Day 2

Teachers' Circle: Counting

To count something it often pays off to find two different ways to count it. This should prove to be a great strategy in next few problems.

1. In how many different ways can you write n as a sum of 1s and 2s? If you prefer more visual representation of this question, you can ask: in how many ways can you cover a strip n squares long by dominos and monominos? The example for $n=4$ is drawn below:

2. How many tilings of $(n+2)$ board use at least one domino?
3. How many tilings of $(2n+1)$- board exist?
4. How many tilings of $(m+n)$-board exist?
5. How many tilings of n-board exist?
6. How many tilings of $(2n-1)$-board exist?
7. How many ways can n numbers be arranged in a list? Express this in 2 different ways thereby obtaining a known identity.
8. How many different ways can we form a k student committee from a class of n students?
9. In how many different ways can we form any size committee from a class of n students?
10. In how many different ways can we form any size committee from a class of n students where one student is designated a chair?
11. How many binary n-tuples are there with no consecutive 0s?
12. How many subsets S does $\{1, 2,\ldots,n\}$ have that contains no consecutive integers?
13. How many tilings of n-board exist where all the tiles have length 2 or more?
14. How many tilings of n-board exist where all the tiles have odd length?

For a change of pace:

1. How many diagonals are there in a convex n-gon?
2. There are 3 rooms in a dormitory, a single, a double, and a quad. How many ways are there to assign 7 people to the rooms?
3. How many 10-digit numbers have at least 2 equal digits?
4. How many ways can you put 2 queens on a chessboard so that they don't attack each other? (Queens attack both on the rows and on the diagonals of a chessboard.)
5. How many ways can you split 14 people into 7 pairs?
6. There are N boys and N girls in a dance class. How many ways are there to pair them all up?
7. Ten points are marked on the plane so that no three of them are in a straight line. How many different triangles can be formed using these 10 points as vertices?
8. A group of soldiers contains 3 officers, 6 sergeants, and 30 privates. How many ways can a team be formed consisting of 1 officer, 2 sergeants, and 20 privates?
9. Ten points are marked on a straight line and 11 on another line, parallel to the first. How many triangles can be formed from these points? How many quadrilaterals?
10. How many ways can you put 10 white and 10 black checkers on the black squares of a checkerboard?

Classroom connections

Can you? How? Why? Would you?[2]

[2] Proofs that really count: A. Benjamin & J. Ouinn