Probability Assignment 1 (12.13.5.12)

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Question

Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Solution

Let X denote the number of sixes obtained after the 6 trials. Clearly, X has the binomial distribution with $n = 6$ and p being the probability of obtaining a six ,

$$p = \frac{1}{6} \tag{1}$$

$$q = 1 - p = \frac{5}{6} \tag{2}$$

Now, since X has the binomial distribution, the cumulative distribution function is given by

$$F_X(r) = Pr(X <= r) \tag{3}$$

$$Pr(X = r) = {}^{n}C_r (p)^r (q)^{n-r} \tag{4}$$

$$\therefore F_X(r) = \sum_{i=0}^{r} {}^{n}C_i p^i q^{n-i} \tag{5}$$

The probability of throwing at most 2 sixes in 6 throws is :

$$F_X(2) = {}^{6}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{6-0} + {}^{6}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{6-1} + {}^{6}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{6-2} \tag{6}$$

$$F_X(2) = \frac{21875}{23328} \tag{7}$$