DA341_project

Rishita Agarwal, Varakala Rajasree, Yoga Venkata Sai Charan Boddapati

2024-11-08

Introduction

This project provides an analysis of global land temperatures by city, with a focus on Århus, Denmark. The analysis includes data cleaning, exploratory data analysis, time series decomposition, and forecasting using ARIMA and ETS models. Models are used to see their performance for this data distribution

Setup

Load Necessary Packages

```
options(repos = c(CRAN = "https://cran.rstudio.com"))
```

```
install.packages("fpp3")
```

```
## Installing package into 'C:/Users/Rishita/AppData/Local/R/win-library/4.4'
## (as 'lib' is unspecified)
```

```
## package 'fpp3' successfully unpacked and MD5 sums checked
##
## The downloaded binary packages are in
##  C:\Users\Rishita\AppData\Local\Temp\RtmpCQiL2B\downloaded_packages
```

```
install.packages("forecast")
```

```
## Installing package into 'C:/Users/Rishita/AppData/Local/R/win-library/4.4'
## (as 'lib' is unspecified)
```

```
## package 'forecast' successfully unpacked and MD5 sums checked
##
## The downloaded binary packages are in
##  C:\Users\Rishita\AppData\Local\Temp\RtmpCQiL2B\downloaded_packages
```

```
library(tsibble)
```

```
## Registered S3 method overwritten by 'tsibble':
##   method               from
##   as_tibble.grouped_df dplyr

##
## Attaching package: 'tsibble'

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, union
```

```
library(fpp3)
```

```
## -- Attaching packages --------------------------------------------- fpp3 1.0.1 --

## v tibble      3.2.1     v ggplot2     3.5.1
## v dplyr       1.1.4     v tsibbledata 0.4.1
## v tidyr       1.3.1     v feasts      0.4.1
## v lubridate   1.9.3     v fable       0.4.1

## -- Conflicts ------------------------------------------------ fpp3_conflicts --
## x lubridate::date()     masks base::date()
## x dplyr::filter()       masks stats::filter()
## x tsibble::intersect()  masks base::intersect()
## x lubridate::interval() masks tsibble::interval()
## x dplyr::lag()          masks stats::lag()
## x tsibble::setdiff()    masks base::setdiff()
## x tsibble::union()      masks base::union()
```

```
library(ggplot2)
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library(zoo)  # for na.approx
```

```
##
## Attaching package: 'zoo'

## The following object is masked from 'package:tsibble':
##
##     index

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
install.packages("readr")
```

```
## Installing package into 'C:/Users/Rishita/AppData/Local/R/win-library/4.4'
## (as 'lib' is unspecified)
```

```
## package 'readr' successfully unpacked and MD5 sums checked
##
## The downloaded binary packages are in
##  C:\Users\Rishita\AppData\Local\Temp\RtmpCQiL2B\downloaded_packages
```

```
library(readr)
```

Read the data

```
climate_data <- read_csv("./GlobalLandTemperaturesByCity.csv")
```

```
## Rows: 8599212 Columns: 7
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr  (4): City, Country, Latitude, Longitude
## dbl  (2): AverageTemperature, AverageTemperatureUncertainty
## date (1): dt
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
climate_data
```

```
## # A tibble: 8,599,212 x 7
##    dt         AverageTemperature AverageTemperatureUnce~1 City  Country Latitude
##    <date>                  <dbl>                    <dbl> <chr> <chr>   <chr>
##  1 1743-11-01               6.07                     1.74 Århus Denmark 57.05N
##  2 1743-12-01              NA                       NA    Århus Denmark 57.05N
##  3 1744-01-01              NA                       NA    Århus Denmark 57.05N
##  4 1744-02-01              NA                       NA    Århus Denmark 57.05N
##  5 1744-03-01              NA                       NA    Århus Denmark 57.05N
##  6 1744-04-01               5.79                     3.62 Århus Denmark 57.05N
##  7 1744-05-01              10.6                      1.28 Århus Denmark 57.05N
##  8 1744-06-01              14.1                      1.35 Århus Denmark 57.05N
##  9 1744-07-01              16.1                      1.40 Århus Denmark 57.05N
## 10 1744-08-01              NA                       NA    Århus Denmark 57.05N
## # i 8,599,202 more rows
## # i abbreviated name: 1: AverageTemperatureUncertainty
## # i 1 more variable: Longitude <chr>
```

Data Cleaning

Handling NA values

Replace NA values with the mean of non-NA values for each city-country pair.

```
climate_data <- climate_data %>%
  group_by(City, Country) %>%
  mutate(
    AverageTemperature = if_else(is.na(AverageTemperature), mean(AverageTemperature, na.rm = TRUE), Ave
    AverageTemperatureUncertainty = if_else(is.na(AverageTemperatureUncertainty), mean(AverageTemperatu
  ) %>%
  ungroup()
```

Create a Tsibble for Time Series Analysis and Group by date, city, country, latitude, and longitude to handle different geographical coordinates

```
climate_tsibble <- climate_data %>%
  group_by(dt, City, Country, Latitude, Longitude) %>%
  summarise(
    AverageTemperature = mean(AverageTemperature, na.rm = TRUE),
    AverageTemperatureUncertainty = mean(AverageTemperatureUncertainty, na.rm = TRUE),
    .groups = 'drop'
  ) %>%
  as_tsibble(index = dt, key = c("City", "Country", "Latitude", "Longitude"))

climate_data # with replaced null values
```

```
## # A tibble: 8,599,212 x 7
##    dt         AverageTemperature AverageTemperatureUnce~1 City  Country Latitude
##    <date>                  <dbl>                    <dbl> <chr> <chr>   <chr>
##  1 1743-11-01               6.07                     1.74 Århus Denmark 57.05N
##  2 1743-12-01               7.70                     1.67 Århus Denmark 57.05N
##  3 1744-01-01               7.70                     1.67 Århus Denmark 57.05N
##  4 1744-02-01               7.70                     1.67 Århus Denmark 57.05N
##  5 1744-03-01               7.70                     1.67 Århus Denmark 57.05N
##  6 1744-04-01               5.79                     3.62 Århus Denmark 57.05N
##  7 1744-05-01              10.6                      1.28 Århus Denmark 57.05N
##  8 1744-06-01              14.1                      1.35 Århus Denmark 57.05N
##  9 1744-07-01              16.1                      1.40 Århus Denmark 57.05N
## 10 1744-08-01               7.70                     1.67 Århus Denmark 57.05N
## # i 8,599,202 more rows
## # i abbreviated name: 1: AverageTemperatureUncertainty
## # i 1 more variable: Longitude <chr>
```

Check for any duplicates that might still exist

```
duplicates <- climate_tsibble %>%
  count(dt, City, Country, Latitude, Longitude) %>%
  filter(n > 1)

if (nrow(duplicates) > 0) {
```

```
  print("Duplicates exist, need further resolution")
} else {
  print(climate_tsibble)
}
```

```
## # A tsibble: 8,599,212 x 7 [1D]
## # Key:       City, Country, Latitude, Longitude [3,510]
##    dt         City     Country Latitude Longitude AverageTemperature
##    <date>     <chr>    <chr>   <chr>    <chr>                   <dbl>
##  1 1743-11-01 A Coruña Spain   42.59N   8.73W                    10.8
##  2 1743-12-01 A Coruña Spain   42.59N   8.73W                    13.1
##  3 1744-01-01 A Coruña Spain   42.59N   8.73W                    13.1
##  4 1744-02-01 A Coruña Spain   42.59N   8.73W                    13.1
##  5 1744-03-01 A Coruña Spain   42.59N   8.73W                    13.1
##  6 1744-04-01 A Coruña Spain   42.59N   8.73W                    13.3
##  7 1744-05-01 A Coruña Spain   42.59N   8.73W                    12.9
##  8 1744-06-01 A Coruña Spain   42.59N   8.73W                    16.4
##  9 1744-07-01 A Coruña Spain   42.59N   8.73W                    18.0
## 10 1744-08-01 A Coruña Spain   42.59N   8.73W                    13.1
## # i 8,599,202 more rows
## # i 1 more variable: AverageTemperatureUncertainty <dbl>
```

Observations

1. We can see that the **index variable** is "dt" (date), which is representing the day wise data.
2. City, Country, Latitude, Longitude are **key variables**, which uniquely represents each row in the data.
3. AverageTemperature and AverageTemperatureUncertainty are the **measured variables** in the tsibble.
4. There are **3510** time series in the above data. # 5. **8,599,212** rows and **7** columns in the tsibble.

Plotting the data

```
arhus_temp <- climate_tsibble %>% filter(City == "Århus", Country == "Denmark")

arhus_temp
```

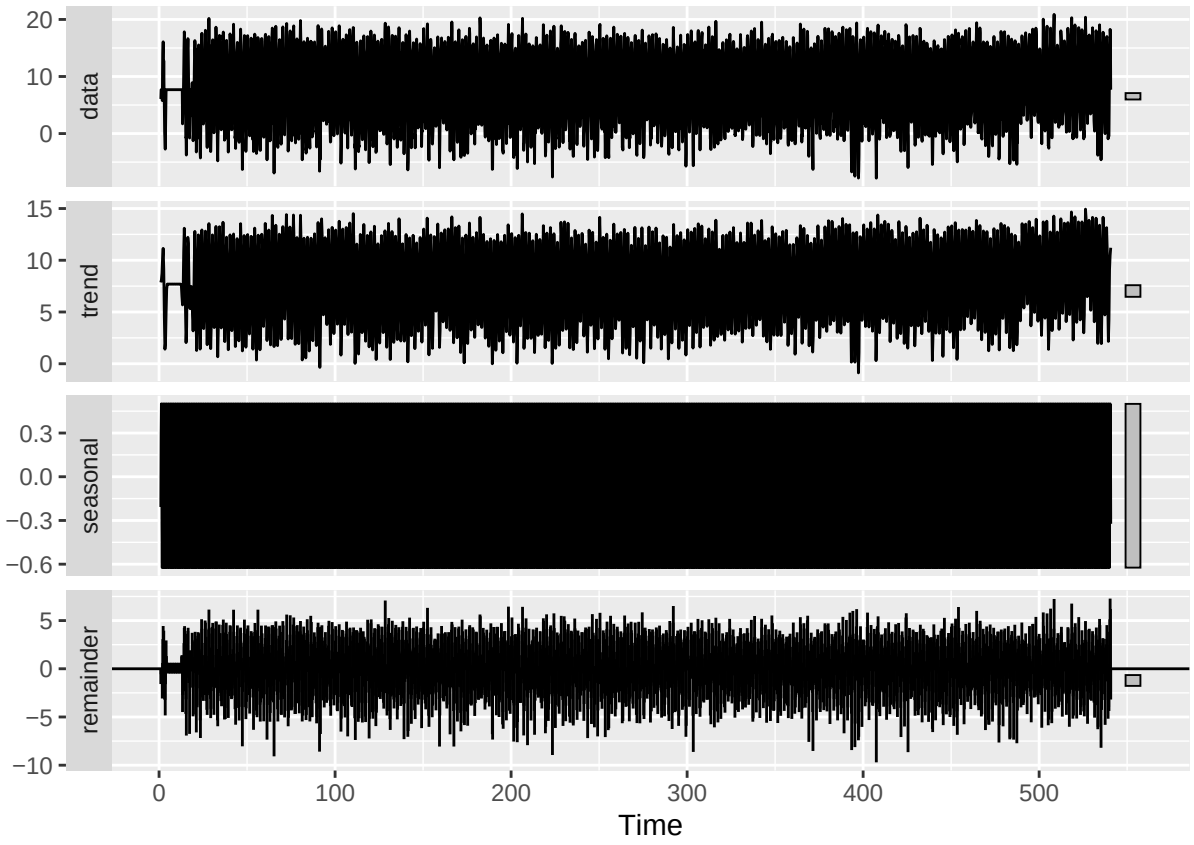
```
## # A tsibble: 3,239 x 7 [1D]
## # Key:       City, Country, Latitude, Longitude [1]
##    dt         City  Country Latitude Longitude AverageTemperature
##    <date>     <chr> <chr>   <chr>    <chr>                   <dbl>
##  1 1743-11-01 Århus Denmark 57.05N   10.33E                   6.07
##  2 1743-12-01 Århus Denmark 57.05N   10.33E                   7.70
##  3 1744-01-01 Århus Denmark 57.05N   10.33E                   7.70
##  4 1744-02-01 Århus Denmark 57.05N   10.33E                   7.70
##  5 1744-03-01 Århus Denmark 57.05N   10.33E                   7.70
##  6 1744-04-01 Århus Denmark 57.05N   10.33E                   5.79
##  7 1744-05-01 Århus Denmark 57.05N   10.33E                  10.6
##  8 1744-06-01 Århus Denmark 57.05N   10.33E                  14.1
##  9 1744-07-01 Århus Denmark 57.05N   10.33E                  16.1
## 10 1744-08-01 Århus Denmark 57.05N   10.33E                   7.70
## # i 3,229 more rows
## # i 1 more variable: AverageTemperatureUncertainty <dbl>
```

Plot for Average temperature of Arhus, Denmark

Decomposing the time series into trend, seasonal and noise

Convert to ts and decompose (trend,seasonal and noise)

```
arhus_ts <- ts(arhus_temp$AverageTemperature, frequency = 6)
decomposed <- stl(arhus_ts, s.window = "periodic")
autoplot(decomposed)
```

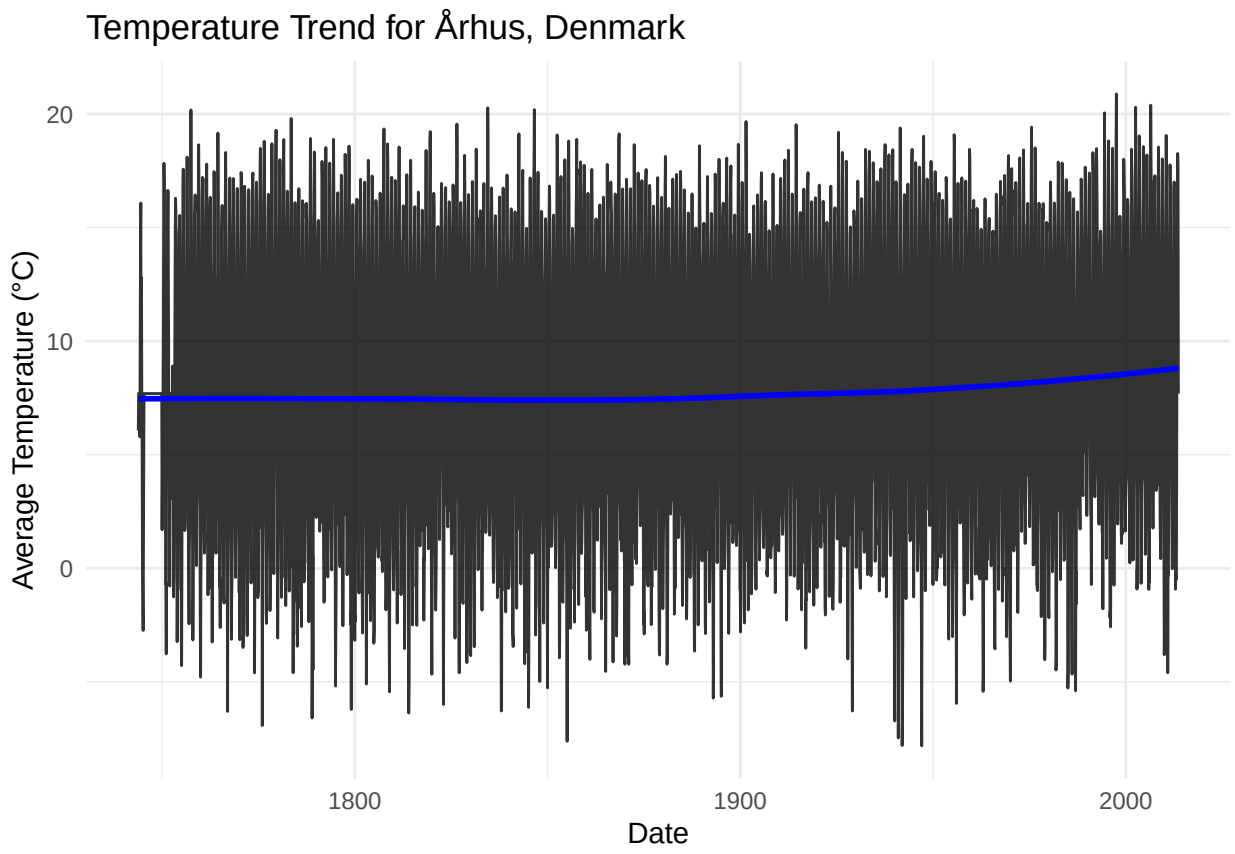


Observations

1. The trend component shows some variability and occasional sharp changes, which might indicate specific years where average temperatures were significantly different from the norm.
2. The residuals show some spikes, indicating months where the actual temperatures were significantly different from what would be predicted by the seasonal and trend components alone.

```
ggplot(arhus_temp, aes(x = dt, y = AverageTemperature)) + geom_line(linewidth = 0.5, alpha = 0.8) +
  # Reduced line size for clarity, adjust opacity
  geom_smooth(se = FALSE, color = "blue", method = "loess") +
  # Smoothed trend line
  labs( title = "Temperature Trend for Århus, Denmark", x = "Date", y = "Average Temperature (°C)" ) +
  theme_minimal() # Cleaner theme
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



Observation

1. Displays a temperature trend for Århus, Denmark, visualized using a plot created with **ggplot**.
2. The smooth blue line provides a general trend, showing how the average temperature changes across the dates. This actually the result of the smoothing operation applied, denoting **almost no trend**.

Model building

Extracting only the AverageTemperature column and converting it to a ts object

```
arhus_ts <- ts(arhus_temp$AverageTemperature, frequency = 12) # assuming monthly data
arhus_ts
```

```
##           Jan       Feb       Mar       Apr       May       Jun       Jul
## 1    6.068000  7.695135  7.695135  7.695135  7.695135  5.788000 10.644000
## 2    4.639000  0.122000 -1.333000 -2.732000  0.129000  4.042000  7.695135
## 3    7.695135  7.695135  7.695135  7.695135  7.695135  7.695135  7.695135
## 4    7.695135  7.695135  7.695135  7.695135  7.695135  7.695135  7.695135
## 5    7.695135  7.695135  7.695135  7.695135  7.695135  7.695135  7.695135
## 6    7.695135  7.695135  7.695135  7.695135  7.695135  7.695135  7.695135
```

```
## 7    7.695135  7.695135  1.699000  3.961000  5.182000  7.197000 10.634000
## 8    7.695135 -0.678000 -0.623000 -3.764000  3.700000  5.267000  7.695135
## 9    7.695135  7.695135 -0.781000  7.695135  3.050000  5.489000  7.695135
## 10   5.637000  2.242000 -1.259000 -0.285000  4.210000  6.765000 10.650000
## 11   2.871000 -3.228000 -0.566000 -0.943000 -0.645000  5.658000 12.266000
## 12   4.141000  1.450000 -3.436000 -4.277000  0.356000  7.591000 10.978000
## 13   3.284000  1.664000  2.011000  2.887000  2.684000  4.208000  9.351000
## 14   1.251000 -0.652000 -2.442000  0.857000  1.727000  7.756000 10.081000
## 15   5.123000  0.325000 -3.154000 -1.142000  1.188000  4.075000 12.793000
## 16   4.106000  0.121000  2.671000  3.162000  3.402000  6.065000  9.503000
## 17   1.885000 -2.658000 -4.787000 -0.386000  0.869000  5.651000 11.112000
## 18   3.447000  1.733000  0.670000  1.186000  4.852000  6.530000 12.109000
## 19   4.229000 -1.163000  1.639000 -0.308000 -0.896000  8.100000 11.219000
## 20   3.598000  0.467000 -3.244000  0.663000  0.501000  4.349000 10.373000
## 21   3.312000  1.644000  0.665000  2.878000  2.159000  5.715000 11.801000
## 22   2.274000 -0.821000 -0.408000 -2.603000  3.120000  6.507000  9.659000
## 23   3.927000 -0.160000 -1.210000 -1.532000  2.475000  8.199000 10.759000
## 24   5.298000 -0.636000 -6.298000  0.070000  1.880000  3.744000  9.561000
## 25   5.817000 -0.099000 -3.122000 -1.230000 -0.348000  5.101000 10.787000
## 26   3.715000  2.568000  0.656000 -0.399000  2.583000  6.177000 10.598000
## 27   2.610000  1.362000 -1.069000  1.045000 -3.143000  4.217000 11.125000
## 28   1.934000  0.642000 -3.483000 -2.997000 -2.872000  2.484000 12.203000
## 29   2.859000  2.664000 -1.882000 -2.959000 -0.867000  4.187000  8.914000
## 30   6.229000  2.422000  1.117000 -0.646000  2.289000  6.826000 12.296000
## 31   4.472000  1.644000 -4.601000 -0.083000  2.110000  6.487000 11.594000
## 32  -1.291000 -1.187000 -0.983000  1.941000  2.908000  5.966000 11.273000
## 33   1.778000  2.153000 -6.918000  0.788000  2.434000  5.912000  9.422000
## 34   4.226000  1.230000 -1.945000 -2.423000  0.782000  4.323000 11.903000
## 35   5.214000  0.994000 -1.863000 -0.922000  1.377000  6.972000 12.309000
## 36   4.296000  3.746000 -0.235000  4.785000  5.427000  8.104000 12.093000
## 37   4.231000  0.197000 -3.069000 -1.354000  4.720000  5.386000 11.492000
## 38   2.106000 -0.090000 -1.272000  0.197000  3.699000  7.094000 11.100000
## 39   4.508000 -0.411000  0.937000 -1.641000 -0.383000  4.148000  9.212000
## 40   1.045000  0.419000 -1.000000  1.384000  0.034000  7.743000 12.143000
## 41   3.580000 -0.868000 -4.596000 -2.132000 -2.915000  3.449000 11.536000
## 42   4.407000 -1.451000 -1.187000 -3.429000 -2.648000  4.388000  9.315000
## 43   4.484000 -0.624000 -1.752000 -1.113000 -2.562000  6.005000  9.793000
## 44  -0.598000  0.472000 -0.142000  2.130000  3.679000  4.598000 10.733000
## 45   2.335000  0.247000  0.544000 -2.351000 -1.211000  6.999000 11.606000
## 46   3.133000 -6.592000 -3.990000  0.562000 -4.458000  4.918000 12.607000
## 47   4.329000  4.851000  2.239000  3.936000  4.550000  3.384000 12.606000
## 48   2.811000  1.630000  3.315000  2.012000  3.957000  7.219000 10.237000
## 49   3.013000  0.403000 -1.492000 -1.430000  1.285000  7.472000 10.447000
## 50   5.031000  1.547000 -0.381000  1.671000  1.717000  5.364000 10.894000
## 51   3.694000  0.962000 -0.070000  1.943000  5.538000  9.133000 11.600000
## 52   3.985000  0.082000 -5.182000 -2.934000 -1.045000  7.757000  9.272000
## 53   3.246000  1.899000  5.096000  1.265000  0.074000  6.752000 10.776000
## 54   2.759000 -2.529000  1.033000  3.822000  2.378000  6.669000 11.615000
## 55   3.283000  1.834000 -0.285000  1.318000  1.431000  7.732000 13.962000
## 56   3.588000 -2.448000 -1.886000 -6.212000 -1.152000  3.192000  8.409000
## 57   5.546000 -2.503000 -3.167000 -2.071000 -2.320000  8.035000 12.313000
## 58   5.093000  1.305000 -0.061000 -1.083000  3.039000  6.813000 14.028000
## 59   4.126000 -0.891000 -2.849000 -0.040000  2.983000  7.138000  9.179000
## 60   3.652000  1.187000 -5.092000 -2.163000  1.309000  8.408000 10.067000
```

```
## 61   2.918000 -1.108000  1.018000 -2.302000 -1.090000  4.541000 12.341000
## 62   1.336000 -3.300000 -3.250000 -2.755000  1.377000  4.670000  8.747000
## 63   2.502000  0.933000  0.751000  0.491000  0.568000  3.359000 11.633000
## 64   4.844000  3.800000  0.349000  0.509000 -0.143000  4.406000 10.676000
## 65   2.968000  1.262000  0.041000 -1.828000 -1.026000  3.312000 12.482000
## 66   2.628000 -2.542000 -5.432000 -0.312000  0.627000  2.433000 12.527000
## 67   2.817000  2.759000 -0.613000 -0.956000  0.210000  4.285000  8.325000
## 68   3.177000  0.674000 -2.409000 -0.127000  4.233000  5.324000 13.692000
## 69   4.788000  1.745000 -1.160000  0.781000 -0.742000  1.941000  9.548000
## 70   2.016000 -3.532000 -0.909000  2.740000  3.013000  6.329000 10.451000
## 71   3.565000  1.318000 -6.369000 -4.907000 -0.834000  7.164000  8.357000
## 72   4.410000  1.221000 -2.496000  1.430000  3.288000  6.434000 11.298000
## 73   3.216000 -0.797000 -0.161000 -2.509000  0.248000  5.070000  8.707000
## 74   2.477000  0.983000  2.748000  3.077000  1.654000  4.272000 10.857000
## 75   5.306000 -2.280000  0.479000  0.358000  2.485000  3.688000 10.675000
## 76   5.196000  1.896000  2.380000  0.859000  2.397000  6.396000 11.834000
## 77   1.247000 -1.969000 -4.663000  0.574000  0.689000  6.789000 11.463000
## 78   2.630000 -1.756000 -1.841000 -0.662000  0.855000  7.866000 10.157000
## 79   5.437000  4.072000  1.269000  3.937000  5.180000  7.717000 12.588000
## 80   6.712000  0.565000 -5.989000 -2.060000  1.957000  4.831000 10.934000
## 81   5.285000  2.790000  2.835000  1.893000  1.820000  6.298000 10.503000
## 82   4.220000  2.525000  2.625000  0.633000  1.232000  6.616000 11.300000
## 83   4.054000  3.066000 -3.069000  2.811000  3.198000  6.080000 11.730000
## 84   3.592000  3.058000 -1.852000 -4.597000  1.580000  7.164000 12.169000
## 85   2.011000  4.011000 -1.284000 -1.544000  2.404000  5.423000 11.416000
## 86   4.358000  1.506000 -2.991000 -4.149000 -0.475000  3.887000 10.885000
## 87   0.412000 -3.751000 -3.791000 -3.842000  2.891000  6.667000 10.673000
## 88   5.425000 -0.660000 -3.451000 -0.204000  1.271000  7.145000 11.033000
## 89   2.514000  2.921000 -0.060000  0.780000  2.614000  6.936000  9.475000
## 90   3.464000  2.137000 -1.840000  1.704000  0.773000  4.524000 13.180000
## 91   4.109000  2.207000  1.579000  1.923000  3.524000  5.615000 12.197000
## 92   3.475000  2.287000  1.457000  2.515000  2.938000  4.932000  9.334000
## 93   2.350000 -0.071000 -0.624000  0.512000  4.195000  5.687000  9.754000
## 94   2.518000  0.641000 -1.289000  0.663000 -0.835000  3.991000  9.355000
## 95   3.591000  0.789000 -6.279000 -6.118000  0.411000  2.751000  9.840000
## 96   2.022000  0.929000 -0.990000 -0.009000 -1.922000  2.549000 10.858000
## 97   4.082000 -0.883000 -0.788000  0.171000  0.743000  6.742000  9.142000
## 98   4.522000 -2.914000 -2.921000 -3.440000  3.170000  6.375000 13.001000
## 99   3.296000  3.327000 -1.754000  1.524000  3.166000  5.819000 12.936000
## 100  1.706000  4.241000  1.427000 -0.687000  0.888000  5.516000  9.589000
## 101  4.656000  4.314000 -0.940000 -4.202000 -0.281000  6.790000 12.010000
## 102  3.521000 -3.712000  0.542000 -6.117000 -3.653000  6.031000  9.739000
## 103  5.572000  1.661000  0.666000  1.502000  4.388000  6.170000 10.320000
## 104  4.660000 -2.937000 -1.848000 -1.622000  1.398000  3.383000 11.060000
## 105  6.288000  1.293000 -4.980000  0.951000  3.047000  6.681000 12.185000
## 106  2.926000  1.661000 -2.410000  1.836000  1.890000  4.830000 11.744000
## 107  2.879000 -1.467000 -5.265000  1.859000  0.032000  5.764000 11.037000
## 108  3.274000  1.743000  0.964000  0.992000  1.301000  6.187000  8.975000
## 109  2.456000  2.583000  1.915000  0.028000  0.981000  3.631000 11.777000
## 110  3.444000  3.038000  2.203000 -3.937000 -3.135000  3.493000 10.368000
## 111  3.599000 -1.430000 -1.494000  0.186000  3.520000  6.209000 11.321000
## 112  1.583000  1.034000 -2.325000 -7.613000 -1.010000  4.260000  8.694000
## 113  3.170000 -2.638000 -0.686000 -0.901000  0.542000  6.012000  9.126000
## 114  0.729000  1.141000 -2.369000  0.466000  1.563000  4.481000 10.737000
```

```
## 115  4.203000  4.873000  0.479000 -1.592000  1.307000  5.311000 10.426000
## 116  0.656000  1.267000  2.340000  2.793000  3.812000  4.793000 12.045000
## 117  3.758000 -1.235000  0.562000 -2.724000  0.011000  4.843000 10.484000
## 118  2.231000 -2.603000 -4.007000  1.476000  3.312000  5.155000  8.323000
## 119  3.487000  2.276000 -1.755000 -1.931000  0.442000  5.513000 12.694000
## 120  4.042000  1.236000  3.265000  3.318000  2.699000  6.713000 10.028000
## 121  5.634000  2.429000 -2.225000 -0.032000  1.256000  5.115000  8.532000
## 122  2.576000  1.059000 -0.349000 -4.537000 -1.075000  6.014000 12.965000
## 123  5.557000  3.283000  3.659000  1.772000 -0.939000  6.127000  9.134000
## 124  2.821000  0.986000 -4.120000  1.868000 -1.044000  4.550000  7.696000
## 125  3.458000 -2.982000 -1.425000  2.795000  3.341000  5.945000 12.843000
## 126  2.937000  2.405000  1.243000  3.304000  0.764000  7.713000  9.880000
## 127  2.116000  0.309000  0.406000 -4.221000  0.457000  6.420000 10.416000
## 128  4.259000 -3.877000 -3.297000 -4.222000  3.519000  3.855000  9.349000
## 129  1.037000 -0.729000  1.708000  1.152000  2.345000  6.695000 11.182000
## 130  5.874000  0.394000  3.089000  0.215000  1.866000  5.036000  8.969000
## 131  4.248000  3.870000  3.247000  1.873000  3.114000  6.711000  9.356000
## 132  2.938000 -2.456000 -1.895000 -2.888000  0.375000  5.437000 11.187000
## 133  0.929000 -0.575000 -0.650000 -0.777000  1.718000  5.933000  9.083000
## 134  0.918000 -2.478000 -0.481000 -0.823000 -0.294000  3.427000  8.622000
## 135  6.325000  1.340000 -0.052000  2.636000  2.830000  7.381000 10.790000
## 136  3.371000 -1.838000 -3.662000 -3.817000  0.390000  3.683000  9.991000
## 137  1.642000 -1.776000 -0.356000  1.503000  2.730000  6.505000 10.334000
## 138  3.137000  0.659000 -4.220000 -3.762000 -1.775000  3.215000 10.076000
## 139  5.033000  2.389000  2.922000  3.006000  4.710000  6.278000 11.285000
## 140  1.722000 -2.020000 -0.897000  1.314000 -0.849000  5.577000 10.760000
## 141  4.875000  1.346000  2.336000  2.312000  3.187000  5.215000 10.238000
## 142  1.828000  1.097000 -1.635000  1.635000  2.455000  6.448000  8.965000
## 143  2.409000  1.619000 -0.542000 -2.227000 -1.142000  5.986000 10.381000
## 144  5.871000 -0.253000 -0.403000  1.114000  2.072000  5.269000 10.170000
## 145  3.007000 -0.259000 -0.298000 -3.094000 -3.642000  2.407000  9.420000
## 146  3.019000  2.344000  0.539000 -2.490000 -0.169000  5.273000 14.743000
## 147  4.981000  0.458000  2.575000  0.323000  3.087000  5.591000 12.776000
## 148  3.778000 -2.515000 -2.873000  1.399000  0.803000  4.693000 10.359000
## 149  2.900000  2.219000 -1.482000 -0.537000  0.649000  5.340000 10.136000
## 150  4.120000 -1.125000 -5.711000 -3.459000  2.847000  6.360000 11.034000
## 151  2.742000  2.669000  0.250000  1.901000  4.450000  8.073000 10.378000
## 152  6.024000  2.517000 -2.854000 -5.632000  0.965000  6.588000 13.070000
## 153  4.050000 -0.017000  0.605000  1.997000  2.975000  5.995000 11.797000
## 154  2.855000 -0.593000 -2.863000 -0.807000  1.969000  5.915000 11.423000
## 155  4.354000  1.973000  3.761000  1.134000  1.467000  5.046000  9.932000
## 156  4.695000  4.094000  1.001000  1.562000  1.709000  5.550000 10.635000
## 157  6.978000 -0.897000 -0.989000 -2.799000  0.592000  4.630000  9.576000
## 158  4.290000  2.785000 -1.507000 -2.406000  1.224000  6.261000 12.181000
## 159  3.163000  0.330000  2.590000 -1.828000  2.106000  5.026000  8.502000
## 160  2.833000 -1.123000 -0.293000  3.228000  4.860000  4.874000 11.589000
## 161  3.828000  0.077000  0.608000 -0.920000  1.302000  6.182000 10.110000
## 162  3.932000  2.210000  0.366000  1.788000  3.197000  4.207000 11.297000
## 163  3.206000  2.200000  1.125000  0.910000  2.051000  6.407000 11.831000
## 164  6.569000 -0.299000 -0.094000 -0.337000  2.746000  5.559000 10.148000
## 165  4.308000  0.460000  0.949000  2.400000  0.985000  4.960000 10.087000
## 166  3.044000  1.368000  0.779000 -1.065000 -0.476000  4.382000  8.707000
## 167  2.468000  0.967000  0.763000  1.861000  3.867000  6.545000 12.338000
## 168  2.360000  2.208000  1.493000  1.834000  2.824000  6.158000 13.201000
```

```
## 169  4.746000  2.747000 -2.277000 -1.132000  4.011000  5.956000 10.175000
## 170  3.585000  4.036000 -0.302000  2.079000  4.296000  6.858000 11.533000
## 171  6.828000  2.543000 -0.347000  3.862000  3.055000  7.907000 10.716000
## 172  3.633000  3.055000 -0.914000  0.543000  0.508000  6.048000 10.099000
## 173  2.094000 -1.830000  2.877000  0.481000  0.258000  6.564000 11.009000
## 174  5.684000  1.485000 -3.518000 -1.346000 -1.394000  3.676000 10.876000
## 175  5.379000  0.285000 -1.036000  0.768000  2.192000  6.481000 12.244000
## 176  4.554000  2.011000  0.822000 -1.624000  1.054000  5.802000 12.341000
## 177  0.106000 -0.858000  0.161000  2.860000  4.990000  6.998000 11.545000
## 178  4.387000  0.926000  2.409000  1.995000  5.295000  8.332000 12.552000
## 179  1.173000  2.246000 -1.608000 -2.057000  1.774000  4.246000 10.524000
## 180  3.698000  2.927000  2.621000 -1.237000  2.562000  4.568000  9.109000
## 181  2.555000 -1.635000 -1.798000 -1.792000 -0.511000  3.798000  9.864000
## 182  5.172000  3.723000  3.376000  2.987000  1.292000  6.840000 12.132000
## 183  1.623000 -1.493000 -0.135000 -0.382000  3.116000  7.028000 10.633000
## 184  4.981000  0.987000  2.069000  0.972000  4.514000  5.292000  8.685000
## 185  1.778000 -3.987000  0.241000  1.580000  1.451000  5.723000 10.409000
## 186  5.478000  0.804000 -2.852000 -6.281000  2.447000  3.247000 10.810000
## 187  5.246000  3.805000  3.233000  0.038000  2.844000  7.008000 11.277000
## 188  5.224000  2.340000 -0.164000 -0.527000 -0.879000  4.665000 12.004000
## 189  5.184000  1.704000  3.366000  1.552000  0.706000  5.886000 11.542000
## 190  4.480000  2.899000 -0.276000  0.161000  3.089000  6.063000 11.327000
## 191  3.190000 -0.332000  2.175000  3.735000  3.854000  6.944000 11.631000
## 192  5.305000  4.534000  0.307000  2.055000  2.597000  6.508000 10.238000
## 193  5.581000  1.061000  1.678000 -0.995000  2.416000  4.778000 11.806000
## 194  4.995000  3.666000 -0.331000 -0.360000  0.579000  6.993000 12.640000
## 195  3.695000 -1.257000  1.556000  2.297000  6.323000  6.728000 10.417000
## 196  7.480000  0.623000  0.560000  3.456000  2.617000  6.473000 11.331000
## 197  4.870000 -0.039000 -4.000000 -6.720000 -0.272000  4.286000 11.922000
## 198  4.926000  0.103000 -7.469000 -4.151000  0.658000  3.744000  9.760000
## 199  2.156000  1.788000 -7.799000 -6.583000 -3.723000  5.179000 10.228000
## 200  4.707000  2.648000 -1.335000  3.912000  5.064000  7.724000 11.914000
## 201  4.219000  1.568000  2.904000  1.209000  1.985000  5.693000  9.685000
## 202  4.091000  2.179000 -1.263000  1.494000  5.134000  7.335000 11.048000
## 203  4.683000  1.585000 -0.110000  0.611000  1.119000  7.652000 12.131000
## 204  4.549000  0.875000 -2.690000 -7.816000 -1.804000  5.310000 13.589000
## 205  3.343000  1.286000 -0.987000 -0.325000  4.135000  8.423000 11.983000
## 206  4.623000  3.567000  2.573000  3.590000  1.879000  7.242000 11.726000
## 207  5.518000  3.444000 -0.712000  1.423000  4.166000  6.743000 12.714000
## 208  3.741000 -0.520000 -0.311000  0.629000  0.034000  5.555000 10.458000
## 209  6.251000  4.464000  0.496000  1.324000  0.688000  7.574000 11.265000
## 210  1.759000  0.508000  0.465000 -0.735000  4.234000  6.767000 11.678000
## 211  6.822000  3.209000 -0.639000 -3.115000  1.474000  5.204000 12.540000
## 212  4.213000  3.140000 -0.941000 -3.007000 -0.319000  4.644000  9.052000
## 213  5.163000  0.606000 -0.424000 -5.946000  1.270000  4.400000 11.130000
## 214  2.946000  3.047000  2.001000  1.988000  2.873000  6.146000  9.975000
## 215  4.568000  1.652000 -1.018000 -2.052000 -1.546000  4.207000  9.873000
## 216  5.737000  1.396000 -0.650000  0.803000  4.243000  7.387000 12.116000
## 217  5.337000  2.229000 -0.657000 -1.359000  2.339000  5.919000 12.214000
## 218  4.693000  2.039000 -0.253000  3.241000  5.742000  7.681000 11.350000
## 219  4.763000 -0.307000  1.803000  1.797000 -0.475000  6.341000  9.406000
## 220  3.665000 -0.763000 -5.417000 -4.765000 -0.327000  5.239000 11.431000
## 221  6.034000 -0.477000  1.030000  0.387000  0.595000  7.112000 11.960000
## 222  5.329000  1.314000  1.084000  0.113000  1.248000  5.943000  9.691000
```

```
## 223  1.002000 -0.870000 -3.440000 -3.546000  2.593000  3.079000 11.278000
## 224  3.835000  1.152000 -0.931000  1.843000  5.139000  5.673000 11.029000
## 225  5.956000  0.581000 -1.445000 -0.471000  3.566000  7.324000 10.035000
## 226  3.429000 -0.309000  0.030000 -3.010000 -0.437000  5.536000 10.107000
## 227  3.765000 -2.045000 -3.488000 -4.962000  0.361000  3.350000 11.343000
## 228  4.093000  1.940000  0.763000  2.384000  1.026000  5.647000 11.901000
## 229  4.196000  4.596000 -1.943000  0.359000  3.029000  6.322000 11.622000
## 230  5.252000  4.064000  1.625000  2.903000  5.321000  5.615000 11.151000
## 231  3.106000  1.095000  2.681000  3.185000  3.782000  7.946000 11.381000
## 232  4.947000  4.062000  4.423000  1.831000  2.964000  5.487000 11.684000
## 233  4.741000  4.059000  0.147000  0.496000  0.182000  5.754000 11.396000
## 234  4.940000 -0.712000 -0.756000 -0.978000  3.452000  4.624000 11.617000
## 235  5.211000  2.317000  1.434000 -2.126000  2.010000  5.199000 11.904000
## 236  7.137000 -1.891000 -4.019000 -3.341000  1.313000  4.975000 10.081000
## 237  4.390000  1.247000 -1.850000 -2.171000  0.625000  6.151000 11.434000
## 238  3.220000  2.457000 -0.157000  0.507000  2.052000  5.748000 12.938000
## 239  4.760000 -4.481000 -4.009000 -0.568000  3.668000  6.469000 10.940000
## 240  6.244000  2.521000  4.535000 -0.501000  3.695000  6.670000 10.953000
## 241  4.227000  1.799000  0.352000  0.408000  1.123000  6.286000 12.375000
## 242  5.891000  2.825000 -5.271000 -4.496000  1.328000  4.517000 11.760000
## 243  1.807000  1.188000 -1.778000 -4.655000  1.774000  4.243000 11.478000
## 244  6.840000  2.079000 -5.391000 -0.127000 -1.569000  6.313000 10.043000
## 245  4.909000  2.688000  3.405000  2.550000  1.729000  5.400000 12.949000
## 246  3.502000  3.197000  5.414000  5.113000  5.804000  6.422000 12.013000
## 247  4.909000  2.328000  4.267000  5.895000  6.565000  8.068000 12.944000
## 248  4.027000  2.547000  2.088000 -0.714000  4.386000  6.804000 10.335000
## 249  5.184000  3.584000  3.148000  3.891000  4.578000  5.868000 13.822000
## 250  4.819000  2.888000  2.276000  1.940000  3.540000  7.851000 13.806000
## 251  2.226000  1.556000  1.995000 -1.780000  3.100000  7.456000 11.847000
## 252  6.231000  4.066000  0.444000  3.706000  3.426000  6.335000 10.545000
## 253  3.416000 -2.165000 -1.880000 -2.574000  0.761000  6.635000  9.339000
## 254  4.644000 -0.715000 -0.742000  2.771000  4.089000  5.984000 10.408000
## 255  4.178000  1.949000  2.170000  5.021000  3.821000  6.329000 12.651000
## 256  1.629000  1.096000  1.749000  1.503000  3.524000  8.065000 11.129000
## 257  5.736000  1.638000  3.065000  3.724000  3.976000  8.321000 13.567000
## 258  7.042000  3.638000  1.918000  0.241000  1.310000  5.890000 12.016000
## 259  5.184000  0.363000  2.673000  4.230000  4.518000  7.595000 13.588000
## 260  3.268000 -0.918000 -0.056000 -0.799000  3.316000  7.140000 11.717000
## 261  6.016000  3.697000 -0.660000  2.155000  3.686000  8.068000 12.435000
## 262  5.058000  3.937000  3.668000  0.615000  1.515000  7.599000 11.417000
## 263  6.214000  2.147000 -0.931000  0.413000 -0.591000  6.229000 12.205000
## 264  7.668000  6.563000  4.625000  1.767000  6.170000  9.174000 11.985000
## 265  4.804000  3.427000  3.953000  4.925000  3.654000  7.758000 13.243000
## 266  5.326000  2.239000  1.173000  0.429000  3.906000  9.481000 12.308000
## 267  6.818000  0.317000 -3.799000 -2.691000  2.429000  7.123000 10.657000
## 268  2.218000 -4.595000  0.098000 -0.307000  3.042000  9.920000 11.952000
## 269  6.953000  3.843000  1.978000 -0.021000  6.050000  6.366000 12.814000
## 270  5.927000 -0.921000 -0.344000 -0.390000 -0.474000  5.514000 12.770000
##          Aug       Sep       Oct       Nov       Dec
## 1  14.051000 16.082000  7.695135 12.781000  7.950000
## 2   7.695135  7.695135  7.695135  7.695135  7.695135
## 3   7.695135  7.695135  7.695135  7.695135  7.695135
## 4   7.695135  7.695135  7.695135  7.695135  7.695135
## 5   7.695135  7.695135  7.695135  7.695135  7.695135
```

```
## 6    7.695135  7.695135  7.695135  7.695135  7.695135
## 7   14.913000 17.831000 16.848000 13.143000  5.251000
## 8   14.429000 16.638000 16.547000 12.177000  7.695135
## 9    7.695135  7.695135  7.695135  7.695135  8.908000
## 10  14.444000 16.293000 15.808000 13.226000  9.529000
## 11  14.835000 15.040000 15.537000 12.023000  9.129000
## 12  16.671000 17.572000 14.833000 11.725000  8.033000
## 13  16.632000 18.103000 15.019000 13.473000  8.355000
## 14  16.435000 20.179000 16.896000 12.893000  4.851000
## 15  15.525000 15.706000 16.438000 11.267000  6.045000
## 16  16.363000 18.645000 17.196000 12.299000  8.314000
## 17  16.334000 17.210000 16.194000 14.510000  7.273000
## 18  17.162000 16.501000 17.794000 14.390000  5.430000
## 19  15.966000 16.334000 13.891000 12.166000  4.046000
## 20  14.920000 17.464000 16.296000 11.749000  7.252000
## 21  12.989000 19.162000 15.503000 11.310000  7.110000
## 22  14.274000 15.238000 15.978000 11.537000  8.121000
## 23  16.351000 18.306000 16.391000 13.039000  7.810000
## 24  13.458000 15.710000 17.182000 14.331000  8.001000
## 25  15.179000 17.159000 16.384000 11.475000  6.684000
## 26  14.661000 16.722000 15.267000 12.947000  5.229000
## 27  14.675000 17.425000 17.239000 14.573000  9.060000
## 28  16.824000 16.344000 14.488000 12.113000  8.178000
## 29  15.192000 16.668000 16.076000 12.777000 10.108000
## 30  14.757000 17.247000 17.400000 13.501000  9.693000
## 31  16.123000 17.003000 16.013000 11.974000  8.160000
## 32  17.367000 18.487000 18.013000 15.632000  9.318000
## 33  16.291000 18.799000 16.971000 13.035000  8.695000
## 34  14.448000 15.922000 16.468000 12.338000  7.844000
## 35  15.900000 18.695000 16.902000 11.777000  4.694000
## 36  14.922000 17.891000 19.289000 14.782000 10.599000
## 37  14.213000 17.245000 17.990000 13.051000  8.931000
## 38  17.548000 17.942000 18.874000 15.195000  7.873000
## 39  15.107000 16.601000 16.008000 13.538000  6.027000
## 40  16.750000 19.803000 17.015000 14.339000  9.291000
## 41  14.818000 16.100000 15.206000 12.756000  6.455000
## 42  15.629000 16.714000 15.292000 12.646000  7.381000
## 43  16.185000 15.418000 16.056000 11.781000  6.549000
## 44  15.184000 16.066000 15.263000 12.995000  9.466000
## 45  16.362000 18.922000 16.064000 14.508000  7.145000
## 46  16.390000 18.329000 18.097000 14.469000  9.398000
## 47  14.578000 15.251000 15.317000 11.955000  7.235000
## 48  14.379000 17.913000 17.148000 12.230000  7.493000
## 49  15.408000 18.521000 16.656000 12.529000  6.887000
## 50  14.525000 17.840000 16.420000 12.114000  9.749000
## 51  16.342000 18.886000 16.071000 12.202000  8.689000
## 52  15.276000 15.971000 16.532000 14.394000 10.285000
## 53  15.162000 17.307000 17.198000 13.630000  7.761000
## 54  14.782000 18.225000 17.178000 14.594000  8.280000
## 55  16.965000 18.586000 17.677000 13.876000  9.243000
## 56  13.757000 16.011000 15.088000 12.380000  7.721000
## 57  12.283000 15.297000 16.249000 13.097000  8.687000
## 58  14.264000 17.126000 15.907000 13.751000  9.256000
## 59  13.951000 14.726000 17.005000 12.669000  9.899000
```

```
## 60  13.774000 17.964000 17.041000 11.388000  7.495000
## 61  15.291000 17.275000 16.556000 14.473000  9.109000
## 62  12.200000 16.193000 15.700000 13.949000  4.926000
## 63  12.976000 15.332000 16.520000 14.368000  8.339000
## 64  14.113000 17.698000 19.344000 10.935000  8.034000
## 65  15.584000 18.684000 17.911000 13.914000  7.734000
## 66  14.435000 16.552000 17.215000 13.307000  7.853000
## 67  14.205000 17.075000 16.441000 14.113000  7.431000
## 68  17.054000 18.549000 16.247000 12.983000  9.521000
## 69  14.411000 14.490000 15.966000 11.513000  9.639000
## 70  14.319000 17.332000 15.621000 12.870000  5.858000
## 71  13.160000 17.965000 15.882000 12.047000  7.277000
## 72  14.489000 15.003000 15.923000 12.071000  8.708000
## 73  14.167000 16.565000 14.380000 12.071000  7.042000
## 74  14.869000 15.763000 14.973000 13.985000  5.205000
## 75  15.919000 18.397000 15.099000 13.388000  8.948000
## 76  16.359000 18.725000 19.225000 14.181000  6.650000
## 77  13.793000 16.513000 15.882000 12.116000  7.528000
## 78  12.962000 14.711000 15.045000 13.944000 10.048000
## 79  15.686000 16.944000 16.438000 12.135000  9.349000
## 80  14.729000 15.501000 16.759000 12.512000  9.417000
## 81  15.350000 16.138000 16.025000 14.682000  7.901000
## 82  14.699000 16.880000 16.282000 13.443000  8.863000
## 83  16.962000 19.556000 18.070000 12.910000  9.384000
## 84  16.001000 16.103000 15.308000 13.926000  8.835000
## 85  16.104000 18.181000 16.100000 12.944000  8.452000
## 86  15.721000 16.451000 14.983000 12.130000  6.681000
## 87  13.906000 17.026000 14.897000 12.116000  8.363000
## 88  15.577000 18.453000 17.500000 11.929000 11.021000
## 89  15.413000 14.608000 15.742000 11.629000  9.056000
## 90  15.264000 16.940000 13.313000 13.075000  8.898000
## 91  15.368000 20.273000 18.867000 13.094000  8.214000
## 92  15.591000 16.746000 15.297000 13.513000  7.831000
## 93  14.926000 15.522000 13.960000 11.028000  8.621000
## 94  14.299000 15.738000 16.350000 12.258000  8.984000
## 95  14.496000 16.741000 14.205000 13.409000  7.401000
## 96  15.137000 17.317000 14.836000 13.352000  9.131000
## 97  13.816000 14.667000 15.827000 12.623000  6.154000
## 98  13.636000 14.281000 15.692000 12.994000  8.034000
## 99  15.106000 15.730000 19.127000 13.325000  7.642000
## 100 14.527000 16.376000 17.524000 13.146000  6.687000
## 101 13.641000 14.733000 14.969000 12.746000  8.281000
## 102 15.272000 17.184000 15.554000 11.920000  7.739000
## 103 16.965000 17.846000 20.193000 14.490000 11.118000
## 104 15.032000 17.439000 17.410000 11.459000  6.936000
## 105 15.715000 15.693000 14.190000 12.379000  8.636000
## 106 12.878000 15.389000 15.182000 12.582000  6.748000
## 107 15.299000 16.832000 16.070000 11.837000  6.424000
## 108 13.494000 15.352000 15.551000 12.673000  9.713000
## 109 15.719000 19.080000 17.675000 13.003000  6.269000
## 110 16.438000 17.247000 15.345000 12.713000  8.659000
## 111 14.799000 17.985000 16.813000 12.837000  8.214000
## 112 14.581000 18.811000 16.214000 12.246000  9.151000
## 113 14.189000 14.964000 14.600000 12.019000  9.406000
```

```
## 114 15.609000 16.680000 18.886000 14.880000 10.268000
## 115 17.558000 17.340000 17.897000 14.822000  8.248000
## 116 16.515000 17.744000 17.384000 12.986000  8.463000
## 117 14.662000 16.509000 14.915000 12.447000  7.582000
## 118 16.804000 17.560000 16.369000 12.300000  9.238000
## 119 13.923000 14.493000 15.384000 13.094000  9.518000
## 120 15.299000 14.846000 16.000000 12.501000 10.089000
## 121 14.339000 16.343000 13.516000 12.616000  6.546000
## 122 13.011000 17.780000 15.559000 14.066000  7.358000
## 123 17.005000 16.037000 15.553000 13.990000  7.488000
## 124 13.667000 15.332000 16.479000 12.792000  8.459000
## 125 15.989000 18.889000 19.133000 13.131000  8.034000
## 126 12.850000 16.701000 15.254000 13.251000  7.495000
## 127 14.141000 17.131000 16.565000 12.147000  6.830000
## 128 13.926000 16.482000 16.713000 11.562000  6.688000
## 129 15.727000 18.652000 15.845000 13.080000  9.551000
## 130 15.329000 17.404000 16.046000 12.721000  7.655000
## 131 14.820000 17.069000 14.884000 13.198000 10.320000
## 132 15.142000 17.234000 17.559000 13.599000  6.076000
## 133 15.637000 16.860000 16.459000 12.296000  8.697000
## 134 14.967000 15.946000 14.884000 10.253000  7.034000
## 135 14.654000 16.082000 17.199000 13.813000  9.718000
## 136 14.102000 15.571000 16.330000 13.120000  8.106000
## 137 15.428000 16.734000 18.136000 14.600000  4.827000
## 138 14.089000 15.852000 14.106000 12.050000  5.557000
## 139 15.070000 17.139000 16.394000 14.103000  8.397000
## 140 15.115000 17.341000 15.702000 12.924000  8.089000
## 141 14.346000 17.476000 16.788000 14.886000  9.094000
## 142 13.977000 16.661000 14.289000 11.595000  5.977000
## 143 14.273000 15.649000 15.604000 13.020000  8.670000
## 144 15.111000 16.484000 15.114000 12.422000  6.512000
## 145 14.503000 14.987000 14.334000 12.322000  6.780000
## 146 18.603000 16.418000 15.184000 11.426000  8.239000
## 147 13.741000 14.548000 15.194000 13.623000  7.895000
## 148 14.896000 17.251000 14.983000 13.401000  9.972000
## 149 13.638000 15.624000 15.352000 12.714000  7.398000
## 150 14.917000 17.357000 16.666000 11.953000  8.906000
## 151 14.924000 18.008000 15.627000 11.345000  6.156000
## 152 15.431000 15.802000 16.076000 13.899000  6.846000
## 153 17.579000 18.060000 15.690000 12.883000  8.242000
## 154 16.158000 17.103000 17.714000 12.482000  7.716000
## 155 14.712000 14.709000 15.554000 12.990000  7.887000
## 156 14.838000 18.667000 16.183000 12.303000  8.307000
## 157 15.888000 16.988000 16.517000 12.984000  8.041000
## 158 14.654000 19.669000 17.131000 13.755000 10.053000
## 159 14.713000 14.357000 13.347000 10.957000  7.032000
## 160 15.186000 15.958000 14.516000 12.743000  7.404000
## 161 14.348000 16.438000 15.752000 12.597000  8.366000
## 162 16.879000 17.417000 15.446000 12.367000  5.337000
## 163 15.975000 16.155000 15.914000 13.175000  8.935000
## 164 13.032000 14.863000 13.685000 11.924000 10.982000
## 165 15.127000 17.360000 15.651000 12.562000  9.067000
## 166 13.788000 15.051000 15.098000 12.388000 10.451000
## 167 16.716000 17.169000 16.510000 13.294000  8.874000
```

```
## 168 14.963000 17.026000 17.979000 13.787000  7.456000
## 169 14.636000 18.409000 15.162000 10.867000  7.303000
## 170 14.611000 16.481000 15.252000 12.980000  8.486000
## 171 15.910000 19.533000 17.418000 13.282000  8.000000
## 172 14.243000 15.360000 15.667000 12.149000  6.071000
## 173 12.773000 16.695000 15.151000 11.747000  7.425000
## 174 16.979000 16.932000 17.421000 13.736000  7.690000
## 175 13.181000 16.140000 16.122000 11.674000  9.294000
## 176 14.101000 16.315000 14.259000 13.131000  6.807000
## 177 15.224000 16.870000 15.134000 12.769000  6.440000
## 178 14.238000 16.154000 15.328000 12.137000  9.589000
## 179 13.665000 15.233000 14.669000 11.756000  5.772000
## 180 11.443000 16.826000 14.370000 12.272000  9.219000
## 181 13.723000 15.462000 15.874000 13.689000  9.285000
## 182 14.694000 19.196000 16.913000 12.331000  7.320000
## 183 15.040000 18.311000 16.565000 13.043000  5.940000
## 184 12.194000 17.925000 17.196000 13.099000  8.415000
## 185 12.106000 14.771000 15.032000 12.337000  8.356000
## 186 12.806000 15.740000 15.301000 13.634000  9.172000
## 187 16.222000 17.050000 16.454000 12.615000  9.030000
## 188 12.879000 16.275000 15.425000 11.008000  7.690000
## 189 14.527000 18.442000 17.187000 13.227000  7.184000
## 190 17.189000 18.363000 16.683000 13.724000  9.174000
## 191 14.930000 17.857000 16.701000 14.919000  9.914000
## 192 15.427000 17.023000 16.411000 13.027000  8.684000
## 193 16.732000 17.590000 16.740000 12.555000  7.068000
## 194 14.972000 17.980000 18.651000 13.766000 10.232000
## 195 14.007000 16.715000 18.204000 14.437000  9.609000
## 196 15.619000 16.932000 18.427000 14.226000  5.529000
## 197 17.019000 16.795000 14.955000 11.452000  7.274000
## 198 15.428000 19.386000 15.481000 12.591000  7.355000
## 199 13.198000 15.759000 16.454000 13.197000  9.537000
## 200 15.502000 16.917000 15.690000 13.469000  9.840000
## 201 13.550000 17.855000 18.447000 12.857000  9.153000
## 202 14.765000 17.862000 17.605000 13.405000  9.978000
## 203 13.819000 17.660000 15.867000 13.731000  6.796000
## 204 16.635000 17.778000 19.022000 15.493000  8.411000
## 205 15.149000 17.142000 16.349000 13.482000  8.160000
## 206 14.563000 17.915000 15.666000 16.127000 10.507000
## 207 15.275000 16.699000 17.462000 13.125000  8.589000
## 208 14.994000 16.051000 16.511000 14.103000  8.878000
## 209 13.048000 16.204000 15.782000 10.826000  6.623000
## 210 17.207000 16.962000 16.113000 13.149000 10.662000
## 211 15.112000 15.372000 15.382000 12.848000  8.809000
## 212 13.664000 19.089000 18.707000 14.534000  8.345000
## 213 14.057000 16.949000 14.014000 13.068000  8.222000
## 214 14.817000 17.189000 16.013000 11.373000  9.332000
## 215 14.691000 17.050000 15.634000 14.222000 10.202000
## 216 15.439000 18.448000 18.003000 13.690000  9.330000
## 217 15.800000 15.625000 16.199000 13.145000  7.534000
## 218 15.847000 15.722000 15.064000 14.466000 11.447000
## 219 13.701000 14.926000 14.243000 12.187000  9.715000
## 220 15.804000 16.265000 16.003000 13.164000  9.170000
## 221 14.379000 15.140000 15.408000 12.570000  7.785000
```

```
## 222 14.787000 14.588000 14.849000 13.836000  9.465000
## 223 16.472000 16.448000 15.219000 12.868000  8.854000
## 224 14.317000 17.034000 16.504000 13.938000 10.278000
## 225 16.387000 15.965000 17.295000 14.029000  8.817000
## 226 16.219000 17.352000 18.170000 14.447000 10.032000
## 227 17.595000 15.570000 16.628000 12.398000  8.847000
## 228 14.350000 16.987000 16.467000 12.735000  9.374000
## 229 14.570000 18.070000 16.017000 11.495000  8.191000
## 230 15.588000 18.419000 16.132000 12.973000  6.705000
## 231 15.092000 15.227000 16.068000 13.556000  6.729000
## 232 14.916000 17.456000 19.422000 14.374000  8.978000
## 233 15.224000 18.512000 17.651000 12.251000  8.279000
## 234 15.677000 16.075000 15.979000 12.011000  9.844000
## 235 15.794000 15.370000 16.058000 11.694000  9.477000
## 236 15.219000 14.824000 15.209000 12.799000  7.690000
## 237 15.834000 17.010000 15.854000 13.649000  7.585000
## 238 14.093000 16.085000 15.815000 13.838000  7.825000
## 239 15.029000 17.888000 17.271000 13.604000  9.698000
## 240 14.811000 17.844000 16.968000 13.294000  9.346000
## 241 14.765000 16.274000 17.110000 11.906000 10.506000
## 242 14.226000 16.555000 15.800000 11.856000 10.020000
## 243 15.506000 16.271000 15.073000 10.866000  9.002000
## 244 12.344000 15.692000 14.466000 12.237000  9.425000
## 245 17.135000 17.067000 15.873000 13.759000  8.264000
## 246 15.521000 17.661000 15.573000 13.604000  9.930000
## 247 15.672000 16.844000 17.402000 12.446000  9.409000
## 248 12.632000 18.301000 17.506000 13.889000  8.789000
## 249 18.477000 17.836000 16.108000 13.398000  6.338000
## 250 14.415000 14.849000 14.570000 11.230000  7.171000
## 251 13.792000 20.052000 17.674000 12.690000  7.938000
## 252 15.073000 17.920000 18.817000 13.390000 11.028000
## 253 14.318000 15.684000 18.492000 12.039000  9.703000
## 254 16.208000 18.919000 20.883000 13.920000  7.711000
## 255 14.502000 15.502000 15.101000 13.706000  8.305000
## 256 14.604000 18.009000 17.299000 16.339000  9.291000
## 257 14.702000 16.244000 15.917000 13.195000 10.984000
## 258 13.944000 18.453000 17.396000 13.206000 11.732000
## 259 16.634000 17.767000 20.299000 14.988000  6.447000
## 260 16.609000 19.040000 18.279000 14.425000  6.621000
## 261 14.496000 15.992000 18.567000 14.116000  9.178000
## 262 14.801000 18.198000 16.267000 14.466000 10.521000
## 263 16.004000 20.387000 17.841000 16.521000 11.868000
## 264 17.072000 16.328000 17.285000 13.133000  8.573000
## 265 15.943000 18.548000 17.044000 13.316000  9.587000
## 266 15.331000 18.023000 17.544000 14.410000  7.524000
## 267 14.989000 19.057000 16.898000 12.966000  8.328000
## 268 16.055000 17.762000 16.747000 14.266000  9.843000
## 269 13.329000 16.746000 17.006000 13.204000  8.320000
## 270 15.223000 18.259000 17.423000  7.695135
```

Now fitting the ARIMA model

```
fit_arima <- auto.arima(arhus_ts)
summary(fit_arima)
```

```
## Series: arhus_ts
## ARIMA(1,0,0)(1,1,0)[12] with drift
##
## Coefficients:
##          ar1     sar1   drift
##       0.3993  -0.5037  0.0000
## s.e.  0.0162   0.0154  0.0032
##
## sigma^2 = 3.845:  log likelihood = -6752.16
## AIC=13512.31   AICc=13512.33   BIC=13536.63
##
## Training set error measures:
##                        ME    RMSE      MAE       MPE     MAPE      MASE
## Training set -0.004118185 1.95625 1.475592 0.2464608 104.9085 0.8047813
##                      ACF1
## Training set -0.01920837
```

Inferences

1. ARIMA(1,0,0) means AR(1) model with no differencing (d = 0) and no MA component (q = 0) implies that each value in the series is directly influenced by its immediate predecessor.
2. (1,1,0)[12] describes the seasonal component of the model, indicating 1 seasonal AR term, 1 order of seasonal differencing (d = 1), and no seasonal MA, across a seasonal period of 12.
3. No significant linear trend in the data as the drift coefficient is 0.
4. Positive correlation with previous value in the series due to positive significant value of ar1 coefficient.
5. Negative correlation with the previous season in the series due to negative significant value of sar1 coefficient.
6. High values of negative log likelihood and AIC errors has been observed.

Fit the Exponential Smoothing model

```
fit_ets <- ets(arhus_ts) #ets - error trend seasonality
summary(fit_ets)
```

```
## ETS(A,Ad,A)
##
## Call:
## ets(y = arhus_ts)
##
##   Smoothing parameters:
##     alpha = 0.5635
##     beta  = 1e-04
##     gamma = 1e-04
##     phi   = 0.9786
```

```
##
##   Initial states:
##     l = 13.1311
##     b = -0.4845
##     s = 0.5426 5.1513 8.3539 9.0459 7.1069 3.2836
##            -1.7816 -5.7904 -7.5659 -8.0483 -6.6035 -3.6946
##
##   sigma:  1.8539
##
##      AIC     AICc      BIC
## 30198.66 30198.87 30308.15
##
## Training set error measures:
##                       ME     RMSE      MAE      MPE    MAPE      MASE
## Training set 0.007808101 1.849025 1.399953 8.471409 87.9858 0.7635283
##                     ACF1
## Training set 0.07267355
```

Inferences

1. ETS(A, Ad, A) - Error (A) is additive error component, Trend(AD) is additive damped trend i.e. trend decreases over time and Seasonality (A) is the additive seasonal component.
2. Alpha is significant implying that the level of the series is quite responsive to the changes in the observation. 3, Beta is very small suggesting that model places very less weight on updating the trend.
3. Gamma is also very small showing less frequent updates on the seasonal pattern.
4. Phi (damping factor) close to 1, suggest slight damping.
5. The values of AIC errors are even higher than ARIMA models, showing a worse fit than that, the model might not be able to explain the variability in the data more efficiently.
6. Since the AIC errors are high and sigma is very low, might indicate overfitting.

Evaluation

```
train <- head(arhus_ts, round(length(arhus_ts) * 0.8))
test <- tail(arhus_ts, round(length(arhus_ts) * 0.2))
```

Forecast on test set

```
forecast_arima <- forecast(fit_arima, h = length(test))
forecast_ets <- forecast(fit_ets, h = length(test))
```

```
test_values <- if (is.data.frame(test)) {
  test$AverageTemperature
} else {
  as.numeric(test)
}
test_values
```

```
##   [1]  9.330000  5.337000  2.229000 -0.657000 -1.359000  2.339000  5.919000
##   [8] 12.214000 15.800000 15.625000 16.199000 13.145000  7.534000  4.693000
##  [15]  2.039000 -0.253000  3.241000  5.742000  7.681000 11.350000 15.847000
```

```
##  [22] 15.722000 15.064000 14.466000 11.447000  4.763000 -0.307000  1.803000
##  [29]  1.797000 -0.475000  6.341000  9.406000 13.701000 14.926000 14.243000
##  [36] 12.187000  9.715000  3.665000 -0.763000 -5.417000 -4.765000 -0.327000
##  [43]  5.239000 11.431000 15.804000 16.265000 16.003000 13.164000  9.170000
##  [50]  6.034000 -0.477000  1.030000  0.387000  0.595000  7.112000 11.960000
##  [57] 14.379000 15.140000 15.408000 12.570000  7.785000  5.329000  1.314000
##  [64]  1.084000  0.113000  1.248000  5.943000  9.691000 14.787000 14.588000
##  [71] 14.849000 13.836000  9.465000  1.002000 -0.870000 -3.440000 -3.546000
##  [78]  2.593000  3.079000 11.278000 16.472000 16.448000 15.219000 12.868000
##  [85]  8.854000  3.835000  1.152000 -0.931000  1.843000  5.139000  5.673000
##  [92] 11.029000 14.317000 17.034000 16.504000 13.938000 10.278000  5.956000
##  [99]  0.581000 -1.445000 -0.471000  3.566000  7.324000 10.035000 16.387000
## [106] 15.965000 17.295000 14.029000  8.817000  3.429000 -0.309000  0.030000
## [113] -3.010000 -0.437000  5.536000 10.107000 16.219000 17.352000 18.170000
## [120] 14.447000 10.032000  3.765000 -2.045000 -3.488000 -4.962000  0.361000
## [127]  3.350000 11.343000 17.595000 15.570000 16.628000 12.398000  8.847000
## [134]  4.093000  1.940000  0.763000  2.384000  1.026000  5.647000 11.901000
## [141] 14.350000 16.987000 16.467000 12.735000  9.374000  4.196000  4.596000
## [148] -1.943000  0.359000  3.029000  6.322000 11.622000 14.570000 18.070000
## [155] 16.017000 11.495000  8.191000  5.252000  4.064000  1.625000  2.903000
## [162]  5.321000  5.615000 11.151000 15.588000 18.419000 16.132000 12.973000
## [169]  6.705000  3.106000  1.095000  2.681000  3.185000  3.782000  7.946000
## [176] 11.381000 15.092000 15.227000 16.068000 13.556000  6.729000  4.947000
## [183]  4.062000  4.423000  1.831000  2.964000  5.487000 11.684000 14.916000
## [190] 17.456000 19.422000 14.374000  8.978000  4.741000  4.059000  0.147000
## [197]  0.496000  0.182000  5.754000 11.396000 15.224000 18.512000 17.651000
## [204] 12.251000  8.279000  4.940000 -0.712000 -0.756000 -0.978000  3.452000
## [211]  4.624000 11.617000 15.677000 16.075000 15.979000 12.011000  9.844000
## [218]  5.211000  2.317000  1.434000 -2.126000  2.010000  5.199000 11.904000
## [225] 15.794000 15.370000 16.058000 11.694000  9.477000  7.137000 -1.891000
## [232] -4.019000 -3.341000  1.313000  4.975000 10.081000 15.219000 14.824000
## [239] 15.209000 12.799000  7.690000  4.390000  1.247000 -1.850000 -2.171000
## [246]  0.625000  6.151000 11.434000 15.834000 17.010000 15.854000 13.649000
## [253]  7.585000  3.220000  2.457000 -0.157000  0.507000  2.052000  5.748000
## [260] 12.938000 14.093000 16.085000 15.815000 13.838000  7.825000  4.760000
## [267] -4.481000 -4.009000 -0.568000  3.668000  6.469000 10.940000 15.029000
## [274] 17.888000 17.271000 13.604000  9.698000  6.244000  2.521000  4.535000
## [281] -0.501000  3.695000  6.670000 10.953000 14.811000 17.844000 16.968000
## [288] 13.294000  9.346000  4.227000  1.799000  0.352000  0.408000  1.123000
## [295]  6.286000 12.375000 14.765000 16.274000 17.110000 11.906000 10.506000
## [302]  5.891000  2.825000 -5.271000 -4.496000  1.328000  4.517000 11.760000
## [309] 14.226000 16.555000 15.800000 11.856000 10.020000  1.807000  1.188000
## [316] -1.778000 -4.655000  1.774000  4.243000 11.478000 15.506000 16.271000
## [323] 15.073000 10.866000  9.002000  6.840000  2.079000 -5.391000 -0.127000
## [330] -1.569000  6.313000 10.043000 12.344000 15.692000 14.466000 12.237000
## [337]  9.425000  4.909000  2.688000  3.405000  2.550000  1.729000  5.400000
## [344] 12.949000 17.135000 17.067000 15.873000 13.759000  8.264000  3.502000
## [351]  3.197000  5.414000  5.113000  5.804000  6.422000 12.013000 15.521000
## [358] 17.661000 15.573000 13.604000  9.930000  4.909000  2.328000  4.267000
## [365]  5.895000  6.565000  8.068000 12.944000 15.672000 16.844000 17.402000
## [372] 12.446000  9.409000  4.027000  2.547000  2.088000 -0.714000  4.386000
## [379]  6.804000 10.335000 12.632000 18.301000 17.506000 13.889000  8.789000
## [386]  5.184000  3.584000  3.148000  3.891000  4.578000  5.868000 13.822000
## [393] 18.477000 17.836000 16.108000 13.398000  6.338000  4.819000  2.888000
```

```
## [400]  2.276000  1.940000  3.540000  7.851000 13.806000 14.415000 14.849000
## [407] 14.570000 11.230000  7.171000  2.226000  1.556000  1.995000 -1.780000
## [414]  3.100000  7.456000 11.847000 13.792000 20.052000 17.674000 12.690000
## [421]  7.938000  6.231000  4.066000  0.444000  3.706000  3.426000  6.335000
## [428] 10.545000 15.073000 17.920000 18.817000 13.390000 11.028000  3.416000
## [435] -2.165000 -1.880000 -2.574000  0.761000  6.635000  9.339000 14.318000
## [442] 15.684000 18.492000 12.039000  9.703000  4.644000 -0.715000 -0.742000
## [449]  2.771000  4.089000  5.984000 10.408000 16.208000 18.919000 20.883000
## [456] 13.920000  7.711000  4.178000  1.949000  2.170000  5.021000  3.821000
## [463]  6.329000 12.651000 14.502000 15.502000 15.101000 13.706000  8.305000
## [470]  1.629000  1.096000  1.749000  1.503000  3.524000  8.065000 11.129000
## [477] 14.604000 18.009000 17.299000 16.339000  9.291000  5.736000  1.638000
## [484]  3.065000  3.724000  3.976000  8.321000 13.567000 14.702000 16.244000
## [491] 15.917000 13.195000 10.984000  7.042000  3.638000  1.918000  0.241000
## [498]  1.310000  5.890000 12.016000 13.944000 18.453000 17.396000 13.206000
## [505] 11.732000  5.184000  0.363000  2.673000  4.230000  4.518000  7.595000
## [512] 13.588000 16.634000 17.767000 20.299000 14.988000  6.447000  3.268000
## [519] -0.918000 -0.056000 -0.799000  3.316000  7.140000 11.717000 16.609000
## [526] 19.040000 18.279000 14.425000  6.621000  6.016000  3.697000 -0.660000
## [533]  2.155000  3.686000  8.068000 12.435000 14.496000 15.992000 18.567000
## [540] 14.116000  9.178000  5.058000  3.937000  3.668000  0.615000  1.515000
## [547]  7.599000 11.417000 14.801000 18.198000 16.267000 14.466000 10.521000
## [554]  6.214000  2.147000 -0.931000  0.413000 -0.591000  6.229000 12.205000
## [561] 16.004000 20.387000 17.841000 16.521000 11.868000  7.668000  6.563000
## [568]  4.625000  1.767000  6.170000  9.174000 11.985000 17.072000 16.328000
## [575] 17.285000 13.133000  8.573000  4.804000  3.427000  3.953000  4.925000
## [582]  3.654000  7.758000 13.243000 15.943000 18.548000 17.044000 13.316000
## [589]  9.587000  5.326000  2.239000  1.173000  0.429000  3.906000  9.481000
## [596] 12.308000 15.331000 18.023000 17.544000 14.410000  7.524000  6.818000
## [603]  0.317000 -3.799000 -2.691000  2.429000  7.123000 10.657000 14.989000
## [610] 19.057000 16.898000 12.966000  8.328000  2.218000 -4.595000  0.098000
## [617] -0.307000  3.042000  9.920000 11.952000 16.055000 17.762000 16.747000
## [624] 14.266000  9.843000  6.953000  3.843000  1.978000 -0.021000  6.050000
## [631]  6.366000 12.814000 13.329000 16.746000 17.006000 13.204000  8.320000
## [638]  5.927000 -0.921000 -0.344000 -0.390000 -0.474000  5.514000 12.770000
## [645] 15.223000 18.259000 17.423000  7.695135
```

Calculate RMSE

```
rmse_arima <- sqrt(mean((forecast_arima$mean - test_values)^2))
rmse_ets <- sqrt(mean((forecast_ets$mean - test_values)^2))
```

```
print(paste("RMSE for ARIMA: ", rmse_arima))
```

```
## [1] "RMSE for ARIMA:  2.22730267505027"
```

```
print(paste("RMSE for ETS: ", rmse_ets))
```

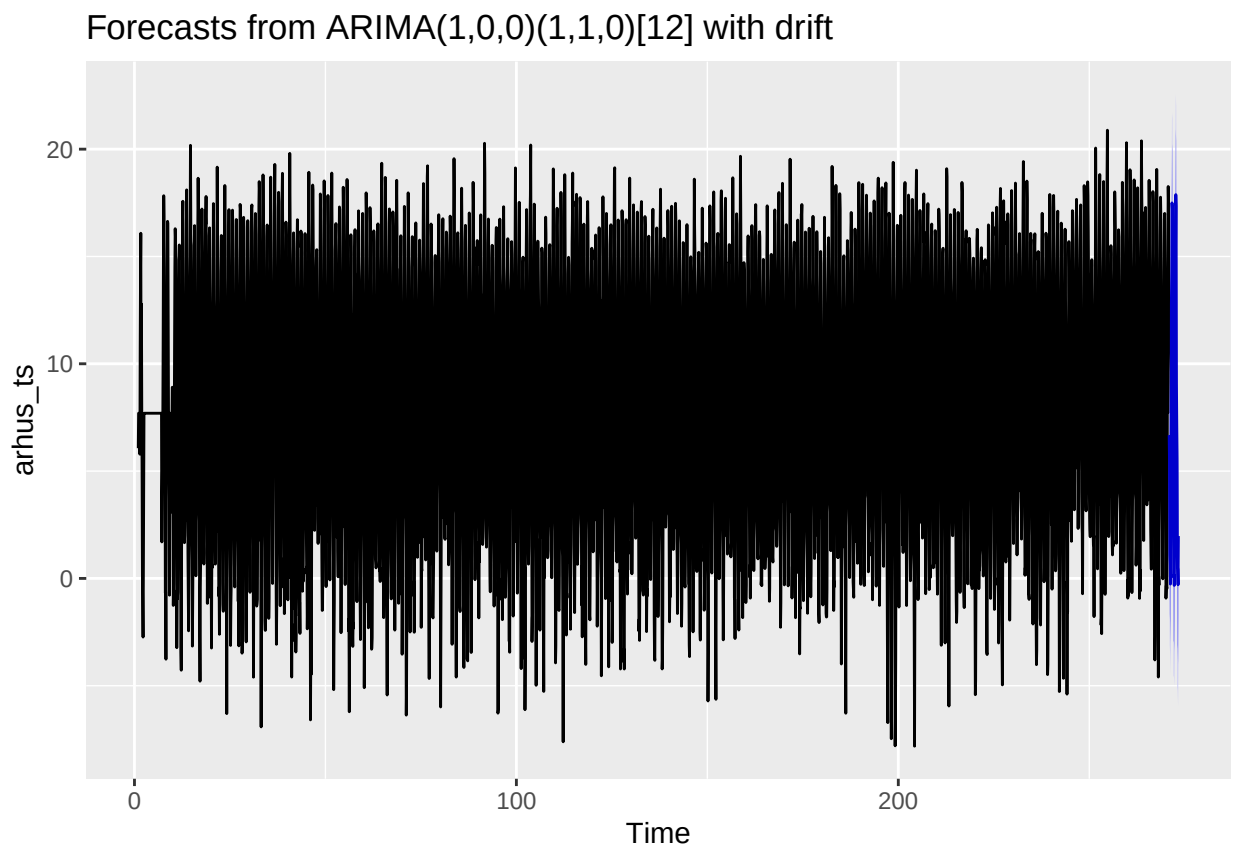
```
## [1] "RMSE for ETS:  3.42223251315167"
```

Lesser RMSE value for ARIMA in comparison to ETS.

Forecasting

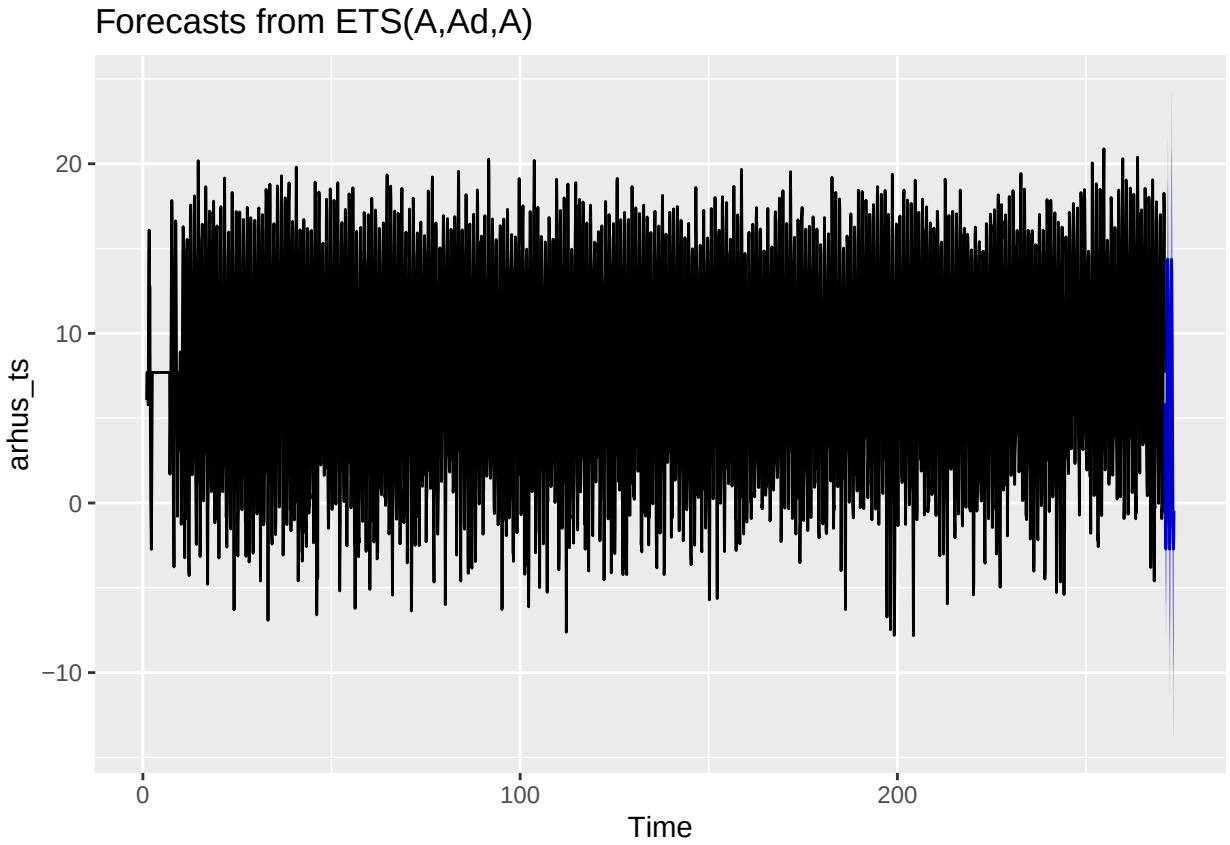
ARIMA forecast

```
future_forecast <- forecast(fit_arima, h = 30)
autoplot(future_forecast)
```



ETS forecast

```
future_ets_forecast <- forecast(fit_ets, h = 30)
autoplot(future_ets_forecast)
```


Forecasts from ETS(A,Ad,A)
arhus_ts
20
10
0
−10
0
100
200
Time