Probability Assignment 2 (11.16.3.7)

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Question A fair coin is tossed four times , and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up.From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution Let X denote the number of heads obtained after the 4 tosses. Clearly, X has the binomial distribution with $n = 4$ and p being the probability of obtaining a head.

$$p = \frac{1}{2} \tag{1}$$

$$q = 1 - p = \frac{1}{2} \tag{2}$$

Now, since X has the binomial distribution,

$$Pr(X = r) = {}^{n}C_{r}(p)^{r}(q)^{n-r} \tag{3}$$

Let Y be the amount obtained after 4 tosses

$$Y = (1 \times X) - (1.5 \times (4 - X)) \tag{4}$$

As Y = Q(X),

$$Pr(Y = Y_0) = \sum_{i} Pr(X = i)\,(\forall i \in [0, 4] : Q(i) = Y_0) \tag{5}$$

The Table 1 shows parameters in the solution along with their definition and Values.

Table 1: PARAMETER DECLARATION

Parameters	Description	Values
n	Number of trials	4
p	probability of sucessful trial	$\frac{1}{2}$
q	probability of unsucessful trial	$\frac{1}{2}$
X	Random variable for the number of heads	0,1,2,3,4
Y	Random variable for amount obtained after 4 trials	-6,-3.5,-1,1.5,4

Now, since X has the binomial distribution, the Probability mass function($Pr(X = r)$) and cummulative distribution function($F_X(r)$) is given by

$$Pr(X = r) = {}^nC_r(p)^r(q)^{n-r} \tag{6}$$

$$F_X(r) = Pr(X \le r) \tag{7}$$

$$\therefore F_X(r) = \sum_{i=0}^{r} {}^nC_i p^i q^{n-i} \tag{8}$$

The Table 2 shows the probability of different amounts of money after four tosses(PMF) and CDF.

S.no	Y	PMF	CDF
1	-6	$\dfrac{1}{16}$	$\dfrac{1}{16}$
2	-3.5	$\dfrac{4}{16}$	$\dfrac{5}{16}$
3	-1	$\dfrac{6}{16}$	$\dfrac{11}{16}$
4	1.5	$\dfrac{4}{16}$	$\dfrac{15}{16}$
5	4	$\dfrac{1}{16}$	1

Table 2: PMF and CDF of Y