

Techniques to Calculate Time Complexity

Once we are able to write the runtime in terms of size of the input (n), we can find the time complexity.

$$\text{For example } T(n) = n^2 \Rightarrow O(n^2)$$

$$T(n) = \log n \Rightarrow O(\log n)$$

Some tricks to calculate complexity

- 1> Drop the constants \div Any thing you might think is $O(3n)$ is $O(n)$
 \rightarrow Better representation
- 2> Drop the non dominant terms \div Any thing you represent as $O(n^2 + n)$ can be written as $O(n^2)$
- 3> Consider all variables which are provided as input \div $O(mn)$ & $O(mnq)$ might exist for some cases!

In most of the cases, we try to represent the runtime in terms of the input which can be more than one in number. For example \div

$$\text{Painting a park of dimension } m \times n \Rightarrow O(mn)$$

Time Complexity – Competitive Practice Sheet

1. Find the time complexity of the func1 function in the program shown in program1.c as follows:

```
#include <stdio.h>

void func1(int array[], int length)
{
    int sum = 0;
    int product = 1;
    for (int i = 0; i < length; i++)
    {
        sum += array[i];
    }

    for (int i = 0; i < length; i++)
    {
        product *= array[i];
    }
}

int main()
{
    int arr[] = {3, 5, 66};
    func1(arr, 3);
    return 0;
}
```

2. Find the time complexity of the func function in the program from program2.c as follows:

```
void func(int n)
{
    int sum = 0;
    int product = 1;
    for (int i = 0; i < n; i++)
    {
        for (int j = 0; j < n; j++)
        {
            printf("%d , %d\n", i, j);
        }
    }
}
```

3. Consider the recursive algorithm above, where the `random(int n)` spends one unit of time to return a random integer which is evenly distributed within the range $[0, n]$. If the average processing time is $T(n)$, what is the value of $T(6)$?

```
int function(int n)
{
    int i;

    if (n <= 0)
    {
        return 0;
    }
    else
    {
        i = random(n - 1);
        printf("this\n");
        return function(i) + function(n - 1 - i);
    }
}
```

4. Which of the following are equivalent to $O(N)$? Why?
- a) $O(N + P)$, where $P < N/9$
 - b) $O(9N - k)$
 - c) $O(N + 8\log N)$
 - d) $O(N + M^2)$
5. The following simple code sums the values of all the nodes in a balanced binary search tree. What is its runtime?

```
int sum(Node node)
{
    if (node == NULL)
    {
        return 0;
    }
    return sum(node.left) + node.value + sum(node.right);
}
```

6. Find the complexity of the following code which tests whether a given number is prime or not?

```
int isPrime(int n){
    if (n == 1){
        return 0;
    }

    for (int i = 2; i * i < n; i++) {
        if (n % i == 0)
            return 0;
    }
}
```

```
    return 1;
}
```

7. What is the time complexity of the following snippet of code?

```
int isPrime(int n){
    for (int i = 2; i * i < 10000; i++) {
        if (n % i == 0)
            return 0;
    }

    return 1;
}
isPrime();
```