CO₂ Dissolution in Water

Table of Contents

0	Intr	oduction	3
	0.A	Henry's Law	3
	0.B	Relation Equations	3
	0.C	Reaction Kinetics	5
1	Tasi	k 1	<i>7</i>
	1.A	Mathematical Task	7
	1.B	Communication Task	8
	1.C	Modelling Task	9
2	Tasi	k 2	10
	2.A	Coding Task	10
	2.B	Validation Task	10
	2.C	Modelling Task	11
3	Tasi	k 3	12
	3.A	Coding Task	12
	3.B	Analysis Task	12
	3.C	Summary Task	13
4	App	endix	14
	4.A	References	14
	4.B	MATLAB Code	15

0 Introduction

0.A Henry's Law

We will be using Henry's Law to mathematically model the solubility of carbon dioxide in seawater. Henry's Law states that for a gaseous mixture that is in contact with a solution, the amount of gas in the mixture that dissolves in the solution is directly proportional to the partial pressure of the gas [1]. Here, partial pressure refers to the pressure contributed solely by the gas in the liquid-gas mixture [1]. For this law to hold true, we assume that the temperature of the system is constant and doesn't change (although in reality, the temperature changes so slowly that its derivative, with respect to time, is close to zero) [4]. Therefore, if the pressure of the gas above the liquid increases, the amount of gas dissolved in the liquid also increases proportionally. Similarly, as the pressure of the gas decreases, the amount of gas dissolved decreases proportionally. This proportionality depends on the Henry's Law constant [2] and can be simplified as shown below:

$$\frac{C_1}{P_1} = \frac{C_2}{P_2} = const.$$

Where C₁ is the concentration of the gas dissolved in the liquid in the system's first state, P₁ is the partial pressure of the gas in the system's first state, C2 is the concentration of the gas dissolved in the liquid in the system's second state and P2 is the partial pressure of the gas in the system's second state [1]. This can therefore be rewritten in the form:

$$\therefore \frac{C}{P} = k$$

Where C is the concentration of the gas dissolved in the liquid, P is the partial pressure of the gas and k is the Henry's Law constant [3].

To apply this to our context of ocean acidification, we will primarily use the following simplifications:

- 1. We will assume that the seawater behaves like pure water. [4]
- 2. We will ignore all other gases in the atmosphere apart from H_2O and CO_2 and assume that the atmosphere is made up of only H₂O and CO₂. [4]

0.B Relation Equations

Henry's Law can be used to give the following relation equations which are said to be valid when the system is in equilibrium [4]:

$$x_1 P_{v,1} = y_1 \phi_1 P \tag{1.1}$$

$$x_1 P_{v,1} = y_1 \phi_1 P$$
 (1.1)
 $x_2 H_{21} = y_2 \phi_2 P$ (1.2)

In the above equations, x_1 refers to the molar fraction of water in liquid phase, x_2 refers to the molar fraction of carbon dioxide in liquid phase, y1 refers to the molar fraction of water in gaseous phase and y_2 refers to the molar fraction of carbon dioxide in gaseous phase.

As for the remaining variables, $P_{\nu,1}$ is the vapour pressure of water and H_{21} is Henry's Constant which we saw earlier. $P_{v,1}$ can be calculated using the following equation [4]:

$$\ln \frac{P_{\nu,1}}{P_c} = (\alpha_1 x + \alpha_2 x^{1.5} + \alpha_3 x^3 + \alpha_4 x^{3.5} + \alpha_5 x^4 + \alpha_6 x^{7.5}) T_r^{-1}$$
(1.3)

In the above equation, $x=1-T_r$, $T_r=T\cdot T_c^{-1}$ and coefficients α_1 to α_6 are given in Table 1 below [4]:

$lpha_1$	-7.85951783
α_2	1.84408259
α_3	-11.7866497
$lpha_4$	22.6807411
α_5	-15.9618719
α_6	1.80122502

The variables x_1 , x_2 , y_1 and y_2 can be modelled in terms of mass fractions which gives us 2 more formulae [4]:

$$\sum_{i} y_i = y_1 + y_2 = 1 \tag{2.1}$$

$$\sum_{i} y_{i} = y_{1} + y_{2} = 1$$

$$\sum_{i} x_{i} = x_{1} + x_{2} = 1$$
(2.1)
(2.2)

 Φ_2 and Φ_1 refer to the fugacity ratio in liquid and gaseous phases, respectively and can be solved with the equations below [4]:

$$\phi_{1} = \frac{\phi_{1}^{v}}{\phi_{1}^{l}}$$

$$\phi_{2} = \phi_{2}^{v}$$
(2.3)
(2.3)

$$\phi_2 = \phi_2^{\nu} \tag{2.3}$$

Also, Henry's Constant can be defined with the following equation [4]:

$$\ln(\frac{H_{21}(T)}{1MPa}) = h_1 + \frac{h_2}{T} + \frac{h_3}{T^2} + \frac{h_4}{T^3}$$
(4)

Here, the variables h_1 , h_2 h_3 and h_4 are given in Table 2 below [4]:

h_1	-6.8346		
h_2	1.2817×10^4		
h_3	-3.7668×10^6		
h_4	2.997×10^{8}		

The Peng-Robinson Equation helps calculate the fugacity ratios and is given in the following equations [4]:

$$\ln \phi^{v} = Z_{v} - 1 - \ln(Z_{v} - B) - \frac{A}{2\sqrt{2}B} \ln(\frac{Z_{v} + (1 + \sqrt{2})B}{Z_{v} + (1 - \sqrt{2})B})$$
(5.1)

$$\ln \phi^{l} = Z_{l} - 1 - \ln(Z_{l} - B) - \frac{A}{2\sqrt{2}B} \ln(\frac{Z_{l} + (1 + \sqrt{2})B}{Z_{l} + (1 - \sqrt{2})B})$$
(5.2)

In the above equations, ϕ^v refers to fugacity ratios in the vapour phase and ϕ^l refers to fugacity ratios in the liquid phase. Also, Z is given by the roots of the following equation [4]:

$$Z^{3} - (1 - B)Z^{2} + (A - 3B^{2} - 2B)Z - (AB - B^{2} - B^{3}) = 0$$
(5.3)

Furthermore, variables A and B can be solved with the equations below [4]:

$$A = \frac{a^*P}{(RT)^2}$$

$$B = \frac{bP}{RT}$$
(5.4)
(5.5)

Where a and b are related in the equations [4]:

$$\frac{aP_c}{(RT_c)^2} = 0.45724\tag{5.6}$$

$$\frac{bP_c}{RT_c} = 0.0778 {(5.7)}$$

Also, a^* is given below [4]:

$$a^* = a\alpha(T_r, w) \tag{5.8}$$

And α is given as [4]:

$$\alpha(T_r, w) = [1 + (0.37464 + 1.54226w - 0.2699w^2)(1 - T_r^{0.5})]^2$$
(5.9)

0.C Reaction Kinetics

Coming to reaction kinetics, we have a system of 6 differential equations [4]:

$$\frac{d}{dt}[CO_2(g)] = -k_1[CO_2(g)] + k_{-1}[CO_2(aq)]$$
(6.1)

$$\frac{d}{dt}[CO_2(aq)] = k_1[CO_2(g)] - (k_{-1} + k_2)[CO_2(aq)] + k_{-2}[H_2CO_3]$$
(6.2)

$$\frac{dt}{dt}[H_2CO_3] = k_2[CO_2(aq)] - (k_{-2} + k_3)[H_2CO_3] + k_{-3}[H^+][HCO_3^-]$$
(6.3)

$$\frac{d}{dt}[H_2CO_3^-] = k_3[H_2CO_3] - k_{-3}[H^+][HCO_3^-] - k_4[HCO_3^-] + k_{-4}[H^+][CO_3^{2-}]$$
(6.4)

$$\frac{d}{dt}[CO_3^{2-}] = k_4[HCO_3^{-}] - k_{-4}[H^+][CO_3^{2-}]$$
(6.5)

$$\frac{d}{dt}[H^+] = k_3[H_2CO_3] - k_{-3}[H^+][HCO_3^-] + k_4[HCO_3^-] - k_{-4}[H^+][CO_3^{2-}]$$
(6.6)

Additionally, the variables k_1 to k_4 and k_{-1} to k_{-4} (s⁻¹) are given in Table 3 below [4]:

k_1	1×10^{10}	k_{-1}	1×10^{10}
k_2	6×10^{-2}	k_{-2}	2×10^{1}
k_3	1×10^{7}	k ₋₃	5×10^{10}
k_4	3×10^{0}	k_{-4}	5×10^{10}

Table 3

And lastly, there are 4 reaction equations as shown below [4]:

$$CO_2(g) \leftrightarrow CO_2(aq)$$
 (R1)

$$CO_{2}(g) \leftrightarrow CO_{2}(aq)$$
 (R1)
 $CO_{2}(aq) + H_{2}O \leftrightarrow H_{2}CO_{3}$ (R2)
 $H_{2}CO_{3} \leftrightarrow H^{+} + HCO_{3}^{-}$ (R3)
 $HCO_{3}^{-} \leftrightarrow H^{+} + CO_{3}^{2-}$ (R4)

$$H_2CO_3 \leftrightarrow H^+ + HCO_3^- \tag{R3}$$

$$HCO_3^- \leftrightarrow H^+ + CO_3^{2-} \tag{R4}$$

1 Task 1

1.A Mathematical Task

Looking at equations 1.1, 1.2, 2.1 and 2.2, we can see that there are 4 key variables: x_1 , x_2 , y_1 and y_2 . To get a matrix in terms of these variables, we must make sure that the equations are all in the correct format so that there are no variables on the right-hand side of the equation. Therefore, the equations can be written as:

$$x_1 P_{v,1} - y_1 \phi_1 P = 0 ag{1.1}$$

$$x_2 H_{21} - y_2 \phi_2 P = 0 ag{1.2}$$

$$y_1 + y_2 = 1 (2.1)$$

$$x_1 + x_2 = 1 (2.2)$$

Now let's separate the coefficients and variables for a moment. By doing this, we can now think of the coefficients as being mapped onto the variables to form an equation. We can do exactly this by using matrices:

$$\begin{bmatrix} P_{v,1} & 0 & -\phi_1 P & 0 \\ 0 & H_{21} & 0 & -\phi_2 P \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$
 (2.3)

As a result, we can solve for x_1 , x_2 , y_1 and y_2 by making them the subject to give:

$$\begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} P_{v,1} & 0 & -\phi_1 P & 0 \\ 0 & H_{21} & 0 & -\phi_2 P \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$
 (2.4)

For the above system of equations to be solvable, the only restriction is the determinant of the 4x4 matrix as it must not be zero. Therefore, if we let the 4x4 matrix be known as A, then $|A| \neq 0$. The determinant of the matrix is given as:

$$|A| = P_{v,1} \begin{vmatrix} H_{21} & 0 & -\phi_2 P \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} - (0) \begin{vmatrix} 0 & 0 & -\phi_2 P \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} + (-\phi_1 P) \begin{vmatrix} 0 & H_{21} & -\phi_2 P \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} - (0) \begin{vmatrix} 0 & H_{21} & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

Since the second and fourth terms are zero, this simplifies to:

$$P_{v,1} \begin{vmatrix} H_{21} & 0 & -\phi_2 P \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} - \phi_1 P \begin{vmatrix} 0 & H_{21} & -\phi_2 P \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\begin{split} &= P_{v,1} \left(H_{21} \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + (-\phi_2 P) \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \right) - \phi_1 P(0 \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - H_{21} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + (-\phi_2 P) \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}) \\ &= P_{v,1} \left(H_{21} \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} - \phi_2 P \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \right) - \phi_1 P \left(-H_{21} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - \phi_2 P \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} \right) \\ &= P_{v,1} (H_{21}(0) - \phi_2 P(-1)) - \phi_1 P(-H_{21}(-1) - \phi_2 P(0)) \\ &= P_{v,1} (\phi_2 P) - \phi_1 P(H_{21}) = P_{v,1} \phi_2 P - \phi_1 PH_{21} = P(P_{v,1} \phi_2 - \phi_1 H_{21}) \end{split}$$

To find instances when the model is not true, we have to assume that |A|=0 so $P(P_{v,1}\phi_2-\phi_1H_{21})=0$. One of the cases that |A|=0 is when P = 0. Therefore, if P=0, pressure is said to be 0 and it implies that no molecules are present – in either phase. However, this is not the case in the real world as it's simply not a vacuum to have a pressure of 0. The other case is when $P_{v,1}\phi_2=\phi_1H_{21}$. In this scenario, the gaseous and liquid phases (i.e. atmosphere and ocean) are in equilibrium so the rate at which CO_2 is being dissolved in the water is the same as the rate at which CO_2 is being released from the surface of the water and $P_{v,1}$ is the pressure of the water at which this equilibrium occurs. This 1:1 ratio is essential in oceans as this determines the amount of CO_2 escaping into the atmosphere from the surface of the water. And as CO_2 is a greenhouse gas, excessive release of CO_2 can cause more heat to be trapped in the atmosphere (due to the greenhouse effect), leading to higher global average temperatures, which in turn, accelerates global warming. Any disruptions to this equilibrium could also lead to changes in habitats and the atmosphere – causing more wildlife to rapidly develop adaptations to withstand such sudden changes, disrupting food chains and global ecosystems as a whole.

1.B Communication Task

From the data at hand, we have results for both temperature (°C) and pressure (kPa). With this, we can first work out T_r using the equation, $T_r = \frac{T}{T_c}$. Since our T values are in °C and T_c is in K, we can it to °C by subtracting 273.15. So, $T_c = 647.096 - 273.15 = 373.946$ °C. Alternatively, the T values in °C can be converted to K by adding 273.15. This value for T_r can then be used to find xusing the equation, $x=1-T_r$. After that, we can rearrange equation 1.3 for $P_{v,1}$ which gives $P_{v,1}=P_c\cdot e^{(\alpha_1x+\alpha_2x^{1.5}+\alpha_3x^3+\alpha_4x^{3.5}+\alpha_5x^4+\alpha_6x^{7.5})T_r^{-1}}$. As for P_c and α_1 - α_6 , we can use P_c = 22.064MPa (= 22064 kPa) and table 1 to substitute the respective values for α_1 to α_6 . This would give us a numerical value for $P_{\nu,1}$. We can then calculate Henry's Constant using equation 4 and rearranging this for $H_{21}(T)$ gives $H_{21}(T) = 1MPa \cdot e^{h_1 + \frac{h_2}{T} + \frac{h_3}{T^2} + \frac{h_4}{T^3}}$ where H_{21} is a function of temperature. Next, if we come to equation 5.6, we can solve for a to give $a = \frac{0.45724(RT_c)^2}{r}$. Since R (8.314 Jmol⁻¹K⁻¹), T_c and P_c are all constants, this gives us a numerical value for a. With this value for a, we can now substitute this into equation 5.4, $A = \frac{a^*P}{(RT)^2}$. However, to find A, we have to find a^* using equation 5.8, where $\alpha(T_r, w)$ is given by equation 5.9. As for values of w, these are constants as $w_{H_2O}=0.348$ and $w_{CO_2}=0.225$. In a similar way to how we found A , we can do this for equation 5.7 and solve for b to give $b = \frac{0.0778RT_c}{P_c}$. And as we have the same constants, this again gives us a numerical value for \emph{b} . With this value for \emph{b} , we can substitute it into equation 5.5, $B = \frac{bP}{RT}$, to find B. Now, we can substitute these values for A and B into the equation 5.3, $Z^3 - (1-B)Z^2 + (A-3B^2-2B)Z - (AB-B^2-B^3) = 0$ and find solutions for Z. Since, it's a cubic equation, there will be 3 roots – either all 3 real roots or only 1 real and 2 imaginary roots (which make up a conjugate pair). In the first case, the smallest of the 3 roots will be known as Z_1 and the largest will be known as Z_n . The second case will only occur for CO_2 and means that the fluid is supercritical and hence can't exist in liquid form in such thermodynamic conditions. We can now use Z_v to find ϕ^v using equation 5.1 and rearranging it to give $\phi^v=e^{Z_v-1-\ln(Z_v-B)-rac{A}{2\sqrt{2}B}\ln(rac{Z_v+(1+\sqrt{2})B}{Z_v+(1-\sqrt{2})B})}$. This can then be repeated and can be used to find ϕ_1^v (which is ϕ^v for water) and ϕ_2^v (which is ϕ^v for carbon dioxide). Similarly, we can use Z_l to find ϕ^l using equation 5.2 and rearrange it to give $\phi^l=e^{Z_l-1-\ln(Z_l-B)-\frac{A}{2\sqrt{2}B}\ln(\frac{Z_l+(1+\sqrt{2})B}{Z_l+(1-\sqrt{2})B})}$. Again, this can be repeated and used to find ϕ_1^l (which is ϕ^l for water). With these values for ϕ_1^v , ϕ_2^v

and ϕ_1^l , we can then use equations 2.3 and 2.2, $\phi_1 = \frac{\phi_1^v}{\phi_1^l}$ and $\phi_2 = \phi_2^v$ to find ϕ_1 and ϕ_2 . After gathering all of these values, we can find x_1, x_2, y_1 and y_2 by inverting the 4x4 matrix and multiplying it by the 4x1 matrix as shown in equation 2.4.

In MATLAB, I would first define the constants T_c, P_c, P, w (acentric factor) and R. I would then make a temperature array in both Celsius for the graph and Kelvin for calculations. Then, after finding T_r and x, I would calculate $P_{v,1}$ and H_{21} . I would then start the cubic equation by calculating a, a*, A, B and α. After substituting in the required values, I'd use the roots and real functions to find Z_v and Z_l for H_2O and CO_2 . And substituting those into equation 5.1 and 5.2, this gives me ϕ_1^v , ϕ_2^v and ϕ_1^l . I would then substitute those into equations 2.3 and 2.2 to find ϕ_1 and ϕ_2 . And finally, I would plot all 4 variables against temperature in Celsius.

1.C Modelling Task

In order to include other gases also present in the atmosphere, the model has to be altered such that it can be generalised for any gas. Therefore, we can change equation 1.1 to become:

$$x_n H_n = y_n \phi_n P \tag{6.1}$$

Here, x_n is the mass fraction of any gas, n, dissolved in water, y_n is the mass fraction of the gas, H_n is Henry's Law constant for the gas, ϕ_n is the fugacity of the gas and P is the system's pressure. To conserve mass, we would also need to change equations 2.1 and 2.2 to:

$$\sum_{n} y_n = y_1 + y_2 \dots + y_n = 1$$
 (6.2)

$$\sum_{n=0}^{\infty} y_n = y_1 + y_2 \dots + y_n = 1$$

$$\sum_{n=0}^{\infty} x_n = x_1 + x_2 \dots + x_n = 1$$
(6.2)

And keeping in mind that we are designing a model, we need some assumptions:

- 1. Seawater continues to behave like pure H_2O .
- 2. The solvent doesn't change (H₂O is constant).
- 3. Each new gas is modelled individually.

As for examples of gases we could add to the system, these could be nitrogen (N_2) , oxygen (O_2) , argon (Ar), etc. As a result, we will need to calculate H_n values for each of these gases which can be done with equation 4. However, the coefficients h_1 through to h_4 will be different and these are values that we will have to know/find. A few more of such coefficients would be the gases' T_c , P_c , w_n (acentric factor). After knowing all of these variables' values, we can then calculate the gases' fugacity using the Peng-Robinson equation, 5.1 and 5.2. To solve for variables x_n and y_n , we can still apply the matrix method seen in Task 1. However, since we have more sets of simultaneous equations, the matrix may appear larger than the previous 4x4. More specifically, adding just one extra gas increases the size of the matrix to 6x6, 2 extra gases increasing it to 8x8, etc. However, doing this computationally will hopefully make it easier to calculate. And to add, we will also need a range of T and P values to serve as data points which we can use to complete the model.

2 Task 2

2.A Coding Task

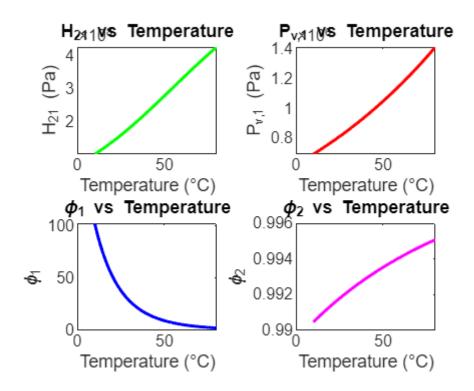


Figure 1. Graphs showing H_{21} , $P_{v,1}$, ϕ_1 and ϕ_2 plotted against Temperature for $T \in [10,80]$ °C

And to note, the number behind H_{21} is $\times 10^8$ and the number behind $P_{v,1}$ is $\times 10^5$.

To do this, I first inputted all the variables into MATLAB and programmed the necessary equations. Then, I got a list of values for $P_{\nu,1}$ and H_{21} for integer values of temperature. After that, I used the roots function to find the relevant max and min real roots then used that to solve for φ_1 and φ_2 . Lastly, I plotted all 4 variables against temperature.

2.B Validation Task

	P = 50 kPa		P = 101.325 kPa		P = 200 kPa	
Temperature	Achieved	Expected	Achieved	Expected	Achieved	Expected
10°C	0.658	0.670	0.948	0.962	1.886	1.900
20°C	0.455	0.479	0.688	0.704	1.372	1.398
30°C	0.221	0.251	0.512	0.531	0.957	1.065
40°C	0.169	0.186	0.389	0.409	0.813	0.835
50°C	0.122	0.135	0.257	0.319	0.645	0.669
60°C	0.082	0.091	0.225	0.247	0.532	0.544
70°C	0.039	0.047	0.164	0.185	0.421	0.444
80°C	0.015	0.006	0.113	0.127	0.319	0.357

From the solubilities that I calculated, most of the data points seem to be as expected except for some anomalies. This could be due to discrepancies during the methodology when the solubilities were worked out in the research paper. However, as the code simulates an ideal environment with the given assumptions that there is only CO_2 in the atmosphere, this could mean that my results are only applicable in ideal environments. Also, my constant values (e.g. H_{21}) might be slightly different.

2.C Modelling Task

To first model the equilibrium concentration of CO_2 in water over time, I first had to clean the data. I did this by finding anomalies in the temperature, pressure and CO_2 concentration columns. I then deleted each row that had an anomaly in either one of those variables. First, I would have to rearrange equation 1.2 for x_2 ($x_2 = \frac{(y_2 \phi_2 P)}{H_{21}}$) and

then after calculating $P_{v,1}$ and H_{21} , substitute those values to find x_2 as y_2 is already given in the dataset.

3 Task 3

3.A Coding Task

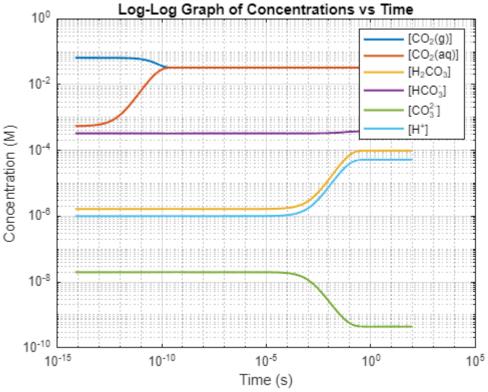


Figure 2: Graphs showing all chemicals involved in equations 6.1 to 6.6 and how they vary with time.

3.B Analysis Task

If we look at the reactions that take place, in R1, there are 2 variables involved: gaseous carbon dioxide and aqueous carbon dioxide. They are also linked by the constants k1 and k_1 where k_1 is the rate of the forward reaction and k_2 is the rate of the backward reaction. Looking at table 3, we can see that the rate of the forward reaction is equal to the rate of the backward reaction and they are both very large. Since all k constants are measured in s⁻¹, this means that the reaction in fact occurs very quickly – almost instantaneously. Coming to R2, we can see that equation 6.2 is involved and links aqueous carbon dioxide, gaseous carbon dioxide and carbonic acid. The constants in the reaction are k_1 , k_2 and k_2 . Since we have already seen k_1 and k_3 , if we focus on k_2 and k_{-2} , we can see that k_2 is relatively smaller which means that the rate of the backward reaction is greater than the rate of the forward reaction. So, it is still quite fast but not as fast as R1. This also means that more CO₂ and H₂O is produced per second than carbonic acid. Coming to R3, this links equation 6.3 and constants k2, k2, k3 and k 3. Now both k3 and k3 have high orders of magnitude which means that they both occur very quickly. But if we were to compare, we can see that k₃ is larger so the forward reaction occurs more which means more H⁺ and HCO₃⁻ is produced per second. Lastly, looking at R4, this links equation 6.4 and constants k_3 , k_4 and k_4 . As k_4 is significantly lower than k.4, this means that the backwards reaction occurs much quicker so more

 HCO_3^- is produced per second. However, the orders of magnitude aren't as big as for other constants so the reaction occurs moderately fast. When I used my programme, I found that the time taken for the system to reach steady-state equilibrium was around 0.018 seconds. Another way to check this is by manually measuring the time difference on the x-axis between the start of the experiment and when the last chemical reaches equilibrium (in this case CO_3^{2-}). Doing this, I got a rough estimate of 0.01 seconds using 10^{-1} and 10^{-14} . This is quite close to my answer in MATLAB so it seems acceptable.

3.C Summary Task

The models mentioned in the phase equilibrium and reaction kinetics sections both describe the behaviour of carbon dioxide in water but they both differ slightly from one another. The phase equilibrium model focuses on the system at a fixed moment in time an only focuses on the balance between gaseous CO_2 and dissolved CO_2 at equilibrium. The model also requires the use of Henry's Law and assumes constant temperature and pressure. It assumes an ideal environment and requires specific conditions for it to work. This can be seen in the assumptions where we treat seawater as pure water and ignore other atmospheric gases or impurities. This approach allows us to predict solubility under equilibrium conditions but only for a fixed moment in time.

On the other hand, the reaction kinetics model uses a model that changes with time – as it does in the real world. It uses a range of reversible reactions and involves many more chemicals including carbonic acid and carbonate ions. The model also uses a system of ordinary differential equations along with constants that describe rates of reactions. This makes it ideal for analysing how each chemical's concentrations change over time. However, unlike the phase equilibrium model, it doesn't need many unknowns as solely the k values and initial conditions were enough.

However, I believe that both models can still be connected. Instead of thinking of the phase equilibrium model as a final or initial state, we can think of it as the range of states of equilibrium that the system goes through by following the reaction kinetics model – much like a quasi-static process (5). Therefore, the reaction kinetics model can be seen as only describing the same process but continuously rather than being done discretely in the phase equilibrium model. Additionally looking solely at the final and initial states, the phase equilibrium model can be used to predict the final state of the reaction kinetics model.

Coming to phenomena, the phase equilibrium model describes the connection of CO_2 between gas and liquid phases at equilibrium and uses Henry's Law, assuming constant conditions. The reaction kinetics model can be used to find out how the chemical reactions vary with time and what occurs, chemically, when CO_2 dissolves in water, producing carbonic acid and carbonate ions, for example. Both models, however, can be used to predict the behaviour of the dissolution of CO_2 in water.

4 Appendix

4.A References

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- [2] Henry's Law Constants [Internet]. Henrys-law.org. 2023. Available from: https://henrys-law.org/henry/index.html (Accessed on: 31/12/2024)
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<u>Thermodynamics Electricity and Magnetism (OpenStax)/03%3A The First Law of Thermodynamics/3.05%3A Thermodynamic Processes</u> (Accessed on: 06/01/2024)

4.B MATLAB Code

alpha1 = -7.8595

```
% Defining known variables
 T_Celsius = linspace(10,80,71) % Temperature in Celsius
T_Celsius = 1 \times 71
                  12
                        13
                               14
                                      15
                                             16
                                                    17
                                                                  19
                                                                         20
                                                                                21 ...
    10
           11
                                                           18
 T Kelvin = T_Celsius + 273.15 % Convert to Kelvin
T_Kelvin = 1 \times 71
  283.1500 284.1500 285.1500 286.1500 287.1500 288.1500 289.1500 ···
 P = 101325 % Pressure in Pa
P = 101325
 Tc_water = 647.096 % Critical temperature of water (K)
Tc_{water} = 647.0960
 Pc_water = 22064000 % Critical pressure of water (Pa)
Pc water = 22064000
 R = 8.314 \% \text{ Units of } J/(\text{mol } K)
R = 8.3140
 Tc_carbon = 304.18 % Kelvin
Tc\_carbon = 304.1800
 Pc_carbon = 7380000 % Pa
Pc\_carbon = 7380000
 Tr_water = T_Kelvin/Tc_water
Tr water = 1 \times 71
              0.4391
                           0.4407
                                      0.4422
                                                  0.4438
                                                             0.4453
                                                                         0.4468 ...
    0.4376
 x = 1-Tr_water
x = 1 \times 71
                           0.5593
                                      0.5578
                                                                         0.5532 ...
    0.5624
               0.5609
                                                  0.5562
                                                             0.5547
 Tr_carbon = T_Kelvin/Tc_carbon
Tr\_carbon = 1 \times 71
    0.9309
              0.9342
                           0.9374
                                      0.9407
                                                  0.9440
                                                             0.9473
                                                                         0.9506 ...
 % Coefficients for Eq. (3)
 alpha1 = -7.85951783
```

15

```
alpha2 = 1.84408259
alpha2 = 1.8441
 alpha3 = -11.7866497
alpha3 = -11.7866
 alpha4 = 22.6807411
alpha4 = 22.6807
 alpha5 = -15.9618719
alpha5 = -15.9619
 alpha6 = 1.80122502
alpha6 = 1.8012
 Pv1 =
Pc_{water.*(exp((alpha1.*x)+(alpha2.*x.^(1.5))+(alpha3.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))}
.5))+(alpha5.*x.^(4))+(alpha6.*x.^(7.5)))./Tr water)
Pv1 = 1 \times 71
10^{6} \times
              0.7009 0.7084 0.7159
                                                 0.7236 0.7313 0.7390 ...
    0.6935
 % Coefficients for Eq. (4)
 h1 = -6.8346
h1 = -6.8346
 h2 = 1.2817*(10^4)
h2 = 12817
 h3 = -3.7668*(10^6)
h3 = -3766800
 h4 = 2.9970*(10^8)
h4 = 299700000
 H21 = 10.^6.*exp(h1+(h2./T_Kelvin)+(h3./(T_Kelvin.^2))+(h4./(T_Kelvin.^3)))
H21 = 1 \times 71
10^{8} \times
    1.0461
              1.0800
                          1.1146
                                      1.1498
                                                 1.1856
                                                             1.2221
                                                                        1.2592 ...
 % H2O Cubic
 a_water = (0.45724.*(R.*Tc_water).^2)./Pc_water
a water = 0.5998
```

```
w water = 0.348
w water = 0.3480
 alpha water = (1+(0.37464+1.54226.*w water-0.2699.*w water.^2).*(1-
sqrt(Tr_water))).^2
alpha_water = 1 \times 71
    1.6833
             1.6807
                          1.6780
                                    1.6754
                                               1.6727
                                                          1.6701
                                                                     1.6675 ...
 a star water = a water.*alpha water
a star water = 1 \times 71
            1.0081
                         1.0065
                                   1.0049
                                               1.0033
                                                          1.0018
                                                                     1.0002 ...
    1.0097
 A_water = (a_star_water.*P)./((R.*T_Kelvin).^2)
A_{water} = 1 \times 71
               0.0183
                         0.0181
                                    0.0180
                                               0.0178
                                                          0.0177
                                                                     0.0175 ...
    0.0185
 b_water = (0.0778.*R.*Tc_water)./Pc_water
b_{water} = 1.8970e-05
 B_water = (b_water.*P)./(R.*T_Kelvin)
B water = 1 \times 71
10^{-3} \times
              0.8136 0.8108
                                    0.8080
                                               0.8051
                                                          0.8023
                                                                     0.7996 ...
    0.8165
 H20_coeff_1 = ones(1,71)
H2O coeff 1 = 1 \times 71
                                                                             1 ...
                        1
                               1
                                     1
                                            1
                                                         1
                                                                1
                                                                      1
                                                   1
 H20_coeff_2 = B_water-1
H20_coeff_2 = 1 \times 71
   -0.9992 -0.9992 -0.9992
                                   -0.9992 -0.9992
                                                         -0.9992
                                                                    -0.9992 ···
 H20_coeff_3 = A_water-(3.*(B_water).^2)-(2.*B_water)
H20_coeff_3 = 1 \times 71
    0.0161
                                                                     0.0159 ...
 H20_coeff_4 = ((B_water).^3)+(B_water).^2-(A_water.*B_water)
H20_coeff_4 = 1 \times 71
10<sup>-4</sup> ×
   -0.1441 -0.1423 -0.1405 -0.1388 -0.1371
                                                         -0.1355
                                                                    -0.1338 ···
 Zv_water = zeros(1, 71)
Zv_water = 1 \times 71
```

17

```
0
        0 0 0
                                0
                                       0
                                              0
                                                     0 0 0
                                                                          0
                                                                                  0 ...
 Zl_water = zeros(1, 71)
Zl water = 1 \times 71
     0
                                         0
                                                0
                                                                     0
                                                                                   0 ...
 for n = 1:71
      cubic_coeffs_H20 = [H20_coeff_1(n), H20_coeff_2(n), H20_coeff_3(n),
H20_coeff_4(n)]
      roots_H20 = roots(cubic_coeffs_H20)
      Zv_{water}(n) = max(roots_{H20})
      Zl_water(n) = min(roots_H20)
 end
cubic\_coeffs\_H20 = 1 \times 4
    1.0000 -0.9992 0.0168 -0.0000
roots H20 = 3 \times 1
    0.9821
    0.0162
    0.0009
Zv_water = 1 \times 71
    0.9821
                    0
                                0
                                             0
                                                       0
                                                                   0
                                                                               0 ...
Zl water = 1 \times 71
10<sup>-3</sup> ×
                                                                    0
    0.9048
                                  0
                                             0
                                                         0
                                                                                0 ...
cubic coeffs H20 = 1 \times 4
    1.0000 -0.9992
                         0.0167
                                      -0.0000
roots_H20 = 3 \times 1
    0.9822
    0.0161
    0.0009
Zv_water = 1 \times 71
    0.9821
               0.9822
                                0
                                             0
                                                       0
                                                                   0
                                                                               0 ...
Zl_water = 1 \times 71
10<sup>-3</sup> ×
              0.9021
                                 0
                                             0
                                                         0
                                                                    0
                                                                                0 ...
    0.9048
cubic\_coeffs\_H20 = 1 \times 4
    1.0000 -0.9992
                         0.0165
                                      -0.0000
roots_H20 = 3 \times 1
    \overline{0.9824}
    0.0159
    0.0009
Zv_water = 1 \times 71
   0.9821
               0.9822
                                                                   0
                          0.9824
                                            0
                                                       0
                                                                                0 ...
Zl water = 1 \times 71
10<sup>-3</sup> ×
                                                                    0
              0.9021
                                                         0
                                                                                0 ...
    0.9048
                           0.8995
                                             0
cubic\_coeffs\_H20 = 1 \times 4
    1.0000 -0.9992
                           0.0164
                                      -0.0000
roots H20 = 3 \times 1
    0.9825
    0.0158
    0.0009
Zv water = 1 \times 71
    0.9821
             0.9822 0.9824 0.9825
                                                       0
                                                                   0
                                                                               0 ...
Zl water = 1 \times 71
10<sup>-3</sup> ×
```

```
0.9048
               0.9021
                           0.8995
                                       0.8969
                                                          0
                                                                      0
                                                                                  0 ...
cubic\_coeffs\_H20 = 1 \times 4
     1.0000 -0.9992
                             0.0162
                                        -0.0000
roots_H20 = 3 \times 1
    0.9827
    0.0156
    0.0009
Zv_water = 1 \times 71
    0.9821
             0.9822
                            0.9824
                                       0.9825
                                                     0.9827
                                                                       0
                                                                                   0 ...
Zl_water = 1 \times 71
10<sup>-3</sup> ×
               0.9021
                                                                       0
                                                                                    0 ...
    0.9048
                             0.8995
                                        0.8969
                                                     0.8944
cubic\_coeffs\_H20 = 1 \times 4
    \overline{1.0000} -0.9992
                             0.0161
                                        -0.0000
roots_H20 = 3 \times 1
    \overline{0.9829}
    0.0155
    0.0009
Zv_water = 1 \times 71
                0.9822
                             0.9824
                                         0.9825
                                                     0.9827
                                                                                    0 ...
    0.9821
                                                                 0.9829
Zl water = 1 \times 71
10<sup>-3</sup> ×
               0.9021
                                                     0.8944
                                                                 0.8918
                                                                                    0 ...
    0.9048
                             0.8995
                                        0.8969
cubic\_coeffs\_H20 = 1 \times 4
    1.0000 -0.9992
                             0.0159
                                        -0.0000
roots_H20 = 3 \times 1
    0.9830
    0.0153
    0.0009
Zv_water = 1 \times 71
                0.9822
                             0.9824
                                       0.9825
                                                     0.9827
                                                                 0.9829
    0.9821
                                                                             0.9830 ...
Zl water = 1 \times 71
10<sup>-3</sup> ×
               0.9021
                             0.8995
                                        0.8969
                                                     0.8944
                                                                 0.8918
                                                                             0.8893 ...
    0.9048
cubic\_coeffs\_H20 = 1 \times 4
                             0.0158
    1.0000 -0.9992
                                        -0.0000
roots_H20 = 3 \times 1
    0.9832
    0.0152
    0.0009
Zv_water = 1 \times 71
    0.9821
                0.9822
                             0.9824
                                         0.9825
                                                     0.9827
                                                                 0.9829
                                                                             0.9830 ...
Zl water = 1 \times 71
10^{-3} \times
                                                     0.8944
    0.9048
               0.9021
                             0.8995
                                        0.8969
                                                                 0.8918
                                                                             0.8893 ...
cubic\_coeffs\_H20 = 1 \times 4
     1.0000 -0.9992
                             0.0157
                                        -0.0000
roots H20 = 3 \times 1
     0.9833
     0.0150
     0.0009
Zv water = 1 \times 71
     0.9821
                  0.9822
                              0.9824
                                             0.9825
                                                           0.9827
                                                                        0.9829
             0.9832
                          0.9833
                                              0
                                                            0
                                                                         0
                                                                                       0
0.9830
0
             0
                          0
                                        0
                                                     0
                                                                   0
                                                                                0
                                        0
0
             0
                          0
                                                     0
                                                                   0
                                                                                0
                          0
             0
Zl water = 1 \times 71
1.0e-03 *
```

```
0.9048 0.9021 0.8995 0.8969 0.8944
                                           0.8918
0
                            0
0
               0
        0
                       0
                                         0
                                                 0
                      0
0
        0
               0
                                        0
                                                 0
      0
               0
0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0155 -0.0000
roots H20 = 3 \times 1
  0.9834
   0.0149
   0.0009
Zv water = 1 \times 71
0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0 0
                                            0
                                                     0
      0
               0
                       0 0
                                        0
                                                0
                      0
0
      0
                0
                                        0
                                                 0
0 0
               0
Zl water = 1 \times 71
1.0e-03 *

      0.9048
      0.9021
      0.8995
      0.8969
      0.8944
      0.8918

      0.8893
      0.8868
      0.8843
      0.8818
      0
      0

                                                     0
       0 0
                       0 0
                                        0
                                                 0
                      0
0 0 0
               0
                               0
                                        0
                                                 0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0154 -0.0000
roots H20 = 3 \times 1
   0.9836
   0.0147
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
                                            0
0.9830 0.9832 0.9833 0.9834 0.9836
                                                      0
      0
               0
                        0
                               0
                                         0
                                                 0
0
      0
                0
                        0
                               0
                                        0
                                                 0
               0
       0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8893 0.8868 0.8843 0.8818 0.8794
                                                      0
       0
               0
                        0
                               0
                                        0
                                                 0
                     0
0
       0
               0
                             0
                                        0
                                                 0
      0
               0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0152 -0.0000
roots H20 = 3 \times 1
```

```
0.9837
   0.0146
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837
                                                    0
0
       0
               0
                       0
                                        0
                               0
                                                0
0
       0
               0
                       0
                               0
                                        0
0
       0
               0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0
0
       0
               0
                       0
                               0
                                        0
                                                0
0
       0
               0
                       0
                               0
                                        0
                                                0
       0
               0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0151 -0.0000
roots H20 = 3 \times 1
   0.9839
   0.0145
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
               0
0
       0
                       0
                               0
                                        0
                                                0
               0
                       0
                               0
                                        0
0
       0
                                                0
       0
               0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0
0
       0
                       0
                               0
                                        0
                                                0
                               0
0
       0
               0
                       0
                                        0
                                                0
       0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0150 -0.0000
roots H20 = 3 \times 1
   0.9840
   0.0143
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
       0.9832 0.9833 0.9834 0.9836 0.9837
0.9830
                                               0.9839
0.9840
                                  0
           0
                  0
                          0
       0
               0
                       0
                               0
                                        0
                                                0
0
0
       0
                0
```

```
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
          0
                  0
                          0
                                  0
                                          0
                                                   0
0
       0
               0
                       0
                               0
                                       0
                                               0
       0
              0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0148 -0.0000
roots H20 = 3 \times 1
   0.9842
   0.0142
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
       0.9842
                  0
                           0
                                  0
                                           0
0
       0
               0
                       0
                               0
                                       0
                                               0
0
       0
               0
Zl_water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698
0.8722
                  0
                           0
                                  0
                                          0
                                                   0
0
       0
               0
                       0
                                       0
                               0
                                               0
       0
              0
cubic\_coeffs\_H20 = 1 \times 4
   1.0000 -0.9992 0.0147 -0.0000
roots H20 = 3 \times 1
   0.9843
   0.0141
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
       0.9842
               0.9843
0.9840
                          0
                                  0
                                          0
0
       0
               0
                       0
                               0
                                       0
                                               0
               0
       0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8893
       0.8868 0.8843 0.8818 0.8794 0.8770
                                               0.8746
0.8722
       0.8698
               0.8674
                          0
                                  0
                                                   0
0
       0
               0
                       0
                               0
                                       0
                                               0
       0
               0
cubic coeffs H20 = 1 \times 4
```

```
1.0000 -0.9992 0.0146 -0.0000
roots H20 = 3 \times 1
   0.9844
   0.0139
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
                             0
0.9840 0.9842 0.9843
                     0.9844
                                    0
                                                0
0
       0
              0
                      0
                          0
                                     0
0
       0
              0
Zl_water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674 0.8651
                             0
0.8722
                                    0
                                                0
       0
              0
                      0
                                    0
0
                             0
                                           0
       0
0
              0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0145 -0.0000
roots H20 = 3 \times 1
  0.9846
   0.0138
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843 0.9844 0.9846
                                    0
                      0
                             0
       0
              0
                                     0
0
      0
              0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
       0.8698 0.8674 0.8651 0.8628
                                    0
                                                0
              0
                             0
0
       0
                      0
                                     0
                                            0
      0
              0
cubic coeffs H2O = 1×4
   1.0000 -0.9992 0.0143 -0.0000
roots H20 = 3 \times 1
   0.9847
   0.0137
  0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843 0.9844 0.9846 0.9847
```

```
0
                                         0
0
    0 0
                                 0
                                                 0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8698
      0.8674
      0.8651
      0.8628
      0.8605

      0
      0
      0
      0
      0

0.8722
                                                   0
        0
                0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0142 -0.0000
roots H20 = 3 \times 1
   0.9848
   0.0136
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843
                        0.9844 0.9846 0.9847
                                                   0.9848
        0
                 0
                         0
                                  0
                                          0
                 0
        0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722 0.8698 0.8674 0.8651 0.8628 0.8605
                                                   0.8582
0
        0
                 0
                        0
                                  0
                                           0
        0
                0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0141 -0.0000
roots H20 = 3 \times 1
   0.9850
   0.0134
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0
      0
      0
      0
      0

                                                   0.9848
0
        0
                 0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698   0.8674   0.8651   0.8628   0.8605
0.8722
```

```
0.8560 0 0
0 0 0
                          0
                                      0
                                              0
                                                         0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0140 -0.0000
roots H20 = 3 \times 1
   0.9851
   0.0133
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843
                         0.9844 0.9846 0.9847
                                                     0.9848
0.9850 0.9851
                 0
                           0
                                    0
0 0
                0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                         0.8651 0.8628 0.8605
0.8722
                                                     0.8582
0.8560 0.8537
                                    0
                          0
                                             0
                 0
   0 0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0138 -0.0000
roots H20 = 3 \times 1
   0.9852
   0.0132
   0.0009
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0
      0
      0

                                                     0.9848
0 0
                0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8722
      0.8698
      0.8674
      0.8651
      0.8628
      0.8605

      0.8560
      0.8537
      0.8515
      0
      0
      0

                                                     0.8582
   0 0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0137 -0.0000
roots H20 = 3 \times 1
   0.9853
   0.0131
   0.0008
Zv water = 1 \times 71
```

```
0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
                      0.9844 0.9846 0.9847
                                             0.9848
0.9840
      0.9842 0.9843
0.9850 0.9851 0.9852 0.9853
                              0
                                      0
                                                  0
               0
       0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                      0.8651 0.8628 0.8605
                                              0.8582
0.8722
0.8560 0.8537 0.8515 0.8493
                              0
                                      0
                                                  0
       0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0136 -0.0000
roots H20 = 3 \times 1
   0.9855
   0.0129
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843
                      0.9844 0.9846 0.9847
                                              0.9848
0.9850 0.9851 0.9852 0.9853 0.9855
                                       9
       0
               0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
      0.8698 0.8674
                      0.8651 0.8628 0.8605
                                              0.8582
0.8560
       0.8537 0.8515 0.8493 0.8471
                                         0
       0
               0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9992 0.0135 -0.0000
roots H20 = 3 \times 1
   0.9856
   0.0128
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837
                                             0.9839
                                             0.9848
0.9840 0.9842 0.9843
                      0.9844 0.9846 0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

      0
               0
Zl_water = 1 \times 71
1.0e-03 *
```

```
0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
      0.8698
             0.8674
                     0.8651
                                            0.8582
                            0.8628 0.8605
       0.8560
                                                0
       0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0134 -0.0000
roots H20 = 3 \times 1
   0.9857
   0.0127
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
                     0.9844 0.9846 0.9847
      0.9842 0.9843
0.9840
                                            0.9848
0.9850 0.9851 0.9852
                     0.9853 0.9855 0.9856
                                            0.9857
       0
              0
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
       0.8698 0.8674
                     0.8651 0.8628 0.8605
                                            0.8582
0.8560
       0.8537 0.8515
                     0.8493 0.8471
                                    0.8449
                                            0.8428
       0
              0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0133 -0.0000
roots H20 = 3 \times 1
   0.9858
   0.0126
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837
                                           0.9839
0.9840
      0.9842 0.9843
                     0.9844 0.9846 0.9847
                                            0.9848
0.9850 0.9851 0.9852
                     0.9853 0.9855 0.9856
                                            0.9857
0.9858
          0
                 0
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
      0.8868 0.8843 0.8818 0.8794 0.8770
0.8893
       0.8698 0.8674
                     0.8651 0.8628 0.8605
0.8722
                                            0.8582
       0.8537 0.8515 0.8493 0.8471 0.8449
0.8560
                                            0.8428
0.8406
       0
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0131 -0.0000
roots H20 = 3 \times 1
   0.9859
```

```
0.0125
     0.0008
Zv water = 1 \times 71
     0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847
      0.9848

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856
      0.9857

0.9858 0.9859
                         0
Zl water = 1 \times 71
1.0e-03 *
     0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8722
      0.8698
      0.8674
      0.8651
      0.8628
      0.8605
      0.8582

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449
      0.8428

                        0
0.8406 0.8385
cubic coeffs H20 = 1 \times 4
     1.0000 -0.9993 0.0130 -0.0000
roots H20 = 3 \times 1
     0.9861
     0.0124
     0.0008
Zv water = 1 \times 71
     0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

                                                                           0.9848
                                                                           0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
    0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8722
      0.8698
      0.8674
      0.8651
      0.8628
      0.8605

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449

                                                                           0.8582
                                                                           0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
     1.0000 -0.9993 0.0129 -0.0000
roots H20 = 3 \times 1
     0.9862
     0.0123
     0.0008
Zv water = 1 \times 71
     0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

                                                                           0.9848
                                                                           0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
```

```
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
       0.8698
             0.8674
                    0.8651 0.8628 0.8605
                                           0.8582
             0.8515 0.8493 0.8471 0.8449 0.8428
0.8560
       0.8537
0.8406 0.8385 0.8364
cubic_coeffs_H20 = 1×4
  1.0000 -0.9993 0.0128 -0.0000
roots H20 = 3 \times 1
  0.9863
  0.0122
  0.0008
Zv water = 1 \times 71
         0.9822 0.9824 0.9825 0.9827 0.9829
  0.9821
      0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9830
0.9840
      0.9842 0.9843
                    0.9844 0.9846
                                   0.9847
                                          0.9848
             0.9852 0.9853 0.9855 0.9856 0.9857
0.9850
       0.9851
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8651 0.8628
0.8722
       0.8698
             0.8674
                                   0.8605
                                          0.8582
0.8560
       0.8537
             0.8515 0.8493 0.8471 0.8449 0.8428
0.8406
      0.8385
             0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0127 -0.0000
roots H20 = 3 \times 1
  0.9864
  0.0120
  0.0008
Zv water = 1 \times 71
          0.9822 0.9824 0.9825 0.9827 0.9829
  0.9821
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843
                    0.9844 0.9846 0.9847
                                          0.9848
      0.9851 0.9852 0.9853 0.9855 0.9856 0.9857
0.9850
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8746
0.8722
       0.8698
             0.8674
                    0.8651 0.8628 0.8605
                                          0.8582
       0.8560
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0126 -0.0000
```

```
roots H20 = 3 \times 1
   0.9865
   0.0119
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
      0.9842 0.9843
                       0.9844 0.9846 0.9847
                                               0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856 0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                       0.8651 0.8628 0.8605
0.8722
                                               0.8582

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449
      0.8428

0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0125 -0.0000
roots H20 = 3 \times 1
   0.9866
   0.0118
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
      0.9842 0.9843
                       0.9844 0.9846 0.9847
0.9840
                                               0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
      0.8698
               0.8674
                       0.8651 0.8628 0.8605
                                               0.8582
               0.8515 0.8493 0.8471 0.8449
0.8560
       0.8537
                                               0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0124 -0.0000
roots H20 = 3 \times 1
   0.9867
   0.0117
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843 0.9844 0.9846 0.9847
                                               0.9848
```

```
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856 0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                        0.8651 0.8628 0.8605
0.8722
                                                  0.8582

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449
      0.8428

0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0123 -0.0000
roots H20 = 3 \times 1
   0.9868
   0.0116
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
       0.9842 0.9843
                        0.9844 0.9846 0.9847
0.9840
                                                  0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856
                                                  0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
       0.8698 0.8674
                        0.8651 0.8628 0.8605
                                                  0.8582
               0.8515 0.8493 0.8471 0.8449
        0.8537
0.8560
                                                  0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0122 -0.0000
roots H20 = 3 \times 1
   0.9869
   0.0115
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

                                                  0.9848
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                        0.8651 0.8628 0.8605
0.8722
```

```
0.8560 0.8537 0.8515 0.8493 0.8471 0.8449 0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0121 -0.0000
roots H20 = 3 \times 1
   0.9870
   0.0114
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
        0.9842 0.9843
                           0.9844 0.9846 0.9847
                                                         0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856
                                                        0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                           0.8651 0.8628 0.8605
0.8722
                                                         0.8582
0.8560 0.8537 0.8515 0.8493 0.8471 0.8449
                                                        0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0120 -0.0000
roots H20 = 3 \times 1
   0.9871
   0.0113
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847
      0.9848

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856
      0.9857

0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8698
      0.8674
      0.8651
      0.8628
      0.8605

      0.8537
      0.8515
      0.8493
      0.8471
      0.8449

0.8722
                                                         0.8582
0.8560 0.8537
                                                        0.8428
0.8406 0.8385 0.8364
cubic coeffs H2O = 1×4
   1.0000 -0.9993 0.0119 -0.0000
roots H20 = 3 \times 1
   0.9873
   0.0112
   0.0008
Zv water = 1 \times 71
```

```
0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
      0.9842 0.9843
                      0.9844
                             0.9846 0.9847
                                             0.9848
0.9850
      0.9851
              0.9852
                      0.9853 0.9855 0.9856
                                             0.9857
      0.9859 0.9861
0.9858
Z1 water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698
              0.8674
                      0.8651
                             0.8628 0.8605
0.8722
                                             0.8582
                      0.8493 0.8471 0.8449
0.8560
       0.8537
              0.8515
                                             0.8428
0.8406
      0.8385
              0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0118 -0.0000
roots H20 = 3 \times 1
   0.9874
   0.0111
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837
                                             0.9839
      0.9842 0.9843
                      0.9844 0.9846 0.9847
0.9840
                                             0.9848
                      0.9853 0.9855 0.9856
0.9850
      0.9851
              0.9852
                                             0.9857
0.9858
       0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8746
0.8722
       0.8698 0.8674
                      0.8651 0.8628 0.8605
                                             0.8582
0.8560
       0.8537 0.8515
                      0.8493 0.8471
                                     0.8449
                                             0.8428
       0.8385
              0.8364
0.8406
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0117 -0.0000
roots H20 = 3 \times 1
   0.9875
   0.0110
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832
              0.9833 0.9834 0.9836 0.9837
                                             0.9839
      0.9842 0.9843
                      0.9844 0.9846 0.9847
0.9840
                                             0.9848
      0.9851
              0.9852
                      0.9853 0.9855 0.9856
0.9850
                                             0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
```

```
0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
                                            0.8582
      0.8698
             0.8674
                     0.8651
                            0.8628 0.8605
      0.8537
             0.8515 0.8493 0.8471 0.8449
                                            0.8428
0.8560
0.8406
       0.8385 0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0116 -0.0000
roots H20 = 3 \times 1
  0.9876
  0.0109
  0.0008
Zv water = 1 \times 71
  0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
                     0.9844 0.9846 0.9847
      0.9842 0.9843
0.9840
                                            0.9848
             0.9852
                     0.9853 0.9855 0.9856
0.9850
      0.9851
                                            0.9857
      0.9859 0.9861
0.9858
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
       0.8698 0.8674
                     0.8651 0.8628 0.8605
                                            0.8582
                     0.8493 0.8471
0.8560
       0.8537 0.8515
                                    0.8449
                                            0.8428
0.8406
      0.8385 0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0115 -0.0000
roots H20 = 3 \times 1
  0.9877
  0.0108
  0.0008
Zv water = 1 \times 71
  0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837
                                           0.9839
0.9840
      0.9842 0.9843
                     0.9844 0.9846 0.9847
                                            0.9848
0.9850 0.9851 0.9852
                     0.9853 0.9855 0.9856
                                            0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8746
                     0.8651 0.8628 0.8605
0.8722
       0.8698 0.8674
                                            0.8582
      0.8537 0.8515 0.8493 0.8471 0.8449
0.8560
                                            0.8428
0.8406
      0.8385 0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0114 -0.0000
roots H20 = 3 \times 1
  0.9878
```

```
0.0107
     0.0008
Zv water = 1 \times 71
     0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847
      0.9848

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856
      0.9857

0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
     0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8722
      0.8698
      0.8674
      0.8651
      0.8628
      0.8605
      0.8582

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449
      0.8428

0.8406
          0.8385 0.8364
cubic coeffs H20 = 1 \times 4
     1.0000 -0.9993 0.0113 -0.0000
roots H20 = 3 \times 1
     0.9879
     0.0106
     0.0008
Zv water = 1 \times 71
     0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

                                                                            0.9848
                                                                            0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
    0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8722
      0.8698
      0.8674
      0.8651
      0.8628
      0.8605

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449

                                                                            0.8582
                                                                            0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
     1.0000 -0.9993 0.0112 -0.0000
roots H20 = 3 \times 1
     0.9879
     0.0106
     0.0008
Zv water = 1 \times 71
     0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

                                                                            0.9848
                                                                            0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
```

```
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
       0.8698
             0.8674
                    0.8651 0.8628 0.8605
                                          0.8582
             0.8515 0.8493 0.8471 0.8449 0.8428
0.8560
       0.8537
0.8406 0.8385 0.8364
cubic_coeffs_H20 = 1×4
  1.0000 -0.9993 0.0111 -0.0000
roots H20 = 3 \times 1
  0.9880
  0.0105
  0.0008
Zv water = 1 \times 71
         0.9822 0.9824 0.9825 0.9827 0.9829
  0.9821
      0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9830
0.9840
       0.9842 0.9843
                    0.9844 0.9846
                                   0.9847
                                          0.9848
             0.9852 0.9853 0.9855 0.9856 0.9857
0.9850
       0.9851
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8651 0.8628
0.8722
       0.8698
             0.8674
                                   0.8605
                                          0.8582
0.8560
       0.8537
             0.8515 0.8493 0.8471 0.8449 0.8428
0.8406
      0.8385 0.8364
cubic coeffs H20 = 1 \times 4
                0.0110 -0.0000
  1.0000 -0.9993
roots H20 = 3 \times 1
  0.9881
  0.0104
  0.0008
Zv water = 1 \times 71
          0.9822 0.9824 0.9825 0.9827 0.9829
  0.9821
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
      0.9842 0.9843
                    0.9844 0.9846 0.9847
                                          0.9848
      0.9851 0.9852 0.9853 0.9855 0.9856 0.9857
0.9850
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8746
0.8722
       0.8698
             0.8674
                    0.8651 0.8628 0.8605
                                          0.8582
       0.8560
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0110 -0.0000
```

```
roots H20 = 3 \times 1
   0.9882
   0.0103
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
      0.9842 0.9843
                       0.9844 0.9846 0.9847
                                               0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856 0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                       0.8651 0.8628 0.8605
0.8722
                                               0.8582

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449
      0.8428

0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0109 -0.0000
roots H20 = 3 \times 1
   0.9883
   0.0102
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
      0.9842 0.9843
                       0.9844 0.9846 0.9847
0.9840
                                               0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
      0.8698 0.8674
                       0.8651 0.8628 0.8605
                                               0.8582
               0.8515 0.8493 0.8471 0.8449
0.8560
       0.8537
                                               0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0108 -0.0000
roots H20 = 3 \times 1
   0.9884
   0.0101
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843 0.9844 0.9846 0.9847
                                               0.9848
```

```
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856 0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698
              0.8674
                       0.8651 0.8628 0.8605
0.8722
                                               0.8582
       0.8560
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0107 -0.0000
roots H20 = 3 \times 1
   0.9885
   0.0100
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
      0.9842 0.9843
                       0.9844 0.9846 0.9847
0.9840
                                               0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856
                                              0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
      0.8698 0.8674
                       0.8651 0.8628 0.8605
                                               0.8582
              0.8515 0.8493 0.8471 0.8449
       0.8537
0.8560
                                              0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0106 -0.0000
roots H20 = 3 \times 1
   0.9886
   0.0099
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

                                               0.9848
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                      0.8651 0.8628 0.8605
0.8722
```

```
0.8560 0.8537 0.8515 0.8493 0.8471 0.8449 0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
    1.0000 -0.9993 0.0105 -0.0000
roots H20 = 3 \times 1
   0.9887
    0.0098
   0.0008
Zv water = 1 \times 71
    0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
        0.9842 0.9843
                            0.9844 0.9846 0.9847
                                                           0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856
                                                           0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                            0.8651 0.8628 0.8605
0.8722
                                                           0.8582

    0.8560
    0.8537
    0.8515
    0.8493
    0.8471
    0.8449

                                                           0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
    1.0000 -0.9993 0.0104 -0.0000
roots H20 = 3 \times 1
   0.9888
   0.0098
   0.0008
Zv water = 1 \times 71
    0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847
      0.9848

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856
      0.9857

0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
    0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8698
      0.8674
      0.8651
      0.8628
      0.8605
      0.8582

      0.8537
      0.8515
      0.8493
      0.8471
      0.8449
      0.8428

0.8722
0.8560 0.8537
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
    1.0000 -0.9993 0.0104 -0.0000
roots H20 = 3 \times 1
    0.9889
    0.0097
    0.0008
Zv water = 1 \times 71
```

```
0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
      0.9842 0.9843
                      0.9844
                             0.9846 0.9847
                                              0.9848
0.9850
      0.9851
              0.9852
                      0.9853 0.9855 0.9856
                                             0.9857
      0.9859 0.9861
0.9858
Z1 water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698
              0.8674
                      0.8651
                             0.8628 0.8605
0.8722
                                              0.8582
                      0.8493 0.8471 0.8449
0.8560
       0.8537
              0.8515
                                             0.8428
0.8406
      0.8385
              0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0103 -0.0000
roots H20 = 3 \times 1
   0.9889
   0.0096
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837
                                             0.9839
      0.9842 0.9843
                      0.9844 0.9846 0.9847
0.9840
                                              0.9848
                      0.9853 0.9855
0.9850
      0.9851
              0.9852
                                     0.9856
                                             0.9857
0.9858
       0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8746
0.8722
       0.8698 0.8674
                      0.8651 0.8628 0.8605
                                              0.8582
0.8560
       0.8537
              0.8515
                      0.8493 0.8471
                                     0.8449
                                              0.8428
       0.8385
              0.8364
0.8406
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0102 -0.0000
roots H20 = 3 \times 1
   0.9890
   0.0095
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832
              0.9833 0.9834 0.9836 0.9837
                                             0.9839
      0.9842 0.9843
                      0.9844 0.9846 0.9847
0.9840
                                              0.9848
      0.9851
              0.9852
                      0.9853 0.9855 0.9856
0.9850
                                             0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
```

```
0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
                                            0.8582
      0.8698
             0.8674
                     0.8651 0.8628 0.8605
      0.8537
             0.8515 0.8493 0.8471 0.8449
                                            0.8428
0.8560
0.8406
       0.8385 0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0101 -0.0000
roots H20 = 3 \times 1
  0.9891
  0.0094
  0.0008
Zv water = 1 \times 71
  0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
                     0.9844 0.9846 0.9847
      0.9842 0.9843
0.9840
                                            0.9848
             0.9852
                     0.9853 0.9855 0.9856
0.9850
      0.9851
                                            0.9857
      0.9859 0.9861
0.9858
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
       0.8698 0.8674
                     0.8651 0.8628 0.8605
                                            0.8582
                     0.8493 0.8471
0.8560
       0.8537 0.8515
                                    0.8449
                                            0.8428
0.8406
      0.8385 0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0100 -0.0000
roots H20 = 3 \times 1
  0.9892
  0.0094
  0.0008
Zv water = 1 \times 71
  0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837
                                           0.9839
0.9840
      0.9842 0.9843
                     0.9844 0.9846 0.9847
                                            0.9848
0.9850 0.9851
             0.9852
                     0.9853 0.9855 0.9856
                                            0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
  0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8746
                     0.8651 0.8628 0.8605
0.8722
       0.8698 0.8674
                                            0.8582
      0.8537 0.8515 0.8493 0.8471 0.8449
0.8560
                                            0.8428
0.8406
      0.8385 0.8364
cubic coeffs H20 = 1 \times 4
  1.0000 -0.9993 0.0100 -0.0000
roots H20 = 3 \times 1
  0.9893
```

```
0.0093
    0.0008
Zv water = 1 \times 71
    0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847
      0.9848

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856
      0.9857

0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
    0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722 0.8698 0.8674
                                  0.8651 0.8628 0.8605
                                                                      0.8582

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449
      0.8428

0.8406
          0.8385
                      0.8364
cubic coeffs H20 = 1 \times 4
    1.0000 -0.9993 0.0099 -0.0000
roots H20 = 3 \times 1
    0.9894
    0.0092
    0.0008
Zv water = 1 \times 71
    0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

                                                                      0.9848
                                                                      0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
    0.9048 0.9021 0.8995 0.8969 0.8944 0.8918

      0.8722
      0.8698
      0.8674
      0.8651
      0.8628
      0.8605

      0.8560
      0.8537
      0.8515
      0.8493
      0.8471
      0.8449

                                                                      0.8582
                                                                      0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
    1.0000 -0.9993 0.0098 -0.0000
roots H20 = 3 \times 1
    0.9894
    0.0091
    0.0008
Zv water = 1 \times 71
    0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839

      0.9840
      0.9842
      0.9843
      0.9844
      0.9846
      0.9847

      0.9850
      0.9851
      0.9852
      0.9853
      0.9855
      0.9856

                                                                      0.9848
                                                                      0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
```

```
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
       0.8698
              0.8674
                     0.8651 0.8628 0.8605
                                            0.8582
             0.8515 0.8493 0.8471 0.8449
0.8560
       0.8537
                                            0.8428
0.8406 0.8385 0.8364
cubic_coeffs_H20 = 1×4
   1.0000 -0.9993 0.0097 -0.0000
roots H20 = 3 \times 1
   0.9895
   0.0090
   0.0008
Zv water = 1 \times 71
          0.9822 0.9824 0.9825 0.9827 0.9829
   0.9821
      0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9830
0.9840
       0.9842 0.9843
                     0.9844 0.9846
                                    0.9847
                                            0.9848
             0.9852 0.9853 0.9855 0.9856 0.9857
0.9850
       0.9851
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8651 0.8628
0.8722
       0.8698
              0.8674
                                    0.8605
                                            0.8582
0.8560
       0.8537
             0.8515 0.8493 0.8471 0.8449 0.8428
0.8406
      0.8385
             0.8364
cubic coeffs H20 = 1 \times 4
                0.0096 -0.0000
   1.0000 -0.9993
roots H20 = 3 \times 1
   0.9896
   0.0090
   0.0008
Zv water = 1 \times 71
          0.9822 0.9824 0.9825 0.9827 0.9829
   0.9821
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
      0.9842 0.9843
                     0.9844 0.9846 0.9847
                                            0.9848
             0.9852 0.9853 0.9855 0.9856 0.9857
0.9850
      0.9851
      0.9859 0.9861
0.9858
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8746
0.8722
       0.8698
              0.8674
                     0.8651 0.8628 0.8605
                                            0.8582
       0.8537
             0.8515 0.8493 0.8471 0.8449 0.8428
0.8560
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0096 -0.0000
```

```
roots H20 = 3 \times 1
   0.9897
   0.0089
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840
      0.9842 0.9843
                      0.9844 0.9846 0.9847
                                             0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856 0.9857
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8698 0.8674
                      0.8651 0.8628 0.8605
0.8722
                                             0.8582
              0.8515 0.8493 0.8471 0.8449 0.8428
       0.8537
0.8560
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0095 -0.0000
roots H20 = 3 \times 1
   0.9898
   0.0088
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
      0.9842 0.9843
                      0.9844 0.9846 0.9847
0.9840
                                             0.9848
0.9850 0.9851 0.9852 0.9853 0.9855 0.9856
0.9858 0.9859 0.9861
Zl water = 1 \times 71
1.0e-03 *
   0.9048 0.9021 0.8995 0.8969 0.8944 0.8918
0.8722
      0.8698
              0.8674
                      0.8651 0.8628 0.8605
                                             0.8582
                    0.8493 0.8471 0.8449
0.8560
       0.8537
              0.8515
                                             0.8428
0.8406 0.8385 0.8364
cubic coeffs H20 = 1 \times 4
   1.0000 -0.9993 0.0094 -0.0000
roots H20 = 3 \times 1
   0.9898
   0.0087
   0.0008
Zv water = 1 \times 71
   0.9821 0.9822 0.9824 0.9825 0.9827 0.9829
0.9830 0.9832 0.9833 0.9834 0.9836 0.9837 0.9839
0.9840 0.9842 0.9843
                      0.9844 0.9846 0.9847
                                             0.9848
```

```
0.9850
          0.9851
                     0.9852
                                0.9853
                                           0.9855
                                                      0.9856
                                                                 0.9857
0.9858
          0.9859
                     0.9861
Zl water = 1 \times 71
1.0e-03 *
    0.9048
               0.9021
                          0.8995
                                     0.8969
                                                0.8944
                                                           0.8918
                     0.8843
0.8893
                                0.8818
          0.8868
                                           0.8794
                                                       0.8770
                                                                 0.8746
0.8722
          0.8698
                                0.8651
                                                      0.8605
                     0.8674
                                           0.8628
                                                                 0.8582
0.8560
          0.8537
                     0.8515
                                0.8493
                                           0.8471
                                                      0.8449
                                                                  0.8428
0.8406
          0.8385
                     0.8364
cubic coeffs H20 = 1 \times 4
             -0.9993
                                    -0.0000
    1.0000
                          0.0093
roots H20 = 3 \times 1
    0.9899
    0.0087
    0.0008
Zv water = 1 \times 71
                                                0.9827
    0.9821
               0.9822
                          0.9824
                                     0.9825
                                                           0.9829
          0.9832
                                0.9834
                                           0.9836
                                                                  0.9839
0.9830
                     0.9833
                                                      0.9837
0.9840
          0.9842
                     0.9843
                                0.9844
                                           0.9846
                                                      0.9847
                                                                  0.9848
0.9850
          0.9851
                     0.9852
                                0.9853
                                           0.9855
                                                      0.9856
                                                                 0.9857
0.9858
          0.9859
                     0.9861
Zl water = 1 \times 71
1.0e-03 *
    0.9048
               0.9021
                          0.8995
                                     0.8969
                                                0.8944
                                                           0.8918
0.8893
          0.8868
                     0.8843
                                0.8818
                                           0.8794
                                                       0.8770
                                                                  0.8746
0.8722
          0.8698
                     0.8674
                                0.8651
                                           0.8628
                                                      0.8605
                                                                  0.8582
0.8560
          0.8537
                      0.8515
                                0.8493
                                           0.8471
                                                      0.8449
                                                                 0.8428
0.8406
          0.8385
                     0.8364
 phi_v_H20 = exp((Zv_water-1)-(log(Zv_water-B_water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zv_water+(1+sqrt(2)).*B_water)./(Zv_wat
er+(1-sqrt(2))*B_water)))
phi v H20 = 1 \times 71
               0.9825
                          0.9827
                                     0.9828
                                                0.9830
                                                           0.9831
    0.9824
          0.9834
                     0.9836
                                0.9837
                                           0.9838
                                                                  0.9841
0.9833
                                                      0.9840
                                                                  0.9850
0.9843
          0.9844
                      0.9845
                                0.9847
                                           0.9848
                                                       0.9849
                                                       0.9858
0.9852
          0.9853
                     0.9854
                                0.9855
                                           0.9857
                                                                 0.9859
0.9860
          0.9861
                     0.9862
 phi_1_H20 = exp((Zl_water-1)-(log(Zl_water-B_water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zl_water+(1+sqrt(2)).*B_water)./(Zl_water)
er+(1-sqrt(2))*B_water)))
phi 1 H2O = 1 \times 71
```

```
0.0110
                                     0.0118
                                                0.0126
                                                           0.0135
    0.0096
               0.0103
0.0144
          0.0154
                     0.0164
                                0.0175
                                           0.0187
                                                      0.0200
                                                                 0.0213
0.0227
          0.0241
                     0.0257
                                0.0273
                                           0.0291
                                                      0.0309
                                                                 0.0328
0.0348
                     0.0392
                                0.0416
                                                                 0.0495
          0.0370
                                           0.0441
                                                      0.0467
0.0524
          0.0554
                     0.0586
 phi_1 = phi_v_H20./phi_1_H20
phi 1 = 1 \times 71
  102.7474
              95.8492
                         89.4641
                                    83.5504
                                               78.0702
                                                         72.9889
68,2747
          63.8988
                     59.8347
                                56.0579
                                           52.5465
                                                      49.2799
                                                                 46.2396
                                           33.8778
                                                      31.8799
43.4083
          40.7704
                     38.3113
                                36.0179
                                                                 30.0137
28.2698
          26.6393
                     25.1142
                                23.6870
                                           22.3508
                                                      21.0992
                                                                 19.9262
18.8266
          17.7951
                     16.8273
 % CO2 Cubic
 a_{carbon} = (0.45724.*(R.*Tc_{carbon}).^2)./Pc_{carbon}
a carbon = 0.3962
 w carbon = 0.225
w carbon = 0.2250
 alpha_carbon = (1+(0.37464+1.54226.*w_carbon-0.2699.*w_carbon.^2).*(1-
sqrt(Tr_carbon))).^2
alpha carbon = 1 \times 71
                                                1.0406
    1.0504
               1.0480
                          1.0455
                                     1.0431
                                                           1.0382
                      1.0309
                                1.0285
1.0357
          1.0333
                                           1.0261
                                                      1.0237
                                                                 1.0213
1.0189
          1.0165
                      1.0142
                                1.0118
                                           1.0094
                                                      1.0071
                                                                 1,0047
                                0.9954
                                           0.9931
1.0024
          1.0001
                     0.9977
                                                      0.9908
                                                                 0.9885
0.9862
          0.9839
                     0.9817
 a star carbon = a carbon.*alpha carbon
a star carbon = 1 \times 71
    0.4162
               0.4153
                          0.4143
                                     0.4133
                                                0.4123
                                                           0.4114
          0.4095
                     0.4085
                                0.4075
                                           0.4066
                                                      0.4056
0.4104
                                                                 0.4047
0.4037
          0.4028
                     0.4019
                                0.4009
                                           0.4000
                                                      0.3991
                                                                 0.3981
                                0.3944
                                                      0.3926
0.3972
          0.3963
                     0.3954
                                           0.3935
                                                                 0.3917
0.3908
          0.3899
                     0.3890
 A_carbon = (a_star_carbon.*P)./((R.*T_Kelvin).^2)
A carbon = 1 \times 71
                                                0.0073
    0.0076
               0.0075
                          0.0075
                                     0.0074
                                                           0.0073
```

0.0072

0.0071

0.0071

0.0070

0.0068

0.0069

0.0069

```
0.0067
          0.0067
                     0.0066
                               0.0066
                                          0.0065
                                                     0.0065
                                                               0.0064
0.0063
          0.0063
                     0.0062
                               0.0062
                                          0.0061
                                                     0.0061
                                                               0.0060
0.0060
          0.0059
                     0.0059
 b carbon = (0.0778.*R.*Tc carbon)./Pc carbon
b carbon = 2.6660e-05
 B carbon = (b_carbon.*P)./(R.*T_Kelvin)
B carbon = 1 \times 71
    0.0011
              0.0011
                         0.0011
                                    0.0011
                                              0.0011
                                                         0.0011
                     0.0011
                               0.0011
                                          0.0011
                                                     0.0011
0.0011
          0.0011
                                                               0.0011
0.0011
                     0.0011
                               0.0011
                                                               0.0011
          0.0011
                                          0.0011
                                                     0.0011
0.0011
          0.0011
                     0.0011
                               0.0011
                                          0.0011
                                                     0.0011
                                                               0.0011
0.0010
          0.0010
                     0.0010
 CO2 coeff 1 = ones(1,71)
C02\_coeff\_1 = 1 \times 71
     1
           1
                        1
                               1
                                     1
                                           1
                                                  1
                                                        1
                                                              1
                                                                     1
                   1
                         1
                               1
                                      1
                                            1
                                                   1
                                                         1
                                                               1
1
      1
            1
                                                                      1
1
      1
            1
                   1
                         1
                               1
                                      1
 CO2\_coeff\_2 = B\_carbon-1
CO2 coeff 2 = 1 \times 71
   -0.9989 -0.9989
                        -0.9989
                                   -0.9989
                                             -0.9989
                                                        -0.9989
0.9989
         -0.9989
                   -0.9989
                              -0.9989
                                         -0.9989
                                                    -0.9989
                                                              -0.9989
-0.9989 -0.9989 -0.9989
                               -0.9989
                                          -0.9989
                                                    -0.9989
                                                               -0.9989
-0.9989
          -0.9989
                     -0.9989
                               -0.9989
                                          -0.9989
                                                     -0.9989
                                                                -0.9989
-0.9990 -0.9990
                     -0.9990
 CO2 coeff 3 = A carbon-(3.*(B carbon).^2)-(2.*B carbon)
CO2 coeff 3 = 1 \times 71
    0.0053
              0.0052
                         0.0052
                                    0.0051
                                              0.0051
                                                         0.0050
                                                     0.0047
0.0049
          0.0049
                     0.0048
                               0.0048
                                          0.0047
                                                                0.0046
0.0046
          0.0045
                     0.0044
                               0.0044
                                          0.0043
                                                     0.0043
                                                               0.0042
0.0042
          0.0041
                     0.0041
                               0.0040
                                          0.0040
                                                     0.0039
                                                               0.0039
0.0039
          0.0038
                     0.0038
 CO2 coeff 4 = ((B carbon).^3)+(B carbon).^2-(A carbon.*B carbon)
CO2 coeff 4 = 1 \times 71
```

```
-0.7210 -0.7111 -0.7013 -0.6916
   -0.7415
              -0.7312
0.6822
         -0.6728 -0.6636
                               -0.6546
                                         -0.6457
                                                     -0.6369
                                                                -0.6283
-0.6198
         -0.6115
                     -0.6033
                                -0.5952
                                            -0.5872
                                                      -0.5794
                                                                 -0.5717
-0.5641
         -0.5566
                     -0.5492
                                 -0.5419
                                            -0.5348
                                                      -0.5278
                                                                  -0.5208
-0.5140
          -0.5073
                     -0.5007
 Zv_carbon = zeros(1, 71)
Zv carbon = 1 \times 71
     0
            0
                  0
                         0
                                0
                                      0
                                             0
                                                   0
                                                          0
                                                                 0
                                                                       0
0
      0
             0
                   0
                          0
                                 0
                                       0
                                              0
                                                    0
                                                           0
                                                                  0
                                                                        0
0
      0
             0
                   0
                          0
                                 0
                                       0
 for n = 1:71
     cubic_coeffs_CO2 = [CO2_coeff_1(n), CO2_coeff_2(n), CO2_coeff_3(n),
CO2_coeff_4(n)]
     roots_CO2 = roots(cubic_coeffs_CO2)
     Zv_carbon(n) = max(real(roots_CO2))
 end
cubic\_coeffs\_CO2 = 1 \times 4
    1.0000 -0.9989
                          0.0053
                                    -0.0000
roots CO2 = 3 \times 1
   0.9935 + 0.0000i
   0.0027 + 0.0006i
   0.0027 - 0.0006i
Zv carbon = 1 \times 71
    0.9935
                     0
                                0
                                           0
                                                      0
                                                                 0
0
          0
                      0
                                 0
                                            0
                                                       0
                                                                  0
0
           0
                      0
                                 0
                                            0
                                                       0
                                                                  0
0
           0
                      0
                                 0
                                            0
                                                       0
                                                                  0
          0
                      0
cubic coeffs CO2 = 1 \times 4
    1.0000 -0.9989
                          0.0052
                                    -0.0000
roots CO2 = 3 \times 1
   0.9936 + 0.0000i
   0.0026 + 0.0006i
   0.0026 - 0.0006i
Zv carbon = 1 \times 71
    0.9935
               0.9936
                                                      0
                                                                 0
                                0
                                           0
                                                       0
                                                                  0
0
          0
                      0
                                 0
                                            0
0
          0
                      0
                                 0
                                            0
                                                       0
                                                                  0
           0
                      0
                                 0
                                            0
                                                       0
                                                                  0
0
0
           0
```

1.0e-05 *

```
cubic coeffs CO2 = 1 \times 4
    1.0000
              -0.9989
                            0.0052
                                      -0.0000
roots CO2 = 3 \times 1
   0.9936 + 0.0000i
   0.0026 + 0.0007i
   0.0026 - 0.0007i
Zv carbon = 1 \times 71
    0.9935
                0.9936
                            0.9936
                                              0
                                                         0
                                                                     0
0
                       0
                                   0
                                               0
                                                           0
                                                                      0
           0
0
           0
                       0
                                   0
                                               0
                                                           0
                                                                       0
0
           0
                       0
                                   0
                                               0
                                                           0
                                                                       0
           0
0
                       0
cubic\_coeffs\_CO2 = 1 \times 4
              -0.9989
                            0.0051
                                       -0.0000
    1.0000
roots CO2 = 3 \times 1
   0.9937 + 0.0000i
   0.0026 + 0.0007i
   0.0026 - 0.0007i
Zv carbon = 1 \times 71
    0.9935
                0.9936
                            0.9936
                                        0.9937
                                                         0
                                                                     0
0
           0
                       0
                                   0
                                               0
                                                           0
                                                                      0
                                               0
0
           0
                       0
                                   0
                                                           0
                                                                       0
0
           0
                       0
                                   0
                                               0
                                                           0
                                                                       0
           0
                       0
cubic\_coeffs\_CO2 = 1 \times 4
    1.0000
               -0.9989
                            0.0051
                                       -0.0000
roots CO2 = 3 \times 1
   0.9938 + 0.0000i
   0.0025 + 0.0008i
   0.0025 - 0.0008i
Zv carbon = 1 \times 71
    0.9935
                0.9936
                            0.9936
                                        0.9937
                                                    0.9938
                                                                     0
0
                                                           0
           0
                       0
                                   0
                                               0
                                                                      0
0
           0
                       0
                                   0
                                               0
                                                           0
                                                                       0
                                                                       0
0
           0
                       0
                                   0
                                               0
                                                           0
           0
                       0
cubic coeffs CO2 = 1 \times 4
    1.0000
               -0.9989
                            0.0050
                                       -0.0000
roots CO2 = 3 \times 1
   0.9938 + 0.0000i
   0.0025 + 0.0008i
   0.0025 - 0.0008i
Zv carbon = 1 \times 71
    0.9935
                                        0.9937
                                                    0.9938
                                                               0.9938
                0.9936
                            0.9936
0
           0
                       0
                                   0
                                               0
                                                           0
                                                                       0
           0
                       0
0
                                   0
                                               0
                                                           0
                                                                       0
0
           0
                       0
                                   0
                                               0
                                                                       0
                                                           0
           0
                       0
cubic\_coeffs\_CO2 = 1 \times 4
```

```
1.0000 -0.9989
                        0.0049 -0.0000
roots CO2 = 3 \times 1
   0.9939 + 0.0000i
   0.0025 + 0.0008i
   0.0025 - 0.0008i
Zv carbon = 1 \times 71
    0.9935
               0.9936
                         0.9936
                                    0.9937
                                               0.9938
                                                         0.9938
0.9939
               0
                                                                     0
                          0
                                     0
                                                0
                                                           0
0
          0
                     0
                                0
                                           0
                                                     0
                                                                0
0
          0
                     0
                                0
                                          0
                                                     0
                                                                0
0
          0
                     0
cubic coeffs CO2 = 1 \times 4
    1.0000 -0.9989
                         0.0049 -0.0000
roots CO2 = 3 \times 1
   0.9940 + 0.0000i
   0.0025 + 0.0009i
   0.0025 - 0.0009i
Zv carbon = 1 \times 71
    0.9935
              0.9936
                         0.9936
                                    0.9937
                                               0.9938
                                                          0.9938
          0.9940
0.9939
                          0
                                     0
                                                0
                                                           0
                                                                     0
0
          0
                     0
                                0
                                           0
                                                     0
                                                                0
          0
                                           0
0
                     0
                                0
                                                     0
                                                                0
          0
                     0
cubic coeffs CO2 = 1 \times 4
    1.0000 -0.9989
                         0.0048
                                   -0.0000
roots CO2 = 3 \times 1
   0.9940 + 0.0000i
   0.0024 + 0.0009i
   0.0024 - 0.0009i
Zv carbon = 1 \times 71
                                    0.9937
                                               0.9938
    0.9935
              0.9936
                         0.9936
                                                          0.9938
0.9939
          0.9940
                     0.9940
                                     0
                                                0
                                                           0
                                                                     0
0
          0
                     0
                                0
                                           0
                                                     0
                                                                0
0
          0
                     0
                                0
                                          0
                                                     0
                                                                0
          0
                     0
cubic coeffs CO2 = 1 \times 4
    1.0000 -0.9989
                        0.0048 -0.0000
roots CO2 = 3 \times 1
   0.9941 + 0.0000i
   0.0024 + 0.0009i
   0.0024 - 0.0009i
Zv carbon = 1 \times 71
    0.9935
              0.9936
                         0.9936
                                    0.9937
                                               0.9938
                                                          0.9938
          0.9940
                     0.9940
                               0.9941
                                                                     0
0.9939
                                                0
                                                           0
0
          0
                     0
                                0
                                           0
                                                     0
                                                                0
0
          0
                                0
                                          0
                                                                0
                     0
                                                     0
          0
                     0
cubic coeffs CO2 = 1 \times 4
    1.0000 -0.9989
                         0.0047 -0.0000
```

```
roots CO2 = 3 \times 1
  0.9942 + 0.0000i
  0.0024 + 0.0009i
  0.0024 - 0.0009i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938
                                              0.9938
0.9939 0.9940 0.9940 0.9941 0.9942
                                               0
                                                         0
        0
                 0
                         0
                                  0
                                            0
                                                     0
                                           0
0
        0
                 0
                         0
                                  0
                                                     0
       0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0047 -0.0000
roots CO2 = 3 \times 1
  0.9942 + 0.0000i
  0.0023 + 0.0010i
  0.0023 - 0.0010i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                         0
                 0
        0
                         0
                                   0
                                            0
                                                     0
0
        0
                 0
                         0
                                  0
                                           0
                                                     0
        0
                 0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0046 -0.0000
roots CO2 = 3 \times 1
  0.9943 + 0.0000i
  0.0023 + 0.0010i
  0.0023 - 0.0010i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
        0
                 0
                         0
                                  0
                                           0
0
        0
                 0
                         0
                                   0
                                           0
                                                     0
       0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0046 -0.0000
roots CO2 = 3 \times 1
  0.9943 + 0.0000i
  0.0023 + 0.0010i
  0.0023 - 0.0010i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                     0.9943
0.9943
                             0
                                   0
            0
                    0
0
        0
                 0
                         0
                                   0
                                            0
                                                     0
        0
                 0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0045 -0.0000
roots CO2 = 3 \times 1
```

```
0.9944 + 0.0000i
  0.0023 + 0.0010i
  0.0023 - 0.0010i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
        0.9944
                     0
                             0
                                   0
                                               0
                                                          0
        0
                 0
                          0
                                   0
                                            0
0
0
        0
                 0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0044 -0.0000
roots CO2 = 3 \times 1
  0.9944 + 0.0000i
  0.0022 + 0.0010i
  0.0022 - 0.0010i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                     0.9943
               0.9944
       0.9944
0.9943
                             0
                                   0
                                               0
                                                          0
                 0
                         0
                                  0
        0
                                            0
                                                     0
        0
                 0
cubic_coeffs_CO2 = 1×4
   1.0000 -0.9989 0.0044 -0.0000
roots CO2 = 3 \times 1
  0.9945 + 0.0000i
  0.0022 + 0.0011i
  0.0022 - 0.0011i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                     0.9943
        0.9944
                 0.9944
                         0.9945
                                   0
0.9943
                                            0
                                                          0
0
        0
                 0
                          0
                                   0
                                            0
                                                     0
0
        0
                 0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0043 -0.0000
roots CO2 = 3 \times 1
  0.9946 + 0.0000i
  0.0022 + 0.0011i
  0.0022 - 0.0011i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942
                                            0.9942
                                                     0.9943
0.9943
        0.9944
                 0.9944
                         0.9945
                                   0.9946
                                                 0
                                                          0
        0
                 0
                          0
                                   0
                                            0
                                                     0
       0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0043 -0.0000
roots CO2 = 3 \times 1
  0.9946 + 0.0000i
```

```
0.0022 + 0.0011i
  0.0022 - 0.0011i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
        0.9944
                 0.9944
                         0.9945
                                  0.9946
                                          0.9946
        0
                 0
                          0
                                  0
                                           0
0
                                                    0
       0
                0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0042 -0.0000
roots CO2 = 3 \times 1
  0.9947 + 0.0000i
  0.0021 + 0.0011i
  0.0021 - 0.0011i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
        0.9944 0.9944
                         0.9945 0.9946 0.9946
                                                    0.9947
        0
                 0
                          0
                                  0
                                           0
0
        0
0
                0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0042 -0.0000
roots CO2 = 3 \times 1
  0.9947 + 0.0000i
  0.0021 + 0.0011i
  0.0021 - 0.0011i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
        0.9944
                 0.9944 0.9945
                                           0.9946
0.9943
                                  0.9946
                                                    0.9947
0.9947
        0
                     0
                                  0
                              0
                                                0
                                                         0
       0
cubic\_coeffs\_CO2 = 1\times4
   1.0000 -0.9989 0.0041 -0.0000
roots CO2 = 3 \times 1
  0.9948 + 0.0000i
  0.0021 + 0.0011i
  0.0021 - 0.0011i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943
        0.9944 0.9944 0.9945
                                  0.9946
                                           0.9946
                                                    0.9947
0.9947
        0.9948
                     0
                              0
                                      0
                                               0
                                                        0
       0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0041 -0.0000
roots CO2 = 3 \times 1
  0.9948 + 0.0000i
  0.0021 + 0.0011i
```

```
0.0021 - 0.0011i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
        0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                     0.9947
0.9947
        0.9948 0.9948
                            0
                                       0
                                                0
        0
0
                 0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0040 -0.0000
roots CO2 = 3 \times 1
  0.9949 + 0.0000i
  0.0020 + 0.0012i
  0.0020 - 0.0012i
Zv carbon = 1 \times 71
            0.9936 0.9936 0.9937 0.9938 0.9938
   0.9935
       0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9939
0.9943
        0.9944
                 0.9944
                         0.9945
                                   0.9946
                                            0.9946
                                                     0.9947
0.9947 0.9948
                0.9948 0.9949
                                       0
                                                0
                                                          0
        0
                 0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0040 -0.0000
roots CO2 = 3 \times 1
  0.9949 + 0.0000i
  0.0020 + 0.0012i
  0.0020 - 0.0012i
Zv carbon = 1 \times 71

    0.9935
    0.9936
    0.9936
    0.9937
    0.9938
    0.9938

0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943
        0.9944 0.9944
                         0.9945 0.9946 0.9946
                                                     0.9947
        0.9948 0.9948 0.9949 0.9949
0.9947
                                                0
                                                          0
0
        0
                 0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0039 -0.0000
roots CO2 = 3 \times 1
  0.9950 + 0.0000i
  0.0020 + 0.0012i
  0.0020 - 0.0012i
Zv carbon = 1 \times 71
            0.9936 0.9936 0.9937 0.9938 0.9938
   0.9935
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                     0.9943
0.9943
        0.9944
                 0.9944
                         0.9945 0.9946
                                           0.9946
                                                     0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                          0
                 0
        0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9989 0.0039 -0.0000
roots CO2 = 3 \times 1
  0.9950 + 0.0000i
  0.0020 + 0.0012i
  0.0020 - 0.0012i
```

```
Zv carbon = 1 \times 71
            0.9936 0.9936 0.9937 0.9938 0.9938
   0.9935
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943
        0.9944 0.9944
                         0.9945 0.9946 0.9946
                                                   0.9947
                 0.9948 0.9949 0.9950
0.9947
        0.9948
                                                   0.9950
        0
                 0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0039 -0.0000
roots CO2 = 3 \times 1
  0.9951 + 0.0000i
  0.0019 + 0.0012i
  0.0019 - 0.0012i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
       0.9944 0.9944
                         0.9945 0.9946 0.9946
0.9943
                                                   0.9947
                       0.9949 0.9949 0.9950
        0.9948 0.9948
0.9947
                                                   0.9950
0.9951
       0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0038 -0.0000
roots CO2 = 3 \times 1
  0.9951 + 0.0000i
  0.0019 + 0.0012i
  0.0019 - 0.0012i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
       0.9944
                0.9944
                         0.9945
                                 0.9946 0.9946
0.9943
                                                   0.9947
                         0.9949 0.9949 0.9950
0.9947
        0.9948
                 0.9948
                                                   0.9950
0.9951 0.9951
                    0
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0038 -0.0000
roots CO2 = 3 \times 1
  0.9952 + 0.0000i
  0.0019 + 0.0012i
  0.0019 - 0.0012i
Zv carbon = 1 \times 71
           0.9936 0.9936 0.9937 0.9938 0.9938
   0.9935
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
                        0.9945 0.9946 0.9946
0.9949 0.9949 0.9950
       0.9944
                0.9944
0.9943
                                                   0.9947
0.9947
        0.9948
                0.9948
                                                    0.9950
0.9951 0.9951 0.9952
cubic_coeffs_CO2 = 1×4
   1.0000 -0.9990 0.0037 -0.0000
roots CO2 = 3 \times 1
  0.9952 + 0.0000i
  0.0019 + 0.0012i
  0.0019 - 0.0012i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0037 -0.0000
roots CO2 = 3 \times 1
  0.9953 + 0.0000i
  0.0018 + 0.0012i
  0.0018 - 0.0012i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0036 -0.0000
roots CO2 = 3 \times 1
  0.9953 + 0.0000i
  0.0018 + 0.0012i
  0.0018 - 0.0012i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0036 -0.0000
roots CO2 = 3 \times 1
  0.9954 + 0.0000i
  0.0018 + 0.0012i
  0.0018 - 0.0012i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0036 -0.0000
roots CO2 = 3 \times 1
  0.9954 + 0.0000i
  0.0018 + 0.0012i
  0.0018 - 0.0012i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
0.9943
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0035 -0.0000
roots CO2 = 3 \times 1
  0.9955 + 0.0000i
  0.0018 + 0.0012i
  0.0018 - 0.0012i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0035 -0.0000
roots CO2 = 3 \times 1
  0.9955 + 0.0000i
  0.0017 + 0.0013i
  0.0017 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0034 -0.0000
roots CO2 = 3 \times 1
  0.9955 + 0.0000i
  0.0017 + 0.0013i
  0.0017 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0034 -0.0000
roots CO2 = 3 \times 1
  0.9956 + 0.0000i
  0.0017 + 0.0013i
  0.0017 - 0.0013i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
0.9943
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0033 -0.0000
roots CO2 = 3 \times 1
  0.9956 + 0.0000i
  0.0017 + 0.0013i
  0.0017 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0033 -0.0000
roots CO2 = 3 \times 1
  0.9957 + 0.0000i
  0.0017 + 0.0013i
  0.0017 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0033 -0.0000
roots CO2 = 3 \times 1
  0.9957 + 0.0000i
  0.0016 + 0.0013i
  0.0016 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0032 -0.0000
roots CO2 = 3 \times 1
  0.9958 + 0.0000i
  0.0016 + 0.0013i
  0.0016 - 0.0013i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
0.9943
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0032 -0.0000
roots CO2 = 3 \times 1
  0.9958 + 0.0000i
  0.0016 + 0.0013i
  0.0016 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0032 -0.0000
roots CO2 = 3 \times 1
  0.9958 + 0.0000i
  0.0016 + 0.0013i
  0.0016 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0031 -0.0000
roots CO2 = 3 \times 1
  0.9959 + 0.0000i
  0.0016 + 0.0013i
  0.0016 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0031 -0.0000
roots CO2 = 3 \times 1
  0.9959 + 0.0000i
  0.0015 + 0.0013i
  0.0015 - 0.0013i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0030 -0.0000
roots CO2 = 3 \times 1
  0.9960 + 0.0000i
  0.0015 + 0.0013i
  0.0015 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0030 -0.0000
roots CO2 = 3 \times 1
  0.9960 + 0.0000i
  0.0015 + 0.0013i
  0.0015 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0030 -0.0000
roots CO2 = 3 \times 1
  0.9960 + 0.0000i
  0.0015 + 0.0013i
  0.0015 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0029 -0.0000
roots CO2 = 3 \times 1
  0.9961 + 0.0000i
  0.0015 + 0.0013i
  0.0015 - 0.0013i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
0.9943
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0029 -0.0000
roots CO2 = 3 \times 1
  0.9961 + 0.0000i
  0.0015 + 0.0013i
  0.0015 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0029 -0.0000
roots CO2 = 3 \times 1
  0.9962 + 0.0000i
  0.0014 + 0.0013i
  0.0014 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0028 -0.0000
roots CO2 = 3 \times 1
  0.9962 + 0.0000i
  0.0014 + 0.0013i
  0.0014 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0028 -0.0000
roots CO2 = 3 \times 1
  0.9962 + 0.0000i
  0.0014 + 0.0013i
  0.0014 - 0.0013i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0028 -0.0000
roots CO2 = 3 \times 1
  0.9963 + 0.0000i
  0.0014 + 0.0013i
  0.0014 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0027 -0.0000
roots CO2 = 3 \times 1
  0.9963 + 0.0000i
  0.0014 + 0.0013i
  0.0014 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0027 -0.0000
roots CO2 = 3 \times 1
  0.9963 + 0.0000i
  0.0014 + 0.0013i
  0.0014 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9990 0.0027 -0.0000
roots CO2 = 3 \times 1
  0.9964 + 0.0000i
  0.0013 + 0.0013i
  0.0013 - 0.0013i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0026 -0.0000
roots CO2 = 3 \times 1
  0.9964 + 0.0000i
  0.0013 + 0.0013i
  0.0013 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0026 -0.0000
roots CO2 = 3 \times 1
  0.9964 + 0.0000i
  0.0013 + 0.0013i
  0.0013 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0026 -0.0000
roots CO2 = 3 \times 1
  0.9965 + 0.0000i
  0.0013 + 0.0013i
  0.0013 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0026 -0.0000
roots CO2 = 3 \times 1
  0.9965 + 0.0000i
  0.0013 + 0.0013i
  0.0013 - 0.0013i
Zv carbon = 1 \times 71
```

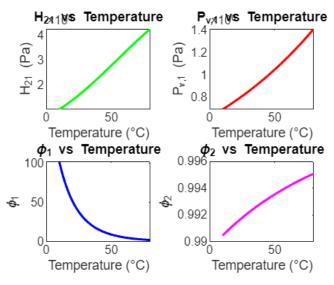
```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0025 -0.0000
roots CO2 = 3 \times 1
  0.9965 + 0.0000i
  0.0013 + 0.0013i
  0.0013 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0025 -0.0000
roots CO2 = 3 \times 1
  0.9966 + 0.0000i
  0.0012 + 0.0013i
  0.0012 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0025 -0.0000
roots CO2 = 3 \times 1
  0.9966 + 0.0000i
  0.0012 + 0.0013i
  0.0012 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0024 -0.0000
roots CO2 = 3 \times 1
  0.9966 + 0.0000i
  0.0012 + 0.0013i
  0.0012 - 0.0013i
Zv carbon = 1 \times 71
```

```
0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942 0.9943
       0.9944 0.9944 0.9945 0.9946 0.9946
0.9943
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9949 0.9950
                                                  0.9950
               0.9952
0.9951 0.9951
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0024 -0.0000
roots CO2 = 3 \times 1
  0.9967 + 0.0000i
  0.0012 + 0.0013i
  0.0012 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0024 -0.0000
roots CO2 = 3 \times 1
  0.9967 + 0.0000i
  0.0012 + 0.0013i
  0.0012 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                  0.9943
0.9943 0.9944 0.9944 0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0023 -0.0000
roots CO2 = 3 \times 1
  0.9967 + 0.0000i
  0.0012 + 0.0013i
  0.0012 - 0.0013i
Zv carbon = 1 \times 71
   0.9935 0.9936 0.9936 0.9937 0.9938 0.9938
0.9939 0.9940 0.9940 0.9941 0.9942 0.9942
                                                 0.9943
0.9943 0.9944 0.9944
                        0.9945 0.9946 0.9946
                                                  0.9947
0.9947 0.9948 0.9948
                        0.9949 0.9949 0.9950
                                                  0.9950
0.9951 0.9951
               0.9952
cubic coeffs CO2 = 1 \times 4
   1.0000 -0.9991 0.0023 -0.0000
roots CO2 = 3 \times 1
  0.9968 + 0.0000i
  0.0012 + 0.0013i
  0.0012 - 0.0013i
Zv carbon = 1 \times 71
```

```
0.9935
               0.9936
                          0.9936
                                     0.9937
                                               0.9938
                                                          0.9938
0.9939
          0.9940
                     0.9940
                                0.9941
                                           0.9942
                                                      0.9942
                                                                 0.9943
0.9943
          0.9944
                     0.9944
                                0.9945
                                           0.9946
                                                      0.9946
                                                                 0.9947
0.9947
          0.9948
                     0.9948
                                0.9949
                                           0.9949
                                                      0.9950
                                                                 0.9950
0.9951
          0.9951
                     0.9952
 phi_v_CO2 = exp((Zv_carbon-1)-(log(Zv_carbon-B_carbon))-
(A carbon./(2*sqrt(2).*B water)).*log((Zv carbon+(1+sqrt(2)).*B carbon)./(Zv
carbon+(1-sqrt(2))*B_carbon)))
phi v CO2 = 1 \times 71
    0.9905
               0.9906
                          0.9907
                                     0.9908
                                               0.9908
                                                          0.9909
0.9910
          0.9911
                     0.9912
                                0.9913
                                           0.9914
                                                      0.9915
                                                                 0.9915
0.9916
          0.9917
                     0.9918
                                0.9919
                                           0.9919
                                                      0.9920
                                                                 0.9921
0.9922
          0.9923
                     0.9923
                                0.9924
                                           0.9925
                                                      0.9925
                                                                 0.9926
0.9927
          0.9928
                     0.9928
 phi_2 = phi_v_CO2
phi 2 = 1 \times 71
    0.9905
               0.9906
                          0.9907
                                     0.9908
                                               0.9908
                                                          0.9909
                     0.9912
0.9910
          0.9911
                                0.9913
                                           0.9914
                                                      0.9915
                                                                 0.9915
0.9916
          0.9917
                     0.9918
                                0.9919
                                           0.9919
                                                      0.9920
                                                                 0.9921
0.9922
          0.9923
                     0.9923
                                0.9924
                                           0.9925
                                                      0.9925
                                                                 0.9926
0.9927
          0.9928
                     0.9928
 % Plotting 4 graphs
 figure;
 % H21 Subplot
 subplot(2, 2, 1);
 plot(T_Celsius, H21, 'g', 'LineWidth', 1.5);
 title('H_2_1 vs Temperature');
 xlabel('Temperature (°C)');
 ylabel('H 2 1 (Pa)');
 % Pv1 Subplot
 subplot(2, 2, 2);
 plot(T_Celsius, Pv1, 'r', 'LineWidth', 1.5);
 title('P_v_,_1 vs Temperature');
 xlabel('Temperature (°C)');
 ylabel('P_v_,_1 (Pa)');
 % phi1 Subplot
 subplot(2, 2, 3);
 plot(T_Celsius, phi_1, 'b', 'LineWidth', 1.5);
```

```
title('\phi_1 vs Temperature');
xlabel('Temperature (°C)');
ylabel('\phi_1');

% phi2 Subplot
subplot(2, 2, 4);
plot(T_Celsius, phi_2, 'm', 'LineWidth', 1.5);
title('\phi_2 vs Temperature');
xlabel('Temperature (°C)');
ylabel('\phi_2');
```



```
%%%%% TASK 2B %%%%%
T_Celsius_split = 10:10:80
```

```
T_Celsius_split = 1×8
10 20 30 40 50 60 70 80
```

```
T_Kelvin_split = T_Celsius_split + 273.15
P1 = 50000
P2 = 101325
P3 = 200000
Tr_water_new = T_Kelvin_split/Tc_water
x_new = 1-Tr_water
Tr_carbon_new = T_Kelvin_split/Tc_carbon
% Coefficients for Eq. (3)
Pv1 new =
Pc_{water.*(exp((alpha1.*x)+(alpha2.*x.^(1.5))+(alpha3.*x.^(3))+(alpha4.*x.^(3))
.5))+(alpha5.*x.^(4))+(alpha6.*x.^(7.5)))./Tr_water_new)
% Coefficients for Eq. (4)
H21 =
10.^6.*exp(h1+(h2./T_Kelvin_split)+(h3./(T_Kelvin_split.^2))+(h4./(T_Kelvin_s
plit.^3)))
% H2O Cubic
```

```
a_water = (0.45724.*(R.*Tc_water).^2)./Pc_water
w water = 0.348
alpha water = (1+(0.37464+1.54226.*w water-0.2699.*w water.^2).*(1-
sqrt(Tr_water_new))).^2
a_star_water = a_water.*alpha_water
A water = (a star water.*P1)./((R.*T Kelvin split).^2)
b_water = (0.0778.*R.*Tc_water)./Pc_water
B water = (b water.*P1)./(R.*T Kelvin split)
H20 coeff 1 = ones(1,71)
H20_coeff_2 = B_water-1
H20_coeff_3 = A_water-(3.*(B_water).^2)-(2.*B_water)
H20_coeff_4 = ((B_water).^3) + (B_water).^2 - (A_water.*B_water)
Zv water = zeros(1, 71)
Zl_water = zeros(1, 71)
for n = 1:71
     cubic_coeffs_H20 = [H20_coeff_1(n), H20_coeff_2(n), H20_coeff_3(n),
H20_coeff_4(n)]
     roots_H20 = roots(cubic_coeffs_H20)
     Zv_{water}(n) = max(roots_{H20})
     Zl_water(n) = min(roots_H20)
end
phi v H20 = exp((Zv water-1)-(log(Zv water-B water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zv_water+(1+sqrt(2)).*B_water)./(Zv_wat
er+(1-sqrt(2))*B_water)))
phi_1_H20 = exp((Zl_water-1)-(log(Zl_water-B_water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zl_water+(1+sqrt(2)).*B_water)./(Zl_wat
er+(1-sqrt(2))*B_water)))
phi_1 = phi_v_H20./phi_l_H20
% CO2 Cubic
a_carbon = (0.45724.*(R.*Tc_carbon).^2)./Pc_carbon
w carbon = 0.225
alpha_carbon = (1+(0.37464+1.54226.*w_carbon-0.2699.*w_carbon.^2).*(1-
sqrt(Tr_carbon))).^2
a_star_carbon = a_carbon.*alpha_carbon
A_carbon = (a_star_carbon.*P)./((R.*T_Kelvin).^2)
b_carbon = (0.0778.*R.*Tc_carbon)./Pc_carbon
B_carbon = (b_carbon.*P)./(R.*T_Kelvin)
CO2 coeff 1 = ones(1,71)
CO2_coeff_2 = B_carbon-1
CO2\_coeff_3 = A\_carbon-(3.*(B\_carbon).^2)-(2.*B\_carbon)
CO2 coeff 4 = ((B \text{ carbon}).^3) + (B \text{ carbon}).^2 - (A \text{ carbon}.*B \text{ carbon})
```

```
Zv_carbon = zeros(1, 71)
    for n = 1:71
                 cubic_coeffs_CO2 = [CO2_coeff_1(n), CO2_coeff_2(n), CO2_coeff_3(n),
  CO2 coeff 4(n)]
                 roots_CO2 = roots(cubic_coeffs_CO2)
                 Zv_carbon(n) = max(real(roots_CO2))
    end
    phi_v_CO2 = exp((Zv_carbon-1)-(log(Zv_carbon-B_carbon))-
  (A_{carbon./(2*sqrt(2).*B_water)).*log((Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*B_carbon)./(Zv_{carbon+(1+sqrt(2)).*
  carbon+(1-sqrt(2))*B_carbon)))
    phi_2 = phi_v_CO2
    % Defining known variables
    T_Celsius = 10:10:80 % Temperature in Celsius
T Celsius = 1 \times 8
            10
                                                                       40
                                                                                           50
                                                                                                               60
                                                                                                                                  70
                                                                                                                                                      80
                             20
                                                   30
    T_Kelvin = T_Celsius + 273.15 % Convert to Kelvin
T Kelvin = 1 \times 8
      283.1500 293.1500 303.1500 313.1500 323.1500 333.1500 343.1500 ···
    Tc_water = 647.096 % Critical temperature of water (K)
Tc\_water = 647.0960
    Pc_water = 22064000 % Critical pressure of water (Pa)
Pc_water = 22064000
    R = 8.314 \% \text{ Units of } J/(\text{mol } K)
R = 8.3140
    Tc_carbon = 304.18 % Kelvin
Tc\_carbon = 304.1800
    Pc_carbon = 7380000 % Pa
Pc carbon = 7380000
    P1 = 50000
P1 = 50000
    P2 = 101325
P2 = 101325
    P3 = 200000
P3 = 200000
```

```
%%%%% TASK 2B %%%%%%
     Tr_water = T_Kelvin/Tc_water
Tr_water = 1 \times 8
              0.4376
                                                  0.4530
                                                                                       0.4685
                                                                                                                             0.4839
                                                                                                                                                                 0.4994
                                                                                                                                                                                                      0.5148
                                                                                                                                                                                                                                           0.5303 ...
     x = 1-Tr_water
x = 1 \times 8
              0.5624
                                         0.5470 0.5315
                                                                                                                           0.5161
                                                                                                                                                                 0.5006
                                                                                                                                                                                                     0.4852
                                                                                                                                                                                                                                      0.4697 ...
     Tr_carbon = T_Kelvin/Tc_carbon
Tr\_carbon = 1 \times 8
              0.9309
                                           0.9637
                                                                                  0.9966
                                                                                                                            1.0295
                                                                                                                                                                 1.0624
                                                                                                                                                                                                      1.0952
                                                                                                                                                                                                                                           1.1281 ...
     % Coefficients for Eq. (3)
     alpha1 = -7.85951783
alpha1 = -7.8595
     alpha2 = 1.84408259
alpha2 = 1.8441
     alpha3 = -11.7866497
alpha3 = -11.7866
     alpha4 = 22.6807411
alpha4 = 22.6807
     alpha5 = -15.9618719
alpha5 = -15.9619
     alpha6 = 1.80122502
alpha6 = 1.8012
     Pv1 =
 Pc_{water.*(exp((alpha1.*x)+(alpha2.*x.^(1.5))+(alpha3.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.
  .5))+(alpha5.*x.^(4))+(alpha6.*x.^(7.5)))./Tr_water)
Pv1 = 1 \times 8
10^6 \times
              0.6935 0.7708 0.8557 0.9486
                                                                                                                                                                1.0502
                                                                                                                                                                                                     1.1613 1.2826 ...
     % Coefficients for Eq. (4)
     h1 = -6.8346
```

h1 = -6.8346

```
h2 = 1.2817*(10^4)
h2 = 12817
 h3 = -3.7668*(10^6)
h3 = -3766800
 h4 = 2.9970*(10^8)
h4 = 299700000
 H21 = 10.^6.*exp(h1+(h2./T_Kelvin)+(h3./(T_Kelvin.^2))+(h4./(T_Kelvin.^3)))
H21 = 1 \times 8
10^{8} \times
    1.0461
                1.4136
                           1.8388
                                       2.3094
                                                  2.8086
                                                                         3.8155 ...
                                                              3.3172
 % H2O Cubic
 a_{\text{water}} = (0.45724.*(R.*Tc_{\text{water}}).^2)./Pc_{\text{water}}
a water = 0.5998
 w water = 0.348
w water = 0.3480
 alpha_water = (1+(0.37464+1.54226.*w_water-0.2699.*w_water.^2).*(1-
sqrt(Tr_water))).^2
alpha_water = 1 \times 8
    1.6833
             1.6570
                           1.6314
                                     1.6064
                                                  1.5819
                                                              1.5580
                                                                         1.5346 ...
 a_star_water = a_water.*alpha_water
a_star_water = 1 \times 8
    1.0097 0.9939
                           0.9785
                                     0.9635
                                                                          0.9205 ...
                                                  0.9488
                                                              0.9345
 A_water = (a_star_water.*P1)./((R.*T_Kelvin).^2)
A water = 1 \times 8
               0.0084
    0.0091
                           0.0077
                                       0.0071
                                                  0.0066
                                                              0.0061
                                                                         0.0057 ...
 b_water = (0.0778.*R.*Tc_water)./Pc_water
b_{water} = 1.8970e-05
 B_water = (b_water.*P1)./(R.*T_Kelvin)
B water = 1 \times 8
10^{-3} \times
    0.4029
                0.3892
                           0.3763
                                       0.3643
                                                  0.3530
                                                              0.3424
                                                                          0.3325 ...
 H20_coeff_1 = ones(1,8)
```

```
H20_coeff_1 = 1 \times 8
                 1 1 1 1 1 1
 H20_coeff_2 = B_water-1
H20_coeff_2 = 1 \times 8
  -0.9996 -0.9996 -0.9996 -0.9996 -0.9997 -0.9997 ...
 H20_coeff_3 = A_water-(3.*(B_water).^2)-(2.*B_water)
H20_coeff_3 = 1 \times 8
    0.0083 0.0076 0.0069 0.0064 0.0059 0.0054
                                                                   0.0050 ...
 H20_coeff_4 = ((B_water).^3) + (B_water).^2 - (A_water.*B_water)
H20_coeff_4 = 1 \times 8
10<sup>-5</sup> ×
   -0.3508 -0.3104 -0.2757 -0.2457 -0.2196 -0.1968 -0.1769 ...
 Zv_{water} = zeros(1, 8)
Zv_water = 1 \times 8
                        0
                               0
                                     0
                                            \cap
                                                  0
    Ω
 Zl_water = zeros(1, 8)
Zl_water = 1 \times 8
    0
                  0
                        0
                               0
                                     0
                                            0
                                                  0
 for n = 1:8
     cubic_coeffs_H20 = [H20_coeff_1(n), H20_coeff_2(n), H20_coeff_3(n),
H20_coeff_4(n)]
     roots_H20 = roots(cubic_coeffs_H20)
     Zv_{water(n)} = max(roots_{H20})
     Zl_water(n) = min(roots_H20)
 end
cubic\_coeffs\_H20 = 1 \times 4
    1.0000 -0.9996 0.0083 -0.0000
roots H20 = 3 \times 1
    0.9912
    0.0079
    0.0004
Zv water = 1 \times 8
    0.9912
                   0
                             0
                                                   0
                                                              0
                                                                        0 ...
Zl water = 1 \times 8
10^{-3} \times
    0.4465
                               0
                                                    0
                                                              0
                                                                          0 ...
cubic\_coeffs\_H20 = 1 \times 4
    1.0000 -0.9996 0.0076 -0.0000
roots H20 = 3 \times 1
    0.9920
    0.0072
    0.0004
Zv_water = 1 \times 8
    0.9912 0.9920
                         0
                                        0
                                                   0
                                                              0
                                                                         0 ...
```

Zl_water = 1×8						
10 ⁻³ × 0.4465	0 4340	0	0	0	0	0
cubic_coeffs_H20		Ŭ	O O	Ŭ.	Ŭ	Ŭ
	-0.9996	0.0069	-0.0000			
roots_H20 = 3×1 0.9926 0.0066						
0.0004						
Zv_water = <i>1×8</i> 0.9912	0.9920	0.9926	0	0	0	0
$Z1_water = 1 \times 8$ $10^{-3} \times$						
0.4465 cubic_coeffs_H20	0.4340 0 = 1×4	0.4224	0	0	0	0
1.0000 roots_H20 = 3×1		0.0064	-0.0000			
0.9932 0.0060 0.0004						
Zv_water = <i>1×8</i> 0.9912	0.9920	0.9926	0.9932	0	0	0
$Zl_water = 1 \times 8$ $10^{-3} \times$						
0.4465		0.4224	0.4117	0	0	0
<pre>cubic_coeffs_H20 1.0000</pre>		0.0059	-0.0000			
roots_H2O = 3×1 0.9937 0.0055 0.0004						
$Zv_water = 1 \times 8$	0.9920	0 0026	0 0022	0 0027	0	0
0.9912 Zl_water = 1×8 10 ⁻³ ×	0.9920	0.9926	0.9932	0.9937	O	0
	0.4340	0.4224	0.4117	0.4018	0	0
	-0.9997	0.0054	-0.0000			
roots_H2O = 3×1 0.9942 0.0050 0.0004						
Zv_water = 1×8 0.9912	0.9920	0.9926	0.9932	0.9937	0.9942	0
$Z1_{water} = 1 \times 8$ $10^{-3} \times$	0.00	0.73520	0.3302	0.000	0.3312	Ç
0.4465 cubic_coeffs_H20		0.4224	0.4117	0.4018	0.3927	0
1.0000		0.0050	-0.0000			
roots_H20 = 3×1 0.9947 0.0046 0.0004 Zv_water = 1×8						
0.9912 Zl_water = <i>1×8</i>	0.9920	0.9926	0.9932	0.9937	0.9942	0.9947
10 ⁻³ × 0.4465	0.4340	0.4224	0.4117	0.4018	0.3927	0.3842
cubic_coeffs_H20) = 1×4 -0.9997	0.0046	-0 0000			
roots_H20 = 3×1 0.9950	J. J. J. J. I	0.0010	0.0000			

```
0.0043
    0.0004
Zv_water = 1 \times 8
    0.9912
              0.9920
                        0.9926
                                  0.9932
                                               0.9937
                                                         0.9942
                                                                    0.9947 ...
Zl_water = 1 \times 8
10<sup>-3</sup> ×
             0.4340
                         0.4224
                                    0.4117
                                               0.4018
                                                         0.3927
    0.4465
                                                                    0.3842 ...
 phi v H20 = exp((Zv water-1)-(log(Zv water-B water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zv_water+(1+sqrt(2)).*B_water)./(Zv_wat
er+(1-sqrt(2))*B_water)))
phi_v_H20 = 1 \times 8
    0.9913
              0.9920
                         0.9927
                                    0.9933
                                               0.9938
                                                         0.9943
                                                                    0.9947 ...
 phi_1_H20 = exp((Zl_water-1)-(log(Zl_water-B_water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zl_water+(1+sqrt(2)).*B_water)./(Zl_wat
er+(1-sqrt(2))*B_water)))
phi 1 H2O = 1 \times 8
                            0.0706
                                        0.1255
                                                   0.2141
    0.0194
                0.0379
                                                               0.3519
0.5590
           0.8609
 phi_1 = phi_v_H20./phi_l_H20
phi 1 = 1 \times 8
               26.1569
                           14.0625
                                                               2.8253
   51.1861
                                        7.9137
                                                   4.6409
1.7793
           1.1558
 % CO2 Cubic
 a_carbon = (0.45724.*(R.*Tc_carbon).^2)./Pc_carbon
a carbon = 0.3962
 w carbon = 0.225
w_carbon = 0.2250
 alpha_carbon = (1+(0.37464+1.54226.*w_carbon-0.2699.*w_carbon.^2).*(1-
sqrt(Tr_carbon))).^2
alpha carbon = 1 \times 8
    1.0504
                1.0261
                            1.0024
                                        0.9794
                                                   0.9570
                                                               0.9352
0.9140
           0.8933
 a_star_carbon = a_carbon.*alpha_carbon
a_star_carbon = 1 \times 8
    0.4162
                0.4066
                            0.3972
                                        0.3881
                                                   0.3792
                                                               0.3706
0.3622
           0.3540
```

```
A_carbon = (a_star_carbon.*P1)./((R.*T_Kelvin).^2)
A carbon = 1 \times 8
    0.0038 0.0034
                       0.0031 0.0029
                                             0.0026
                                                       0.0024
0.0022 0.0021
 b_{carbon} = (0.0778.*R.*Tc_{carbon})./Pc_{carbon}
b_carbon = 2.6660e-05
 B_carbon = (b_carbon.*P1)./(R.*T_Kelvin)
B carbon = 1 \times 8
1.0e-03 *
    0.5662 0.5469 0.5289 0.5120
                                            0.4962
                                                       0.4813
0.4672 0.4540
 CO2\_coeff_1 = ones(1,8)
C02\_coeff\_1 = 1 \times 8
     1
        1
                 1
                       1
                             1
                                   1
                                         1
                                                1
 CO2_coeff_2 = B_carbon-1
C02 coeff 2 = 1 \times 8
   -0.9994 -0.9995 -0.9995 -0.9995 -0.9995
0.9995 -0.9995
 CO2 coeff 3 = A carbon-(3.*(B carbon).^2)-(2.*B carbon)
CO2 coeff 3 = 1 \times 8
    0.0026 0.0023 0.0021
                                                       0.0015
                                  0.0018
                                            0.0016
0.0013
        0.0011
 CO2_coeff_4 = ((B_carbon).^3)+(B_carbon).^2-(A_carbon.*B_carbon)
C02\_coeff\_4 = 1 \times 8
1.0e-05 *
   -0.1806 -0.1572 -0.1374 -0.1203 -0.1057 -0.0931 -
0.0821 -0.0726
```

```
Zv_{carbon} = zeros(1, 8)
Zv carbon = 1 \times 8
     0
                  0
                         0
                                0
                                      0
                                             0
                                                    0
            0
 for n = 1:8
     cubic_coeffs_CO2 = [CO2_coeff_1(n), CO2_coeff_2(n), CO2_coeff_3(n),
CO2_coeff_4(n)]
     roots_CO2 = roots(cubic_coeffs_CO2)
     Zv carbon(n) = max(real(roots CO2))
 end
cubic\_coeffs\_CO2 = 1 \times 4
    1.0000 -0.9994 0.0026 -0.0000
roots CO2 = 3 \times 1
   0.9968 + 0.0000i
   0.0013 + 0.0003i
   0.0013 - 0.0003i
Zv carbon = 1 \times 8
    0.9968
                              0
                                          0
                                                    0
                                                                 0
           0
cubic coeffs CO2 = 1 \times 4
    1.0000 -0.9995 0.0023 -0.0000
roots CO2 = 3 \times 1
   0.9971 + 0.0000i
   0.0012 + 0.0005i
   0.0012 - 0.0005i
Zv carbon = 1 \times 8
    0.9968
               0.9971
                         0
                                        0
                                                   0
                                                                 0
           0
cubic\_coeffs\_CO2 = 1 \times 4
    1.0000 -0.9995
                          0.0021 -0.0000
roots CO2 = 3 \times 1
   0.9974 + 0.0000i
   0.0010 + 0.0006i
   0.0010 - 0.0006i
Zv carbon = 1 \times 8
    0.9968
               0.9971
                         0.9974
                                        0
                                                    0
                                                                 0
cubic\_coeffs\_CO2 = 1 \times 4
    1.0000 -0.9995
                          0.0018 -0.0000
roots CO2 = 3 \times 1
   0.9976 + 0.0000i
   0.0009 + 0.0006i
   0.0009 - 0.0006i
Zv carbon = 1 \times 8
```

```
0.9968
             0.9971
                         0.9974
                                                   0
                                                               0
                                   0.9976
cubic coeffs CO2 = 1 \times 4
    1.0000 -0.9995
                         0.0016
                                  -0.0000
roots_CO2 = 3 \times 1
   0.9979 + 0.0000i
   0.0008 + 0.0006i
   0.0008 - 0.0006i
Zv carbon = 1 \times 8
    0.9968
               0.9971
                         0.9974
                                    0.9976
                                               0.9979
                                                               0
cubic coeffs CO2 = 1 \times 4
    1.0000
            -0.9995
                         0.0015
                                   -0.0000
roots CO2 = 3 \times 1
   0.9981 + 0.0000i
   0.0007 + 0.0006i
   0.0007 - 0.0006i
Zv carbon = 1 \times 8
    0.9968
              0.9971
                         0.9974
                                    0.9976
                                               0.9979
                                                          0.9981
0
          0
cubic\_coeffs\_CO2 = 1 \times 4
    1.0000
             -0.9995
                         0.0013
                                   -0.0000
roots CO2 = 3 \times 1
   0.9982 + 0.0000i
   0.0006 + 0.0006i
   0.0006 - 0.0006i
Zv carbon = 1 \times 8
    0.9968
              0.9971
                         0.9974
                                   0.9976
                                               0.9979
                                                          0.9981
0.9982
                0
cubic coeffs CO2 = 1 \times 4
                                  -0.0000
    1.0000 -0.9995
                         0.0011
roots CO2 = 3 \times 1
   0.9984 + 0.0000i
   0.0006 + 0.0006i
   0.0006 - 0.0006i
Zv carbon = 1 \times 8
    0.9968
               0.9971
                         0.9974
                                    0.9976
                                               0.9979
                                                          0.9981
0.9982
         0.9984
 phi v CO2 = exp((Zv carbon-1) - (log(Zv carbon-B carbon)) -
(A_carbon./(2*sqrt(2).*B_water)).*log((Zv_carbon+(1+sqrt(2)).*B_carbon)./(Zv_
carbon+(1-sqrt(2))*B_carbon)))
phi v CO2 = 1 \times 8
               0.9957
                                    0.9965
    0.9953
                         0.9961
                                               0.9968
                                                          0.9971
0.9973 0.9976
```

```
phi_2 = phi_v_CO2
phi 2 = 1 \times 8
   0.9953
             0.9957
                       0.9961
                                   0.9965
                                             0.9968
                                                       0.9971
0.9973 0.9976
 val1 = Pv1./1000
val1 = 1 \times 8
1.0e+03 *
    0.6935 0.7708 0.8557 0.9486 1.0502 1.1613
1.2826 1.4151
 val2 = (-phi_1.*P1)./1000
val2 = 1 \times 8
1.0e+03 *
   -2.5593 -1.3078 -0.7031 -0.3957 -0.2320 -0.1413 -
0.0890 -0.0578
 val3 = H21./1000
val3 = 1 \times 8
1.0e+05 *
    1.0461 1.4136 1.8388 2.3094 2.8086 3.3172
3.8155 4.2845
 val4 = (-phi_2.*P1)./1000
val4 = 1 \times 8
  -49.7646 -49.7871 -49.8069 -49.8246 -49.8404 -49.8545 -
49.8671 -49.8785
 matrix1 = [val1(1), 0, val2(1), 0; 0, val3(1), 0, val4(1); 0, 0, 1, 1; 1, 1,
0, 0];
 matrix2 = [0;0;1;1];
 matrix3 = inv(matrix1)*matrix2;
 result1 = (matrix3(2,1)*1000);
 matrix4 = [val1(2), 0, val2(2), 0; 0, val3(2), 0, val4(2); 0, 0, 1, 1; 1, 1,
0, 0];
```

```
matrix5 = inv(matrix4)*matrix2;
 result2 = (matrix5(2,1)*1000);
 matrix6 = [val1(3), 0, val2(3), 0; 0, val3(3), 0, val4(3); 0, 0, 1, 1; 1, 1,
0, 0];
 matrix7 = inv(matrix6)*matrix2;
 result3 = (matrix7(2,1)*1000);
 matrix8 = [val1(4), 0, val2(4), 0; 0, val3(4), 0, val4(4); 0, 0, 1, 1; 1, 1,
0, 0];
 matrix9 = inv(matrix8)*matrix2;
 result4 = (matrix9(2,1)*1000);
 matrix10 = [val1(5), 0, val2(5), 0; 0, val3(5), 0, val4(5); 0, 0, 1, 1; 1,
1, 0, 0];
 matrix11 = inv(matrix10)*matrix2;
 result5 = (matrix11(2,1)*1000);
 matrix12 = [val1(6), 0, val2(6), 0; 0, val3(6), 0, val4(6); 0, 0, 1, 1; 1,
1, 0, 0];
 matrix13 = inv(matrix12)*matrix2;
 result6 = (matrix13(2,1)*1000);
 matrix14 = [val1(7), 0, val2(7), 0; 0, val3(7), 0, val4(7); 0, 0, 1, 1; 1,
1, 0, 0];
 matrix15 = inv(matrix14)*matrix2;
 result7 = (matrix15(2,1)*1000);
 matrix16 = [val1(8), 0, val2(8), 0; 0, val3(8), 0, val4(8); 0, 0, 1, 1; 1,
1, 0, 0];
 matrix17 = inv(matrix16)*matrix2;
 result8 = (matrix17(2,1)*1000);
 allresults = [result1; result2; result3; result4; result5; result6; result7;
result8]
allresults = 8 \times 1
    0.3469
    0.1446
   -0.0588
   -0.3016
   -0.6262
   -1.0866
   -1.7569
   -2.7419
```

% Defining known variables

```
T_Celsius = 10:10:80 % Temperature in Celsius
T Kelvin = T Celsius + 273.15 % Convert to Kelvin
Tc water = 647.096 % Critical temperature of water (K)
Pc_water = 22064000 % Critical pressure of water (Pa)
R = 8.314 \% \text{ Units of } J/(\text{mol } K)
Tc carbon = 304.18 % Kelvin
Pc_carbon = 7380000 % Pa
P1 = 50000
P2 = 101325
P3 = 200000
%%%%% TASK 2B %%%%%
Tr_water = T_Kelvin/Tc_water
x = 1-Tr_water
Tr_carbon = T_Kelvin/Tc_carbon
% Coefficients for Eq. (3)
alpha1 = -7.85951783
alpha2 = 1.84408259
alpha3 = -11.7866497
alpha4 = 22.6807411
alpha5 = -15.9618719
alpha6 = 1.80122502
Pv1 =
Pc_{water.*(exp((alpha1.*x)+(alpha2.*x.^(1.5))+(alpha3.*x.^(3))+(alpha4.*x.^(3))
.5))+(alpha5.*x.^(4))+(alpha6.*x.^(7.5)))./Tr_water)
% Coefficients for Eq. (4)
h1 = -6.8346
h2 = 1.2817*(10^4)
h3 = -3.7668*(10^6)
h4 = 2.9970*(10^8)
H21 = 10.^6.*exp(h1+(h2./T_Kelvin)+(h3./(T_Kelvin.^2))+(h4./(T_Kelvin.^3)))
% H2O Cubic
a_water = (0.45724.*(R.*Tc_water).^2)./Pc_water
w water = 0.348
alpha_water = (1+(0.37464+1.54226.*w_water-0.2699.*w_water.^2).*(1-
sqrt(Tr water))).^2
a_star_water = a_water.*alpha_water
A_water = (a_star_water.*P2)./((R.*T_Kelvin).^2)
b_water = (0.0778.*R.*Tc_water)./Pc_water
B_water = (b_water.*P2)./(R.*T_Kelvin)
H20 coeff 1 = ones(1,8)
H20 coeff 2 = B water-1
H20_coeff_3 = A_water-(3.*(B_water).^2)-(2.*B_water)
H20\_coeff\_4 = ((B\_water).^3) + (B\_water).^2 - (A\_water.*B\_water)
```

```
Zv_{water} = zeros(1, 8)
Zl water = zeros(1, 8)
for n = 1:8
     cubic_coeffs_H20 = [H20_coeff_1(n), H20_coeff_2(n), H20_coeff_3(n),
H20 coeff 4(n)
     roots_H20 = roots(cubic_coeffs_H20)
     Zv_{water(n)} = max(roots_{H20})
     Zl water(n) = min(roots H20)
end
phi_v_H20 = exp((Zv_water-1)-(log(Zv_water-B_water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zv_water+(1+sqrt(2)).*B_water)./(Zv_wat
er+(1-sqrt(2))*B_water)))
phi_1_H20 = exp((Zl_water-1)-(log(Zl_water-B_water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zl_water+(1+sqrt(2)).*B_water)./(Zl_wat
er+(1-sqrt(2))*B_water)))
phi_1 = phi_v_H20./phi_1_H20
% CO2 Cubic
a_carbon = (0.45724.*(R.*Tc_carbon).^2)./Pc_carbon
w_{carbon} = 0.225
alpha_carbon = (1+(0.37464+1.54226.*w_carbon-0.2699.*w_carbon.^2).*(1-
sqrt(Tr_carbon))).^2
a_star_carbon = a_carbon.*alpha_carbon
A_carbon = (a_star_carbon.*P2)./((R.*T_Kelvin).^2)
b carbon = (0.0778.*R.*Tc carbon)./Pc carbon
B_carbon = (b_carbon.*P2)./(R.*T_Kelvin)
CO2\_coeff_1 = ones(1,8)
CO2 coeff 2 = B carbon-1
CO2\_coeff_3 = A\_carbon-(3.*(B\_carbon).^2)-(2.*B\_carbon)
CO2\_coeff_4 = ((B\_carbon).^3) + (B\_carbon).^2 - (A\_carbon.*B\_carbon)
Zv_carbon = zeros(1, 8)
for n = 1:8
     cubic_coeffs_CO2 = [CO2_coeff_1(n), CO2_coeff_2(n), CO2_coeff_3(n),
CO2_coeff_4(n)]
     roots_CO2 = roots(cubic_coeffs_CO2)
     Zv_carbon(n) = max(real(roots_CO2))
end
phi_v_CO2 = exp((Zv_carbon-1)-(log(Zv_carbon-B_carbon))-
(A_carbon./(2*sqrt(2).*B_water)).*log((Zv_carbon+(1+sqrt(2)).*B_carbon)./(Zv_
carbon+(1-sqrt(2))*B carbon)))
phi_2 = phi_v_CO2
val1 = Pv1./1000
```

```
val2 = (-phi_1.*P2)./1000
 val3 = H21./1000
 val4 = (-phi 2.*P2)./1000
 matrix1 = [val1(1), 0, val2(1), 0; 0, val3(1), 0, val4(1); 0, 0, 1, 1; 1, 1,
0, 0];
 matrix2 = [0;0;1;1];
 matrix3 = inv(matrix1)*matrix2;
 result1 = (matrix3(2,1)*1000);
 matrix4 = [val1(2), 0, val2(2), 0; 0, val3(2), 0, val4(2); 0, 0, 1, 1; 1, 1,
0, 0];
 matrix5 = inv(matrix4)*matrix2;
 result2 = (matrix5(2,1)*1000);
 matrix6 = [val1(3), 0, val2(3), 0; 0, val3(3), 0, val4(3); 0, 0, 1, 1; 1, 1,
0, 0];
 matrix7 = inv(matrix6)*matrix2;
 result3 = (matrix7(2,1)*1000);
 matrix8 = [val1(4), 0, val2(4), 0; 0, val3(4), 0, val4(4); 0, 0, 1, 1; 1, 1,
0, 0];
 matrix9 = inv(matrix8)*matrix2;
result4 = (matrix9(2,1)*1000);
 matrix10 = [val1(5), 0, val2(5), 0; 0, val3(5), 0, val4(5); 0, 0, 1, 1; 1,
1, 0, 0];
 matrix11 = inv(matrix10)*matrix2;
 result5 = (matrix11(2,1)*1000);
matrix12 = [val1(6), 0, val2(6), 0; 0, val3(6), 0, val4(6); 0, 0, 1, 1; 1,
1, 0, 0];
 matrix13 = inv(matrix12)*matrix2;
 result6 = (matrix13(2,1)*1000);
matrix14 = [val1(7), 0, val2(7), 0; 0, val3(7), 0, val4(7); 0, 0, 1, 1; 1,
1, 0, 0];
 matrix15 = inv(matrix14)*matrix2;
 result7 = (matrix15(2,1)*1000);
matrix16 = [val1(8), 0, val2(8), 0; 0, val3(8), 0, val4(8); 0, 0, 1, 1; 1,
1, 0, 0];
 matrix17 = inv(matrix16)*matrix2;
result8 = (matrix17(2,1)*1000);
 allresults = [result1; result2; result3; result4; result5; result6; result7;
result8]
```

```
% Defining known variables
T Celsius = 10:10:80 % Temperature in Celsius
T_Kelvin = T_Celsius + 273.15 % Convert to Kelvin
Tc_water = 647.096 % Critical temperature of water (K)
Pc water = 22064000 % Critical pressure of water (Pa)
R = 8.314 \% \text{ Units of } J/(\text{mol } K)
Tc_carbon = 304.18 % Kelvin
Pc carbon = 7380000 % Pa
P1 = 50000
P2 = 101325
P3 = 200000
%%%%% TASK 2B %%%%%
Tr_water = T_Kelvin/Tc_water
x = 1-Tr_water
Tr_carbon = T_Kelvin/Tc_carbon
% Coefficients for Eq. (3)
alpha1 = -7.85951783
alpha2 = 1.84408259
alpha3 = -11.7866497
alpha4 = 22.6807411
alpha5 = -15.9618719
alpha6 = 1.80122502
Pv1 =
Pc_{water.*(exp((alpha1.*x)+(alpha2.*x.^(1.5))+(alpha3.*x.^(3))+(alpha4.*x.^(3))
.5))+(alpha5.*x.^(4))+(alpha6.*x.^(7.5)))./Tr_water)
% Coefficients for Eq. (4)
h1 = -6.8346
h2 = 1.2817*(10^4)
h3 = -3.7668*(10^6)
h4 = 2.9970*(10^8)
H21 = 10.^6.*exp(h1+(h2./T_Kelvin)+(h3./(T_Kelvin.^2))+(h4./(T_Kelvin.^3)))
% H2O Cubic
a_water = (0.45724.*(R.*Tc_water).^2)./Pc_water
w water = 0.348
alpha_water = (1+(0.37464+1.54226.*w_water-0.2699.*w_water.^2).*(1-
sqrt(Tr_water))).^2
a_star_water = a_water.*alpha_water
A_water = (a_star_water.*P3)./((R.*T_Kelvin).^2)
b_water = (0.0778.*R.*Tc_water)./Pc_water
B_water = (b_water.*P3)./(R.*T_Kelvin)
H20_coeff_1 = ones(1,8)
H20_coeff_2 = B_water-1
H20_coeff_3 = A_water-(3.*(B_water).^2)-(2.*B_water)
```

```
H20_coeff_4 = ((B_water).^3)+(B_water).^2-(A_water.*B_water)
Zv water = zeros(1, 8)
Zl_water = zeros(1, 8)
for n = 1:8
     cubic_coeffs_H20 = [H20_coeff_1(n), H20_coeff_2(n), H20_coeff_3(n),
H20_coeff_4(n)]
     roots H20 = roots(cubic coeffs H20)
     Zv_{water(n)} = max(roots_{H20})
     Zl_water(n) = min(roots_H20)
end
phi_v_H20 = exp((Zv_water-1)-(log(Zv_water-B_water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zv_water+(1+sqrt(2)).*B_water)./(Zv_wat
er+(1-sqrt(2))*B water)))
phi_l_H20 = exp((Zl_water-1)-(log(Zl_water-B_water))-
(A_water./(2*sqrt(2).*B_water)).*log((Zl_water+(1+sqrt(2)).*B_water)./(Zl_wat
er+(1-sqrt(2))*B_water)))
phi_1 = phi_v_H20./phi_1_H20
% CO2 Cubic
a_carbon = (0.45724.*(R.*Tc_carbon).^2)./Pc_carbon
w_carbon = 0.225
alpha_carbon = (1+(0.37464+1.54226.*w_carbon-0.2699.*w_carbon.^2).*(1-
sqrt(Tr_carbon))).^2
a_star_carbon = a_carbon.*alpha_carbon
A_carbon = (a_star_carbon.*P3)./((R.*T_Kelvin).^2)
b_carbon = (0.0778.*R.*Tc_carbon)./Pc_carbon
B_carbon = (b_carbon.*P3)./(R.*T_Kelvin)
CO2\_coeff_1 = ones(1,8)
CO2 coeff 2 = B carbon-1
CO2 coeff 3 = A carbon-(3.*(B carbon).^2)-(2.*B carbon)
CO2_coeff_4 = ((B_carbon).^3)+(B_carbon).^2-(A_carbon.*B_carbon)
Zv_carbon = zeros(1, 8)
for n = 1:8
     cubic_coeffs_CO2 = [CO2_coeff_1(n), CO2_coeff_2(n), CO2_coeff_3(n),
CO2_coeff_4(n)]
     roots_CO2 = roots(cubic_coeffs_CO2)
     Zv_carbon(n) = max(real(roots_CO2))
end
phi_v_CO2 = exp((Zv_carbon-1)-(log(Zv_carbon-B_carbon))-
(A_carbon./(2*sqrt(2).*B_water)).*log((Zv_carbon+(1+sqrt(2)).*B_carbon)./(Zv_
carbon+(1-sqrt(2))*B_carbon)))
phi_2 = phi_v_CO2
```

```
val1 = Pv1./1000
val2 = (-phi 1.*P3)./1000
val3 = H21./1000
val4 = (-phi_2.*P3)./1000
matrix1 = [val1(1), 0, val2(1), 0; 0, val3(1), 0, val4(1); 0, 0, 1, 1; 1, 1,
0, 0];
matrix2 = [0;0;1;1];
matrix3 = inv(matrix1)*matrix2;
result1 = (matrix3(2,1)*1000);
matrix4 = [val1(2), 0, val2(2), 0; 0, val3(2), 0, val4(2); 0, 0, 1, 1; 1, 1,
0, 0];
matrix5 = inv(matrix4)*matrix2;
result2 = (matrix5(2,1)*1000);
matrix6 = [val1(3), 0, val2(3), 0; 0, val3(3), 0, val4(3); 0, 0, 1, 1; 1, 1,
0, 0];
matrix7 = inv(matrix6)*matrix2;
result3 = (matrix7(2,1)*1000);
matrix8 = [val1(4), 0, val2(4), 0; 0, val3(4), 0, val4(4); 0, 0, 1, 1; 1, 1,
0, 0];
matrix9 = inv(matrix8)*matrix2;
result4 = (matrix9(2,1)*1000);
matrix10 = [val1(5), 0, val2(5), 0; 0, val3(5), 0, val4(5); 0, 0, 1, 1; 1,
1, 0, 0];
matrix11 = inv(matrix10)*matrix2;
result5 = (matrix11(2,1)*1000);
matrix12 = [val1(6), 0, val2(6), 0; 0, val3(6), 0, val4(6); 0, 0, 1, 1; 1,
1, 0, 0];
matrix13 = inv(matrix12)*matrix2;
result6 = (matrix13(2,1)*1000);
matrix14 = [val1(7), 0, val2(7), 0; 0, val3(7), 0, val4(7); 0, 0, 1, 1; 1,
1, 0, 0];
matrix15 = inv(matrix14)*matrix2;
result7 = (matrix15(2,1)*1000);
matrix16 = [val1(8), 0, val2(8), 0; 0, val3(8), 0, val4(8); 0, 0, 1, 1; 1,
1, 0, 0];
matrix17 = inv(matrix16)*matrix2;
result8 = (matrix17(2,1)*1000);
allresults = [result1; result2; result3; result4; result5; result6; result7;
result8]
```

```
% Import the dataset
data = ProjectData; % Replace 'your_dataset.csv' with the actual file name
% Initial table containing all data points
initialTable = table(data.site_code, data.measurement_id, data.TIME, ...
                      data.TEMP, data.Press ATM, data.CO2EQ PPM, ...
                      'VariableNames', {'SiteCode', 'Measurement', 'Time',
                                        'Temperature', 'Pressure',
'CO2ConcWater'});
% Remove duplicate rows
uniqueTable = unique(initialTable, "rows");
% Clipping data columns for easier handling
TempClipped = uniqueTable.Temperature;
PressureClipped = uniqueTable.Pressure;
CO2WaterClipped = uniqueTable.CO2ConcWater;
% Detecting outliers in specified variables
detection_Pressure = isoutlier(PressureClipped, "quartiles");
detection_Temperature = isoutlier(TempClipped, "quartiles");
detection_CO2Water = isoutlier(CO2WaterClipped, "quartiles");
% Removing rows with outliers
% Create logical index to remove rows where any of the selected variables
have outliers
outlierIndex = detection_Pressure | detection_Temperature |
detection_CO2Water;
% Create a cleaned table by removing rows with outliers
cleanTable = uniqueTable(~outlierIndex, :);
% Adding a 'Year' column for easier analysis
cleanTable.Year = year(datetime(cleanTable.Time, 'InputFormat', 'yyyy-MM-
dd''T''HH:mm:ss''Z'''));
% Save the cleaned dataset
writetable(cleanTable, 'cleaned_dataset.csv');
disp(size(cleanTable))
    154437
```

```
data = cleanTable
```

 $data = 154437 \times 7 table$

	SiteCode	Measurement	Time	Temperature
1	GBRWIS	1	"2020-10-31T10:00:00Z"	24.3132
2	GBRWIS	1	"2021-03-27T10:00:00Z"	26.9895
3	GBRWIS	1	"2021-09-21T10:00:00Z"	22.4750
4	GBRWIS	1	"2022-05-16T12:00:00Z"	24.8867
5	GBRWIS	1	"2022-11-02T04:00:00Z"	24.4562
6	GBRWIS	1	"2023-05-01T10:00:00Z"	25.6385
7	GBRWIS	1	"2023-10-11T06:00:00Z"	23.0590
8	GBRWIS	2	"2020-10-31T12:00:00Z"	24.1960
9	GBRWIS	2	"2021-03-27T12:00:00Z"	27.0938
10	GBRWIS	2	"2021-09-21T12:00:00Z"	22.3367
11	GBRWIS	2	"2022-05-16T14:00:00Z"	24.8710
12	GBRWIS	2	"2022-11-02T06:00:00Z"	24.4706
13	GBRWIS	2	"2023-05-01T12:00:00Z"	25.6113
14	GBRWIS	2	"2023-10-11T08:00:00Z"	22.8557
15	GBRWIS	3	"2020-10-31T14:00:00Z"	24.1836
16	GBRWIS	3	"2021-03-27T14:00:00Z"	27.0388
17	GBRWIS	3	"2021-09-21T14:00:00Z"	22.1666
18	GBRWIS	3	"2022-05-16T16:00:00Z"	24.8712
19	GBRWIS	3	"2022-11-02T08:00:00Z"	24.2090
20	GBRWIS	3	"2023-05-01T14:00:00Z"	25.5656
21	GBRWIS	3	"2023-10-11T10:00:00Z"	22.8102
22	GBRWIS	4	"2020-10-31T16:00:00Z"	24.1438
23	GBRWIS	4	"2021-03-27T16:00:00Z"	27.1254
24	GBRWIS	4	"2021-09-21T16:00:00Z"	21.9194
25	GBRWIS	4	"2022-05-16T18:00:00Z"	24.8447
26	GBRWIS	4	"2022-11-02T10:00:00Z"	24.1262
27	GBRWIS	4	"2023-05-01T16:00:00Z"	25.5886
28	GBRWIS	4	"2023-10-11T12:00:00Z"	22.9046
29	GBRWIS	5	"2020-10-31T18:00:00Z"	24.1152
30	GBRWIS	5	"2021-03-27T18:00:00Z"	27.1007
31	GBRWIS	5	"2021-09-21T18:00:00Z"	21.9535
32	GBRWIS	5	"2022-05-16T20:00:00Z"	24.8342
33	GBRWIS	5	"2022-11-02T12:00:00Z"	24.0226
34	GBRWIS	5	"2023-05-01T18:00:00Z"	25.5826
35	GBRWIS	5	"2023-10-11T14:00:00Z"	23.3348
36	GBRWIS	6	"2020-10-31T20:00:00Z"	24.0574
37	GBRWIS	6	"2021-03-27T20:00:00Z"	26.9823
38	GBRWIS	6	"2021-09-21T20:00:00Z"	21.9551

	SiteCode	Measurement	Time	Temperature
39	GBRWIS	6	"2022-05-16T22:00:00Z"	24.8936
40	GBRWIS	6	"2022-11-02T14:00:00Z"	24.2494
41	GBRWIS	6	"2023-05-01T20:00:00Z"	25.5685
42	GBRWIS	6	"2023-10-11T16:00:00Z"	22.9298
43	GBRWIS	7	"2020-10-31T22:00:00Z"	24.1220
44	GBRWIS	7	"2021-03-27T22:00:00Z"	26.9796
45	GBRWIS	7	"2021-09-21T22:00:00Z"	21.9069
46	GBRWIS	7	"2022-05-17T00:00:00Z"	24.9454
47	GBRWIS	7	"2022-11-02T16:00:00Z"	24.2046
48	GBRWIS	7	"2023-05-01T22:00:00Z"	25.5693
49	GBRWIS	7	"2023-10-11T18:00:00Z"	22.8290
50	GBRWIS	8	"2020-11-01T00:00:00Z"	24.2227
51	GBRWIS	8	"2021-03-28T00:00:00Z"	27.0428
52	GBRWIS	8	"2021-09-22T00:00:00Z"	21.9473
53	GBRWIS	8	"2022-05-17T02:00:00Z"	24.9087
54	GBRWIS	8	"2022-11-02T18:00:00Z"	24.1167
55	GBRWIS	8	"2023-05-02T00:00:00Z"	25.5798
56	GBRWIS	8	"2023-10-11T20:00:00Z"	22.7747
57	GBRWIS	9	"2020-11-01T02:00:00Z"	24.9186
58	GBRWIS	9	"2021-03-28T02:00:00Z"	27.4048
59	GBRWIS	9	"2021-09-22T02:00:00Z"	22.0762
60	GBRWIS	9	"2022-05-17T04:00:00Z"	25.1657
61	GBRWIS	9	"2022-11-02T20:00:00Z"	24.0587
62	GBRWIS	9	"2023-05-02T02:00:00Z"	25.6358
63	GBRWIS	9	"2023-10-11T22:00:00Z"	22.8217
64	GBRWIS	10	"2020-11-01T04:00:00Z"	25.4500
65	GBRWIS	10	"2021-03-28T04:00:00Z"	28.2432
66	GBRWIS	10	"2021-09-22T04:00:00Z"	22.3441
67	GBRWIS	10	"2022-05-17T06:00:00Z"	24.9786
68	GBRWIS	10	"2022-11-02T22:00:00Z"	24.0738
69	GBRWIS	10	"2023-05-02T04:00:00Z"	25.7073
70	GBRWIS	10	"2023-10-12T00:00:00Z"	22.9473
71	GBRWIS	11	"2020-11-01T06:00:00Z"	24.6910
72	GBRWIS	11	"2021-03-28T06:00:00Z"	27.2303
73	GBRWIS	11	"2021-09-22T06:00:00Z"	22.1390
74	GBRWIS	11	"2022-05-17T08:00:00Z"	24.9591
75	GBRWIS	11	"2022-11-03T00:00:00Z"	24.0673
76	GBRWIS	11	"2023-05-02T06:00:00Z"	25.5942

	SiteCode	Measurement	Time	Temperature
77	GBRWIS	11	"2023-10-12T02:00:00Z"	24.2813
78	GBRWIS	12	"2020-11-01T08:00:00Z"	24.6383
79	GBRWIS	12	"2021-03-28T08:00:00Z"	27.2084
80	GBRWIS	12	"2021-09-22T08:00:00Z"	22.0102
81	GBRWIS	12	"2022-05-17T10:00:00Z"	24.9529
82	GBRWIS	12	"2022-11-03T02:00:00Z"	24.2072
83	GBRWIS	12	"2023-05-02T08:00:00Z"	25.6123
84	GBRWIS	12	"2023-10-12T04:00:00Z"	23.4801
85	GBRWIS	13	"2020-11-01T10:00:00Z"	24.4941
86	GBRWIS	13	"2021-03-28T10:00:00Z"	27.1524
87	GBRWIS	13	"2021-09-22T10:00:00Z"	22.0491
88	GBRWIS	13	"2022-05-17T12:00:00Z"	24.9093
89	GBRWIS	13	"2022-11-03T04:00:00Z"	24.2029
90	GBRWIS	13	"2023-05-02T10:00:00Z"	25.5825
91	GBRWIS	13	"2023-10-12T06:00:00Z"	23.0891
92	GBRWIS	14	"2020-11-01T12:00:00Z"	24.4879
93	GBRWIS	14	"2021-03-28T12:00:00Z"	27.1884
94	GBRWIS	14	"2021-09-22T12:00:00Z"	22.0046
95	GBRWIS	14	"2022-05-17T14:00:00Z"	24.8798
96	GBRWIS	14	"2022-11-03T06:00:00Z"	24.1388
97	GBRWIS	14	"2023-05-02T12:00:00Z"	25.5450
98	GBRWIS	14	"2023-10-12T08:00:00Z"	22.9383
99	GBRWIS	15	"2020-11-01T14:00:00Z"	24.3364
100	GBRWIS	15	"2021-09-22T14:00:00Z"	21.9002

T_Celsius = data.Temperature % Temperature in Celsius

```
T_Celsius = 154437×1
    24.3132
    26.9895
    22.4750
    24.8867
    24.4562
    25.6385
    23.0590
    24.1960
    27.0938
    22.3367
    :
```

T_Kelvin = T_Celsius + 273.15 % Convert to Kelvin

```
T_Kelvin = 154437×1
  297.4632
  300.1395
  295.6250
  298.0367
  297.6062
  298.7885
  296.2090
  297.3460
  300.2438
  295.4867
 P = data.Pressure * 100; % Pressure in Pa (assuming Pressure in atmosphere,
so multiply by 100)
 y2 = data.CO2ConcWater; % The equilibrium CO2 concentration in water (ppm)
 Tc_water = 647.096 % Critical temperature of water (K)
Tc water = 647.0960
 Pc_water = 22064000 % Critical pressure of water (Pa)
Pc_water = 22064000
 R = 8.314 \% \text{ Units of } J/(\text{mol } K)
R = 8.3140
 Tc_carbon = 304.18 % Kelvin
Tc\_carbon = 304.1800
 Pc_carbon = 7380000 % Pa
Pc carbon = 7380000
 Tr_water = T_Kelvin/Tc_water
Tr\_water = 154437 \times 1
    0.4597
    0.4638
    0.4568
    0.4606
    0.4599
    0.4617
    0.4578
    0.4595
    0.4640
    0.4566
 x = 1-Tr_water
x = 154437 \times 1
    0.5403
    0.5362
    0.5432
    0.5394
```

```
0.5405
                   0.5360
                   0.5434
       Tr_carbon = T_Kelvin/Tc_carbon
Tr\_carbon = 154437 \times 1
                   0.9779
                   0.9867
                   0.9719
                   0.9798
                   0.9784
                   0.9823
                   0.9738
                   0.9775
                   0.9871
                   0.9714
       % Coefficients for Eq. (3)
       alpha1 = -7.85951783
alpha1 = -7.8595
       alpha2 = 1.84408259
alpha2 = 1.8441
       alpha3 = -11.7866497
alpha3 = -11.7866
       alpha4 = 22.6807411
alpha4 = 22.6807
       alpha5 = -15.9618719
alpha5 = -15.9619
       alpha6 = 1.80122502
alpha6 = 1.8012
      Pv1 =
   Pc_{water.*(exp((alpha1.*x)+(alpha2.*x.^(1.5))+(alpha3.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.*x.^(3))+(alpha4.
   .5))+(alpha5.*x.^(4))+(alpha6.*x.^(7.5)))./Tr_water)
Pv1 = 154437×1
10<sup>5</sup> ×
                   8.0649
                   8.2933
                   7.9112
```

0.5401 0.5383 0.5422

```
8.1134
    8.0770
    8.1773
    7.9598
    8.0550
    8.3023
    7.8998
 % Coefficients for Eq. (4)
 h1 = -6.8346
h1 = -6.8346
 h2 = 1.2817*(10^4)
h2 = 12817
 h3 = -3.7668*(10^6)
h3 = -3766800
 h4 = 2.9970*(10^8)
h4 = 299700000
 H21 = 10.^6.*exp(h1+(h2./T_Kelvin)+(h3./(T_Kelvin.^2))+(h4./(T_Kelvin.^3)))
H21 = 154437 \times 1
10<sup>8</sup> ×
    1.5906
    1.7054
    1.5139
    1.6149
    1.5966
    1.6470
    1.5381
    1.5856
    1.7099
    1.5082
 % CO2 Cubic
 a_{carbon} = (0.45724.*(R.*Tc_{carbon}).^2)./Pc_{carbon}
a carbon = 0.3962
 w_{carbon} = 0.225
w_carbon = 0.2250
 alpha_carbon = (1+(0.37464+1.54226.*w_carbon-0.2699.*w_carbon.^2).*(1-
sqrt(Tr_carbon))).^2
alpha_carbon = 154437×1
    \frac{1}{1.0158}
    1.0095
```

```
1.0144
    1.0154
    1.0126
    1.0188
    1.0161
    1.0092
    1.0205
 a_star_carbon = a_carbon.*alpha_carbon
a_star_carbon = 154437 \times 1
    0.4025
    0.4000
    0.4042
    0.4020
    0.4024
    0.4013
    0.4037
    0.4026
    0.3999
    0.4044
       :
 A_carbon = (a_star_carbon.*P)./((R.*T_Kelvin).^2)
A_{carbon} = 154437 \times 1
10<sup>-3</sup> ×
    0.6638
    0.6508
    0.6778
    0.6609
    0.6631
    0.6585
    0.6765
    0.6651
    0.6505
    0.6798
 b_carbon = (0.0778.*R.*Tc_carbon)./Pc_carbon
b_carbon = 2.6660e-05
 B_carbon = (b_carbon.*P)./(R.*T_Kelvin)
B carbon = 154437 \times 1
10^{-3} \times
    0.1087
    0.1082
    0.1099
    0.1086
    0.1087
    0.1087
    0.1100
    0.1089
    0.1083
    0.1101
```

1.0202

```
:
 CO2_coeff_1 = ones(length(T_Kelvin),1)
C02_coeff_1 = 154437 \times 1
      1
      1
      1
      1
      1
      1
      1
      1
      1
 CO2\_coeff\_2 = B\_carbon-1
C02_coeff_2 = 154437 \times 1
   -0.9999
   -0.9999
   -0.9999
   -0.9999
   -0.9999
   -0.9999
   -0.9999
   -0.9999
   -0.9999
   -0.9999
 CO2\_coeff_3 = A\_carbon-(3.*(B\_carbon).^2)-(2.*B\_carbon)
CO2_coeff_3 = 154437×1
10<sup>-3</sup> ×
    0.4463
    0.4343
    0.4580
    0.4436
    0.4456
    0.4411
    0.4564
    0.4473
    0.4340
    0.4596
 CO2_coeff_4 = ((B_carbon).^3)+(B_carbon).^2-(A_carbon.*B_carbon)
C02_coeff_4 = 154437 \times 1
10<sup>-7</sup> ×
   -0.6036
   -0.5873
   -0.6240
   -0.5999
   -0.6026
```

-0.5975 -0.6232

```
-0.6055
-0.5870
-0.6273
```

```
Zv_carbon = zeros(length(T_Kelvin),1)
```

```
Zv_carbon = 154437×1
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
0
```

```
for n = 1:length(T_Kelvin);
    cubic_coeffs_CO2 = [CO2_coeff_1(n), CO2_coeff_2(n), CO2_coeff_3(n),
CO2_coeff_4(n)];
    roots_CO2 = roots(cubic_coeffs_CO2);
    Zv_carbon(n) = max(real(roots_CO2));
end
phi_v_CO2 = exp((Zv_carbon-1)-(log(Zv_carbon-B_carbon))-
(A_carbon./(2*sqrt(2).*B_water)).*log((Zv_carbon+(1+sqrt(2)).*B_carbon)./(Zv_
carbon+(1-sqrt(2))*B carbon)))
phi_2 = phi_v_CO2
% % Extract Temperature, Pressure, and CO2 Equilibrium concentration (y2)
from the dataset
% T_Celsius = data.Temperature; % Temperature in Celsius (from your dataset)
% T_Kelvin = T_Celsius + 273.15; % Convert to Kelvin
% P = data.Pressure * 100; % Pressure in Pa (assuming Pressure in
atmosphere, so multiply by 100)
% y2 = data.CO2ConcWater; % The equilibrium CO2 concentration in water (ppm)
% % Constants for CO2 and water
% Tc water = 647.096; % Critical temperature of water (K)
% Pc_water = 22064000; % Critical pressure of water (Pa)
% Tc_carbon = 304.18; % Critical temperature of CO2 (K)
% Pc carbon = 7380000; % Critical pressure of CO2 (Pa)
% R = 8.314; % Universal gas constant in J/(mol K)
% % Reduced temperature for water and CO2
% Tr water = T Kelvin / Tc water;
% x = 1 - Tr_water; % Deviation from critical temperature
```

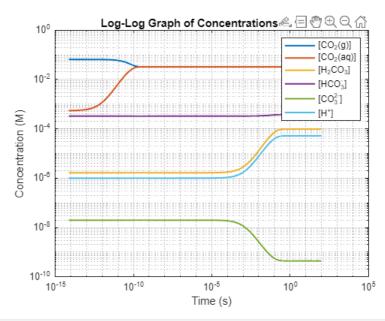
```
%
% Tr carbon = T Kelvin / Tc carbon; % Reduced temperature for CO2
% % CO2 Equations
% h1 = -6.8346;
\% h2 = 1.2817 * 10^4;
% h3 = -3.7668 * 10^6;
% h4 = 2.9970 * 10^8;
% H21 = 10.^6 .* exp(h1 + (h2 ./ T_Kelvin) + (h3 ./ (T_Kelvin .^ 2)) + (h4
./ (T_Kelvin .^ 3)));
%
% % CO2 Cubic Equation for Compressibility Factor
% a_carbon = (0.45724 .* (R .* Tc_carbon) .^ 2) ./ Pc_carbon;
% w_{carbon} = 0.225;
% alpha carbon = (1 + (0.37464 + 1.54226 .* w carbon - 0.2699 .* w carbon .^
2) .* (1 - sqrt(Tr_carbon))) .^ 2;
% a_star_carbon = a_carbon .* alpha_carbon;
% A_carbon = (a_star_carbon .* P) ./ ((R .* T_Kelvin) .^ 2);
%
% b_carbon = (0.0778 .* R .* Tc_carbon) ./ Pc_carbon;
% B_carbon = (b_carbon .* P) ./ (R .* T_Kelvin);
%
% % Fugacity coefficient for CO2 (phi_2)
\% \text{ phi}_2 = \exp((Pv1 - 1) - (\log(Pv1 - B_carbon)) - (A_carbon ./ (2 * sqrt(2)))
.* B_carbon)) .* ...
       log((Pv1 + (1 + sqrt(2)) * B_carbon) ./ (Pv1 + (1 - sqrt(2)) *
%
B_carbon)));
% % Now solve for x_2 using the equation: x_2 = (y_2 * phi_2 * P) / H21
% x2 = (y2 .* phi_2 .* P) ./ H21;
% % Plotting the results
% figure;
% subplot(3, 1, 1);
% plot(data.TIME, T_Celsius, 'r');
% xlabel('Time');
% ylabel('Temperature (°C)');
% title('Temperature vs Time');
%
% subplot(3, 1, 2);
% plot(data.TIME, P, 'g');
% xlabel('Time');
% ylabel('Pressure (Pa)');
% title('Pressure vs Time');
% subplot(3, 1, 3);
% plot(data.TIME, x2, 'b');
% xlabel('Time');
```

```
% ylabel('CO₂ Concentration in Water (ppm)');
% title('CO<sub>2</sub> Concentration in Water (x2) vs Time');
% % Save the results to a new table
% results = table(data.TIME, T_Celsius, P, x2, 'VariableNames', {'Time',
'Temperature', 'Pressure', 'CO2ConcWater'});
% writetable(results, 'modelled_CO2_concentration.csv');
%
% Constants
k1 = 1e10;
k2 = 6e-2;
k3 = 1e7;
k4 = 3;
k_neg1 = 1e10;
k neg2 = 2e1;
k_neg3 = 5e10;
k_neg4 = 5e10;
% Initial conditions
y0 = [0.065, 5.41e-4, 1.64e-6, 3.28e-4, 1.97e-8, 1e-6]; % [CO2(g)],
[CO2(aq)], [H2CO3], [HCO3^-], [CO3^2-], [H+]
% Time for simulation
tspan = [0 100];
% Combining the equations
carbon_system = @(t, y) [
    -k1*y(1) + k_neg1*y(2); % d[CO2(g)]/dt
    k1*y(1) - (k_neg1 + k2)*y(2) + k_neg2*y(3); % d[CO2(aq)]/dt
    k2*y(2) - (k_neg2 + k3)*y(3) + k_neg3*y(6)*y(4); % d[H2CO3]/dt
    k3*y(3) - k_neg3*y(6)*y(4) - k4*y(4) + k_neg4*y(6)*y(5); % d[HCO3^-]/dt
    k4*y(4) - k_neg4*y(6)*y(5); % d[CO3^2-]/dt
    k3*y(3) - k_neg3*y(6)*y(4) + k4*y(4) - k_neg4*y(6)*y(5); % d[H+]/dt
];
% Set tolerances
options = odeset('AbsTol', 1e-12, 'RelTol', 1e-6);
% Solving the equations
[t, y] = ode15s(@(t, y) carbon_system(t, y), tspan, y0, options);
% Results
CO2_g = y(:,1); % [CO2(g)]
CO2_aq = y(:,2); % [CO2(aq)]
H2CO3 = y(:,3); % [H2CO3]
HCO3_minus = y(:,4); % [HCO3^-]
CO3_2_minus = y(:,5); % [CO3^2-]
H_{plus} = y(:,6); % [H+]
```

```
% Plot all chemicals on a log-log graph
figure;
loglog(t, CO2_g,'DisplayName', '[CO_2(g)]', 'LineWidth', 1.5); hold on;
loglog(t, CO2_aq, 'DisplayName', '[CO_2(aq)]', 'LineWidth', 1.5);
loglog(t, H2CO3, 'DisplayName', '[H_2CO_3]', 'LineWidth', 1.5);
loglog(t, HCO3_minus,'DisplayName', '[HCO_3^-]', 'LineWidth', 1.5);
loglog(t, CO3_2_minus,'DisplayName', '[CO_3^2^-]', 'LineWidth', 1.5);
loglog(t, H_plus, 'DisplayName', '[H^+]', 'LineWidth', 1.5);

% Add labels and legend
xlabel('Time (s)');
ylabel('Concentration (M)');
title('Log-Log Graph of Concentrations vs Time');
legend('show'); % Display legend

% Improve plot appearance
grid on;
hold off;
```



```
% Calculate the rate of change for each species
dCO2_g_dt = diff(CO2_g) ./ diff(t);
dCO2_aq_dt = diff(CO2_aq) ./ diff(t);
dH2CO3_dt = diff(H2CO3) ./ diff(t);
dHCO3_minus_dt = diff(HCO3_minus) ./ diff(t);
dCO3_2_minus_dt = diff(CO3_2_minus) ./ diff(t);
dH_plus_dt = diff(H_plus) ./ diff(t);
% Plot the rate of change for each species
figure;
plot(t(2:end), dCO2_g_dt, 'DisplayName', 'd[CO2(g)]/dt');
```

```
hold on;
plot(t(2:end), dCO2_aq_dt, 'DisplayName', 'd[CO2(aq)]/dt');
plot(t(2:end), dH2CO3_dt, 'DisplayName', 'd[H2CO3]/dt');
plot(t(2:end), dHCO3_minus_dt, 'DisplayName', 'd[HCO3^-]/dt');
plot(t(2:end), dCO3_2_minus_dt, 'DisplayName', 'd[CO3^2-]/dt');
plot(t(2:end), dH_plus_dt, 'DisplayName', 'd[H+]/dt');
xlabel('Time (s)');
ylabel('Rate of Change');
title('Rate of Change of Species vs Time');
legend('show');
grid on;
hold off;
```

