# Assignment 8

## Task 1:

Probability of sensor S in Maine, P(M) = 0.05

Probability of sensor S in Sahara, P(S) = 1 – 0.05 = 0.95

Let, X = probability of getting a daily high temperature of 80 degrees or more.

Y = probability of getting a daily high temperature less than 80 degrees.

P(X|M) = probability of getting a daily high temperature of 80 degrees or more in Maine.

P(X|M) = 0.2

P(Y|M) = probability of getting a daily high temperature less than 80 degrees or more in Maine.

P(Y|M) = 1 – 0.2 = 0.8

P(X|S) = = probability of getting a daily high temperature of 80 degrees or more in Sahara.

P(X|S) = 0.9

P(Y|S) = = probability of getting a daily high temperature less than 80 degrees or more in Sahara.

P(Y|S) = 1 - 0.9 = 0.1

a. Let P(M|Y) = probability that the sensor is placed in Maine given a daily high temperature under 80 degrees.
   P(M|Y) = [(P(M) * P(Y|M)) / P(Y)]
   Here, P(Y) = P(Y|M) * P(M) + (Y|S) * P(S)
   = 0.8 * 0.05 + 0.1 * 0.95 = 0.135

Therefore, $P(M|Y) = [(P(M) * P(Y|M)) / P1(Y)]$

$= [(0.05 * 0.8) / 0.135]$

$= 0.2963$

The probability that the sensor is placed in Maine given the first email you got from sensor S indicates a daily high under 80 degrees is **29.63%.**

b. $P2(M) = [(P(Y|M) * P(M)) / P1(Y)]$

$= [(0.8 * 0.05) / 0.135] = 0.2963$

$P2(S) = [(P(Y|S) * P1(S)) / P1(Y)]$

$= [(0.1 * 0.95) / 0.135] = 0.704$

$P2(Y) = P(Y|S) * P2(S) + P(Y|M) * P2(M)$

$= 0.1 * 0.704 + 0.8 * 0.2963 = 0.3074$

The probability that the second email also indicates a daily high under 80 degrees if the first email indicates a daily high under 90 degrees is **30.74%.**

c. $P3(M) = [(P(Y|M) * P(M)) / P2(Y)]$

$= [(0.8 * 0.2963) / 0.3074] = 0.771$

$P3(S) = [(P(Y|S) * P(S)) / P2(Y)]$

$= [(0.1 * 0.704) / 0.3074] = 0.229$

$P3(Y) = P(Y|S) * P3(S) + P(Y|M) * P3(M)$

$= 0.1 * 0.229 + 0.8 * 0.771 = 0.6397$

The probability that the first three emails all indicate daily highs under 80 degrees is,

$P1(Y) * P2(Y) * P3(Y) = 0.135 * 0.3074 * 0.6397$

$= \mathbf{0.02654}$

## Task 2:

a. As A is independent and it has 5 values, we have to store all of them. Also, B1,……,B10 is dependent on A, so for single variable we have to store 7 values. Therefore, total number of values we need to store In joint distribution table for these 11 variables will be = **5 * (7^10) values.**

b. For each variable of B, we need to store 5 * 6 = 30 values; as we can calculate the $7^{th}$ value from 1 – total of 6 values. Similarly, for variable A we need to store only 4 values as we can calculate $5^{th}$ value. For each variables we only need to store 30 * 10 + 4 = **304 values.**

## Task 3:

P(A=T, B=T, C=T) = 0.048

P(A=T, B=T, C=F) = 0.196

P(A=T, B=F, C=T) = 0.192

P(A=T, B=F, C=F) = 0.084

P(A=F, B=T, C=T) = 0.012

P(A=F, B=T, C=F) = 0.294

P(A=F, B=F, C=T) = 0.048

P(A=F, B=F, C=F) = 0.126

a. P(A|B) = P(A=T, B=T, C=T)+ P(A=T, B=T, C=F)
P(A=T, B=F, C=T) + P(A=T, B=F, C=F)
P(A=F, B=T, C=T) + P(A=F, B=T, C=F)

P(A=F, B=F, C=T) + P(A=F, B=F, C=F)
= 0.048 + 0.196
0.192 + 0.084
0.012 + 0.294
0.048 + 0.126
= 0.244/0.982    0.276/0.982    0.288/0.982
0.174/0.982

b. P(A|B, C) = P(A=T, B=T, C=T)    P(A=T, B=F, C=T)
P(A=T, B=T, C=F)    P(A=T, B=F, C=F)
P(A=F, B=T, C=T)    P(A=F, B=F, C=T)
P(A=F, B=T, C=F)    P(A=F, B=F, C=F)
= 0.048    0.192    0.196    0.084
0.012    0.048    0.294    0.126

c. P(A, C|B) = P(A=T, B=T, C=T)+ P(A=T, B=T, C=F)
+ P(A=F, B=T, C=T) + P(A=F, B=T, C=F)
P(A=T, B=F, C=T) + P(A=T, B=F, C=F)
+ P(A=F, B=F, C=T) + P(A=F, B=F, C=F)
= 0.048 + 0.196 + 0.012 + 0.294
0.192 + 0.084 + 0.048 + 0.126
= 0.55 0.45

d. Given B, is A conditionally independent of C?
For A to be conditionally independent of C,
P(A|B, C) = P(A|B)

From b, we have
P(A|B, C) = 0.048    0.192    0.196    0.084

0.012 0.048 0.294 0.126

From a, we have

P(A|B) = 0.244/0.982 0.276/0.982 0.288/0.982 0.174/0.982

We see that P(A|B, C) != P(A|B). Therefore, A is not conditionally independent of C.