Probability Assignment 2 (11.16.3.7)

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Question A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution Let X denote the number of heads obtained after the 4 tosses. Clearly, X has the binomial distribution with n = 4 and p being the probability of obtaining a head.

$$p = \frac{1}{2} \tag{1}$$

$$q = 1 - p = \frac{1}{2} \tag{2}$$

Now, since X has the binomial distribution,

$$Pr(X = r) = {}^{n}C_{r}(p)^{r}(q)^{n-r}$$
 (3)

Let Y be the amount obtained after 4 tosses

$$Y = (1 \times X) - (1.5 \times (4 - X)) \tag{4}$$

As Y = Q(X),

$$Pr(Y = Y_0) = \sum_{i} Pr(X = i) (\forall i \in [0, 4] : Q(i) = Y_0)$$
 (5)

The Table 1 shows the declaration of random variable.

The Table 2 shows the probability of different amounts of money after four tosses.

Table 1: RANDOM VARIABLE DECLARATION

Random Variable	Value of the random variable	Event
X	0	number of heads tossed is zero
	1	number of heads tossed is one
	2	number of heads tossed is two
	3	number of heads tossed is three
	4	number of heads tossed is four
Y	-6.00	amount obtained when number of heads tossed is zero
	-3.50	amount obtained when number of heads tossed is one
	-1.00	amount obtained when number of heads tossed is two
	1.50	amount obtained when number of heads tossed is three
	4	amount obtained when number of heads tossed is four

Table 2: Probability of Amounts after 4 tosses

S.no	Y	Pr(Y)
1	-6	1/16
2	-3.5	4/16
3	-1	6/16
4	1.5	4/16
5	4	1/16