∴ The process of confining the vibrations of the electric vector of light waves to one direction.

10

POLARIZATION

10.1 INTRODUCTION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion *i.e.*, whether the light waves are longitudinal or transverse, or whether the vibrations are linear, circular or torsional. The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

10.2 POLARIZATION OF TRANSVERSE WAVES

Let a rope *AB* be passed through two parallel slits S_1 and S_2. The rope is attached to a fixed point at *B* [Fig. 10.1(*a*)]. Hold the end *A* and

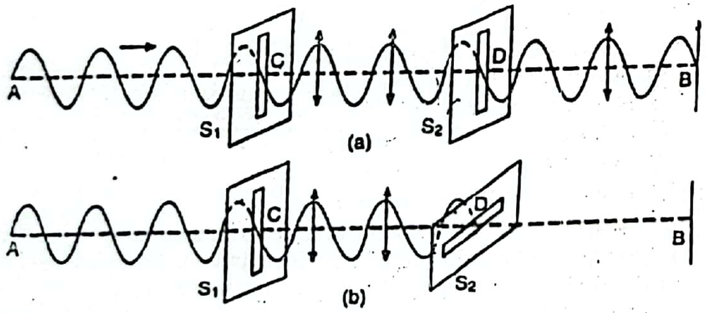

Fig. 10.1

move the rope up and down perpendicular to *AB*. A wave emerges along *CD* and it is due to transverse vibrations parallel to the slit S_1. The slit S_2 allows the wave to pass through it when it is parallel to S_1. It is observed that the slit S_2 does not allow the wave to pass through it when it is at right angles to the slit S_1 [Fig. 10.1(*b*)].

If the end *A* is moved in a circular manner, the rope will show circular motion up to the slit S_1. Beyond S_1, it will show only linear vibrations parallel to the slit S_1, because the slit S_1 will stop the other components. If S_1 and S_2 are at right angles to each other the rope will not show any vibration beyond S_2.

If longitudinal waves are set up by moving the rope forward and backward along the string, the waves will pass through S_1 and S_2 irrespective of their position.

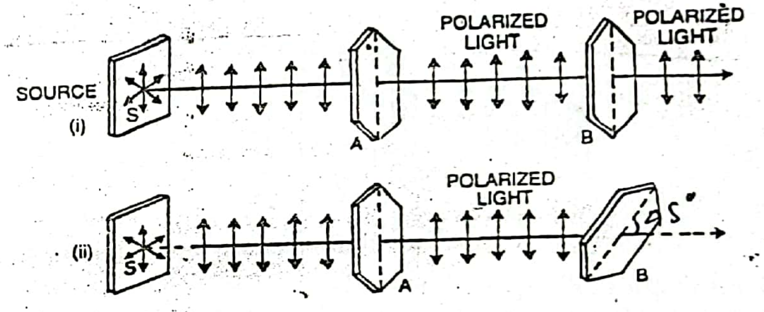

Fig. 10.2

A similar phenomenon has been observed in light when it passes through a tourmaline crystal.

Let light from a source *S* fall on a tourmaline crystal *A* which is cut parallel to its axis (Fig. 10.2). The crystal *A* will act as the slit S_1. The light is slightly coloured due to the natural colour of the crystal. On rotating the crystal *A*, no remarkable change is noticed. Now place the crystal *B* parallel to *A*.

(1) Rotate both the crystals together so that their axes are always parallel. No change is observed in the light coming out of *B* [Fig. 10.2 (*i*)].

(2) Keep the crystal *A* fixed and rotate the crystal *B*. The light transmitted through *B* becomes dimmer and dimmer. When *B is at right angles to A*, no light emerges out of *B* [Fig. 10.2 (*ii*)].

If the crystal *B* is further rotated, the intensity of light coming out of it gradually increases and is maximum again when the two crystals are parallel.

This experiment shows conclusively that light is not propagated as longitudinal or compressional waves. If we consider the propagation of light as a longitudinal wave motion then no extinction of light should occur when the crystal *B* is rotated.

It is clear that after passing through the crystal *A*, the light waves vibrate only in one direction. Therefore light coming out of the crystal *A* is said to be **polarized** because it has acquired the property of one **sidedness** with regard to the direction of the rays.

This experiment proves that light waves are transverse waves, otherwise light coming out of *B* could never be extinguished by simply rotating the crystal *B*.

10.3 PLANE OF POLARIZATION

When ordinary light is passed through a tourmaline crystal, the light is polarized and vibrations are confined to only one direction perpendicular to the direction of propagation of light. This is plane polarized light and

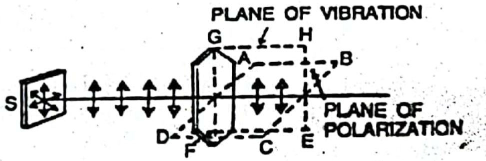

Fig. 10.3

it has acquired the property of one sidedness. The plane of polarization is that plane in which no vibrations occur. The plane *ABCD* in Fig. 10.3 is the plane of polarization. The vibrations occur at right angles to the plane of polarization and the plane in which vibrations occur is known as plane of vibration. The plane *EFGH* in Fig. 10.3 is the plane of vibration.

Ordinary light from a source has very large number of wavelengths. Moreover, the vibrations may be linear, circular or elliptical. From our idea of wave motion, circular or elliptical vibrations consist of two linear vibrations at right angles to each other and having a phase difference of $\frac{\pi}{2}$.

Therefore any vibration can be resolved into two component vibrations at right angles to each other. As light waves are transverse waves the vibrations can be resolved into two planes *xx'* and *yy'*

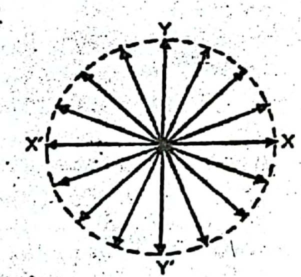

Fig. 10.4

at right angles to each other and also perpendicular to the direction of propagation of light (Fig. 10.4).

In Fig. 10.5(*i*), the vibrations of the particles are represented parallel (arrow heads) and perpendicular to the plane of the paper (dots).

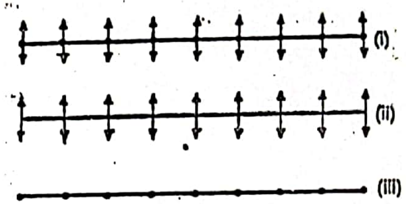

Fig. 10.5

In Fig. (10.3) (*ii*) the vibrations are shown only parallel to the plane of the paper. In Fig. (10.5) (*iii*) the vibrations are represented only perpendicular to the plane of the paper.

10.4 POLARIZATION BY REFLECTION

Polarization of light by reflection from the surface of glass was discovered by Malus in 1808. He found that polarized light is obtained when ordinary light is reflected by a plane sheet of glass. Consider the light incident along the path *AB* on the glass surface (Fig. 10.6). Light is

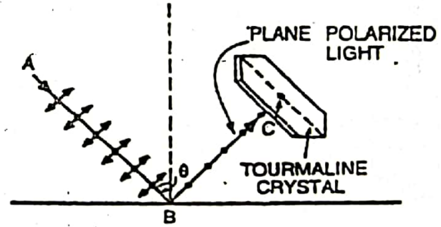

Fig. 10.6

reflected along *BC*. In the path of *BC*, place a tourmaline crystal and rotate it slowly. It will be observed that light is completely extinguished only at one particular angle of incidence. This angle of incidence is equal to 57.5° for a glass surface and is known as the polarizing angle. Similarly polarized light by reflection can be produced from water surface also.

The production of polarized light by glass is explained as follows. The vibrations of the incident light can be resolved into components parallel to the glass surface and perpendicular to the glass surface. Light due to the components parallel to the glass surface is reflected whereas light due to the components perpendicular to the glass surface is transmitted.

Thus, the light reflected by glass is plane polarized and can be detected by a tourmaline crystal.

The polarized light has been analysed by using another mirror by Biot.

10.5 BIOTS POLARISCOPE

It consists of two glass plates M_1 and M_2 (Fig. 10.7). The glass plates are painted black on their back surfaces so as to avoid any reflection and this also helps in absorbing refracted light. A beam of unpolarized light AB is incident at an angle of about 57.5° on the first glass surface at B and is reflected along BC (Fig. 10.8). This light is again reflected at 57.5° by the second glass plate M_2 placed parallel to the first. The glass plate M_1 is known as the polarizer and M_2 as the analyser.

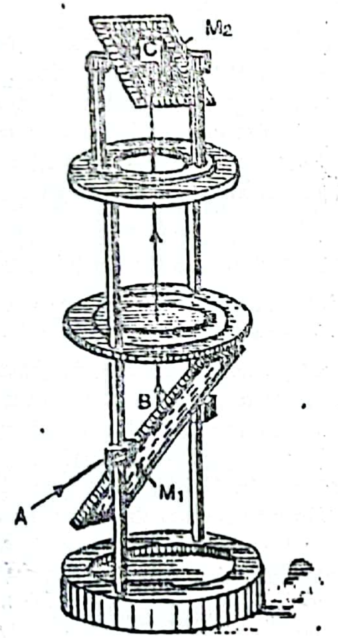

Fig. 10.7

When the upper plate M_2 is rotated about BC, the intensity of the reflected beam along CD decreases and becomes zero for 90° rotation of M_2. Remember, the rotation of the plate M_2 about BC, keeps the angle of incidence constant and it does not change with the rotation of M_2. Thus we find that light travelling along BC is plane polarized.

When the mirror M_2 is rotated further it is found that the intensity of CD becomes maximum at 180°, minimum at 270° and again maximum at 360°.

The above experiment proves that when light is incident at an angle ... reflected light consists of waves in which

the displacements are confined to a certain direction at right angles to the ray and we get polarized light by reflection.

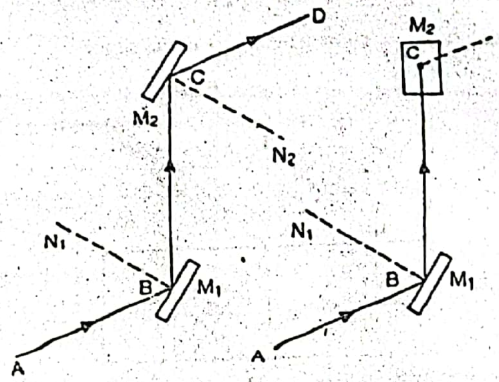

Fig. 10.8

10.6 BREWSTER'S LAW

In 1811, Brewster performed a number of experiments to study the polarization of light by reflection at the surfaces of different media.

He found that ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium at a particular angle known as the **angle of polarization.**

He was able to prove that the tangent of the angle of polarization is numerically equal to the refractive index of the medium. Moreover, the reflected and the refracted rays are perpendicular to each other.

Suppose, unpolarized light is incident at an angle equal to the polarizing angle on the glass surface. It is reflected along BC and refracted along BD (Fig. 10.9).

From Snell's law

$$\mu = \frac{\sin i}{\sin r} \qquad ...(i)$$

From Brewster's law

$$\mu = \tan i = \frac{\sin i}{\cos i} \qquad ...(ii)$$

Comparing (*i*) and (*ii*)

$$\cos i = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$\therefore \quad i = \frac{\pi}{2} - r, \quad \text{or} \quad i + r = \frac{\pi}{2}$$

As $i + r = \frac{\pi}{2}$, $\angle CBD$ is also equal to $\frac{\pi}{2}$. Therefore, the reflected and the refracted rays are at right angles to each other.

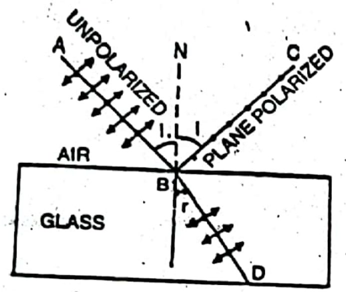

Fig. 10.9

From Brewster's law, it is clear that for crown glass of refractive index 1.52, the value of i is given by

$$i = \tan^{-1}(1.52) \quad \text{or} \quad i = 56.7°$$

However, 57° is an approximate value for the polarizing angle for ordinary glass. For a refractive index of 1.7 the polarising angle is about 59.5° *i.e.*, the polarizing angle is not widely different for different glasses.

As the refractive index of a substance varies with the wavelength of the incident light, the polarizing angle will be different for light of different wavelengths. Therefore, polarization will be complete only for light of a particular wavelength at a time *i.e.*, for monochromatic light.

It is clear that the light vibrating in the plane of incidence is not reflected along BC [Fig. 10.9]. In the reflected beam the vibrations along BC cannot be observed, whereas vibrations at right angles to the plane of incidence can contribute for the resultant intensity. Thus, we get plane polarized light along BC. The refracted ray will have both the vibrations (*i*) in the plane of incidence and (*ii*) at right angles to the plane of incidence. But it is richer in vibrations in the plane of incidence. Hence it is partially plane-polarized.

10.7 BREWSTER WINDOW

One of the important applications of Brewster's law and Brewster's angle is in the design of a glass window that enables 100% transmission of light. Such a type of window is used in lasers and it is called a **Brewster window**,

When an ordinary beam of light is incident normally on a glass window, about 8% of light is lost by reflection on its two surfaces and about 92% intensity is transmitted. In the case of a gas laser filled with mirrors outside the windows, light travels through the window about a hundred times. In this way the intensity of the final beam is about 3×10^{-4} because $(0.92)^{100} = 3 \times 10^{-4}$. It means the transmitted beam has practically no intensity.

To overcome this difficulty, the window is tilted so that the light beam is incident at Brewster's angle. After about hundred transmissions, the final beam will be plane polarized.

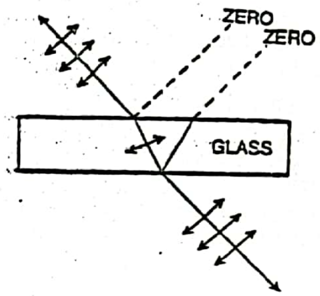

Fig. 10.10

The light component vibrating at right angles to the plane of incidence is reflected. After about 100 reflections at the Brewster window, the transmitted beam will have 50% of the intensity of the incident beam and it will be completely plane polarized. The net effect of this type of arrangement is that half the amount of light intensity has been discarded and the other half is completely retained. Brewster's windows are used in gas lasers.

10.8 POLARIZATION BY REFRACTION

It is found that at a single glass surface or any similar transparent medium, only a small fraction of the incident light is reflected.

For glass ($\mu = 1.5$) at the polarizing angle, 100% of the light vibrating parallel to the plane of incidence is transmitted whereas for the perpendicular vibrations only 85% is transmitted and 15% is reflected. Therefore, if we use a **pile of plates** and the beam of ordinary light is incident at the polarizing angle on the pile of plates, some of the vibrations perpendicular to the plane of incidence are reflected by the first plate and the rest are transmitted through it. When this beam of light is reflected by the second plate, again some of the vibrations perpendicular to the ane of incidence are reflected by it and the rest are transmitted. The proc- s continues and when the beam has traversed about 15 or 20 plates, the ansmitted light is completely free from the vibrations at right angles to e plane of incidence and is having vibrations only in the plane of in- dence. Thus, we get plane-polarized light by refraction with the help of pile of plates, the vibrations being in the plane of incidence as shown Fig. 10.11.

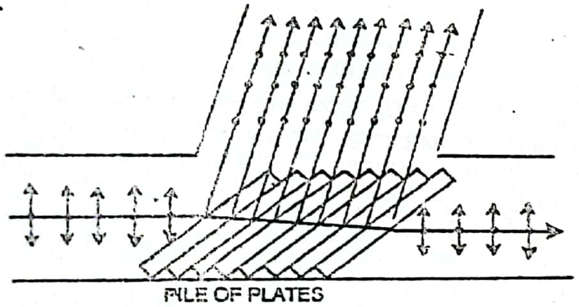

Fig. 10.11

The pile of plates consists of number of glass plates (microscope over slips) and are supported in a tube of suitable size and are inclined an angle of 32.5° to the axis of the tube. A beam of monochromatic ght is allowed to fall on the pile of plates at the polarizing angle. The ansmitted light is polarized perpendicular to the plane of incidence and n be examined by a similar pile of plates which works as an analyser.

Note. (i) If light is polarized perpendicular to the plane of incidence, means vibrations are in the plane of incidence.

(ii) If light is polarized in the plane of incidence, it means vibrations e perpendicular to the plane of incidence.

0.9 MALUS LAW

When a beam of light, polarized by reflection at one plane surface allowed to fall on the second plane surface at the polarizing angle the tensity of the twice reflected beam varies with the angle between the lanes of the two surfaces. In the Biot's polariscope it was found that the tensity of the twice reflected beam is maximum when the two planes e parallel and zero when the two planes are at right angles to each other. he same is also true for the twice transmitted beam from the polarizer

and analyser. The law of Malus states that the **intensity of the polarized light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polarizer.** In the case of the Biot's polariscope this angle is between the two reflecting planes.

The proof of the law is based on the fact that any polarized vibration may be resolved into two rectangular components : (i) parallel to the plane of transmission of the analyser (ii) at right angles to it.

Let $OP = a$ be the amplitude of the vibrations transmitted or reflected by the polarizer and θ is the angle between the planes of the polarizer and the analyser (Fig. 10.12).

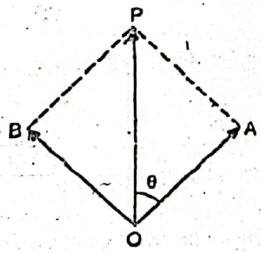

Fig. 10.12

Resolve OP into two components,

(i) $a \cos\theta$ along OA and

(ii) $a \sin\theta$ along OB.

Only the $a \cos\theta$ component is transmitted through the analyser.

∴ Intensity of the transmitted light through the analyser

$$E_1 = (a\cos\theta)^2 = a^2\cos^2\theta.$$

But $\quad E = a^2$

where E is the intensity of incident polarized light

$$\therefore \quad E_1 = E\cos^2\theta, \text{ and } E_1 \propto \cos^2\theta$$

When $\quad \theta = 0$ *i.e.*, the two planes are parallel

$$E_1 = E, \text{ because } \cos 0 = 1$$

When $\quad \theta = \frac{\pi}{2}$ the two planes are at right angles to each other

$$\therefore \quad E_1 = E\left(\cos\frac{\pi}{2}\right)^2 = 0.$$

Example 10.1. *If the plane of vibration of the incident beam makes an angle of 30° with the optic axis, compare the intensities of extraordinary and ordinary light.*

Intensity of the extraordinary ray

$$I_E = A^2\cos^2\theta$$

Intensity of the ordinary ray

$$I_0 = A^2 \sin^2 \theta$$

$$\frac{I_E}{I_0} = \frac{A^2 \cos^2 \theta}{A^2 \sin^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta}$$

Here $\theta = 30°$

$$\therefore \quad E = \frac{I_E}{I_0} = 3$$

10.10 DOUBLE REFRACTION

Erasmus Bartholinus discovered, in 1669, that when a ray of light is refracted by a crystal of calcite it gives two refracted rays. This phenomenon is called **double refraction**. Calcite or Iceland spar is crystallised calcium carbonate ($Ca\,CO_3$) and was found in large quantities in Iceland as very large transparent crystals. Due to this reason calcite is also known as Iceland spar. It crystallises in many forms and can be reduced by cleavage or breakage into a rhombohedron, bounded by six parallelograms with angles equal to 102° and 78° (more accurately 101° 55′ and 78° 5′).

Optic Axis. At two opposite corners A and H, of the rhombohedron all the angles of the faces are obtuse [Fig. 10.13 (*a*)]. These corners A and H are known as the blunt corners of the crystal. A line drawn through A making equal angles with each of the three edges gives the direction of the optic axis. In fact any line parallel to this line is also an optic axis. Therefore, optic axis is not a line but it is a direction. Moreover, it is not defined by joining the two blunt corners. Only in a special case, when the three edges of the crystal are equal, the line joining the two blunt corners A and H coincides with the crystallographic axis of the crystal and it gives the direction of the optic axis [Fig. 10.13 (*b*)]. If a ray of light is incident along the optic axis or in a direction parallel to the optic axis, then

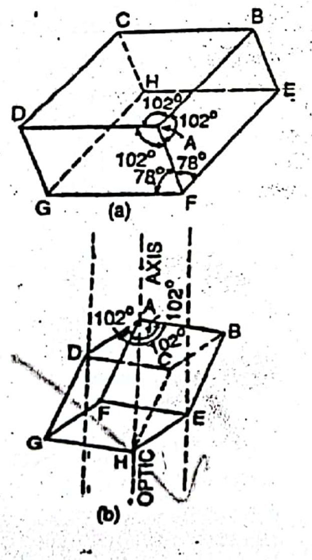

Fig. 10.13

it will not split into two rays. **Thus, the phenomenon of double refraction is absent when light is allowed to enter the crystal along the optic axis.**

The phenomenon of double refraction can be shown with the help of the following experiment :

Mark an ink dot on a piece of paper. Place a calcite crystal over this dot on the paper. Two images will be observed. Now rotate the crystal slowly as shown in Fig. 10.14 (*i*). Place your eye vertically above the crystal. It is found that one image remains stationary and the second image rotates with the rotation of the crystal. The stationary image is known as the ordinary image while the second one is known as the extraordinary image.

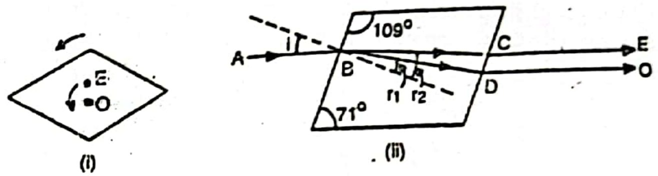

Fig. 10.14

When a ray of light AB is incident on the calcite crystal making an angle of incidence = i, it is refracted along two paths inside the crystal, (*i*) along BC making an angle of refraction = r_2 and (*ii*) along BD making an angle of refraction = r_1. These two rays emerge out along DO and CE which are parallel [Fig. 10.14 (*ii*)].

The ordinary ray has a refractive index $\mu_0 = \dfrac{\sin i}{\sin r_1}$ and the extraordinary ray has a refractive index $\mu_e = \dfrac{\sin i}{\sin r_2}$. It is found that the ordinary ray obeys the laws of refraction and its refractive index is constant. In the case of the extraordinary ray, its refractive index varies with the angle of incidence and it is not fixed.

In the case of calcite $\mu_0 > \mu_e$ because r_1 is less than r_2 [Fig. 10.14 (*ii*)]. Therefore the **velocity of light for the ordinary ray inside the crystal will be less** compared to the velocity of light for the extraordinary ray. In calcite, the extraordinary ray travels faster as compared to the ordinary ray. Moreover, the velocity of the extraordinary ray is different in different directions because its refractive index varies with the angle of incidence.

It has been found that both the rays are plane polarized. The vibrations of the ordinary ray are perpendicular to the principal section of the crystal while the vibrations of the extraordinary ray are in the plane of the principal section of the crystal. Thus, the two rays are **plane polarised**, their vibrations being at right angles to each other.

Special Cases. (1) It should be remembered that a ray of light is not split up into ordinary and extraordinary components when it is incident on calcite parallel to its optic axis. In this case, the ordinary and the extraordinary rays travel along the same direction with the same velocity.

(2) When a ray of light is incident perpendicular to the optic axis on the caloric crystal, the ray of light is not split up into ordinary and extraordinary components. It means that the ordinary and the extraordinary rays travel in the same direction but with **different velocities**.

10.11 PRINCIPAL SECTION OF THE CRYSTAL

A plane which contains the optic axis and is perpendicular to the opposite faces of the crystal is called the **principal section** of the crystal. As a crystal has six faces, therefore, for every point there are three principal sections. A principal section always cuts the surface of a calcite crystal in a parallelogram with angles 109° and 71°

10.12 PRINCIPAL PLANE

A plane in the crystal drawn through the optic axis and the ordinary ray is defined as the principal plane of the ordinary ray, Similarly, a plane in the crystal drawn through the optic axis and the extraordinary ray is defined as the principal plane of the extraordinary ray. In general, the two planes do not coincide. In a particular case, when the plane of incidence is a principal section then the principal section of the crystal and the principal planes of the ordinary and the extraordinary rays coincide.

10.13 NICOL PRISM

It is an optical device used for producing and analysing plane polarized light. It was invented by William Nicol, in 1828, who was an expert in cutting and polishing gems and crystals. We have discussed that when a beam of light is transmitted through a calcite crystal, it breaks up into two rays : (1) the ordinary ray which has its vibrations perpendicular to the principal section of the crystal and (2) the extraordinary ray which has its vibrations parallel to the principal section.

The **nicol prism** is made in such a way that it eliminates one of the two rays by total internal reflection. It is generally found that the ordinary ray is eliminated and, only the extraordinary ray is transmitted through the prism.

A calcite crystal whose length is three times its breadth is taken. Let $A'BCDEFG'H$ represent such a crystal having A' and G' as its blunt corners and $A'CG'E$ is one of the principal sections with $\angle A'CG' = 70°$.

The faces $A'BCD$ and $EFG'H$ are ground in such a way that the angle ACG becomes = 68° instead of 71°. The crystal is then cut along the plane $AKGL$ as shown in Fig. 10.15. The two cut surfaces are grounded and polished optically flat and then cemented together by Canada balsam whose refractive index lies between the refractive indices for the ordinary and the extraordinary rays for calcite.

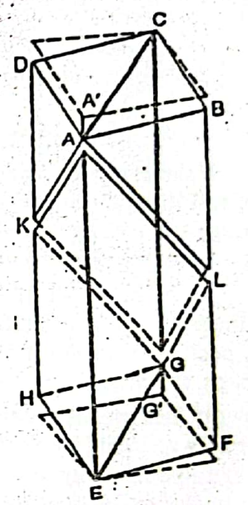

Fig. 10.15

Refractive index for the ordinary

$$\mu_0 = 1.658$$

Refractive index for Canada balsam

$$\mu_B = 1.55$$

Refractive index for the extraordinary $\mu_E = 1.486$

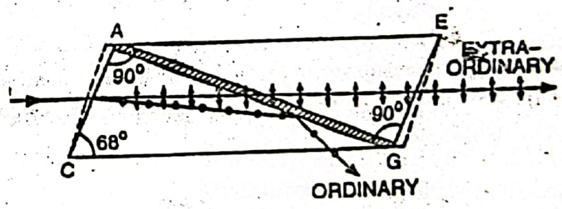

Fig. 10.16

In Fig. 10.16, the section $ACGE$ of the crystal is shown, The diagonal AC represents the Canada balsam layer in the plane $ALGK$ of Fig. 10.15.

It is clear that Canada balsam acts as a rarer medium for an ordinary ray and it acts as a denser medium for the extraordinary ray. Therefore, when the ordinary ray passes from a portion of the crystal into the layer of Canada balsam it passes from a denser to a rarer medium. When the angle of incidence is greater than the critical angle, the ray is totally internally reflected and is not transmitted. The extraordinary ray is not affected and is therefore transmitted through the prism. The working of the prism is clear from the following cases :-

(1) Refractive index for ordinary ray with respect to Canada balsam

$$= \mu = \frac{1.658}{1.550}$$

$$\therefore \quad \sin\theta = \frac{1}{\mu} = \frac{1.550}{1.658}$$

$$\therefore \quad \theta = 69^\circ$$

If the angle of incidence for the ordinary ray is more than the critical angle, it is totally internally reflected and only the extraordinary ray passes through the nicol prism. Therefore, a ray of unpolarized light on passing through the nicol prism in this position becomes plane-polarized.

(2) If the angle of incidence is less than the critical angle for the ordinary ray, it is not reflected and is transmitted through the prism. In this position both the ordinary and the extraordinary rays are transmitted through the prism.

(3) The extraordinary ray also has a limit beyond which it is totally internally reflected by the Canada balsam surface. The refractive index for the extraordinary ray = 1.486 when the extraordinary ray is travelling at right angles to the direction of the optic axis. If the extraordinary ray travels along the optic axis, its refractive index is the same as that of the ordinary ray and it is equal to 1.658. Therefore, depending upon the direction of propagation of the extraordinary ray μ_e lies between 1.486 and 1.658. Therefore for a particular case μ_e may be more than 1.55 and the angle of incidence will be more than the critical angle. Then, the extraordinary ray will also be totally internally reflected at the Canada balsam layer. The sides of the nicol prism are coated with black paint to absorb the ordinary rays that are reflected towards the sides by the Canada balsam layer.

10.14 NICOL PRISM AS AN ANALYSER

Nicol prism can be used for the production and detection of plane-polarizer light.

When two nicol prisms P_1 and P_2 are placed adjacent to each other as shown in Fig. 10.17 (*i*), one of them acts as a polarizer and the other acts as an analyser. Fig. 10.17 (*i*) shows the position of two parallel nicols and only the extraordinary ray passes through both the prisms.

If the second prism P_2 is gradually rotated, the intensity of the extraordinary ray decreases in accordance with Malus Law and when

the two prisms are crossed [*i.e.*, when they are at right angles to each other, Fig. 10.16 (*ii*), then no light comes out of the second prism P_2. It means that light coming out of P_1 is plane polarized. When the polarized extraordinary ray enters the prism P_2 in this positions it acts as

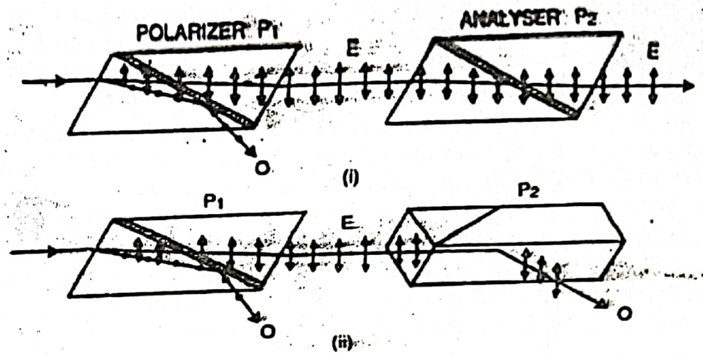

Fig. 10.17

an ordinary ray and is totally internally reflected by the Canada balsam layer and so no light comes out of P_2. Therefore, the prism P_1 produces plane-polarized light and the prism P_2 detects it.

Hence P_1 and P_2 are called the polarizer and the analyser respectively. The combination of P_1 and P_2 is called a polariscope.

10.15 HUYGENS EXPLANATION OF DOUBLE REFRACTION IN UNIAXIAL CRYSTALS

Huygens explained the phenomenon of double refraction with the help of his principle of secondary wavelets. A point source of light in a double refracting medium is the origin of two wavefronts. For the ordinary ray, for which the velocity of light is the same in all directions the wavefront is spherical. For the extraordinary ray, the velocity varies with the direction and the wavefront is an ellipsoid of revolution. **The velocities of the ordinary and the extraordinary rays are the same along the optic axis.**

Consider a point source of light S in a calcite crystal [Fig. 10.18.(*a*)]. The sphere is the wave surface for the ordinary ray and the ellipsoid is the wave surface for the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface. Such crystals are known as negative crystals. For crystals like quartz, which are known as positive crystals, the extraordinary wave surface lies within the ordinary wave surface [Fig. 10.18 (*b*)].

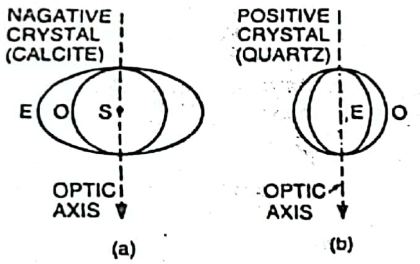

Fig. 10.18

(1) For the **negative uniaxial** crystals, $\mu_0 > \mu_E$. The velocity of the extraordinary ray varies as the radius vector of the ellipsoid. It is least and equal to the velocity of the ordinary ray along the optic axis but it is maximum at right angles to the direction of the optic axis.

(2) For the **positive uniaxial** crystals $\mu_E > \mu_0$. The velocity of the extraordinary ray is least in a direction at right angles to the optic axis. It is maximum and is equal to the velocity of the ordinary ray along the optic axis. Hence, from Huygens' theory, the wavefronts or surfaces in uniaxial crystals are a sphere and an ellipsoid and there are two points where these two wavefronts touch each other. The direction of the line joining these two points (Where the sphere and the ellipsoid touch each other) is the **optic axis**.

10.16 OPTIC AXIS IN THE PLANE OF INCIDENCE AND INCLINED TO THE CRYSTAL SURFACE

(*a*) **Oblique incidence.** *AB* is the incident plane wavefront of the rays falling obliquely on the surface *MN* of the negative crystal. The crystal is cut so that the optic axis is in the plane of incidence and is in the direction shown in Fig. 10.19. O_1 is the spherical secondary wavefront for the ordinary ray and E_1 is the ellipsoidal secondary wavefront for the extraordinary ray. *CP* is the tangent meeting the spherical wavefront at *P* and *CQ* is the tangent meeting the ellipsoidal wavefront at *Q*.

According to Huygens' construction, by the time the incident wave reaches from *B* to *C*, the ordinary ray travels the distance *AP* and the extraordinary ray travels the distance *AQ*. Suppose, the velocity of light

in air is V_a and the velocities of light for the ordinary ray along *AP* and the extraordinary ray along *AQ* are V_0 and V_e respectively. In this case,

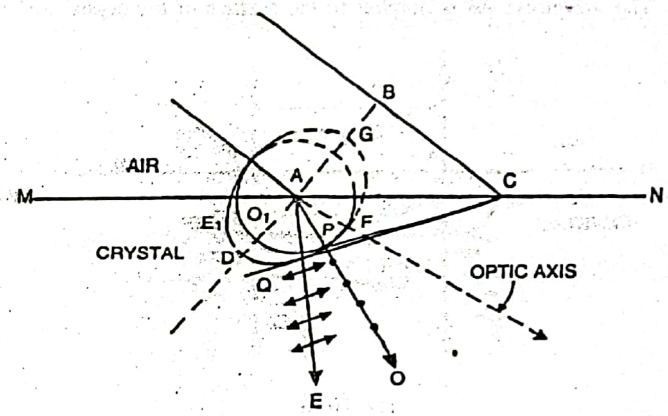

Fig. 10.19

$$\frac{BC}{V_a} = \frac{AP}{V_0} = \frac{AQ}{V_e} \qquad ...(i)$$

Therefore
$$AP = \frac{BC.V_0}{V_a} = \frac{BC}{\mu_0} \qquad ...(ii)$$

and
$$AQ = \frac{BC.V_e}{V_a} = \frac{BC}{\mu_e} \qquad ...(iii)$$

Here, μ_0 and μ_e are the refractive indices for the ordinary and the extraordinary rays along *AP* and *AQ* respectively. In Fig. 10.19, *CP* and *CQ* are the ordinary and the extraordinary refracted plane wavefronts respectively in the crystal. Therefore, the ordinary and the extraordinary rays travel with different velocities along different direction. Here, the semi-major axis of the ellipsoid is $\frac{BC}{\mu_E}$ and the semi-minor axis is $\frac{BC}{\mu_0}$, where μ_E is the principal refractive index for the extraordinary ray and

$$\mu_E < \mu_e < \mu_0$$

Note. The direction *AE* of the extraordinary ray is not perpendicular to the tangent *CQ*, whereas the direction *AO* of the ordinary ray is perpendicular to the tangent *CP*.