**1 Set From Hydrodynamics.
**And practice all maths which were given by sir during class.

CHAPTER - 6

Hydrodynamics

6.1 Introduction (Self)

The subject of hydrodynamics (Greek Hydro, water; dynamics, force) deals with the motion of fluids under forces. To begin with we shall consider only a so-called **ideal fluid**, that is, one which is incompressible and which has no internal friction or viscosity.

Line of flow : The path followed by an element of a moving fluid is called a line of flow. In general, the velocity of the elements changes in both magnitude and direction along its line of flow.

Steady flow : If every element passing through a given point follows the same line of flow as that of the preceding elements, the flow is said to be steady or stationary. Velocity, Pressure & other properties can't be functions of time.

Stream line : A streamline is defined as a curve whose tangent, at any point, is in the direction of the fluid velocity at that point. In steady flow, the stream lines coincide with the lines of flow.

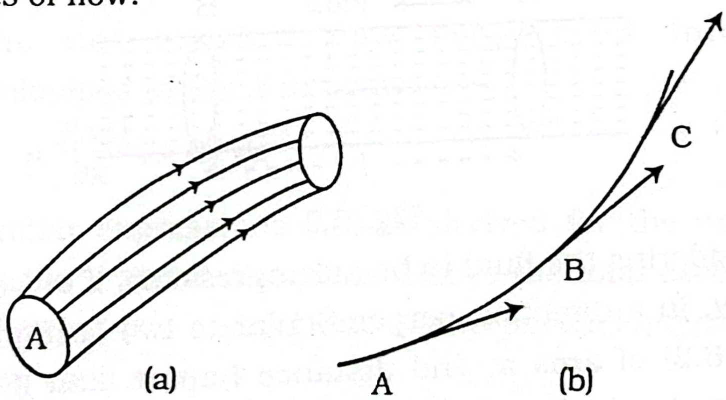

Fig. 6.1

Tube of flow : If we construct all the streamlines passing through the periphery of the element of area, such as the area A in fig. 6.1 (a) those lines enclose a tube called a flow tube or tube of flow. From the definition of a stream line, no fluid can cross the side walls of a tube of flow; in steady flow there can be no mixing of the fluids in different flow tubes.

The simplest type of fluid flow is a homogeneous flow, in which all the flow tubes are strainght and parallel, and the velocity is the same in each.

6.2 Rate of flow of a fluid

Fluids, as we know, include both liquids and gases, their chief characteristic being that they cannot permanently withstand any shearing stresses, howsoever small.

We shall concern ourselves here only with *ideal* liquids and gases, i.e. with liquids which are *perfectly mobile* (zero viscosity) and *incompressible* (zero compressibility or infinite bulk modulus) and with gases which perfectly obey *Boyle's* and *Charle's laws*.

The rate of flow of a fluid is defined as the volume of it that flows across any section of a pipe in unit time. It is really the volume rate of flow of the fluid or its discharge, usually represented by the letter Q or V.

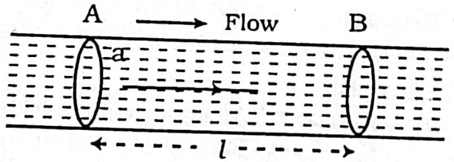

Fig. 6.2

Considering the fluid to be incompressible, if its velocity of flow be v, in a direction perpendicular to two sections A and B, (Fig. 6.2) of area a, and distance l apart, and if t be the time taken by the liquid to flow from A to B. We have $vt = l$.

Obviously, the volume of liquid flowing through the section AB, in this time, is equal to the cylindrical column AB $= l \times a$, or $= vt \times a$. This, therefore is the volume of liquid flowing across the section in time t.

$\therefore$ volume rate of flow of liquid × discharge

$$Q = vt \times \frac{a}{t} = v \times a$$

= velocity of liquid × area of cross-section of the tube.

When measured in cubic metre per sec. (MKS or SI system), the unit is the m^3/sec (briefly m^3s^{-1})

Sometimes, the rate of flow of a liquid is also expressed in terms of the mass of the liquid flowing across any section in unit time and is referred to as its mass rate of flow. Thus,

mass rate of flow of liquid = mass of liquid flowing across any section per unit time

= velocity of liquid × area of cross-section × density of liquid $= v \times a \times \rho$.

6.3 The equation of Continuity

If we consider any fixed closed surface in a moving fluid then, in general, fluid flows into the volume enclosed by the surface at some points, and flows out at other points. The equation of continuity is a mathematical statement of the fact that the net rate of flow of mass inward across any closed surface is equal to the rate of increase of the mass within the surface.

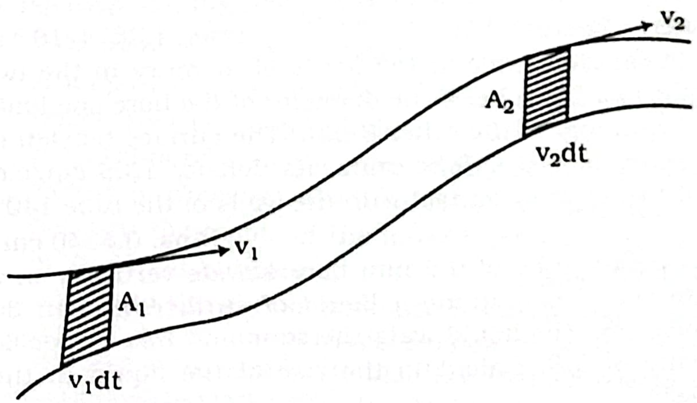

Fig. 6.3

For an incompressible fluid in steady flow : The equation takes the following form. The previous fig. represents a portion of a tube of flow, between two fixed cross sections of areas A_1 and A_2. Let v_1 and v_2 be the speeds at these sections. There is no flow across the side walls of tube. The volume of the fluid that will flow into the tube across A_1 in a time dt is that contained in the short cylindrical element of base A_1 and height v_1 dt or is A_1v_1 dt. If the density of the fluid is ρ, the mass flowing in is ρA_1v_1 dt. the volume between A_1 and A_2 is constant, and since the flow is steady the mass flowing out equals that flowing in. Hence

$$\rho A_1 v_1 dt = \rho A_2 v_2 dt$$

or, $A_1v_1 = A_2v_2$ ... ... (6.1)

and the product Av is constant along any given tube of flow. It follows that when the cross section of a tube decreases, as in the fig. 3 the velocity increases.

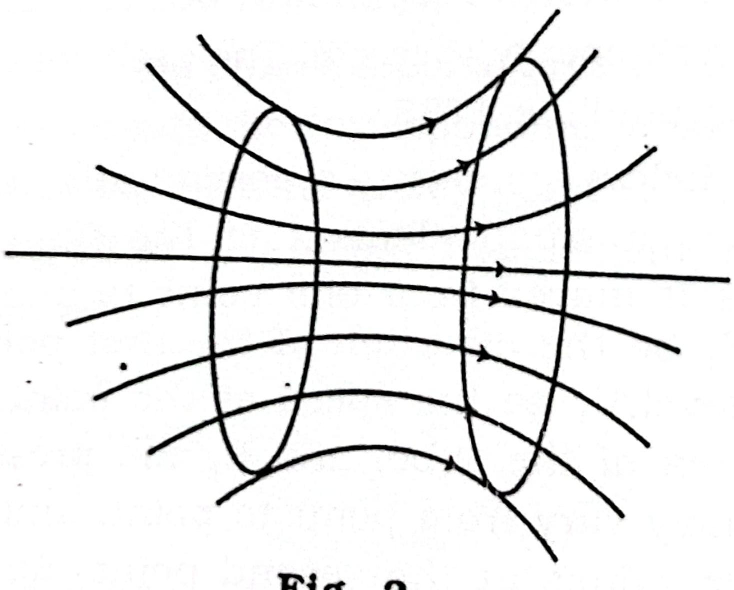

Fig. 3

and ρ is the volume density of charge at the point, then the law of conservation of electric charge requires that,

$$\vec{\nabla} \cdot \vec{V} + \frac{\partial \rho}{\partial t} = 0 \qquad \ldots \qquad \ldots \qquad (6.2)$$

6.5 Bernoulli's equation

When an incompressible fluid flows along a flow tube of varying cross section its velocity changes, that is, it accelerates or decelerates. It must therefore be acted on by a resultant force, and this means that the pressure must vary along the flow tube even though the elevation does not change. For two points at different elevations, the pressure difference depends not only on the difference in level but also on the difference between the velocities at the points. The general expression for the pressure difference can be obtained directly from Newton's Second law, but it is simpler to make use of the work-energy theorem. The problem was first solved by Daniel Bernoulli in 1738.

Fig. 6.5 below represents a portion of a tube of flow. We are to follow a small element of the fluid, indicated by shading, as it moves from one point to another along the tube. Let Y_1 be the elevation of the first point above some reference level, V_1 be the speed at the point, A_1 the cross-sectional area of the tube, and P_1 the pressure. All those quantities may vary from point to point, and Y_2, V_2, Y_2 and P_2 are their values at the second point. Since the fluid is under pressure at all points, inward forces, shown by the heavey arrows, are exerted against both faces of the element. As the element moves from the first point to the second, positive work is done by the force acting on the left face; and negative work by the force acting on its right face.

The net work, or the difference between these quantities, equals the change in kinetic energy of the element plus the change in its potential energy.

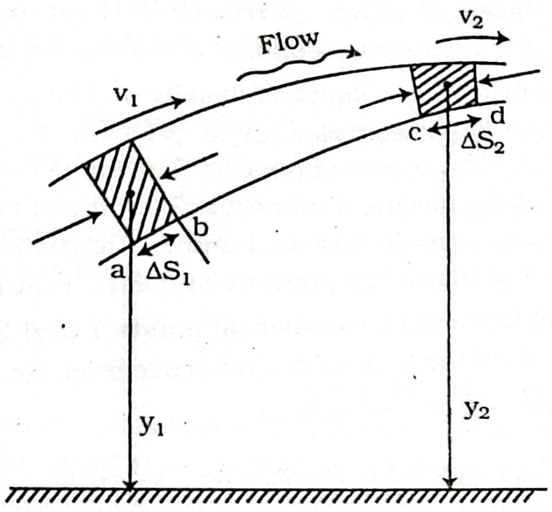

Fig. 6.5

If A represents the cross-sectional area of the tube at any point and P represents the corresponding pressure, the force against a face of the element at any point is PA. The work of the force acting on the left face of the element, in the motion in the diagram, is

$$\int_a^c F_s ds = \int_a^c PAds$$

Where ds is a short distance measured along the tube of flow. The limits of integration are from a to c, since these are the initial and final positions of the left face. This integral nay be written

$$\int_a^c PAds = \int_a^b PAds + \int_b^c PAds$$

Similarly, the work of the force acting on the right face of the element is

$$\int_b^d PAds = \int_b^c PAds + \int_c^d PAds$$

The net work is

$$\text{Net work} = \int_a^b PAds + \int_b^c PAds - \int_b^c PAds - \int_c^d PAds$$

$$= \int_a^b PAds - \int_c^d PAds$$

The distances from a to b and from c to d are sufficiently small so that the pressures and areas may be considered constant along their extent. Then

$$\int_a^b PAds = P_1A_1\Delta S_1; \qquad \int_c^d PAds = P_2A_2\Delta S_2$$

But $A_1\Delta S_1 = A_2\Delta S_2 = V$, where V is the volume of the element. Hence

$$\text{Net work} = (P_1 - P_2)V \quad \ldots \quad \ldots \quad (6.3)$$

Let ρ be the density of the fluid and m the mass of the element. Then

$V = m/\rho$ and equation (6.3) becomes

Net work $= (P_1 - P_2)m/\rho$

We now equate the net work to the sum of the changes in potential and kinetic energy of the element.

$$(P_1 - P_2)m/\rho = (mgy_2 - mgy_1) + \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right)$$

$$\text{or,} \quad (P_1 - P_2) = \rho g(y_2 - y_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\text{or,} \quad P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$$

Since the subscripts 1 and 2 refer to any two points along the tube of flow.

$$P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant} \quad \ldots \quad \ldots \quad (6.4)$$

This is know as Bernoulli's equation.

Just Names

6.6 Applications of Bernoullis equation

(1) The equations of hydrostatics are special cases of Bernoulli's equation, when the velocity is everywhere zero. Take the Bernoulli's equation :

$$P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$$

or, $P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Putting $v_1 = v_2 = 0$ the above equations reduces to.

$P_1 + \rho g y_1 = P_2 + \rho g y_2$

e.g $P_1 - P_2 = \rho g(y_2 - y_1)$

Let us apply this equation to a liquid in an open vessel, such as shown in fig. 6.6. Take point 1 at any level and P represents the pressure at this point. Take point 2 at the top where the pressure is atmospheric, P_a. Then

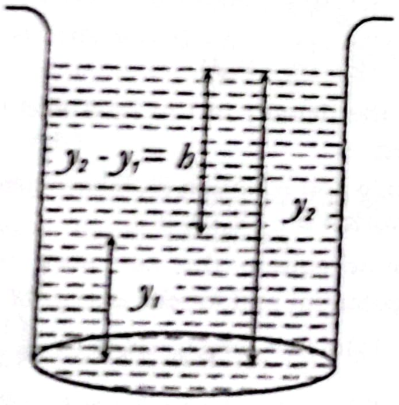

Fig. 6.6

$P - P_a = \rho g(y_2 - y_1)$

or, $P = P_a + \rho g(y_2 - y_1)$

e.g. $P = P_a + \rho gh$ ... ... (6.5)

where $y_1 - y_2 = h$.

(2) Speed of efflux : Torricelli's theorem :

Fig. 6.7 represents a tank of cross-sectional area A_1 filled to a depth h with a liquid of density ρ. The space above the top of the liquid contains air at pressure P and the liquid flows out of an orifice of area A_2. Let us consider the entire volume of the moving fluid as a single tube of flow, and let v_1 and v_2 be the speeds at point 1 and 2. The quantity v_2 is called the speed of efflux. The pressure at point 2 is atmospheric, P_a. Applying Bernoulli's equation to points 1 and 2, and taking the bottom of the tank as our reference level, we get,

$$P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$$

or, $P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

or, $P + \rho gh + \frac{1}{2}\rho v_1^2 = P_a + \frac{1}{2}\rho v_2^2$ $[\because y_2 = 0]$

or, $v_2^2 = v_1^2 + 2\frac{P - P_a}{\rho} + 2gh$

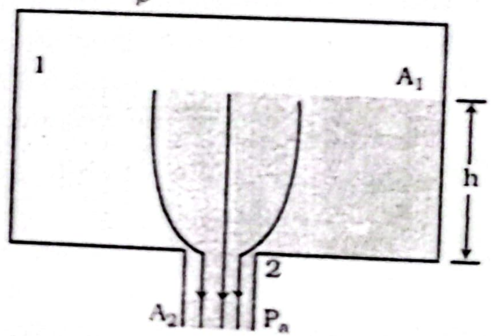

Fig. 6.7

Suppose the tank is open to the atmosphere, so that $P = P_a$ and $P - P_a = 0$

Suppose also that $A_1 >> A_2$. Then v_1^2 is very much less than v_2^2 and can be neglected and thus the above equations reduces to

$v_2 = \sqrt{2gh}$ ... ... (6.5)

That is, the speed of efflux of a liquid through an orifice is the same as that acquired by any body in falling freely through a height h equal to the height. This is Torricelli's theorem.

(3) The venturi tube illustrasted in fig. 6.8 consists of a constriction or throat inserted in a pipeline and having properly designed tapes at inlet and outlet to avoid turbulence. Bernoulli's equation, applied to the wide and to the constricted portions of the pipe, becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

[$\therefore \gamma_1 = \gamma_2$ the pipe being horizontal]

From the equation of continuity, the speed v_2 is greater than the speed v_1 and hence the pressure P_2 in the throat is less than the pressure P_1. Thus a net force to the right acts to accelerate the fluid as it enters the throat, and a net force to the left decelerates it as it leaves. The pressures P_1 and P_2 can be measured by attaching vertical side tubes as shown in the diagram.

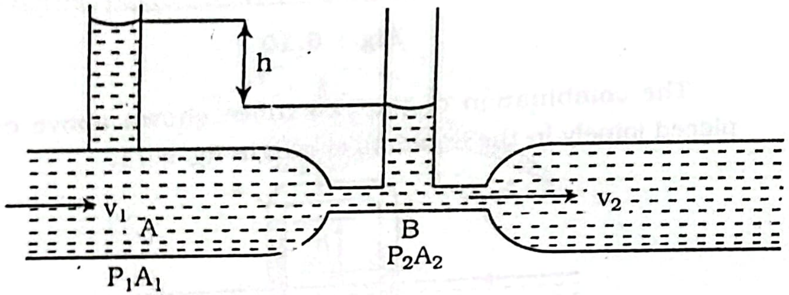

Fig : 6.8

From a knowledge of thses pressures and the cross-sectional areas A_1 and A_2, the velocities and the mass rate of flow can be computed when used for this purpose, the device is called a **venturimeter**.

Now

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or, $P_1 - P_2 = \rho/2\,(v_2^2 - v_1^2) = \rho v_1^2/2\,(v_2^2/v_1^2 - 1)$ ... ... (1)

From equation of continuity,

$$A_1 v_1 = A_2 v_2$$

or, $v_2/v_1 = A_1/A_2$

Equation (1) comes out to be

$$P_1 - P_2 = \frac{\rho v_1^2}{2}\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$= \frac{\rho v_1^2}{2A_2^2}(A_1^2 - A_2^2)$$

or, $v_1^2 = \dfrac{2A_2^2}{\rho} \cdot \dfrac{P_1 - P_2}{A_1^2 - A_2^2}$

or, $v_1 = A_2\sqrt{\dfrac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$ ... ... (6.7)

$\therefore$ Volume of fluid passing per sec through A e.g. the volume of liquid flowing per sec through the pipe

$$A_1 v_1 = A_1 A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

$$= A_1 A_2 \sqrt{\frac{2hg\rho}{\rho(A_1^2 - A_2^2)}} \quad \text{.........} (6.8)$$

(4) **Pitot tube :** - Sometimes it is necessary to measure the velocity of a gas which flows through a pipe or tube. An instrument designed for the above purpose is known as the pitot tube. The working principle is the same as in the case of a venturimeter.

Construction :- It consists of a bend manometer tube so arranged that the opening of one of its arms is parallel to th direction in which gas flows while the opening of the other is perpendicular to the direction of flow.

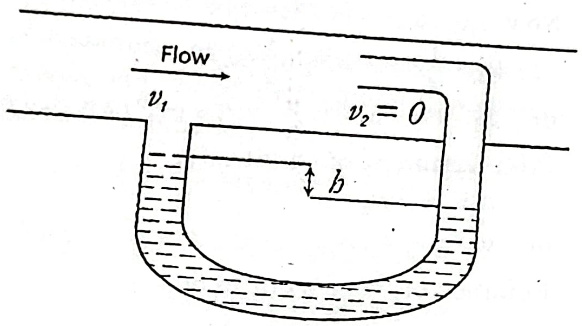

Fig : 6.9

The pressure in the arm whose opening is parallel to the direction of flow is the fame as the pressure of the gas that flows in the tube. The pressure in the other arm can be calculated on the principle of Bernoulli's theorem.

The manometer is made of glass.

Theory : $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y^2$

or, $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \rho g y_2$

$[\because y_1 = y_2 \text{ and } v_2 = 0]$

$\therefore \quad P_1 - P_2 = \frac{1}{2}\rho v_1^2$

$h_1\rho_m g - h_2\rho_m g = \frac{1}{2}\rho v_1^2 \qquad \therefore v_1 = \sqrt{\frac{2h\rho_m g}{\rho}}$

(5) **Pitot tube (alternative) :**

Another device for the measurement of the speed of fluids is given below (Fig. 6.10) in which one of the opening is perpendicular to the flow and the other opening faces the fl w. As a result the levels of the liquids in the two arms are separated by a height is

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \rho g y_2$

$[\because v_2 = 0]$

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 \qquad \left[\begin{array}{l} y_1 = y_2 \\ h \text{ same}\end{array}\right.$

$\frac{1}{2}\rho v_1^2 = P_2 - P_1 = h_2\rho g - h_1\rho g$

$\frac{1}{2}\rho v_1^2 = (h_2 - h_1)\rho g$

$\frac{1}{2}\rho v_1^2 = h\rho g$

$\frac{1}{2} v^2 = gh$

$v = \sqrt{2gh}$

$v\alpha = \alpha\sqrt{2gh}$

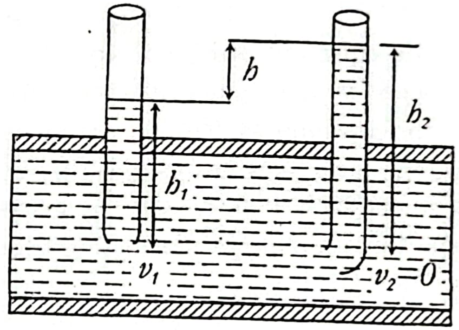

Fig : 6.10

The combination of the two tubes shown above can be placed joinely in the liquid as shown in fig. 6.11.

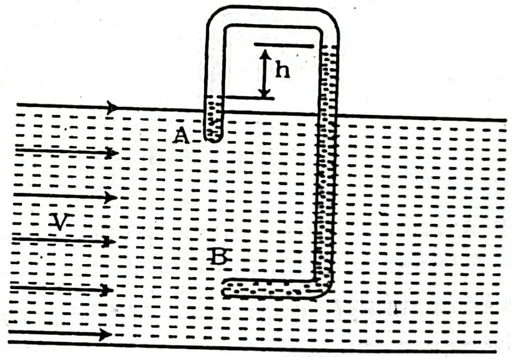

Fig : 6.11

6. Carburettor

The reduced pressure at a constriction finds a number of technical applications of which carburettor is one of these.

Carburettor is a chamber where air and petrol vapours are missed in an internal combustion engine. Air is allowed to enter through a nozzle with large velocity. The pressure is lowered and the petrol is sucked up into the chamber. The petrol vaporizes quickly. The mixed vapours of petrol and air are fed into the cylinder of the internal combustion engine.

7. Lift of an Aeroplane

Wings of an aeroplane are made tapering as illustrate in Fig : 6.12. The upper surface is made convex and the lower surface is made concave. Due to this shape of the wing, the air currents at the top have a large velocity than at the bottom. As a result the pressure above the surface of the wing is less as compared to the lower surface of the wing. This difference of pressure is helpful for the vertical lift of the plane. The flow lines around the air craft wing are shown in the figure.

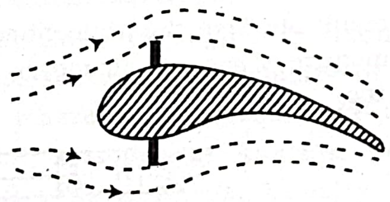

Fig. 6.12

(8) The Steam Injector : It is a simple device to accelerate the ejection of the exhaust steam from the cylinder of a steam engine, and consists of a tube A, (Fig. 6.13), narrowing down into a nozzle N at its lower end, inside another tube B, having a side-tube, C, Which is conected to the cylinder of the engine.

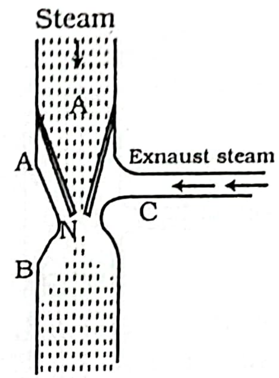

Fig. 6.13

A jet of steam is introduced into A, and as it issues out of the nozzle, N, its velocity is considerably increased, resulting in a corresponding fall in pressure there, and the steam from the engine-cylinder thus rushes into this region of reduced pressure, whence, it is ejected out through the lower end of B.

(9) The filter pump

It is also based on the same principle and is used to reduce the pressure in a vessel. Here, a stream of water from a tap, flowing through a tube A, (Fig. 6.14), issues out in the form of a jet from its narrow orifice O, which results in a great rise in its velocity and proportionate fall into its pressure, which is thus soon reducced to a value below that of the atmosphere. The air from the vessel, connected through a side-tube B to this region of reduced pressure, then rushes into it, and is carried away by the stream of water as it flows down through C.

In this way, the pressure in the vessel is ultimately reduced to just a little above the vapour pressure of water, in a comparatively very short time.

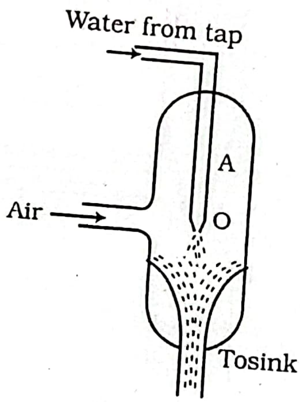

Fig. 6.14

If the inlet water tube be a twisted, instead of a straight one, the exhaustion proceeds more rapidly, due to the rotating water-jet in the tube breaking up more readily and mixing up easily with the incoming air from the vessel.

(10) The attracted disc Paradox. The following is a simple and interesting experiment, which the student may well try for amusement at a small gathering at home.

DE. is a flat cardboard disc (Fig. 6.15), over which is placed another flat disc BC, fitted with a tube A, the opening of which is in flush with BC.

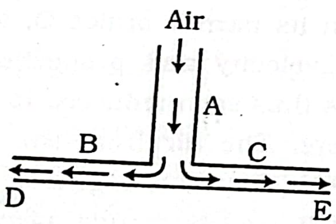

Fig. 6.15

On blowing air down through A, on to DE, the latter, instead of being blown away from BC, as one might ordinarily expect, sticks on to it more and more closely, and might even be lifted up a little.

This seeming paradox is, however, easily explained. For, as the air from A rushes through the narrow space in between BC and DE, its velocity increases and consequently the pressure there decreases, so that it soon falls below the atmospheric pressure on DE, which thus pushes it up towards BC.

(11) Blowing of Roofs

Due to wind, storm or cyclone, the roofs are blown off. When a high velocity wind blows over the roof, there is considerable lowering of pressure on the roof. As the pressure on the lower side of the roof is higher, roofs are easily blown off without damaging the walls of the building.

(12) Bunsen Burner

This is another example for the fall of pressure due to increase of velocity.

In a bunsen burner, the gas enters the base and comes out of the nozzle N. As the cross-section of the nozzle is very small, the velocity of the gas is highly increased Consequently, pressure just above the nozzle N is reduced (Fig. 6.16). As a result air from the atmosphere rushes into the burner. This mixture of gas and air moves up the burner and burns at the top.

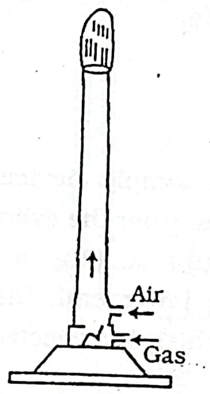

Fig. 6.16

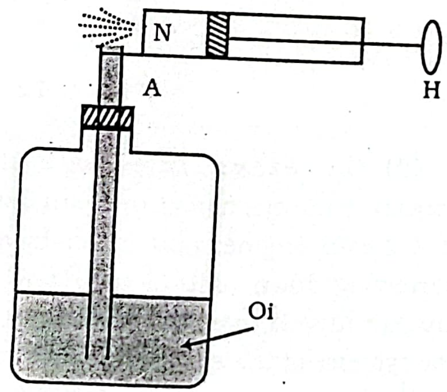

Fig. 6.17

13) Atomiser or Sprayer

An atomizer is shown in Fig. 6.17. The oil is kept in a can and it is fitted with a vertical tube A. When the handle of the pump pushes the piston, air comes out through the nozzle N with a high velocity. As the velocity increases, the pressure just outside N decreases and the oil in the tube A is pushed up due to the atmospheric pressure acting on the surface of the liquid in the can. When the oil reaches the top of the tube A, it is sprayed out by the escaping air through N.

WORKED EXAMPLES

6.1. A water filled tank has an orifice in one vertical wall of the tank near the bottom. If the height of the water level from the centre of the orifice upto the water surface is 3.2 metres. Calculate the velocity of the issuing water through the orifice.

We know $v = \sqrt{2gh}$

Here $g = 9.8\ m/s^2$, $h = 3.2$ m

$\therefore v = \sqrt{2 \times 9.8 \times 3.2} = 7.92$ m/s.

6.2. Water flows through a horizontal pipe line of different cross-section. The velocity of flow is 0.3 m/s at a point where the pressure is 0.04 m of Hg. Calculate the pressure at another point where the velocity of flow is 0.4 m/s. Density of water = 10^3 Kg/m³.

From Bernoulli's equation : $P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$

We can write

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

The pipe being horizontal, $y_1 = y_2$

or, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

or, $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

Here, $P_1 = 0.04$ m of Hg $= 0.04 \times 13.6 \times 9.8\ N/m^2$

$V_1 = 0.3$ m/s

$V_2 = 0.4$ m/s

$\therefore P_2 = 0.04 \times 13.6 \times 10^3 \times 9.8 \times \frac{1}{2} \times 10^3 \times (0.3^2 - 0.4^2)$

$= 5296.2\ N/m^2$

6.3. A horizontal tube has different areas of cross-section at two points A and B. The diameter at A is 0.04 m and the diameter at B is 0.02 m. Two manometer limbs are fixed at A and B. When a liquid of density 900 kg/m³ flows through, the difference in pressure between the manometer limbs is 0.08 m. Calculate the rate of flow of the liquid through the tube.

$$\text{Rate of flow} = A_1 v_1 = A_1 A_2 \sqrt{\frac{2hg\rho}{\rho(A_1^2 - A_2^2)}}$$

Here, $A_1 = \pi(0.02)^2 = 4\pi \times 10^{-4}\ m^2$

$A_2 = \pi(0.01)^2 = \pi \times 10^{-4}\ m^2$

$h = 0.09$ m $= 0.09$ m of the liquid column

$= 0.09 \times 900 \times 9.8\ N/m^2$

$\rho = 900\ kg/m^3$

$$\therefore \text{Rate of flow} = 4\pi \times 10^{-4} \times \pi \times 10 \sqrt{\frac{2 \times 0.08 \times 900 \times 9.80}{900(16\pi^2 \times 10^{-8} - \pi^2 \times 10^{-8})}}$$

$= 4.062 \times 10^{-4}\ m^3/s$

6.4. Calculate the speed at which the velocity head of a stream of water is eaual to 0.50 m of Hg.

Here, $h = 0.50$ m of Hg

$= 0.5 \times 13.6$ m of water

$\therefore v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.5 \times 13.6}$

$= 11.54$ m/s

6.5. Air is streaming past a horizontal air plane wing such that the speed is 100 m/s over the upper surface and 80 m/s at the lower surface. If the density of air is 1.3 kg/m³, find the difference of pressure between the top and the bottom of the wing. If the wing is 12 m long and has an average width of 2.5 m. Calculate the gross lift of the wing.

Bernoulli's equation : $P + \rho gy + \frac{1}{2}\rho v^2$ = constant

gives,

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or, $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$

Cross-lift of the wing

$$L = (P_1 - P_2)A = \frac{1}{2}\rho(v_2^2 - v_1^2)A$$

Here, $v_2 = 100$ m/s

$v_1 = 80$ m/s

∴ Lift $= \frac{1}{2} \times 1.3 \times (100^2 - 80^2) \times 30$ [∵ A= 12×2.5 = 30 m²]

= 70200 N

6.6. Water flows through a horizontal pipe line of varying cross-section. At two points A and B, the diameters are 0.8 m and 0.4 m respectively. The difference of pressure between the two points A and B is 1.5 m of water column. Calculate the volume of water flowing per second

Volume of water following per second

$$= A_1A_2\sqrt{\frac{2(P_1-P_2)}{\rho(A_1^2-A_2^2)}}$$

$$= A_1A_2\sqrt{\frac{2h\rho g}{\rho(A_1^2-A_2^2)}}$$

$$= A_1A_2\sqrt{\frac{2gh}{A_1^2-A_2^2}}$$

We have, $A_1 = \pi\left(\frac{d_1}{2}\right)^2$

$= \pi\left(\frac{0.8}{2}\right)^2$

$= 0.16\,\pi$ m²

$A_2 = \pi\left(\frac{d_2}{2}\right)^2 = \pi\left(\frac{0.4}{2}\right)^2 = 0.04\,\pi$ m²

$A_1^2 - A_2^2 = 0.12\,\pi^2$

∴ Rate of flow $= 0.16\pi \times 0.04\pi \times \sqrt{\frac{2\times1.5}{8\times0.12\pi^2}}$

$= 35.54 \times 10^{-3}$ m³/s.

6.7. A venturimeter has a pipe having diameter of 0.2 m and a throat having diameter of 0.15 m. The levels of water columns in the two limbs differs by 0.1 m. Calculate the amount of water discharged through the pipe in half an hour. Density of water = 10³kg/m³.

Rete of flow $= A_1A_2\sqrt{\frac{2(P_1-P_2)}{\rho(A_1^2-A_2^2)}}$

$= A_1A_2\sqrt{\frac{2hg\rho}{\rho(A_1^2-A_2^2)}}$

$= A_1A_2\sqrt{\frac{2gh}{A_1^2-A_2^2}}$

$A_1^2 - A_2^2 = \frac{\pi d_1^2}{4} - \frac{\pi d_2^2}{4}$

$= \frac{\pi(0.2)^2}{4} - \frac{\pi(0.15)^2}{4}$

$= \frac{\pi}{100}\text{ m}^2 - \frac{9\pi}{1600}\text{ m}^2$

$= \frac{175\pi^2}{256\times10^4}$

∴ $A_1^2 = \frac{\pi}{100}$ m² and $A_2^2 = \frac{9\pi}{1600}$ m² & h = 0.1 m

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$\therefore$ Rate of flow $= \left(\frac{\pi}{100}\right)\left(\frac{9\pi}{1600}\right)\sqrt{\frac{2\times9.8\times256\times10^4}{175\pi^2}}$

$= 0.02991 \text{ m}^3/\text{s}$

Therefore, amount of water discharged in half an hour

$= 0.02991\times\frac{1}{2}\times60\times60$

$= 53.86 \text{ m}^3.$

6.8. In a horizontal pipe line of uniform area of cross-section, the pressure drops by 8 N/m² between two point 2 km apart. Calculate the change in kinetic energy per kg of the oil through the two points. Density of oil = 800 kg/m³

From Bernoulli's equation : $P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$

We can write

$$P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$$

Since the pipe line is horizontal, $y_1 = y_2$

$$\therefore \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\therefore \quad \frac{P_1 - P_2}{\rho} = \frac{1}{2}(v_1^2 - v_2^2)$$

Here, $P_1 - P_2 = 8 \text{ N/m}^2$

$\therefore$ Change of KE per kg $= \frac{8}{800} = 10^{-2}$ J/kg

6.9. When a liquid falls freely through an orifice show that for the range of the liquid to be maximum the orifice must be at half of the height of the liquid column. (Fig. 6.18)

According to Bernoulli's theorem, $P+\rho gy+\frac{1}{2}\rho v^2 = \text{constant}$

As P is the same at the free surface and at the orifice,

$$P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$$

$$\rho gy + \frac{1}{2}\rho v^2 = \text{constant}$$

or, $gy + \frac{1}{2}v^2 = \text{constant}$

$\therefore \quad \frac{v^2}{2} + gy = \text{constant}$

At A, $v = 0$, $y = h_1$

At B, $y = 0$.

$$\therefore \quad gh_1 = \frac{v^2}{2} = \text{or } v = \sqrt{2gh_1} \quad \ldots \quad \ldots \quad \ldots \quad \text{(i)}$$

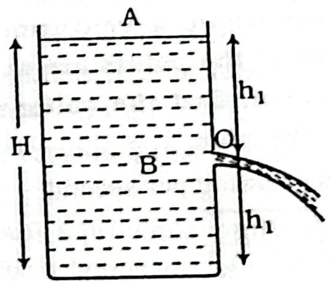

Fig. 6.18

The horizontal range,

$R = v \times t$

$$R = v \times \sqrt{\frac{2h_2}{g}}$$

$$R = \left(\sqrt{2gh_1}\right)\sqrt{\frac{2h_2}{g}}$$

$$R = 2\sqrt{h_1 h_2} \quad \ldots \quad \ldots \quad \ldots \quad \text{(ii)}$$

But $h_2 = H - h_1$

$$R = 2\sqrt{h_1(H-h_1)}$$

$$R^2 = 4h_1H - 4h_1^2$$

Differentiating

$$2R\left(\frac{dR}{dh_1}\right) = 4H - 8h_1$$

For R to be maximum, $\frac{dR}{dh_1} = 0$

$\therefore \quad 4H - 8h_1 = 0$

$$h_1 = \frac{H}{2}$$

and $h_2 = \frac{H}{2}$

Therefore, for the range of the liquid to be maximum, the orifice must be at half the height of the liquid column.

6.10. If the diameters of a pipe are 10 cm. and 6 cm. at the points where a venturimeter is connected and the pressures at the points are shown to differ by 5 cm. of water column, find the volume of water per sec flowing through the pipe.

Volume of water flowing per second

$$= A_1A_2\sqrt{\frac{2hg}{A_1^2 - A_2^2}}$$

Here, $A_1 = \pi(5)^2 = 25\pi \text{ cm}^2$

$A_2 = \pi(3)^2 = 9\pi \text{ cm}^2$

$h = 5 \text{ cm}.$

$$\therefore \text{Reqd. Volume } = 25\pi\times9\pi\sqrt{\frac{2\times5\times981}{(25\pi)^2 - (9\pi)^2}}$$

$$= 25\times9\times\pi\sqrt{\frac{9810}{544}}$$

$= 3002$ c.c.

$= 3.0002$ litres

$= 3\times10^{-3} \text{ m}^3$

6.11. Calculate the velocity of efflux of Kerosene oil from a tank in which the pressure is 50 lbs. wt. per sq. inch above the atmospheric pressure. The density of Kerosene is 48 lbs/cft.