

# Assignment 2 - ML

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## SECTION A

## Section A.

1(a) let us consider

D: event that the company issues a dividend.

$\sim D$ : the event that the company doesn't issue a dividend.

P: profit increase percentage.

It's given that

$$P(D) = 0.8 \text{ and } P(\sim D) = 0.2$$

If dividends are issued

Profit Increase.

$$\hookrightarrow \text{Avg increase} = 10\%$$

$$\hookrightarrow \text{Variance} = 36\% \\ (\sigma^2)$$

Likelihood

$$\hookrightarrow P(P=x|D) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2(\sigma^2)}}$$

$$P(P=4|D) = \frac{1}{\sqrt{2\pi(36)}} e^{-\frac{(4-10)^2}{2(36)}}$$

Likelihood of a 4%  
incr given the comp. issues a dividend.

$$P(P=4|D) = \frac{1}{6\sqrt{2\pi}} e^{-1/2}$$

$$P(P=4|D) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{(4-0)^2}{2(36)}}$$

If the dividends were not issued  
 $\hookrightarrow$  Avg profit increase = 0%

$$\Rightarrow P(P=4\% | ND) = \frac{1}{6\sqrt{2\pi}} e^{-2/4}.$$

ivideo!

Applying Baye's Theorem:

$$P(D | P=4\%) = \frac{P(P=4\% | D) \cdot P(D)}{P(P=4\%)}$$

$$P(P=4\% | D) = \frac{1}{6\sqrt{2\pi}} e^{-1/2} = \frac{0.606}{6\sqrt{2\pi}} = 0.0404$$

$$P(P=4\% | ND) = \frac{1}{6\sqrt{2\pi}} e^{-2/4} = \frac{0.18}{15} = 0.053$$

$$P(P=4\%) = ((0.404) \times (0.8)) + ((0.2) \times (0.053))$$

$$P(P=4\%) = 0.4292$$

Thus,

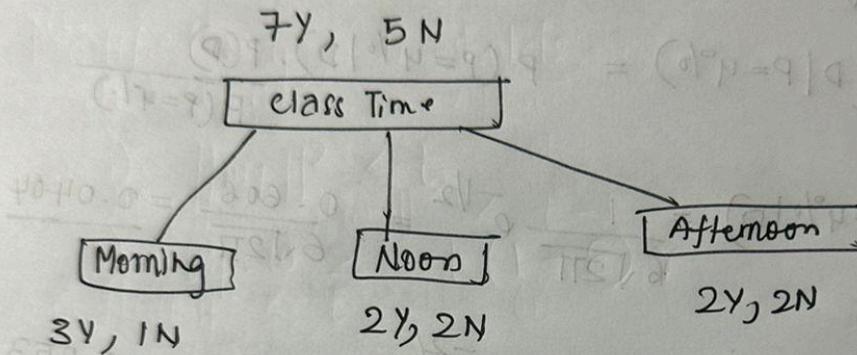
$$P(D | P=4\%) = \frac{(0.8) \times (0.0404)}{(0.4292)} = 0.75$$

Thus, the company has a 75% prob that it will issue a dividend this year according to the trends given.

1(b). Building the decision tree for the table using ID3

Using the ID3 info gain,  
top-down.

Starting with class time:



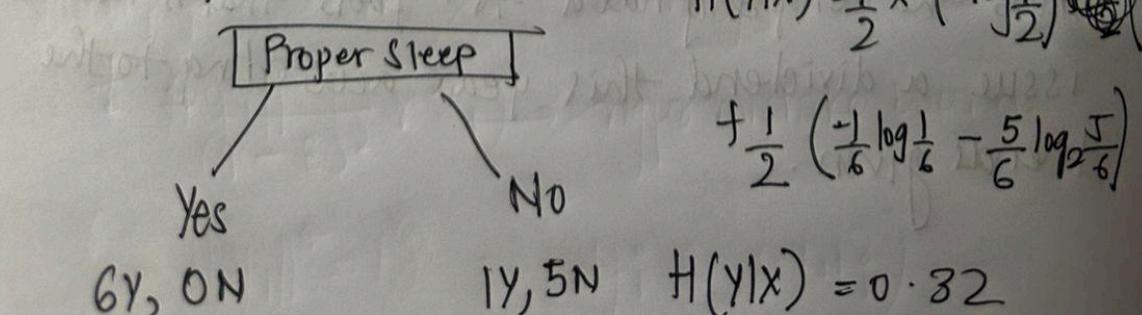
$$IG = H(Y) - H(Y|X)$$

$$\text{lowest } H(Y|X) = \sum_{r=1}^m \frac{|S_r|}{|S|} \times (\text{cross-entropy of each node})$$

For class time:

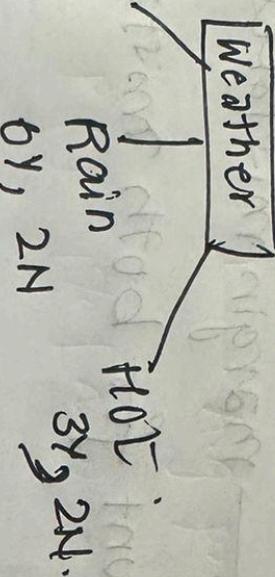
$$H(Y|X) = 0.93$$

For Proper Sleep:



Q:

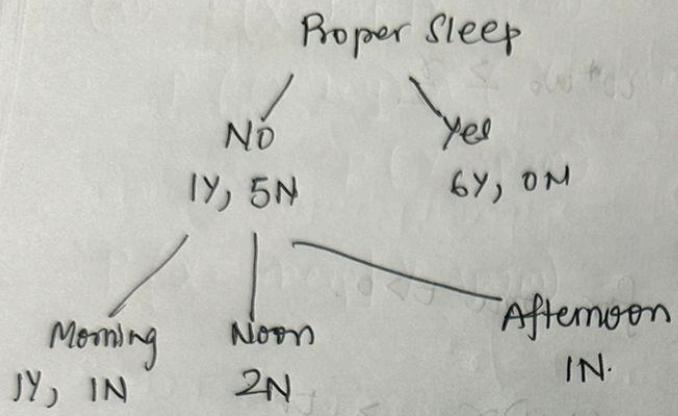
for weather:



$$\frac{5}{12} \left( \frac{1}{5} \log \frac{1}{5} - \frac{4}{5} \log \frac{4}{5} \right) + \frac{5}{12} \left( \frac{2}{3} \log \frac{2}{5} - \frac{2}{3} \log \frac{2}{5} \right) + \frac{2}{12}(0) = \underline{\underline{0.7}}$$

Best split: Mod. Proper sleep > lowest entropy

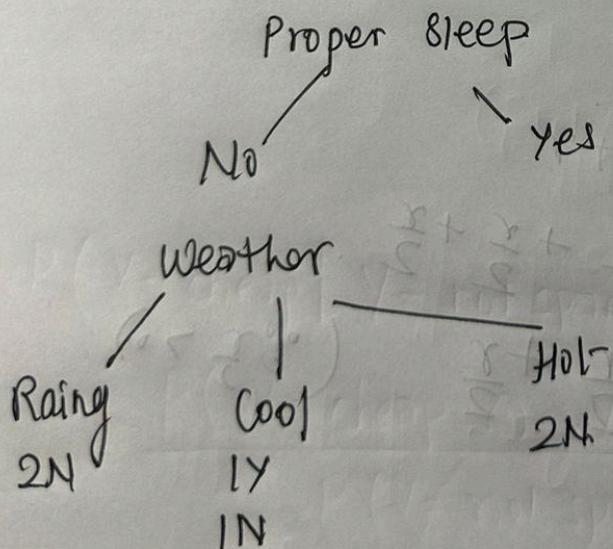
To find 2<sup>nd</sup> best split we use the same method.



Again checking the  $H(Y|X) = -\frac{1}{3} \left( \frac{1}{2} \log \frac{1}{2} + \frac{1}{2} \log \frac{1}{2} \right)$

$$H(Y|X) = \frac{1}{3}$$

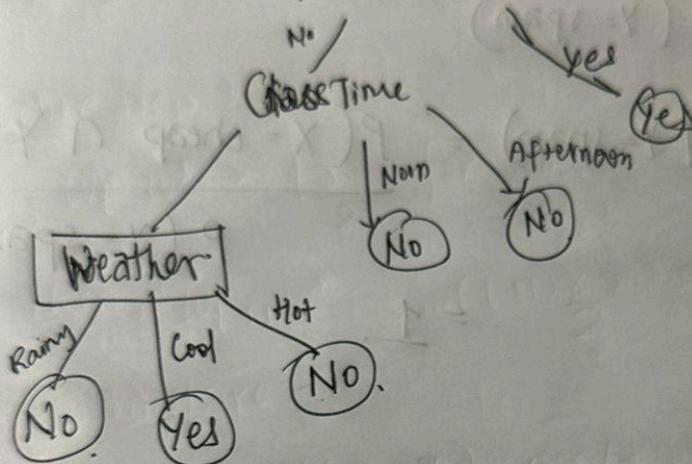
For weather:



which will give us the exact same distribution as the last split.

∴ we take class time as the splitting attribute and the remaining attribute is weather so the tree is

Had Proper Sleep



1(c). Given:-

- (i)  $S \rightarrow$  labelled dataset, that is linearly separable by a margin.
- (ii) All examples normalized to Euclidean length of 1.
- (iii) Update on classification mistakes & margin mistake  
 $(\text{margin} < \frac{\gamma}{2}) \rightarrow$  special perceptron algorithm.

To prove:

- (i) No. of updates is almost  $\frac{1}{\gamma^2}$
- (ii) Derive mistake bound when margin threshold is  $(1-\epsilon)r$ .

~~(iii)~~  $w_t \rightarrow w_t$  of vector after t updates

$w^* \rightarrow$  optimal wt. vector  
(given  $\|w^*\| = 1$ )

$x \rightarrow$  input vector ( $\|x\| = 1$ )

$y \rightarrow$  true label ( $y \in \{-1, 1\}$ )

Proof:-

On each update:

$$w_{t+1} = w_t + yx, \|x\| = 1.$$

On squaring

$$\|w_{t+1}\|^2 = \|w_t + yx\|^2$$

$$\|w_{t+1}\|^2 = \|w_t\|^2 + \|yx\|^2 + 2yxw_t$$

Now, since we assumed points are misclassified.

Thus,

$$y_i x_i w^* < 0$$

$$\|w_{t+1}\|^2 \leq \|w_t\|^2 + 1.$$

Now,

Induction!

$$\|w_t\|^2 - \|w_0\|^2 \leq t-1$$

$$\|w_t\|^2 \leq t \quad \text{--- ①}$$

Now, to prove:

$$w_t \cdot w \geq \frac{t\gamma}{2} = \|w\| \cos(\theta)$$

when  $t=0$ ;  $w_0 = y_1 x_1$

Also,  $y_1 (w^* x_1) \geq \gamma$ . Given by margin.

So  $y_1 (w^* x_1) \geq \gamma$

Multiplying both sides with  $y_1$ ,

$$\|y_1\|^2 (w^* x_1) \geq y_1 \gamma$$

$$(\because \|y_1\|^2 = 1) \Rightarrow y_1 \in \{-1, 1\}$$

$$\therefore w^*x_i = y_i \gamma$$

$$w_0 w^* \geq y_i x_i w^*$$

$$w_0 w^* \geq y_i^2 \gamma$$

$$w_0 w^* \geq r > 0 \Rightarrow w^* w_0 \geq \frac{\gamma}{2}.$$

Inductive step:

$$w_t w^* \geq \frac{t\gamma}{2}, \text{ for some } t > 0.$$

$$\text{then, } w_{t+1} w^* \geq \frac{(t+1)\gamma}{2} > \frac{t\gamma}{2}.$$

Margin mistake:-

$$0 < y(w_t \cdot x) < \gamma/2 \quad (\text{given})$$

$$w_{t+1} = w_t + yx.$$

$$(w_{t+1}) w^* \geq w_t w^* + r \quad [\because y(w^* x) \geq r]$$

$$\text{Also, } w_t w^* \geq \frac{t\gamma}{2}.$$

$$w_{t+1} w^* \geq \frac{t\gamma}{2} + r$$

$$\geq \frac{t\gamma}{2} + \frac{r}{2} + \frac{\gamma}{2}$$

$$\geq \frac{t\gamma}{2} + \frac{r}{2} \quad (\because \frac{\gamma}{2} > 0)$$

$$\text{thus } \Rightarrow w_{t+1} w^* \geq (t+1) \frac{\gamma}{2}$$

hence proved.

$$\text{Thus } \Rightarrow w_t w^* \geq \frac{t\gamma}{2} \quad \text{--- Q.E.D}$$

~~(each)~~

Using Cauchy-Swartz Inequality:

$$\Rightarrow (a \cdot b)^2 \leq \|a\|^2 \|b\|^2$$

$$\Rightarrow (w_t \cdot w^*)^2 \leq \|w_t\|^2 \cdot \|w^*\|^2$$

$$\Rightarrow (w_t \cdot w^*)^2 \leq (t^{-1}) \quad \boxed{\|w_t\|^2 < t}$$

Also,  $w_t \cdot w^* \geq t \frac{\gamma}{2}$  from eqn Q

$$\text{so, } \left(\frac{t\gamma}{2}\right)^2 \leq t \cdot \frac{t^2\gamma^2}{4} \leq t. \Rightarrow \boxed{t \leq 4/\gamma^2} \quad (\because \gamma^2 > 0)$$

We have to account for  $\frac{t}{\gamma^2}$  classification mistakes  
 $\frac{t}{\gamma^2}$  margin mistakes

we'll multiply by 2 to account for both mistakes

$$t \leq \frac{8}{\gamma^2}$$

Modified margin thresholds:-

$$\text{i) } \|w_t\|^2 \leq t$$

$$\text{ii) } w_t \cdot w \geq t(1-\epsilon)\gamma$$

Proof for i)

Each update incr.  $w w^*$  by at least  $(1-\epsilon)\gamma$ .

Thus after  $t$  updates:

$$w_t \cdot w^* \geq t(1-\epsilon)\gamma$$

Applying Cauchy-Swartz:

$$[t(1-\epsilon)\gamma]^2 \leq t$$

$$t \leq \frac{1}{(1-\epsilon)^2} \gamma^2 \rightarrow \text{new mistake bound.}$$

1(d) For Spam:

$$\textcircled{a} \quad P(X = \text{buy} | Y = \text{spam})$$

$$\frac{P((X = \text{buy}) \cap (Y = \text{spam}))}{P(Y = \text{spam})} = 1$$

$$P(X = \text{cheap} | Y = \text{spam}) = \frac{P(X = \text{cheap} \cap Y = \text{spam})}{P(Y = \text{spam})}$$

= 1.

For Not Spam:

$$P(X = \text{buy} | Y = \text{Not spam}) = \frac{P(X = \text{buy} \cap Y = \text{not spam})}{P(Y = \text{not spam})}$$

$$= \frac{1/4}{2/4} \\ = 1/2.$$

$$P(X = \text{cheap} | Y = \text{not spam})$$

$$\frac{P(X = \text{cheap} \cap Y = \text{not spam})}{P(Y = \text{not spam})} = \frac{1/4}{2/4} = 1/2$$

(b) To compute the posterior probabilities.

Given that :  $X = \text{cheap} \Rightarrow \text{not buy}$

First checking spam: we just need to check the numerator as denominator is same for both

$$\begin{aligned} P(Y=1 | X=\text{cheap, not buy}) &= P(Y=\text{spam}) \times P(X=\text{cheap} | Y=\text{spam}) \\ &\quad \times P(X=\text{not buy} | Y=\text{spam}) \\ &= \frac{1}{2} \times \frac{1}{2} \times (1-D) = 0. \end{aligned}$$

Checking not spam:

$$\begin{aligned} P(Y=\text{Not spam} | X=\text{cheap, Not buy}) &= P(Y=\text{not spam}) \times P(X=\text{cheap} | Y=\text{not spam}) \\ &\quad \times P(X=\text{not buy} | Y=\text{not spam}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= 1/8 \end{aligned}$$

(c) The prob. with prob. is that if a conditional prob. is zero, the entire expression becomes zero showing lack of generalization and a complete aversion to unseen data.

To approach to solve this is to use

$$m\text{-estimate} = c : \text{number of class } = P(A_i|c) = \frac{N_{ic} + m}{N_c + m}$$

$$P(A_i|c) = \frac{N_{ic} + 1}{N_c + c} \leftarrow \text{Laplace estimate}$$

$N_{ic}$  is the no. of times  $A_i$  and  $c$  occurs and  $n_c$  is the no. of times  $c$  occurs.

## SECTION B

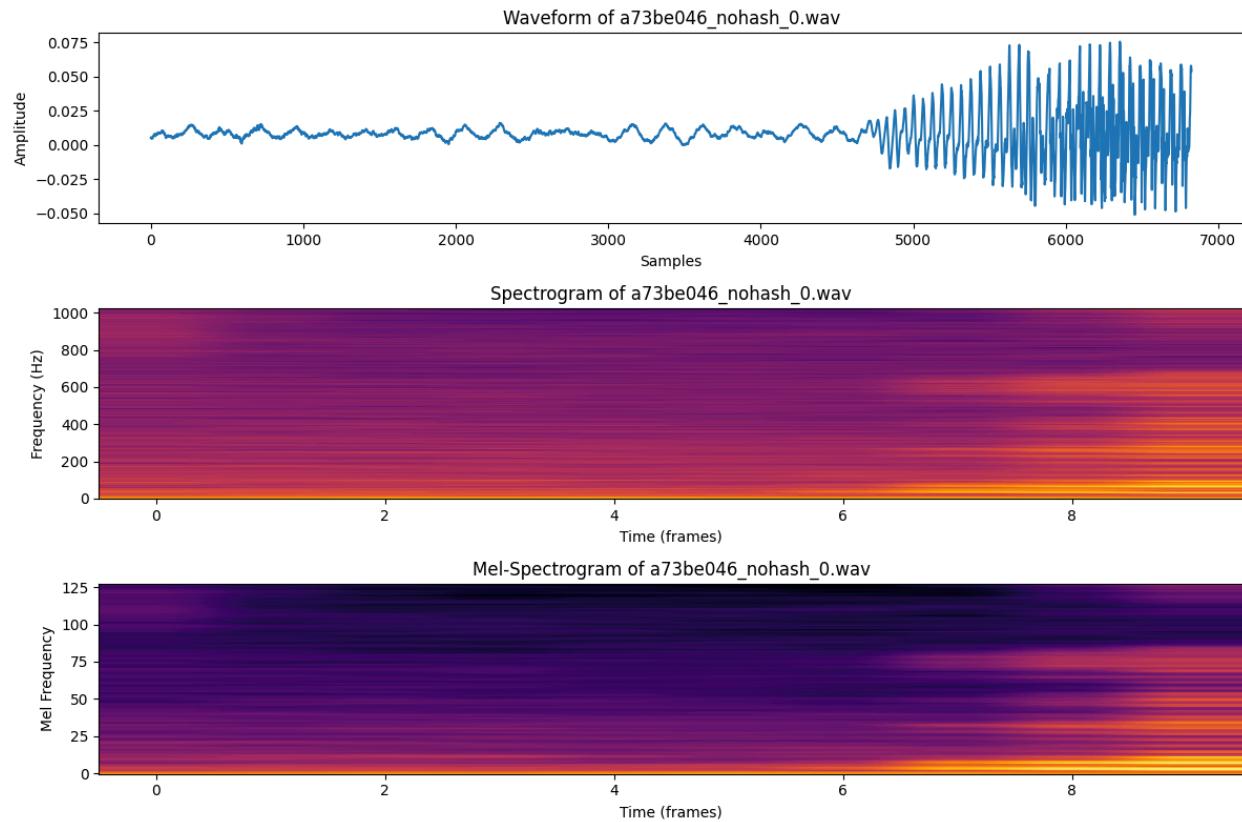
1 (a). Statistical summary of the data is given below:

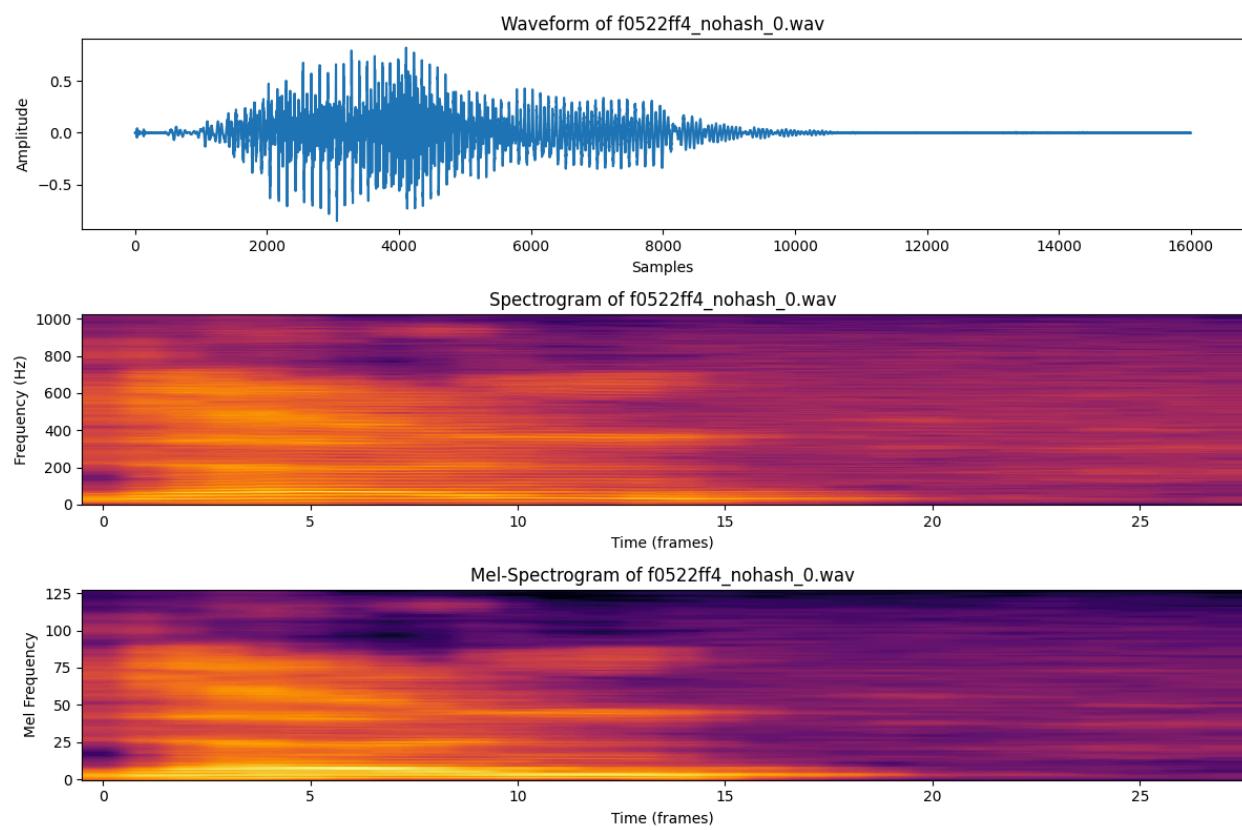
Class	mean	median	min	max	std
_background_noise_	-0.004955	-1.775734e-05	-0.029636	-0.000007	0.012091
backward	0.000144	-2.651215e-07	-0.077356	0.088151	0.004145
bed	0.000097	-2.174734e-07	-0.077014	0.088236	0.006772
bird	-0.000143	3.795624e-07	-0.230286	0.088332	0.010779
cat	-0.000265	-2.574921e-07	-0.211095	0.087865	0.011951
dog	-0.000425	-1.331314e-07	-0.230329	0.088878	0.012447
down	0.000050	-3.643036e-07	-0.208074	0.088093	0.006024
eight	-0.000052	-3.471374e-07	-0.208647	0.331275	0.008099
five	-0.000010	2.784729e-07	-0.230202	0.088356	0.007253
follow	-0.000194	2.185822e-06	-0.157871	0.017067	0.005703
forward	0.000120	4.287720e-06	-0.009457	0.088124	0.002684
four	-0.000172	-3.814697e-08	-0.230293	0.062128	0.007559
go	-0.000184	1.323799e-08	-0.160621	0.064560	0.005164
happy	-0.000244	1.029968e-07	-0.230300	0.085434	0.011628
house	0.000146	-7.057190e-08	-0.155956	0.085782	0.005475
learn	-0.000004	1.346588e-06	-0.155392	0.087803	0.004846
left	0.000048	-2.632141e-07	-0.210939	0.088600	0.006803
marvin	-0.000342	4.005432e-07	-0.230317	0.085572	0.010735
nine	-0.000002	-6.961823e-07	-0.076602	0.088251	0.003824
no	0.000019	2.250671e-07	-0.230324	0.084674	0.006008
off	0.000044	-8.392334e-08	-0.119272	0.084558	0.004421
on	0.000047	-3.523421e-07	-0.209007	0.088544	0.005009
...					
yes	1.000000	0.067037	4044		
zero	1.000000	0.057204	4052		

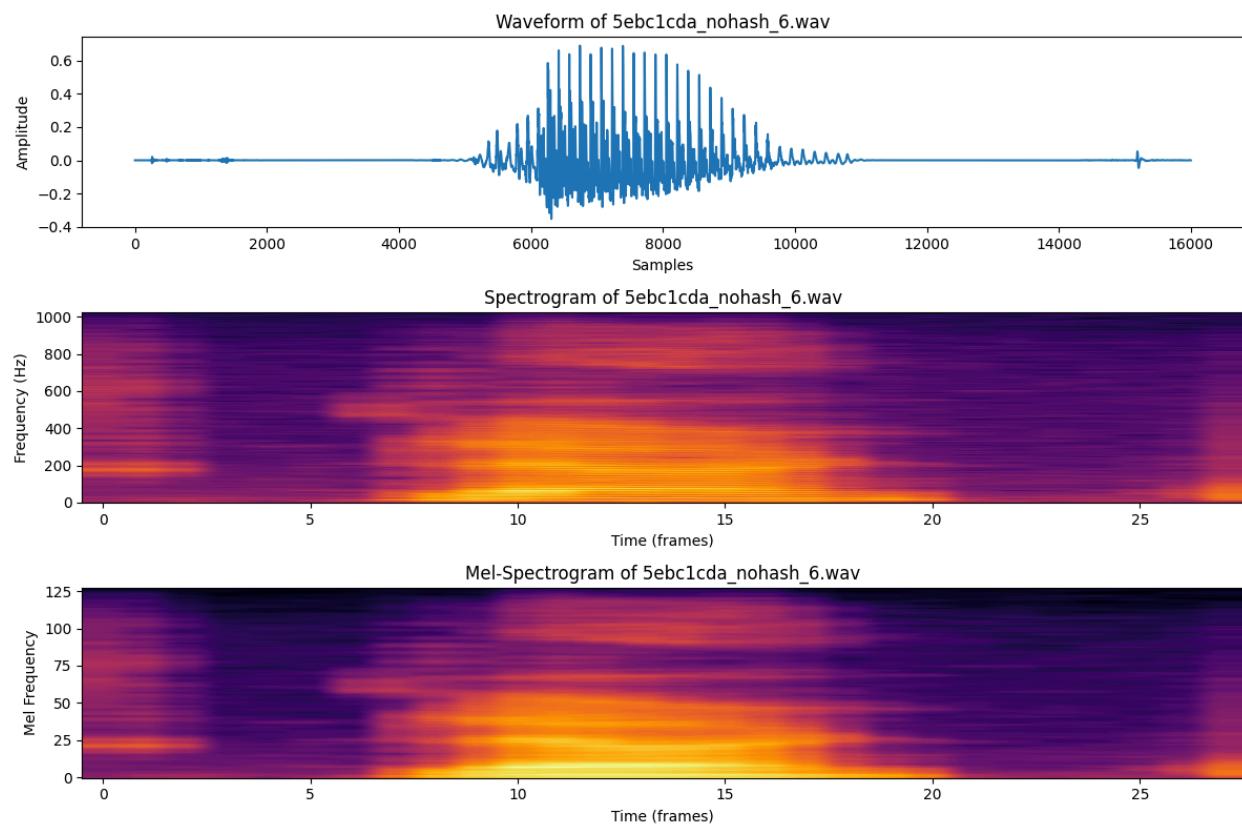
The mean amplitude is close to zero, which implies the audio has been normalized.

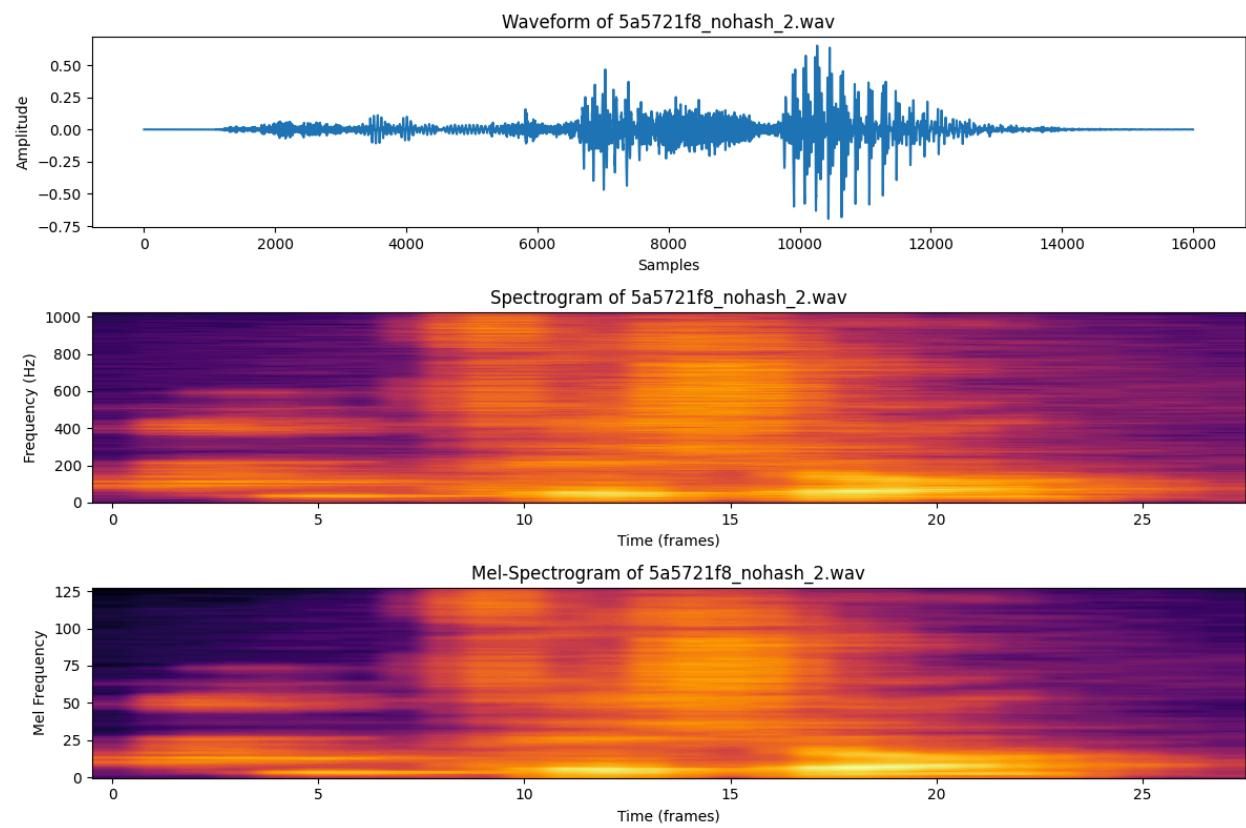
The std dev is quite low indicating the data is quite consistent.

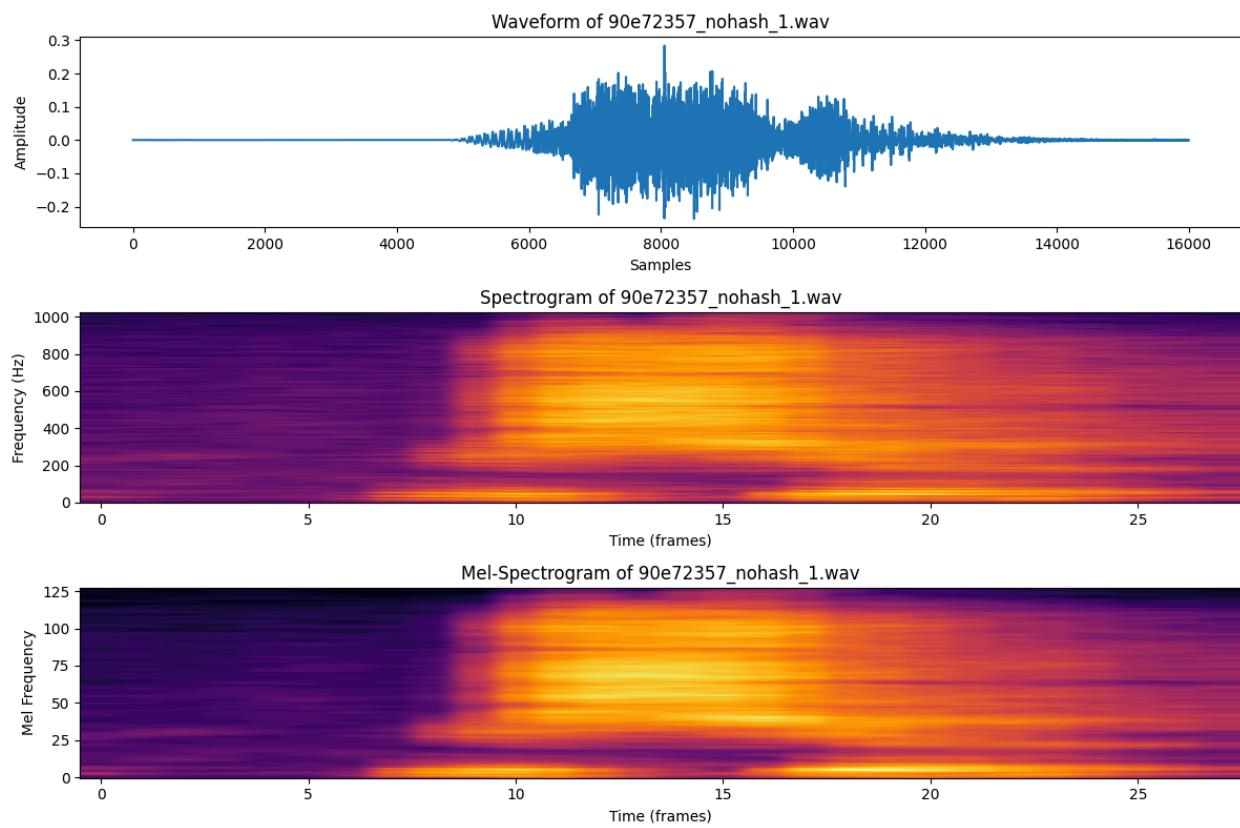
## 1 (b). Plots

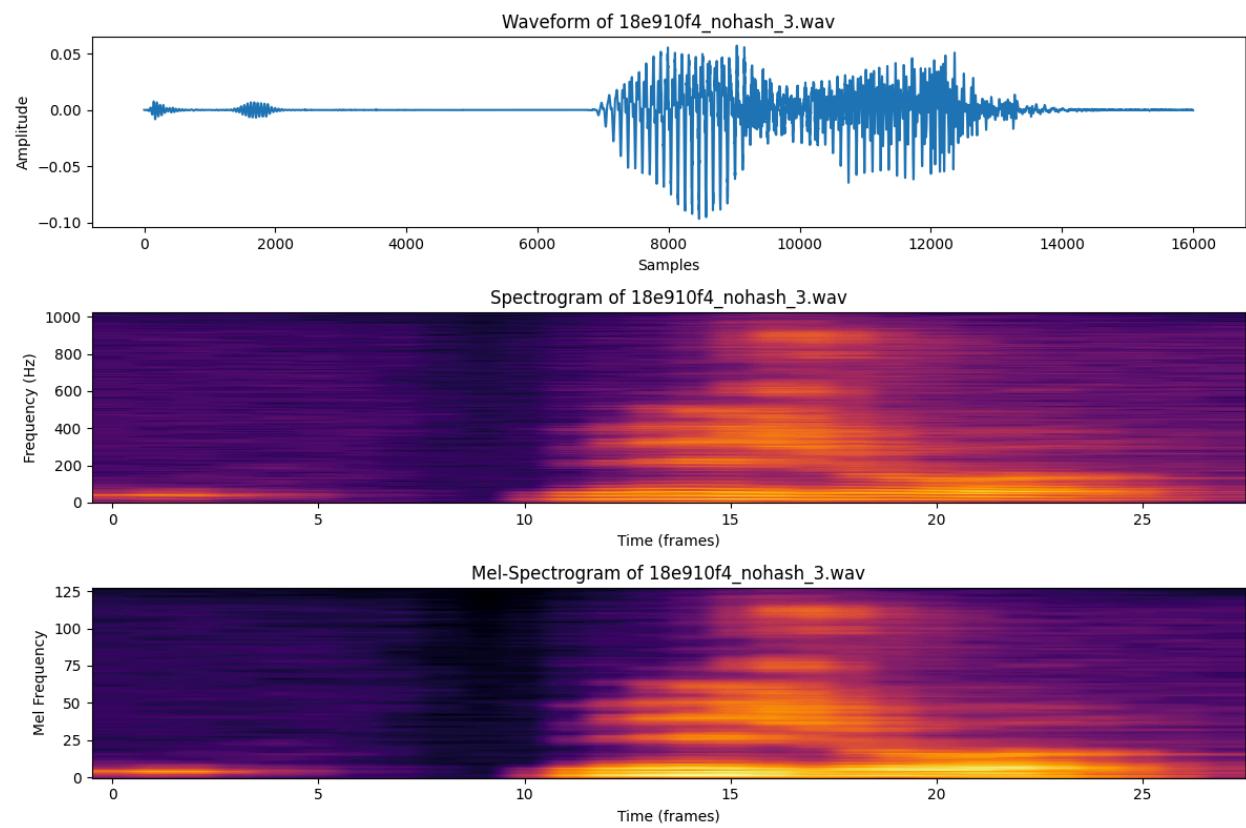


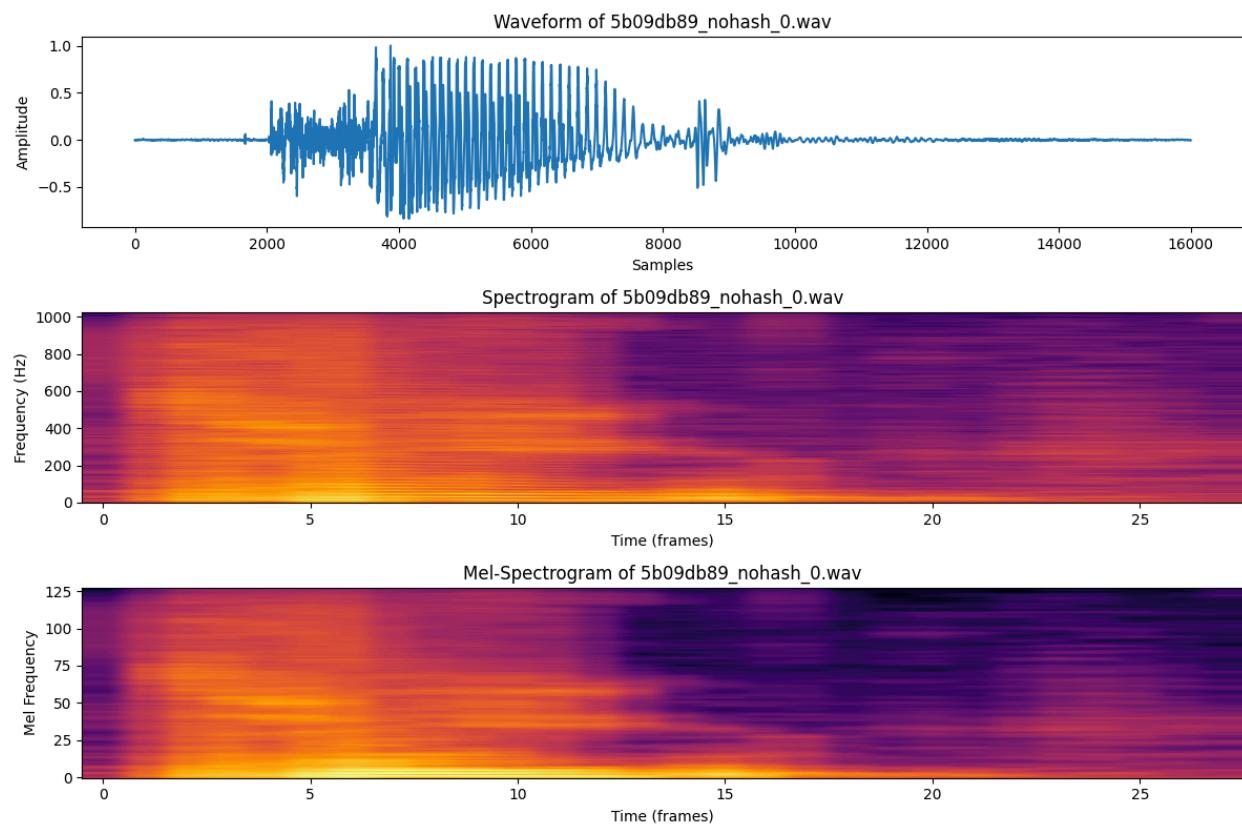


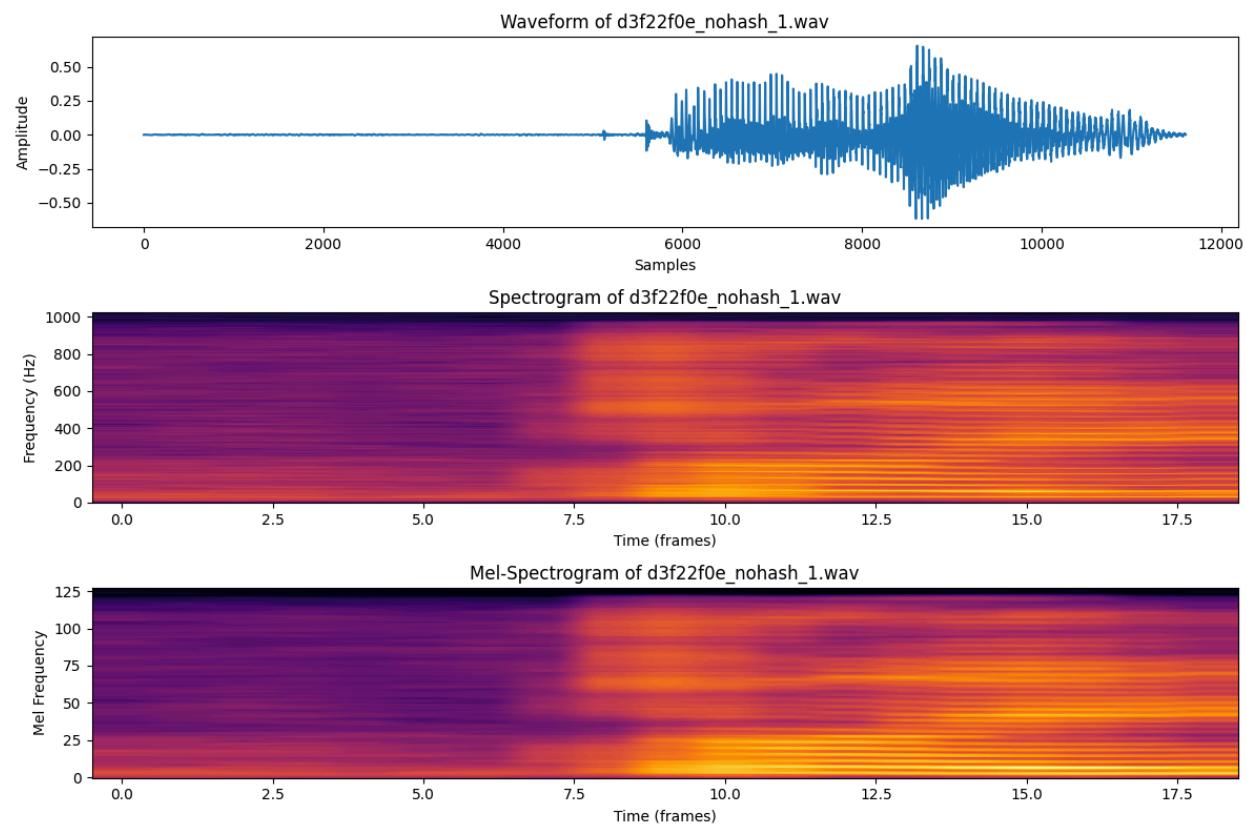


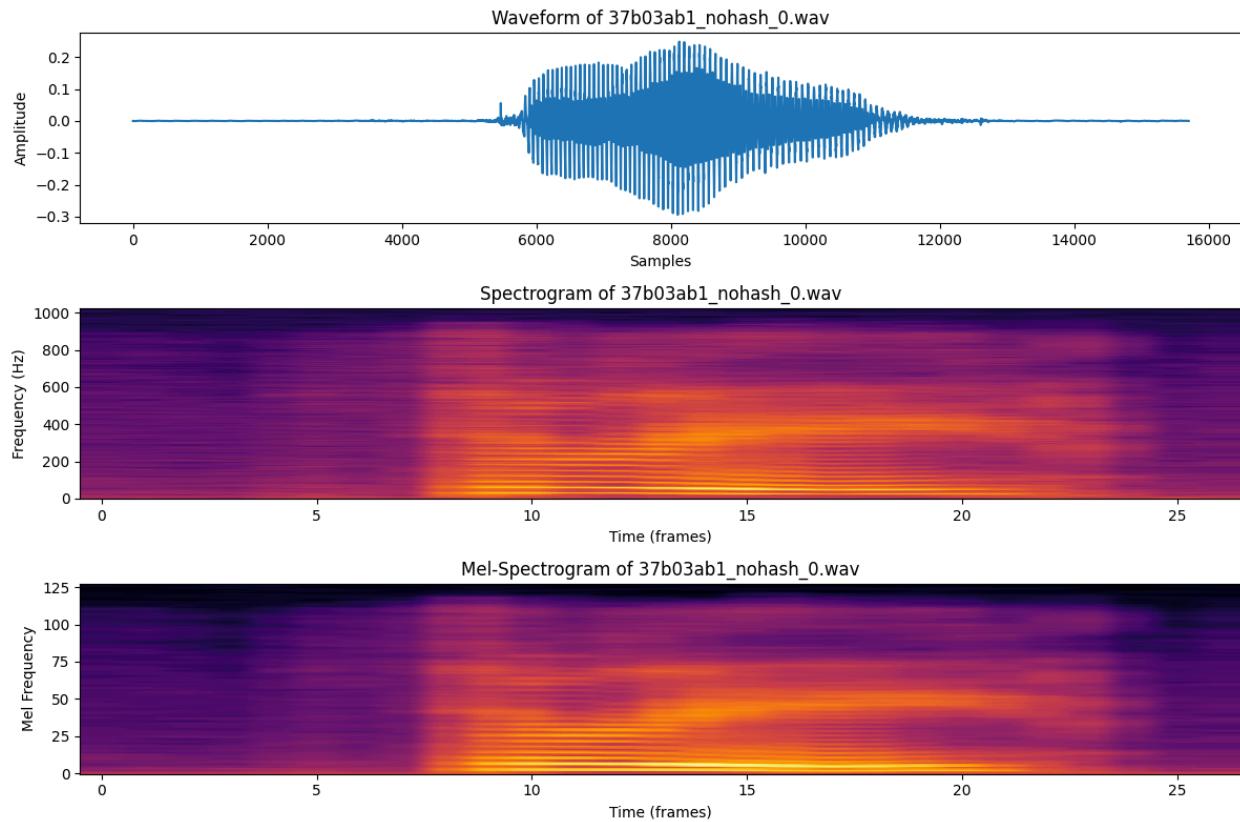












Observations:

The amplitude fluctuates over time.

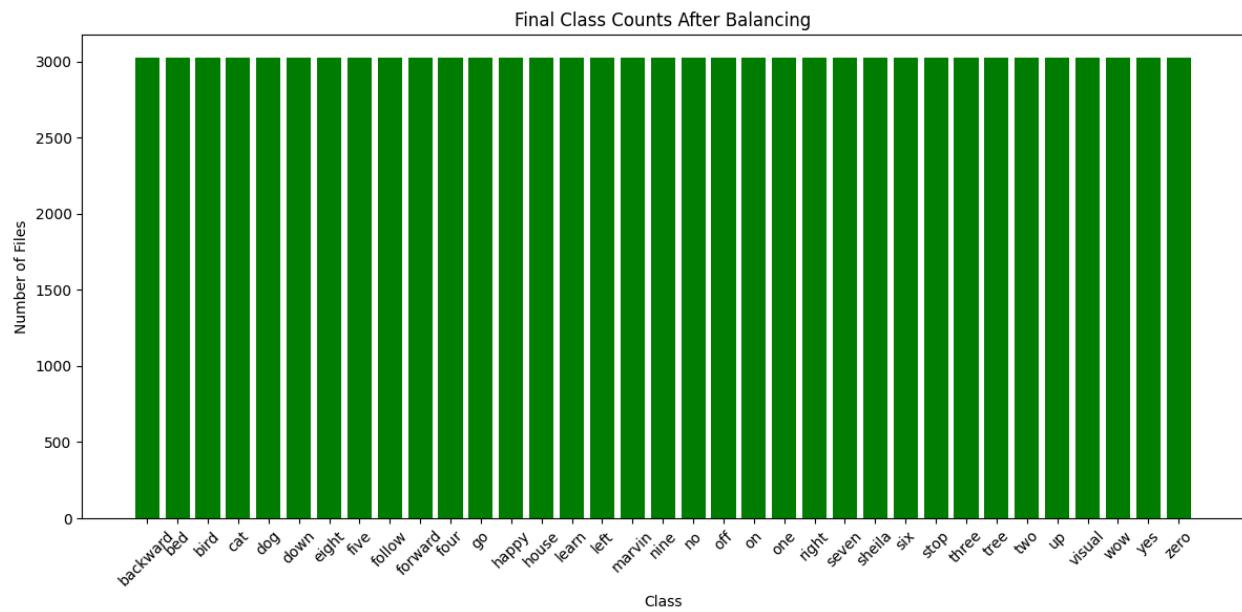
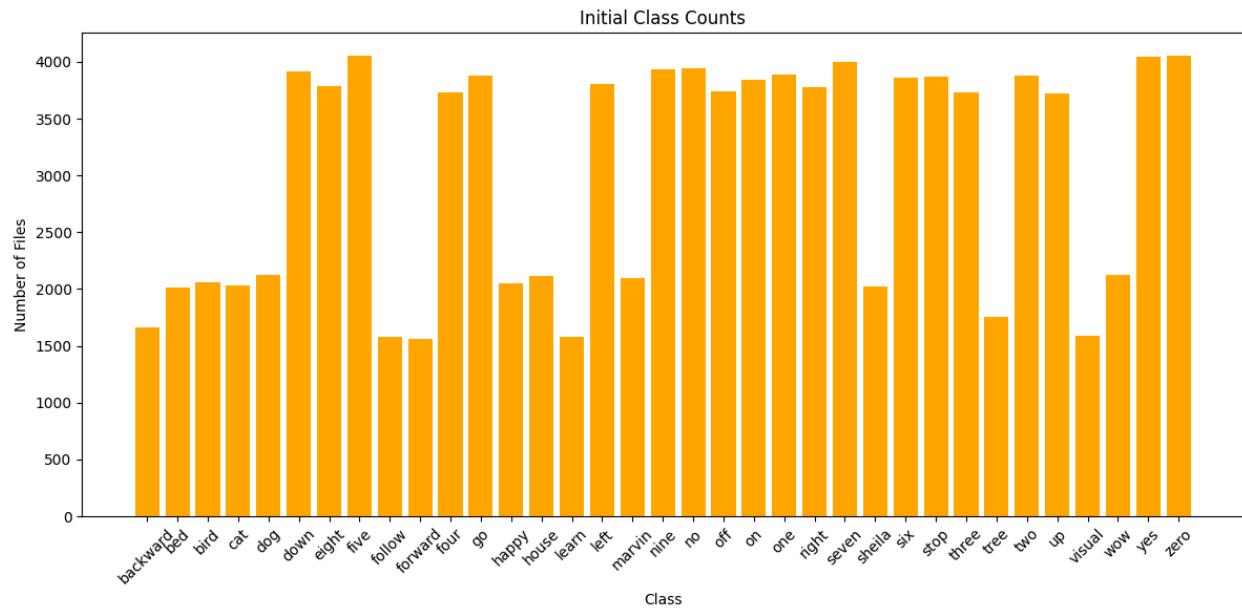
The waveform has a clear periodic pattern.

The frequency is considerably high. The frequency content appears to vary slightly over time, possibly indicating a changing pitch or timbre.

The frequency content appears to vary slightly over time, possibly indicating a changing pitch or timbre.

### 1 (c).Imbalances

There were imbalances and this was dealt with by using undersampling and oversampling all classes to avg number of files in a class.



1 (d). Data cleaning was done by removing silent segments.

2. Feature extraction

Features used:

MFCCs capture the short-term power spectrum of sound and are thus useful in speech

recognition and classification.

Spectral Contrast measures the difference in amplitude between peaks and valleys in the sound spectrum, reflecting the timbral texture of the audio.

Spectral Rolloff is the frequency below which a certain percentage of the total spectral energy lies, often used to distinguish between harmonic and non-harmonic content.

Spectral Centroid is the frequency below which a certain percentage of the total spectral energy lies, often used to distinguish between harmonic and non-harmonic content.

Zero crossing rate is the rate at which the signal changes sign, indicating how often the audio signal crosses the zero amplitude level.

Root Mean Square energy provides a measure of the audio signal's power, reflecting its loudness over time.

### 3. Model

Random Forest performed best with 52.86% accuracy.

Hyperparameter tuning helped improve accuracy. The parameters were tried and tested by trial and error. The best performing parameters were:

n\_estimators = 200

max\_depth = 25

min\_samples\_split = 2

Naive Bayes performed poorly with Accuracy: 26.78%.

Perceptron was not used as it is only useful for binary classification.

## SECTION C

1.(a) the number of images per class, the distribution of image sizes

Number of images per class:

label

sitting	840
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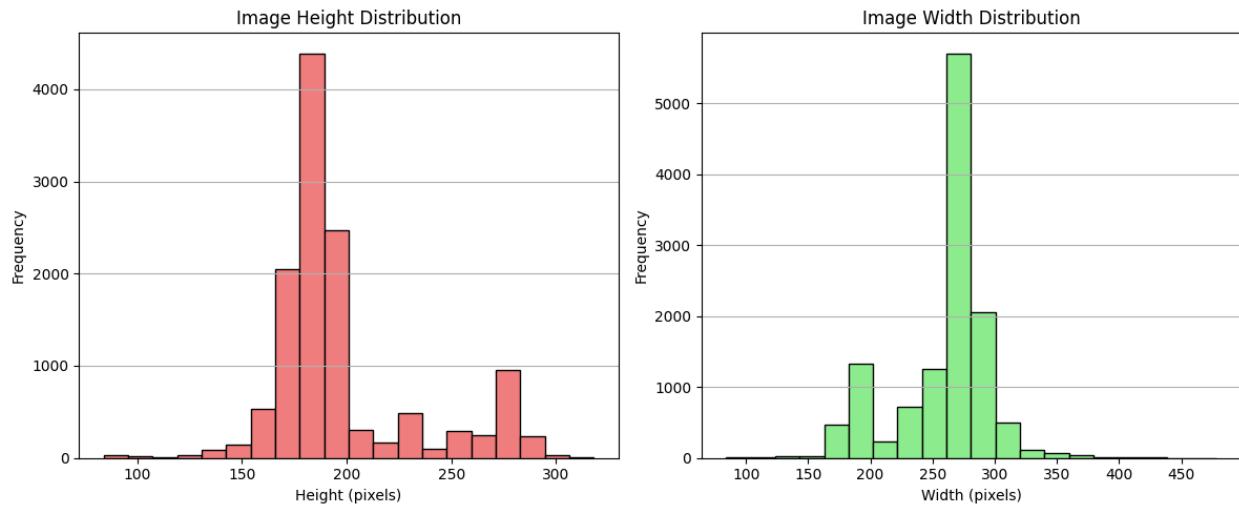
```
using_laptop      840
hugging          840
sleeping         840
drinking         840
clapping          840
dancing          840
cycling          840
calling          840
laughing         840
eating            840
fighting          840
listening_to_music 840
running          840
texting          840
```

Name: count, dtype: int64

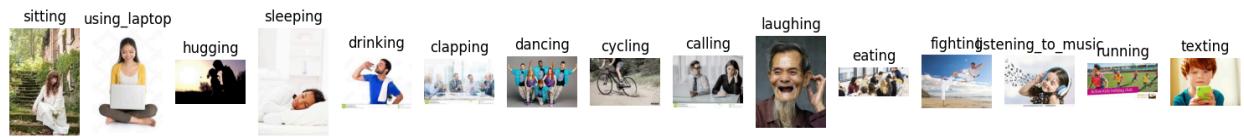


Image size distribution:

	Height	Width
count	12601.000000	12601.000000
mean	196.574399	260.379652
std	35.280124	39.917997
min	84.000000	84.000000
25%	181.000000	254.000000
50%	183.000000	275.000000
75%	194.000000	276.000000
max	318.000000	478.000000



1.(b)



1 . (c)

Class distribution imbalance:

label

sitting            840

using\_laptop    840

hugging          840

sleeping        840

drinking        840

clapping        840

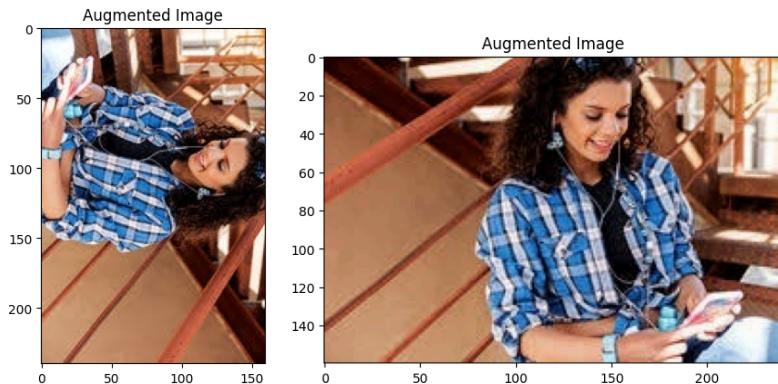
dancing        840

cycling        840

```
calling      840
laughing    840
eating       840
fighting     840
listening_to_music 840
running      840
texting      840
```

Name: count, dtype: int64

All classes are balanced with 840 images.



Data augmentation was not required as classes were balanced.

## 2. Feature Extraction:

Used features:

HOG - captures the distributions of localized gradients by computing a histogram using pixel intensities. Helps in object detection by using structural information.

Color histogram - captures the distribution of colors in an image by converting it to HSV color space and calculating histograms of associated values. Helps in object detection.

LBP - Local binary patterns help in feature extractions by using differences in localized pixel intensities. This helps in texture classification and facial recognition.

Gabor features - captures information about spatial structure and orientation of images, helping with texture analysis and edge detection.

GLCM - Gray Level Co-occurrence Matrix measures the frequency of pixel intensity combinations and calculates properties like contrast, energy, and homogeneity. It is useful in texture analysis and medical image classification

ORB (Oriented FAST and Rotated BRIEF) detects and describes local features like corners and edges. It is particularly useful in object recognition as it can deal with rotations, etc.

Other features I tried but were not useful:

Canny Edges - did not improve the feature set. Probably because Gabor already provides that functionality.

Hessian Matrix - This was very computationally intensive and didn't contribute to my accuracy.

### 3. Model Selection and Implementation

Random forest model was the best performing model. The highest accuracy recorded was 36.27 %.

Naive Bayes did not perform well with a low accuracy of 18.73%

Hyper parameter tuning helped improve accuracy of Random Forest Model. This was tested using GridSearchCV and also trial and error. Calculated estimates were based on the references below.

The accuracies for each set of hyperparameters were saved to a txt file.

```
hyperparameters = {'random_state':42, 'max_depth':16, 'n_estimators':7000, 'n_jobs':-1, 'min_samples_split':5, 'min_samples_leaf':2}
```

Perceptron was not trained as it is only useful for binary classification.

This was the hyperparameter set that performed the best.

## **REFERENCES:**

[Random forests - Machine Learning.](#)

[RandomForestClassifier — scikit-learn 1.5.2 documentation](#)

[Depth of Trees](#)