

Parser Rule:

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line      : build EOF;
LEFT_PAREN : '(';
RIGHT_PAREN : ')';
build     : (LEFT_PAREN build RIGHT_PAREN)*;
```

Informal proof of correctness:

It suffices to prove the following two things:

- 1) Every valid balanced parentheses (BP) can be described by the recursive definition of build.
- 2) Any string created from build must be a BP.

We proceed inductively, by proving (1) and (2) on BP's of length $2k$, for all non-negative integers k . It is trivial to prove that BP's cannot have odd length.

Base case: $k = 0$. 1) clearly holds because the length 0 BP is the empty string, and since any grammar α^* implies 0 or more α s, (1) holds here. Similarly, the only length-0 string that can be created from build is the empty string, which is a BP.

We can also easily establish (1) and (2) for base cases $k=1$ and $k=2$.

Inductive step: Suppose (1) and (2) hold for length- $2n$ cases, $n \geq 2$. We will establish that they hold for length- $2(n+1)$ string, S . The proof follows easily from observing construction of build. S must be a concatenation of more than 1 smaller builds (in which case, S is valid, for a concatenation of BP's necessarily results in a BP) or S is of the format *left_paren build right_paren*. Our inductive assumption assures us that *build* is valid, hence S must be valid as well. This proves (2).

(1) follows from noting that any BP must be a concatenation of smaller BP's, or must otherwise be a smaller BP wrapped in left and right parentheses. This is precisely the definition of build.