CMJ 1230 SOLUTION

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1. Solution

For a triangle with side lengths $(a, b, b+1)\, a, b \in \mathbb{Z}$, this result in a integer perimeter and a integer area since at most one value is even. Given that these values are relatively prime, these are primitive Heronian triangles. We now claim that the equivalent rectangle will have sides $\left(b+1, \frac{a-1}{2}\right)$. Since the area and perimeters of the triangle and rectangle are equivalent we can rewrite the semi-perimeter in the following manner,

$$S = \frac{a+b+(b+1)}{2} = \frac{2b+a+1}{2}$$

$$(b+1)\left(\frac{a-1}{2}\right) = \sqrt{(S)(S-a)(S-b)(S-c)}$$

$$(b+1)\left(\frac{a-1}{2}\right) = \sqrt{\left(\frac{2b+a+1}{2}\right)\left(\frac{2b-a+1}{2}\right)\left(\frac{a+1}{2}\right)\left(\frac{a-1}{2}\right)}$$

Using the values generated by the equation and running a regression model we have two expressions that generate values for side lengths a and b respectively.

$$a_n = 5 + 6n + 2n^2$$

$$b_n = 5 + 9n + 5n^2 + n^3$$

For any $n \in \mathbb{Z}$, the two expressions for a_n and b_n described above can be used to generate a and b values for the triangle, and the last side length is $(b+1)$. The corresponding equivalent rectangles have dimensions $\left(b+1, \frac{a-1}{2}\right)$. For an infinite domain of $n \geq 0$, where $n \in \mathbb{Z}$, infinite cases of primitive Heronian triangles can be generated using the expressions for a and b in terms of n, showing that infinitely many primitive Heronian triangles with equivalent rectangles exist.

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