MCS2 Linear Algebra (Assignment 2)

October 2021

1 Questions

1. Show that the transformation $T(x_1, x_2) = (4x_1 - 2x_2, 3|x_2|)$ is not linear.

2. Let $T : R^3 \to R^2$ projects each point (x_1, x_2, x_3) onto the x_1x_2 plane. Find the matrix representation of the linear transformation T

3. Find the kernel and range of the following linear transformations:
(i) Differential operator $D : P^3 \to P^2$ defined by $D(p(x)) = \frac{d}{dx}p(x)$, P^i is a polynomial of degree i.
(ii) $S : P^1 \to R$, $S(p(x)) = \int_0^1 p(x)dx$
(iii) $T : M_{22} \to M_{22}$, $T(A) = A^T$, M_{22} is a $2 \otimes 2$ matrix and T is the transpose of the matrix.

4. Let $B = [v_1, v_2, ...v_n]$ be a basis of a vector space V and let $u_1, u_2....u_n$ be vectors in V. Then show that $[u_1, u_2......u_n]$ is linearly independent in V iff $[[u_1]_B, [u_2]_B.....[u_n]_B]$ is linearly independent in R^n.

5. Let B and C be bases for P_2 (set of all real polynomials upto degree 2) . If $B = [x, 1 + x, 1 - x + x^2]$ and the change of basis matrix from $B$to C is $P_{C \leftarrow B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ Then find the basis C.

6. A linear transformation $T : V \to V$ is given. If possible find a basis C for V such that the matrix $[T]_C$ of T with respect to C is diagonal.
(a) $T : R^2 \to R^2$ defined by $T \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -4b \\ a + 5b \end{bmatrix}$
(b) $T : P_2 \to P_2$ defined by $T(p(x)) = p(3x + 2)$.

7. Define V as the vector space of all polynomials in x of degree < 3 over R. Define a set $B = \{x^2, x, 1\}$. Define a transformation T as

$$\begin{aligned} T(x^2) &= x + m \\ T(x) &= (m - 1)x \\ T(1) &= x^2 + m \end{aligned}$$

Answer the following:

(a) Prove that B is a basis.

(b) Show that T is a linear transformation.

(c) Find the matrix representation of T relative to the given basis.

(d) Find kernel(T) for all values of m.

(e) Find the image of T for all values of m.

8. Find the coordinate vector of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ with respect to the Basis $B = [E_{22}, E_{21}, E_{12}, E_{11}]$ of $M_{2\times 2}$(set of all 2×2 matrices) (E_{ij} are the standard basis vectors).