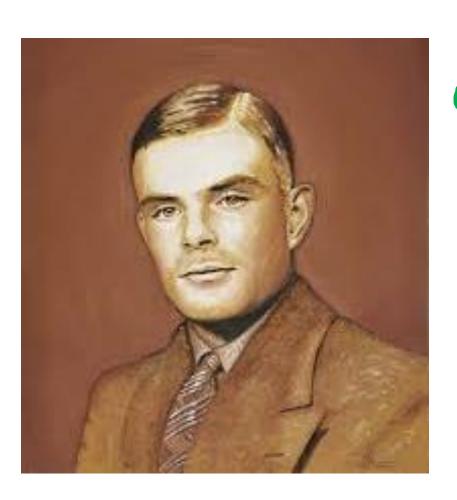


Introduction to Machine Learning

Sarath Chandar A P

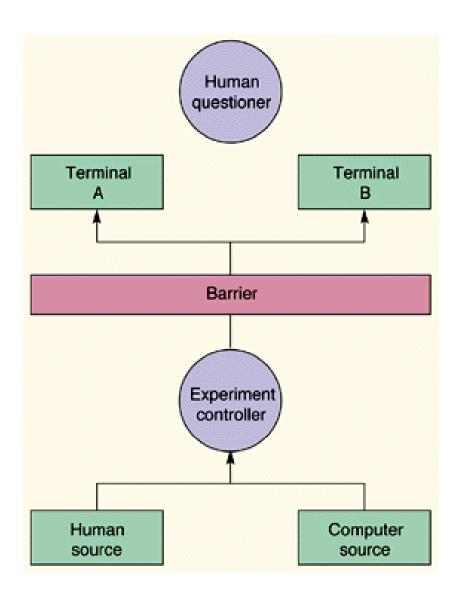
IBM Research India



Can Machines think?

Alan M Turing (1950)

Turing Test

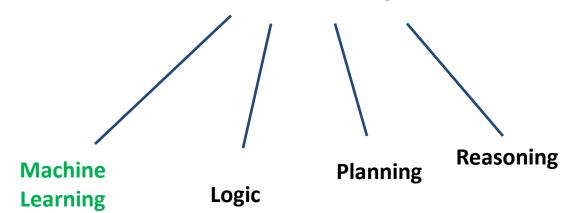


Turing Test

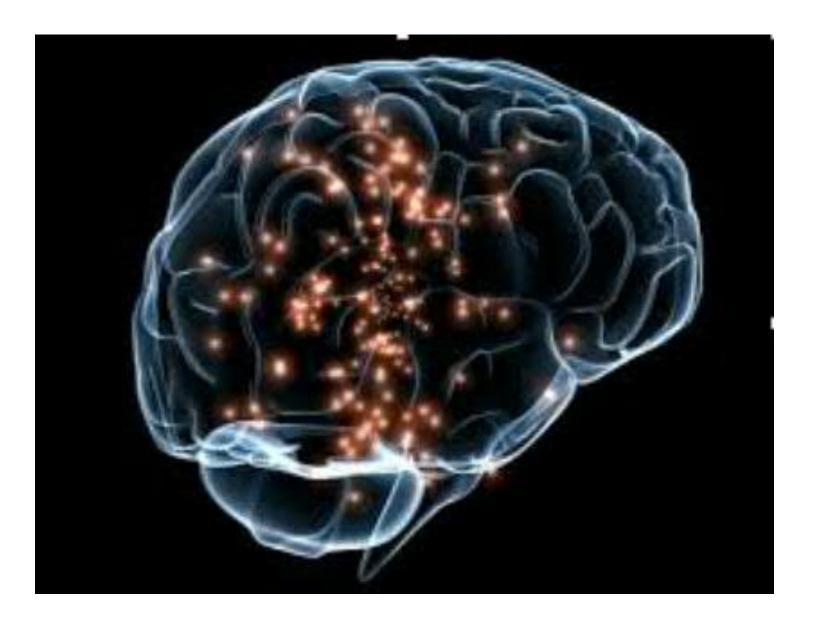
Can Machines think???



Artificial Intelligence



1950....



The human brain, a mystery to itself!!

Identify the male face









Compute

546734555 * 12056385 = ???

What is going wrong??

Learning Algorithm

A Biologically Inspired Learning Algorithm

Artificial Neuron / Perceptron

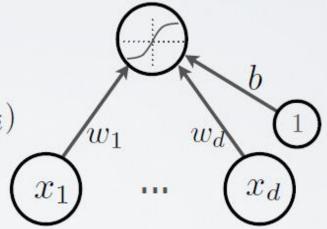
Neuron pre-activation (or input activation):

$$a(\mathbf{x}) = b + \sum_{i} w_i x_i = b + \mathbf{w}^{\mathsf{T}} \mathbf{x}$$

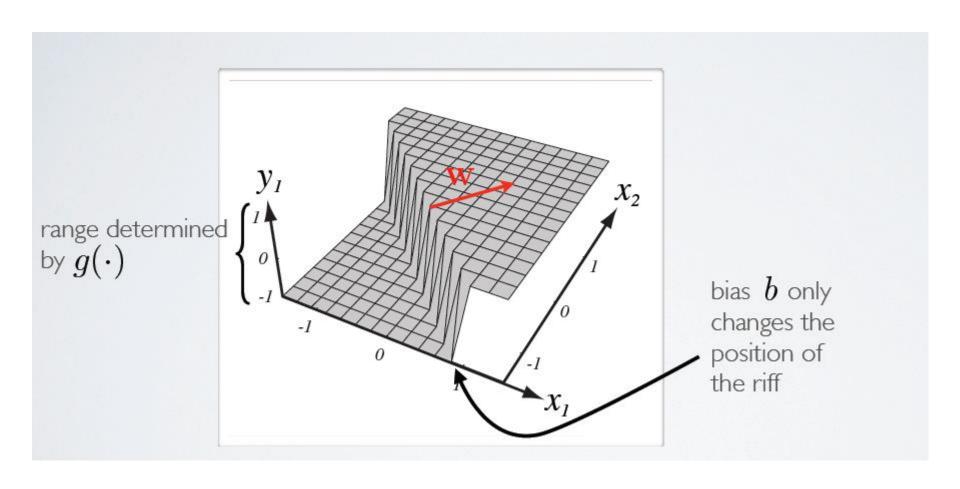
Neuron (output) activation

$$h(\mathbf{x}) = g(a(\mathbf{x})) = g(b + \sum_{i} w_i x_i)$$

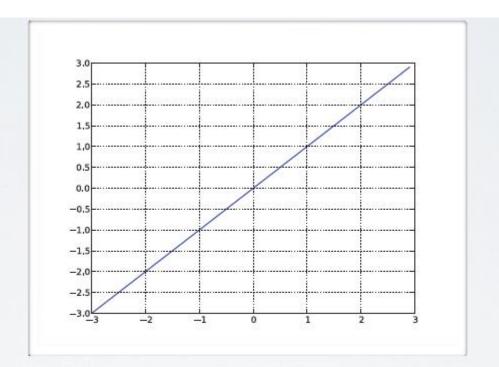
- · W are the connection weights
- b is the neuron bias
- $g(\cdot)$ is called the activation function



Decision Boundary

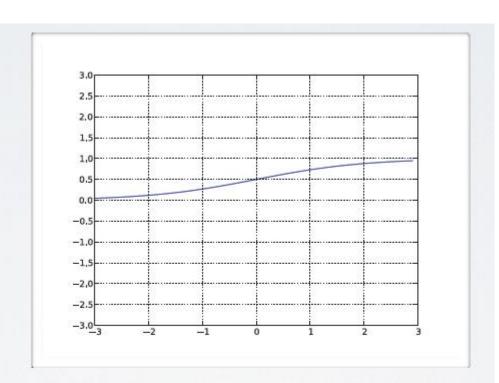


- Performs no input squashing
- Not very interesting...



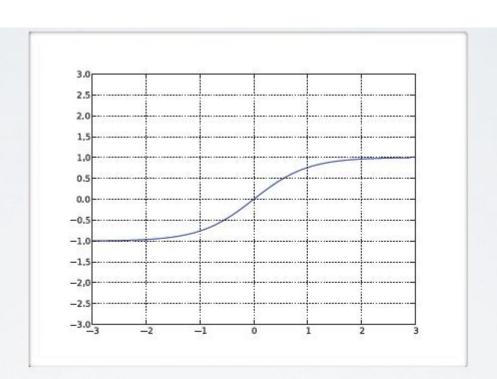
$$g(a) = a$$

- Squashes the neuron's pre-activation between 0 and I
- Always positive
- Bounded
- Strictly increasing



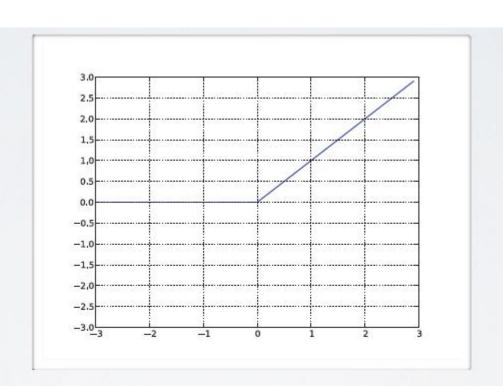
$$g(a) = \operatorname{sigm}(a) = \frac{1}{1 + \exp(-a)}$$

- Squashes the neuron's pre-activation between
 - -I and I
- Can be positive or negative
- Bounded
- Strictly increasing



$$g(a) = \tanh(a) = \frac{\exp(a) - \exp(-a)}{\exp(a) + \exp(-a)} = \frac{\exp(2a) - 1}{\exp(2a) + 1}$$

- Bounded below by 0 (always non-negative)
- Not upper bounded
- Strictly increasing
- Tends to give neurons with sparse activities

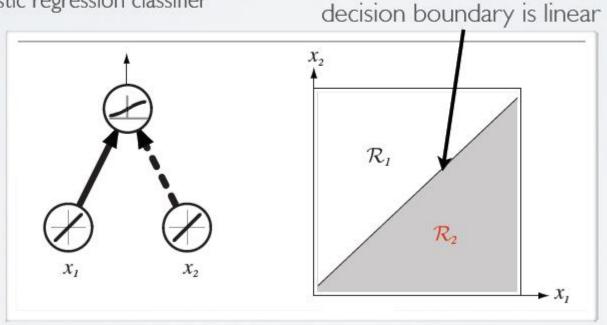


$$g(a) = reclin(a) = max(0, a)$$

What can a Perceptron actually do?

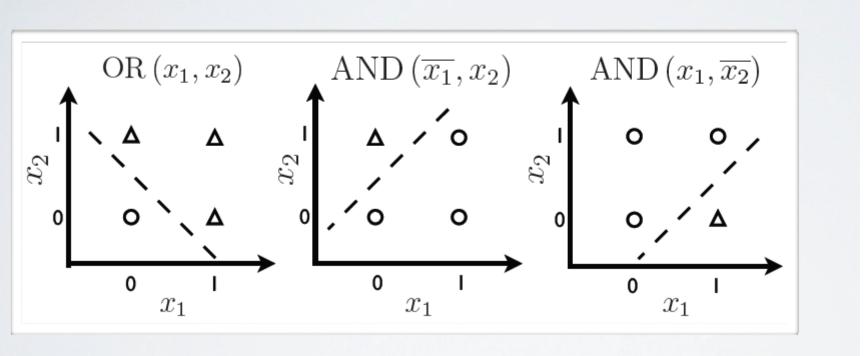
- · Could do binary classification:
 - with sigmoid, can interpret neuron as estimating $p(y=1|\mathbf{x})$
 - also known as logistic regression classifier
 - if greater than 0.5, predict class 1
 - otherwise, predict class 0

(similar idea can apply with tanh)

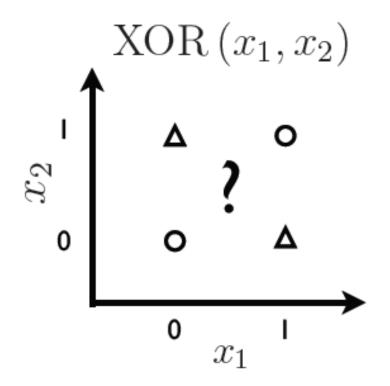


What can a Perceptron actually do?

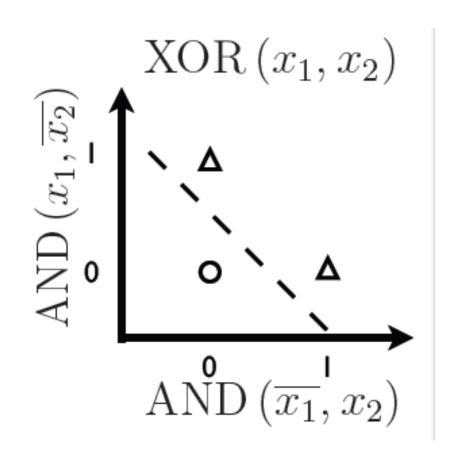
Can solve linearly separable problems



Can't solve non-linearly separable problems



Unless the i/p is transformed in a better representation



Neural Network

Hidden layer pre-activation:

$$\mathbf{a}(\mathbf{x}) = \mathbf{b}^{(1)} + \mathbf{W}^{(1)}\mathbf{x}$$
$$\left(a(\mathbf{x})_i = b_i^{(1)} + \sum_j W_{i,j}^{(1)} x_j\right)$$

Hidden layer activation:

$$\mathbf{h}(\mathbf{x}) = \mathbf{g}(\mathbf{a}(\mathbf{x}))$$

Output layer activation:

$$f(\mathbf{x}) = o\left(b^{(2)} + \mathbf{w}^{(2)^{\mathsf{T}}} \mathbf{h}^{(1)} \mathbf{x}\right) \underbrace{x_1}_{\text{output activation function}}^{t, j} \dots \underbrace{x_j}_{\text{output activation function}}^{t, j} \dots \underbrace{x_j}_{\text{output activation function}}^{t, j}$$

 (\mathbf{x})

 $h(\mathbf{x})_i$

Neural Network

- · For multi-class classification:
 - we need multiple outputs (I output per class)
 - $oldsymbol{\cdot}$ we would like to estimate the conditional probability $p(y=c|\mathbf{x})$
- · We use the softmax activation function at the output:

$$\mathbf{o}(\mathbf{a}) = \operatorname{softmax}(\mathbf{a}) = \left[\frac{\exp(a_1)}{\sum_c \exp(a_c)} \dots \frac{\exp(a_C)}{\sum_c \exp(a_c)}\right]^\top$$

- strictly positive
- > sums to one
- Predicted class is the one with highest estimated probability

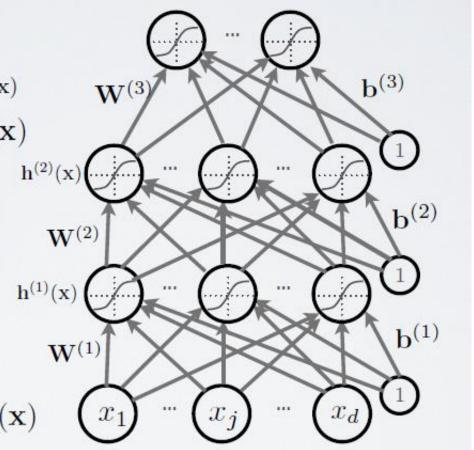
Neural Network

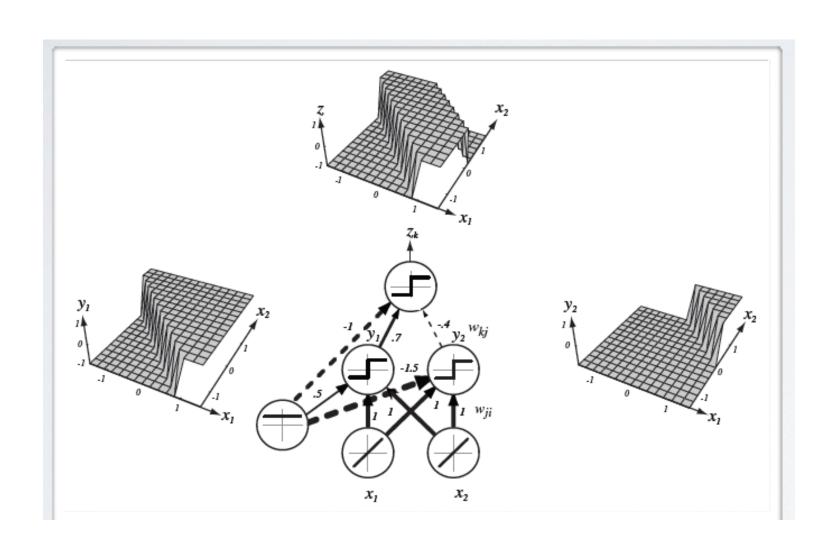
- Could have L hidden layers:
 - layer pre-activation for k>0 $(\mathbf{h}^{(0)}(\mathbf{x}) = \mathbf{x})$ $\mathbf{a}^{(k)}(\mathbf{x}) = \mathbf{b}^{(k)} + \mathbf{W}^{(k)}\mathbf{h}^{(k-1)}(\mathbf{x})$
 - ▶ hidden layer activation (k from 1 to L):

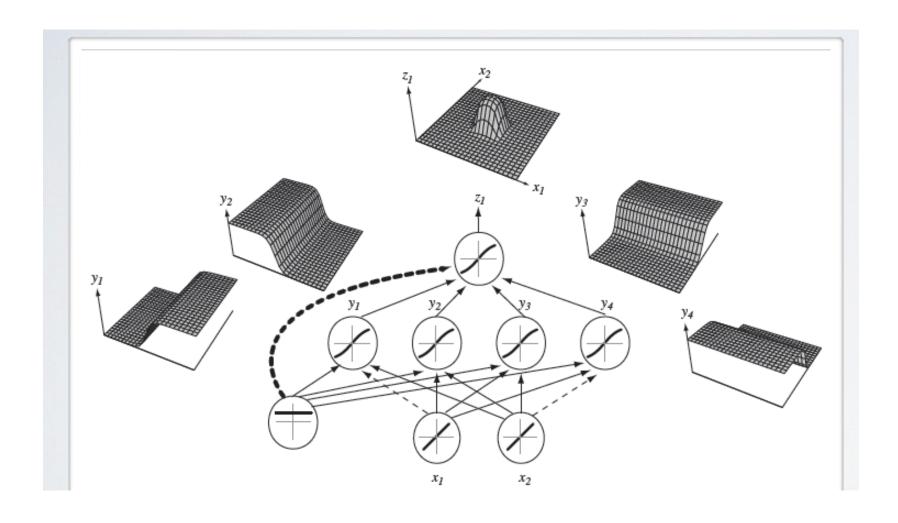
$$\mathbf{h}^{(k)}(\mathbf{x}) = \mathbf{g}(\mathbf{a}^{(k)}(\mathbf{x}))$$

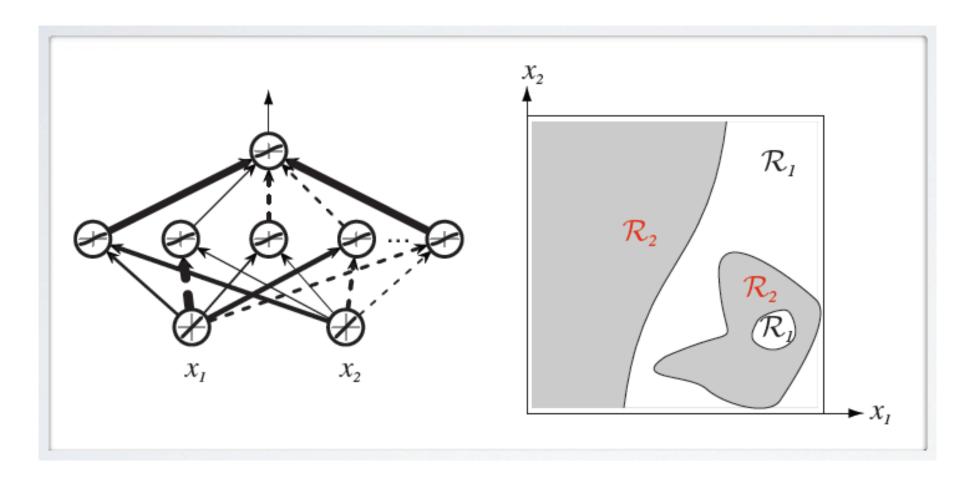
• output layer activation (k=L+1):

$$\mathbf{h}^{(L+1)}(\mathbf{x}) = \mathbf{o}(\mathbf{a}^{(L+1)}(\mathbf{x})) = \mathbf{f}(\mathbf{x})$$



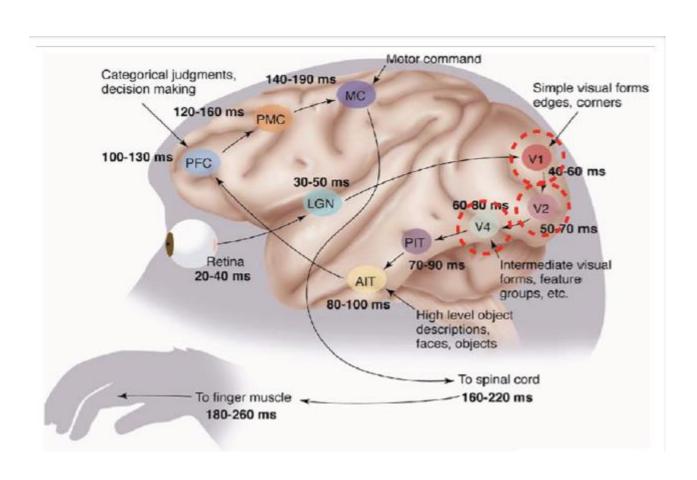


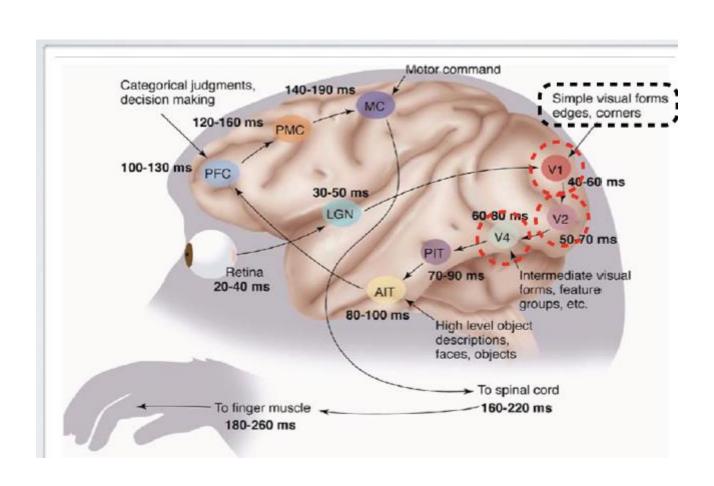


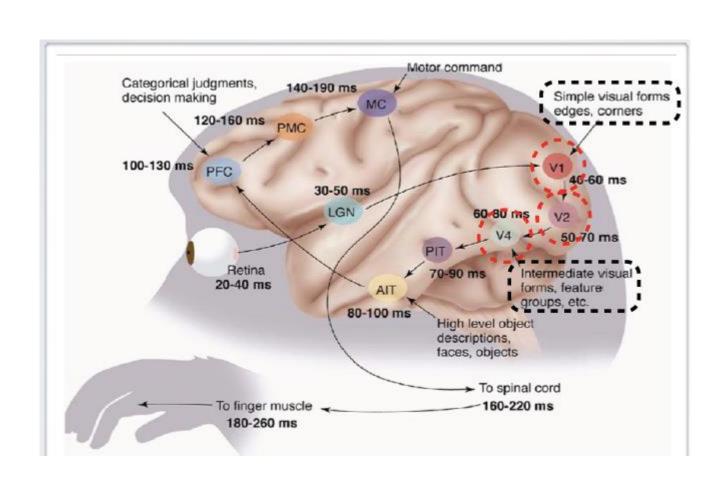


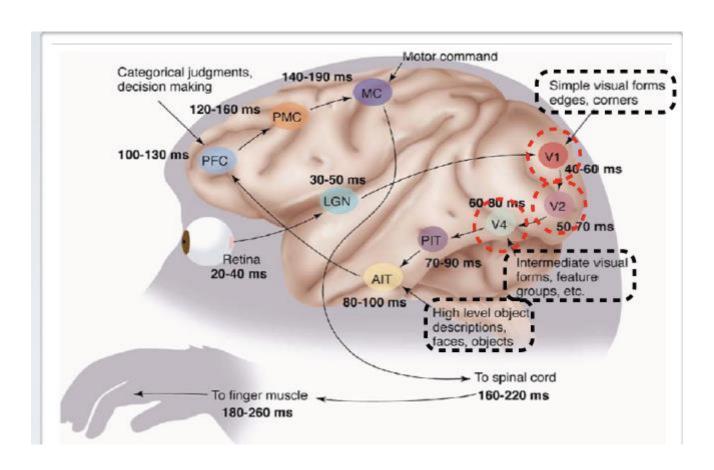
- · Universal approximation theorem (Hornik, 1991):
 - "a single hidden layer neural network with a linear output unit can approximate any continuous function arbitrarily well, given enough hidden units"
- The result applies for sigmoid, tanh and many other hidden layer activation functions

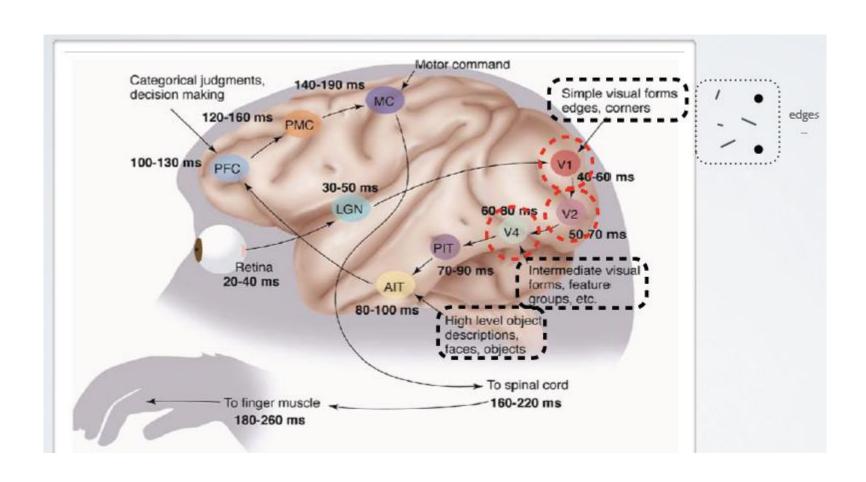
 This is a good result, but it doesn't mean there is a learning algorithm that can find the necessary parameter values!

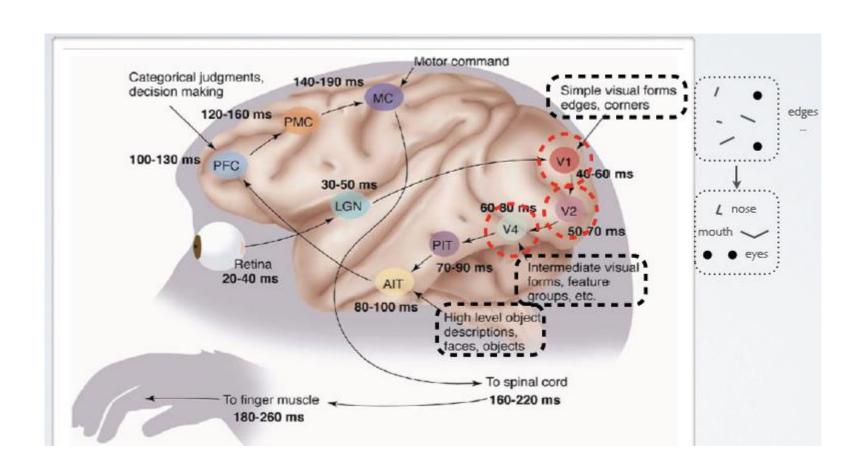


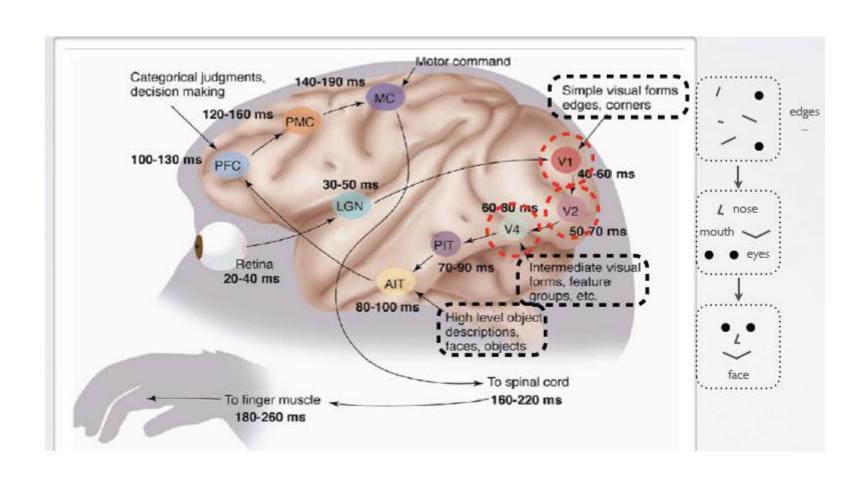




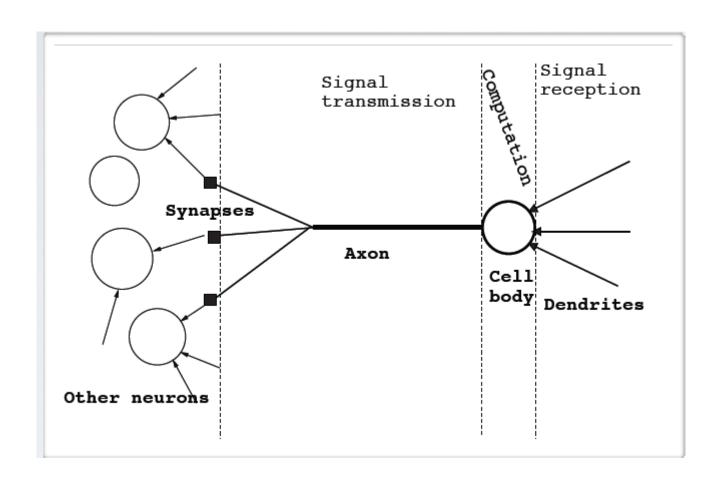




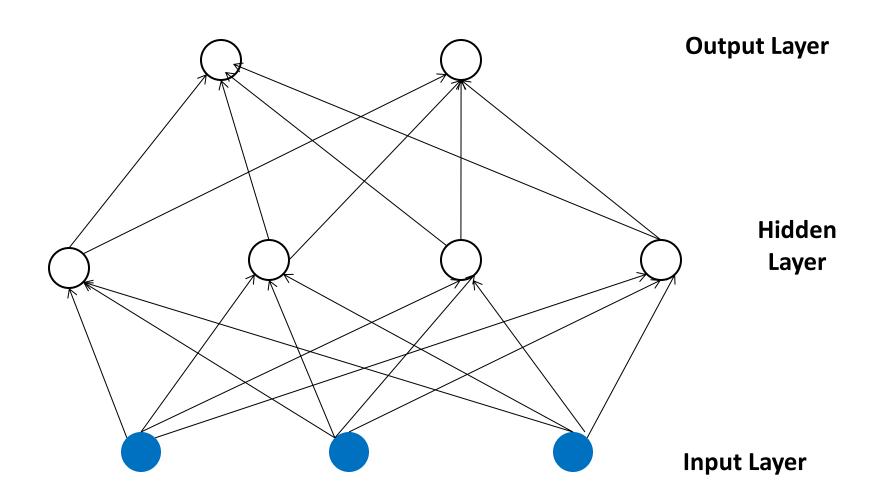


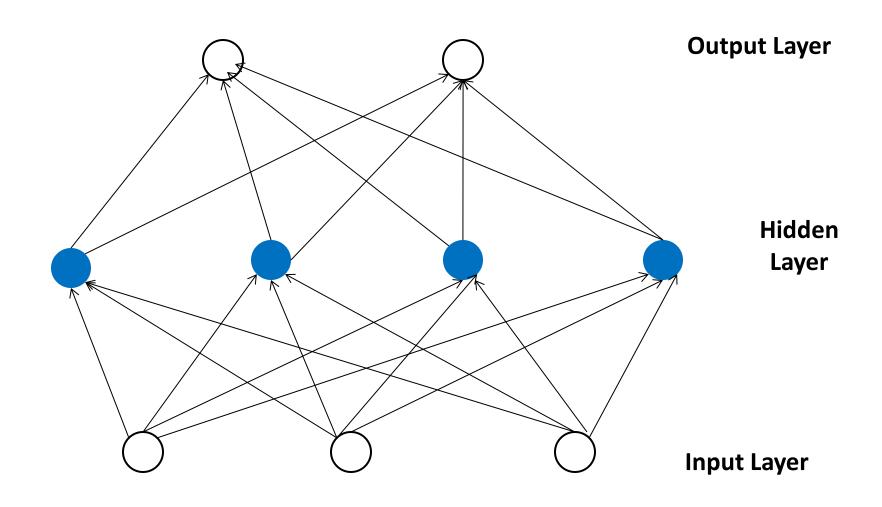


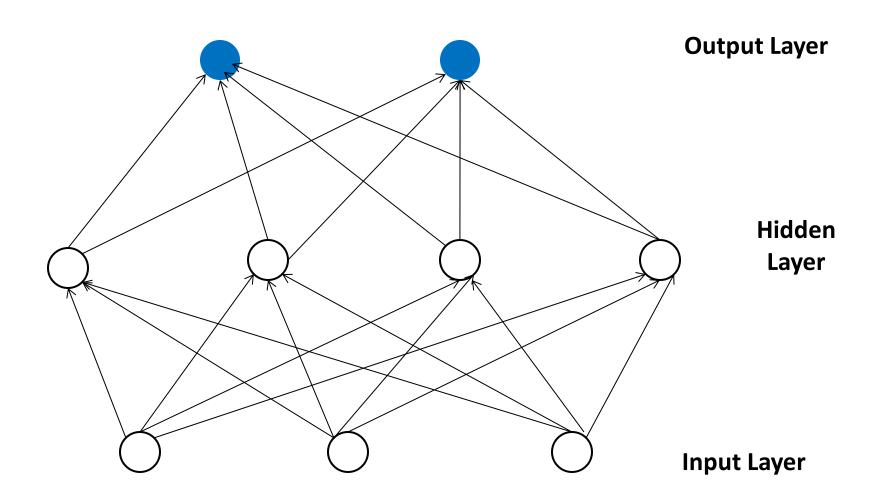
Biological Neuron

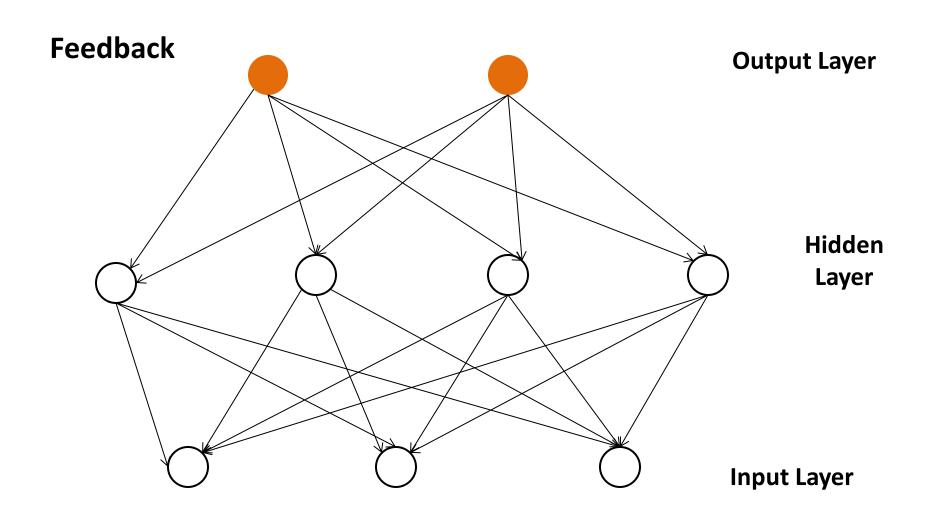


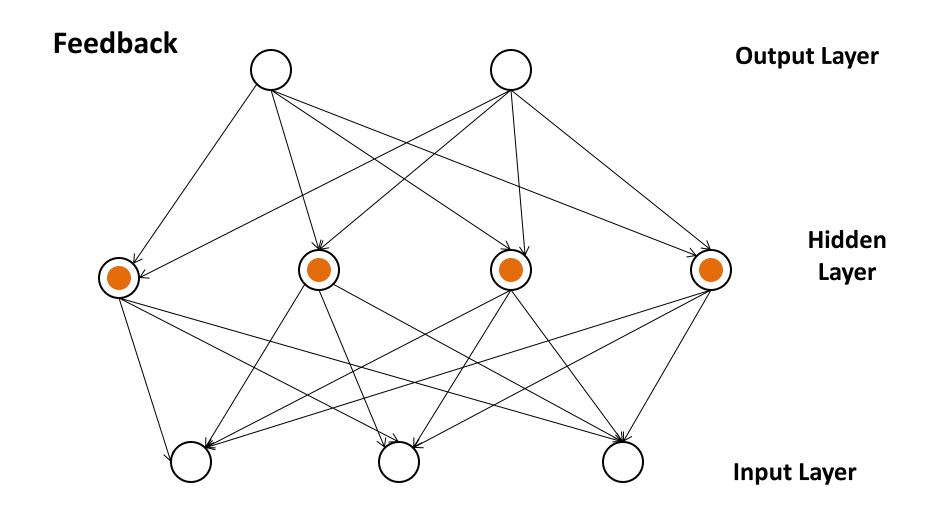
Training Neural Network - Backpropagation

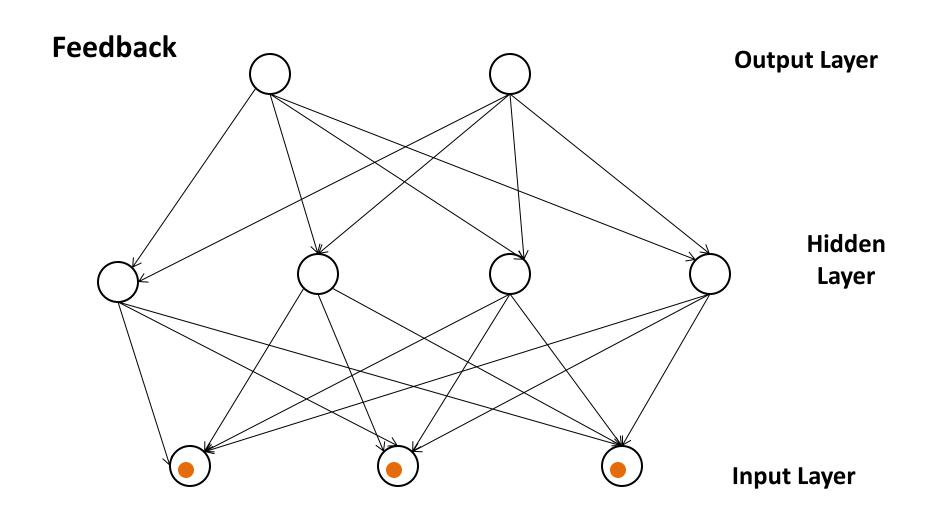




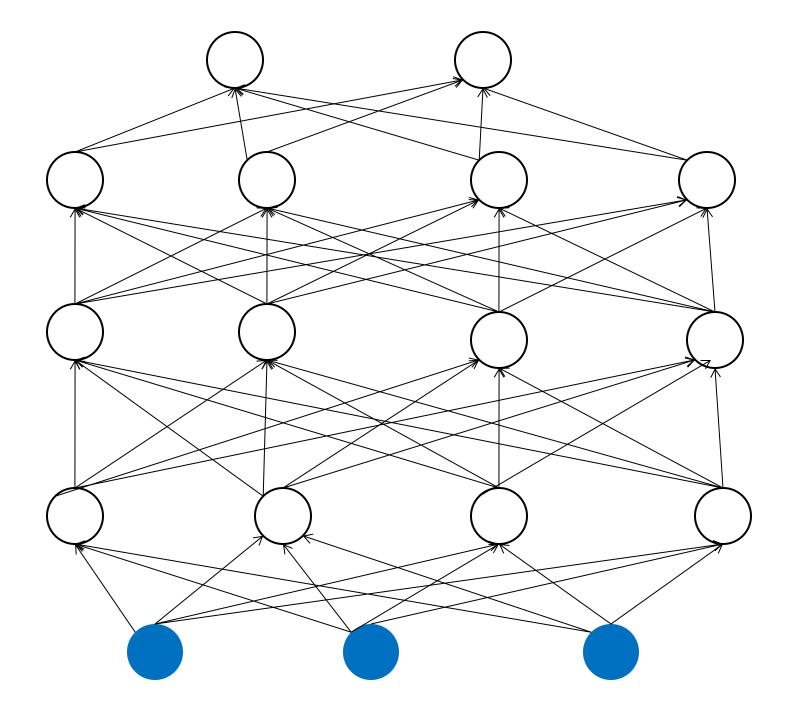


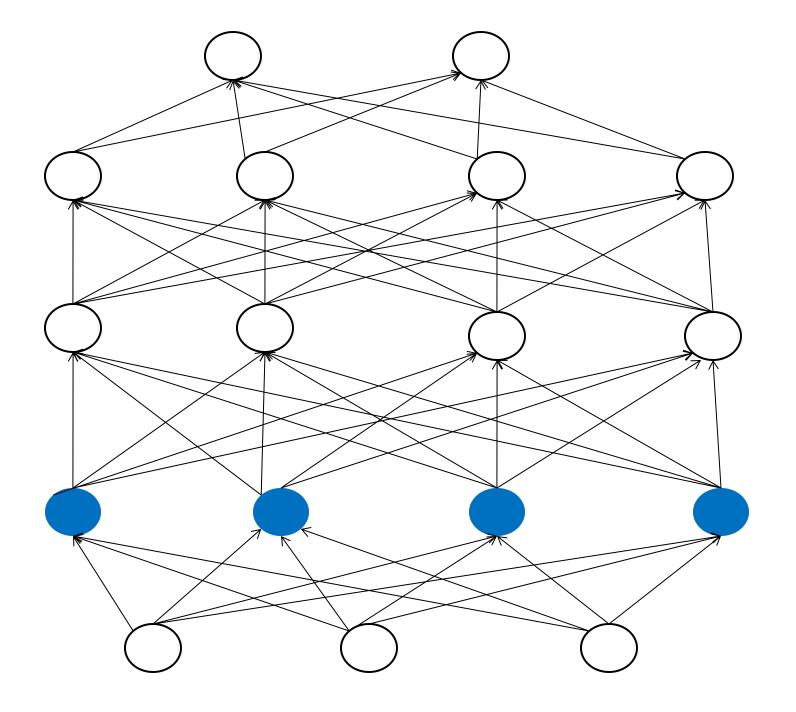


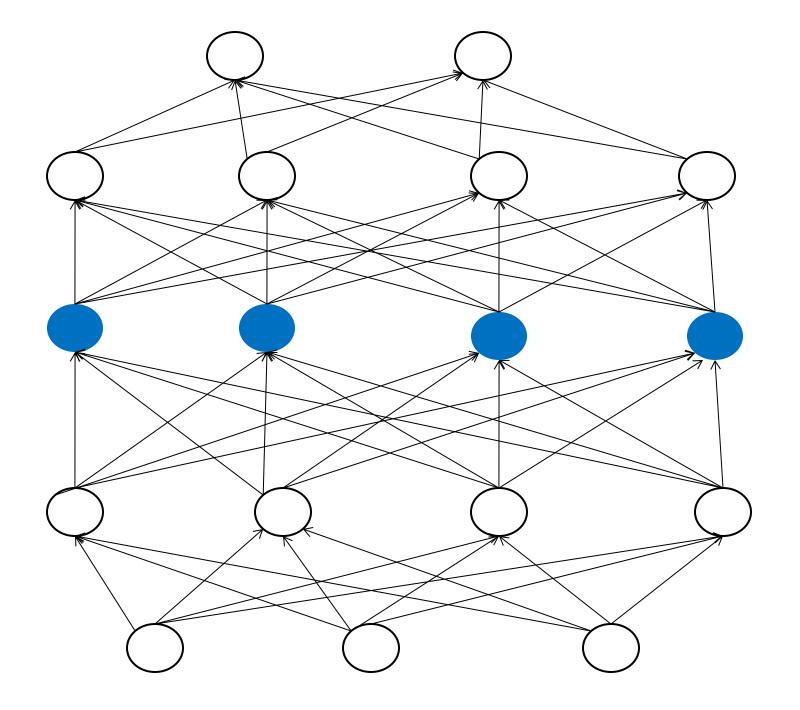


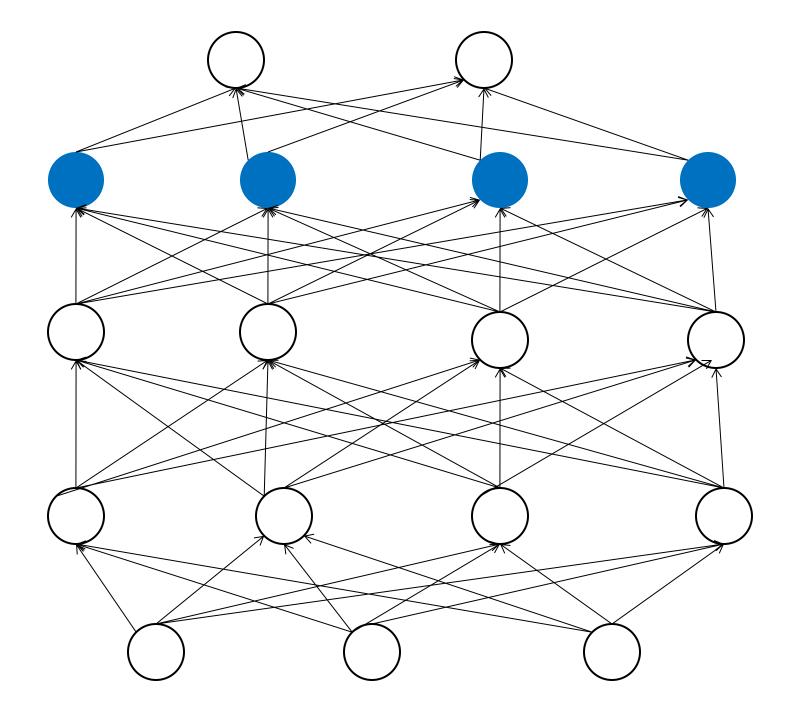


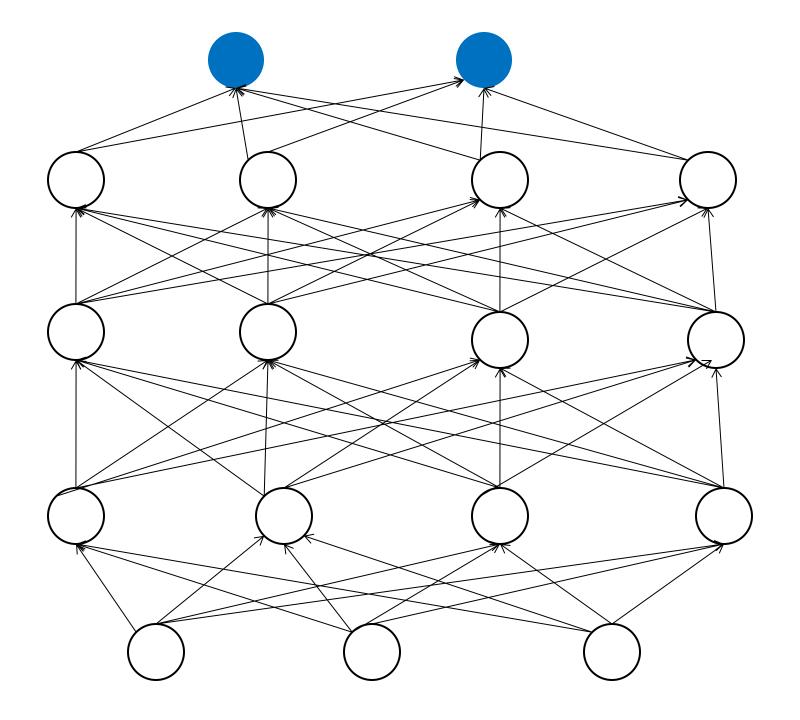
Issue in Back-propagation

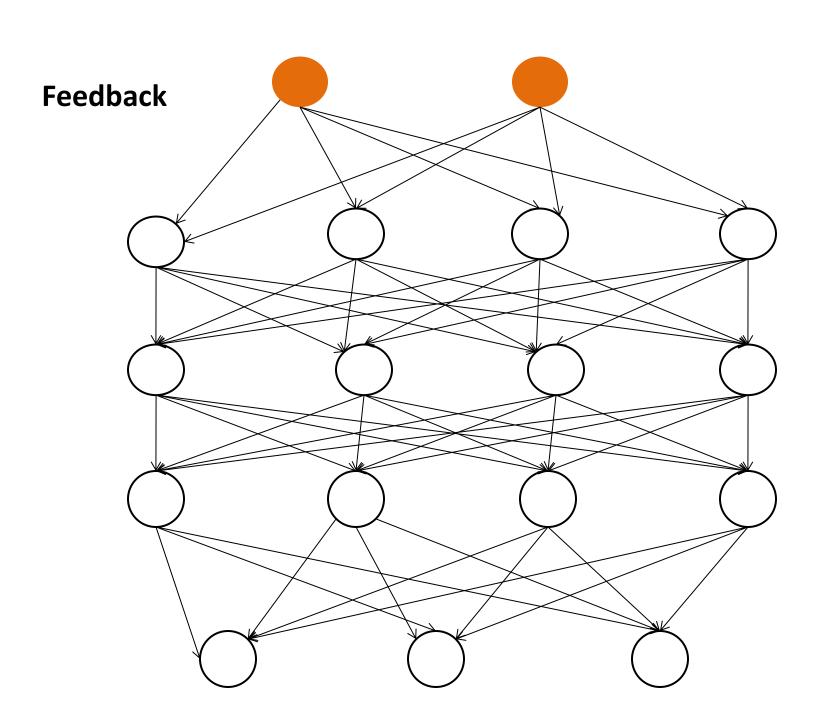


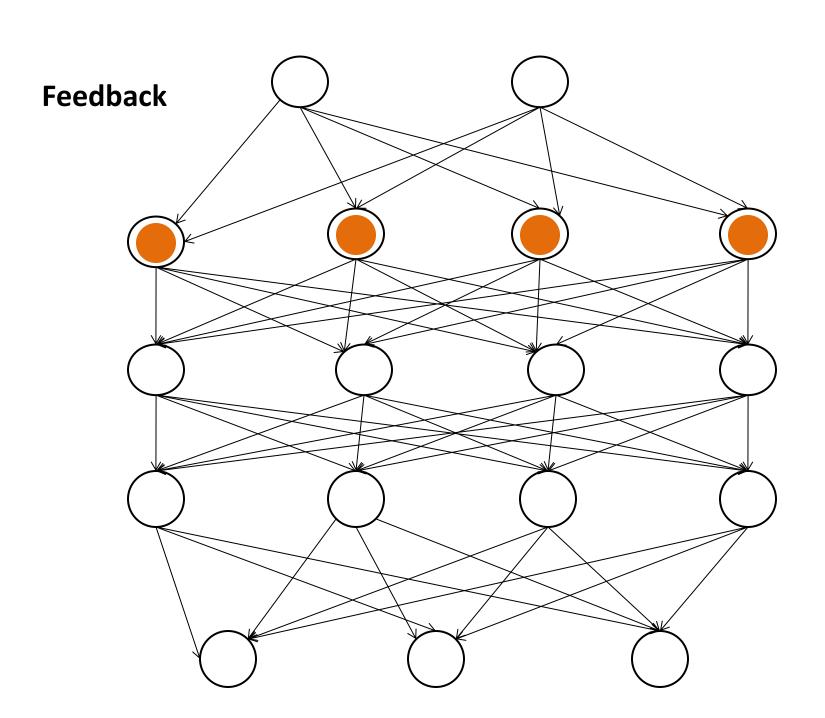


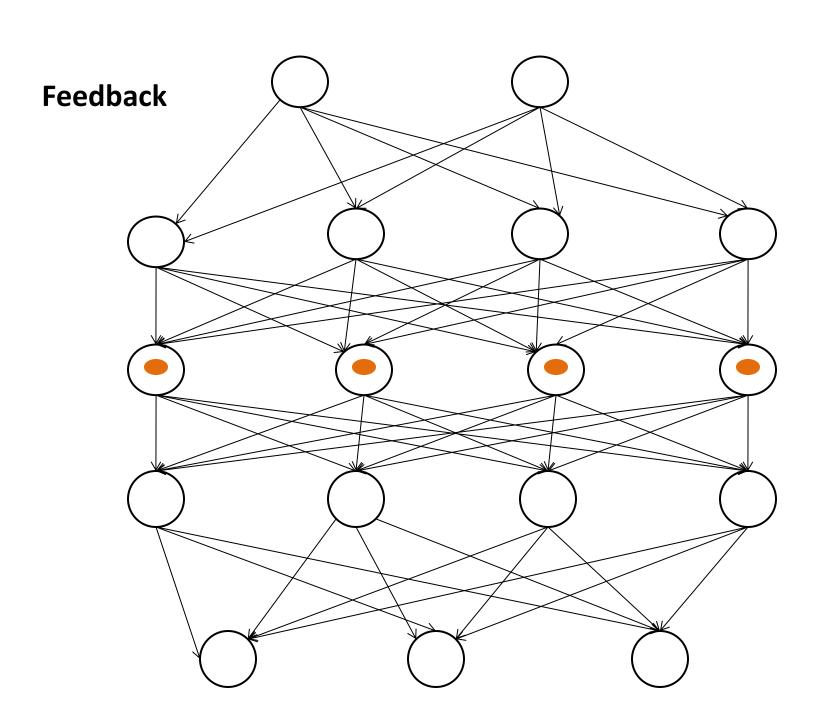


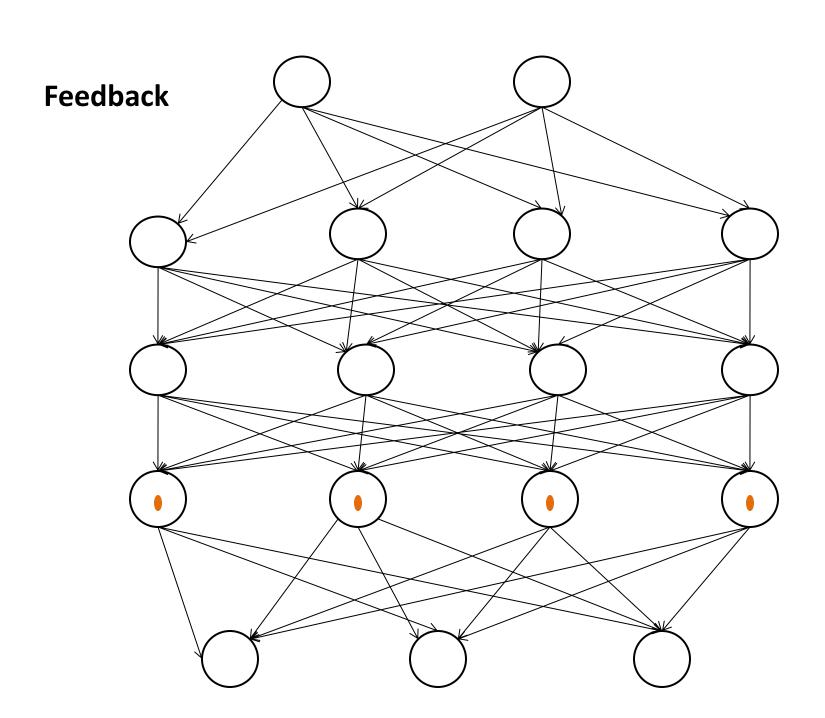


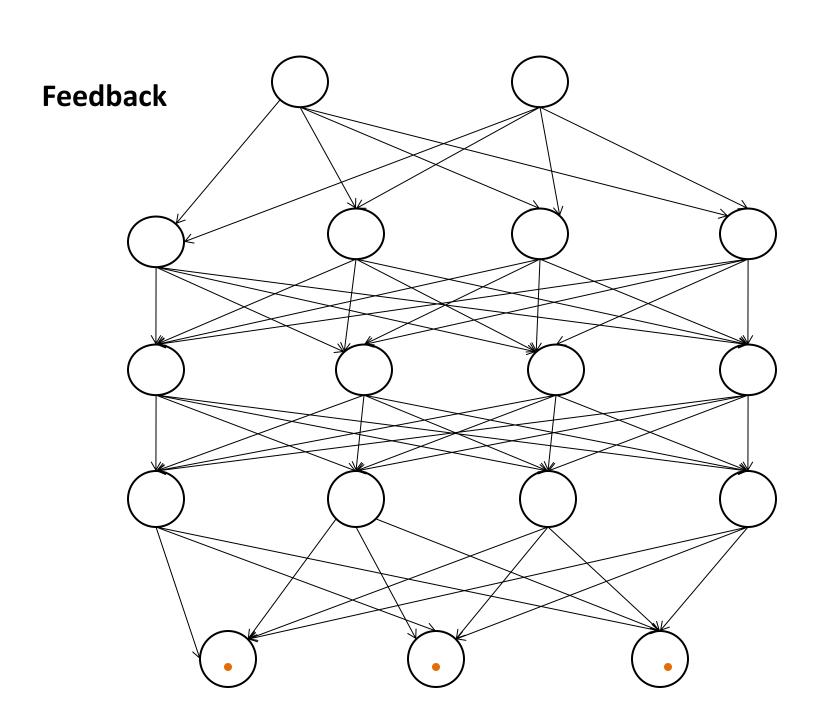






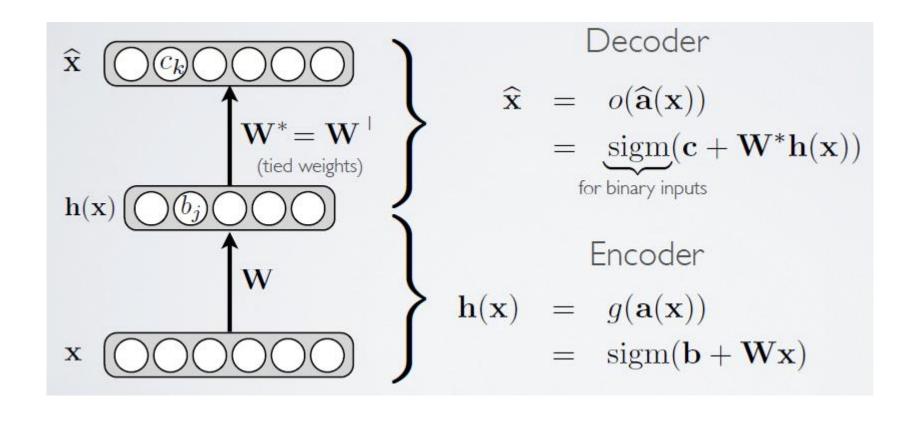


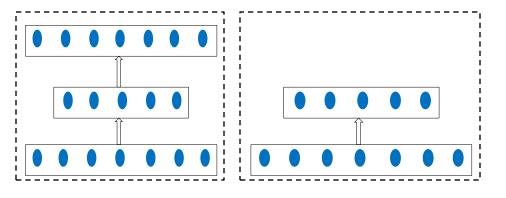


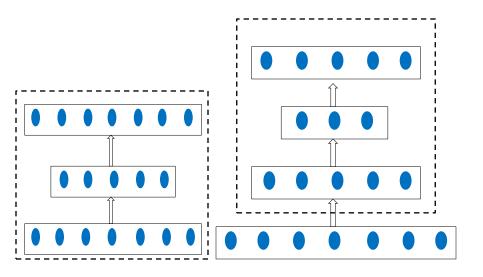


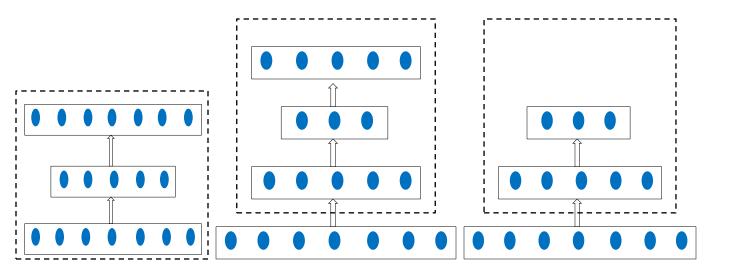
How to Solve this??

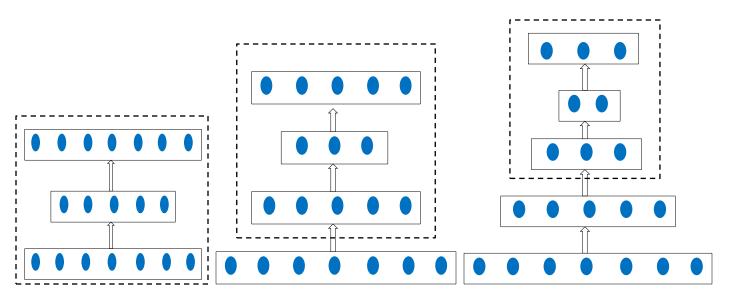
Auto encoder

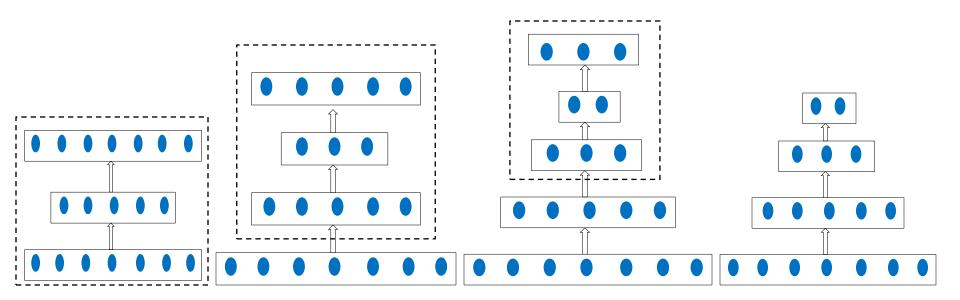


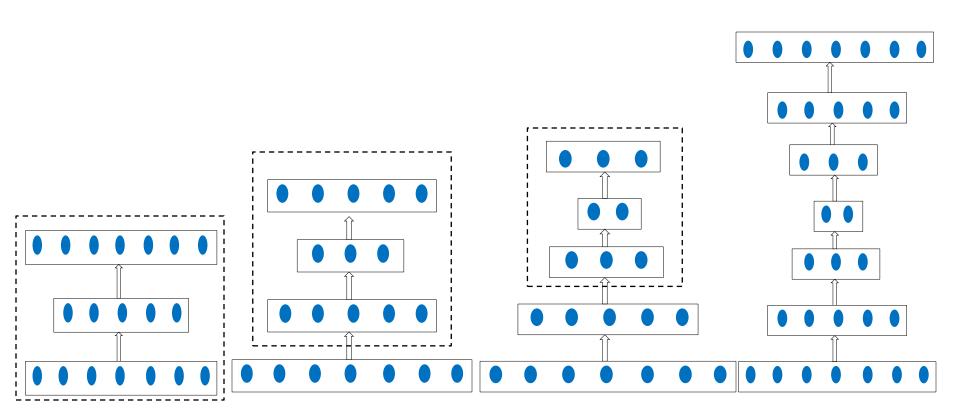


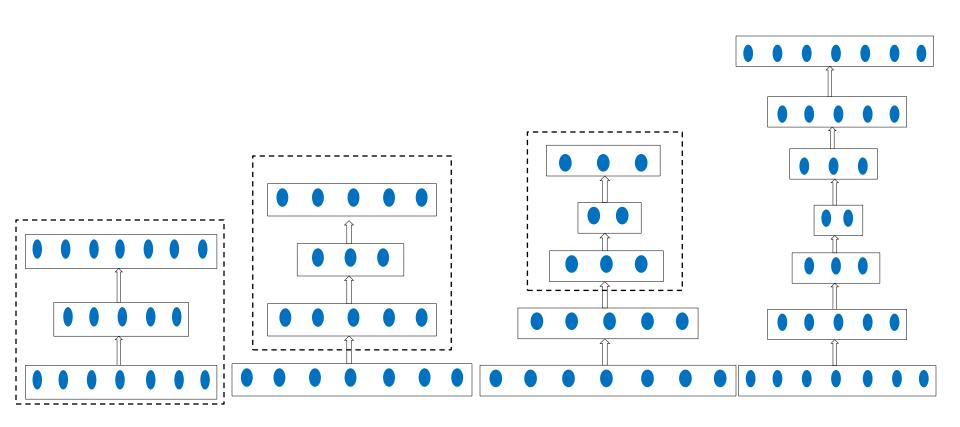












Greedy layerwise pretraining

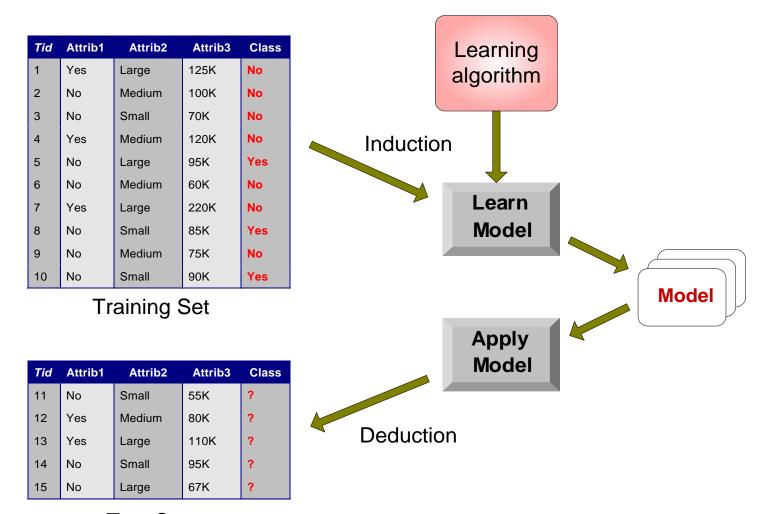
Fine-tuning

Classification

Classification: Definition

- Given a collection of records (training set)
 - Each record contains a set of attributes, one of the attributes is the class.
- Find a model for class attribute as a function of the values of other attributes.
- Goal: <u>previously unseen</u> records should be assigned a class as accurately as possible.
 - A test set is used to determine the accuracy of the model. Usually, the given data set is divided into training and test sets, with training set used to build the model and test set used to validate it.

Illustrating Classification Task



Test Set

Examples of Classification Task

- Predicting tumor cells as benign or malignant
- Classifying credit card transactions as legitimate or fraudulent



- Classifying secondary structures of protein as alpha-helix, beta-sheet, or random coil
- Categorizing news stories as finance, weather, entertainment, sports, etc

Other Classification Techniques...

- Decision Tree Based Methods
- Rule Based Methods
- Memory Based Reasoning
- Naïve Bayes and Bayesian Belief Networks
- Support Vector Machines

Other Classification Techniques...

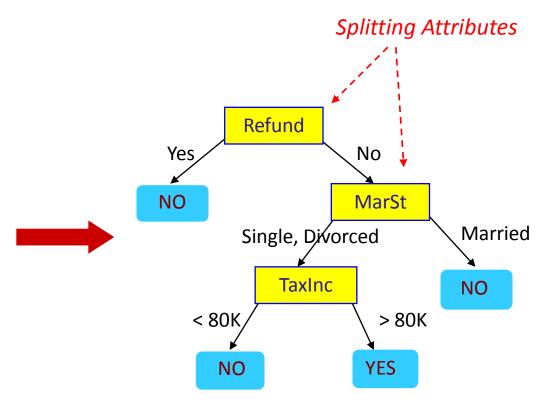
- Decision Tree Based Methods
- Rule Based Methods
- Memory Based Reasoning
- Naïve Bayes and Bayesian Belief Networks
- Support Vector Machines

Decision Tree Based Methods

Example of a Decision Tree

categorical continuous

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



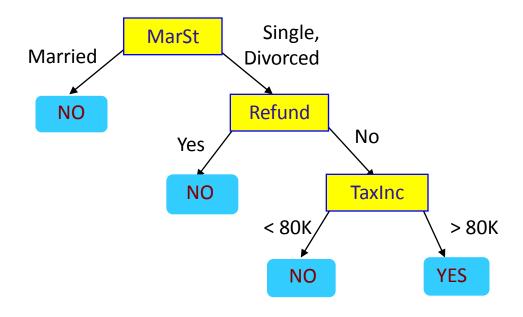
Training Data

Model: Decision Tree

Another Example of Decision Tree

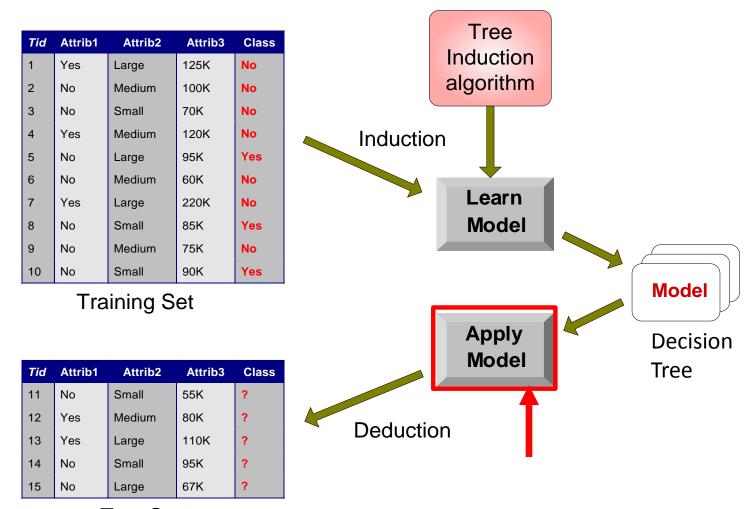
categorical continuous

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
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3	No	Single	70K	No
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5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



There could be more than one tree that fits the same data!

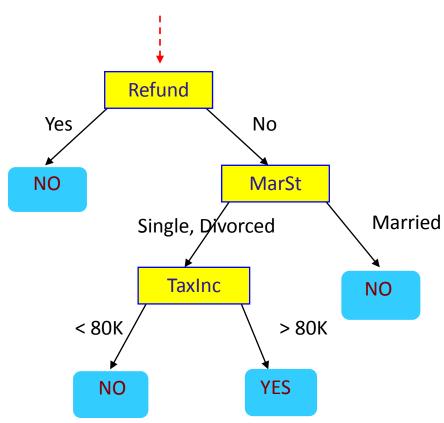
Decision Tree Classification Task



Test Set

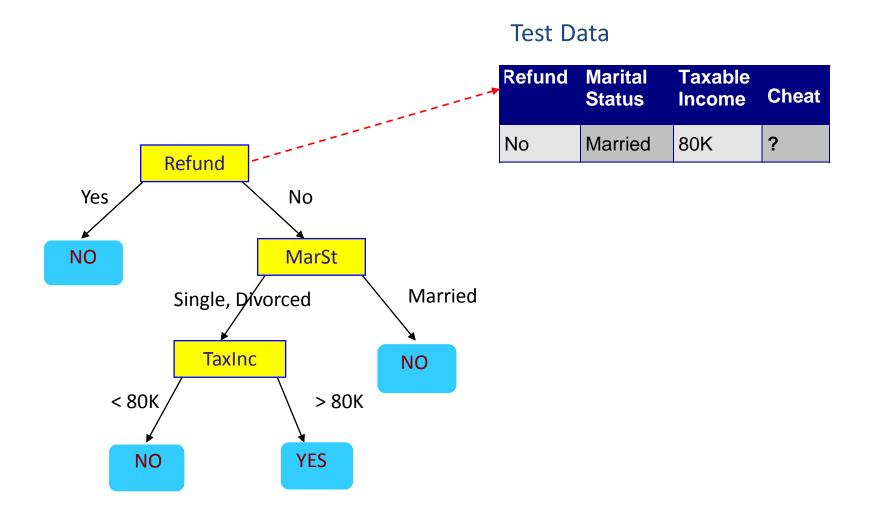
Apply Model to Test Data

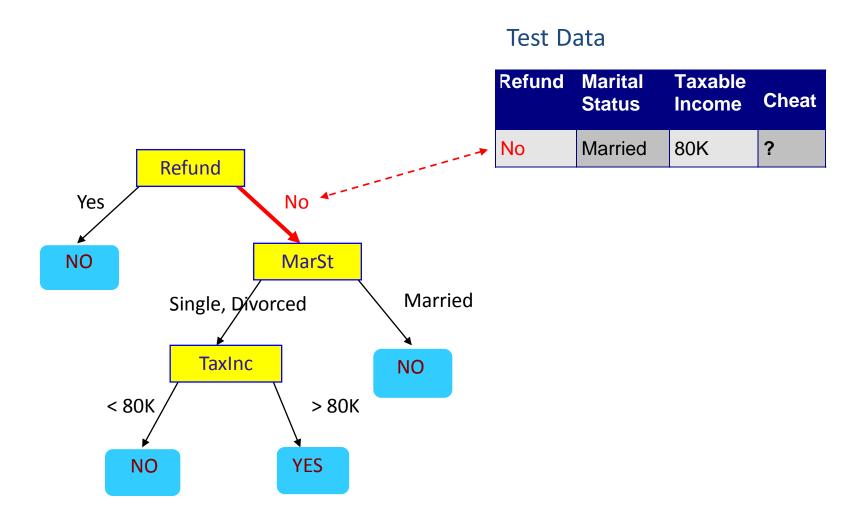
Start from the root of tree.

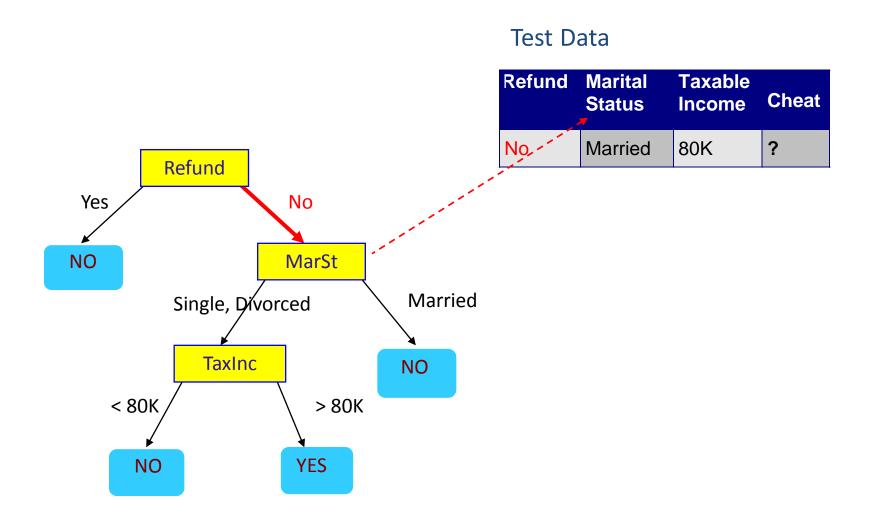


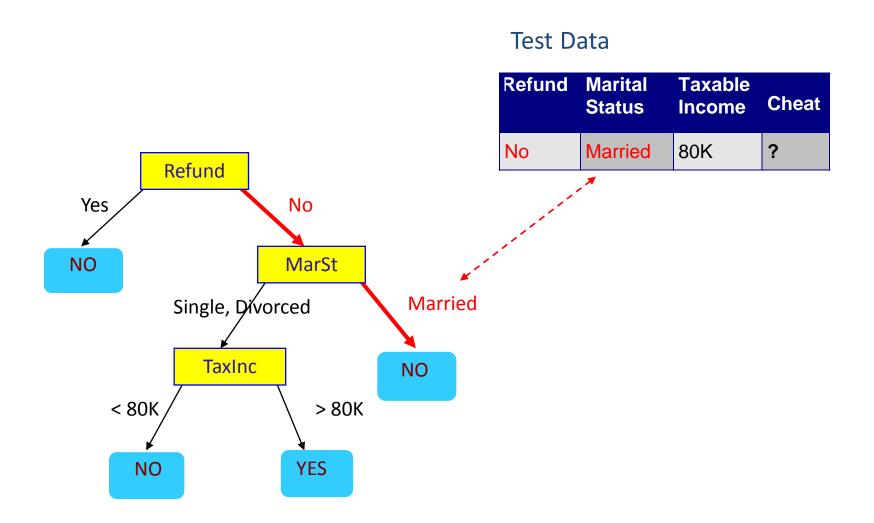
Test Data

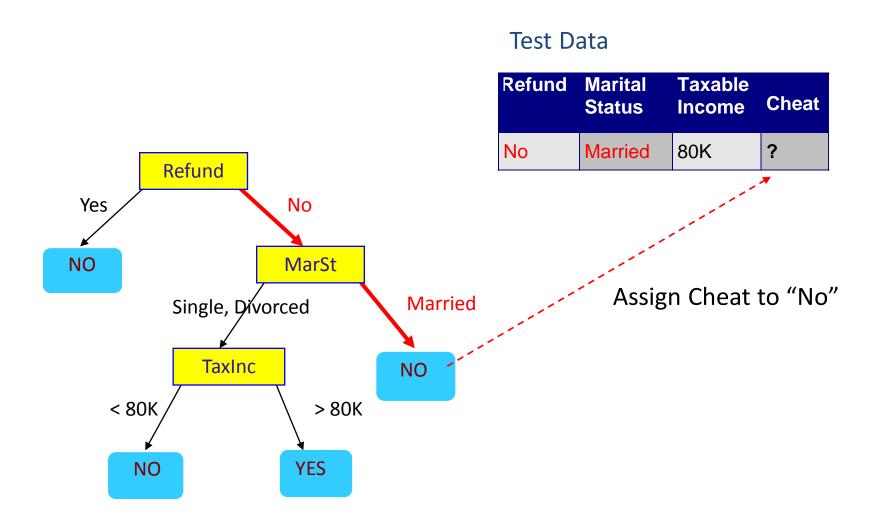
Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



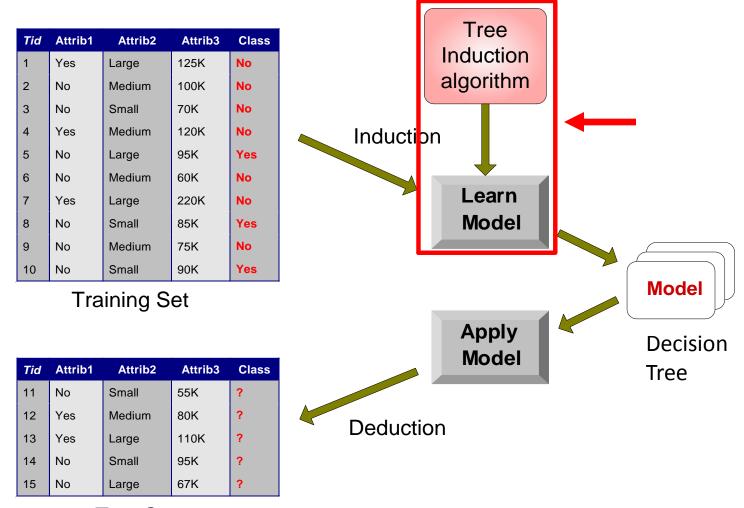








Decision Tree Classification Task



Test Set

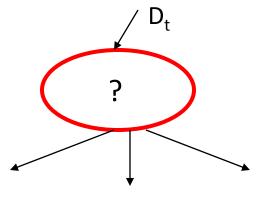
Decision Tree Induction

- Many Algorithms:
 - Hunt's Algorithm (one of the earliest)
 - CART
 - ID3, C4.5
 - SLIQ,SPRINT

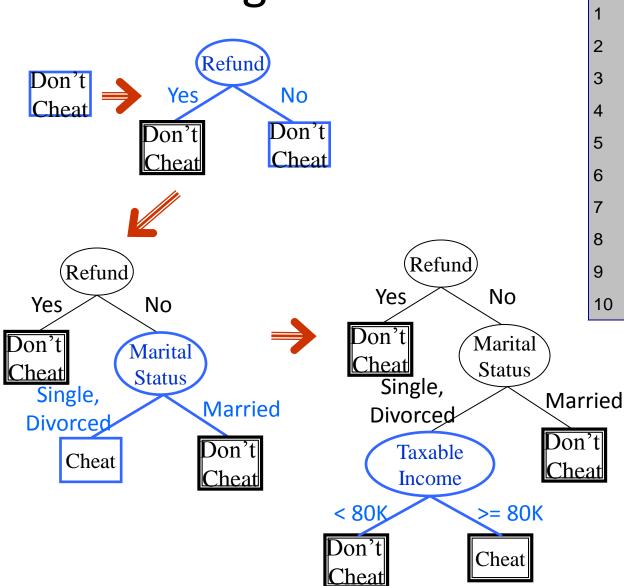
General Structure of Hunt's Algorithm

- Let D_t be the set of training records that reach a node t
- General Procedure:
 - If D_t contains records that belong the same class y_t, then t is a leaf node labeled as y_t
 - If D_t is an empty set, then t is a leaf node labeled by the default class,
 Y_d
 - If D_t contains records that belong to more than one class, use an attribute test to split the data into smaller subsets. Recursively apply the procedure to each subset.

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Hunt's Algorithm



Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
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3	No	Single	70K	No
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5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Tree Induction

- Greedy strategy.
 - Split the records based on an attribute test that optimizes certain criterion.

- Issues
 - Determine how to split the records
 - How to specify the attribute test condition?
 - How to determine the best split?
 - Determine when to stop splitting

Tree Induction

- Greedy strategy.
 - Split the records based on an attribute test that optimizes certain criterion.

- Issues
 - Determine how to split the records
 - How to specify the attribute test condition?
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 - Determine when to stop splitting

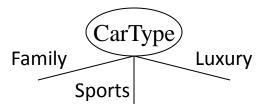
How to Specify Test Condition?

- Depends on attribute types
 - Nominal
 - Ordinal
 - Continuous

- Depends on number of ways to split
 - 2-way split
 - Multi-way split

Splitting Based on Nominal Attributes

Multi-way split: Use as many partitions as distinct values.

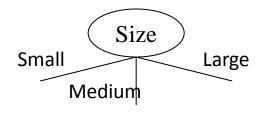


Binary split: Divides values into two subsets.
 Need to find optimal partitioning.

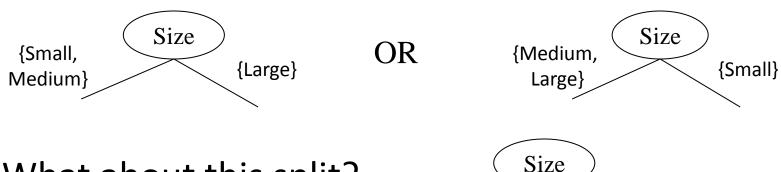


Splitting Based on Ordinal Attributes

Multi-way split: Use as many partitions as distinct values.



Binary split: Divides values into two subsets.
 Need to find optimal partitioning.



{Small.

Large}

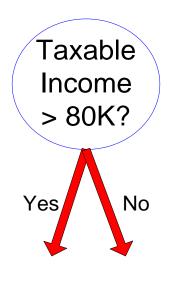
{Medium}

What about this split?

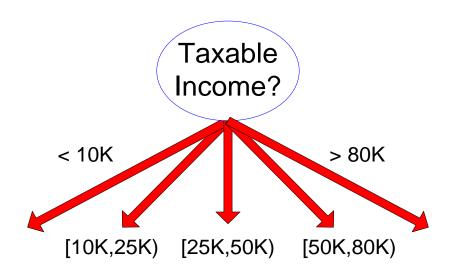
Splitting Based on Continuous Attributes

- Different ways of handling
 - Discretization to form an ordinal categorical attribute
 - Static discretize once at the beginning
 - Dynamic ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
 - Binary Decision: (A < v) or (A ≥ v)
 - consider all possible splits and finds the best cut
 - can be more compute intensive

Splitting Based on Continuous Attributes



(i) Binary split



(ii) Multi-way split

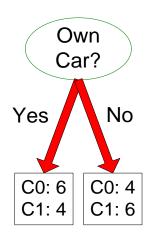
Tree Induction

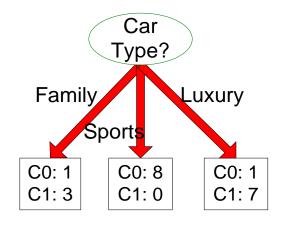
- Greedy strategy.
 - Split the records based on an attribute test that optimizes certain criterion.

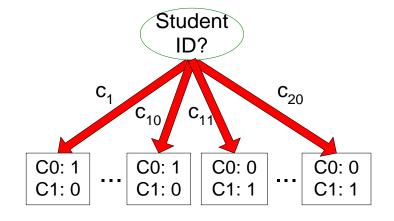
- Issues
 - Determine how to split the records
 - How to specify the attribute test condition?
 - How to determine the best split?
 - Determine when to stop splitting

How to determine the Best Split

Before Splitting: 10 records of class 0, 10 records of class 1







Which test condition is the best?

How to determine the Best Split

- Greedy approach:
 - Nodes with homogeneous class distribution are preferred
- Need a measure of node impurity:

C0: 5 C1: 5

Non-homogeneous,

High degree of impurity

C0: 9

C1: 1

Homogeneous,

Low degree of impurity

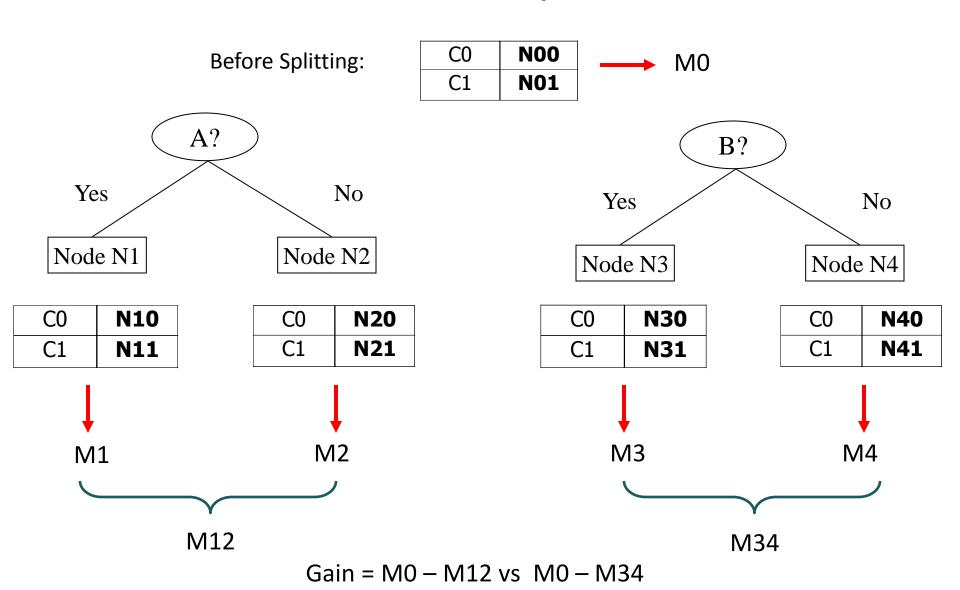
Measures of Node Impurity

• Gini Index

Entropy

Misclassification error

How to Find the Best Split



Measure of Impurity: GINI

Gini Index for a given node t :

$$GINI(t) = 1 - \sum_{j} [p(j|t)]^{2}$$

(NOTE: $p(j \mid t)$ is the relative frequency of class j at node t).

- Maximum $(1 1/n_c)$ when records are equally distributed among all classes, implying least interesting information
- Minimum (0.0) when all records belong to one class, implying most interesting information

Gini=	0.000
C2	6
C1	0

Gini=	0.278
C2	5
C1	1

C1	2	
C2	4	
Gini=0.444		

C1	3	
C2	3	
Gini=0.500		

Examples for computing GINI

$$GINI(t) = 1 - \sum_{j} [p(j|t)]^{2}$$

C1	0
C2	6

$$P(C1) = 0/6 = 0$$
 $P(C2) = 6/6 = 1$
 $Gini = 1 - P(C1)^2 - P(C2)^2 = 1 - 0 - 1 = 0$

$$P(C1) = 1/6$$
 $P(C2) = 5/6$
 $Gini = 1 - (1/6)^2 - (5/6)^2 = 0.278$

C1	2
C2	4

$$P(C1) = 2/6$$
 $P(C2) = 4/6$
Gini = 1 - $(2/6)^2$ - $(4/6)^2$ = 0.444

Splitting Based on GINI

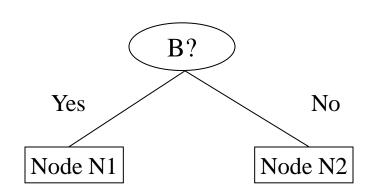
- Used in CART, SLIQ, SPRINT.
- When a node p is split into k partitions (children), the quality of split is computed as,

$$GINI_{split} = \sum_{i=1}^{k} \frac{n_i}{n} GINI(i)$$

where, n_i = number of records at child i, n = number of records at node p.

Binary Attributes: Computing GINI Index

- Splits into two partitions
- Effect of Weighing partitions:
 - Larger and Purer Partitions are sought for.



	Parent
C1	6
C2	6
Gini = 0.500	

Gini(N1)

$$= 1 - (5/6)^2 - (2/6)^2$$

= 0.194

Gini(N2)

$$= 1 - (1/6)^2 - (4/6)^2$$

= 0.528

	N1	N2
C1	5	1
C2	2	4
Cini_0 222		

Gini(Children)

= 0.333

Categorical Attributes: Computing Gini Index

- For each distinct value, gather counts for each class in the dataset
- Use the count matrix to make decisions

Multi-way split

	CarType		
	Family Sports Luxury		
C1	1	2	1
C2	4	1	1
Gini	0.393		

Two-way split (find best partition of values)

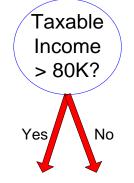
	CarType		
	{Sports, Luxury} {Family}		
C1	3	1	
C2	2	4	
Gini	0.400		

	CarType					
	{Sports}	{Family, Luxury}				
C 1	2	2				
C2	1 5					
Gini	0.419					

Continuous Attributes: Computing Gini Index

- Use Binary Decisions based on one value
- Several Choices for the splitting value
 - Number of possible splitting valuesNumber of distinct values
- Each splitting value has a count matrix associated with it
 - Class counts in each of the partitions, A< v and A ≥ v
- Simple method to choose best v
 - For each v, scan the database to gather count matrix and compute its Gini index
 - Computationally Inefficient! Repetition of work.

Tid	Refund	Marital Status	Taxable Income	Cheat	
1	Yes	Single	125K	No	
2	No	Married	100K	No	
3	No	Single	70K	No	
4	Yes	Married	120K	No	
5	No	Divorced	95K	Yes	
6	No	Married	60K	No	
7	Yes	Divorced	220K	No	
8	No	Single	85K	Yes	
9	No	Married	75K	No	
10	No	Single	90K	Yes	



Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
 - Sort the attribute on values
 - Linearly scan these values, each time updating the count matrix and computing gini index
 - Choose the split position that has the least gini index

Sorted Values
Split Positions

Cheat		No		No		N	0	Yes Yes Ye					es	No			No		No		No	
	Taxable Income																					
		60	70 75 85 90 95 100 120 125 220																			
	5	5	6	5 5	7	2	8	0	8	7	9	2	9	7	11	10	12	22	17	72	23	0
	<=	^	V =	>	\=	>	\=	^	<=	>	<=	^	<=	^	<=	^	<=	^	<=	^	<=	>
Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
Gini	0.4	20	0.4	400	0.3	375	0.3	43	0.4	17	0.4	100	<u>0.3</u>	<u>300</u>	0.3	343	0.3	75	0.4	00	0.4	20

Alternative Splitting Criteria based on INFO

Entropy at a given node t:

$$Entropy(t) = -\sum_{j} p(j|t) \log p(j|t)$$

(NOTE: $p(j \mid t)$ is the relative frequency of class j at node t).

- Measures homogeneity of a node.
 - Maximum ($log n_c$) when records are equally distributed among all classes implying least information
 - Minimum (0.0) when all records belong to one class, implying most information
- Entropy based computations are similar to the GINI index computations

Examples for computing Entropy

$$Entropy(t) = -\sum_{j} p(j | t) \log_{2} p(j | t)$$

C1	0
C2	6

$$P(C1) = 0/6 = 0$$
 $P(C2) = 6/6 = 1$
 $Entropy = -0 log 0 - 1 log 1 = -0 - 0 = 0$

$$P(C1) = 1/6$$
 $P(C2) = 5/6$
Entropy = $-(1/6) \log_2 (1/6) - (5/6) \log_2 (1/6) = 0.65$

C1	2
C2	4

$$P(C1) = 2/6$$
 $P(C2) = 4/6$
 $Entropy = -(2/6) log_2(2/6) - (4/6) log_2(4/6) = 0.92$

Splitting Based on INFO...

Information Gain:

$$GAIN_{split} = Entropy(p) - \left(\sum_{i=1}^{k} \frac{n_i}{n} Entropy(i)\right)$$

Parent Node, p is split into k partitions; n_i is number of records in partition i

- Measures Reduction in Entropy achieved because of the split. Choose the split that achieves most reduction (maximizes GAIN)
- Used in ID3 and C4.5
- Disadvantage: Tends to prefer splits that result in large number of partitions, each being small but pure.

Splitting Based on INFO...

Gain Ratio:

$$GainRATIO_{split} = \frac{GAIN_{split}}{SplitINFO} SplitINFO = -\sum_{i=1}^{k} \frac{n_i}{n} \log \frac{n_i}{n}$$

Parent Node, p is split into k partitions n_i is the number of records in partition i

- Adjusts Information Gain by the entropy of the partitioning (SplitINFO). Higher entropy partitioning (large number of small partitions) is penalized!
- Used in C4.5
- Designed to overcome the disadvantage of Information Gain

Splitting Criteria based on Classification Error

Classification error at a node t :

$$Error(t) = 1 - \max_{i} P(i \mid t)$$

- Measures misclassification error made by a node.
 - Maximum $(1 1/n_c)$ when records are equally distributed among all classes, implying least interesting information
 - Minimum (0.0) when all records belong to one class, implying most interesting information

Examples for Computing Error

$$Error(t) = 1 - \max_{i} P(i \mid t)$$

C1	0
C2	6

$$P(C1) = 0/6 = 0$$
 $P(C2) = 6/6 = 1$
 $Error = 1 - max(0, 1) = 1 - 1 = 0$

Error =
$$1 - \max(0, 1) = 1 - 1 = 0$$

$$P(C1) = 1/6$$
 $P(C2) = 5/6$

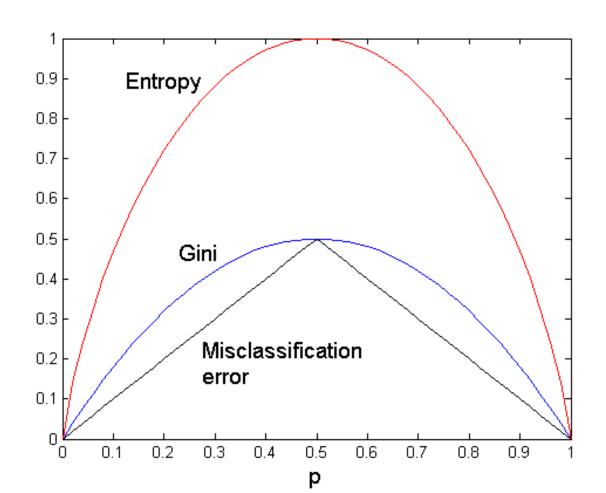
Error =
$$1 - \max(1/6, 5/6) = 1 - 5/6 = 1/6$$

$$P(C1) = 2/6$$
 $P(C2) = 4/6$

Error =
$$1 - \max(2/6, 4/6) = 1 - 4/6 = 1/3$$

Comparison among Splitting Criteria

For a 2-class problem:



Tree Induction

- Greedy strategy.
 - Split the records based on an attribute test that optimizes certain criterion.

- Issues
 - Determine how to split the records
 - How to specify the attribute test condition?
 - How to determine the best split?
 - Determine when to stop splitting

Stopping Criteria for Tree Induction

 Stop expanding a node when all the records belong to the same class

 Stop expanding a node when all the records have similar attribute values

Early termination

Decision Tree Based Classification

Advantages:

- Inexpensive to construct
- Extremely fast at classifying unknown records
- Easy to interpret for small-sized trees
- Accuracy is comparable to other classification techniques for many simple data sets

Instance Based Classifiers

Instance Based Classifier

Set of Stored Cases

Atr1	 AtrN	Class
		A
		В
		В
		С
		A
		С
		В

- Store the training records
- Use training records to predict the class label of unseen cases

Unseen Case

Atr1	 AtrN

Instance Based Classifier

Examples:

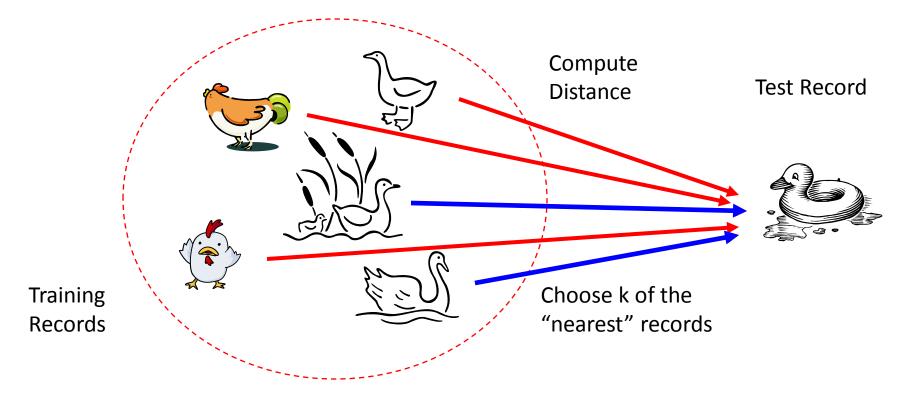
- Rote-learner
 - Memorizes entire training data and performs classification only if attributes of record match one of the training examples exactly

- Nearest neighbor
 - Uses k "closest" points (nearest neighbors) for performing classification

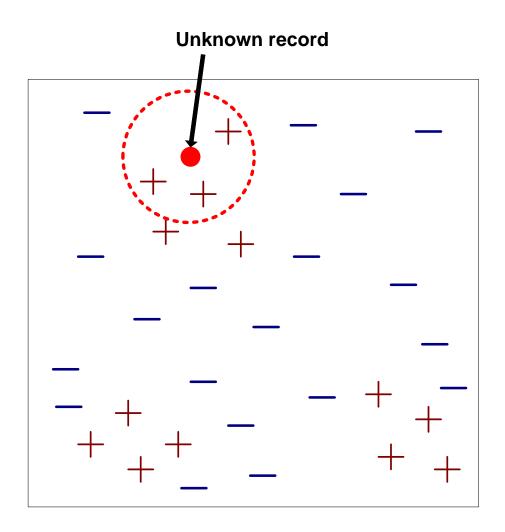
Nearest Neighbor Classifiers

Basic idea:

 If it walks like a duck, quacks like a duck, then it's probably a duck

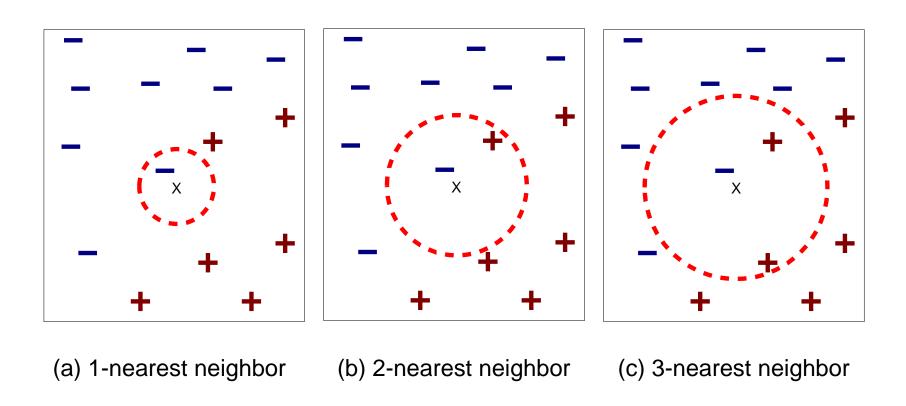


Nearest Neighbor Classifiers



- Requires three things
 - The set of stored records
 - Distance Metric to compute distance between records
 - The value of k, the number of nearest neighbors to retrieve
- To classify an unknown record:
 - Compute distance to other training records
 - Identify k nearest neighbors
 - Use class labels of nearest neighbors to determine the class label of unknown record (e.g., by taking majority vote)

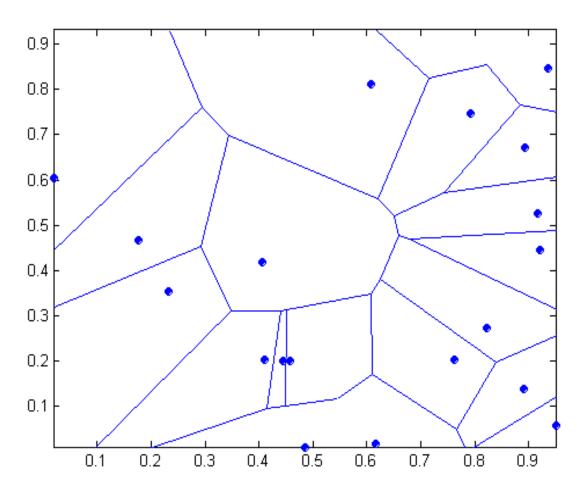
Definition of Nearest Neighbor



K-nearest neighbors of a record x are data points that have the k smallest distance to x

1 nearest neighbor

Voronoi Diagram



Nearest Neighbor Classification

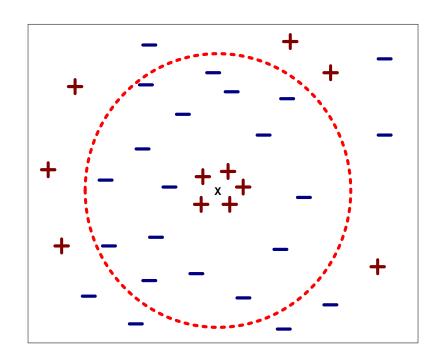
- Compute distance between two points:
 - Euclidean distance

$$d(p,q) = \sqrt{\sum_{i} (p_{i} - q_{i})^{2}}$$

- Determine the class from nearest neighbor list
 - take the majority vote of class labels among the knearest neighbors
 - Weigh the vote according to distance
 - weight factor, w = 1/d²

Nearest Neighbor Classification..

- Choosing the value of k:
 - If k is too small, sensitive to noise points
 - If k is too large, neighborhood may include points from other classes



Nearest Neighbor Classification..

Scaling issues

 Attributes may have to be scaled to prevent distance measures from being dominated by one of the attributes

– Example:

- height of a person may vary from 1.5m to 1.8m
- weight of a person may vary from 90lb to 300lb
- income of a person may vary from \$10K to \$1M

Nearest Neighbor Classification..

- Problem with Euclidean measure:
 - High dimensional data
 - curse of dimensionality
 - Can produce counter-intuitive results

Solution: Normalize the vectors to unit length

Bayes Classifier

Bayes Classifier

- A probabilistic framework for solving classification problems
- Conditional Probability: $P(C \mid A) = \frac{P(A, C)}{P(A)}$

$$P(A \mid C) = \frac{P(A,C)}{P(C)}$$

Bayes theorem:

$$P(C \mid A) = \frac{P(A \mid C)P(C)}{P(A)}$$

Example of Bayes Theorem

Given:

- A doctor knows that meningitis causes stiff neck 50% of the time
- Prior probability of any patient having meningitis is 1/50,000
- Prior probability of any patient having stiff neck is 1/20
- If a patient has stiff neck, what's the probability he/she has meningitis?

$$P(M \mid S) = \frac{P(S \mid M)P(M)}{P(S)} = \frac{0.5 \times 1/50000}{1/20} = 0.0002$$

Bayesian Classifiers

Consider each attribute and class label as random variables

- Given a record with attributes (A₁, A₂,...,A_n)
 - Goal is to predict class C
 - Specifically, we want to find the value of C that maximizes $P(C \mid A_1, A_2,...,A_n)$
- Can we estimate P(C| A₁, A₂,...,A_n) directly from data?

Bayesian Classifiers

- Approach:
 - compute the posterior probability P(C | A_1 , A_2 , ..., A_n) for all values of C using the Bayes theorem

$$P(C \mid A_{1}A_{2}...A_{n}) = \frac{P(A_{1}A_{2}...A_{n} \mid C)P(C)}{P(A_{1}A_{2}...A_{n})}$$

- Choose value of C that maximizes $P(C \mid A_1, A_2, ..., A_n)$
- Equivalent to choosing value of C that maximizes $P(A_1, A_2, ..., A_n | C) P(C)$
- How to estimate $P(A_1, A_2, ..., A_n \mid C)$?

Naïve Bayes Classifier

- Assume independence among attributes A_i when class is given:
 - $P(A_1, A_2, ..., A_n | C) = P(A_1 | C_j) P(A_2 | C_j)... P(A_n | C_j)$
 - Can estimate $P(A_i | C_j)$ for all A_i and C_j .
 - New point is classified to C_j if $P(C_j)$ Π $P(A_i | C_j)$ is maximal.

How to Estimate Probabilities from Data?

Tid	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

• Class: $P(C) = N_c/N$

$$-$$
 e.g., $P(No) = 7/10$, $P(Yes) = 3/10$

For discrete attributes:

$$P(A_i \mid C_k) = |A_{ik}|/N_{c_k}$$

- where |A_{ik}| is number of instances having attribute A_i and belongs to class C_k
- Examples:

How to Estimate Probabilities from Data?

- For continuous attributes:
 - Discretize the range into bins
 - one ordinal attribute per bin
 - violates independence assumption
 - Two-way split: (A < v) or (A > v)
 - choose only one of the two splits as new attribute
 - Probability density estimation:
 - Assume attribute follows a normal distribution
 - Use data to estimate parameters of distribution (e.g., mean and standard deviation)
 - Once probability distribution is known, can use it to estimate the conditional probability P(A_i|c)

How to Estimate Probabilities from Data?

Tid	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No
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6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Normal distribution:

$$P(A_{i} \mid c_{j}) = \frac{1}{\sqrt{2\pi\sigma_{ij}^{2}}} e^{-\frac{(A_{i} - \mu_{ij})^{2}}{2\sigma_{ij}^{2}}}$$

- One for each (A_i,c_i) pair
- For (Income, Class=No):
 - If Class=No
 - sample mean = 110
 - sample variance = 2975

$$P(Income = 120 \mid No) = \frac{1}{\sqrt{2\pi}(54.54)}e^{\frac{-(120-110)^2}{2(2975)}} = 0.0072$$

Example of Naïve Bayes Classifier

Given a Test Record:

$$X = (Refund = No, Married, Income = 120K)$$

naive Bayes Classifier:

```
P(Refund=Yes|No) = 3/7
P(Refund=No|No) = 4/7
P(Refund=Yes|Yes) = 0
P(Refund=No|Yes) = 1
P(Marital Status=Single|No) = 2/7
P(Marital Status=Divorced|No)=1/7
P(Marital Status=Married|No) = 4/7
P(Marital Status=Single|Yes) = 2/7
P(Marital Status=Divorced|Yes)=1/7
P(Marital Status=Married|Yes) = 0
```

For taxable income:

If class=No: sample mean=110

sample variance=2975

If class=Yes: sample mean=90

sample variance=25

```
• P(X|Class=No) = P(Refund=No|Class=No)
 \times P(Married|Class=No)
 \times P(Income=120K|Class=No)
 = 4/7 \times 4/7 \times 0.0072 = 0.0024
```

Naïve Bayes Classifier

- If one of the conditional probability is zero,
 then the entire expression becomes zero
- Probability estimation:

Original :
$$P(A_i \mid C) = \frac{N_{ic}}{N_c}$$

Laplace :
$$P(A_i | C) = \frac{N_{ic} + 1}{N_c + c}$$

m - estimate :
$$P(A_i \mid C) = \frac{N_{ic} + mp}{N_c + m}$$

c: number of classes

p: prior probability

m: parameter

Example of Naïve Bayes Classifier

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

A: attributes

M: mammals

N: non-mammals

$$P(A|M) = \frac{6}{7} \times \frac{6}{7} \times \frac{2}{7} \times \frac{2}{7} = 0.06$$

$$P(A \mid N) = \frac{1}{13} \times \frac{10}{13} \times \frac{3}{13} \times \frac{4}{13} = 0.0042$$

$$P(A \mid M)P(M) = 0.06 \times \frac{7}{20} = 0.021$$

$$P(A \mid N)P(N) = 0.004 \times \frac{13}{20} = 0.0027$$

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

P(A|M)P(M) > P(A|N)P(N)

=> Mammals

Naïve Bayes (Summary)

- Robust to isolated noise points
- Handle missing values by ignoring the instance during probability estimate calculations
- Robust to irrelevant attributes

- Independence assumption may not hold for some attributes
 - Use other techniques such as Bayesian Belief Networks (BBN)

Wait!!

Not all problems in real life are classification problems!!

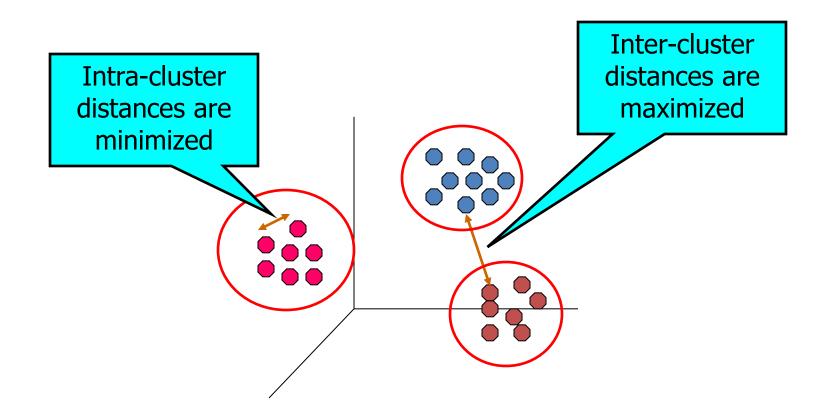
What about these ??

- Group related documents for browsing.
- Group genes and proteins that have similar functionality.
- Group stocks with similar price fluctuations.
- Reduce the size of large data sets.

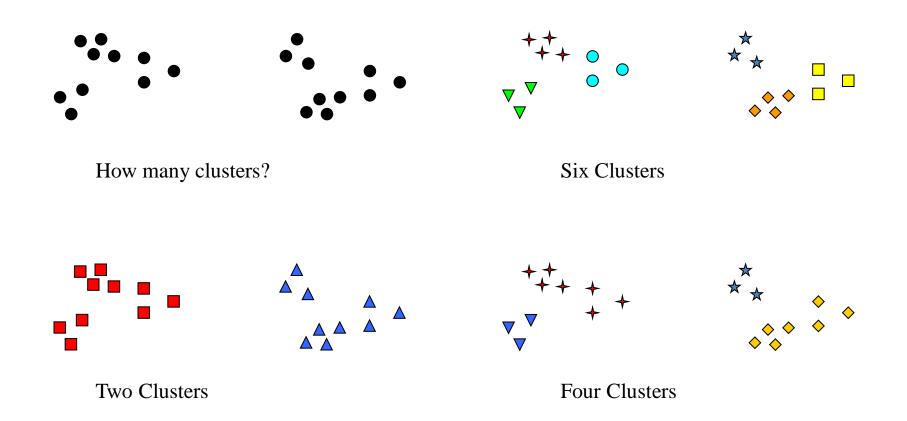
Clustering

Cluster Analysis

 Finding groups of objects such that the objects in a group will be similar (or related) to one another and different from (or unrelated to) the objects in other groups



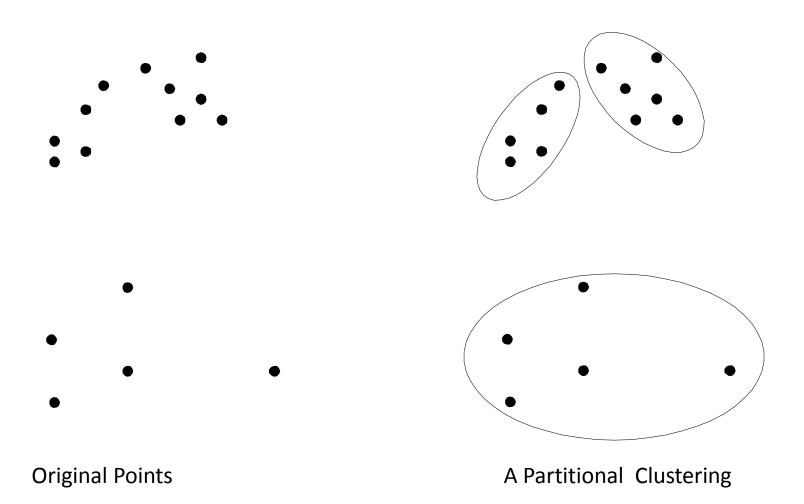
Notion of a Cluster can be Ambiguous



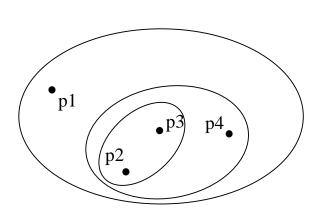
Types of Clustering

- A clustering is a set of clusters
- Important distinction between hierarchical and partitional sets of clusters
- Partitional Clustering
 - A division data objects into non-overlapping subsets (clusters) such that each data object is in exactly one subset
- Hierarchical clustering
 - A set of nested clusters organized as a hierarchical tree

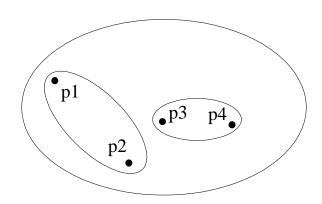
Partitional Clustering



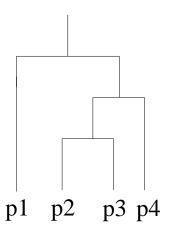
Hierarchical Clustering



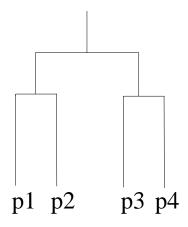
Traditional Hierarchical Clustering



Non-traditional Hierarchical Clustering



Traditional Dendrogram

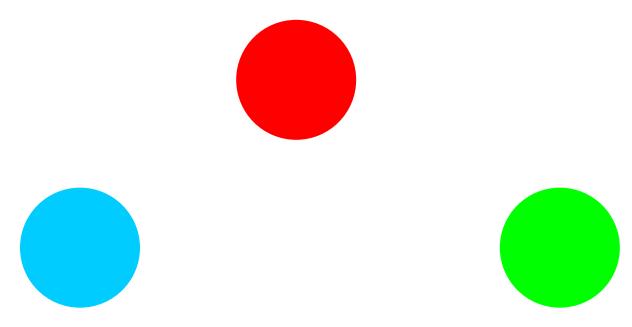


Non-traditional Dendrogram

Types of clusters: Well Seperated

Well-Separated Clusters:

 A cluster is a set of points such that any point in a cluster is closer (or more similar) to every other point in the cluster than to any point not in the cluster.



3 well-separated clusters

Types of Clusters: Center-Based

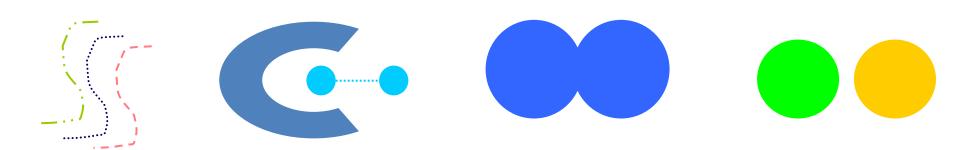
Center-based

- A cluster is a set of objects such that an object in a cluster is closer (more similar) to the "center" of a cluster, than to the center of any other cluster
- The center of a cluster is often a centroid, the average of all the points in the cluster, or a medoid, the most "representative" point of a cluster



Types of Clusters: Contiguity Based

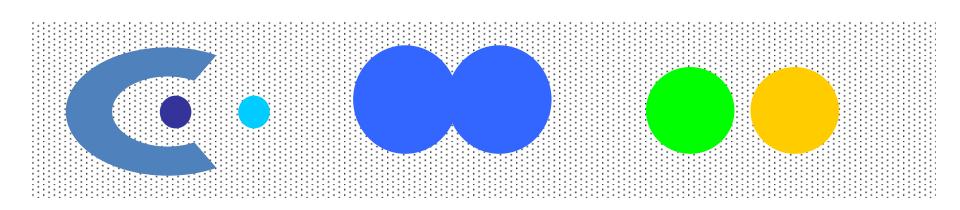
- Contiguous Cluster (Nearest neighbor or Transitive)
 - A cluster is a set of points such that a point in a cluster is closer (or more similar) to one or more other points in the cluster than to any point not in the cluster.



Types of Clusters: Density-Based

Density-based

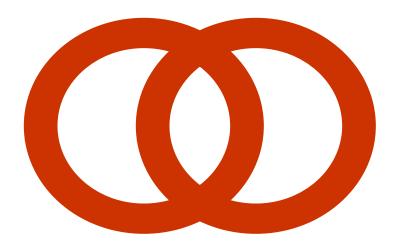
- A cluster is a dense region of points, which is separated by lowdensity regions, from other regions of high density.
- Used when the clusters are irregular or intertwined, and when noise and outliers are present.



Types of Clusters: Conceptual clusters

- Shared Property or Conceptual Clusters
 - Finds clusters that share some common property or represent a particular concept.

.



Clustering Algorithms

- K-means and its variants
- Density-based clustering
- Hierarchical clustering

K-means clustering

- Partitional clustering approach.
- Each cluster is associated with a centroid (center point)
- Each point is assigned to the cluster with the closest centroid
- Number of clusters, K, must be specified
- The basic algorithm is very simple

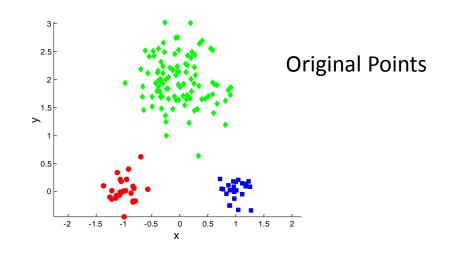
Algorithm 1 Basic K-means Algorithm.

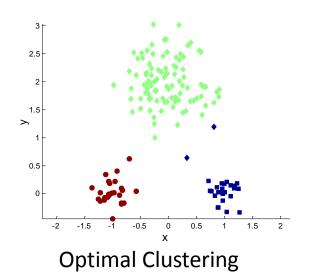
- 1: Select K points as the initial centroids.
- 2: repeat
- 3: Form K clusters by assigning all points to the closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: **until** The centroids don't change

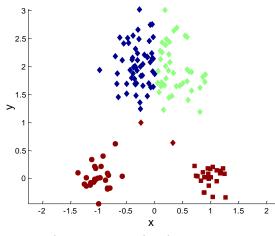
K-means clustering: Details

- Initial centroids are often chosen randomly.
 - Clusters produced vary from one run to another.
- The centroid is (typically) the mean of the points in the cluster.
- 'Closeness' is measured by Euclidean distance, cosine similarity, correlation, etc.
- K-means will converge for common similarity measures mentioned above.
- Most of the convergence happens in the first few iterations.
 - Often the stopping condition is changed to 'Until relatively few points change clusters'
- Complexity is O(n * K * I * d)
 - n = number of points, K = number of clusters,
 I = number of iterations, d = number of attributes

Two different K-means Clusterings

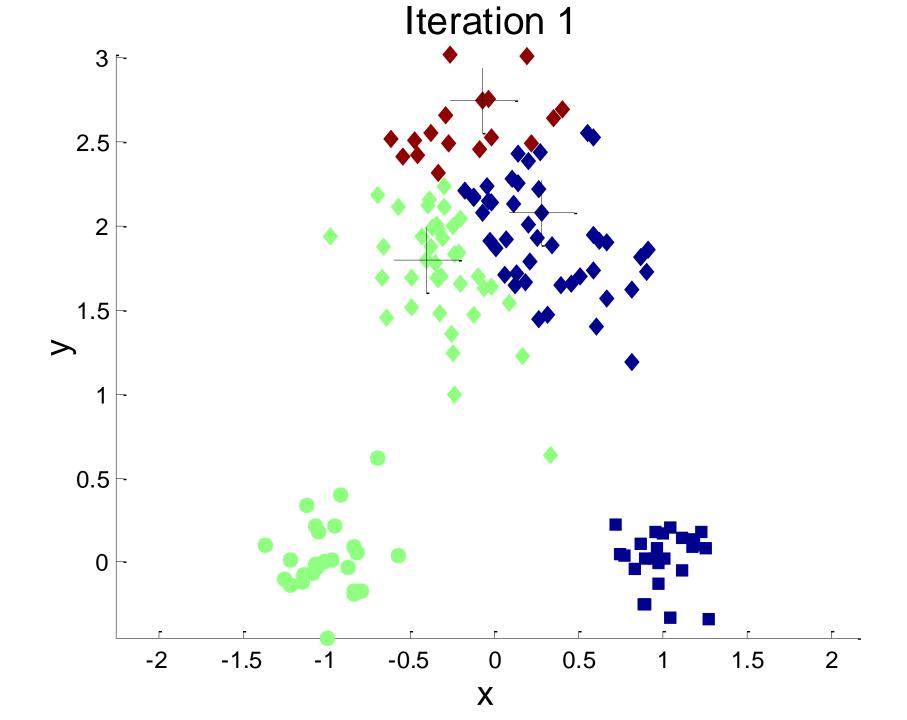


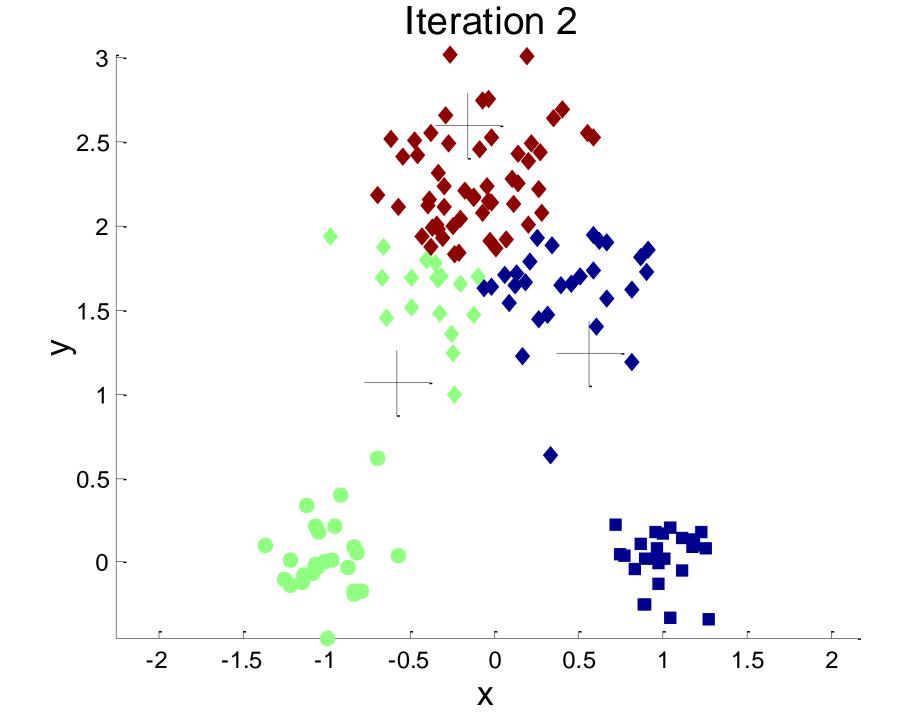


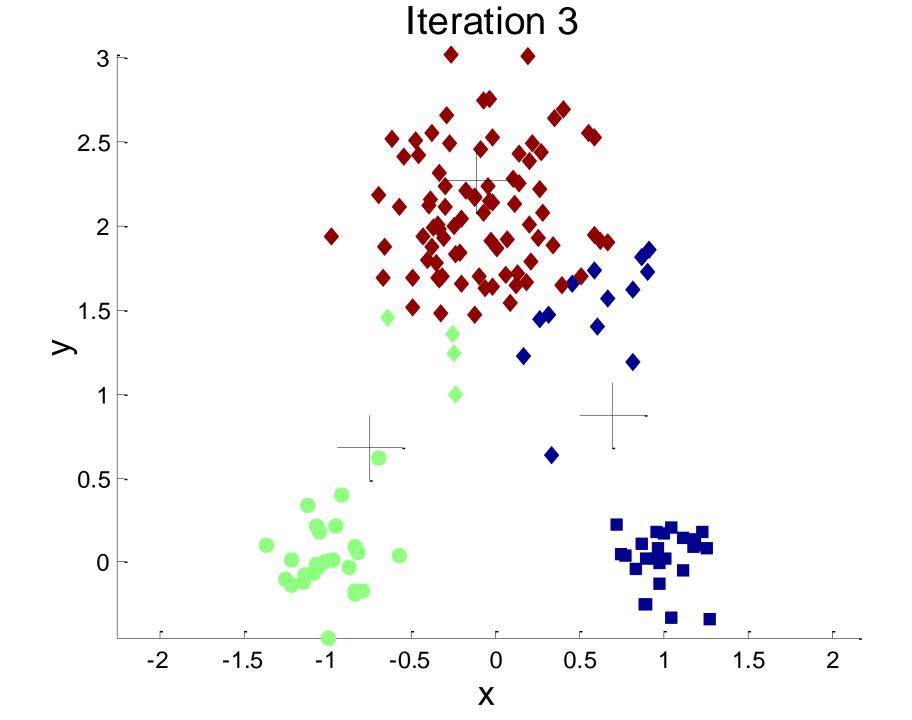


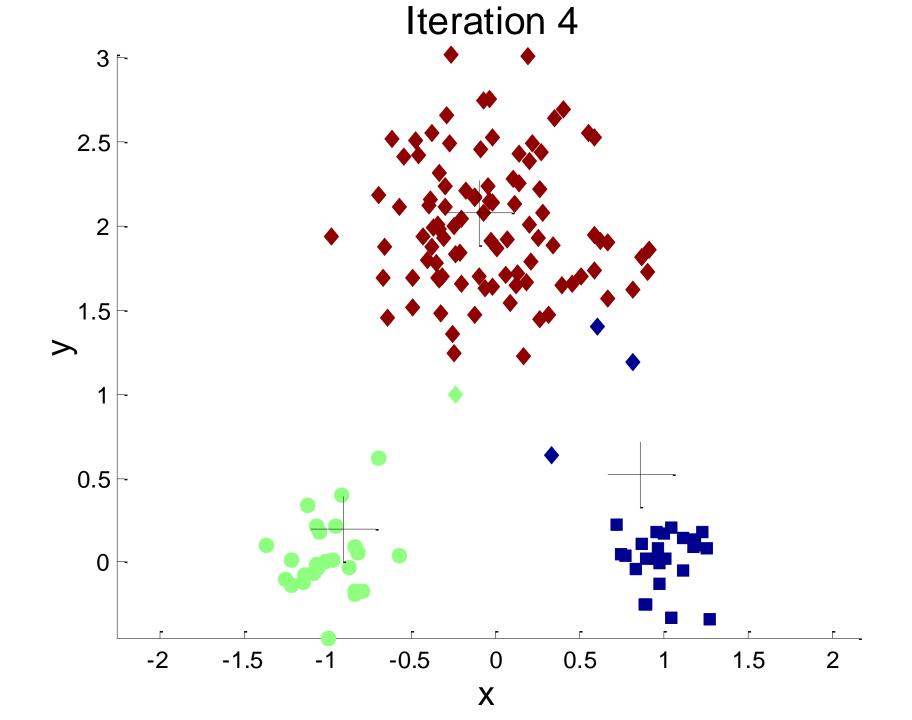
Sub-optimal Clustering

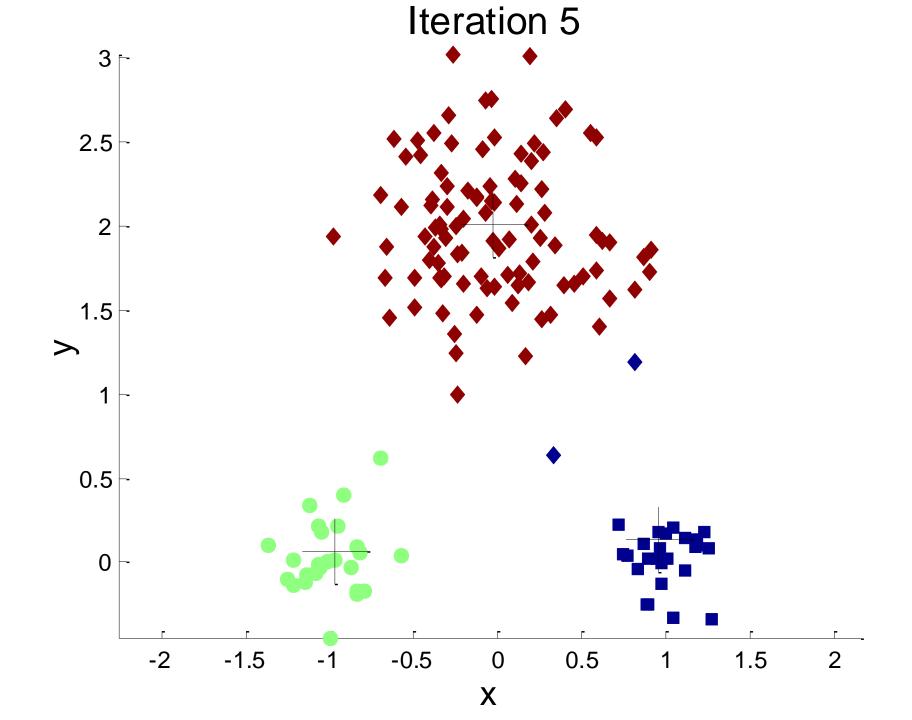
An Example

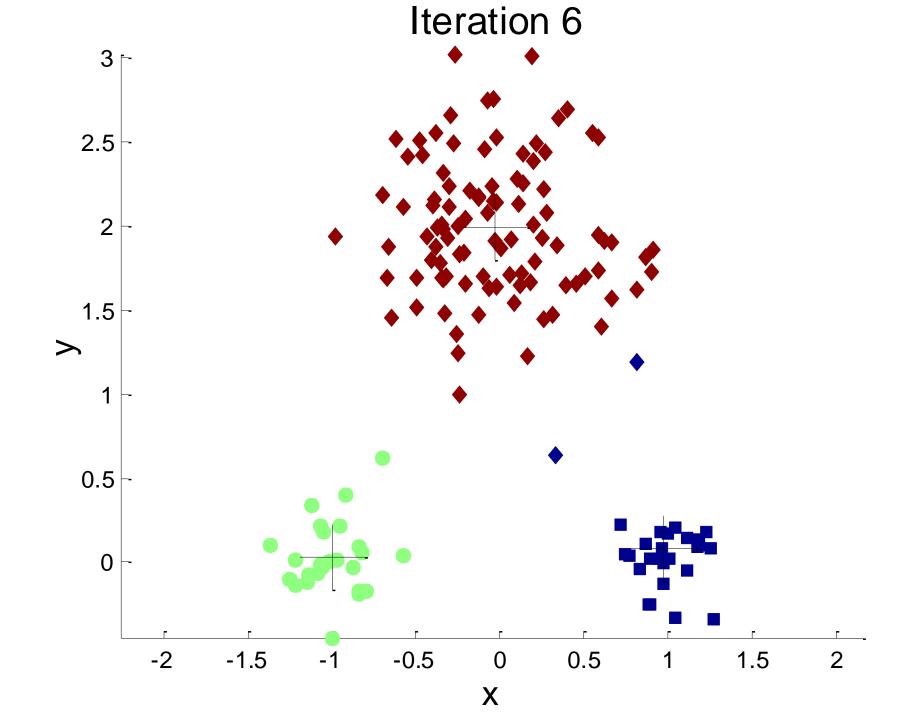












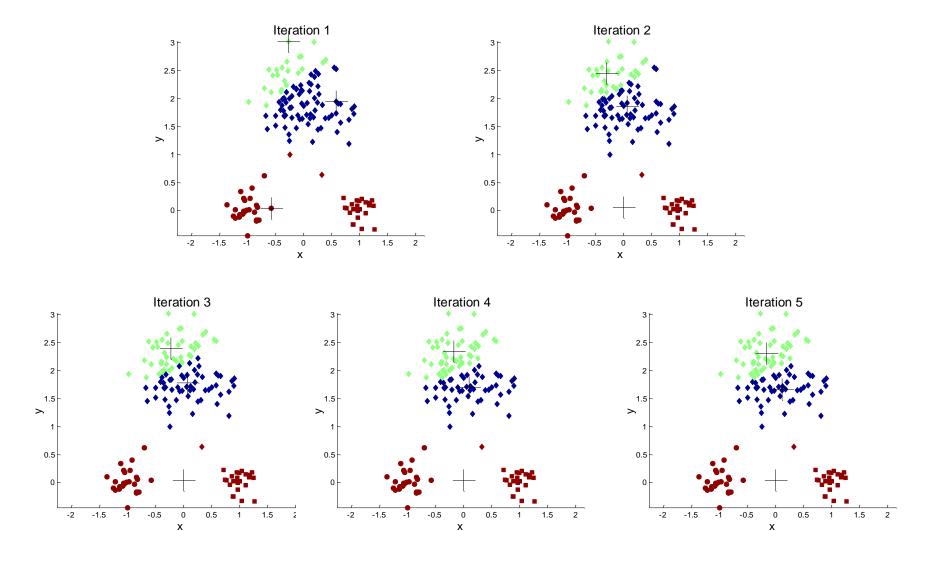
Evaluating K-means clusters

- Most common measure is Sum of Squared Error (SSE)
 - For each point, the error is the distance to the nearest cluster
 - To get SSE, we square these errors and sum them.

$$SSE = \sum_{i=1}^{K} \sum_{x \in C_i} dist^2(m_i, x)$$

- x is a data point in cluster C_i and m_i is the representative point for cluster C_i
 - can show that m_i corresponds to the center (mean) of the cluster
- Given two clusters, we can choose the one with the smallest error
- One easy way to reduce SSE is to increase K, the number of clusters
 - A good clustering with smaller K can have a lower SSE than a poor clustering with higher K

Importance of choosing Initial centroids...

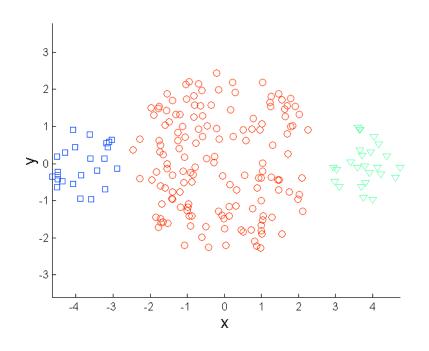


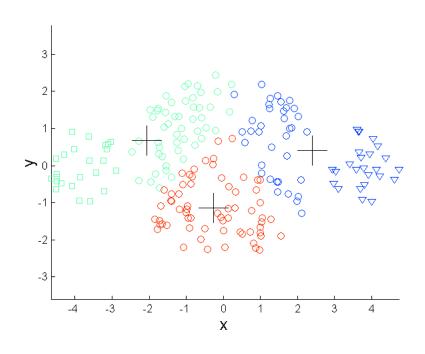
Limitations of K-means

- K-means has problems when clusters are of differing
 - Sizes
 - Densities
 - Non-globular shapes

K-means has problems when the data contains outliers.

Limitations of K-means: Differing Sizes

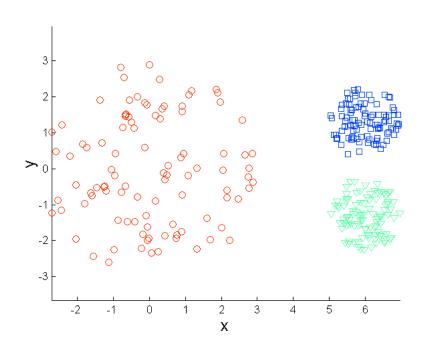


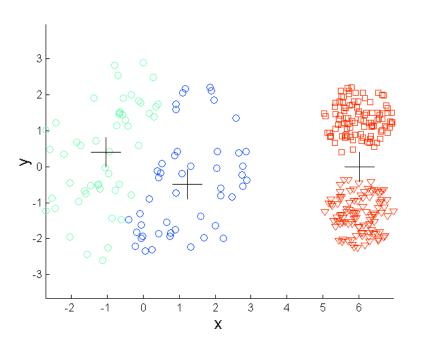


Original Points

K-means (3 Clusters)

Limitations of K-means: Differing Density

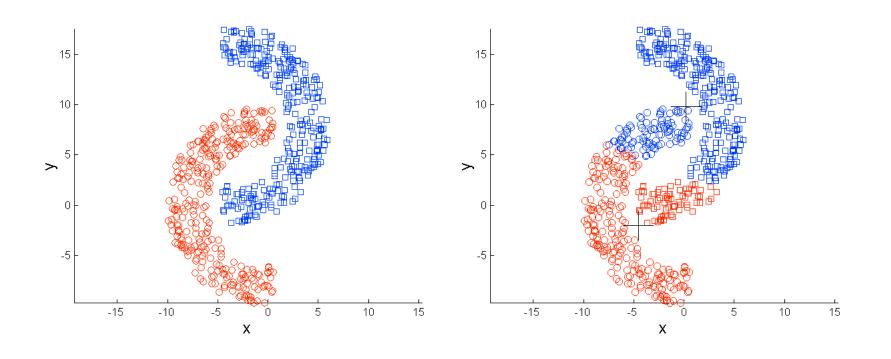




Original Points

K-means (3 Clusters)

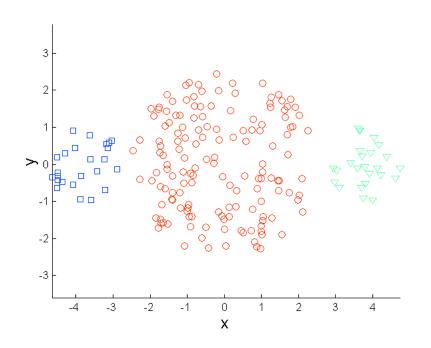
Limitations of K-means: Non-globular Shapes

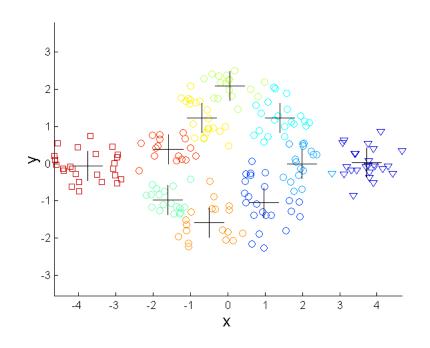


Original Points

K-means (2 Clusters)

Overcoming K-means Limitations





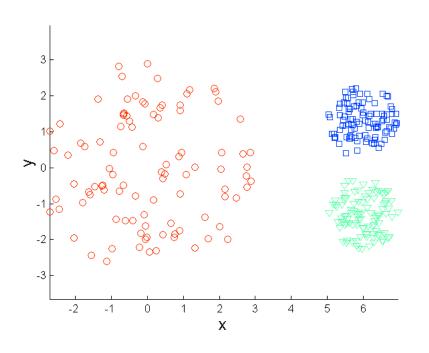
Original Points

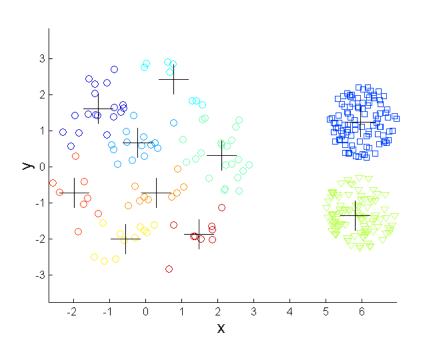
K-means Clusters

One solution is to use many clusters.

Find parts of clusters, but need to put together.

Overcoming K-means Limitations

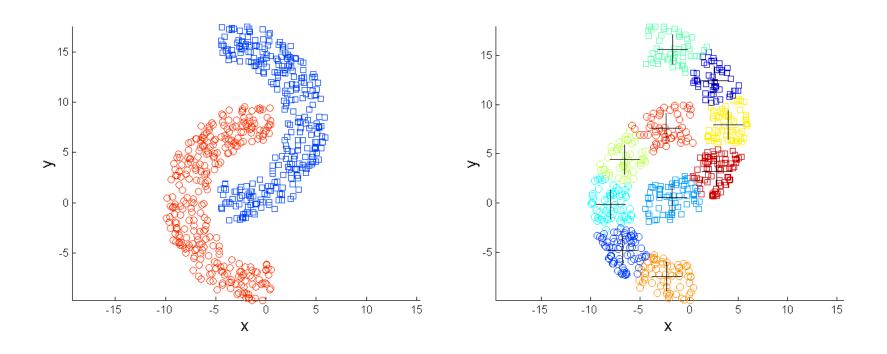




Original Points

K-means Clusters

Overcoming K-means Limitations



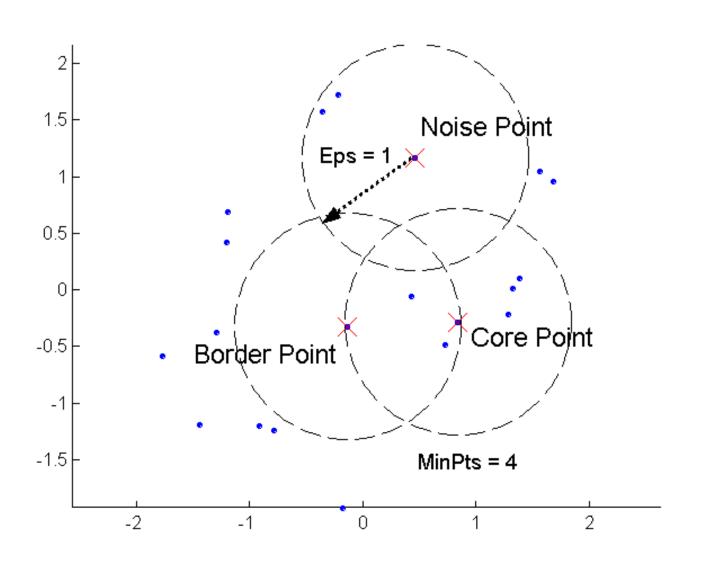
Original Points

K-means Clusters

DBSCAN

- DBSCAN is a density-based algorithm.
 - Density = number of points within a specified radius (Eps)
 - A point is a core point if it has more than a specified number of points (MinPts) within Eps
 - These are points that are at the interior of a cluster
 - A border point has fewer than MinPts within Eps, but is in the neighborhood of a core point
 - A noise point is any point that is not a core point or a border point.

DBSCAN: Core, Border, and Noise Points

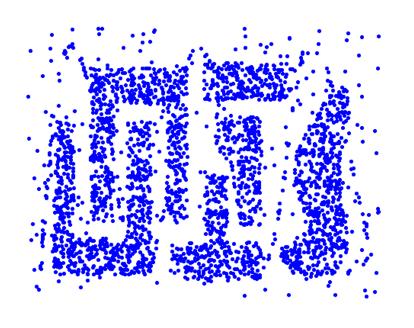


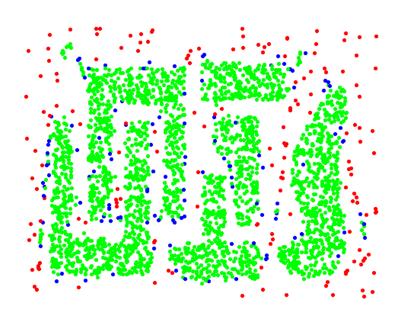
DBSCAN Algorithm

- Eliminate noise points
- Perform clustering on the remaining points

```
current\_cluster\_label \leftarrow 1
for all core points do
  if the core point has no cluster label then
    current\_cluster\_label \leftarrow current\_cluster\_label + 1
    Label the current core point with cluster label current_cluster_label
  end if
  for all points in the Eps-neighborhood, except i^{th} the point itself do
    if the point does not have a cluster label then
       Label the point with cluster label current\_cluster\_label
    end if
  end for
end for
```

DBSCAN: Core, Border and Noise Points



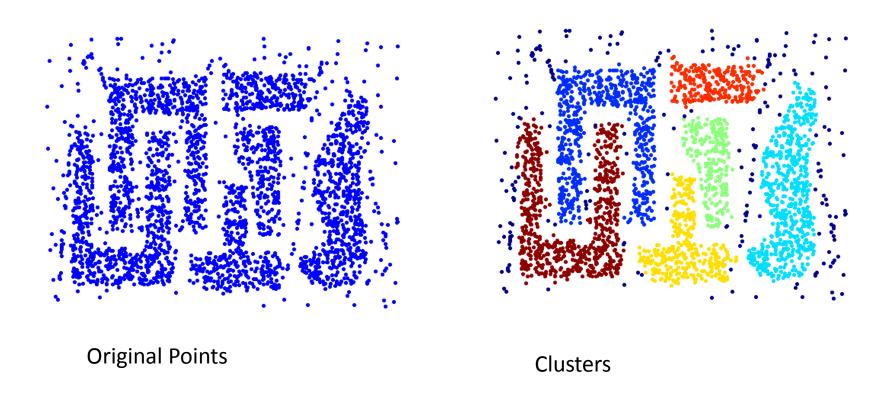


Original Points

Point types: core, border

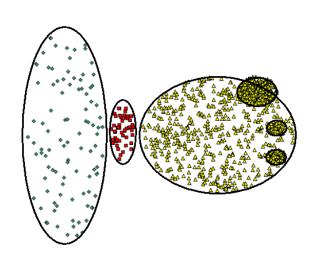
and noise

When DBSCAN Works Well



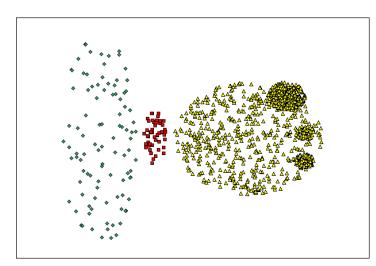
- Resistant to Noise
- Can handle clusters of different shapes and sizes

When DBSCAN Does NOT Work Well

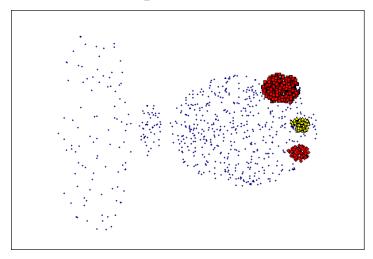


Original Points

- Varying densities
- High-dimensional data



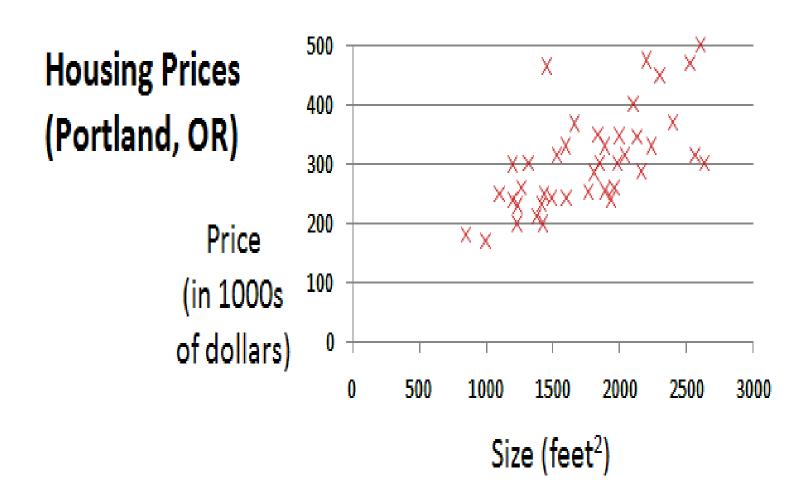
(MinPts=4, Eps=9.75).



(MinPts=4, Eps=9.92)

Yet Another type of problem..

Predict my house's price!



Price Prediction

Training set of
housing prices
(Portland, OR)

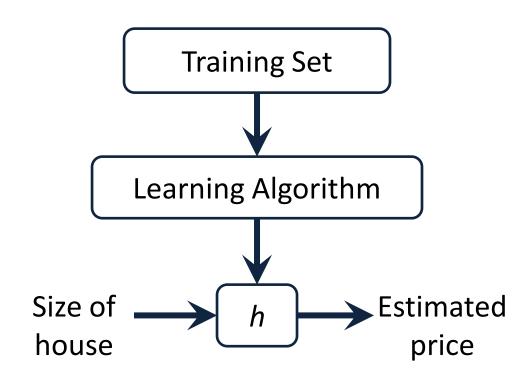
Size in feet ² (x)	Price (\$) in 1000's (y)
2104	460
1416	232
1534	315
852	178
•••	•••

Notation:

m = Number of training examples

x's = "input" variable / features

y's = "output" variable / "target" variable



	Trai	ini	inσ	Set
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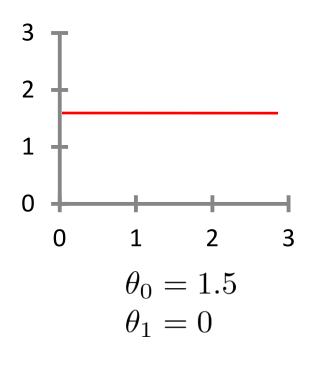
Size in feet ² (x)	Price (\$) in 1000's (y)
2104	460
1416	232
1534	315
852	178
•••	•••

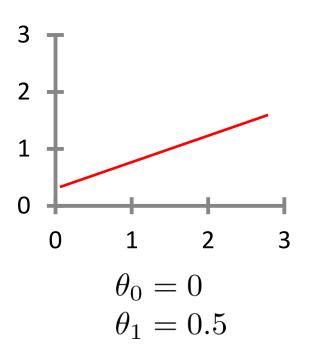
Hypothesis: $h_{\theta}(x) = \theta_0 + \theta_1 x$

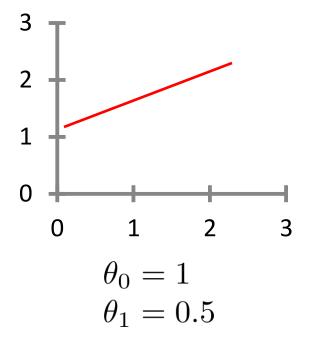
 θ_i 's: Parameters

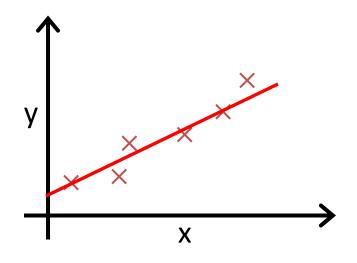
How to choose θ_i 's ?

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$









Idea: Choose θ_0, θ_1 so that $h_{\theta}(x)$ is close to y for our training examples (x,y)

Linear Regression

Hypothesis:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Parameters:

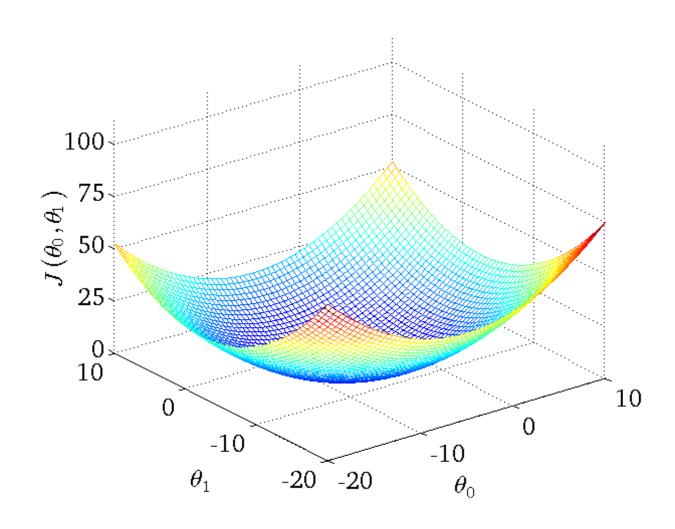
$$\theta_0, \theta_1$$

Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_{\theta}(x^{(i)}) - y^{(i)} \right)^2$$

Goal:
$$\underset{\theta_0,\theta_1}{\text{minimize}} J(\theta_0,\theta_1)$$

Linear Regression

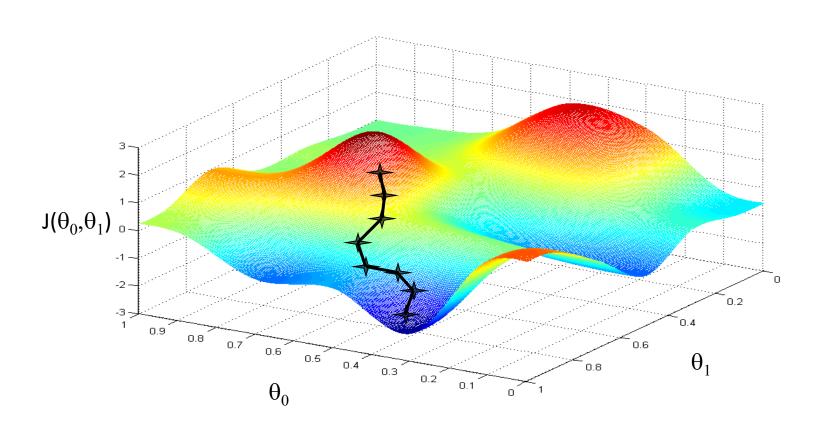


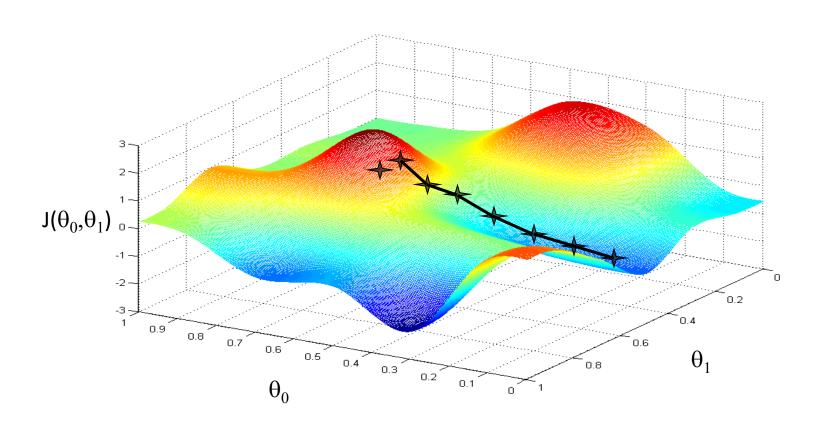
Have some function $J(\theta_0, \theta_1)$

Want
$$\min_{\theta_0,\theta_1} J(\theta_0,\theta_1)$$

Outline:

- Start with some θ_0, θ_1
- Keep changing $heta_0, heta_1$ to reduce $J(heta_0, heta_1)$ until we hopefully end up at a minimum





Gradient descent algorithm

```
repeat until convergence { \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad \text{(simultaneously update } j = 0 \text{ and } j = 1) }
```

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.

If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

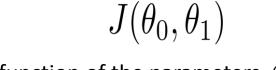
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

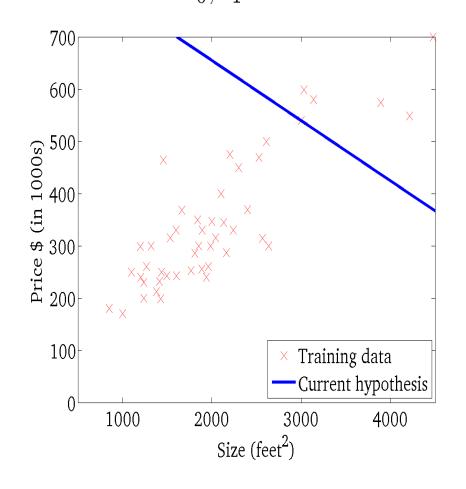
As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.

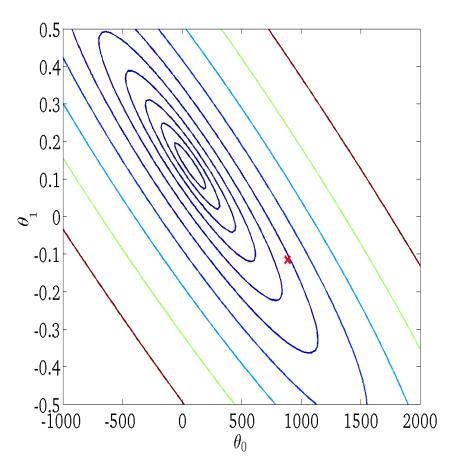
Gradient descent algorithm

```
repeat until convergence { \theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m \left( h_\theta(x^{(i)}) - y^{(i)} \right) \quad \text{update} \\ \theta_0 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m \left( h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)} \quad \text{simultaneously}  }
```

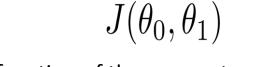
 $h_{ heta}(x)$

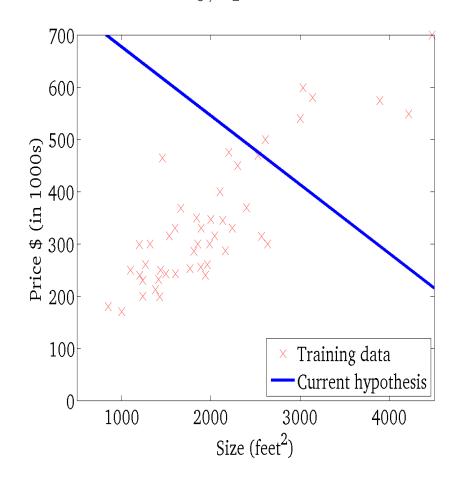


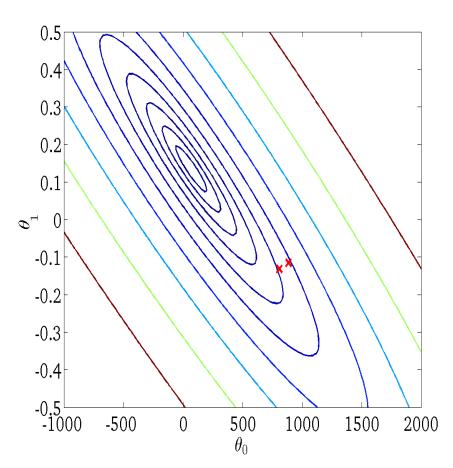




 $h_{ heta}(x)$

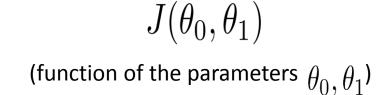


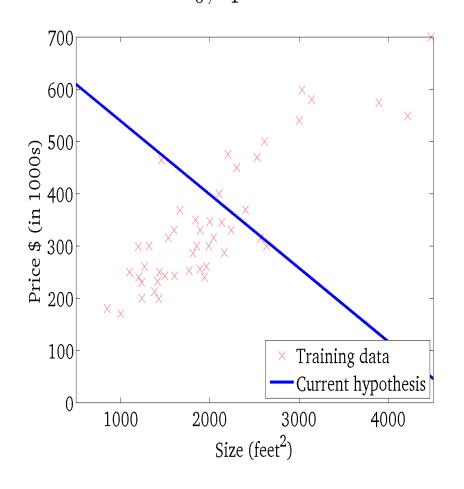


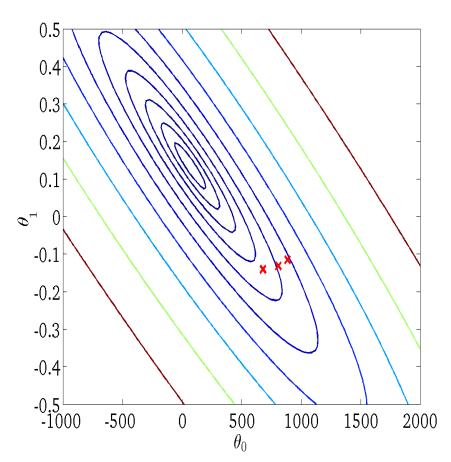


 $h_{ heta}(x)$

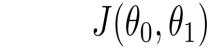
(for fixed θ_0, θ_1 , this is a function of x)

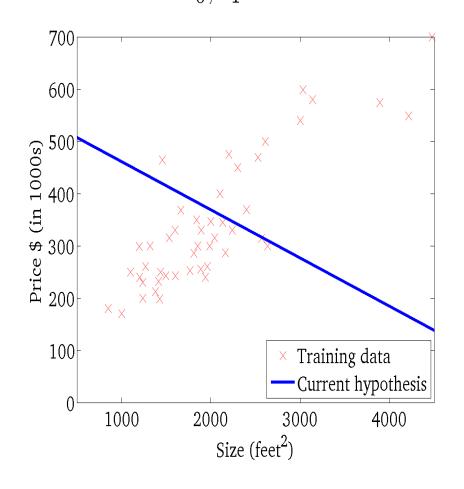


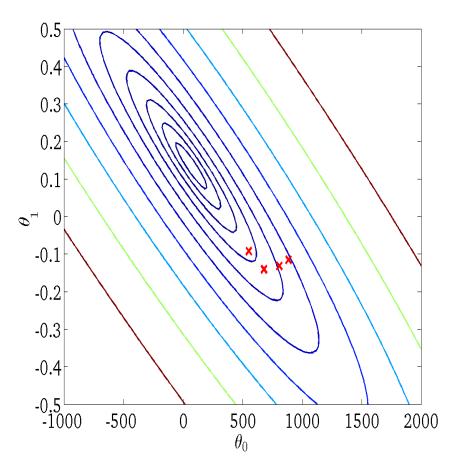




 $h_{\theta}(x)$

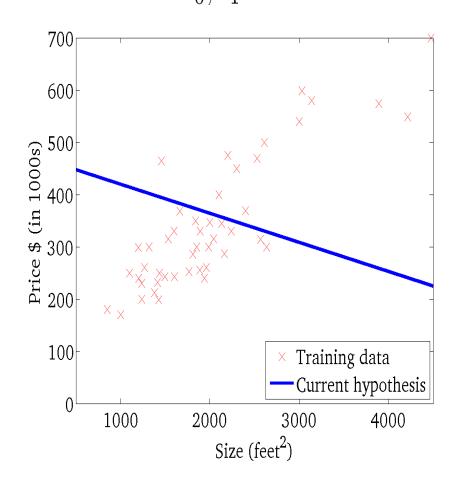


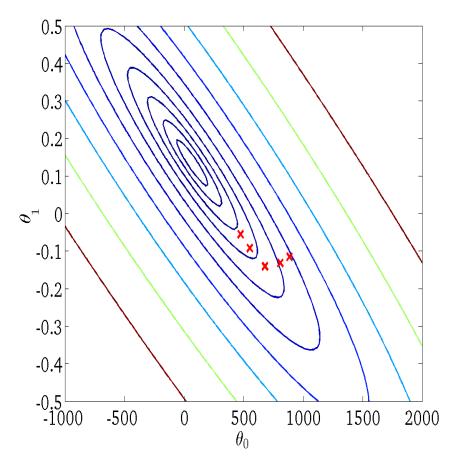




 $h_{\theta}(x)$

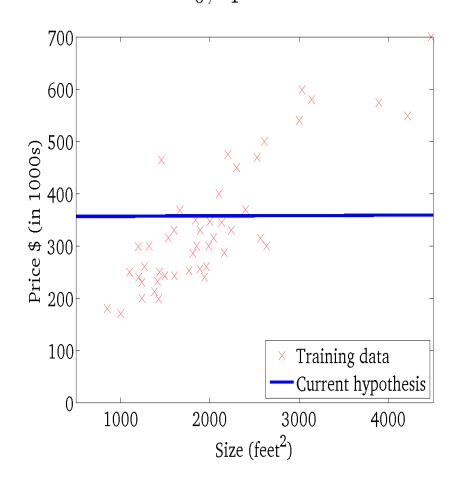


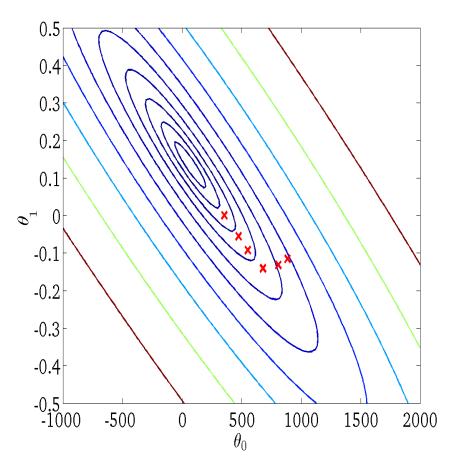




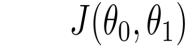
 $h_{\theta}(x)$

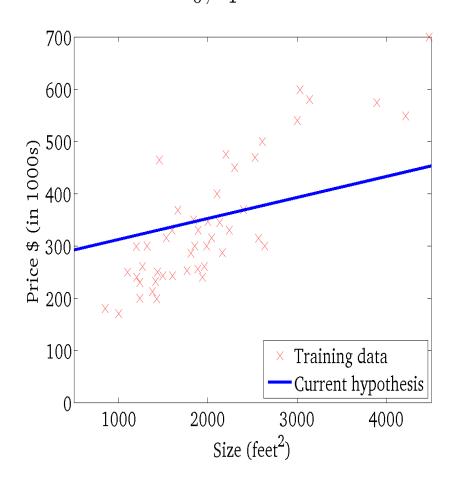


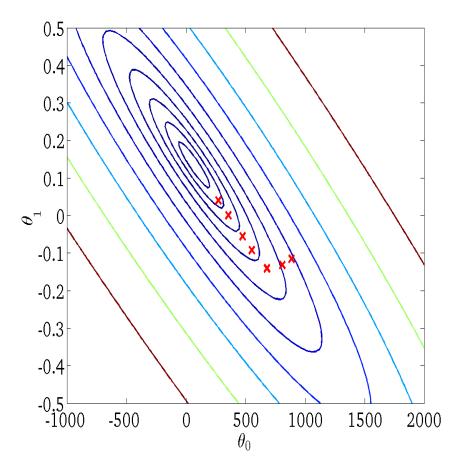




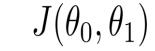
 $h_{\theta}(x)$

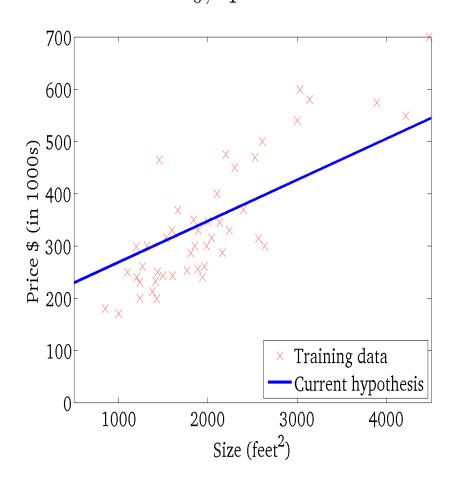


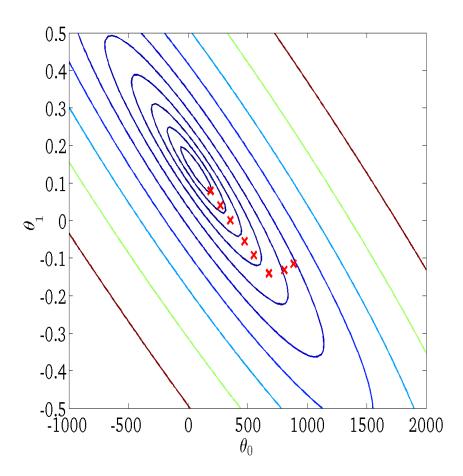




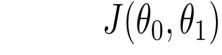
 $h_{\theta}(x)$

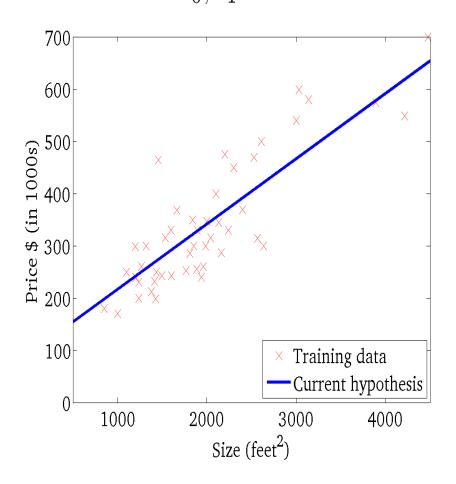


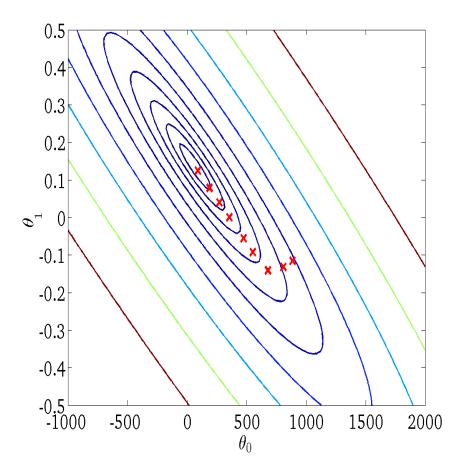




 $h_{\theta}(x)$









Why Pre-process

- We need *clean* data to produce good results
- Noisy data
 - Errors in measurement
 - 'Temperature: 76' (Faulty thermometer)
 - Errors in transcription/transmission
 - 'Sex: E'
- Missing Values
 - Not recorded
 - Not considered relevant
 - Change in DB design
 - Not accessible
 - No comments
 - Privacy considerations
- Redundant data
 - Derive from other fields (to facilitate presentation)
 - Irrelevant to task at hand

Noise in Data

- Unknown encoding
 - Sex: E
- Out of range values
 - Temperature: 1004
 - Age: 105
- Inconsistent entries
 - DoB: 10-Feb-2003; Age: 30
- Inconsistent formats
 - DoB: 11-Feb-1984; DoJ: 2/11/2007

Noise removal

- Consistency checks
 - Can catch many transcription errors
- Canonicalization
 - Convert to standard format
- Domain knowledge
 - Paediatric data => Age most probably 10.5
- Statistical methods

How to Handle Noisy Data?

Binning

- first sort data and partition into (equal-frequency) bins
- then one can smooth by bin means, smooth by bin median, smooth by bin boundaries, etc.

Regression

- smooth by fitting the data into regression functions
- Clustering
 - detect and remove outliers
- Combined computer and human inspection
 - detect suspicious values and check by human (e.g., deal with possible outliers)

Missing Values

- Change in technology
 - New medical tests/equipment
 - Legacy databases
- Change in DB design
- Change in focus
- Malfunctioning equipment
- No comments
- Not relevant
 - Patient came for eye exam temperature not recorded
- Inconsistency removed

Handling Missing Values

- Attribute values
 - Infer missing values
- Class labels
 - Ignore data points
 - Semi-supervised learning

Missing Attribute Values

- Ignore it
 - Not a good option, especially if there is a paucity of data
- Manually fill it
 - Tedious; time consuming; expensive
 - Sometimes essential medical data
- Treat it as a special value
 - Missing;?
- Infer it automatically

Inferring missing attribute values

X	5	4	2	3	4	?	9	4	0
Y	I	I	II	II	I	II	I	I	II

- Mean 3.875
 - Truncated mean 3.667
 - Median 4
 - Mode 4
- Conditioned mean
 - On class labels relatively cheap 1.667
 - One all known attributes expensive; but best use of data

Inferring Missing Values – Cont.

- Conditioning on all known attributes
 - Might bias data samples heavily, leading to poor performance of most machine learning algorithms
 - Regression
 - Ignore attribute if fit is "too good"
 - Expectation- Maximization
 - Iterate till a resulting model fits observed data well
 - Local optima

Imputation

- Filling in missing values apriori
 - Statistics literature
- Simple imputation
 - Mean, class conditioned mean etc.
- Full information imputation
- Multiple imputation
 - Monte-Carlo method
 - Several samples
 - Combine output of classifiers

Summary

Summary

- Machine Learning
 - Supervised Learning
 - Classification
 - Regression
 - Unsupervised Learning
 - Clustering
 - Reinforcement Learning (Not covered)

Acknowledgements

- Most of the slides are taken from
 - Hugo Larochelle
 - Tan, Steinbach and Kumar
 - Andrew Ng
- Thanks to Subhashree for the animations.

Some Standard Text Books

- Machine Learning by Tom Mitchell.
- Pattern Recognition and Machine Learning by Chris. Bishop.
- Machine Learning a Probabilistic Perspective by Kevin Murphy

Slides will be available in the following link:

www.cse.iitm.ac.in/~sarath/mlw

It will be online from tomorrow.

Thank You