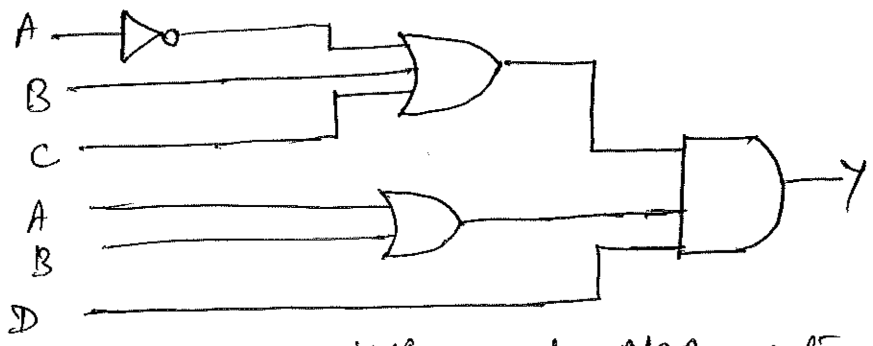
$$Y = A+B+\bar{C}$$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

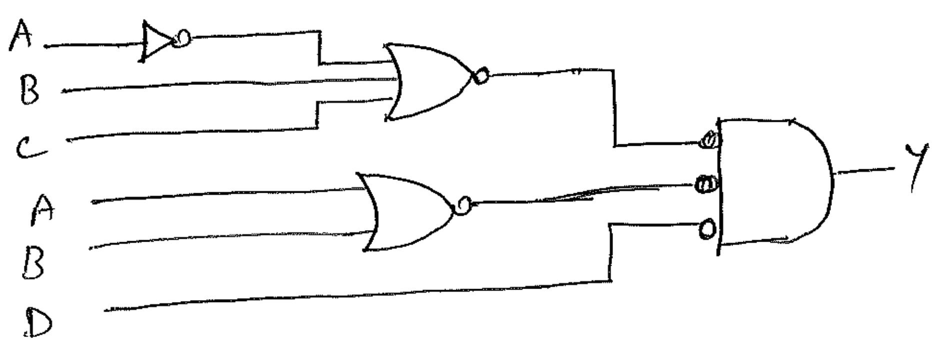
2) Implement the following Boolean function using only NOR gates. $Y = (\bar{A}+B+C)(A+B)D$

Soln:-

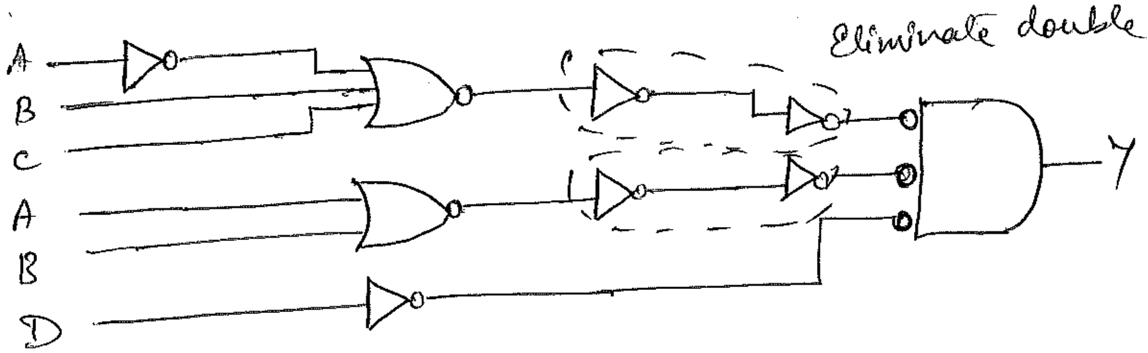
$$Y = (\bar{A}+B+C)(A+B)D$$

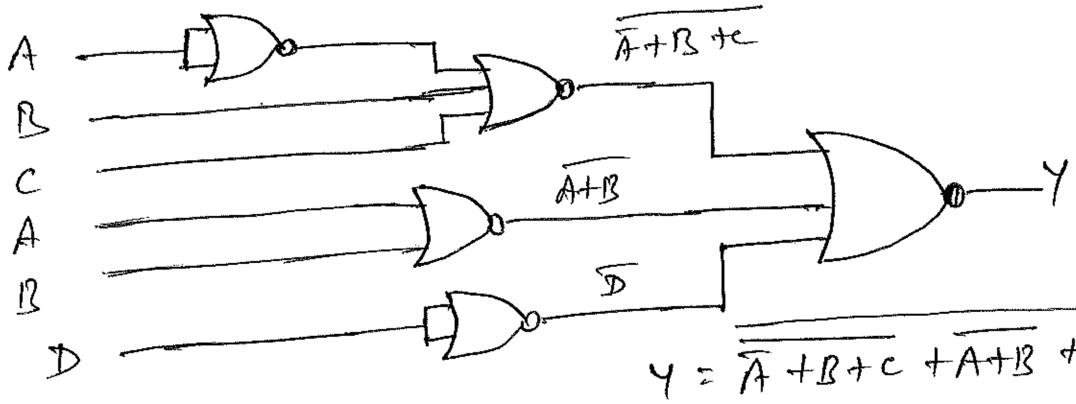


using only NOR gates



Eliminate double inversion





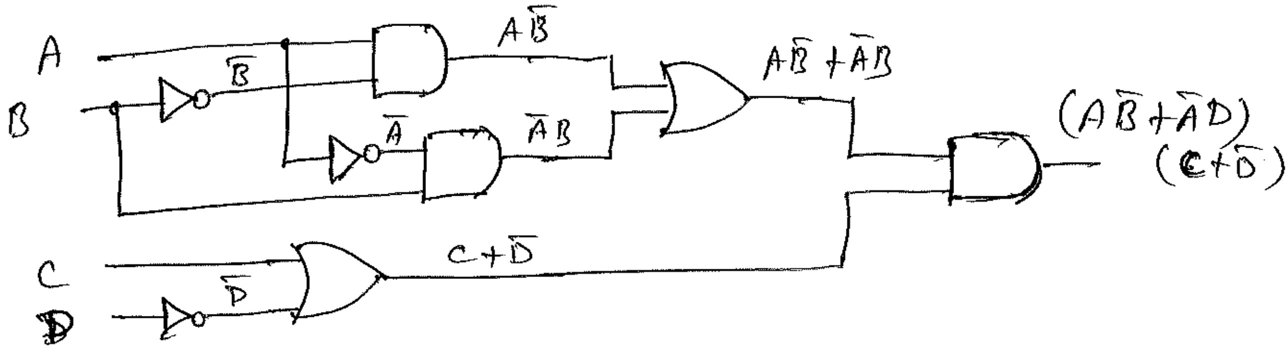
$$Y = \overline{\overline{\bar{A}+B+C} + \overline{A+B} + \bar{D}}$$

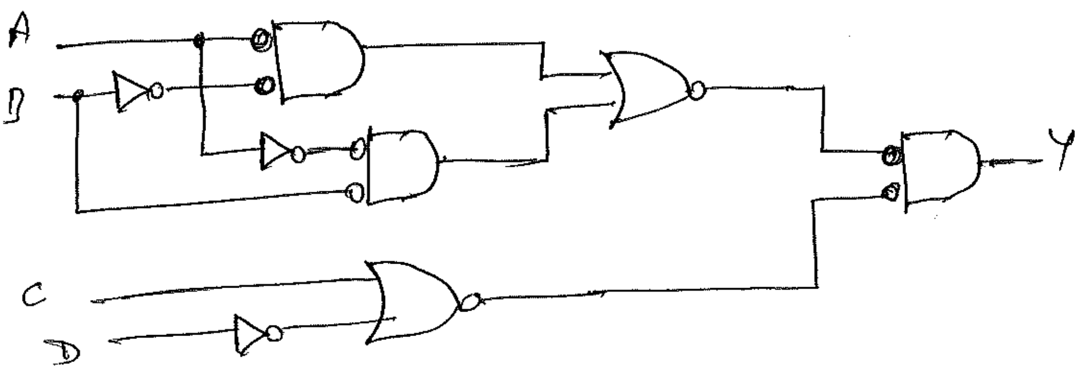
$$= \left(\overline{\overline{\bar{A}+B+C}}\right)\left(\overline{\overline{A+B}}\right)\left(\overline{\bar{D}}\right)$$

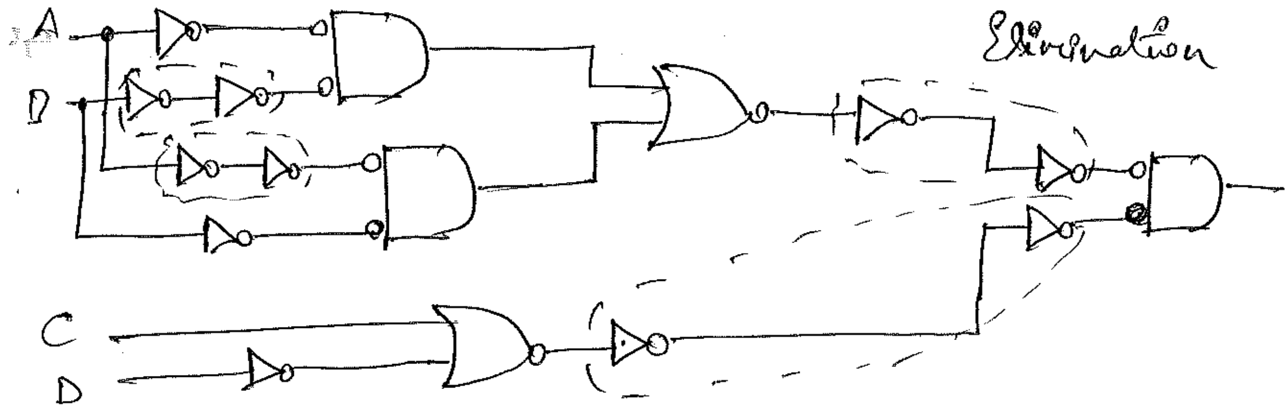
$$= (\bar{A}+B+C)(A+B)D$$

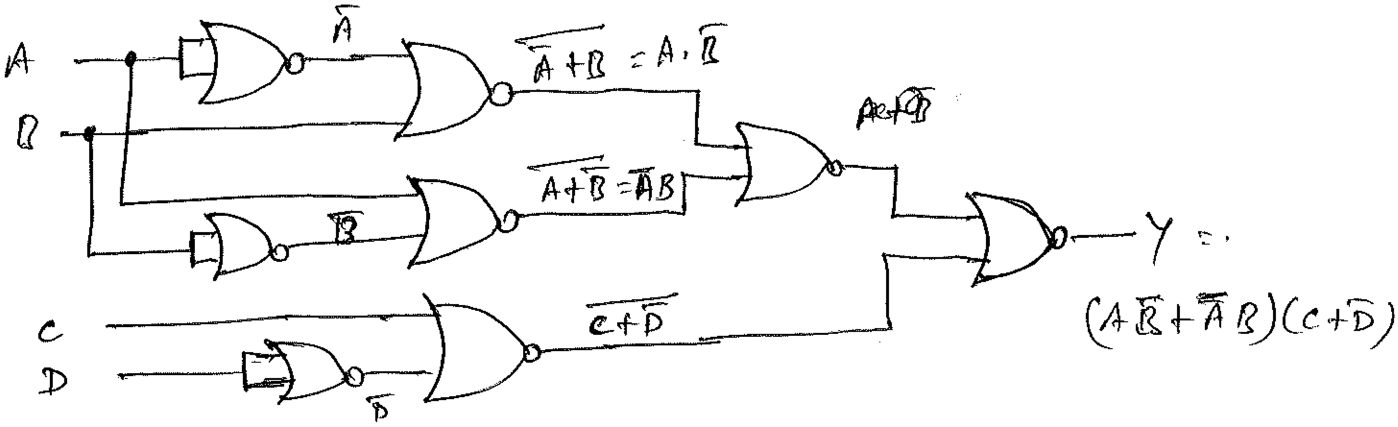
3) Implement the following Boolean function using NOR gates. $Y = (A\bar{B} + \bar{A}B)(C+\bar{D})$

Soln:







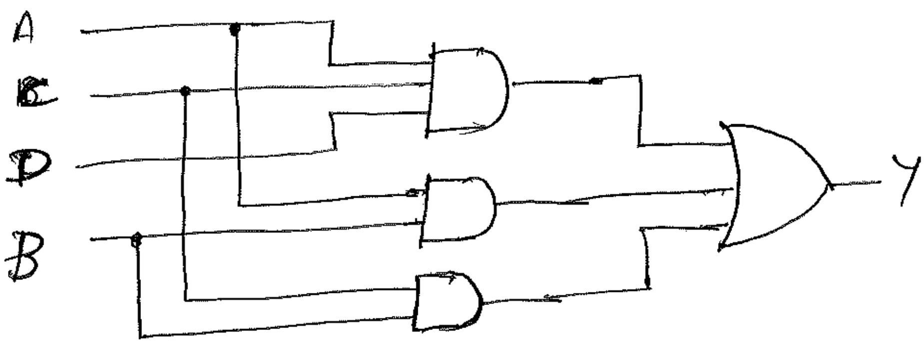


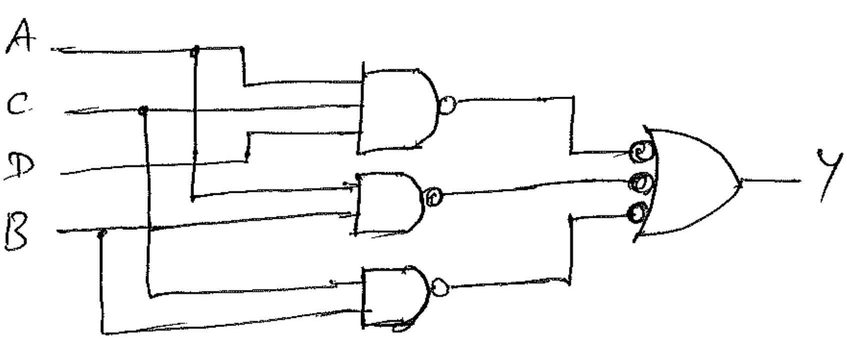
3) Implement the following Boolean Expression using NAND gates only $F = A(CD+B)+BC$

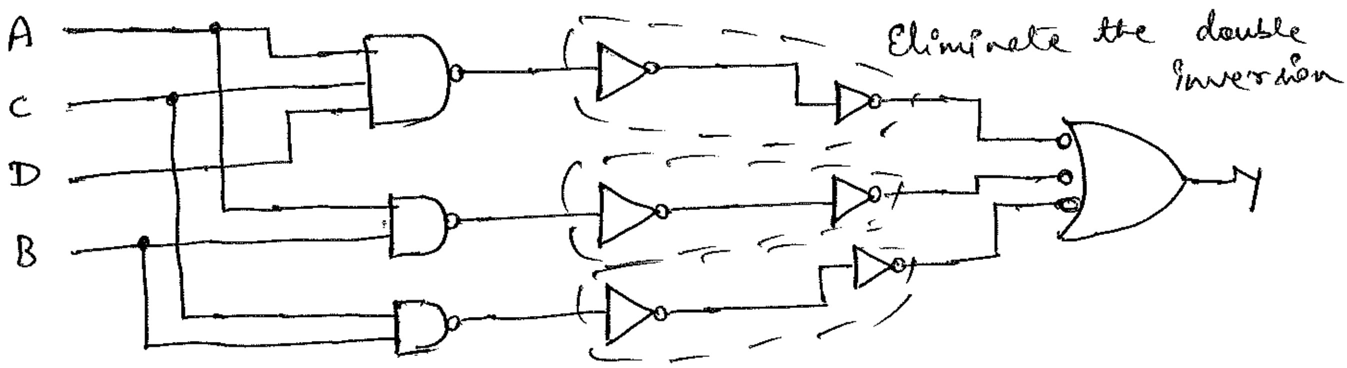
Soln.

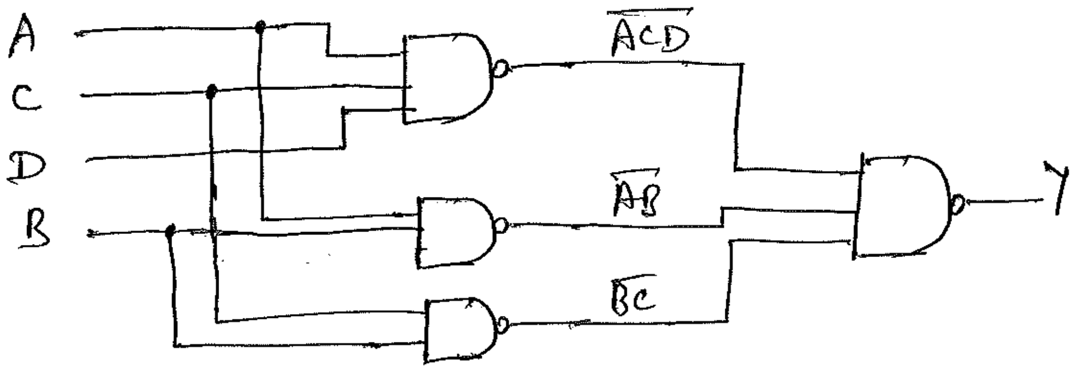
$$F = A(CD+B)+BC$$

$$= ACD + AB + BC$$









$$Y = \overline{\overline{ACD} \cdot \overline{AB} \cdot \overline{BC}}$$

$$= \overline{\overline{ACD}} + \overline{\overline{AB}} + \overline{\overline{BC}}$$

$$= ACD + AB + BC$$

$$Y = A(CD+B) + BC$$

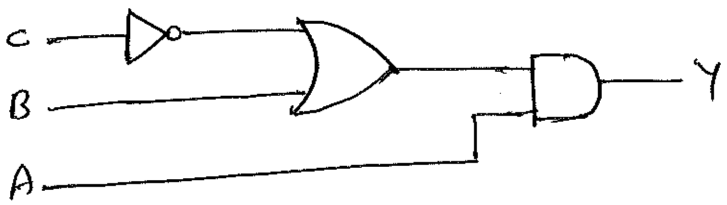
4) Implement the following Boolean Equation using only NAND gates $Y = AB + CDE + F$

5) Implement the following using NAND gates only $Y = (a+c)(\bar{b}+\bar{d})(\bar{a}+\bar{b}+\bar{c})$

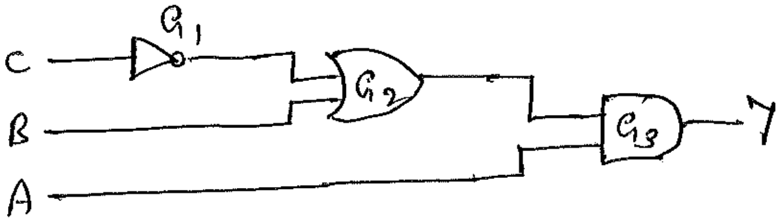
→ Analysis of combinational circuits :-

- check the output of initial gate.
- check the output of next exceeding gate, write the Expression
- Write the final Expression.

1) Analyze the following circuit by writing its Boolean Expression.



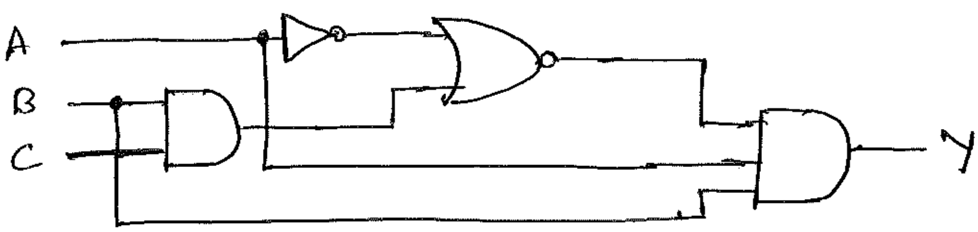
Soln.



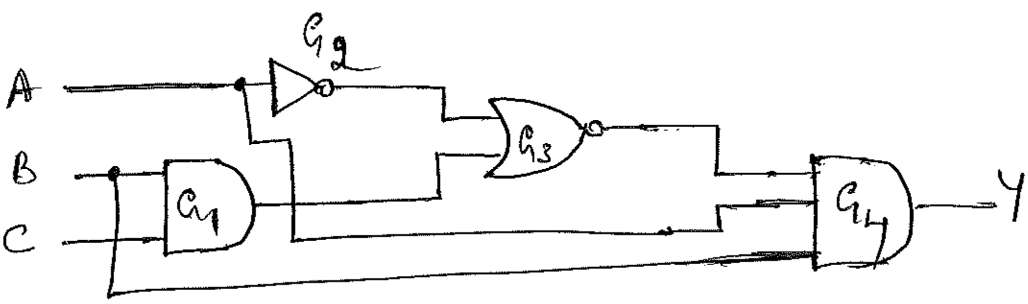
$G_1 = \overline{C}$, $G_2 = (B+\overline{C})$, $G_3 = A(B+\overline{C})$

$Y = A(B+\overline{C})$

2) Analyze the following circuit by writing its Boolean Expression.



Soln:-

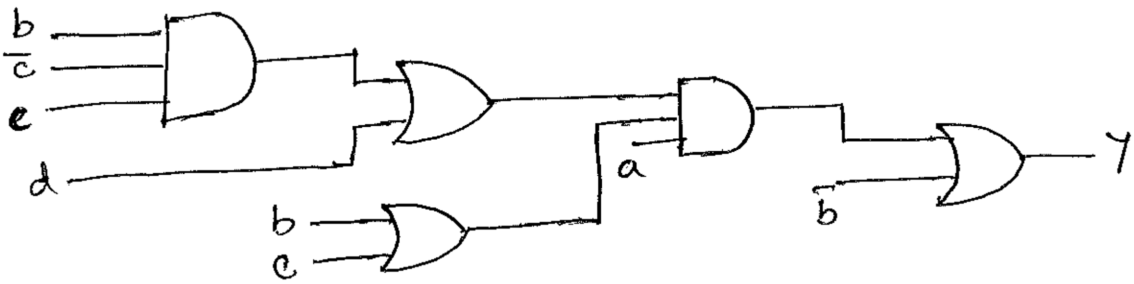


$G_1 = BC$, $G_2 = \overline{A}$, $G_3 = \overline{G_2 + G_1}$, $G_4 = AB\,G_3$

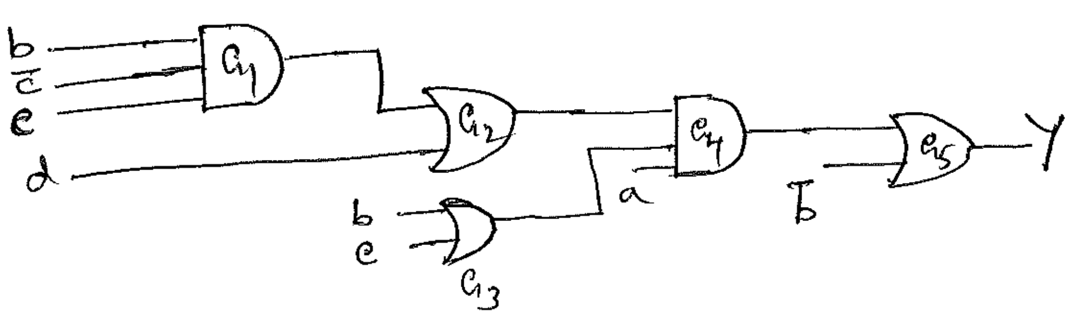
$= \overline{\overline{A}+BC}$ $\qquad = AB(\overline{\overline{A}+BC})$

$$G_4 = \overline{(\overline{A}+\overline{BC})}AB$$
$$= (\overline{\overline{A}}\cdot\overline{\overline{BC}})AB = A(\overline{B}+\overline{C})AB$$
$$= AB(\overline{B}+\overline{C}) = \cancel{AB\overline{B}}^{\,0} + AB\overline{C} = \underline{\underline{AB\overline{C}}}$$

3) Analyze the following circuit by writing its Boolean Expression.



Soln :-

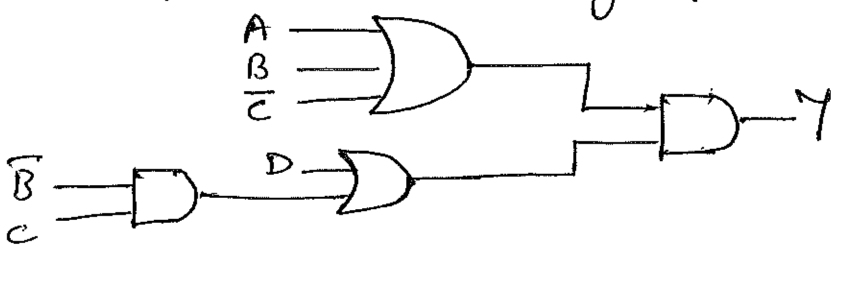


$G_1 = b\overline{c}e$, $G_2 = (b\overline{c}e+d)$, $G_3 = (b+e)$

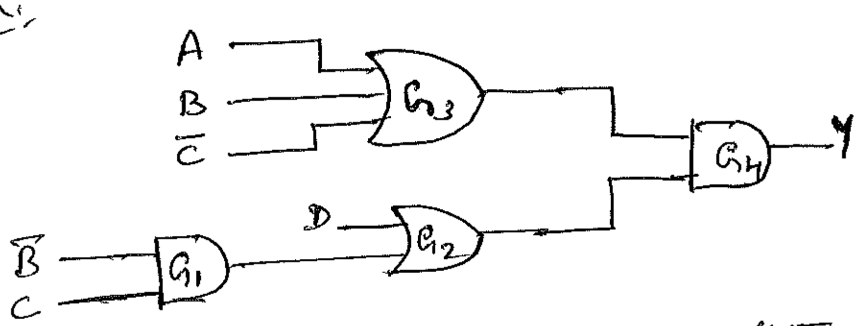
$$G_4 = (b\overline{c}e+d)(b+e)(a)$$
$$G_5 = (b\overline{c}e+d)(b+e)(a)+(\overline{b})$$

$$Y = ((b\overline{c}e+d)(b+e)a+\overline{b})$$
$$= (ab\overline{c}e+ad)(b+e)+\overline{b}$$
$$= ab\overline{c}e+ab\overline{c}e+abd+ade+\overline{b}$$
$$= \underline{\underline{ab\overline{c}e+abd+ade+\overline{b}}}$$

4) Analyze the following gate combinational network.



Soln.,



$G_1 = \overline{B}C$, $G_2 = \overline{B}C + D$, $G_3 = \cancel{AB\overline{C}}$, $G_4 = (A + B + \overline{C})(\overline{B}C + D)$

$= A + B + \overline{C}$

$$Y = A\overline{B}C + AD + \cancel{B\overline{B}C} + BD + \cancel{\overline{C}\overline{B}C} + \overline{C}D$$

$$Y = A\overline{B}C + AD + BD + \overline{C}D$$

→ **Synthesis of combinational circuits** :-

Synthesis involves the design of a combinational circuit resulting in its schematic diagram or gate diagram. Given the either the truth table or the Boolean Expression, we can write down the schematic diagram.

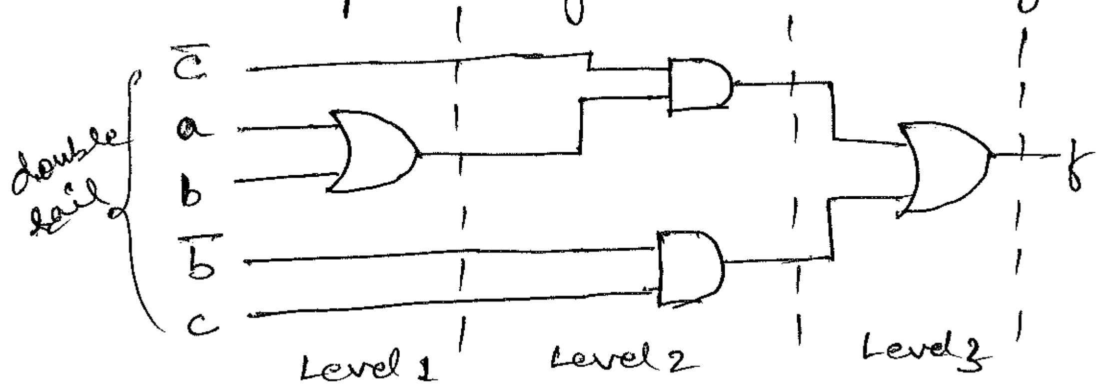
* For given or obtained gate circuit, when all the variables and their complements are available, the situation is described as Double rail logic
* For given or obtained gate circuit when all the complements are not available, the situation is referred to as single rail logic.

NOTE:- Whenever, the complement and the uncomplemented form of the inputs are used, then circuit is called as double rail logic.

1) Synthesis the following Boolean Expression.

$f(a,b,c,d) = \bar{c}(a+d) + \bar{b}c$

Soln. Implementing double rail logic

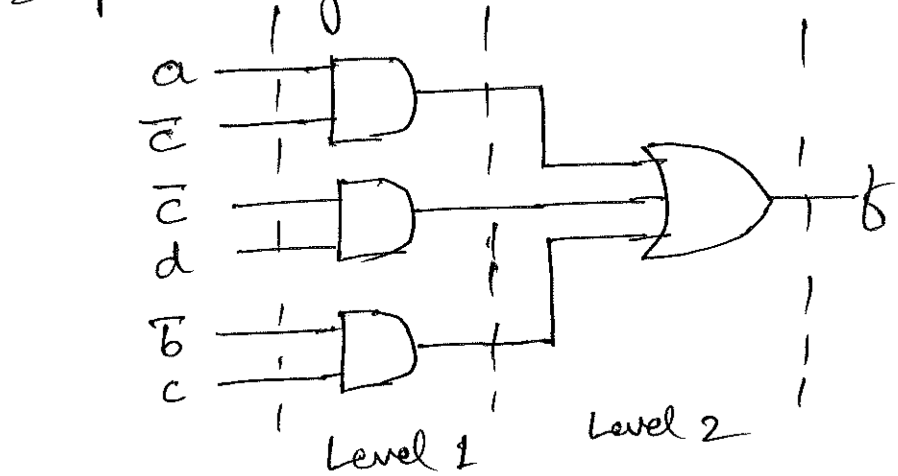


This requires a three-level gatting circuit.

Convert f into its Sop form

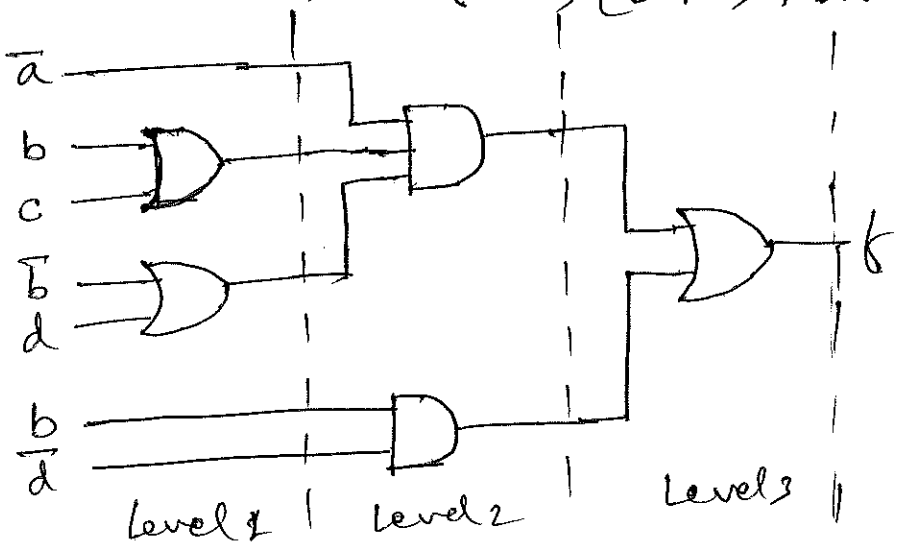
$f = a\bar{c} + \bar{c}d + \bar{b}c$

Implementing this to two level gating circuit.



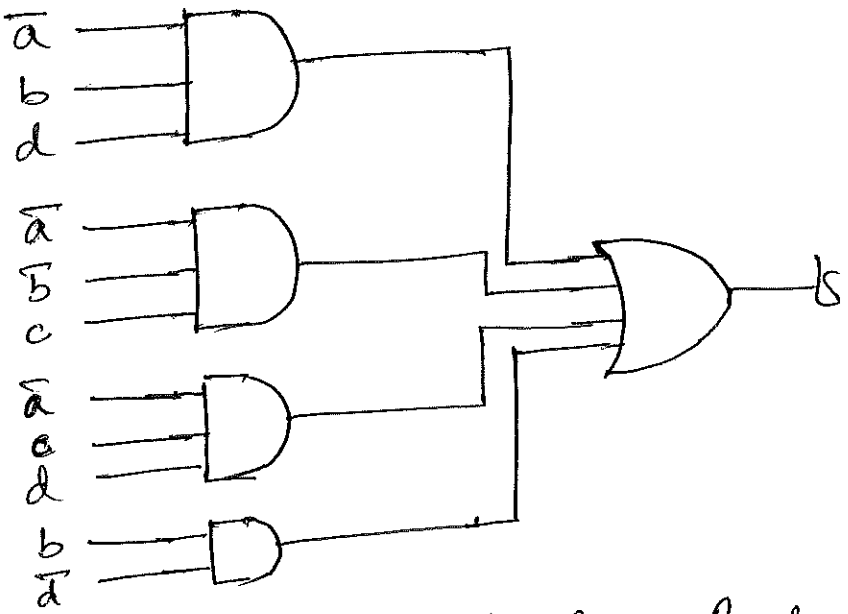
2) Synthesis the following Boolean Expression.

Soln: $f(a,b,c,d) = \bar{a}(b+c)(\bar{b}+d) + b\bar{d}$



Convert f into its SOP form

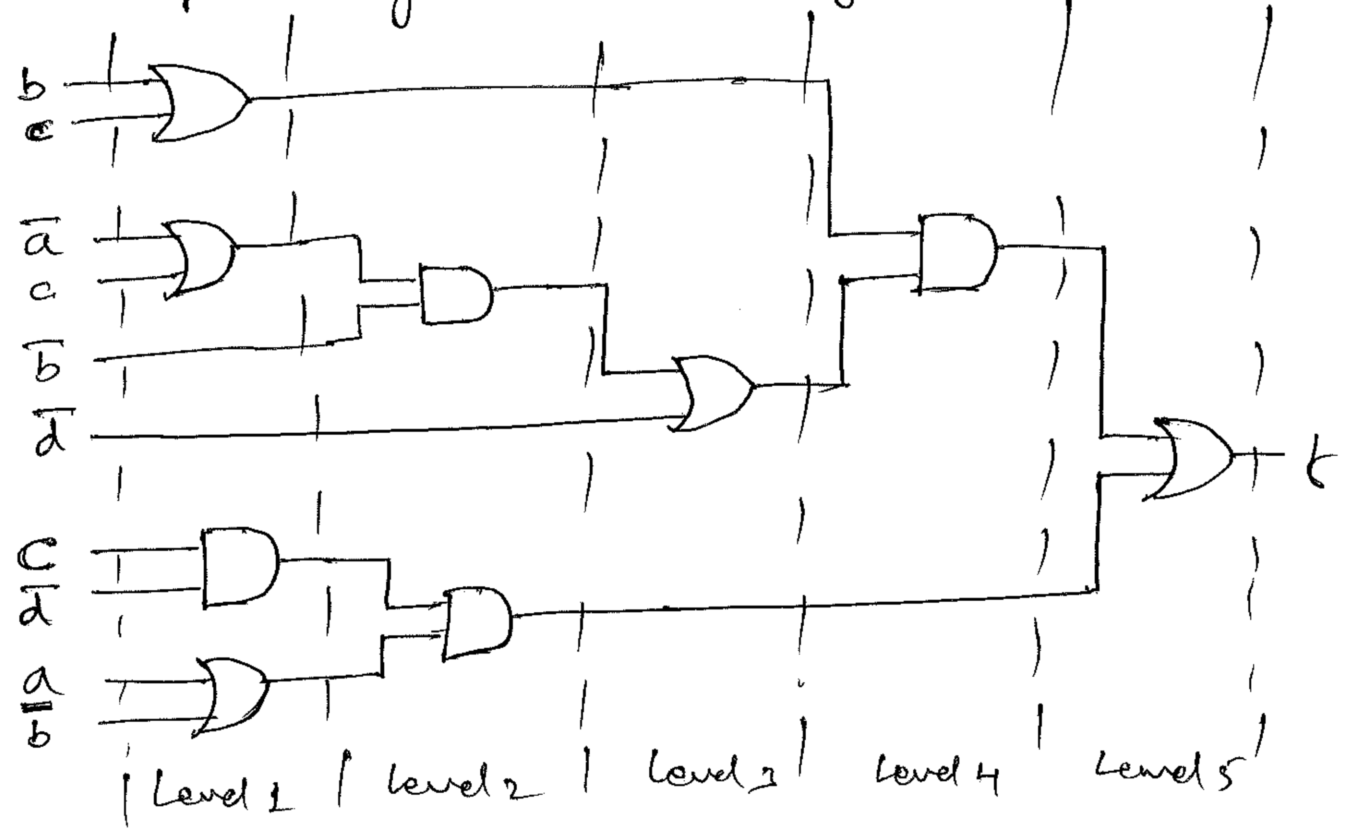
$$f = (\bar{a}b + \bar{a}c)(\bar{b} + d) + b\bar{d}$$
$$= \bar{a}\cancel{b\bar{b}} + \bar{a}bd + \bar{a}\bar{b}c + \bar{a}cd + b\bar{d}$$
$$= \bar{a}bd + \bar{a}\bar{b}c + \bar{a}cd + b\bar{d}$$



3) Synthesis the following Boolean Expression

$$f(a,b,c,d) = (b+c)(\bar{d} + (\bar{a}+c)\bar{b}) + c\bar{d}(a+\bar{b})$$

Soln. Implementing double rail logic.



$$f(a,b,c,d) = (b+c)(\bar d + \bar a\bar b + c\bar b) + c\bar d a + c\bar b\bar d$$

$$= b\bar d + \cancel{a\bar b b}^{0} + \cancel{c b\bar b}^{0} + c\bar d + \bar a\bar b c + c\bar b + c\bar d a + c\bar b\bar d$$

$$= \bar d(b + c\bar b) + c\bar d(1+a) + \bar b c(\bar a + 1)$$

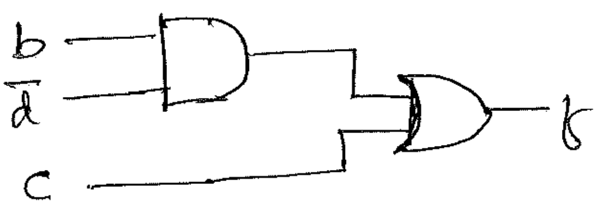
$$= \bar d(b+c) + cd + \bar b c$$

$$= b\bar d + c\bar d + cd + \bar b c$$

$$= b\bar d + c(d+\bar d) + \bar b c$$

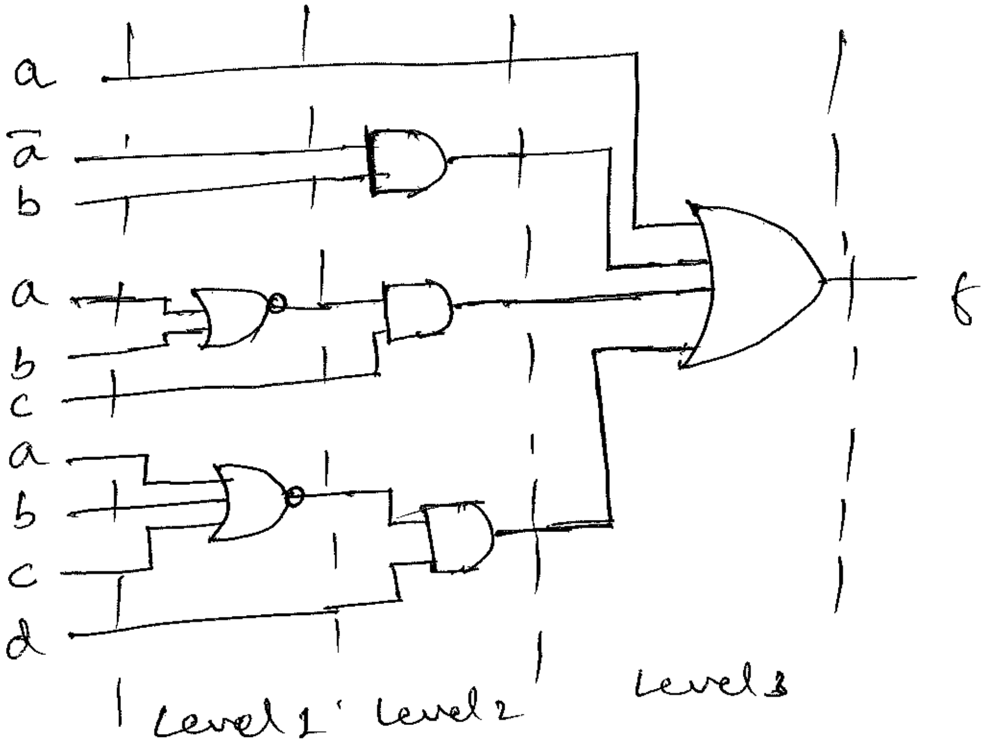
$$= b\bar d + c + \bar b c = b\bar d + c(1+\bar b)$$

$$= b\bar d + c$$



4) $f(a,b,c,d) = a + \bar a b + \overline{(a+b)}c + \overline{(a+b+c)}d$

Soln



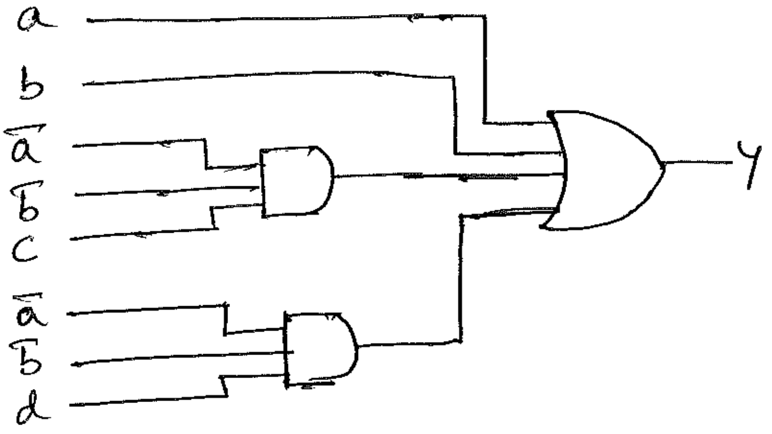
$$f(a,b,c,d) = a + \bar a b + \overline{(a+b)}c + \overline{(a+b+c)}d$$

$$= a + b + (\bar a \cdot \bar b)c + (\bar a\,\bar b\,\bar c)d$$

$$= a + b + \bar a\bar b(c + \bar c d)$$

$$= a + b + \bar a\bar b\,(c+d)$$

$$= a + b + \bar a\bar b c + \bar a\bar b d$$



5) Design a combinational circuit using four variables if both MSB and LSB is high output goes high, assuming double rail logic.