MCS2 Linear Algebra

October 2021

1 Questions

1. Find out whether the following are a vector spaces or not. If not, list all the axioms that fail to hold :
   (a) The set of rational numbers with usual addition and multiplications
   (b) The set of all skew symmetric $n \times n$ matrices with the usual matrix addition and scalar multiplication.
   (c) The set of all upper triangular 2×2 matrices with the usual matrix addition and scalar multiplication.

2. Denote by $R[0,1]$ the set of all continuous real valued functions with domain $[0,1]$, i.e.,

   $$R[0,1]\{allfunctions\ f : [0,1] \to R,\ suchthat\ f\ iscontinuous\}.$$

   Show that $R[0,1]$ forms a vector space over R.

3. In the space $R[0,1]$, define the vectors f, g, and h by $f(x) = x$, $g(x) = e^x$ and $h(x) = e^{-x}$ for $0 \leq x \leq 1$. Use the definition of linear independence to show that the functions f, g, and h are linearly independent.

4. Let V be a vector space of $n \times n (n \geq 2)$ matrices over an arbitrary field F. Which of the following sets of matrices A in V are subspaces of V?
   (a) Set of all invertable matrices A.
   (b) Set of all non-invertable matrices A.
   (c) All A such that $AB = BA$ for a fixed matrix B in V.
   (d) All idempotent matrices A

5. **S**uppose V is a vector space of all functions $f : R \to R$. Which of the following sets of functions are subspaces of V?
   (a) all continuous functions.
   (b) f such that $f(x^2) = f(x)^2$.

(c) $f(3) = 1 + f(-5)$

6. Let V and W be vector spaces over a field F. Let $Z = [(v, w), v \in V, w \in W]$. Prove that Z is a vector space over the field F with the operations: $(v_1, w_1) + (v_2, w_2) = (v_1 + v_2, w_1 + w_2)$ and $c(v_1, w_1) = (cv_1, cw_1)$.

7. Let V be a vector space and let $S_1 \subseteq S_2 \subseteq V$. Show that if S_2 is linearly independent then S_1 is linearly independent.

8. Show that a subset W of a vector space V is a subspace of V if and only if $span(W) = W$.