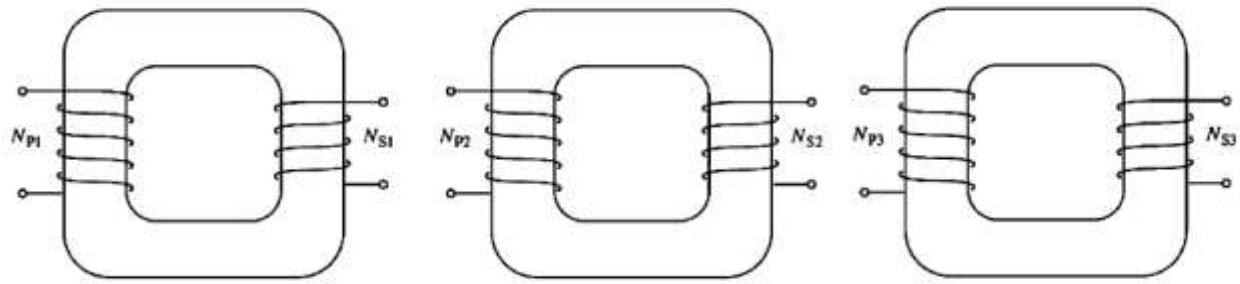
Three-phase Transformers:

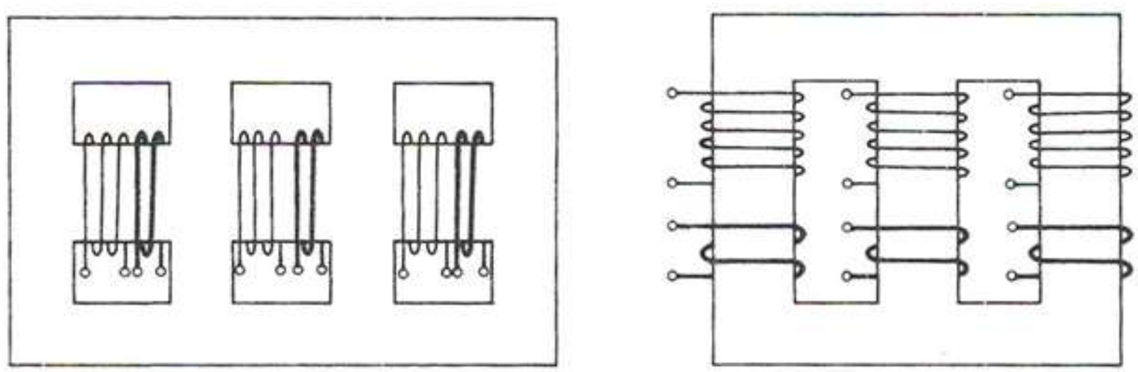
Almost all the major power generation and distribution systems in the world today are three-phase ac systems. Since three-phase systems play such an important role in modern life, it is necessary to understand how transformers are used in them. Transformers for three-phase circuits can be constructed in one of two ways.

One approach is simply to take three single-phase transformers and connect them in a three-phase bank.



A three-phase transformer bank composed of independent transformers.

An alternative approach is to make a three-phase transformer consisting of three sets of windings wrapped on a common core. Two possible types of transformer construction are shown below.



(a) (b)

Basic construction of three-phase transformers: (a) shell type; (b) core type

The construction of a single three-phase transformer is the preferred practice today, since it is lighter, smaller, cheaper, and slightly more efficient. Furthermore, since all three phases are in one tank, the wye or delta connections can be made internally, reducing the number of external high-voltage connections from six to three.

The older construction approach was to use three separate transformers. That approach had the advantage that each unit in the bank could be replaced individually in the event of trouble, but that does not outweigh the advantages of a combined three phase unit for most applications. However, there are still a great many installations consisting of three single-phase units in service.

Initial cost, cost of operation, cost of spares, cost of repairs, cost of downtime, space requirement, need for uninterrupted operation in case of fault occurrence - all these are to be considered to make a choice between these two.

The Per-Unit System for Three-Phase Transformers

The per-unit system of measurements applies just as well to three-phase transformers as to single-phase transformers.

If the total base voltampere value of the transformer bank is called S_{base}, then the base voltampere value of one of the transformers $S_{1\emptyset,base}$ is given by

$$S_{1\emptyset,base} = \frac{S_{base}}{3}$$

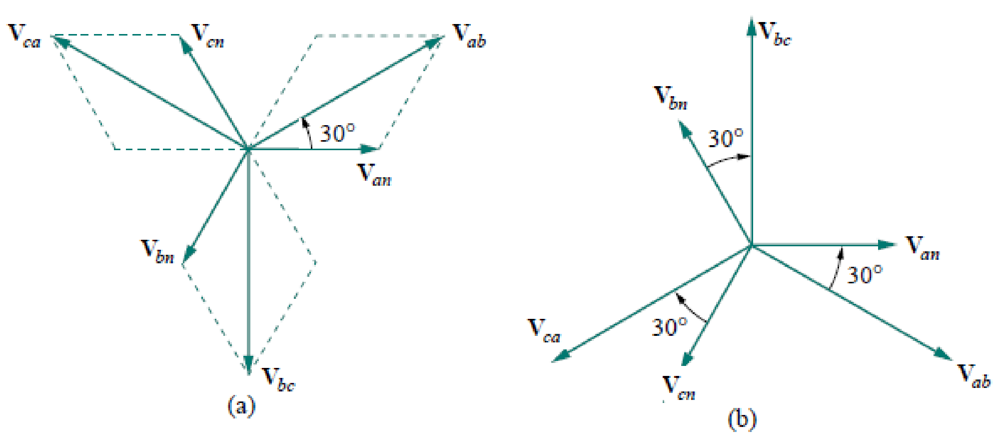
The base phase current and impedance of the transformer are expressed as:

$$I_{\emptyset,base} = \frac{S_{1\emptyset,base}}{V_{\emptyset,base}} = \frac{S_{base}}{3\,V_{\emptyset,base}}$$

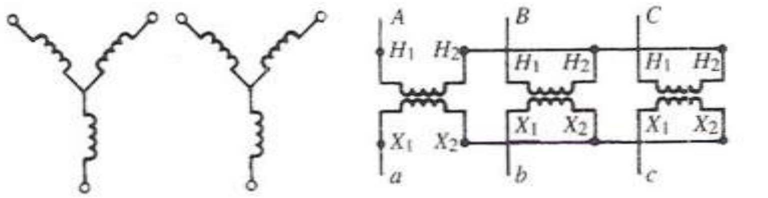
$$Z_{base} = \frac{\left(V_{\emptyset,base}\right)^2}{S_{1\emptyset,base}} = \frac{3\left(V_{\emptyset,base}\right)^2}{S_{base}}$$

Paralleling three-phase transformer banks:

In a Y-connected system, the line voltage leads corresponding phase voltage by 30° for a-b-c sequence**,** and lags by 30° for a-c-b sequence. Due to this, there is a phase shift between primary and secondary line voltages in case of Y-Δ and Δ-Y banks. In the United States, it is customary to make the secondary voltage lag the primary voltage by 30°. Although this is the standard, it has not always been observed, and older installations must be checked very carefully before a new transformer is paralleled with them, to make sure that their phase angles match.

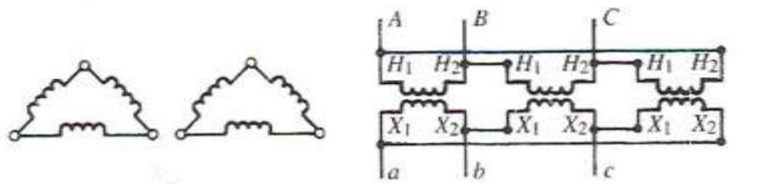


Phase relationship between phase voltage and line voltage in a balanced Y connected network for (a) a-b-c sequence and (b) a-c-b sequence



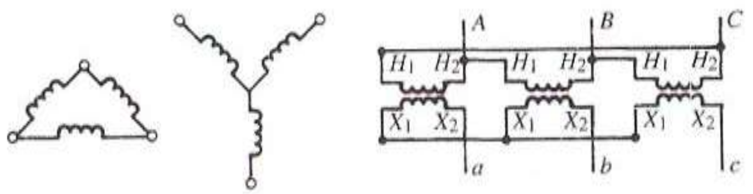
For Y-Y bank,

$$\boldsymbol{V_{an}} = \frac{1}{a}\boldsymbol{V_{AN}} \;\; ; \;\; \boldsymbol{V_{ab}} = \frac{1}{a}\boldsymbol{V_{AB}}$$



For Δ-Δ bank,

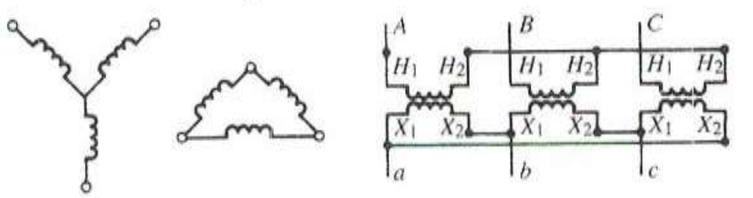
$$\boldsymbol{V_{ab}} = \frac{1}{a}\boldsymbol{V_{AB}}$$



For Δ-Y bank,

$$\boldsymbol{V_{an}} = \frac{1}{\sqrt{3}a}\boldsymbol{V_{AB}}$$

$$\boldsymbol{V_{ab}} = \sqrt{3}\boldsymbol{V_{an}}\angle 30° = \frac{1}{a}\boldsymbol{V_{AB}}\angle 30°$$



For Y-Δ bank,

$$\boldsymbol{V_{AB}} = \sqrt{3}\boldsymbol{V_{AN}}\angle 30°$$

$$\boldsymbol{V_{ab}} = \frac{\sqrt{3}}{a}\boldsymbol{V_{AN}} = \frac{1}{a}\boldsymbol{V_{AB}}\angle -30°$$

There is an angular displacement, called phase shift, between the corresponding primary and secondary line voltages in the Y-Δ bank and also in the Δ-Y bank. There is no phase shift between corresponding primary and secondary line voltages in a Y-Y, Δ-Δ or V-V bank. Because of the phase shift inherent in Y-Δ and Δ-Y banks, they must not be paralleled with Y-Y, Δ-Δ or V-V banks. Doing so would cause large circulating currents and severe overheating of the windings.

The Y-Y connection has two very serious problems and this is why, seldom used.

- If loads on the transformer secondaries are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.
- Third-harmonic voltages can be very large, even larger than the fundamental voltage.

The Y-Δ and Δ-Y connection has no problem with third-harmonic components in its voltages, since they are consumed in a circulating current on the Δ side. This connection is also more stable with respect to unbalanced loads, since the Δ partially redistributes any imbalance that occurs. A phase shift of 30° between the primary and secondary line voltages must be taken into consideration before paralleling. The Δ-Δ connection has no phase shift associated with it and no problems with unbalanced loads or harmonics. And this bank will continue to operate if one of the phases is removed for maintenance.

The simple rule which normally dominates the selection of a three-phase transformer is the network where it is to be installed. The preferable connection for the unit transformers are YNd i.e low voltage side (generator side) is connected in Delta and high voltage side (transmission side) is connected as Wye)

From transmission network to the distribution network, the preferable connection is Dyn. (transmission side in Delta and Distribution Side in Wye). Even in the distribution networks, the transformers are Dyn. But here besides the above benefits, it has an extra advantage that power can be supplied to both three-phase and single phase loads by the distributors.

Three-phase Transformation using two Transformers

In addition to the standard three-phase transformer connections, there are ways to perform three-phase transformation with only two transformers. One such technique is ***open-delta connection*** or ***V-V connection***. It provides a convenient means for inspection, maintenance, testing and replacing of transformers one at a time, with only a brief power interruption. Transformers selected for delta-delta or open-delta connection must have the same turns ratio and same per unit impedance in order to share the load equally.

Suppose a transformer bank composed of separate transformers has a damaged phase that must be removed for repair. Open delta bank will still remain able to operate successfully, although at a lower capacity.

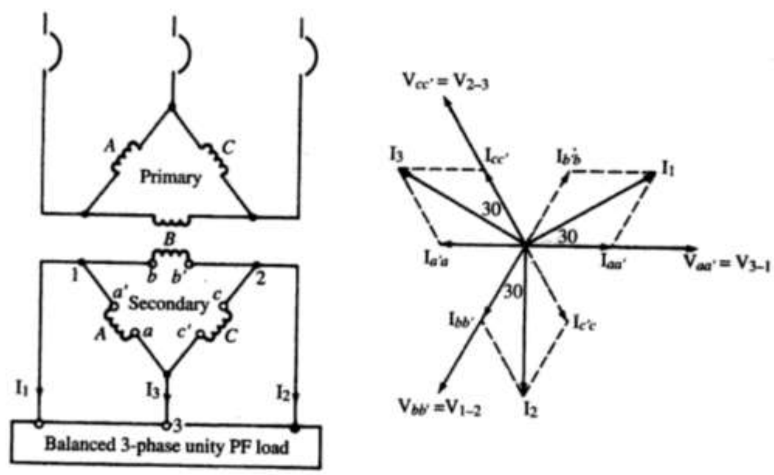
The open-delta connection is also used to provide three-phase service in applications where a future increase in load is expected. The increase may be accommodated by adding the third transformer to the bank at a later date.

Considering balanced three-phase, the phase voltages will be same in magnitude and $120°$ angle apart in phase.
Considering a-b-c phase sequence

$$\boldsymbol{V_{3-1}} = \boldsymbol{V_{aa'}} = V_p\angle 0°$$

$$\boldsymbol{V_{1-2}} = \boldsymbol{V_{bb'}} = V_p\angle -120°$$

$$\boldsymbol{V_{2-3}} = \boldsymbol{V_{cc'}} = V_p\angle{-240°} = V_p\angle 120°$$



Delta-Delta bank and corresponding phasor diagram

Now if transformer A is removed,

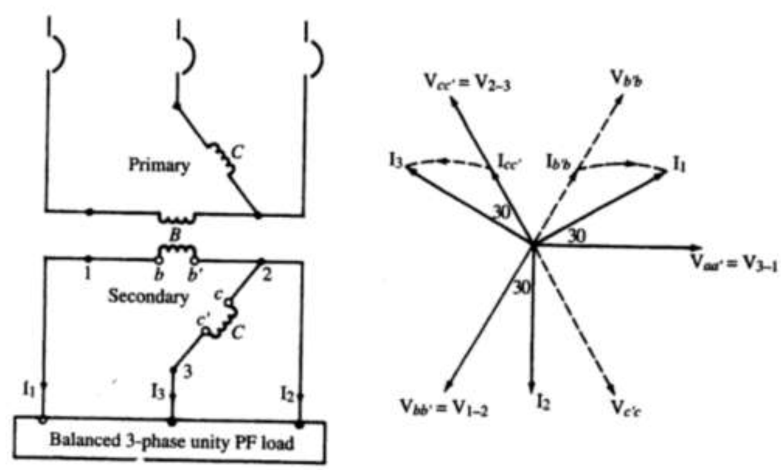
$$\boldsymbol{V_{1-3}} = \boldsymbol{V_{1-2}} + \boldsymbol{V_{2-3}}$$

$$\boldsymbol{V_{1-3}} = V_p\angle -120° + V_p\angle 120°$$

$$\boldsymbol{V_{1-3}} = V_p\angle 180°$$

$$\therefore \boldsymbol{V_{3-1}} = V_p\angle 0°$$

This is exactly the same voltage that would be present if the third transformer were still there. Therefore the removed phase (phase A in this case) is termed as ***ghost phase***.



V-V bank and corresponding phasor diagram

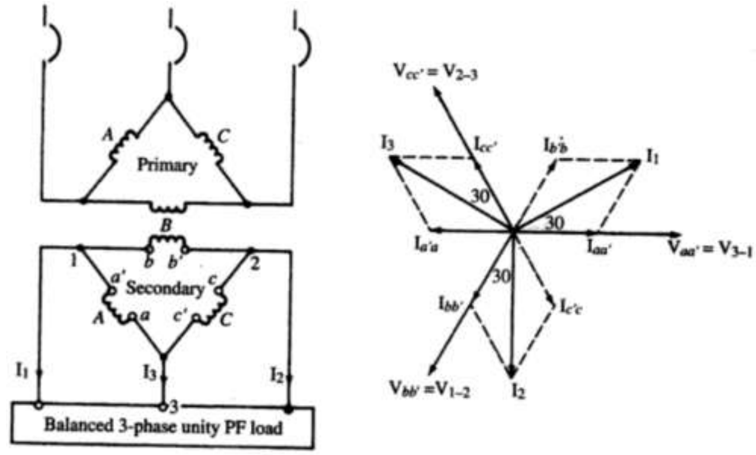
For $\Delta{-}\Delta$ operation, the line currents can be expressed in terms of phase currents.

$$\boldsymbol{I_1 = I_{aa'} + I_{b'b}}$$
$$\boldsymbol{I_2 = I_{bb'} + I_{c'c}}$$
$$\boldsymbol{I_3 = I_{cc'} + I_{a'a}}$$

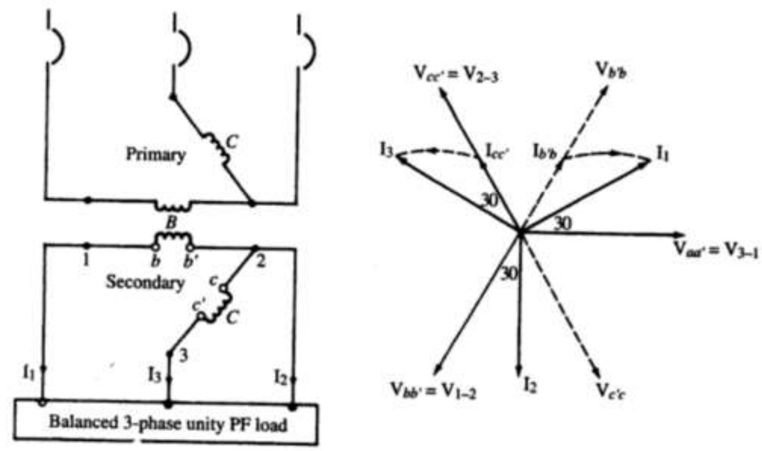
Since the three secondary line voltages in V-V operation remain same as in $\Delta{-}\Delta$, and the load impedance has not changed, line currents must remain same as well. To keep the line currents unchanged, the coil currents in the remaining two transformers increase in magnitude by $\sqrt{3}$ and also shift in phase by 30° to match the corresponding line currents. This may lead to overheating of transformer windings. Therefore bank current and bank apparent power must be rerated.

For $\Delta{-}\Delta$ operation, the line currents can be expressed in terms of phase currents.

$$\boldsymbol{I_1 = I_{b'b}}$$
$$\boldsymbol{I_2 = I_{bb'} + I_{c'c}}$$
$$\boldsymbol{I_3 = I_{cc'}}$$



Delta-Delta bank and corresponding phasor diagram



V-V bank and corresponding phasor diagram

For V-V operation,

$$P_B = V_{bb'} I_{bb'} \cos(30°) = \frac{\sqrt{3}}{2} V_\varphi I_\varphi \qquad P_C = V_{cc'} I_{cc'} \cos(-30°) = \frac{\sqrt{3}}{2} V_\varphi I_\varphi$$
$$Q_B = V_{bb'} I_{bb'} \sin(30°) = \frac{1}{2} V_\varphi I_\varphi \qquad Q_C = V_{cc'} I_{cc'} \sin(-30°) = -\frac{1}{2} V_\varphi I_\varphi$$
$$S_B = \sqrt{(P_B)^2 + (Q_B)^2} = V_\varphi I_\varphi \qquad S_C = \sqrt{(P_C)^2 + (Q_C)^2} = V_\varphi I_\varphi$$

$$P_{T,(V-V)} = P_B + P_c = \sqrt{3}\, V_\varphi I_\varphi$$
$$Q_{T,(V-V)} = Q_B + Q_c = 0$$

$$S_{T,(V-V)} = \sqrt{(P_{T,(V-V)})^2 + (Q_{T,(V-V)})^2} = \sqrt{3}\, V_\varphi I_\varphi$$

In case of $\Delta{-}\Delta$ operation, the phase voltage and phase currents are in phase.
Therefore, $P_A = P_B = P_C = V_\varphi I_\varphi$ and $Q_A = Q_B = Q_c = 0$. Hence, $S_{T,(\Delta-\Delta)} = P_{(T,\Delta-\Delta)} = 3V_\varphi I_\varphi$

$$S_{V-V,rated} = \frac{S_{\Delta-\Delta,rated}}{\sqrt{3}}$$

Although resistive load has been considered to reduce complicacy in analysis, the relation in terms of apparent power (as written above) holds for loads with any power factor.

Transformer Ratings and Related Problems

Transformers have four major ratings: apparent power, voltage, current, and frequency.

The voltage rating of a transformer serves two functions. One is to protect the winding insulation from breakdown due to an excessive voltage applied to it. The second function is related to the magnetization curve and magnetization current of the transformer.

The principal purpose of the apparent power rating of a transformer is that, together with the voltage rating, it sets the current flow through the transformer windings. The current flow is important because it controls the heating of the transformer coils. Overheating the coils of a transformer drastically shortens the life of its insulation. The actual voltampere rating of a transformer may be more than a single value. In real transformers, there may be a voltampere rating for the transformer by itself, and another (higher) rating for the transformer with forced cooling. The key idea behind the power rating is that the hot-spot temperature in the transformer windings must be limited to protect the life of the transformer.

If a steady state sinusoidal voltage $v(t)$ is applied to the transformer's primary winding, the flux established in the core can be found from Faraday's Law.

$$v(t) = \mathrm{V_m}\ \sin\omega t$$

$$\emptyset(t) = \frac{1}{N_p}\int v(t)\,dt = \frac{1}{N_p}\int \mathrm{V_m}\ \sin\omega t\,dt = -\frac{\mathrm{V_m}}{\omega N_p}\cos\omega t$$

Hence, the maximum core flux depends on both voltage and frequency.

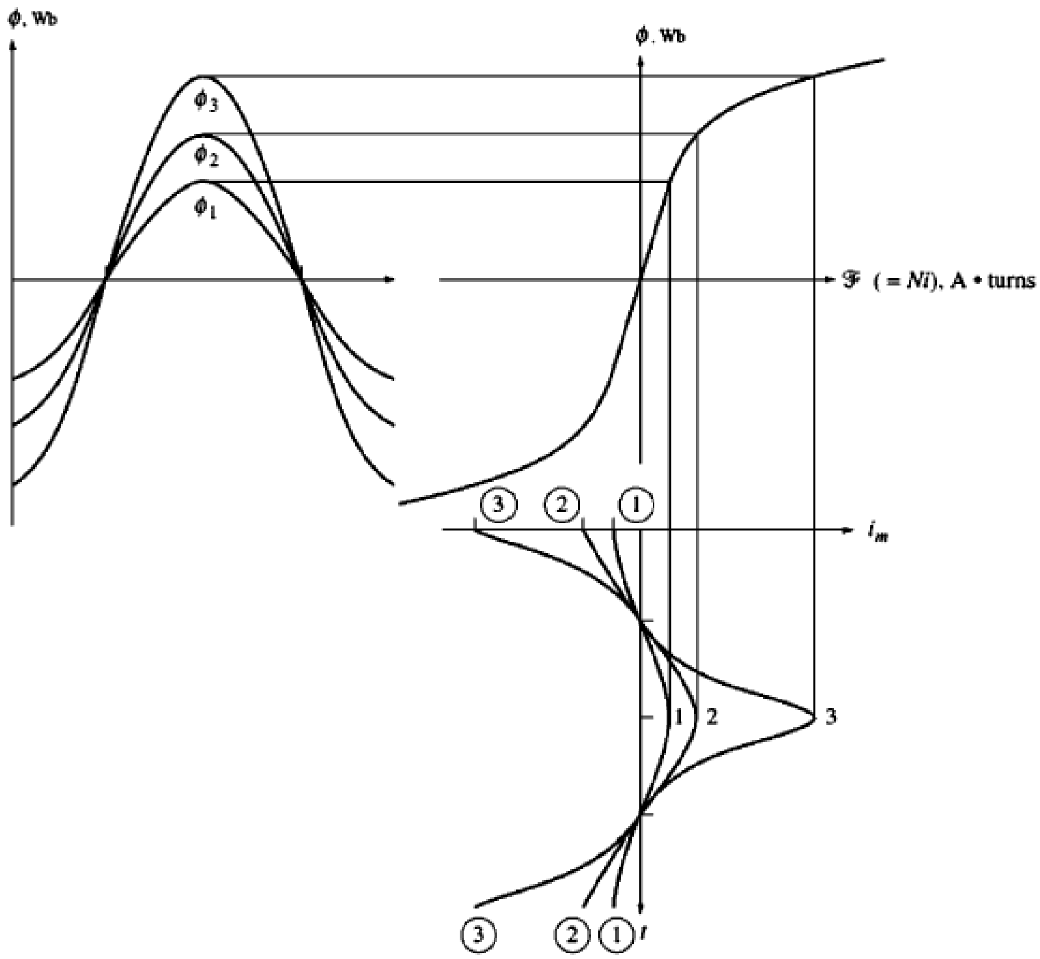
$$\Phi_{\max} = \frac{\mathrm{V_m}}{\omega N_p} = \frac{\mathrm{V_m}}{2\pi f N_p} \Rightarrow \quad \Phi_{\max} \propto \frac{\mathrm{V_m}}{f}$$

We know, magnetization curve for ferromagnetic material shows saturation when all of the magnetic domains of the material are oriented in the direction of the applied magnetomotive force. Saturation begins at the start of the knee region and is essentially complete when the curve starts to flatten. Transformers and AC machines are operated in the linear region and lower end of the knee.

If the applied voltage *v(t)* is increased, the resulting maximum flux in the core will also increase. However, a small increase in flux will require an even greater increase in magnetization current *due to saturation*. As the voltage increases, the high-magnetization currents soon become unacceptable due to overheating of the primary coils. The maximum applied voltage (and therefore the rated voltage) is set by the maximum acceptable magnetization current in the core.

Thus, if a 60-Hz transformer is to be operated on 50 Hz, its applied voltage must also be reduced by one-sixth or the peak flux in the core will be too high. This reduction in applied voltage with frequency is called **derating**. Similarly, a 50-Hz transformer may be operated at a 20 percent higher voltage on 60 Hz if this action does not cause insulation problems. Mathematically,

$$\frac{V_{new}}{V_{old}} = \frac{f_{new}}{f_{old}}$$



The effect of the peak flux in a transformer core upon the required magnetization current.

If a transformer's voltage is reduced for any reason (e.g., if it is operated at a lower frequency than normal), then the transformer's voltampere rating must be reduced by an equal amount. If this is not done, then the current in the transformer's windings will exceed the maximum permissible level and cause overheating. So while operating a 60 Hz transformer on a 50 Hz system, its kVA rating should be adjusted according to the following equation.

$$\frac{S_{new}}{S_{old}} = \frac{V_{new}}{V_{old}} \times \frac{I_{new}}{i_{old}} = \frac{f_{new}}{f_{old}} \times 1 = \frac{f_{new}}{f_{old}}$$

Harmonics in Transformer Exciting Current

The nonlinear characteristics of ferromagnetic cores used in transformers cause the magnetizing current to be nonsinusoidal even though the mutual flux is sinusoidal.

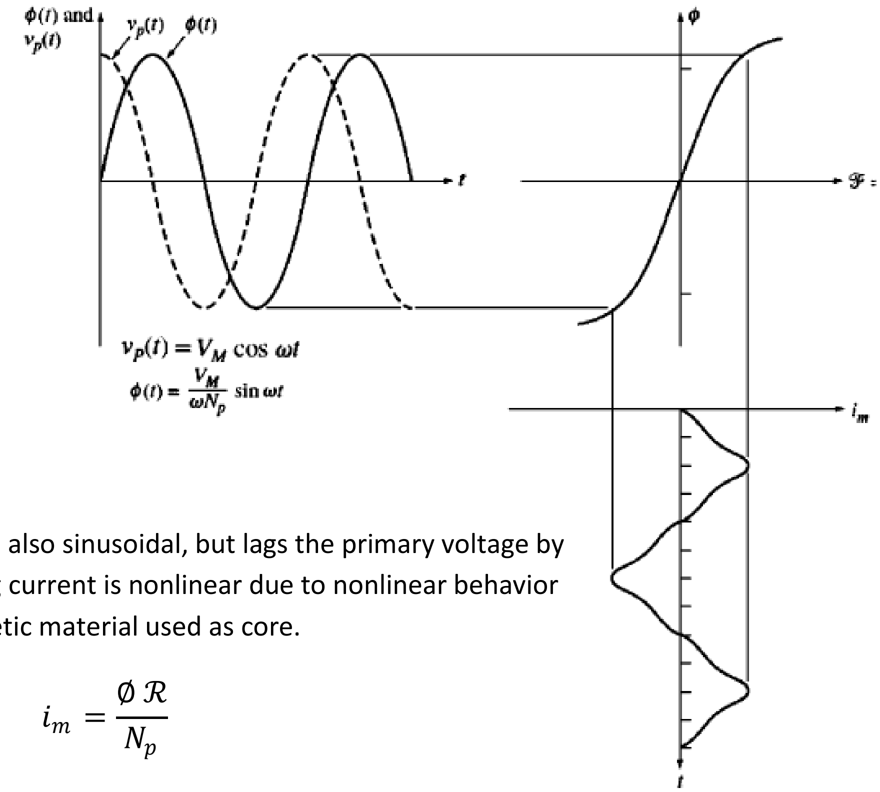
If the primary voltage is given by,

$$v_p(t) = \mathrm{V_M}\ \cos\omega t$$

The resulting flux is given by

$$\emptyset(t) = \frac{1}{N_p}\int v_p(t)\ dt$$

$$= \frac{\mathrm{V_m}}{\omega N_p}\sin\omega t$$

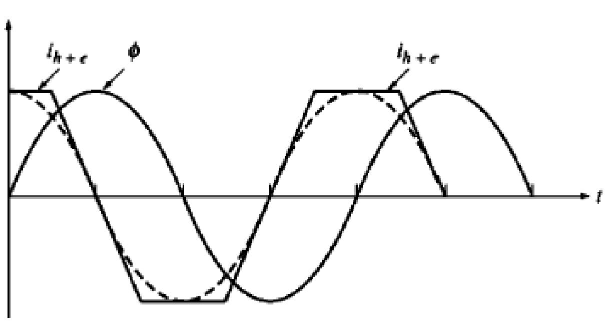


This means the resulting flux is also sinusoidal, but lags the primary voltage by 90°. However, the magnetizing current is nonlinear due to nonlinear behavior (reluctance) of the ferromagnetic material used as core.

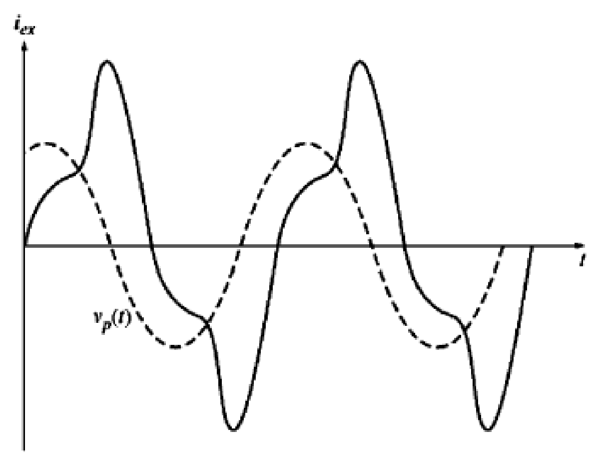
$$i_m = \frac{\emptyset\ \mathcal{R}}{N_p}$$

The nonsinusoidal magnetizing current caused by the nonlinear characteristics of ferromagnetic core

Core-loss current is nonlinear because of the nonlinear effects of hysteresis and its fundamental component is in-phase with the voltage applied to the core. Excitation current (no-load current) is just the sum of magnetization current and the core-loss current. Magnetizing current is much higher than the core-loss current; therefore, for all practical purposes, exciting current and magnetizing current may be used interchangeably.



The core-loss current in a transformer.



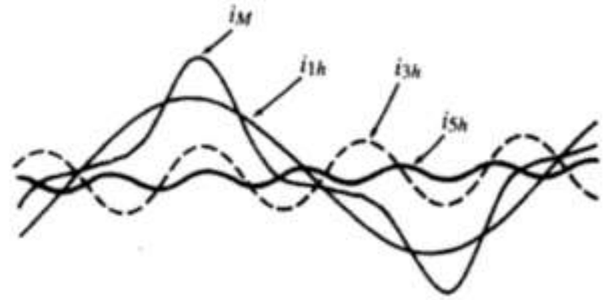
The total excitation current in a transformer.

Any periodic non-sinusoidal wave can be expressed as a sum of weighted and shifted sinusoids according to Fourier series.

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

For a wave with zero average value, $a_0 = 0$ For a half-wave odd symmetric wave, only odd harmonics are present.

The figure shows fundamental and first three harmonics present in the magnetizing current. The other harmonics are of small magnitudes and not dominant.

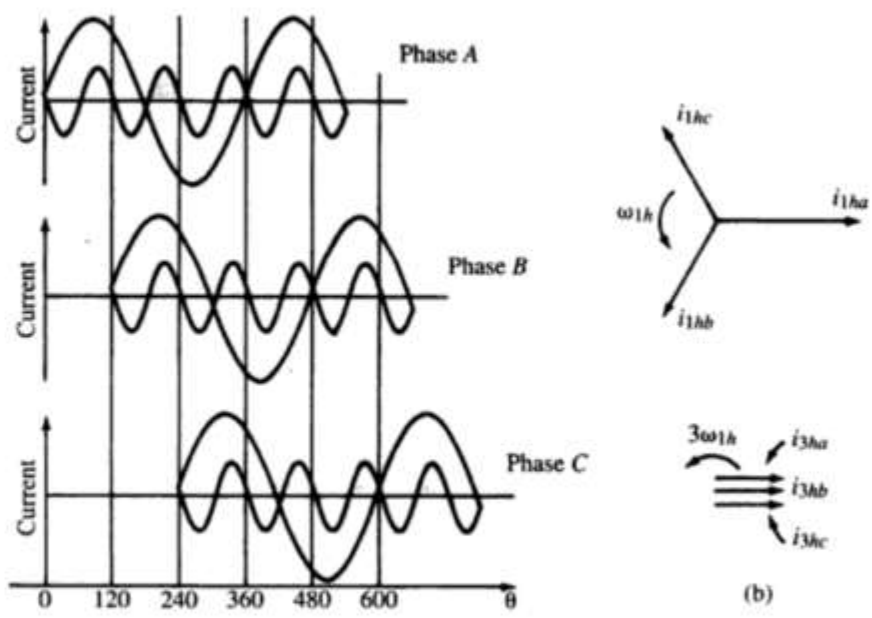


Magnetizing current and its first three harmonics.

Harmonic Suppression in Three-phase Connections

Magnetizing current that produces a sinusoidal flux and hence a sinusoidal output voltage is itself nonsinusoidal, containing many odd harmonic components. Suppressing any one of the harmonic components will result in a nonsinusoidal flux and hence a nonsinusoidal secondary voltage.

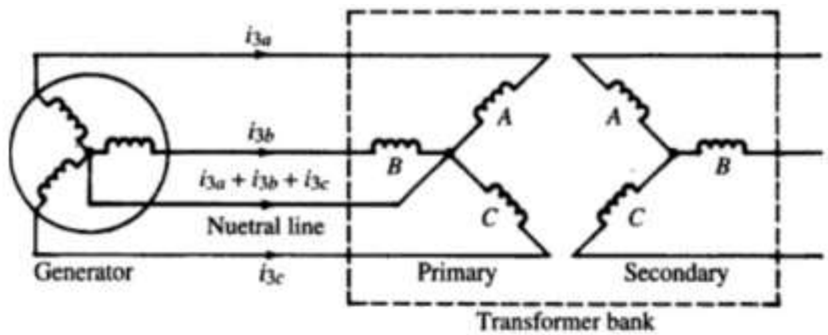
Third-harmonic currents and their multiples, called triplen harmonics, have zero phase sequence and thus are not three-phase quantities. The three third harmonics (one from each transformer) are in phase with each other. Although other triplen harmonics, such as the ninth and the fifteenth are present; their magnitudes are quite small compared to the magnitude of the third harmonic. All these components add up, and as a result, a very large third-harmonic component is observed in the secondary.



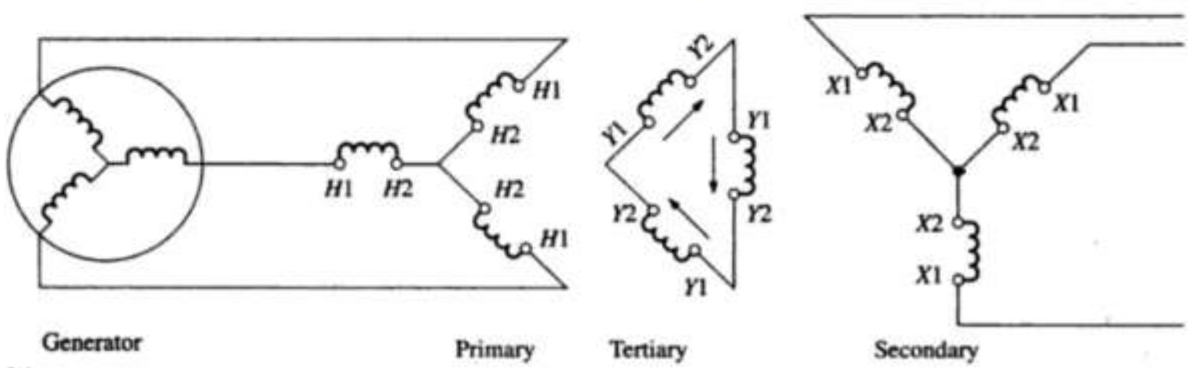
(a) Waves of the fundamental and the third harmonic
(b) Phasors of the fundamental and the third harmonic

Y-Y bank: Triplen harmonic currents of each phase of a three-phase wye-connected transformer bank are all in phase, all going in or all going out. Third harmonic voltages can be large, even larger than the fundamental voltage itself. The problem can be solved using one of two techniques.

Solidly ground the neutrals of the transformers, especially the neutral of the primary winding. This connection permits the additive third-harmonic components to cause a current flow in the neutral instead of building up large voltages.



Wye-wye bank with a neutral connection to primary; triplen harmonic components will flow in the neutral

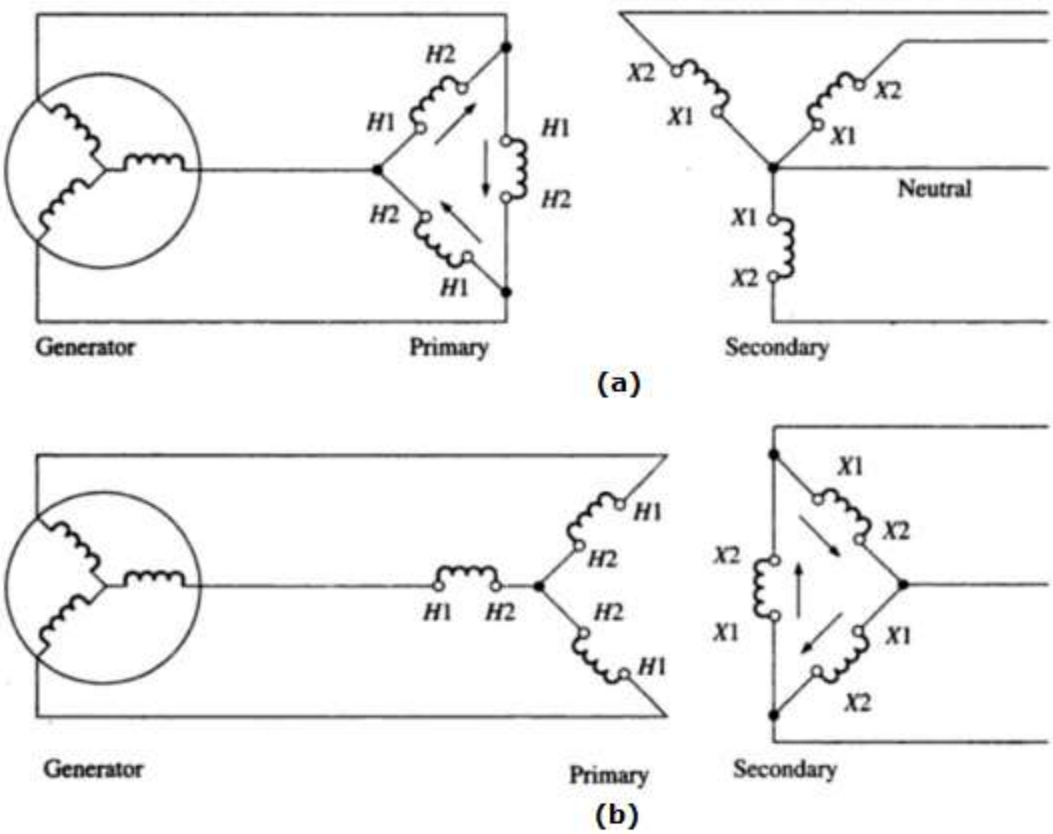


Wye-wye bank with no neutral connection; a delta connected tertiary coil is used to provide a path for triplen harmonic components

Add a third (tertiary) winding connected in Δ to the transformer bank. If a third Δ-connected winding is added to the transformer, then the third- harmonic components of voltage in the Δ will add up, causing a circulating current flow within the winding. This suppresses the third-harmonic components of voltage in the same manner as grounding the transformer neutrals.

The Δ-connected tertiary windings need not even be brought out of the transformer case, but they often are used to supply lights and auxiliary power within the substation where it is located. The tertiary windings must be large enough to handle the circulating currents, so they are usually made about one-third the power rating of the two main windings.

Delta-Wye Bank: Figure shows a Δ-Y transformer bank with Y-connected source. There can be no neutral connection to the primary. However, it has no problem with third-harmonic components because circulating triplen-harmonic currents in the delta enables a sinusoidal output voltage.



Third-harmonic currents circulating in the closed loop formed by the delta (a) Delta-Wye bank (b) Wye-Delta bank with no neutral to the generator

Wye-Delta Bank: Figure shows a Y-Δ bank with no neutral connection to the generator. In this case, the path for triplen harmonic currents is the delta-connected secondary.

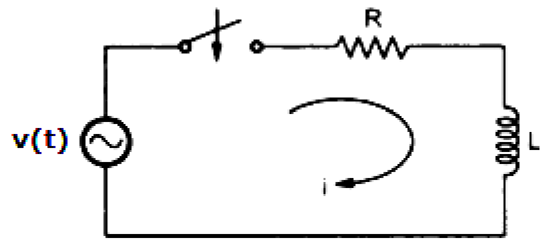
Delta-Delta Bank: Delta-delta banks have no problems with harmonics due to close loop in both primaries and secondaries.

Harmonic currents in power lines interfere with telephone communications by introducing an objectionable hum. Also, the power system is subject to overvoltages caused by possible series resonance between the capacitive reactances of the lines and the leakage reactances of the transformer at a harmonic frequency. The third harmonic is the principal trouble maker and that’s why needs to treated with caution.

Inrush Current

When a transformer is initially connected to a source of AC voltage, there may be a substantial surge of current through the primary winding called *inrush current*. The magnitude of the in-rush current depends on the magnitude and phase angle of the voltage wave at the instant the switch is closed, the magnitude and direction of the residual flux in the ferromagnetic core, and the type and magnitude of the load connected to the secondary. The high in-rush current must be taken into consideration when selecting fuses and/or circuit breakers.

When a switch is closed connecting an ac source to an R-L series circuit, the current will have a source-free response (transient component), and a forced response (steady-state component).



$$L\frac{di}{dt} + iR = V_m sin(\omega t + \phi)$$

$$\frac{di}{dt} + i\frac{R}{L} = \frac{V_m}{L} sin(\omega t + \phi)$$

$$i_{tr} = Ae^{-\frac{R}{L}t}$$

$$i_{ss} = I_m sin(\omega t + \phi - \theta)$$

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} = tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i(t) = i_{tr} + i_{ss} = Ae^{-\frac{R}{L}t} + I_m sin(\omega t + \phi - \theta)$$

$$i(0) = 0 = A + I_m sin(\phi - \theta) => A = -I_m sin(\phi - \theta)$$

$$i(t) = I_m \left[sin(\omega t + \phi - \theta) - sin(\phi - \theta)\, e^{-\frac{R}{L}t}\right]$$

The transient term has a damping factor associated with it and hence, decays exponentially within a few cycles. However, during first few cycles current may vary from 0 to $2I_m$ depending on the switching time.

The transient term will be zero when $\phi - \theta = 0, \pi, 2\pi,$ etc. If the R-L branch is highly inductive, the ratio of ωL to R is large, thereby causing θ to approach $\pi/2$ as an upper limit. In such cases, the transient term is zero when ϕ is approximately equal to $\pi/2, 3\pi/2, 5\pi/2,$ etc. Physically this means that, zero transient effect takes place in highly inductive circuits when the circuit is energized at points of approximately maximum voltage on the voltage wave. On the contrary, the transient term is maximum when ϕ is approximately equal to $0, \pi, 2\pi,$ etc.

We know that the rate of change of instantaneous flux in a transformer core is proportional to the instantaneous voltage drop across the primary winding. Alternatively, the voltage waveform is the derivative of the flux waveform, and the flux waveform is the integral of the voltage waveform. In a continuously-operating transformer, these two waveforms are phase-shifted by 90^o (Fig. 1). Since flux (Φ) is proportional to the magnetomotive force in the core, and the mmf is proportional to winding current, the current waveform will be in-phase with the flux waveform, and both will be lagging the voltage waveform by 90^o.

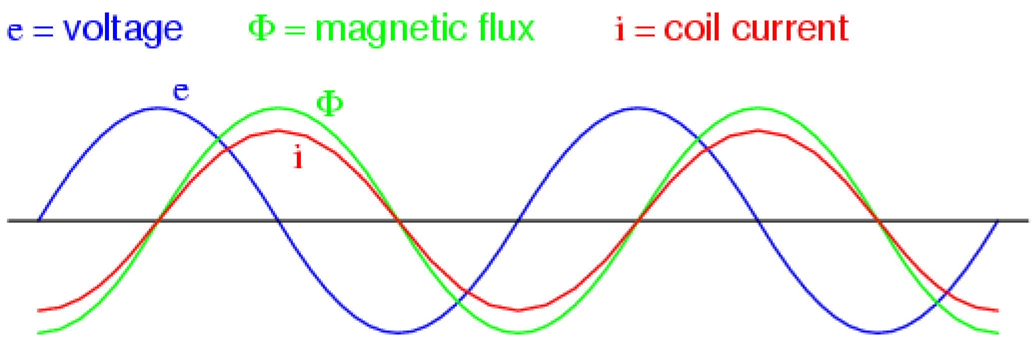


Fig 1: Continuous steady-state operation: Magnetic flux, like current, lags applied voltage by 90^o.

Let us suppose that the primary winding of a transformer is suddenly connected to an AC voltage source at the exact moment in time when the instantaneous voltage is at its positive peak value. In order for the transformer to create an opposing voltage drop to balance against this applied source voltage, a magnetic flux of rapidly increasing value must be generated. The result is that winding current increases rapidly, but actually no more rapidly than under normal conditions (Figure 2).

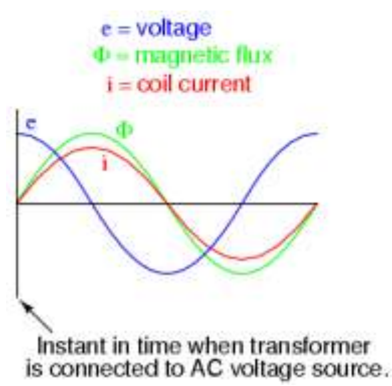
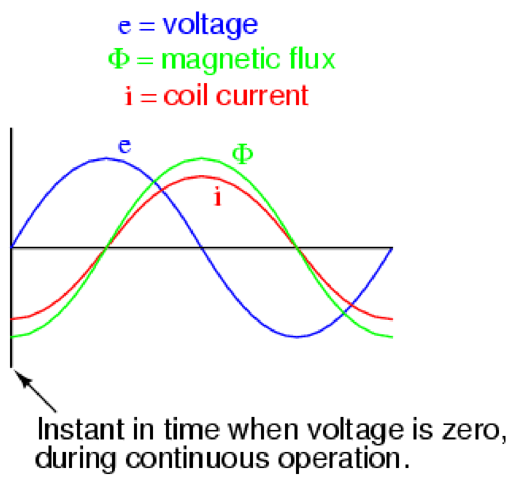


Fig 2. ***Connecting transformer to line at AC volt peak: Flux increases rapidly from zero, same as steady-state operation.***

Both core flux and coil current start from zero and build up to the same peak values experienced during continuous operation. Thus, there is no "surge" or "inrush" or current in this scenario (Fig. 2).

Alternatively, let us consider what happens if the transformer's connection to the AC voltage source occurs at the exact moment in time when the instantaneous voltage is at zero. During continuous operation (when the transformer has been powered for quite some time), this is the point in time where both flux and winding current are at their negative peaks, experiencing zero rate-of-change ($d\Phi/dt = 0$ and $di/dt = 0$). As the voltage builds to its positive peak, the flux and current waveforms build to their maximum positive rates-of-change, and on upward to their positive peaks as the voltage descends to a level of zero.



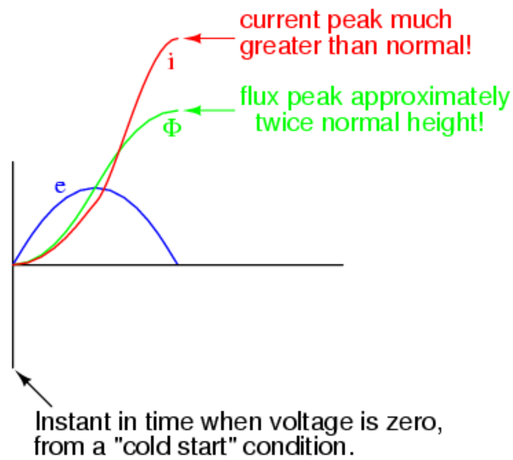


Fig 3. Starting at e=0 V is not the same as running continuously shown in Fig. 1. These expected waveforms are incorrect. Φ and i should start at zero.

Fig 4. Starting at e=0 V, both Φ and i starts from zero. Φ increases to twice the normal value. Current also increases to twice the normal value for an unsaturated core, or considerably higher in case of saturation.

A significant difference exists, however, between continuous-mode operation and the sudden starting condition assumed in this scenario. During continuous operation, the flux and current levels were at their negative peaks when voltage was at its zero point; in a transformer that has been sitting idle, however, both magnetic flux and winding current should start at *zero*. When the magnetic flux increases in response to a rising voltage, it will increase from zero upward, not from a previously negative (magnetized) condition as we would normally have in a transformer that's been powered for awhile. Thus, in a transformer that's just “starting,” the flux will reach approximately twice its normal peak magnitude as it “integrates” the area under the voltage waveform's first half-cycle (Fig. 4).

In an ideal transformer, the magnetizing current would rise to approximately twice its normal peak value as well, generating the necessary mmf to create this higher-than-normal flux. However, most transformers aren't designed with enough of a margin between normal flux peaks and the saturation limits to avoid saturating in a condition like this, and so the core will almost certainly saturate during this first half-cycle of voltage. During saturation, disproportionate amounts of mmf are needed to generate magnetic flux. This means that winding current, which creates the mmf to cause flux in the core, will disproportionately rise to a value *easily exceeding* twice its normal peak (Fig. 4).

This is the mechanism causing inrush current in a transformer's primary winding when connected to an AC voltage source. The magnitude of the inrush current strongly depends on the exact time that electrical connection to the source is made. If the transformer happens to have some residual magnetism in its core at the moment of connection to the source, the inrush could be even more severe. Inductive loads increase the in-rush, whereas the resistive loads and capacitive loads decrease the in-rush.