MCS2 Linear Algebra (Assignment 3)

October 2021

1 Questions

1. (a) Using Gaussian elimination, solve for x, y and z in

$$x + 3y + 5z = 14, 2x - y - 3z = 3, 4x + 5y - z = 7$$

   (b) Using Gauss-Jordan elimination, solve for x, y and z in

$$y + z = 4, 3x + 6y - 3z = 3, -2x - 3y + 7z = 10$$

   (c) Using Gauss-Jordan elimination, solve for x, y and z in

$$\sqrt{2}x + y + 2z = 1, \sqrt{2}y - 3z = -\sqrt{2}, -y + \sqrt{2}z = 1$$

2. Use Cramer's rule to solve the given linear system
   $x + y - z = 1$
   $x + y + z = 2$
   $x - y = 3.$

3. Use Cramer's rule to solve the given linear system
   $2x + y - 3z = 1$
   $y + z = 1$
   $z = 1.$

4. Use Cramer's rule to solve the given linear system
   $x + y = 1$
   $x - y = 2.$

5. Let λ_i $(i = 1, 2, ..n)$ are complete set of eigenvalues (repetitions included) of the $n \times n$ matrix A. Prove that $det(A) = \Pi_{i=1}^{n} \lambda_i$

6. Prove that $det(AB) = det(BA)$

7. If A is idempotent find all possible values of $det(A)$.

8. Find the (a) characteristic equations (b) eigenvalues (c) basis for the eigen space of the eigen values (d) algebraic multiplicity and geometric multiplicity of the eigen values of the follwing matrices

(a) $\begin{pmatrix} 1 & 3 \\ -2 & 6 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

9. Show that given vectors forms an orthogonal basis for R^3

(a) $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $v_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(b) $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $v_3 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$, $v_4 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

10. Find the inverse of following matrices using Gauss Jordan Method,

(a) $A = \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}$

(b) $B = \begin{pmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{pmatrix}$