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CHAPTER-4

ELASTICITY

4.1 Introduction

When a Crystalline body*, in statical, equilibrium, is subjected to an external force, the force tends to displace the particles within the body (the particles may be atoms, ions or molecules) from their equilibrium postitions and as a result the body is deformed. However, the altered condition of the body is opposed by a restoring force which builds up to a value that balances the external force. When the applied force is released, the restoring force brings the particles back to their equilibrium position and consequently the original state of the body is achieved. This property by virtue of which a body regains its original state when the deforming force is withdrawn is what is known as **elasticity.**

The presence of a restoring force can be best understood by examining the force between two neighbouring particles as a function of interparticle separation as shown in fig. 4.1.

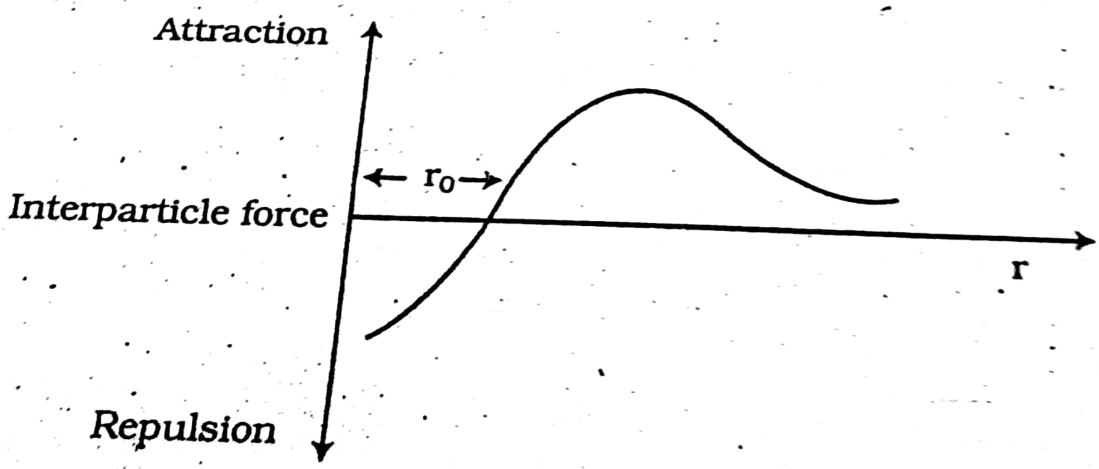

Fig : 4.1

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* Solids whose atoms are arranged in a regular and periodic manner are called crystalline solids. Example-NaCl, quartz, Cu, Ag

At equilibrium the particle separation is r_0 when the force of repulsion equals the force of attraction and so the resultant force is zero. When they are displaced from the equilibrium position the force between the particles is not zero any more. The restoring force will be attractive if the particle separation is increased and repulsive if particle separation is decreased. In either case the restoring force always opposes the external force applied.

4.2 Stress and strain

A strain is a measure of deformation; a stress measures the effectiveness of a force in producing deformation, stress is always defined as the force exerted per unit area of the body. Strain is generally defined as deformation or distortion in the body per unit dimension.

While discussing stress-strain relationships in different materials it is necessary to divide them into two main classes called **isotropic** and **anisotropic**. In isotropic materials the strain at any point is independent of direction, or the elastic properties of the materials are the same in every direction. For anisotropic materials the elastic properties at any point are different in different directions about the point. Quartz and many other crystalline substances are anisotropic. They also show different optical properties in different directions.

Crystalline substances are built up from a characteristic unit cell with atoms of the substance so placed as to outline the shape of the cell. Large single crystals having a great number of unit cells in orderly arrangement can be made. It has been found that the elastic, thermal, and magnetic properties of these single crystals depend on the direction relative to the crystal axes. Most solids are polycrystalline, i.e., they consist of microscopic crystals, called **grains**, which are joined together at their boundaries. If the grains are

Isotropic → which have the same properties at all points and in all directions oriented at random in the solid, then, the variation in properties with crystal direction averages out and the polycrystalline solid behaves as an isotropic substance. By suitable metallurgical procedures it is possible to orient the grains in some polycrystalline substances more or less in the same direction. Such substances show decidedly different properties in different directions, i.e., they are anisotropic. There are a number of technological applications for such substances. The analysis of the relationship between stress and strain for anisotropic materials is relatively complicated and must be left to more advanced treatises. Here we shall limit the discussion to isotropic materials.

4.3 Different kinds of stress

Stress is generally divided into three classes :

(1) Tensile or Longitudinal Stress : It is the stretching force acting per unit area of the section of the solid acting in the direction of its length.

(2) Compressive or volume Stress : It is the uniform force acting per unit area on the faces of a cube.

(3) Shearing or Tangential Stress : It is the force acting tangentially per unit area on the body.

4.4 Different kinds of Strain

Strain produced in a body by the application of external forces acting on it may be divided into three kinds according as they consist of a change in length and in volume or a change in shape only.

(1) Longitudinal or Tensile Strain : It is the change of length per unit length in the direction of the length of a rod whose length is very large in comparison with its dimensions.

$$\text{Longitudinal Strain} = \frac{l}{L}$$

where L is the original length of the rod and l the change produced in this length.

(2) Volume Strain : or Bulk strain The change in volume without any change in shape may be produced in all the three states of matter when a body in any one of those states is subjected to uniform pressure acting normally everywhere on its surface. Thus strain is measured by the change in volume per unit volume of the body. If V be the original volume of the body and dv, the change produced in this volume, then

$$\text{Volume Strain} = \frac{dv}{V}$$

(3) Shearing Strain : The shape of the body may be altered in various ways viz. (1) by the tension or stretching (2) by flexure or bending (3) by torsion or twisting.

4.5 Hooke's law

For small deformation and up to the elastic limit the stress acting on the body is proportional to the strain produced in the body and the relation between the stress and the strain is represented by a straight line. The term elastic limit means the largest deformation which does not leave permanent distortion.

4.6 Behaviour of a Strained wire

The stress-strain curve (Fig 4.2) shows how the strain change with the stress. When the stress is zero, the strain is zero also and the curve passes through O. From O upto the point A, Hook's law of linear relationship holds as the strain increases proportionally with the gradual increase of stress. The point A corresponds to the proportional limit, With further increase of stress the strain increases at a faster ratio than that between O and A and the upper point B is reached. The point B is called the **Elastic limit or yield point.**

If the deforming force is continued for a long time the shape of the body will be permanently altered even though the stress be less than the limit of elasticity. This phenomenon is referred to as **elastic fatigue**.

So. $\frac{\text{stress}}{\text{strain}}$ = a constant = Modulus or coefficient of Elasticity.

For different natures of strain there are different moduli of elasticity.

4.7 Elastic Constants

There are four kinds of elastic constants of a solid;

(1) Young's Modulus or Tensile Elasticity.

(2) Bulk Modulus or volume Elasticity.

(3) Shear Elasticity or simply Rigidity.

(4) Poisson's Ratio.

Young's Modulus :

If a force F acts along the length L of a wire of cross section A and produces an elongation l. Within elastic limit, the ratio of longitudinal stress to longitudinal strain is called the Young's modulus of the material of the wire.

The longitudinal stress $= \frac{F}{A} = \frac{F}{\pi r^2}$

where r is the radius of the wire.

The longitudinal strain $= \frac{l}{L}$

$$Y = \text{Young's Modulus} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$= \frac{F/A}{l/L} = \frac{FL}{lA} = \frac{FL}{\pi r^2 l}$$

$$= \frac{MgL}{\pi r^2 l}\ \text{N/m}^2.$$

where M = mass of the body.

Bulk Modulus or coefficient of volume Elasticity

The ratio of stress to strain in the body which undergoes a change in volume only (without any change in shape) is known as Bulk Modulus. Here the stress acting on the body is the pressure and the strain, the change in volume per unit volume of the body.

If, by the pressure p, i.e., the force per unit area on the body, a change in volume, v is produced in the body of volume V, then

$$K \text{ (Bulk Modulus)} = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$= \frac{P}{v/V} = \frac{PV}{v}$$

In the language of calculus it is,

$$K = -\frac{dp}{dv/v} = -v\frac{dp}{dv}$$

'—' sign because an increase of pressure is accompanied by a decrease in volume.

The reciprocal of the bulk modulus is called **compressibility**,

i.e. $C = \frac{1}{k}$

Rigidity Modulus

When a body suffers a shearing strain by the application of tangential force on its surface the modulus of rigidity is defined as the ratio of the tangential force per unit area to the angular deformation produced.

Let equal forces act at the middle point of the faces AEFB and HDCG along the respective faces as shown in the figure 4.3. These forces may be termed tangential forces as they act parallel to the face to which they are applied. The effect of these forces on the cube is to alter its shape and also to rotate it as they form a couple. To prevent the rotational motion let two other forces, each of the same magnitude as before, be applied through the middle points of the faces AEHD and BCGF along the faces in directions as shown in the figure. The system of forces now consist of two equal and opposite couples. Their only effect on the cube is to alter its shape, so that the face ABCD or its parallel, becomes a rhombus A'B'CD from a square. This type of strain is called a shear and is measured by the deformation of the angle ADC i.e. by the angle ADA', which is called the angle of shear.

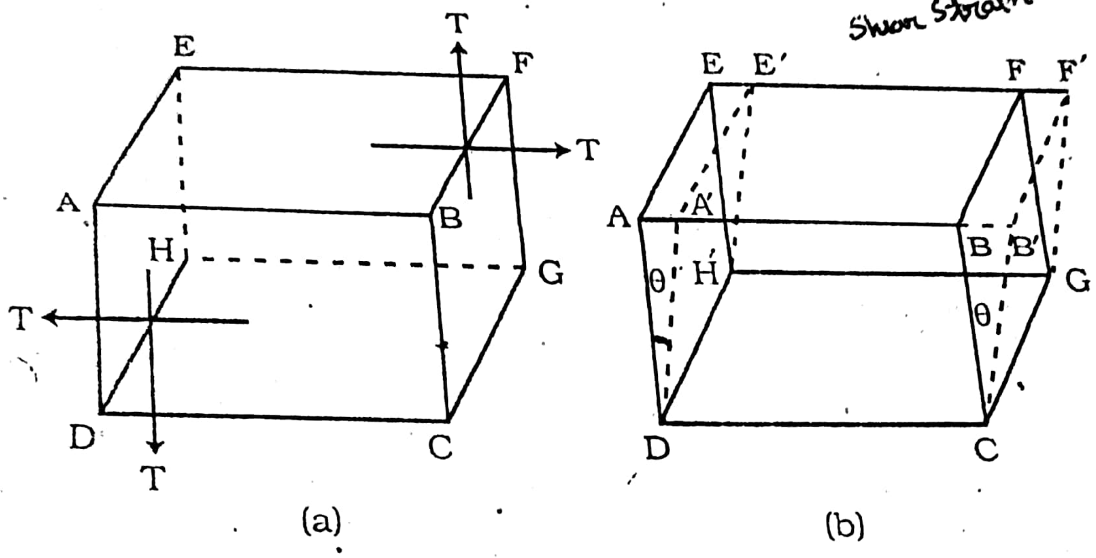

Fig. 4.3

Poisson's Ratio

When a body undergoes a longitudinal extension, it experiences a contraction side ways. This lateral contraction is directly proportional to the longitudinal extension when the longitudinal extension is small.

The ratio of the lateral strain to longitudinal strain is called **Poisson's ratio**. Thus if a wire of circular section having a length L and diameter D, is subject to a longitudinal stress which stretches it by an amount l, the longitudinal strain = l/L. If the consequant decrease in diameter amounts to d, the lateral strain = d/D. Poisson's ratio

$$(\sigma) = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{d/D}{l/L} = \frac{dL}{Dl}$$

4.8 Shear is equivalent to an extension and an equal compression in mutually perpendicular directions

We have seen that when a cube is sheared its volume is unchanged but its shape is altered, the thickness of the block remaining unchanged.

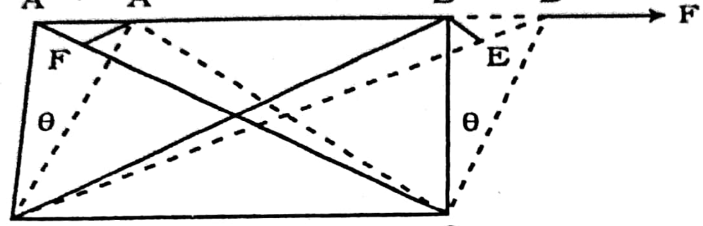

Fig : 4.4

In the figure the diagonal BD has increased to length DB′ and at the same time AC has been diminished to A′C.

Since the amount of shear is extremely small, this extension and compression may be expressed in terms of θ.

If BE be drawn perpendicularly to DB′,

$EB' = BB' \cos BB'E = BB' \cos 45° = \frac{BB'}{\sqrt{2}}$ Since the triangles BEB′ and AFA′ are right-angled 45° triangles. If AB $= l$, $BD = DE = l\sqrt{2}$

Extension strain along BD $= \frac{EB'}{DB} = \frac{BB'}{\sqrt{2}} \frac{1}{l\sqrt{2}} = \frac{BB'}{2l}$

But $\theta = \frac{BB'}{l}$ $\because \tan\theta = \theta$, when θ is small.

Extension strain = θ/2

Similarly, there is a compression strain along AC

$\therefore$ Compression strain $= \frac{AF}{AC} = \frac{AA'}{AC\sqrt{2}}$ $= \frac{AA' \cos 45°}{AC} = \frac{AA'}{l\sqrt{2} \cdot \sqrt{2}} = \frac{AA'}{2l} = \frac{\theta}{2}$

$= \frac{l\theta}{l\sqrt{2}} = \frac{\theta}{2}$

Thus simple shear θ is equivalent to an extension and an equal compression at right angles to each other, each of value θ/2.

4.9 Relation between Young's modulus (Y), Bulk modulus (K), Rigidity modulus (η) and poisson's ration (σ)

Let a unit cube of a substance be under the action of tangential stresses as shown in Fig. 4.5 (a).

The result will be to distort the cube so that the faces ABCD becomes a rhombus A'B'CD (fig. 4.5b)

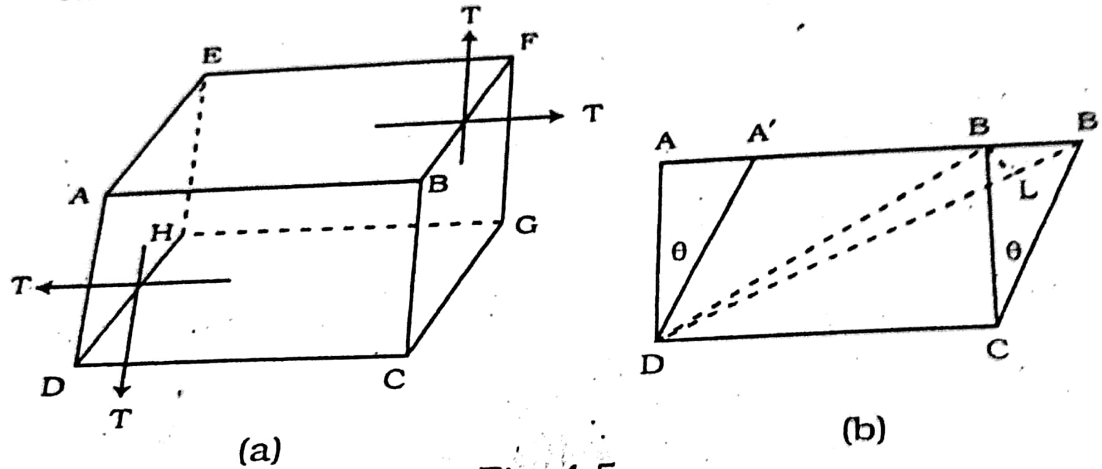

Fig: 4.5

There is only a change of shape but the size remains the same since the area ABCD is equal to that of A'B'CD. The body is unchanged in dimension in a direction perpendicular to ABCD.

The forces applied to the cube must be in equilibrium among themselves i.e., they must have no tendency to set the cube in motion either linear or rotational.

The forces T, T acting on the faces EFBA and HGCD have no tendency to communicate linear motion to the cube, on the other hand they constitute a torque which tends to set the cube in rotation and so to produce equilibrium an equal and opposite torque T, T on the faces ADHE and BCGF must be applied.

Thus the forces T, T, T, T acting on the above faces are sufficient to produce a shear.

Then according to Hook's law we have

$$\eta = \frac{T}{\theta}$$

where T is the tangential stress and θ, the strain.

A shear may be regarded as a combination of an extension together with a contraction perpendicular to extensions.

For in fig. 4.5b the diagonal DB becomes a length DB' and if BL be drawn perpendicular to DB' then the extension strain along DB is

$$\frac{DB'-DB}{DB} = \frac{LB'}{DB} = \frac{BB'\cos 45^\circ}{\sqrt{2}\,BC} = \frac{BB'}{\sqrt{2}} \times \frac{1}{\sqrt{2}\,BC}$$

$$= \frac{BB'}{2BC} = \frac{\theta}{2} = \left(\frac{1}{2}\right)\left(\frac{T}{\eta}\right) \qquad \left[\because \frac{BB'}{BC} = \tan\theta \equiv \theta \text{ when } \theta \text{ is small}\right]$$

Now the stretch in DB may be regarded as compounded of a part due to stretching force T in DB i.e. T/Y and a part due to compressive force T in CA i.e. σ(T/Y) where σ is the poisson's Ratio.

[Note :-

$$\text{Young's modulus } Y = \frac{\text{Longitudinal Stress T}}{\text{Longitudinal Strain}} = \frac{T}{\text{Long-strain}}$$

$$\text{Long strain} = \frac{T}{Y}$$

$$\text{Again Poisson's Ratio, } \sigma = \frac{\text{lateral contraction strain}}{\text{Longitudinal strain}}$$

$$\text{Lateral Contraction Strain} = \sigma\left(\frac{T}{Y}\right)$$]

$$\therefore \quad \frac{1}{2}\frac{T}{\eta} = \frac{T}{Y} + \sigma\frac{T}{Y} = \frac{T}{Y}(1+\sigma)$$

i.e. $Y = 2\eta\,(1+\sigma)$ ... ... (1)

If the unit cube be subjected to a uniform pressure P over each face, the strain in this case is $\frac{dv}{V}$, where, dv is the diminution in volume and V, the original volume.

But by Hooke's Law

$$\frac{\text{Stress}}{\text{Strain}} = \text{Bulk modulus K}$$

$$\therefore \quad \frac{dV}{V} = \frac{P}{K}$$

Let each side of the unit cube become $1-\alpha$, where α is the contraction. Then the volume of the cube $= (1-\alpha)^3 = 1 - 3\alpha$ (approx.)

$$\therefore \quad \frac{dV}{V} = 3\alpha = \frac{P}{K}$$

$$\therefore \quad \alpha = \frac{P}{3K}$$

Again, in the direction perpendicular to one pair of faces the pressure P on these faces produces a compression P/Y and the pressures on the other two pair of faces produce stretches each equal to $\sigma(P/Y)$.

$$\therefore \quad \frac{P}{3K} = \left(\frac{P}{Y}\right) - \left(\frac{2\sigma P}{Y}\right) = \left(\frac{P}{Y}\right)(1-2\sigma)$$

$$\therefore \quad Y = 3K(1-2\sigma) \qquad \dots \quad \dots \quad \dots \quad (2)$$

From (1) and (2)

$$\sigma = \frac{3K-2\eta}{6K+2\eta}$$

$$Y = \frac{9K\eta}{3K+\eta}$$

We have from (1) and (2)

$$Y = 2\eta(1+\sigma)$$

$$Y = 3k(1-2\sigma)$$

$$\therefore \quad 2\eta(1+\sigma) = 3k(1-2\sigma) \qquad \dots \quad \dots \quad \dots \quad (3)$$

Here η and K are positive quantities

$$\text{or,} \quad \frac{3k}{2\eta} = \frac{1+\sigma}{1-2\sigma} \qquad \dots \quad \dots \quad \dots \quad (4)$$

Limiting Values of σ

(a) If the Poisson's ratio is a positive quantity, as K and η are always positive,

$$(1-2\sigma) > 0$$

$$2\sigma < 1 \text{ or } \sigma < 0.5$$

(b) If the Poisson's ratio is a negative quantity, for η and K to be positive,

$$(1+\sigma) > 0$$

$$\sigma > -1$$

If means the value σ lies between - 1 and + 0.5

However, in actual practice the value of σ cannot be negative because the body does not expand laterally when it expands longitudinally. Also when $\sigma = 0.5$, it means that there is no change in volume and the body is completely incompressible. Therefore, it is not possible. In practice, the value of σ for most of the isotropic substances is between 0.2 and 0.4.

4.10 Alternative Method

The relation between Y, η and K can also be obtained with

$$\text{or} \quad 3 = Y\left[\frac{1}{3K} + \frac{1}{\eta}\right] \quad \ldots \quad (v)$$

$$\frac{3}{Y} = \frac{1}{3K} + \frac{1}{\eta} \quad \ldots \quad \ldots$$

4.11 Work done per unit volume in a strain

In order to deform a body, work must be done by the applied force. The energy so spent is stored up in the body and is called the energy of strain. When the applied forces are removed, the stress disappears and the energy of strain appears as heat.

Let us consider the work done during the three cases of strain.

(i) Elongation Strain : (Stretch of a wire). Let F be the force applied to a wire fixed at the upper end. Then, clearly, for a small increase in length dl of the wire, the work done will be equal to F. dl and, therefore, during the whole stretch of the wire from 0 to l

work done $= \int_0^l F.dl.$

Now, Young's modulus for the material of the wire, i.e.

$$Y = \frac{F.L}{a.l} \qquad [\because Y = \frac{F/a}{l/L}]$$

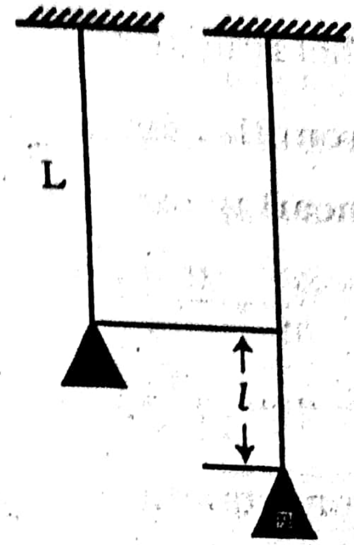

Fig : 4.7

where L is the original length, l, the increase in length, a, the cross sectional area of the wire, and F, the force applied.

And. $\therefore F = Y.a.l/L.$

Therefore, work done during the stretch of the wire from 0 to l is given by

$$W = \int_0^l F dl = \frac{Ya}{L}\int_0^l l.dl = \frac{Ya}{L}\cdot\frac{l^2}{2} = \frac{1}{2}\frac{Yal}{L}.l$$

But $\frac{Y.a.l}{L} = F$, the force applied.

Hence $W = \frac{1}{2}F.l = \frac{1}{2}$ stretching force × stretch.

$\therefore$ Work done per unit volume $= \frac{1}{2}Fl \times \frac{1}{La}$

[$\because$ volume of the wire = L x a.]

$= \frac{1}{2} \times \frac{F}{a} \times \frac{l}{L} = \frac{1}{2}$ stress × strain.

[$\because \frac{F}{a}$ = stress. and $\frac{l}{L}$ = strain]

(ii) Volume strain : Consider a cube of volume V, area of cross section a and length L. Let P be the stress applied. Then, over an area a, the force applied is P.a, and therefore, work done for a small movement dx, in the direction of P., is equal to P.a.dx. Now, a dx, is equal to dv. the small change in volume. Thus, work done for a change in volume dv is equal to pdv

And therefore, total work done for the whole change in volume, from 0 to v, is given by

$$\text{Work done } W = \int_0^v P.dv.$$

Fig. 4.8

Now, $K = \frac{P.V}{v}$, so that, $p = \frac{K.v}{V}$,

where V is the original volume and K, the Bulk Modulus

$$\therefore \quad W = \int_0^v \frac{K.v}{V}.dv = \frac{K}{V}\int_0^v v.dv = \frac{K}{V}.\frac{1}{2}v^2.$$

$$= \frac{1}{2}.\frac{Kv}{V}.v$$

$$= \frac{1}{2}P.v$$

$$= \frac{1}{2} \times \text{Stress x change in volume.}$$

Or, Work done per unit volume $= \frac{1}{2}\frac{P.v}{V} = \frac{1}{2} \times$ stress x strain.

(iii) Shearing strain : Consider a cube of side L, (Fig.4.9). with its lower face DC fixed, and let F be the tangential force applied to its upper face in the plane of AB, so that the face ABCD is distorted into the position A'B'CD, i.e., the cube is sheared through an angle θ. Let the displacement AA' be equal to BB' = l. Then, work done for the whole of the displacement from θ to l

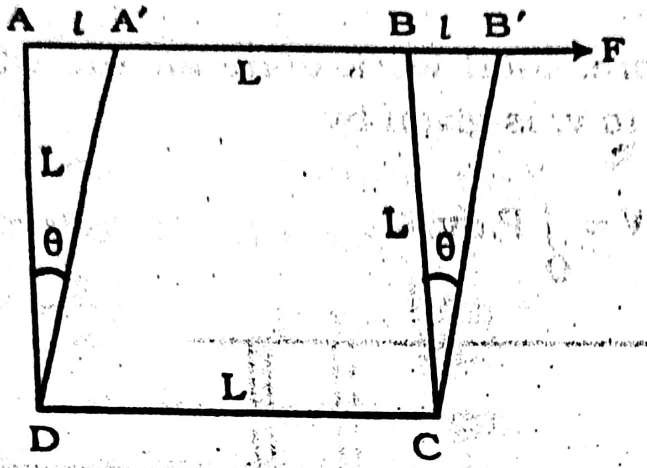

Fig. 4.9

$$W = \int_0^l F.dl.$$

Now, $\eta = \frac{F}{a.\theta}$, or $F = \eta.a.\theta$ and $a = L^2$; also $\theta = \frac{l}{L}$ where L is the length of each edge of the cube.

So, $F = \eta.L^2.\frac{l}{L} = \eta.L.l$

∴ Work done during the whole stretch from 0 to 1, i. e.,

$$W = \int_0^l \eta.L.l.dl = \frac{1}{2}\eta.L.l^2 = \frac{1}{2}.\frac{F}{Ll}.L.l^2 \; [\because \eta = \frac{F}{a\,\theta} = \frac{F}{L^2}\frac{L}{l} = \frac{F}{L.l}]$$

$$= \frac{1}{2}F.l = \frac{1}{2} \text{ tangential force x displacement.}$$

∴ work done per unit volume $= \frac{1}{2}F.l/L^3$

[∵ volume of the cube $= L^3$]

$$= \frac{1}{2} \times \frac{F}{L^2} \times \frac{l}{L}$$

$$= \frac{1}{2} \times \frac{F}{a} \times \theta = \frac{1}{2} \times \text{stress} \times \text{strain}.$$

Thus, we see that, in any kind of strain, work done per unit volume is equal to $\frac{1}{2}$ stress × strain.

4.12 Determination of Young's Modulus by Searle's Apparatus

the same material length

4.13 Glass in more elastic than Rubber

Glass is more elastic than rubber. This is because, for a given stress, the corresponding strain in glass is much less than that produced in the case. of rubber. Suppose for rubber the young modulus is given by

$$Y_r = \frac{FL}{Al_r}$$

And for glass it is $Y_g = \frac{FL}{Al_g}$

For both rubber & glass, FL and A remaining constant as $l_r > l_g$

$\therefore Y_g > Y_r$

i.e Glass is more elastic than rubber.

4.14 Poisson's Ratio of Rubber

A long rubber tube of about 1 metre in length is suspended vertically and a weight hanger is fixed at its lower end (Fig. 4.12). The rubber tube is completely filled with air free water so that water rises in the graduated glass tube C where water meniscuses is visible.

As soon as a suitable weight is placed in the weight hanger, the length of the rubber tube increases and consequently its area of cross section decreases. The internal volume of the tube also increases, causing a fall in the level of water in the tube C. The distance through which the pointer P moves is measured with the help of a travelling microscope. Suppose, increase in the length of the tube = dL

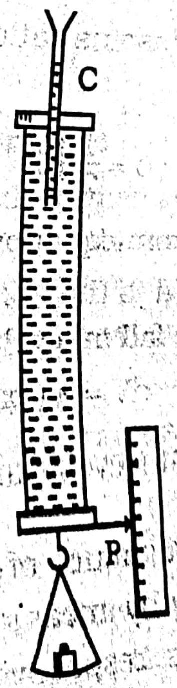

Fig. 4.12

Initial volume = V

Initial length = L

4.5 : The young's modulus of a metal is 2×10^{11} N/m^2 and its breaking stress is 1.078×10^{9} N/m^2. Calculate the maximum amount of energy per unit volume which can be stored in the metal when stretched.

Here, $Y = 2\times10^{11}\ N/m^2$

Maximum Stress $= 1.078\times10^{9}\ N/m^2$

Energy stored per unit volume $= \frac{1}{2}\times stress \times strain$

$$= \left[\frac{1}{2}\times 1.078\times 10^{9}\right]\left[\frac{Stress}{Y}\right]$$

$$= \frac{1.078\times10^{9}\times 1.078\times10^{9}}{2\times2\times10^{11}}$$

$$= 2.90\times10^{6}\ J/m^3$$

4.6 : Calculate the work done in streching a wire of 2 mm^2 cross section and 4 m long through 0.1 mm. $Y = 2\times10^{11}\ N/m^2$

Here $Y = 2\times10^{11}\ Nm^{-2}$

$L = 4m$

$l = 0.1\ mm = 10^{-4}\ m$

$a = 2\ mm^2 = 2\times10^{-6} m^2$

Strain $= \frac{l}{L} = \frac{10^{-4}}{4} = 2.5\times10^{-5}$

Stress $=$ Y×Strain

Stress $= 2\times10^{11}\times2.5\times10^{-5}$

Stress $= 0.5\times10^{-5}\ N/m^2$

Volume of the wire $= aL = 2\times10^{-6}\times4 = 8\times10^{-6} m^3$

4.

metal

2m

Work done per unit volume $= \frac{1}{2} \times$ Stress $\times$ Volume

$\therefore \quad W = \frac{1}{2} \times 0.5\times10^{-5} \times 2.5\times10^{-5} \times 8\times10^{-6}$ Joules

$W = 5 \times 10^{-4}$ J

4.7 : Find the work done in stretching a uniform metal wire of area of cross section $10^{-6}m^2$ and length 2m through 2×10^{-3}m. Given, $Y = 2\times10^{-11}$ N/m^2.

Here $\quad Y = 2\times10^{11}$ N/m^2

$L = 2$m

$a = 10^{-6}$m^2

$l = 2\times10^{-3}$ m

$$\text{Strain} = \frac{l}{L} = \frac{2\times10^{-3}}{2}$$

$$\text{Stress} = Y\times\text{Strain}$$

$$\text{Stress} = \frac{2\times10^{11}\times2\times10^{-3}}{2} \text{ N/m}^2$$

$$\text{Stress} = 0.5\times10^{-5} \text{ N/m}^2$$

Volume of the wire $= aL = 10^{-6}\times2$

Work done per unit volume $= \frac{1}{2} \times$ Stress $\times$ Strain

Total work done $= \frac{1}{2} \times$ Stress $\times$ Strain $\times$ Volume

$$W = \frac{1}{2}\times \frac{2\times10^{11}\times2\times10^{-3}}{2} \times \left(\frac{2\times10^{-3}}{2}\right) \times 2\times10^{-6}$$

$W = 0.2$ J

4.8 : The modulus of rigidity and Possion's ratio of the material of a wire are 2.87×10^{10} N/m^2 and 0.379 respectively. Find the value of young's modulus of the material of the wire. Also calculate the bulk madulus of the material of the wire

Hence $\quad \eta = 2.87\times10^{10}$ N/m^2

$\sigma = 0.379$

$$\eta = \frac{Y}{2(1+\sigma)}$$

$$Y = 2\eta(1+\sigma)$$

$$= 2.87\times10^{10}\,(1+0.379)$$

$$= 7.915\times10^{10}\ \mathrm{N/m^2}$$

Again $Y = 3K(1-2\sigma)$

$$K = \frac{Y}{3(1-2\sigma)}$$

$$= \frac{7.915\times10^{10}}{3(1-2\times0.379)}$$

$$= 10.9\times10^{10}\ \mathrm{N/m^2}$$

4.9 :A steel wire 10m long and 2 mm in diameter is fived to two rigid supports. Calculate the increase in tension when the temperature falls by 10°C

Given $\alpha = 12\times10^{-6}/°C$

$Y = 2\times10^{10}\ \mathrm{N/m^2}$

Here $\alpha = 12\times10^{-6}/°C$

$Y = 2\times10^{10}\ \mathrm{N/m^2}$

$l = 12\times10^{-6}/°C$

$L = 10m$

$$r = \frac{2\times10^{-3}}{2}\,m = 10^{-3}\,m$$

Increase in length $\quad l = L\,\alpha\,t$

$$\text{Strain} = \frac{l}{L} = \frac{L\,\alpha\,t}{L} = \alpha t$$

$$\text{Stress} = Y\times\text{Strain}$$

$$= Y\,\alpha\,t$$

Area of cross-section $= \pi r^2$

Increase in tension = Stress × area of cross-section

$$= Y\,\alpha\,t\times\pi r^2 N$$

$$= \frac{2\times10^{11}\times12\times10^{-6}\times10\times22\times(10^{-3})^2}{7}$$

$$= 75.43\ N$$

4.10 : Calculate the Poission's ratio for the material given Y = 12.25×10^{10} N/m^2 and η = 4.55×10^{10} N/m^2. Also calculate bulk modulus for the wire.

We have, $\eta = \dfrac{Y}{2(1+\sigma)}$

$\therefore \quad \sigma = \left(\dfrac{Y}{2\eta}\right) - 1$

Here $\quad Y = 12.25\times10^{10}\ N/m^2$

$\eta = 4.55\times10^{10}\ N/m^2$

$\therefore \quad \sigma = \left(\dfrac{12.25\times10^{10}}{2\times4.55\times10^{10}}\right) - 1$

$\sigma = 1.347 - 1$

$\sigma = 0.347$

Again, $\quad Y = 3K(1-2\sigma)$

$K = \dfrac{Y}{3(1-2\sigma)}$

$= \dfrac{12.25\times10^{10}}{3(1-2\times0.347)}$

$= 1.33\times10^{10}\ N/m^2$

4.11 : A steel rod of length 5 m is fixed rigidly between two supports. The coefficient of linear expansion of steel = 12×10^{-6}/°C. Calculate the stress in the rod for an increase in temperature of 40°C. The Young's modulus of elasticicy of steel 2×10^{11} N/m^2.

Here $\quad \alpha = 12\times10^{-6}/°C$

$Y = 2\times10^{11}\ N/m^2$

$t = 40°C$

$L = 5m$

Increase in length $\quad l = L\alpha t$

Strain $= \dfrac{l}{L} = \dfrac{L\alpha t}{L}$

$= \alpha t$

Stress $= Y \times \text{Strain} = Y\alpha t$

$= 2\times10^{11}\times12\times10^{-6}\times40$

$= 9.6\times10^{7}\ N/m^2$

4.12 : A uniform steel rod of 2 sq mm cross-section is heated from 0°C to 20°C. Find the compressive force which must be exerted to prevent it from expanding. Find also the energy stored per unit volume. $\alpha = 12\times10^{-6}/°C$ and $Y = 2\times10^{11} Nm^{2}$

Here $\alpha = 12\times10^{-6}/°C$

$Y = 2\times10^{11}\ N/m^2$

$t = 20°C$

$a = 2sq\ mm^2 = 2\times10^{-6}\ m^2$

Increase in length $l = L\alpha t$

Strain $= \frac{l}{L} = \frac{L\alpha t}{L}$

$= \alpha t$

Stress $= Y \times Strain = Y\alpha t$

Froce = Stress × area of cross-section

$= Y\alpha t \times a$

$= 10^{11} \times 12 \times 10^{-6} \times 20 \times 2 \times 10^{-6}$

$= 96\ N$

Energy stored per unit volume

$= \frac{1}{2}$ stress × strain $= \frac{1}{2} \times Y\alpha t \times \alpha t$

$= \frac{2\times10^{11}\times 12\times10^{-6}\times(20)^2}{2}$

$= 5260\ J/m^3$

4.13 : A steel wire of diameter 2 mm supports a load which is sufficient to keep the wire taut at 10° C. Calulate the additional load that will be required to restore the length of the wire to its initial value when the temperature falls to 0°C. Given $Y = 2\times10^{11}\ N/m^2$, $\alpha = 12\times10^{-6}/°C$.

Diameter 2 mm $= 2\times10^{-3}$ m

Area of corss-section

$a = \frac{\pi d^2}{4} = \frac{22\times10^{-6}}{7} m^2.$

Decrease in length = $L\alpha t$

Strain $= \frac{L\alpha t}{L} = \alpha t$

Stress $= Y \times$ Strain $= Y\alpha t$

Force required to maintain the original length

$= $ Stress $\times$ area of cross-section

$= Y\alpha t \times a$

$$= \frac{2\times10^{11}\times12\times10^{-6}\times10\times22\times10^{-6}}{7} N$$

$$= \frac{44\times12}{7\times9.8} \text{ kg wt}$$

$=7.696$ kg-wt

4.14 Calculate the longest length of a steel wire hat can hang vertically without breaking. Breaking ress for steel 8.1×10^{8} N/m² and density of steel 1×10^{3} kg/m³.

Let L be the longest length of the wire

rea of cross section = a

ass of the wire = $La\rho$

eigth of the wire $= (La\rho)g$

ess on the wire $= \frac{(La\rho)g}{a} = L\rho g$

$L\rho g = 8.1\times10^{8}$

$$= \frac{8.1\times10^{3}}{\rho g}$$