Asymptotic Notation or Function Grow

$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \ldots < 2^n < 3^n < \ldots n^n$$

Big-oh ----- Upper bound

Big-omega ------ Lower bound

Theta -------- Average bound

Big-oh

The function $f(n) = O(g(n))$ iff $\exists$ +ve constant c and n_0

such that $f(n) \le c * g(n) \;\; \forall \; n \ge n_0$

eg: $f(n) = 2n + 3$

$2n + 3 \le \underbrace{10}_{c}\underbrace{n}_{g(n)} \quad n \ge 1$ $\qquad \therefore f(n) = O(n)$

$\uparrow$ $f(n)$

Big-oh

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$2n + 3 \le$

Big-oh

The function $f(n) = O(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \leq c * g(n) \;\; \forall \; n \geq n_0$

eg: $f(n) = 2n+3$

$$2n+3 \leq 2n^2 + 3n^2$$

$$2n+3 \leq 5n^2 \quad n \geq 1$$

$\therefore f(n) = O(n)$

$$1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \dots < 2^n < 3^n < \dots < n^n$$

Lower bound — avg bound — Upper bound

Big-oh

eg: $f(n) = 2n+3$

$$2n+3 \leq 2n^2 + 3n^2$$

$$\underbrace{2n+3}_{f(n)} \leq \underbrace{5n^2}_{c\,g(n)} \quad n \geq 1$$

✓ $\therefore f(n) = O(n)$

✓ $f(n) = O(n^2)$

✓ $f(n) = O(2^n)$

✗ $\cancel{f(n) = O(\log n)}$

Omega

The function $f(n) = \Omega(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \geq c * g(n)\ \forall\ n \geq n_0$

eg: $f(n) = 2n+3$

$2n+3 \geq 1 \times n\ \forall\ n \geq 1$

$\ldots gn < \sqrt{n} < n < n\log n < n^2 < n^3 < \ldots < 2^n < 3^n < \ldots < n^n$

...er bound   avg bound   upper bound

Omega

The function $f(n) = \Omega(g(n))$ iff $\exists$ +ve constants c and n_0

such that $f(n) \geq c * g(n)\ \forall\ n \geq n_0$

eg: $f(n) = 2n+3$

$\longrightarrow \therefore \boxed{f(n) = \Omega(n)}$

$2n+3 \geq 1 \times \log n\ \forall\ n \geq 1$ $f(n) = \Omega(\log n)$

$f(n)$ c $g(n)$ $\times\ f(n) = \Omega(n^2)$

$$n < \sqrt{n} < n < n\log n < n^2 < n^3 < \dots < 2^n < 3^n < \dots < n^n$$

bound avg bound upper bound

Theta Notation

The function $f(n) = \Theta(g(n))$ iff $\exists$ +ve constants c_1, c_2 and n_0

such that $c_1 * g(n) \le f(n) \le c_2 * g(n)$

eg: $f(n) = 2n + 3$

$$\underbrace{1}_{c_1} \times \underbrace{n}_{g(n)} \le \underbrace{2n+3}_{f(n)} \le \underbrace{5}_{c_2} \times \underbrace{n}_{g(n)}$$

$\therefore \underline{f(n) = \Theta(n)}$

https://www.youtube.com/watch?v=A03oI0znAoc