To determine the value of g, Acceleration due to gravity, by means of a compound pendulum.

Theory:- Compound pendulum is a rigid body of any shape free to turn about a horizontal axis. In Figure G is the centre of gravity of the pendulum of mass M, which performs oscillations about a horizontal axis through O. When the pendulum is at an angle θ to the vertical, the equation of motion of the pendulum is $I\omega = Mgl \sin\theta$ where ω is the angular acceleration produced, l is the distance OG and I is the moment of inertia of the pendulum about the axis of oscillation. For small amplitude of vibrations, $\sin\theta = \theta$ so that,

$I\omega = Mgl\theta$

Hence the motion is simple harmonic, with period of vibrations;

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

If K is the radius of gyration of the pendulum about an axis through G parallel to the axis of oscillation through O, from the parallel axes theorem,

$I = M(K^2 + l^2)$ and so,

$$T = 2\pi\sqrt{\frac{K^2 + l^2}{gl}} = 2\pi\sqrt{\frac{\frac{K^2}{l} + l}{g}} \quad \text{———} (1)$$

Since . . . [illegible] given by .

$T = 2\pi\sqrt{\frac{L}{g}}$ the period of rigid body (compound pendulum) is the same as that of a simple pendulum of length of length

$$L = \frac{k^2 + l^2}{l} \quad \text{(ii)}$$

This length L is known as the length of the simple equivalent pendulum. The expression from L can be written as a quadratic in (l). Thus from (2)

$$l^2 - lL + k^2 = 0 \quad \text{(iii)}$$

This gives two values of l (l_1 and l_2) for which the body has equal time of vibration from the theory of quadratic equation.

$l_1 + l_2 = L$ and $l_1 l_2 = k^2$.

As the sum and products of two roots are positive, the two roots are both positive. This means that there are two positions of the centre of suspension on the other side of the C.G. about which the time periods (T) will again be the same. Thus, there are altogether four points, two on either side of the C.G., about which the time periods of the pendulum are same.

The distance between two such points, asymmetrically situated on either side of the C.G, will be the length (L) of the simple equivalent pendulum. If the length OG in Figure is l_1 and we measure the length $GS = \frac{K^2}{l_1}$ along OG produced, then obviously $\frac{K^2}{l_1} = l_2$ or $OS = OG + GS = l_1 + l_2 = L$. The period of oscillation about either O or S is the same. The point S is called the centre of oscillations. The points O and S are

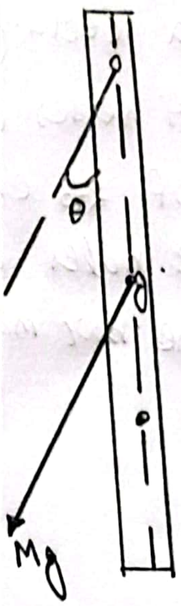


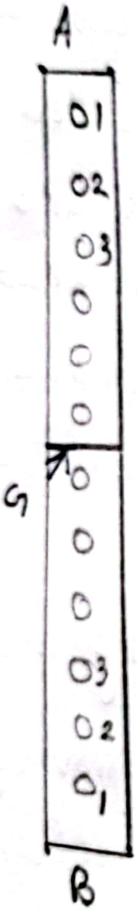
Figure.

interchangeable when the body oscillates about O or S, the time period is the same. If this period of oscillation is T, then from the expression $T = 2\pi\sqrt{\frac{L}{g}}$ we get, $g = 4\pi^2 \frac{L}{T^2}$

By finding L graphically, and determining the value of the period T, the acceleration due to gravity (g) at the place of the experiment can be determined.

Apparatus: A bar pendulum, a small metal wedge, a beam compass, a spirit level, a telescope with cross-wires in the eye piece, stop-watch and a wooden prism with metal edge.

Description of aparatus:- The aparatus ordinarily used in the labratory is a rectangular bar AB of brass about 1 meter long. A series of holes is drilled along the bar at intervals of 2-3 cm. By inserting the metal wege S in one of the holes and placing the wedge on the support S_1S_2, the bar may be made to oscillate.



Procedure:- (I) Find out the centre of gravity G of the bar by balancing it on the wooden prism.

(II) Put a chalk mark on the line AB of the bar. Insert the metal wedge in the first hole in the bar towards A and place the wedge on the support S_1S_2 so that the bar can turn around S.

(III) Place a telescope at a distance of about a meter from the bar and focus the cross wires and rotate the collar of the tube till the cross wires form distinct cross. Next focus the telescope on the bar and see that the point of inter-section of the cross-wires coincides with the chalk mark along the line AB of the bar.

(iv) Set the bar to oscillate taking care to see that the amplitude of oscillations is more than not 5°. Note the time for 50 oscillations by counting the oscillations when the line AB passes the inter-section of the cross-wires in the same direction.

(v) Measure the length from the end A of the bar to the top of the first hole into the point of suspension of the pendulum.

(vi) In the same way, suspend the bar at holes 2,3 and each time note times for 50 oscillations. Also measure distances from the end A for each hole.

(vii) When the middle part of the bar is passed, it will turn round so that the end B is now on the top. But continue measuring distances from the point of suspension to the end A.

(viii) Now calculate the time-period T from the time recorded for 50 oscillations.

(ix) On a nice and large graph paper plot a curve with length as abscissa and period T as ordinate with the origin at the middle of the paper along the abscissa.

(x) Through the point on the graph paper corresponding to the centre of gravity of the bar, draw a vertical line. Draw a second line ABCD along the abscissa. AC or BD is the length of the equivalent simple pendulum $L = l_1 + \frac{K^2}{l_1}$. $AG = l_1$ and $GC = \frac{K^2}{l_1} = l_2$, C being the centre of oscillation.

Similarly, $GD = l_1$ and $GB = \frac{K^2}{l_1} = l_2$, B being the centre of oscillation. From this $g = 4\pi^2 \frac{L}{T^2}$ can be calculated.

(xi) By drawing another line A'B'C'D calculate another value of g.

ID-2102006

EXP-To determine the value of g acceleration due to gravity by means of a compound pendulum.

At top	Hole No.	Distance from A	Time for 20 oscillations	Mean time	Mean Period T.
End A	1	5	1) 31.31 2) 31.32 3) 31.33	31.32	1.5666
	2	10	1) 30.58 2) 30.64 3) 30.57	30.597	1.52985
	3	15	1) 30.15 2) 30.13 3) 30.14	30.140	1.507
	4	20	1) 29.64 2) 29.61 3) 29.69	29.647	1.4825
	5	25	1) 29.94 2) 29.93 3) 29.96	29.94	1.437
	6	30	1) 30.67 2) 30.69 3) 30.68	30.680	1.534
	7	35	1) 32.32 2) 32.26 3) 32.37	32.317	1.61585
	8	40	1) 36.91 2) 36.82 3) 36.99	36.907	1.84535
	9	45	1) 49.60 2) 49.63 3) 49.72	49.65	2.4825

12.10.23

At the Top	Hole no	Distance From A	Time for 20 Oscillation	Mean time	Mean Period T
End B	1	5	1) 31.5 2) 31.55 3) 31.65	31.5667	1.5783
	2	10	1) 30.83 2) 30.85 3) 30.91	30.863	1.54316
	3	15	1) 30.40 2) 30.45 3) 30.43	30.4267	1.5213
	4	20	1) 30.06 2) 30.02 3) 30.08	30.053	1.5026
	5	25	1) 30.34 2) 30.32 3) 30.37	30.343	1.517167
	6	30	1) 31.08 2) 31.11 3) 31.18	31.123	1.55616
	7	35	1) 33.32 2) 33.38 3) 33.41	33.37	1.6685
	8	40	1) 38.62 2) 38.83 3) 38.77	38.74	1.937
	9	45	1) 51.52 2) 51.77 3) 52.65	51.98	2.599

From graph,

Length $AC = 54.5$ cm, $BD = 55.5$ cm

mean length, $L = \frac{AC + BD}{2} = \frac{55.5 + 54.5}{2} = 55$ cm.

Corresponding time Period from the graph $T = 1.55$ sec

$\therefore g = \frac{4\pi^2 L}{T^2} = \frac{4 \times 3.1416^2 \times 55}{1.55^2} = 903.78$ cm per sec^2.


Roll No. 2102006
3
TIME Period (T) in second
0.5
1
1.5
2
2.5
A
B
C
D
G
0
5 10 15 20 25 30 35 40 45
45 40 35 30 25 20 15 10 5
Distance of knife-edge from fixed end (d) in cm.
8 Inch x 10 Inch
MICRO

Result

From graph,

Length AC = 54.5 cm, BD = 55.5 cm.

$$\text{mean length}, L = \frac{AC+BD}{2} = \frac{55.5+54.5}{2} = 55 \text{ cm}.$$

Corresponding time period from graph $T = 1.55$ sec.

$$g = \frac{4\pi^2 L}{T^2} = \frac{4 \times 3.1416^2 \times 55}{1.55^2} = 903.78 \text{ cm per sec}^2$$

$$= 9.03 \text{ ms}^{-2}.$$

Error percentage:

Actual value of gravity, $g = 9.8 \text{ ms}^{-2}$

Experimental value of gravity, $g = 9.0378 \text{ ms}^{-2}$

So,

$$\text{Percentage of Error} = \frac{|\text{Experimental value} - \text{theoritical value}|}{\text{theorical value}}$$

$$= \frac{|9.038 - 9.8|}{9.8} \times 100\%$$

$$= 7.778\%$$

$\therefore$ percentage of error $= 7.778\%$.

Discussion:- (I) Distances are to be measured from the end A or the point G, preferably from A.

(II) In measuring time an accurate stop watch should be used.

(III) Oscillations should be counted whenever the line of the bar crosses the intersecting point of the cross-wires in the same direction

(IV) Graph paper used should have sharp lines and accurate squares. also large curves.

(V) Amplitude of oscillations must not be more than 5°.

(VI) Error due to the yielding of support, air resistance and irregular knife-edge should be avoided.

(VII) Determining of the position of G only helps us to understand that $AG = l_1$ and $GC = \frac{K^2}{l_1} = l_2$ and is not necessary for determining the value g.

(IX) For the lengths corresponding to the points A, B, C and D the period is the same.