Name: Y Rishita Reddy
Sec: I

Roll no: 47

Tutorial-4

(1.) $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Sol^n: $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a \geq 1,\ b > 1$

On comparing

$a = 3,\ b = 2,\ f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n^2)$

(2.) $T(n) = 4T(n/2) + n^2$

Sol^n: $a \geq 1,\ b > 1$

$a = 4,\ b = 2,\ f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

(3.) $T(n) = T(n/2) + 2^n$

Sol^n: $a = 1$

$b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 c = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

(4.) $T(n) = 2^n T(n/2) + n^n$

Sol^n: $a = 2^n$

$b = 2,\ f(n) = n^n$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c \Rightarrow n^c$

$\therefore f(n) = n^c$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

(5.) $T(n) = 16T\left(\frac{n}{4}\right) + n$

Sol^n: $a = 16,\ b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

(6.) $T(n) = 2T(n/2) + n\log n$

Sol^n: $a = 2,\ b = 2$

$f(n) = n\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Hence, $n\log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n\log n)$

(7.) $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

Sol^n: $a=2,\ b=2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

(8.) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Sol^n: $a=2,\ b=4,\ f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

(9.) $T(n) = 0.5\,T\left(\frac{n}{2}\right) + 1/n$

Sol^n: $a = 0.5,\ b = 2$

Since acc. to Master Theorem $a \geq 1$, but here a is 0.5

So, we cannot apply Master theorem.

(10.) $T(n) = 16T\left(\frac{n}{4}\right) + n!$

Sol^n: $a=16,\ b=4,\ f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

Now, $n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \theta(n!)$

(11.) $4T(n/2) + \log n$

Sol^n: $a=4,\ b=2,\ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

Since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n^c)$

$= \theta(n^2)$

(12.) $T(n) = sqrt(n)T(n/2) + \log n$

Sol^n: $a = \sqrt{n},\ b = 2$

$\therefore c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(f(n))$

$= \theta(\log(n))$

(13.) $T(n) = 3T(n/2) + n$

Sol^n: $a=3$, $b=2$, $f(n)=n$

$c = \log_b a = \log_2 3 = 1.5849$

$\Rightarrow n^c = n^{1.5849}$

$\therefore n < n^{1.5849}$

$\Rightarrow f(n) < n^c$

$\therefore T(n) = \Theta(n^{1.5849})$

(14.) $T(n) = 3T(n/3) + \text{sqrt}(n)$

Sol^n: $a=3$, $b=3$

$c = \log_b a = \log_3 3 = 1$

$\therefore n^c = n^1 = n$

As, $\text{sqrt}(n) < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

(15.) $T(n) = 4T(n/2) + cn$

Sol^n: $a=4$, $b=2$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$\therefore cn < n^2$ (for any constant)

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

(16.) $T(n) = 3T(n/4) + n\log n$

Sol^n: $a=3$, $b=4$, $f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \Theta(n\log n)$

(17.) $T(n) = 3T(n/3) + n/2$

Sol^n: $a=3$, $b=3$

$c = \log_a b = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

As, $n/2 \le n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

(18.) $T(n) = 6T(n/3) + n^2\log n$

$a=6$, $b=3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2\log n$

$\therefore T(n) = \Theta(n^2\log n)$

(19.) $T(n) = 4T(n/2) + n\log n$

Sol^n: $a = 4$, $b = 2$, $f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \Theta(n^2)$

(20.) $T(n) = 64\,T(n/8) - n^2\log n$

Sol^n: $a = 64$, $b = 8$

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$\therefore n^c = n^2$

$\therefore n^2\log n > n^2$

$\therefore T(n) = \Theta(n^2\log n)$

(21.) $T(n) = 7T(n/3) + n^2$

Sol^n: $a = 7$, $b = 3$, $f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$\Rightarrow n^{1.7712} < n^2$

$\therefore T(n) = \Theta(n^2)$

(22.) $T(n) = T(n/2) + n(2 - \cos n)$

Sol^n: $a = 1$, $b = 2$

$c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \Theta(n(2 - \cos n))$