Integrating, $2x+C=v+\frac{b-a}{2}\log(v^2-ab)$

or $2x+C=x+y+\frac{1}{2}(b-a)\log[(x+y)^2-ab]$ etc.

Ex. 8. $\frac{dy}{dx}=(x+y)^2$. [Gauhati 62; Delhi 62; Raj. 62]

Hint. Put $x+y=v$ etc.

2·4. Homogeneous Differential Equations. [Poona 61 (S)]

An equation of the form $\frac{dy}{dx}=\frac{f_1(x,y)}{f_2(x,y)}$ in which $f_1(x,y)$ and $f_2(x,y)$ are homogeneous functions* of x and y of the same degree can be reduced to an equation in which variables are separable by putting $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}$.

The following few examples will illustrate the method.

Ex. 1. *Solve* $(x^2+y^2)\,dx+2xy\,dy=0$.

Solution. We have $\frac{dy}{dx}=-\frac{x^2+y^2}{2xy}$ (homogeneous).

Putting $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}$, the equation becomes

$$v+x\frac{dv}{dx}=-\frac{x^2+x^2v^2}{2x.vx}=-\frac{1+v^2}{2v}$$

or $$x\frac{dv}{dx}=-\frac{1+v^2}{v}-v=-\frac{1+3v^2}{2v} \text{ (variable separable).}$$

$$\therefore \quad \frac{dx}{x}=-\frac{2v}{1+3v^2}\,dv.$$

Integrating, $\log x+\frac{1}{3}\log(1+3v^2)=\log C$

or $x(1+3v^2)^{1/3}=C$ or $x(1+3y^2/x^2)^{1/3}=C$.

Ex. 2. *Solve* $x^2y\,dx-(x^3+y^3)\,dy=0$. [Agra B Sc. 54]

Solution. We have $\frac{dy}{dx}=\frac{x^2y}{x^3+y^3}$ (homogeneous).

Putting $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}$, the equation becomes

$$v+x\frac{dv}{dx}=\frac{v}{1+v^3} \quad \text{or} \quad x\frac{dv}{dx}=\frac{v}{1+v^3}-v=-\frac{v^4}{1+v^3}$$

or $$\frac{dx}{x}=-\frac{1+v^3}{v^4}\,dv=-\left[\frac{1}{v^4}+\frac{1}{v}\right]dv.$$

Integrating, $\log x=\frac{1}{3v^3}-\log v+C$; $\log vx=\frac{1}{3v^3}+C$

*A function $f(x,y)$ is called homogeneous of degree n, if $f(\lambda x,\lambda y)=\lambda^n f(x,y)$.

or $\log y = \frac{x^3}{3y^3} + C$ as $v = \frac{y}{x}$.

Ex. 3. *Solve* $\frac{dy}{dx} = \frac{y^3 + 3x^2y}{x^3 + 3xy^2}$. [Lucknow Pass 60]

Solution. Putting $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$x\frac{dv}{dx} = \frac{dy}{dx} - v = \frac{v^3 + 3v}{1 + 3v^2} - v = \frac{2v(1-v^2)}{1+3v^2}$$

or $$\frac{2\,dx}{x} = \frac{1+3v^2}{2v(1-v^2)}\,dv = \left(\frac{1}{v} - \frac{2}{1+v} + \frac{2}{1-v}\right)dv.$$

Integrating,

$$2\log x = \log v - 2\log(1-v) - 2\log(1+v) + \log C$$

or $x^2(1-v)^2(1+v)^2 = Cv$. Put $v = y/x$ etc.

Ex. 4. *Solve* $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.

[Delhi Hons. 66; Cal. Hons. 61, 56; Osmania 60; Gujrat 61]

Solution. The equation is $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ [homogeneous].

Putting $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{v^2}{v-1} \quad \text{or} \quad x\frac{dv}{dx} = \frac{v^2}{v-1} - v$$

or $x\frac{dv}{dx} = \frac{v}{v-1}$ or $\frac{dx}{x} = \frac{v-1}{v}\,dv$

or $\frac{dx}{x} = \left(1 - \frac{1}{v}\right)dv$.

Integrating, $\log x = v - \log v + \log c$

or $\log xv = v + \log c$ or $xv = ce^v$

or $y = ce^{y/x}$ as $y = vx$.

Ex. 5. *Solve* $(x^2 + y^2)\,dy = xy\,dx$. [Nagpur T.D.C. 1961]

Hint. Homogeneous. Put $y = vx$. Ans $y = Ce^{x^2/2y^2}$

Ex. 6. *Solve the following homogeneous equations:*

(i) $y(y^2 - 2x^2)\,dx + x(2y^2 - x^2)\,dy = 0$.

[Karnatak B.Sc. (Sub.) 1960]

(ii) $\frac{1}{2x}\frac{dy}{dx} + \frac{x+y}{x^2+y^2} = 0$. [Lucknow Pass 1955]

(iii) $\frac{dy}{dx} + \frac{y(x+y)}{x^2} = 0$. Ans. $x^2y = c^2(y + 2x)$

[Poona 1964; Nag. 58; Kerala 61; Vikram 61]

(iv) $x^2y\,dx - x^3\,dy = y^3\,dy$. Ans $\log y = \frac{x^2}{3y^2} + C$.

(v) $(x^2-y^2)\dfrac{dy}{dx}=xy$.

(vi) $(x+y)^2=xy\dfrac{dy}{dx}$. [Poona 1964]

(vii) $x\dfrac{dy}{dx}-y=\sqrt{(x^2+y^2)}$. [Sagar. 1963; Cal. Hons. 62; Raj. 56]
Ans. $x^2+y^2=(Cx^2-y)^2$.

(Cf. Ex: 6 P. 10)

Ex. 7. $\left(x\cos\dfrac{y}{x}+y\sin\dfrac{y}{x}\right)y=\left(y\sin\dfrac{y}{x}-x\cos\dfrac{y}{x}\right)x\dfrac{dy}{dx}$
[Cal. Hons 1962]

or $x\cos\dfrac{y}{x}(y\,dx+x\,dy)=y\sin\dfrac{y}{x}(x\,dy-y\,dx)$.
[Raj. 1959; Cal. Hons. 61, 55; Delhi 68, 61]

Solution. The equation is $\dfrac{dy}{dx}=\dfrac{y\,(y\sin y/x+x\cos y/x)}{x\,(y\sin y/x-x\cos y/x)}$.

Putting $y=vx$, $x\dfrac{dv}{dx}=\dfrac{dy}{dx}-v=\dfrac{2v\cos v}{v\sin v-\cos v}$

or $\left(\tan v-\dfrac{1}{v}\right)dv=2\dfrac{dx}{x}$, *i.e.*, $\log\dfrac{\sec v}{v}=\log C+2\log x$

or $\sec(y/x)=Cxy$ is the solution.

Ex. 8. *Solve* $\left(x\sin\dfrac{y}{x}\right)\dfrac{dy}{dx}=\left(y\sin\dfrac{y}{x}-x\right)$ [Delhi Pass 67]

Solution. Equation is $\dfrac{dy}{dx}=\dfrac{y}{x}-\operatorname{cosec}\dfrac{y}{x}$.

Putting $y=vx$. $\dfrac{dy}{dx}=x\dfrac{dv}{dx}+v$.

Equation reduces to $\sin v\;dv=-\dfrac{dx}{x}$.

Integrating, $-\cos v=-\log Cx$

or $\cos\dfrac{y}{x}=\log Cx$ is the solution.

Ex. 9. *Solve* $(x^2+2xy-y^2)\,dx+(y^2+2xy-x^2)\,dy=0$.
[Gujrat B.Sc. (Prin.) 1961]

Solution. $\dfrac{dy}{dx}=-\dfrac{x^2+2xy-y^2}{y^2+2xy-x^2}$. Put $y=vx$.

$$\therefore\quad v+x\frac{dv}{dx}=-\frac{1+2v-v^2}{v^2+2v-1}.$$

$$x\frac{dv}{dx}=-\frac{1+2v-v^2}{v^2+2v-1}-v=-\frac{v^3+v^2+v+1}{v^2+2v-1}.$$

$$\therefore\quad \frac{dx}{x}=-\frac{v^2+2v-1}{v^3+v^2+v+1}\,dv=-\frac{v^2+2v-1}{(v+1)(v^2+1)}\,dv$$

$$=\left(\frac{1}{v+1}-\frac{2v}{v^2+1}\right)dv.$$

Integrating, $\log x=\log(v+1)-\log(v^2+1)+\log C$

or $\dfrac{x}{v^2+1}=C(v+1)$ or $\dfrac{x}{y^2/x^2+1}=C\left(\dfrac{y}{x}+1\right)$

Ex. 10. *Solve* $2y^3\,dx+(x^2-3y^2)\,x\,dy=0.$

[Bombay B.Sc. (Sub.) 1962]

Solution. Proceed yourself.

2·5. Equation Reducible to Homogeneous Form.

An equation of the type $\dfrac{dy}{dx}=\dfrac{ax+by+c}{a'x+b'y+c'}$, when $\dfrac{a}{a'}\neq\dfrac{b}{b'}$ can be reduced to homogeneous form as follows :

Put $x=X+h$, $y=Y+k$; then $\dfrac{dy}{dx}=\dfrac{dY}{dX}$, where X, Y are new variables and h, k are arbitrary constants. The equation now becomes

$$\frac{dY}{dX}=\frac{aX+bY+(ah+bk+c)}{a'X+b'Y+(a'h+b'k+c')}$$

We choose the constants h and k in such a way that

$$ah+bk+c=0,\quad a'h+b'k+c'=0.$$

With this substitution the differential equation reduces to $\dfrac{dY}{dX}=\dfrac{aX+bY}{a'X+b'Y}$ which is a homogeneous equation in X, Y and can be solved by putting $Y=vX$ as earlier.

Special Case. When $\dfrac{a}{a'}=\dfrac{b}{b'}=\dfrac{1}{m}$ (say), then the differential equation can be written as

$$\frac{dy}{dx}=\frac{ax+by+c}{m(ax+by)+c'}.$$

Put $ax+by=v$, so that $a+b\dfrac{dy}{dx}=\dfrac{dv}{dx}$.

(1) then becomes $\dfrac{1}{b}\left(\dfrac{dv}{dx}-a\right)=\dfrac{v+c}{mv+c'}$ in which variables can be separated.

Ex. 1. *Solve* $\dfrac{dy}{dx}=\dfrac{x+2y-3}{2x+y-3}$.

[Vikram 60]

Solution. Put $x=X+h$, $y=Y+k$, where h, k are some constants; then $\dfrac{dy}{dx}=\dfrac{dY}{dX}$. The given equation then becomes

$$\frac{dY}{dX}=\frac{X+2Y+(h+2k-3)}{2X+Y+(2h+k-3)}.$$

Now choose h, k such that $h+2k-3=0$ and $2h+k-3=0$. Solving these we get $h=1, k=1$.

$\therefore \quad \frac{dY}{dX}=\frac{X+2Y}{2X+Y}$ homogeneous in X and Y.

Put $Y=vX$, so that $\frac{dY}{dX}=v+X\frac{dv}{dX}$.

$$\therefore \quad v+X\frac{dv}{dX}=\frac{X+2vX}{2X+vX}=\frac{1+2v}{2+v}, \quad i.e., \quad X\frac{dv}{dX}=\frac{1+2v}{2+v}-v$$

or $\quad \frac{dX}{X}=\frac{2+v}{1-v^2}\,dv=\left(\frac{1}{1-v^2}+\frac{v}{1-v^2}\right)dv.$

Integrating, $\log X=2.\tfrac{1}{2}\log\frac{1+v}{1-v}-\tfrac{1}{2}\log(1-v^2)+\log C$

or $\quad X=C\,\frac{1+v}{1-v}\cdot\frac{1}{\sqrt{(1-v^2)}}=\frac{C\sqrt{(1+v)}}{(1-v)^{3/2}}$

or $$X^2(1-v)^3=C^2(1+v)$$

or $$X^2\left(1-\frac{Y}{X}\right)^3=C^2\left(1+\frac{Y}{X}\right) \text{ as } v=\frac{Y}{X}$$

or $$(X-Y)^3=C^2(X+Y) \text{ but } x=X+1,\ y=Y+1.$$

$\therefore \quad (x-y)^3=C^2(x+y-2)$ is the required solution.

Ex. 2. *Solve* $(3x-7y-3)\frac{dy}{dx}=3y-7x+7.$ **[Raj. M.Sc. 61]**

Solution. $\frac{dy}{dx}=\frac{3y-7x+7}{3x-7y-3}.$

Put $x=X+h$, $y=Y+k$, where h, k are some constants. Then $\frac{dy}{dx}=\frac{dY}{dX}$. And the given equation becomes

$$\frac{dY}{dX}=\frac{3Y-7X+(3k-7h+7)}{3X-7Y+(3h-7k-3)}.$$

Choose h, k such that $3h-7k-3=0$ and $3k-7h+7=0$, which give $h=1$, $k=0$.

$\therefore \quad \frac{dY}{dX}=\frac{3Y-7X}{3X-7Y}$ [homogeneous].

Put $Y=vX$, $\frac{dY}{dX}=v+X\frac{dv}{dX}$.

$$\therefore \quad v+X\frac{dv}{dX}=\frac{3vX-7X}{3X-7vX}=\frac{3v-7}{3-7v}$$

or $$X\frac{dv}{dX}=\frac{3v-7}{3-7v}-v=\frac{7(v^2-1)}{3-7v}$$

or $$\frac{7\,dX}{X}=\frac{3-7v}{(v^2-1)}\,dv=-\left(\frac{2}{v-1}+\frac{5}{v+1}\right)dv.$$

Integrating, $7 \log X = -2 \log (v-1) - 5 \log (v+1) + \log C$

or $\quad X^7 (v-1)^2 (v+1)^5 = C$

or $\quad X^7 \left(\frac{Y}{X} - 1\right)^2 \left(\frac{Y}{X} + 1\right)^5 = C$ as $Y = vX$

or $\quad (Y-X)^2 (Y+X)^5 = C$

or $\quad (y-x+1)^2 (y+x-1)^5 = C$ as $x = X+1$, $y = Y+0$.

Ex. 3. *Solve* $(2x+y+3) \dfrac{dy}{dx} = x+2y+3.$

[Karnatak B.Sc. (Princ.) 60]

Solution. $\dfrac{dy}{dx} = \dfrac{x+2y+3}{2x+y+3}.$

Put $x = X+h$, $y = Y+k$, where h, k are constants.

$$dx = dX, \quad dy = dY; \quad \therefore \frac{dy}{dx} = \frac{dY}{dX}$$

$$\therefore \frac{dY}{dX} = \frac{X+2Y+(h+2k+3)}{2X+Y+(2h+k+3)}.$$

Choose h, k such that $h+2k+3=0$, $2h+k+3=0$. Solving these, we get $h=-1$, $k=-1$.

$\therefore \dfrac{dY}{dX} = \dfrac{X+2Y}{2X+Y}$. Put $Y = vX$, $\dfrac{dY}{dX} = v + X\dfrac{dv}{dX}$.

$\therefore v + X\dfrac{dv}{dX} = \dfrac{X+2vX}{2X+vX}$ or $X\dfrac{dv}{dX} = \dfrac{1+2v}{2+v} - v$

or $\dfrac{dX}{X} = \dfrac{2+v}{1-v^2}\, dv = \left(\dfrac{\frac{3}{2}}{1-v} + \dfrac{\frac{1}{2}}{1+v}\right) dv.$

Integrating, $2 \log X = -3 \log (1-v) + \log (1+v) + \log C$

or $\quad X^2 \dfrac{(1-v)^3}{1+v} = C$ or $X^2 \dfrac{(1-Y/X)^3}{(1+Y/X)} = C$

or $\quad (X-Y)^3 = C(X+Y)$; where $x = X-1$, $y = Y-1$

or $\quad (X-y)^3 = C(x+y-2)$ is the solution.

Ex. 4. *Solve* $(2x-2y+5) \dfrac{dy}{dx} = x-y+3.$

[Sagar 63; Agra B.Sc. 61, 52]

Solution. The equation is $\dfrac{dy}{dx} = \dfrac{x-y+3}{2(x-y)+5}.$

Put $x - y = v$, so that $1 - \dfrac{dy}{dx} = \dfrac{dv}{dx}$ or $\dfrac{dy}{dx} = 1 - \dfrac{dv}{dx}.$

$\therefore$ The equation becomes

$$1 - \frac{dv}{dx} = \frac{v+3}{2v+5} \quad \text{or} \quad \frac{dv}{dx} = 1 - \frac{v+3}{2v+5} = \frac{v+2}{2v+5}.$$

or $\quad dx = \dfrac{2v+5}{v+2}\, dv = \left(2 + \dfrac{1}{v+2}\right) dv$, separating the variables.

Integrating, $x = 2v + \log(v+2) + C$,

$$x = 2(x-y) + \log(x-y+2) + C \text{ as } v = x - y$$

or $2y - x = \log(x-y+2) + C$ is the required solution.

Ex. 5. *Solve* $\dfrac{dy}{dx} = \dfrac{6x-4y+3}{3x-2y+1}$

[Poona 64; Karnatak B.Sc. (Princ.) 61]

Solution. Put $3x-2y=v$, *i.e.*, $3 - 2\dfrac{dy}{dx} = \dfrac{dv}{dx}$

$$\therefore \quad \frac{dv}{dx} = 3 - 2\frac{2v+3}{v+1} = -\frac{v+3}{v+1}.$$

$$\therefore \quad dx = -\frac{v+1}{v+3}\,dv = -\left(1 - \frac{2}{v+3}\right) dv.$$

Integrating, $x = -v + 2\log(v+3) + C$

or $x = (2y-3x) + 2\log(3x-2y+3) + C$

or $2x - y = \log(3x-2y+3) + \frac{1}{2}C$ is the solution.

Ex. 6. *Solve* $(6x-4y+1)\dfrac{dy}{dx} = (3x-2y+1)$.

[Karnatak B.Sc. (Sub.) 61]

Solution. $\dfrac{dy}{dx} = \dfrac{3x-2y+1}{2(3x-2y)+1}$. Put $3x-2y=v$.

$$\therefore \quad \frac{dv}{dx} = 3 - 2\frac{dy}{dx} = 3 - 2\frac{v+1}{2v+1} = \frac{4v+1}{2v+1}$$

or $dx = \dfrac{2v+1}{4v+1}\,dv$ or $2\,dx = \left(1 + \dfrac{1}{4v+1}\right) dv$ etc.

Ex. 7. *Solve the following equations :*

(i) $(2x+y+1)\,dx + (4x+2y-1)\,dy = 0$. [Gujrat B.Sc. (Princ.) 61]

(ii) $\dfrac{dy}{dx} = \dfrac{6x-2y-7}{3x-y+4}$. [Luck. Pass 56]

(iii) $(2x-5y+3)\,dx - (2x+4y-6)\,dy = 0$. [Delhi Hons. 61]

(iv) $\dfrac{dy}{dx} = \dfrac{y-x+1}{y-x-5}$ [Poona 62; Nag. 62]

(v) $\dfrac{dy}{dx} = \dfrac{3x-4y-2}{3x-4y-3}$.

(vi) $(3y+2x+4)\,dx - (4x+6y+5)\,dx = 0$. [Cal. Hons 63]

(vii) $(2x-5y+3)\,dx - (2x+4y-6)\,dy = 0$. [Karnatak 63]

(viii) $(x-y-2)\,dx + (x-2y-3)\,dy = 0$. [Delhi Hons. 65]

(ix) $(4x+2y+1)\,dy = (2x+y+3)\,dx$. [All. 66]

[Delhi Pass 67]

Hint. In (*i*) put $2x+y=v$, in (*ii*) put $3x-y=v$ and (*iii*) can be reduced to homogeneous form as usual. In (*ix*) putting $v=2x+y$, variables can be separated.

Ex. 8. *Solve* $2y\dfrac{dy}{dx}=\dfrac{x+y^2}{x+4y^2}$ **[Bombay B.Sc. 61]**

Solution. Put $y^2=v$, $2y\dfrac{dy}{dx}=\dfrac{dv}{dx}$.

$\therefore\quad \dfrac{dv}{dx}=\dfrac{x+v}{x+4v}$ [homogeneous]. Now put $v=xz$ etc.

2·6. A particular case

A differential equation of the form

$$\frac{dy}{dx}=\frac{ax+by+c}{-bx+hy+k'}$$

in which coefficient of y in the numerator is equal to the coefficient of x in the denominator with sign changed, can be integrated as follows :

The equation (1) can be written as

$$-b\,(x\,dy+y\,dx)+(hy+k)\,dy-(ax+c)\,dx=0.$$

Integrating, we get $-bxy+(\frac{1}{2}hy^2+ky)-(\frac{1}{2}ax^2+cx)=A$.

Ex. 1. *Solve* $\dfrac{dy}{dx}+\dfrac{ax+hy+g}{hx+by+f}=0.$

[Raj. B.Sc. 66; Agra B.Sc. 57; Delhi B.A. 57; Raj. M.Sc. 62]

Solution. The equation can be written as

$$(hx+by+f)\,dy+(ax+hy+g)\,dx=0$$

or $h\,(x\,dy+y\,dx)+(by+f)\,dy+(ax+g)\,dx=0.$

Integrating, $hxy+\frac{1}{2}by^2+fy+\frac{1}{2}ax^2+gx=A$

or $ax^2+2hxy+by^2+2fy+2gx+c=0$, writing $c=-2A$.

Ex. 2. *Solve* $\dfrac{dy}{dx}=\dfrac{2x-y+1}{x+2y-3}$ **[Agra B Sc. 59; Nag. 53 (S)]**

Solution. Here coefficient of y in numerator is equal to coefficient of x in the denominator with sign changed. Hence write it as

$$(x+2y-3)\,dy-(2x-y+1)\,dx=0$$

or $(x\,dy+y\,dx)+(2y-3)\,dy-(2x+1)\,dx=0.$

Integrating, $xy+y^2-3y-x^2-x=C.$

Ex. 3. *Solve* $(2x-y+1)\,dx+(2y-x-1)\,dy=0.$

[Bombay B Sc. (Sub.) 61; Poona 61]

Solution. The equation is of above type. Hence [illegible] regrouping, we have

$(2x+1)\,dx+(2y-1)\,dy$ [illegible] $dx+x\,d$[illegible]

Integrating, $(x^2+x)+(y^2-y)$ [illegible]

which is the solution.

Ex. 4. *Solve* $\dfrac{dy}{dx}+\dfrac{\text{[illegible]}x+3y+1}{3x+4y-1}=0.$ **[Delhi Hons. 60]**

Solution. The equation is of the above type and can be written as $(3x+4y-1)\ dy+(2x+3y+1)\ dx=0$,

i.e., $3\,(x\ dy+y\ dx)+(4y-1)\ dy+(2x+1)\ dx=0$.

Integrating, $3xy+2y^2-y+x^2+x=C$ is the solution.

2.7. Linear Differential Equations

[Poona 63, 61; Nagpur 62, 61; Guj 61]

A differential equation of the form

$$\frac{dy}{dx}+Py=Q,$$

where P, Q are functions of x or constants, is called the *linear differential equation of the first order.*

To solve this equation, multiply both the sides by $e^{\int P\,dx}$

Then it becomes $e^{\int P\,dx}\cdot\dfrac{dy}{dx}+Py\ e^{\int P\,dx}=Qe^{\int P dx}$.

or $\dfrac{d}{dx}\,[ye^{\int P\,dx}]=Q\ e^{\int P\,dx}$.

Integrating both the sides, w.r.t. x, we get

$$ye^{\int P\,dx}=\int [Qe^{\int Pdx}]dx+C,$$

which is the required solution.

Integrating factor (I.F.). It will be noticed that for solving (1), we multiplied it by a factor $e^{\int P\,dx}$ and the equation became readily (directly) integrable. Such a factor is called the integrating factor.

Note. Sometimes a differential equation takes linear form if we *regard x as dependent variable and y as independent variable.* The equation can then be put as $\dfrac{dx}{dy}+Px=Q$, where P, Q are functions of y or constants.

The integrating factor in this case is $e^{\int P\,dy}$ and solution is

$$xe^{\int P\,dy}=\int [Qe^{\int P\,dy}]\ dy+C.$$

(See Ex. 1 to 4 pages 21 and 22).

Ex. 1. *Solve* $(1-x^2)\dfrac{dy}{dx}-xy=1$.

[Delhi 68 : Nag. 61]

Solution. The equation can be written as

$$\frac{dy}{dx}-\frac{x}{1-x^2}\ y=\frac{1}{1-x^2}$$

This is now expressed in the linear form