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Assignment-3

1.(a)

$$x + 3y + 5z = 14$$
$$2x - y - 3z = 3$$
$$4x + 5y - z = 7$$

Gaussian Elimination $\longrightarrow$

Augmented matrix $\left[\begin{array}{ccc:c} 1 & 3 & 5 & 14 \\ 2 & -1 & -3 & 3 \\ 4 & 5 & -1 & 7 \end{array}\right]$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 3 & 5 & 14 \\ 0 & -7 & -13 & -25 \\ 0 & -7 & -21 & -49 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 3 & 5 & 14 \\ 0 & -7 & -13 & -25 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$\hookrightarrow$ Row-echelon matrix

$-8z = -4 \Rightarrow \boxed{z=3}$

$-7y - 13z = -25$

$-7(y) + 13(-4) = -25$

$7y = -14 \Rightarrow \boxed{y=-2}$

$x + 3y + 5z = 14$

$x + 3(-2) + 5(3) = 14$

$\Rightarrow \boxed{x=5}$

$\therefore$ Solution is -

$x = 5$; $y = -2$, $z = 3$

1(b)

$$y + z = 4$$
$$3x + 6y - 3z = 3$$
$$-2x - 3y + 7z = 10$$

Gauss-Jordan elimination →

Augmented matrix = $\left[\begin{array}{ccc|c} 0 & 1 & 1 & 4 \\ 3 & 6 & -3 & 3 \\ -2 & -3 & 7 & 10 \end{array}\right]$

$R_2 \leftrightarrow R_1$

$$\sim \left[\begin{array}{ccc|c} 3 & 6 & -3 & 3 \\ 0 & 1 & 1 & 4 \\ -2 & -3 & 7 & 10 \end{array}\right]$$

$R_1 \to R_1 + R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 13 \\ 0 & 1 & 1 & 4 \\ -2 & -3 & 7 & 10 \end{array}\right]$$

$R_3 \to R_3 + 2R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 13 \\ 0 & 1 & 1 & 4 \\ 0 & 3 & 15 & 36 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 13 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 12 & 24 \end{array}\right]$$

$R_3 \to R_3 / 12$.

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 13 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 \to R_1 - 3R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\boxed{x = -1, \quad y = 2, \quad z = 2}$ Solution

1.(c) Equation -

$$\sqrt{2}x + y + 2z = 1$$
$$\sqrt{2}y - 3z = -\sqrt{2}$$
$$-y + \sqrt{2}z = 1$$

Gauss-Jordan elimination -

$$\left[\begin{array}{ccc|c} \sqrt{2} & 1 & 2 & 1 \\ 0 & \sqrt{2} & -3 & -\sqrt{2} \\ 0 & -1 & \sqrt{2} & 1 \end{array}\right]$$

$R_3 \to \sqrt{2}R_3 + R_2$

$$\sim \left[\begin{array}{ccc|c} \sqrt{2} & 1 & 2 & 1 \\ 0 & \sqrt{2} & -3 & -\sqrt{2} \\ 0 & 0 & -1 & -0 \end{array}\right]$$

$R_1 \to R_1 + 2R_3$, $R_2 \to R_2 - 3R_3$

$$\sim \left[\begin{array}{ccc|c} \sqrt{2} & 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 & -\sqrt{2} \\ 0 & 0 & -1 & 0 \end{array}\right]$$

$R_2 \to R_2 / \sqrt{2}$, $R_3 \to (-1) * R_3$

$$\sim \left[\begin{array}{ccc|c} \sqrt{2} & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$\sim \left[\begin{array}{ccc|c} \sqrt{2} & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 \to R_1/\sqrt{2}$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \sqrt{2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\boxed{x = \sqrt{2} \qquad y = -1 \qquad z = 0}$$ Solution

2. Equations

$$x + y - z = 1$$
$$x + y + z = 2$$
$$x - y = 3$$

Cramer's Rule $\rightarrow$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1(1) - 1(-1) - 1(-1-1)$$
$$= 4$$

$$x = \frac{D_x}{D} = \frac{1}{4}\begin{vmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 3 & -1 & 0 \end{vmatrix}$$

$$= \frac{1}{4}\left[1(1) - 1(-3) - 1(-2-3)\right]$$

$$= 9/4$$

$$y = \frac{D_y}{D} = \frac{1}{4}\begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 3 & 0 \end{vmatrix}$$

$$= \frac{1}{4}\left[1(-3) - 1(-1) - 1(3-2)\right] - \frac{-3}{4}$$

$$z = \frac{D_z}{D} = \frac{1}{4}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 3 \end{vmatrix}$$

$$= \frac{1}{4}\left[1(5) - 1(1) + 1(-2)\right] = \frac{1}{2}$$

$$\therefore \boxed{x = 9/4,\; y = -3/4,\; z = 1/2}$$ Solution

3. Equations

$$2x + y - 3z = 1$$
$$y + z = 1$$
$$z = 1$$

Cramer's rule →

$$D = \begin{vmatrix} 2 & 1 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2(1-0) + 0 + 0 = 2$$

$$x = \frac{D_x}{D} = \frac{1}{2}\begin{vmatrix} 1 & 1 & -3 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(1) - 1(1-1) - 3(-1)\right] = 2$$

$$y = \frac{D_y}{D} = \frac{1}{2}\begin{vmatrix} 2 & 1 & -3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(1-1) + 0 + 0\right] = 0$$

$$z = \frac{D_z}{2} = \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}\left[2(1) + 0 + 0\right] = 1$$

$$\boxed{x = 2,\; y = 0,\; z = 1}$$ Solution

4. Equations : $x+y=1$
$x-y=2$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1(-1) - 1(1) = -2$$

$$x = \frac{D_x}{D} = \frac{-1}{2} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$$
$$= \frac{1}{-2}\left[1(-1) - 1(2)\right] = \frac{3}{2}$$

$$y = \frac{D_y}{D} = \frac{-1}{2} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= \frac{1}{-2}\left[1(2) - 1(1)\right] = \frac{-1}{2}$$

$$\boxed{x = \frac{3}{2}, \; y = \frac{-1}{2}}$$ Solution

5. Let, $\lambda_i \ (i=1,2,\dots,n)$ are complete set of eigen values of $A_{n\times n}$. Prove that $\det(A) = \prod_{i=1}^{n} \lambda_i$

Given, $\lambda_1, \lambda_2, \dots, \lambda_n$ are eigen values of A.

λ_i are roots of charecteristic equation $|A - \lambda I| = 0$

$\therefore \; |A - \lambda I| = (\lambda_1 - \lambda)(\lambda_2 - \lambda) \dots (\lambda_n - \lambda)$

put $\lambda = 0$

$|A - 0.I| = (\lambda_1 - 0)(\lambda_2 - 0) \dots (\lambda_n - 0)$

$|A| = \lambda_1 . \lambda_2 \dots \lambda_n$

$$\boxed{\det = \prod_{i=1}^{n} \lambda_i}$$

6. To prove $\det(AB) = \det(BA)$

$$LHS = \det(AB)$$
$$= \det(A) \cdot \det(B)$$
$$= \det(B) \cdot \det(A)$$

[$\det(A)$ and $\det(B)$ are real numbers, and real numbers are commutative under multiplication]

$$= \det(BA)$$
$$= RHS$$

7. If A is idempotent, then $A^2 = A$

Hence,
$$|A^2| = |A|$$
$$\Rightarrow |A \cdot A| = |A|$$
$$\Rightarrow |A \cdot A| - |A| = 0$$
$$\Rightarrow |A| \cdot |A| - |A| = 0$$
$$\Rightarrow |A|(|A| - 1) = 0$$
$$\Rightarrow \boxed{|A| = 0} \quad \text{or} \quad \boxed{|A| = 1}$$

Possible values of $|A|$ are 0 and 1.

8.(a) Given, $A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$

Characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ -2 & 6-\lambda \end{vmatrix} = 0$$
$$(1-\lambda)(6-\lambda) + 6 = 0$$
$$\Rightarrow \lambda^2 - 7\lambda + 6 + 6 = 0$$
$$\Rightarrow \boxed{\lambda^2 - 7\lambda + 12 = 0} \rightarrow \text{Characteristic eqn}$$
$$\Rightarrow (\lambda - 3)(\lambda - 4) = 0$$
$$\Rightarrow \boxed{\lambda = 3, 4} \rightarrow \text{Eigen values}$$

At $\lambda = 4$

$$(A - \lambda I) = (A - 4I)$$
$$= \begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix}$$
$$\sim \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow -x + y = 0$$
$$\Rightarrow \boxed{x = y} = k \quad \text{(say)}$$

General solution is $\begin{bmatrix} k \\ k \end{bmatrix}$

$\therefore$ Basis of eigen space for $\lambda = 4$ is $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

$\therefore$ Algebraic multiplicity $= 1$

$\therefore$ Geometric multiplicity $= \dim (V_{\lambda=4}) = 1$

At $\lambda = 3$ $\quad (A - \lambda I) = (A - 3I)$

$$= \begin{bmatrix} -2 & 3 \\ -2 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 3 \\ 0 & 0 \end{bmatrix}$$

Thus, $-2x + 3y = 0 \Rightarrow 2x = 3y$

$\therefore$ Basis of eigen space for $\lambda = 3$ is $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$

$\therefore$ Algebraic multiplicity $= 1$

Geometric multiplicity $= \dim (V_{\lambda=3}) = 1$

8.(b)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Characteristic equation $\Rightarrow$ $|A-\lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ -1 & -1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)\left[(\lambda+1)(\lambda-1) - 1\right] - 2\left[(\lambda-1) - 0\right] + 0 = 0$

$\lambda^2 - 2 - \lambda^3 + 2\lambda - 2\lambda + 2 = 0$

$\Rightarrow \boxed{\lambda^2 - \lambda^3 = 0} \longrightarrow$ Characteristic equation

$\lambda^2(1-\lambda) = 0$

$\boxed{\lambda = 0, 0, 1} \longrightarrow$ Eigen values

At $\lambda = 0$ $\quad (A-\lambda I) = (A - 0\cdot I) = A$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$$\sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x + 2y = 0$, $\quad y + z = 0$

$y = -z$

put $z = k$ (say)

$\therefore y = -k$

$x + 2(-k) = 0$

$x = 2k$

General solution is $\begin{bmatrix} 2k \\ -k \\ k \end{bmatrix}$

$\therefore$ Basis of eigen space for $\lambda = 0$ is $\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$

$\therefore$ Algebraic multiplicity $= 2$

$\therefore$ Geometric multiplicity $= \dim(V_{\lambda=0}) = 1$

<u>At $\lambda = 1$</u>

$$(A - \lambda I) = (A - I)$$

$$= \begin{bmatrix} 0 & 2 & 0 \\ -1 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2$$

$$= \begin{bmatrix} -1 & -2 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_3 \to R_3 - \frac{R_2}{2}, \quad R_1 \to (-1) * R_1$$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \to R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $x - z = 0$ and $2y = 0$

$x = z = k$ (say) $\qquad y = 0$

General solution is $\left\{ \begin{bmatrix} k \\ 0 \\ k \end{bmatrix} \right\}$

$\therefore$ Basis of eigen space for $\lambda = 1$ is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\therefore$ Algebraicc multiplicity $= 0$

$\therefore$ Geometric multiplicity $= \dim(V_{\lambda=1}) = 1$

8(c) $$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Characteristic equation for A is $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(\lambda-1)\lambda - 1\right] - 0 + 1\left[0 - (1-\lambda)\right] = 0$$

$$(1-\lambda)(\lambda^2 - \lambda - 1) - 1 + \lambda = 0$$

$$\lambda^2 - \lambda - 1 - \lambda^3 + \lambda^2 + \lambda - 1 + \lambda = 0$$

$$\Rightarrow \boxed{\lambda^3 - 2\lambda^2 - \lambda + 2 = 0} \rightarrow \text{Characteristic equation}$$

$$(\lambda-1)(\lambda^2 - \lambda - 2) = 0 \Rightarrow (\lambda-1)(\lambda+1)(\lambda-2) = 0$$

$$\boxed{\lambda = 1, -1, 2} \rightarrow \text{Eigen values}$$

At $\lambda = 1$: $(A - \lambda I) = (A - I)$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$R_3 \leftrightarrow R_2$, $R_2 \leftrightarrow R_1$

$$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$, $R_1 \to R_1 + R_2$

$$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $x+y=0$ and $z=0$

$x=-y$

put $(y=k)$

$x=-k$

$\therefore$ General solution is $\begin{bmatrix} -k \\ k \\ 0 \end{bmatrix}$

Basis of eigen space for $\lambda=1$ is $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

$\therefore$ Algebraic multiplicity $=1$

$\therefore$ Geometric multiplicity $=\dim(V_{\lambda=1})=1$

<u>At $\lambda=-1$</u>

$$(A-\lambda I) = (A+I)$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_1 \to R_1/2$

$$\sim \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 2 & 1 \\ 0 & 1 & 1/2 \end{bmatrix}$$

$R_3 \to R_3 - \frac{R_2}{2}$

$$\sim \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore\ x + \frac{z}{2} = 0$ and $2y + z = 0$

$\Rightarrow \boxed{x = -z/2}$ and $\boxed{y = -z/2}$

put $z = 2k$ (say)

$\therefore\ x = -k$ and $y = -k$

$\therefore$ General solution is $\begin{bmatrix} -k \\ -k \\ 2k \end{bmatrix}$

$\therefore$ Basis of eigen space for $\lambda = -1$ is $\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$

Algebraic multiplicity $= 1$
Geometric multiplicity $= 1$

<u>At $\lambda = 2$</u> $\quad (A - \lambda I) = (A - 2I)$

$$= \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$\sim \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore\ -x + z = 0$ and $-y + z = 0$

$x = z$ and $y = z$

put $z = k$ (say) $\quad \therefore\ x = y = z = k$

General solution is $\begin{bmatrix} k \\ k \\ k \end{bmatrix}$

$\therefore$ Basis of eigen space for $\lambda = 2$ is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

$\therefore$ Algebraic multiplicity $= 1$
Geometric multiplicity $= 1$

9. (a) If the set of vectors $\{V_1, V_2, \ldots, V_n\}$ are orthogonal then it means $V_i \cdot V_j = 0 \quad \forall\, i, j \in 1, 2, \ldots, n$
and $i \neq j$

$V_1 \cdot V_2 = 1(1) + 0(2) - 1(1) = 1 - 1 = 0$
$V_1 \cdot V_3 = 1(1) + 0(-1) - 1(1) = 1 - 1 = 0$
$V_1 \cdot V_4 = 1(1) + 0(1) - 1(1) = 1 - 1 = 0$
$V_2 \cdot V_3 = 1(1) + 2(-1) + 1(1) = 2 - 2 = 0$
$V_2 \cdot V_4 = 1(1) + 2(1) + 1(1) = 4 \neq 0$
$V_3 \cdot V_4 = 1(1) - 1(1) + 1(1) = 1 \neq 0$

$\therefore$ Vectors are not mutually orthogonal and hence does not form orthogonal basis.

(b) $V_1 \cdot V_2 = 1(1) + 1(-1) + 1(0) = 0$
$V_1 \cdot V_3 = 1(1) + 1(1) + 1(-2) = 2 - 2 = 0$
$V_1 \cdot V_4 = 1(1) + 1(2) + 1(3) = 6 \neq 0$
$V_2 \cdot V_3 = 1(1) - 1(1) + 0(-2) = 0$
$V_2 \cdot V_4 = 1(1) - 1(2) + 0(3) = -1 \neq 0$
$V_3 \cdot V_4 = 1(1) + 1(2) - 2(3) = -3 \neq 0$

$\therefore$ Vectors are not mutually orthogonal and hence does not form orthogonal basis.

10.(a) $A = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

Inverse using Gauss Jordan Method →

Augment given matrix with identity matrix $= \left[\begin{array}{cc|cc} -2 & 4 & 1 & 0 \\ 3 & -1 & 0 & 1 \end{array}\right]$

$R_1 \to R_1/(-2)$

$$\sim \left[\begin{array}{cc|cc} 1 & -2 & -1/2 & 0 \\ 3 & -1 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$

$$\sim \left[\begin{array}{cc|cc} 1 & -2 & -1/2 & 0 \\ 0 & 5 & 3/2 & 1 \end{array}\right]$$

$R_2 \to R_2/5$

$$\sim \left[\begin{array}{cc|cc} 1 & -2 & -1/2 & 0 \\ 0 & 1 & 3/10 & 1/5 \end{array}\right]$$

$R_1 \to R_1 + 2R_2$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 1/10 & 2/5 \\ 0 & 1 & 3/10 & 1/5 \end{array}\right]$$

Identity matrix

$$\therefore A^{-1} = \begin{bmatrix} 1/10 & 2/5 \\ 3/10 & 1/5 \end{bmatrix}$$

10 (b) $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{bmatrix}$

Inverse using Gauss Jordan method $\rightarrow$

Augment given matrix with identity matrix $= \left[\begin{array}{ccc|ccc} 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$

$R_1 \rightarrow R_1/2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3/2 & 0 & 1/2 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3/2 & 0 & 1/2 & 0 & 0 \\ 0 & -7/2 & -1 & -1/2 & 1 & 0 \\ 0 & -3 & -1 & -1 & 0 & 1 \end{array}\right]$$

$R_2 \rightarrow R_2 * (-2/7)$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 2/7 & 1/7 & -2/7 & 0 \\ 0 & -3 & -1 & -1 & 0 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 + 3R_2$, $R_1 \rightarrow R_1 - \frac{3}{2}R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -3/7 & 2/7 & 3/7 & 0 \\ 0 & 1 & 2/7 & 1/7 & -2/7 & 0 \\ 0 & 0 & -1/7 & -4/7 & -6/7 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 * (-7)$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -3/7 & 2/7 & 3/7 & 0 \\ 0 & 1 & 2/7 & 1/7 & -2/7 & 0 \\ 0 & 0 & 1 & 4 & 6 & -7 \end{array}\right]$$

$R_1 \to R_1 + \frac{3}{7} R_3$, $R_2 \to R_2 - \frac{2}{7} R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 3 & -3 \\ 0 & 1 & 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & 4 & 6 & -7 \end{array}\right]$$

Identity Matrix

$$\therefore B^{-1} = \begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 4 & 6 & -7 \end{bmatrix}$$