প্রশ্নমালা–7

1(a) : (i). $\int_0^{\pi/2} \sin^7 x\, dx = \dfrac{\Gamma(4)\,\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{9}{2}\right)} = \dfrac{3.2.1.\,\Gamma\left(\frac{1}{2}\right)}{2.\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}$

$= \dfrac{16}{35}.$

(ii). $\int_0^{\pi} \sin^9 x\, dx = 2\int_0^{\pi/2} \sin^9 x\, dx = 2.\dfrac{\Gamma(5)\,\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{11}{2}\right)}$

$= \dfrac{4.3.2.1.\,\Gamma\left(\frac{1}{2}\right)}{\frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\,\Gamma\left(\frac{1}{2}\right)} = \dfrac{128}{315}.$

(iii). $\int_0^{\pi} \sin^{11} x\, dx = 2\int_0^{\pi/2} \sin^{11} x\, dx = \dfrac{2\Gamma(6)\,\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{13}{2}\right)}$

$= \dfrac{2.5.4.3.2.1\,\Gamma\left(\frac{1}{2}\right)}{2.\frac{11}{2}\cdot\frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)} = \dfrac{512}{693}.$

(iv). $\int_0^{\pi/2} \cos^6 x\, dx = \dfrac{\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\,\Gamma(4)} = \dfrac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\,\sqrt{\pi}}{2.3.2.1} = \dfrac{5\pi}{32}$

(v). $\int_0^{\pi/2} \cos^7 x\, dx = \dfrac{\Gamma(4)\,\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{9}{2}\right)} = \dfrac{3.2.1.\,\Gamma\left(\frac{1}{2}\right)}{2.\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma\left(\frac{1}{2}\right)} = \dfrac{16}{35}.$

(vi). $I = \int_0^{\pi} \cos^7 x\, dx \cdots (1)$

বা $I = \int_0^{\pi} \{\cos(\pi - x)\}^7\, dx = -\int_0^{\pi} \cos^7 x\, dx \cdots (2)$

$\therefore (1) + (2) \Rightarrow 2I = \int_0^{\pi} \cos^7 x\, dx - \int_0^{\pi} \cos^7 x\, dx = 0.$

$\Rightarrow I = 0.$

(vii). $I = \int_0^{\pi/2} (a^2 \cos^2 x + b^2 \sin^2 x)\, dx$

$$= \frac{a^2 \Gamma\left(\frac{3}{2}\right) \Gamma\left(\frac{1}{2}\right)}{2\Gamma(2)} + \frac{b^2 \Gamma\left(\frac{3}{2}\right) \Gamma\left(\frac{1}{2}\right)}{2\,\Gamma(2)}$$

$$= \frac{a^2 . \frac{1}{2}\sqrt{\pi}\,\sqrt{\pi}}{2.1} + b^2 \frac{\frac{1}{2} . \sqrt{\pi}\,\sqrt{\pi}}{2.1} = \frac{1}{4}\pi\,(a^2 + b^2).$$

(viii). ধরি $I = \int_0^{\pi} \cos^n x\, dx \cdots (1)$

বা $I = \int_0^{\pi} \{\cos(\pi - x)\}^n\, dx$

বা $I = \int_0^{\pi} (-\cos x)^n\, dx \cdots (2)$

বা $I = -\int_0^{\pi} \cos^n x\, dx \cdots (3)$, যদি n বিজোড় পূর্ণ সংখ্যা হয়।

$(1) + (3) \Rightarrow 2I = 0 \Rightarrow I = 0$

$\therefore$ n বিজোড় পূর্ণসংখ্যা হইলে $\int_0^{\pi} \cos^n x\, dx = 0$

এখন (2) নং হইতে পাই, $I = \int_0^{\pi} (-\cos x)^n\, dx$

$= \int_0^{\pi} \cos^n x\, dx$, যদি n জোড় পূর্ণসংখ্যা হয়।

$= 2\int_0^{\pi/2} \cos^n x\, dx$, $[\because f(\pi - x) = f(x)]$.

$$\text{বা } I = \frac{2\Gamma\left(\frac{n+1}{2}\right) \Gamma\left(\frac{1}{2}\right)}{2\,\Gamma\left(\frac{n+2}{2}\right)}$$

অর্থাৎ $\int_0^{\pi} \cos^n x\, dx = \dfrac{\Gamma\left(\frac{n+1}{2}\right)\sqrt{\pi}}{\Gamma\left(\frac{n+2}{2}\right)}$ যদি n জোড়পূর্ণ সংখ্যা হয়।

(i). $$\int_0^{\pi/2} \sin^5 x \cos^6 x \, dx = \frac{\Gamma(3)\,\Gamma\left(\frac{7}{2}\right)}{2\,\Gamma\left(\frac{13}{2}\right)}$$

$$= \frac{2.1\,\Gamma\left(\frac{7}{2}\right)}{2.\,\frac{11}{2}\cdot\frac{9}{2}\cdot\frac{7}{2}\,\Gamma\left(\frac{7}{2}\right)} = \frac{8}{693}.$$

ii. $$\int_0^{\pi/2} \sin^6 x \cos^8 x \, dx = \frac{\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{9}{2}\right)}{2(8)}$$

$$= \frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\,.\,\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}{2.7.6.5.4.3.2.1} = \frac{5\pi}{4096}.$$

iii. $$\int_0^{\pi/2} \cos^6 x \sin^3 x \, dx = \frac{\Gamma\left(\frac{7}{2}\right)\Gamma(2)}{2\Gamma\left(\frac{11}{2}\right)} = \frac{\Gamma\left(\frac{7}{2}\right).1}{2.\,\frac{9}{2}\cdot\frac{7}{2}\,\Gamma\left(\frac{7}{2}\right)} = \frac{2}{63}$$

iv). $$\int_0^{\pi/2} \sin^5\theta \cos^4\theta \, d\theta = \frac{\Gamma(3)\,\Gamma\left(\frac{5}{2}\right)}{2\,\Gamma\left(\frac{11}{2}\right)} = \frac{2.1.\,\Gamma\left(\frac{5}{2}\right)}{2.\,\frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\,\Gamma\left(\frac{5}{2}\right)} = \frac{8}{315}.$$

v). $$\int_0^{\pi/2} \sin^2\theta \cos^3\theta \, d\theta = \frac{\Gamma\left(\frac{3}{2}\right)\Gamma(2)}{2\Gamma\left(\frac{7}{2}\right)} = \frac{\Gamma\left(\frac{3}{2}\right).\,1}{2.\,\frac{5}{2}\cdot\frac{3}{2}\,\Gamma\left(\frac{3}{2}\right)} = \frac{2}{15}.$$

vi). $$\int_0^{\pi/2} \sin^4\theta \cos^8\theta \, d\theta = \frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{9}{2}\right)}{2\,\Gamma(7)}$$

$$= \frac{\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\,.\,\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}{2.6.5.4.3.2.1} = \frac{7\pi}{2048}.$$

vii). $$\int_0^{\pi/2} \cos^4\theta \sin^3\theta \, d\theta = \frac{\Gamma\left(\frac{5}{2}\right)\Gamma(2)}{2\Gamma\left(\frac{9}{2}\right)} = \frac{\Gamma\left(\frac{5}{2}\right).1}{2.\,\frac{7}{2}\cdot\frac{5}{2}\,\Gamma\left(\frac{5}{2}\right)} = \frac{2}{35}$$

(viii). $I = \int_0^{\pi} \sin^6 x \cos^4 x \, dx = 2 \int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$

$[\because f(\pi - x) =$

$$= \frac{2\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{5}{2}\right)}{2\Gamma(6)} = \frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}{5.4.3.2.1} = \frac{3\pi}{256}.$$

(ix). $I = \int_0^{\pi} \sin^3 x \cos^2 x \, dx = 2 \int_0^{\pi/2} \sin^3 x \cos^2 x \, dx$

$\therefore f(\pi - x) = f(x)$

$$= \frac{2\Gamma(2)\,\Gamma\left(\frac{3}{2}\right)}{2\,\Gamma\left(\frac{7}{2}\right)} = \frac{1.\,\Gamma\left(\frac{3}{2}\right)}{\frac{5}{2}\cdot\frac{3}{2}\cdot\Gamma\left(\frac{3}{2}\right)} = \frac{4}{15}.$$

(x). $I = \int_0^{\pi} \sin^3 x \cos^3 x \, dx \cdots (1)$

বা $I = \int_0^{\pi} \{\sin(\pi - x)\}^3 \{\cos(\pi - x)\}^3 \, dx$

$= -\int_0^{\pi} \sin^3 x \cos^3 x \, dx \cdots (2)$

$\therefore (1) + (2) \Rightarrow 2I = \int_0^{\pi} \sin^3 x \cos^3 x \, dx - \int_0^{\pi} \sin^3 x \cos^3 x \, dx = 0.$

$\Rightarrow I = 0.$

(x). ধরি $\frac{1}{2}x = z$ তবে $dx = 2dz$

সীমা ঃ যদি $x = 0$ হয়, তবে $z = 0$

যদি $x = 2\pi$ হয়, তবে $z = \pi.$

$\therefore I = 2\int_0^{\pi} \sin^4 z \cos^5 z \, dz \cdots (1)$

বা $I = 2\int_0^{\pi} \{\sin(\pi - z)\}^4 \{\cos(\pi - z)\}^5 \, dz$

$\therefore I = -2\int_0^{\pi} \sin^4 z \cos^5 z \, dz \cdots (2).$

$\therefore (1) + (2) \Rightarrow 2I = 2\int_0^{\pi} \sin^4 z \cos^5 z \, dz - 2\int_0^{\pi} \sin^4 z \cos^5 z \, dz$

$= 0$

$\Rightarrow I = 0.$

(xii). $I = \int_0^{\pi}\left(2\sin^2\frac{x}{2}\right)^2 dx = 4\int_0^{\pi}\sin^4\frac{x}{2}\,dx$

ধরি $\frac{x}{2} = z$ তবে $dx = 2\,dz$

সীমা ঃ যদি $x = 0$ হয়, তবে $z = 0$

যদি $x = \pi$ হয়, তবে $z = \frac{\pi}{2}$

$$\therefore I = 4.2\int_0^{\pi/2}\sin^4 z\,dz = 8\int_0^{\pi/2}\sin^4 x\,dx = 8\frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(3)}$$

$$= \frac{8.\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{2.2.1} = \frac{3\pi}{2}.$$

(xiii). $I = \int_0^{\pi/2}\frac{\sin^n x}{\cos^n x}dx = \int_0^{\pi/2}\sin^n x\cos^{-n}x\,dx$

$$= \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1-n}{2}\right)}{2\Gamma\left(\frac{n-n+2}{2}\right)} = \frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1-n}{2}\right)$$

c(i). $\int_0^{\pi/2}\cos^4 x\sin 3x\,dx = \int_0^{\pi/2}\cos^4 x(3\sin x - 4\sin^3 x)\,dx$

$$= 3\int_0^{\pi/2}\cos^4 x\sin x\,dx - 4\int_0^{\pi/2}\cos^4 x\sin^3 x\,dx$$

$$= \frac{3\Gamma\left(\frac{5}{2}\right)\Gamma(1)}{2\Gamma\left(\frac{7}{2}\right)} - \frac{4.\Gamma\left(\frac{5}{2}\right)\Gamma(2)}{2\left(\frac{9}{2}\right)} = \frac{3\Gamma\left(\frac{5}{2}\right)1}{2.\frac{5}{2}\Gamma\left(\frac{5}{2}\right)} - \frac{4\Gamma\left(\frac{5}{2}\right)1}{2\frac{7}{2}\cdot\frac{5}{2}\Gamma\left(\frac{5}{2}\right)}$$

$$= \frac{3}{5} - \frac{8}{35} = \frac{13}{35}.$$

(ii). $I = \int_0^{\pi/4}\cos^3 2x\sin^4 4x\,dx = \int_0^{\pi/4}\cos^3 2x(2\sin 2x\cos 2x)^4\,dx$

$= 16\int_0^{\pi/4}\cos^7 2x\sin^4 2x\,dx$; ধরি $2x = z$ তবে $dx = \frac{1}{2}dz$

সীমা ঃ যদি $x = 0$ হয়, তবে $z = 0$

যদি $x = \frac{\pi}{4}$ হয়, তবে $z = \frac{\pi}{2}$

$$\therefore I = \frac{16}{2} \cdot \int_0^{\pi/2} \cos^7 z \sin^4 z \, dz = 8 . \frac{\Gamma(4)\Gamma\left(\frac{5}{2}\right)}{2\Gamma\left(\frac{13}{2}\right)}$$

$$= 8 . \frac{3.2.1\, \Gamma\left(\frac{5}{2}\right)}{2 . \frac{11}{2} \cdot \frac{9}{2} \cdot \frac{7}{2} \cdot \frac{5}{2} \cdot \Gamma\left(\frac{5}{2}\right)} = \frac{128}{1155}.$$

2(i). $I = \int_0^{\pi} x \sin^2 x \, dx \cdots (1)$

বা $I = \int_0^{\pi} (\pi - x) \{\sin(\pi - x)\}^2 \, dx$

$= \int_0^{\pi} (\pi - x) \sin^2 x \, dx \cdots (2)$

$\therefore (1) + (2) \Rightarrow 2I = \pi \int_0^{\pi} \sin^2 x \, dx = 2\pi \int_0^{\pi/2} \sin^2 x \, dx$

বা $2I = \frac{2\pi\, \Gamma\left(\frac{3}{2}\right) \Gamma\left(\frac{1}{2}\right)}{2\Gamma(2)} = \frac{2\pi . \frac{1}{2} \sqrt{\pi} . \sqrt{\pi}}{2.1} = \frac{\pi^2}{2}$

$\therefore I = \frac{\pi^2}{4}.$

(ii). $I = \int_0^{\pi} x \sin^6 x \cos^4 x \, dx \cdots (1)$

বা $I = \int_0^{\pi} (\pi - x) \{\sin(\pi - x)\}^6 \{\cos(\pi - x)\}^4 \, dx$

$= \int_0^{\pi} (\pi - x) \sin^6 x \cos^4 x \, dx \cdots (2)$

$\therefore (1) + (2) \Rightarrow 2I = \pi \int_0^{\pi} \sin^6 x \cos^4 x \, dx$

$\therefore I = \frac{\pi}{2} \int_0^{\pi} \sin^6 x \cos^4 x \, dx$

$= \frac{\pi}{2} \cdot \frac{3\pi}{256}$; b(viii) নং দেখুন।

(iii). $I = \int_0^{\pi} x \sin x \cos^2 x \, dx \cdots (1)$

বা $I = \int_0^{\pi} (\pi - x) \{\sin(\pi - x)\} \{\cos(\pi - x)\}^2 \, dx$

$\therefore I = \int_0^{\pi} (\pi - x) \sin x \cos^2 x \, dx \cdots (2)$

$\therefore (1) + (2) \Rightarrow 2I = \pi \int_0^{\pi} \sin x \cos^2 x \, dx$

$$= 2\pi \int_0^{\pi/2} \sin x \cos^2 x \, dx$$

$$\text{বা } I = \pi . \frac{\Gamma(1)\, \Gamma\left(\frac{3}{2}\right)}{2\Gamma\left(\frac{5}{2}\right)} = \frac{\pi\, \Gamma\left(\frac{3}{2}\right)}{2 . \frac{3}{2} . \Gamma\left(\frac{3}{2}\right)} = \frac{\pi}{3}.$$

3(i). ধরি $x = \sin\theta$ তবে $dx = \cos\theta \, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \sin^2\theta \, (\cos^2\theta)^{1/2} \cos\theta \, d\theta = \int_0^{\pi/2} \sin^2\theta \cos^2\theta \, d\theta$$

$$= \frac{\Gamma\left(\frac{3}{2}\right) \Gamma\left(\frac{3}{2}\right)}{2\, \Gamma(3)} = \frac{\frac{1}{2} . \sqrt{\pi} . \frac{1}{2} \sqrt{\pi}}{2.2} = \frac{\pi}{16}.$$

(ii). ধরি $x = \sin\theta \Rightarrow dx = \cos\theta \, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \sin^2\theta \, (\cos^2\theta)^{7/2} \cos\theta \, d\theta = \int_0^{\pi/2} \sin^2\theta \cos^8\theta \, d\theta$$

$$= \frac{\Gamma\left(\frac{3}{2}\right) \Gamma\left(\frac{9}{2}\right)}{2\, \Gamma(6)} = \frac{\frac{1}{2} \sqrt{\pi} . \frac{7}{2} . \frac{5}{2} . \frac{3}{2} . \frac{1}{2} . \sqrt{\pi}}{2.5.4.3.2.1} = \frac{7\pi}{512}.$$

(iii). ধরি $x = \sin\theta$ তবে $dx = \cos\theta \, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \sin^3\theta\, (\cos^2\theta)^{5/2} \cos\theta\, d\theta = \int_0^{\pi/2} \sin^3\theta \cos^6\theta\, d\theta$$

$$= \frac{\Gamma(2)\,\Gamma\left(\frac{7}{2}\right)}{2\,\Gamma\left(\frac{11}{2}\right)} = \frac{1.\,\Gamma\left(\frac{7}{2}\right)}{2\,.\,\frac{9}{2}\,.\,\frac{7}{2}\,\Gamma\left(\frac{7}{2}\right)} = \frac{2}{63}.$$

(iv). $I = \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_0^1 = 0 + \frac{1}{2}\sin^{-1}1 = \frac{\pi}{4}.$

(v). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$.

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \sin^2\theta\, (\cos^2\theta)^{5/2} \cos\theta\, d\theta = \int_0^{\pi/2} \sin^2\theta \cos^6\theta\, d\theta$$

বা $I = \dfrac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{7}{2}\right)}{2\Gamma(5)} = \dfrac{\frac{1}{2}\sqrt{\pi}.\,\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}{2.4.3.2.1} = \dfrac{5\pi}{256}.$

(vi). $I = \int_0^1 x^3(1-x)^3\, dx = \int_0^1 x^{4-1}(1-x)^{4-1}\, dx$

$$= \beta(4, 4) = \frac{\Gamma(4)\,\Gamma(4)}{\Gamma(8)} = \frac{3.\,2.1.\,\Gamma(4)}{7.\,6.\,5.\,4.\,\Gamma(4)} = \frac{1}{140}.$$

(vii). ধরি $x = a\sin\theta$ তবে $dx = a\cos\theta\, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = a\sin\theta \Rightarrow \theta = 0$

যদি $x = a$ হয়, তবে $a = a\sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} a^4 \sin^4\theta\, (a^2\cos^2\theta)^{1/2}\, a\cos\theta\, d\theta$$

$$= a^6 \int_0^{\pi/2} \sin^4\theta \cos^2\theta\, d\theta$$

$$= \frac{a^6\,\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{3}{2}\right)}{2\Gamma(4)} = \frac{a^6\,\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\,.\,\frac{1}{2}\cdot\sqrt{\pi}}{2.\,3.\,2.\,1} = \frac{\pi a^6}{32}.$$

(viii). ধরি $x = a\sin\theta$ তবে $dx = a\cos\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = a\sin\theta \Rightarrow \theta = 0$

যদি $x = a$ হয়, তবে $a = a\sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} a^7 \sin^7\theta\, (a^2\cos^2\theta)^{1/2}\, a\cos\theta\, d\theta = a^9 \int_0^{\pi/2} \sin^7\theta \cos^2\theta\, d\theta$$

$$= \frac{a^9\,\Gamma(4)\,\Gamma\left(\frac{3}{2}\right)}{2\Gamma\left(\frac{11}{2}\right)} = \frac{a^9.3.2.1.\,\Gamma\left(\frac{3}{2}\right)}{2.\frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\,\Gamma\left(\frac{3}{2}\right)} = \frac{16a^9}{315}.$$

(ix). ধরি $x = 2\sin^2\theta$ তবে $dx = 4\sin\theta\cos\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = 2\sin\theta \Rightarrow \theta = 0$

যদি $x = 2$ হয়, তবে $2 = 2\sin^2\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} (2\sin^2\theta)^{5/2}\,(2\cos^2\theta)^{1/2}.\, 4\sin\theta\cos\theta\, d\theta$$

$$= 32\int_0^{\pi/2} \sin^6\theta\cos^2\theta\, d\theta = \frac{32.\,\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{3}{2}\right)}{2\Gamma(5)}$$

$$= \frac{32.\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}\,.\,\frac{1}{2}\cdot\sqrt{\pi}}{2.\,4.\,3.\,2.1} = \frac{5\pi}{8}.$$

(x). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \sin^4\theta\,(\cos^2\theta)^{3/2}\cos\theta\, d\theta = \int_0^{\pi/2}\sin^4\theta\cos^4\theta\, d\theta$$

$$= \frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{5}{2}\right)}{2\Gamma(5)} = \frac{\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\,.\,\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}{2.4.3.\,2.\,1} = \frac{3\pi}{256}.$$

(xi). $I = \int_0^{2a} x^2\sqrt{2ax - x^2}\, dx = \int_0^{2a} x^2\, x^{1/2}\sqrt{2a - x}\, dx$

ধরি $x = 2a\sin^2\theta$ তবে $dx = 4a\sin\theta\cos\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = 2a\sin^2\theta \Rightarrow \theta = 0$

যদি $x = 2a$ হয়, তবে $2a = 2a\sin^2\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} (2a\sin^2\theta)^{5/2}\sqrt{2a\cos^2\theta}.\ 4a\sin\theta\cos\theta\, d\theta$$

$$= 32a^4\int_0^{\pi/2}\sin^6\theta\cos^2\theta\, d\theta$$

$$= \frac{32a^4\,\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{3}{2}\right)}{2\Gamma(5)} = \frac{32a^4.\frac{5}{2}.\frac{3}{2}.\frac{1}{2}\sqrt{\pi}\,\frac{1}{2}\sqrt{\pi}}{2.4.3.2.1} = \frac{5\pi a^4}{8}$$

(xiii). $I = \int_0^2 x^3\sqrt{2x-x^2}\,dx = \int_0^2 x^{7/2}\sqrt{2-x}\,dx$

ধরি $x = 2\sin^2\theta$ তবে $dx = 4\sin\theta\cos\theta\,d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = 2\sin^2\theta \Rightarrow \theta = 0$

যদি $x = 2$ হয়, তবে $2 = 2\sin^2\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2}(2\sin^2\theta)^{7/2}\sqrt{2\cos^2\theta}\ 4\sin\theta\cos\theta\,d\theta$$

$$= 64\int_0^{\pi/2}\sin^8\theta\cos^2\theta\,d\theta = \frac{64.\,\Gamma\left(\frac{9}{2}\right)\Gamma\left(\frac{3}{2}\right)}{2\Gamma(6)}$$

$$= \frac{64.\frac{7}{2}.\frac{5}{2}.\frac{3}{2}.\frac{1}{2}\sqrt{\pi}.\frac{1}{2}\sqrt{\pi}}{2.5.4.3.2.1} = \frac{7\pi}{8}.$$

(xiii). $I = \int_0^{2a} x^m\sqrt{2ax-x^2}\,dx = \int_0^{2a} x^{(2m+1)/2}\sqrt{2a-x}\,dx$

ধরি $x = 2a\sin^2\theta$ তবে $dx = 4a\sin\theta\cos\theta\,d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = 2a\sin^2\theta \Rightarrow \theta = 0$

যদি $x = 2a$ হয়, তবে $2a = 2a\sin^2\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2}(2a\sin^2\theta)^{(2m+1)/2}\sqrt{2a\cos^2\theta}\ 4a\sin\theta\cos\theta\,d\theta$$

$$= 2(2a)^{m+2}\int_0^{\pi/2}\sin^{2m+2}\theta\cos^2\theta\,d\theta$$

$$= \frac{2(2a)^{m+2}\,\Gamma\left(\frac{2m+3}{2}\right)\Gamma\left(\frac{3}{2}\right)}{2\Gamma\left(\frac{2m+6}{2}\right)}$$

$$= \frac{(2a)^{m+2}\,\Gamma\left(\frac{2m+3}{2}\right)\Gamma\left(\frac{3}{2}\right)}{\Gamma(m+3)}.$$

(xiv). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$I = \int_0^{\pi/2} \frac{\sin^6\theta \cos\theta\, d\theta}{\sqrt{\cos^2\theta}} = \int_0^{\pi/2} \sin^6\theta\, d\theta = \frac{\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(4)} = \frac{5\pi}{32}.$$

(xv). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \frac{\sin\theta.\cos\theta\, d\theta}{\sqrt{\cos^2\theta}} = \int_0^{\pi/2} \sin\theta\, d\theta = \frac{\Gamma(1)\,\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{3}{2}\right)} = 1.$$

(xvi). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \frac{\sin^n\theta.\cos\theta\, d\theta}{\sqrt{\cos^2\theta}} = \int_0^{\pi/2} \sin^n\theta\, d\theta$$

$$= \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{n+2}{2}\right)} = \frac{\Gamma\left(\frac{n+1}{2}\right)\sqrt{\pi}}{2\Gamma\left(\frac{n+2}{2}\right)}.$$

(xvii). ধরি $x^2 = \sin\theta$ তবে $2x\, dx = \cos\theta\, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$I = \frac{1}{2}\int_0^{\pi/2} \frac{\sin^4\theta.\cos\theta\, d\theta}{\sqrt{\cos^2\theta}} = \frac{1}{2}\int_0^{\pi/2} \sin^4\theta\, d\theta = \frac{1}{2}\cdot\frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(3)}$$

$$= \frac{1}{2}\cdot\frac{\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\,\sqrt{\pi}}{2.2.1} = \frac{3\pi}{32}.$$

(xviii). ধরি $x = a\sin\theta$ তবে $dx = a\cos\theta\, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = a\sin\theta \Rightarrow \theta = 0$

যদি $x = a$ হয়, তবে $a = a\sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$I = \int_0^{\pi/2} \frac{a^4\sin^4\theta\; a\cos\theta\, d\theta}{\sqrt{a^2\cos^2\theta}} = a^4 \int_0^{\pi/2} \sin^4\theta\, d\theta = \frac{a^4\,\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\,\Gamma(3)}$$

$$= \frac{a^4 . \frac{3}{2} . \frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{2.2.1} = \frac{3\pi a^4}{16}$$

(xix). ধরি $x = 2a\sin^2\theta$ তবে $dx = 4a\sin\theta\cos\theta\, d\theta$.

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = 2a\sin^2\theta \Rightarrow \theta = 0$

যদি $x = 2a$ হয়, তবে $2a = 2a\sin^2\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \frac{(2a\sin^2\theta)^{9/2}\, 4a\sin\theta\cos\theta\, d\theta}{\sqrt{2a\cos^2\theta}}$$

$$= (2a)^4\, 4a \int_0^{\pi/2} \sin^{10}\theta\, d\theta = 64a^5 . \frac{\Gamma\left(\frac{11}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(6)}$$

$$= \frac{64a^5 . \frac{9}{2} . \frac{7}{2} . \frac{5}{2} . \frac{3}{2} . \frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{2.5.4.3.2.1} = \frac{63\pi a^5}{8}.$$

(xx). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \frac{\sin^2\theta(2 - \sin^2\theta)\cos\theta\, d\theta}{\sqrt{\cos^2\theta}} = \int_0^{\pi/2} [2\sin^2\theta - \sin^4\theta]\, d\theta$$

$$= 2.\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\,\Gamma(2)} - \frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\,\Gamma(3)} = \frac{\frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{1} - \frac{\frac{3}{2} . \frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{2.2}$$

$$= \frac{\pi}{2} - \frac{3\pi}{16} = \frac{5\pi}{16}.$$

(xxi). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2} \frac{\sin^2\theta\,(2 - \sin\theta)\cos\theta\, d\theta}{\sqrt{\cos^2\theta}} = \int_0^{\pi/2} [2\sin^2\theta - \sin^3\theta]\, d\theta$$

$$= \frac{2\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(2)} - \frac{\Gamma(2)\,\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{5}{2}\right)} = \frac{2.\frac{1}{2}\cdot\sqrt{\pi}\,\sqrt{\pi}}{2.1} - \frac{1\,\Gamma\left(\frac{1}{2}\right)}{2.\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma\left(\frac{1}{2}\right)}$$

$$= \frac{\pi}{2} - \frac{2}{3} = \frac{1}{6}(3\pi - 4).$$

(xxii). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1/2$ হয়, তবে $1/2 = \sin\theta \Rightarrow \theta = \frac{\pi}{6}$

$$\therefore I = \int_0^{\pi/6} \frac{\cos\theta\, d\theta}{(1 - 2\sin^2\theta)\sqrt{\cos^2\theta}} = \int_0^{\pi/6} \frac{d\theta}{1 - 2\sin^2\theta}$$

$$= \int_0^{\pi/6} \frac{\sec^2\theta\, d\theta}{\sec^2\theta - 2\tan^2\theta} = \int_0^{\pi/6} \frac{\sec^2\theta\, d\theta}{1 - \tan^2\theta}$$

ধরি $\tan\theta = t$ তবে $\sec^2\theta\, d\theta = dt$

সীমা : যদি $\theta = 0$ হয়, তবে $t = \tan 0 = 0$

যদি $\theta = \frac{\pi}{6}$ হয়, তবে $t = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$$\therefore I = \int_0^{1/\sqrt{3}} \frac{dt}{1 - t^2} = \frac{1}{2}\left[\ln\frac{1 + t}{1 - t}\right]_0^{1/\sqrt{3}} = \frac{1}{2}\ln\frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}$$

$$= \frac{1}{2}\ln\frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{1}{2}\ln(2 + \sqrt{3}).$$

(xxiii). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = \sin\theta \Rightarrow \theta = 0$

যদি $x = 1/2$ হয়, তবে $1/2 = \sin\theta \Rightarrow \theta = \frac{\pi}{6}$

$$\therefore I = \int_0^{\pi/6} \frac{\cos\theta\, d\theta}{(\cos^2\theta)^2} = \int_0^{\pi/6} \sec^3\theta\, d\theta$$

$$= \int_0^{\pi/6} \sqrt{1+\tan^2\theta}.\ \sec^2\theta\, d\theta$$

ধরি $\tan\theta = t$ তবে $\sec^2\theta\, d\theta = dt$

সীমা ঃ যদি $\theta = 0$ হয়, তবে $t = \tan 0 = 0$

যদি $\theta = \frac{\pi}{6}$ হয়, তবে $t = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$$\therefore I = \int_0^{1/\sqrt{3}} \sqrt{1+t^2}\, dt = \left[\frac{t\sqrt{1+t^2}}{2} + \frac{1}{2} ln\,(t+\sqrt{1+t^2}\right]_0^{1/\sqrt{3}}$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\sqrt{1+\frac{1}{3}} + \frac{1}{2} ln\left(\frac{1}{\sqrt{3}} + \sqrt{1+\frac{1}{3}}\right)$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\cdot\frac{2}{\sqrt{3}} + \frac{1}{2} ln\left(\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\right)$$

$$= \frac{1}{3} + \frac{1}{2} ln\sqrt{3}.$$

(xxiv). ধরি $x = \sin\theta$ তবে $dx = \cos\theta\, d\theta$

সীমা ঃ যদি $x = \frac{1}{2}$ হয়, তবে $\frac{1}{2} = \sin\theta \Rightarrow \theta = \frac{\pi}{6}$

যদি $x = 1$ হয়, তবে $1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_{\pi/6}^{\pi/2} \frac{\cos\theta\, d\theta}{\sin\theta\sqrt{\cos^2\theta}} = \int_{\pi/6}^{\pi/2} \text{casec}\theta\, d\theta$$

$$= -\left[ln(\text{cos ec}\theta + \cot\theta)\right]_{\pi/6}^{\pi/2}$$

$$= -[ln\,(1+0) - ln(2+\sqrt{3})] = ln(2+\sqrt{3}) = -ln(2-\sqrt{3}).$$

(xxv). ধরি $x = 3\sec\theta$ তবে $dx = 3\sec\theta\tan\theta\, d\theta$

সীমা ঃ যদি $x = 3$ হয়, তবে $3 = 3\sec\theta \Rightarrow \theta = 0$

যদি $x = 5$ হয়, তবে $5 = 3\sec\theta \Rightarrow \theta = \sec^{-1}\frac{5}{3}$

$$\therefore I = \int_0^\theta \frac{3\sec\theta \tan\theta \, d\theta}{9\sec^2\theta \sqrt{3^2 \tan^2\theta}}$$

$$= \frac{1}{9}\int_0^\theta \cos\theta \, d\theta = \frac{1}{9}[\sin\theta)]_1^\theta$$

$$= \frac{1}{9}\sin\theta = \frac{1}{9}\sqrt{1-\cos^2\theta}$$

$$= \frac{1}{9}\sqrt{1-\frac{9}{25}} = \frac{1}{9}\cdot\frac{4}{5} = \frac{4}{45}.$$

(xxvi). ধরি $I = \int_0^1 \frac{dx}{(1-x^n)^{1/n}}$.

ধরি $x^n = \sin^2\theta$, বা $x = (\sin\theta)^{2/n}$ তবে $dx = \frac{2}{n}(\sin\theta)^{2/n-1}\cos\theta \, d\theta.$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \sin^2\theta \Rightarrow \theta = 0$

যদি $x = 1$ হয়, তবে $1 = \sin^2\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \frac{2}{n}\int_0^{\pi/2} \frac{\sin^{2/n-1}\theta \cos\theta \, d\theta}{(\cos^2\theta)^{1/n}}$$

$$= \frac{2}{n}\int_0^{\pi/2} \sin^{2/n-1}\theta \cos^{1-2/n}\theta \, d\theta$$

$$= \frac{2}{n}\cdot\frac{\Gamma\left\{\frac{1}{2}\left(\frac{2}{n}-1+1\right)\right\}\Gamma\left\{\frac{1}{2}\left(1-\frac{2}{n}+1\right)\right\}}{2\Gamma\left\{\frac{1}{2}\left(\frac{2}{n}-1+1-\frac{2}{n}+2\right)\right\}}$$

$$= \frac{1}{n}\frac{\Gamma\left(\frac{1}{n}\right)\Gamma\left(1-\frac{1}{n}\right)}{\Gamma(1)} = \frac{\Gamma\left(1+\frac{1}{n}\right)\cdot\Gamma\left(1-\frac{1}{n}\right)}{1}$$

(i). ধরি $x = a\cos\theta$ তবে $dx = -a\sin\theta \, d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = a\cos\theta \Rightarrow \theta = \frac{\pi}{2}$

যদি $x = a$ হয়, তবে $a = a\cos\theta \Rightarrow \theta = 0$

$$I = -\int_{\pi/2}^0 \sqrt{\frac{a(1+\cos\theta)}{a(1-\cos\theta)}} \, a\sin\theta \, d\theta$$

$$= a\int_0^{\pi/2}\sqrt{\frac{2\cos^2\theta/2}{2\sin^2\theta/2}}\cdot 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,d\theta$$

$$= 2a\int_0^{\pi/2}\cos^2\frac{\theta}{2}\,d\theta = a\int_0^{\pi/2}(1+\cos\theta)\,d\theta = a[\theta+\sin\theta]_0^{\pi/2}$$

$$= a\left(\frac{\pi}{2}+1\right).$$

(ii). ধরি $x^2 = a^2\cos\theta$ তবে $2x\,dx = -a^2\sin\theta\,d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = a^2\cos\theta \Rightarrow \theta = \frac{\pi}{2}$

যদি $x = a$ হয়, তবে $a^2 = a^2\cos\theta \Rightarrow \theta = 0$

$$\therefore I = -\frac{a^2}{2}\int_{\pi/2}^{0}\sqrt{\frac{a^2(1-\cos\theta)}{a^2(1+\cos\theta)}}\cdot\sin\theta\,d\theta$$

$$= \frac{a^2}{2}\int_0^{\pi/2}\sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}}\,2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,d\theta$$

$$= a^2\int_0^{\pi/2}\sin^2\frac{\theta}{2}\,d\theta = \frac{a^2}{2}\int_0^{\pi/2}(1-\cos\theta)\,d\theta$$

$$= \frac{a^2}{2}[\theta-\sin\theta]_0^{\pi/2}$$

$$= \frac{a^2}{2}\left[\frac{\pi}{2}-1\right] = \frac{1}{4}a^2[\pi-2].$$

5(i). ধরি $x = \tan\theta$ তবে $dx = \sec^2\theta\,d\theta$

সীমা ঃ যদি $x = 0$ হয়, তবে $0 = \tan\theta \Rightarrow \theta = 0$

যদি $x = \infty$ হয়, তবে $\infty = \tan\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \int_0^{\pi/2}\frac{\tan^6\theta\sec^2\theta\,d\theta}{(\sec^2\theta)^6} = \int_0^{\pi/2}\frac{\sin^6\theta}{\cos^6\theta}\cos^{10}\theta\,d\theta$$

$$= \int_0^{\pi/2}\sin^6\theta\cos^4\theta\,d\theta = \frac{\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{5}{2}\right)}{2\Gamma(6)} = \frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}{2.5.4.3.2.1}$$

$$= \frac{3\pi}{512}.$$

(ii). ধরি $x^3 = \tan\theta$ তবে $3x^2\, dx = \sec^2\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = \tan\theta \Rightarrow \theta = 0$

যদি $x = \infty$ হয়, তবে $\infty = \tan\theta \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore I = \frac{1}{3}\int_0^{\pi/2} \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^{7/2}} = \frac{1}{3}\int_0^{\pi/2} \cos^5\theta\, d\theta$$

$$= \frac{1}{3}\cdot\frac{\Gamma(3)\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{7}{2}\right)} = \frac{2.1.\,\Gamma\left(\frac{1}{2}\right)}{3.\,2.\,\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\,\Gamma\left(\frac{1}{2}\right)} = \frac{8}{45}.$$

(iii). ধরি $x = \tan\theta$ তবে $dx = \sec^2\theta\, d\theta$.

সীমা : যদি $x = 0$ হয়, তবে $0 = \tan\theta \Rightarrow \theta = 0$

যদি $x = \infty$ হয়, তবে $\infty = \tan\theta \Rightarrow \theta = \frac{\pi}{2}$.

$$\therefore I = \int_0^{\pi/2} \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^n} = \int_0^{\pi/2} \cos^{2n-2}\theta\, d\theta$$

$$= \frac{\Gamma\left\{\frac{1}{2}(2n-2+1)\right\}\Gamma\left(\frac{1}{2}\right)}{2.\Gamma\left\{\frac{1}{2}(2n-2+2)\right\}}$$

$$= \frac{\Gamma\left(n-\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(n)} = \frac{\sqrt{\pi}\,\Gamma\left(n-\frac{1}{2}\right)}{2\Gamma(n)}.$$

(iv). ধরি $x = \tan\theta$ তবে $dx = \sec^2\theta\, d\theta$

সীমা : যদি $x = 0$ হয়, তবে $0 = \tan\theta \Rightarrow \theta = 0$

যদি $x = \infty$ হয়, তবে $\infty = \tan\theta \Rightarrow \theta = \frac{\pi}{2}$

$$I = \int_0^{\pi/2} \frac{\tan^2\theta\, \sec^2\theta\, d\theta}{(\sec^2\theta)^{n+1/2}} = \int_0^{\pi/2} \frac{\sin^2\theta}{\cos^2\theta}\, \frac{\sec^2\theta\, d\theta}{\sec^{2n+1}\theta}$$

বা $I = \int_0^{\pi/2} \sin^2\theta\, \cos^{2n-3}\theta\, d\theta$

$$= \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left\{\frac{1}{2}(2n-3+1)\right\}}{2\Gamma\left\{\frac{1}{2}(2+2n-3+2)\right\}} = \frac{\frac{1}{2}\sqrt{\pi}\,\Gamma(n-1)}{2\Gamma\left(n+\frac{1}{2}\right)}.$$