



Maratona de Programação

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"Fibonacci with a Twist"

- <http://www.spoj.pl/problems/FIBTWIST/>
- Variação da sequência de Fibonacci:
 - $ft_0 = 0$
 - $ft_1 = 1$
 - $ft_n = ft_{n-1} + ft_{n-2} + (n-1)$
 - 0 1 2 5 10 19 34 59
- Dados "n" (entre 0 e 10^9) e "M" (entre 100 e 10^9):
 - Obter $ft_n \bmod M$



Algoritmos: Produto Matricial

- Usa a identidade:

- $T_n = \begin{bmatrix} n-1 \\ 1 \\ ft_{n+1} \\ ft_n \end{bmatrix} A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

- $T_{n+1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot T_n = A \cdot T_n$

- $T_n = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}^n \cdot T_0 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}^n \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = A^n \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$



Algoritmos: Produto Matricial

```
int fibtwist( n, m )
{
    
$$A \leftarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad R \leftarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

    while( n > 0 )
    {
        if( ( n & 1 ) == 1 ) R = A x R mod m;
        A = A x A mod m;
        n >>= 1;
    }
    return -R[3][0] + R[3][1] + R[3][2];
}
```