Probability Assignment 2 (11.16.3.7)

Pallala Rishitha (BT22BTECH11011)

Question A fair coin is tossed four times , and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up.From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution Let X denote the number of heads obtained after the 4 tosses. Clearly, X has the binomial distribution with $n = 4$ and p being the probability of obtaining a head.

$$p = \frac{1}{2} \tag{1}$$

$$q = 1 - p = \frac{1}{2} \tag{2}$$

Now, since X has the binomial distribution,

$$Pr\left(X = r\right) = {}^{n}C_{r}\left(p\right)^{r}\left(q\right)^{n-r} \tag{3}$$

Let Y be the amount obtained after 4 tosses

$$Y = \left(1 \times X\right) - \left(1.5 \times \left(4 - X\right)\right) \tag{4}$$

As Y = Q(X),

$$Pr\left(Y = Y_0\right) = \sum_{i} Pr\left(X = i\right)\left(\forall i \in [0,4] : Q\left(i\right) = Y_0\right) \tag{5}$$

The Table 1 shows the probability of different amounts of money after four tosses.

Table 1: Probability of Amounts after 4 tosses

X	Y	Pr(Y)
0	-6	1/16
1	-3.5	4/16
2	-1	6/16
3	1.5	4/16
4	4	1/16