(Q1) Solve the following recurrences by giving an upper bound:

(a)
$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1. \\ 2 \cdot T(n/2 + 17) + n & \text{otherwise.} \end{cases}$$

(b)
$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n/2) + 1 & \text{otherwise} \end{cases}$$

(c)
$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 3 \cdot T(n/2) + \log n & \text{otherwise} \end{cases}$$

(d)
$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 4 \cdot T(n/2) + n^2 & \text{otherwise} \end{cases}$$

(e)
$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 8 \cdot T(n/2) + n^2 & \text{otherwise} \end{cases}$$

(f)
$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 4 \cdot T(n/2) + n^2 \log n & \text{otherwise} \end{cases}$$

(g)
$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 2 \cdot T(n/2) + n/\log n & \text{otherwise} \end{cases}$$

(h)
$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ T(n/3) + T(2n/3) + n & \text{otherwise} \end{cases}$$

(i)
$$T(n) = \begin{cases} 1, & \text{if } n = 1. \\ T(n/5) + T(7n/10) + n, & \text{otherwise.} \end{cases}$$

(j)
$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ T(n/5) + T(2n/3) + n & \text{otherwise} \end{cases}$$

(k)

$$T(n) = \begin{cases} \sqrt{n} \cdot T(\sqrt{n}) + n & \text{if } n \geq 1; \\ 1 & \text{if } n = 0. \end{cases}$$

(Q2) Let $A[1..n]$ and $B[1..n]$ be two arrays of distinct integers sorted in increasing order. Give an efficient algorithm ($O(\log n)$) to find the median of the $2n$ elements in both A and B. Derive the time complexity. Implement the algorithm in any programming language.

(Q3) Let $A[1..n]$ be an array of n integers and x an integer. Derive a divide and conquer algorithm to find the frequency of x in A, i.e the number of times x appears in A. What is the time complexity of your algorithm? Implement the algorithm in any programming language.

(Q4) Derive the time complexity of $Select$ algorithm if the group size is 9. Implement the $RandomizedSelect$. Count the number of elements in each recursion and number of total recursion in different runs of the $RandomizedSelect$ varying the number of elements n and value of k. Plot graph to see how it matches with theoretical complexity bounds.

References

[Cormen] T. Cormen, C. Leiserson, R. Rivest, Introduction to Algorithms, Chapters 1-4.