প্রশ্নমালা–5(B)

1(i). $y = (x-1)(x-2)(x-3)$

$\Rightarrow ln y = ln(x-1) + ln(x-2) + ln(x-3)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}$$

$$\therefore \frac{dy}{dx} = y\left[\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}\right].$$

(ii). $y = e^{x^2\sqrt{1-x^2}}$

$$\therefore \frac{dy}{dx} = e^{x^2\sqrt{1-x^2}} \cdot \frac{d}{dx}[x^2\sqrt{(1-x^2)}]$$

$$= e^{x^2\sqrt{1-x^2}}\left[x^2\frac{d}{dx}\{\sqrt{(1-x^2)}\} + \sqrt{(1-x^2)}\frac{d}{dx}(x^2)\right]$$

$$= e^{x^2\sqrt{1-x^2}}\left[x^2\frac{(-2x)}{2\sqrt{(1-x^2)}} + \sqrt{(1-x^2)}.\ 2x\right]$$

$$= e^{x^2\sqrt{1-x^2}}\left[\frac{-x^3}{\sqrt{(1-x^2)}} + 2x\sqrt{(1-x^2)}\right]$$

$$= x.e^{x^2\sqrt{1-x^2}}[2\sqrt{(1-x^2)} - x^2/\sqrt{(1-x^2)}].$$

(iii). $y = e^{\sin x (ln x)^2}$

$$\frac{dy}{dx} = e^{\sin x(ln x)^2}\frac{d}{dx}[\sin x(ln x)^2]$$

$$= e^{\sin x(ln x)^2}\left[\sin x\frac{d}{dx}(ln x)^2 + (ln x)^2\frac{d}{dx}(\sin x)\right]$$

$$= e^{\sin x(ln x)^2}\left[\sin x\frac{2 ln x}{x} + (ln x)^2.\ \cos x\right]$$

$$= e^{\sin x(ln x)^2}\ ln x[(2/x)\sin x + ln x.\ \cos x].$$

(iv). $y = x(a+x^2)\sqrt{(a^2-x^2)}$

$$\Rightarrow ln y = ln x + ln(a+x^2) + \frac{1}{2}ln(a^2-x^2)$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{2x}{a + x^2} + \frac{1}{2}\cdot\frac{(-2x)}{a^2 - x^2}$$

$$\therefore \frac{dy}{dx} = y\left[\frac{1}{x} + \frac{2x}{a + x^2} - \frac{x}{a^2 - x^2}\right].$$

2(i). $y = \frac{(a^2 + x^2)^{1/2}}{x} \Rightarrow \ln y = \frac{1}{2}\ln(a^2 + x^2) - \ln x$

$$\frac{d}{dx}(\ln y) = \frac{1}{2}\frac{d}{dx}\{\ln(a^2 + x^2)\} - \frac{d}{dx}(\ln x)$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\cdot\frac{2x}{(a^2 + x^2)} - \frac{1}{x} = \frac{x^2 - a^2 - x^2}{x(a^2 + x^2)}$$

$$\therefore \frac{dy}{dx} = \frac{y(-a^2)}{x(a^2 + x^2)} = \frac{\sqrt{(a^2 + x^2)}\,(-a^2)}{x.x(a^2 + x^2)} = \frac{-a^2}{x^2\sqrt{(a^2 + x^2)}}.$$

(ii). $y = \frac{1 - x}{(1 - x^3)^{1/2}} \Rightarrow \ln y = \ln(1 - x) - \frac{1}{2}\ln(1 - x^3)$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\{\ln(1 - x)\} - \frac{1}{2}\frac{d}{dx}\{\ln(1 - x^3)\}$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{-1}{1 - x} - \frac{1}{2}\cdot\frac{(-3x^2)}{1 - x^3} = \frac{3x^2}{2(1 - x^3)} - \frac{1}{1 - x}$$

বা $\frac{1}{y}\frac{dy}{dx} = \frac{3x^2 - 2(1 + x + x^2)}{2(1 - x^3)} = \frac{x^2 - 2x - 2}{2(1 - x^3)}$

$$\therefore \frac{dy}{dx} = \frac{y(x^2 - 2x - 2)}{2(1 - x^3)} = \frac{(1 - x)\,(x^2 - 2x - 2)}{(1 - x^3)^{1/2}\,2(1 - x^3)}$$

$$= \frac{(1 - x)\,(x^2 - 2x - 2)}{2(1 - x^3)^{3/2}}.$$

(iii). প্রশ্ন সংশোধন : $y = \left(\frac{4 + 3x}{4 - 3x}\right)^{1/2}$ এর পরিবর্তে $y = \left(\frac{4 + 3x}{4 - 3x}\right)^{1/3}$ হইবে।

$$y = \left(\frac{4 + 3x}{4 - 3x}\right)^{1/3} \Rightarrow \ln y = \frac{1}{3}[\ln(4 + 3x) - \ln(4 - 3x)]$$

$$\frac{d}{dx}(\ln y) = \frac{1}{3}\frac{d}{dx}[\ln(4 + 3x) - \ln(4 - 3x)]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3}\left[\frac{3}{4 + 3x} + \frac{3}{4 - 3x}\right] = \frac{8}{(4 + 3x)\,(4 - 3x)}$$

$$\therefore \frac{dy}{dx} = y.\frac{8}{(16 - 9x^2)} = \left(\frac{4 + 3x}{4 - 3x}\right)^{1/3}\frac{8}{(16 - 9x^2)}.$$

(iv). $y=\left[\dfrac{(x-1)\ (x-2)}{(x-3)\ (x-4)}\right]^{1/2}$

$\Rightarrow ln y=\dfrac{1}{2}\,[ln(x-1)+ln(x-2)-ln(x-3)-ln(x-4)].$

$\therefore \dfrac{1}{y}\dfrac{dy}{dx}=\dfrac{1}{2}\left[\dfrac{1}{x-1}+\dfrac{1}{x-2}-\dfrac{1}{x-3}-\dfrac{1}{x-4}\right]$

বা $\dfrac{1}{y}\dfrac{dy}{dx}=\dfrac{1}{2}\left[\dfrac{2x-3}{(x-1)\ (x-2)}-\dfrac{2x-7}{(x-3)\ (x-4)}\right]$

$\therefore \dfrac{dy}{dx}=\dfrac{y}{2}\left[\dfrac{-4x^2+20x-22}{(x-1)\ (x-2)\ (x-3)\ (x-4)}\right]$

$=\left[\dfrac{(x-1)\ (x-2)}{(x-3)\ (x-4)}\right]^{1/2}\ \dfrac{(-2x^2+10x-11)}{(x-1)\ (x-2)\ (x-3)\ (x-4)}.$

(v). $y=\left[\dfrac{x(1+x^2)}{e^x(1-x^2)}\right]^{3/2}$

$\Rightarrow ln y=\dfrac{3}{2}\,[ln x+ln(1+x^2)-ln e^x-ln(1-x^2)]$

বা $ln y=\dfrac{3}{2}\,[ln x+ln(1+x^2)-ln(1-x^2)-x]$

$\therefore \dfrac{1}{y}\dfrac{dy}{dx}=\dfrac{3}{2}\left[\dfrac{1}{x}+\dfrac{2x}{1+x^2}+\dfrac{2x}{1-x^2}-1\right]$

$\therefore \dfrac{dy}{dx}=\dfrac{3y}{2}\left[\dfrac{1}{x}+\dfrac{2x}{(1+x^2)}+\dfrac{2x}{1-x^2}-1\right]$

$=\dfrac{3}{2}\left[\dfrac{x(1+x^2)}{e^x(1-x^2)}\right]^{3/2}\left[\dfrac{1}{x}+\dfrac{2x}{1+x^2}+\dfrac{2x}{1-x^2}-1\right].$

(vi). $y=\dfrac{(x^2-1)^{3/2}}{(x^2-4)^{1/2}}\Rightarrow ln y=\dfrac{3}{2}\,ln(x^2-1)-\dfrac{1}{2}\,ln(x^2-4)$

$\therefore \dfrac{1}{y}\dfrac{dy}{dx}=\dfrac{3}{2}\cdot\dfrac{2x}{x^2-1}-\dfrac{1}{2}\cdot\dfrac{2x}{x^2-4}=\dfrac{3x}{x^2-1}-\dfrac{x}{x^2-4}.$

$\dfrac{dy}{dx}=y\left[\dfrac{2x^3-11x}{(x^2-1)\ (x^2-4)}\right]=\dfrac{(x^2-1)^{3/2}}{(x^2-4)^{1/2}}\cdot\dfrac{(2x^3-11x)}{(x^2-1)\ (x^2-4)}$

$=x(2x^2-11)\ (x^2-1)^{1/2}/(x^2-4)^{3/2}.$

(vii). $y = \dfrac{(x^2+1)^3}{(x^3-1)^2} \Rightarrow \ln y = 3\ln(x^2+1) - 2\ln(x^3-1)$

$$\therefore \frac{1}{y}\frac{dy}{dx} = 3.\frac{2x}{x^2+1} - \frac{2.3x^2}{x^3-1} = \frac{6x}{x^2+1} - \frac{6x^2}{x^3-1}$$

$$\therefore \frac{dy}{dx} = \frac{y(-6x-6x^2)}{(x^2+1)(x^3-1)} = \frac{(x^2+1)^3}{(x^3-1)^2}\frac{(-6x-6x^2)}{(x^2+1)(x^3-1)}$$

$$= -6x(1+x)(x^2+1)^2/(x^3-1)^3.$$

(viii). $y = \left(\dfrac{3-x^2}{x^2+1}\right)^{1/2} \Rightarrow \ln y = \dfrac{1}{2}[\ln(3-x^2) - \ln(x^2+1)]$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{-2x}{3-x^2} - \frac{2x}{x^2+1}\right] = \frac{1}{2}\left[\frac{-8x}{(3-x^2)(x^2+1)}\right]$$

$$\therefore \frac{dy}{dx} = y\frac{(-4x)}{(3-x^2)(x^2+1)}, = \frac{(3-x^2)^{1/2}}{(x^2+1)^{1/2}}\frac{(-4x)}{(3-x^2)(x^2+1)}$$

$$= -4x/(3-x^2)^{1/2}(x^2+1)^{3/2}.$$

(ix). $y = \left(\dfrac{x}{1+\sqrt{(1-x^2)}}\right)^n$

$$\Rightarrow \ln y = n[\ln x - \ln(1+\sqrt{(1-x^2)})]$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = n\left[\frac{1}{x} - \frac{1}{1+\sqrt{(1-x^2)}}\frac{d}{dx}\{1+\sqrt{(1-x^2)}\}\right]$$

বা $\dfrac{dy}{dx} = ny\left[\dfrac{1}{x} - \dfrac{1}{\{1+\sqrt{(1-x^2)}\}}\cdot\dfrac{-2x}{2\sqrt{(1-x^2)}}\right]$

$$\frac{dy}{dx} = ny\left[\frac{1}{x} + \frac{x}{\{1+\sqrt{(1-x^2)}\}\sqrt{(1-x^2)}}\right]$$

$$= ny\left[\frac{\sqrt{(1-x^2)}+1-x^2+x^2}{x\{1+\sqrt{(1-x^2)}\}\sqrt{(1-x^2)}}\right] = \frac{ny[1+\sqrt{(1-x^2)}]}{x[1+\sqrt{(1-x^2)}]\sqrt{(1-x^2)}}$$

$$= ny\frac{1}{x\sqrt{(1-x^2)}} = \left(\frac{x}{1+\sqrt{(1-x^2)}}\right)^n\frac{n}{x\sqrt{(1-x^2)}}.$$

(x). $y = \left(\dfrac{x}{x+\sqrt{(a^2-x^2)}}\right)^n$

$$\Rightarrow \ln y = n[\ln x - \ln\{x+\sqrt{(a^2-x^2)}\}]$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = n\left[\frac{1}{x} - \frac{1}{x + \sqrt{(a^2 - x^2)}}\left\{1 - \frac{2x}{2\sqrt{(a^2 - x^2)}}\right\}\right]$$

বা $$\frac{dy}{dx} = ny\left[\frac{1}{x} - \frac{1}{x + \sqrt{(a^2 - x^2)}}\left\{\frac{\sqrt{(a^2 - x^2)} - x}{\sqrt{(a^2 - x^2)}}\right\}\right]$$

$$= ny\left[\frac{1}{x} - \frac{\sqrt{(a^2 - x^2)} - x}{\{x + \sqrt{(a^2 - x^2)}\}\sqrt{(a^2 - x^2)}}\right]$$

$$= ny\left[\frac{x\sqrt{(a^2 - x^2)} + a^2 - x^2 - x\sqrt{(a^2 - x^2)} + x^2}{x\{x + \sqrt{(a^2 - x^2)}\}\sqrt{(a^2 - x^2)}}\right]$$

$$= n\left(\frac{x}{x + \sqrt{(a^2 - x^2)}}\right)^n \frac{a^2}{x\{x + \sqrt{(a^2 - x^2)}\}\sqrt{(a^2 - x^2)}}$$

$$= na^2 x^{n-1}/\sqrt{(a^2 - x^2)}\ \{(x + \sqrt{(a^2 - x^2)}\}^{n+1}.$$

(xi). $$y = \left(\frac{x^3 + 1}{\sqrt{x^2 - x + 1}}\right)^4$$

$$\Rightarrow \ln y = 4[\ln(x^3 + 1) - \frac{1}{2}\ln(x^2 - x + 1)]$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = 4\left[\frac{3x^2}{x^3 + 1} - \frac{1}{2}\cdot\frac{2x - 1}{x^2 - x + 1}\right]$$

বা $$\frac{dy}{dx} = 4y\left[\frac{6x^4 - 6x^3 + 6x^2 - 2x^4 - 2x + x^3 + 1}{2(x^3 + 1)(x^2 - x + 1)}\right]$$

$$= 2\left(\frac{x^3 + 1}{\sqrt{(x^2 - x + 1)}}\right)^4\left[\frac{4x^4 - 5x^3 + 6x^2 - 2x + 1}{(x^3 + 1)(x^2 - x + 1)}\right].$$

(xii). $$y = \frac{3 + 5\cos x}{5 + 3\cos x}$$

$$\Rightarrow \ln y = \ln(3 + 5\cos x) - \ln(5 + 3\cos x)$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{-5\sin x}{3 + 5\cos x} + \frac{3\sin x}{5 + 3\cos x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\sin x[-25 - 15\cos x + 9 + 15\cos x]}{(3 + 5\cos x)(5 + 3\cos x)}$$

বা $$\frac{dy}{dx} = y.\frac{(-16)\sin x}{(3 + 5\cos x)(5 + 3\cos x)}$$

$$= \frac{(3 + 5\cos x)}{5 + 3\cos x}\cdot\frac{(-16)\sin x}{(3 + 5\cos x)(5 + 3\cos x)}$$

$$= -16\sin x/(5 + 3\cos x)^2.$$

(xiii). $y = \dfrac{x\cos 2x}{e^{3x}\sqrt{(1+x^2)}}$

$\Rightarrow \ln y = \ln x + \ln(\cos 2x) - 3x\ln e - \dfrac{1}{2}\ln(1+x^2)$

$\therefore \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x} - \dfrac{2\sin 2x}{\cos 2x} - 3 - \dfrac{1}{2}\cdot\dfrac{2x}{1+x^2}$

বা $\dfrac{dy}{dx} = y\left[\dfrac{1}{x} - 2\tan 2x - 3 - \dfrac{x}{1+x^2}\right]$.

(xiv). $y = \dfrac{x\sin^{-1}x}{\sqrt{(1-x^2)}}$

$\Rightarrow \ln y = \ln x + \ln(\sin^{-1}x) - \dfrac{1}{2}\ln(1-x^2)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x} + \dfrac{1}{\sin^{-1}x}\cdot\dfrac{1}{\sqrt{(1-x^2)}} - \dfrac{1}{2}\dfrac{1}{(1-x^2)}(-2x)$

বা $\dfrac{dy}{dx} = y\left[\dfrac{1}{x} + \dfrac{1}{\sqrt{(1-x^2)}\sin^{-1}x} + \dfrac{x}{1-x^2}\right]$

$= y\left[\dfrac{(1-x^2)\sin^{-1}x + x\sqrt{(1-x^2)} + x^2\sin^{-1}x}{x(1-x^2)\sin^{-1}x}\right]$

$= \dfrac{x\sin^{-1}x}{\sqrt{(1-x^2)}}\cdot\dfrac{\sin^{-1}x + x\sqrt{(1-x^2)}}{x(1-x^2)\sin^{-1}x}$

$= \{\sin^{-1}x + x\sqrt{(1-x^2)}\}/(1-x^2)^{3/2}$.

(xv). $y = \dfrac{x^n}{\log_a x} \Rightarrow \ln y = n\ln x - \ln(\log_a x)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{n}{x} - \dfrac{1}{\log_a x}\cdot\dfrac{1}{x}\log_a e$

বা $\dfrac{dy}{dx} = y\left[\dfrac{n}{x} - \dfrac{\log_a e}{x\log_a x}\right] = \dfrac{x^n}{\log_a x}\left[\dfrac{n}{x} - \dfrac{1}{x\ln x}\right]$.

(xvi). $y = \dfrac{x\cos^{-1}x}{\sqrt{(1-x^2)}}$

$\Rightarrow \ln y = \ln x + \ln(\cos^{-1}x) - \dfrac{1}{2}\ln(1-x^2)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\frac{1}{\cos^{-1}x}\cdot\frac{(-1)}{\sqrt{(1-x^2)}}-\frac{1}{2}\cdot\frac{1}{(1-x^2)}(-2x)$$

$$\frac{dy}{dx}=y\left[\frac{1}{x}-\frac{1}{\sqrt{(1-x^2)}\cos^{-1}x}+\frac{x}{1-x^2}\right]$$

$$=y\left[\frac{(1-x^2)\cos^{-1}x-x\sqrt{(1-x^2)}+x^2\cos^{-1}x}{x(1-x^2)\cos^{-1}x}\right]$$

$$=\frac{x\cos^{-1}x}{\sqrt{(1-x^2)}}\cdot\frac{\cos^{-1}x-x\sqrt{(1-x^2)}}{x(1-x^2)\cos^{-1}x}$$

$$=\{(\cos^{-1}x-x\sqrt{(1-x^2)}\}/(1-x^2)^{3/2}.$$

(xvii). $y=\dfrac{x\ln x}{\sqrt{(1+x^2)}}$

$$\Rightarrow \ln y=\ln x+\ln(\ln x)-\frac{1}{2}\ln(1+x^2)$$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\frac{1}{\ln x}\cdot\frac{1}{x}-\frac{1}{2}\cdot\frac{1}{(1+x^2)}2x$$

বা $\dfrac{dy}{dx}=y\left[\dfrac{1}{x}+\dfrac{1}{x\ln x}-\dfrac{x}{1+x^2}\right]$

$$=y\left[\frac{(1+x^2)\ln x+1+x^2-x^2\ln x}{x(1+x^2)\ln x}\right]$$

$$=\frac{x\ln x}{\sqrt{(1+x^2)}}\cdot\frac{\{\ln x+(1+x^2)\}}{x(1+x^2)\ln x}=\frac{(1+x^2)+\ln x}{(1+x^2)^{3/2}}.$$

3(i). $y=(x^x)^x\Rightarrow \ln y=x\ln x^x$

বা $\ln y=x.x\ln x=x^2\ln x$

$$\therefore\frac{d}{dx}(\ln y)=\frac{d}{dx}(x^2\ln x)$$

বা $\dfrac{1}{y}\dfrac{dy}{dx}=x^2.\dfrac{1}{x}+2x\ln x$

বা $\dfrac{dy}{dx}=y[x+2x\ln x]=(x^x)^x.\ x(1+2\ln x).$

(ii). $y=x^{x^{x\cdots\infty}}$

বা $y=x^y$, যেহেতু $y=x^{x^{\cdots\infty}}$

$\Rightarrow ln y = y\, ln x$

$\therefore \frac{d}{dx}(ln y) = \frac{d}{dx}(y\, ln x)$

$\therefore \frac{1}{y}\frac{dy}{dx} = y.\frac{1}{x} + ln x \frac{dy}{dx}$

বা $\left(\frac{1}{y} - ln x\right)\frac{dy}{dx} = \frac{y}{x}$

বা $\frac{(1 - y\, ln x)}{y}\frac{dy}{dx} = \frac{y}{x}$

$\therefore \frac{dy}{dx} = \frac{y^2}{x(1 - y\, ln x)}$.

(iii). $y = x^{1/x} \Rightarrow ln y = \frac{1}{x} ln x;\ \frac{d}{dx}(ln y) = \frac{d}{dx}\left(\frac{1}{x} ln x\right)$

বা $\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}.\frac{1}{x} - \frac{1}{x^2} ln x = \frac{1}{x^2}(1 - ln x)$

বা $\frac{dy}{dx} = y.\frac{1}{x^2}[1 - ln x] = x^{1/x}.\frac{1}{x^2}[1 - ln x]$.

(iv). $y = (2x+3)^{2x+3} \Rightarrow ln y = (2x+3)\, ln(2x+3)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\frac{1}{y}\frac{dy}{dx} = (2x+3).\frac{2}{(2x+3)} + 2.ln(2x+3)$

$\therefore \frac{dy}{dx} = y[2 + 2ln(2x+3)] = 2y[1 + ln(2x+3)]$.

(v). $y = (x^2+2)^{2-x} \Rightarrow ln y = (2-x)\, ln(x^2+2)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\frac{1}{y}\frac{dy}{dx} = (2-x).\frac{2x}{(x^2+2)} - 1.ln(x^2+2)$

$\therefore \frac{dy}{dx} = y\left[\frac{2x(2-x)}{x^2+2} - ln(x^2+2)\right]$.

(vi). $y = x^{1+x+x^2} \Rightarrow ln y = (1+x+x^2)ln x$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\frac{1}{y}\frac{dy}{dx} = (1+x+x^2)\frac{1}{x} + (1+2x)ln x$

$\therefore \frac{dy}{dx} = y\left[\frac{1+x+x^2}{x} + (1+2x)ln x\right]$.

(vii). $y = 3^{\sqrt{(1+x+x^2)}} \Rightarrow \ln y = \sqrt{(1+x+x^2)}.\ \ln 3$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2\sqrt{(1+x+x^2)}}(1+2x).\ \ln 3$$

$$\therefore \frac{dy}{dx} = y.\frac{(1+2x)\ \ln 3}{2\sqrt{(1+x+x^2)}} = 3^{\sqrt{(1+x+x^2)}}.\frac{(1+2x)\ \ln 3}{2\sqrt{(1+x+x^2)}}.$$

(viii). $y = \left(\frac{3+x}{5+x}\right)^{8+2x}$

$$\Rightarrow \ln y = (8+2x)\ [\ln(3+x) - \ln(5+x)]$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = (8+2x)\left[\frac{1}{3+x} - \frac{1}{5+x}\right] + 2[\ln(3+x) - \ln(5+x)]$$

বা $\frac{1}{y}\frac{dy}{dx} = 2(4+x).\frac{2}{(3+x)\ (5+x)} + 2\ln\frac{3+x}{5+x}$

$$\therefore \frac{dy}{dx} = 2y\left[\frac{2(4+x)}{(3+x)\ (5+x)} + \ln\frac{3+x}{5+x}\right].$$

(ix). $y = \left(\frac{x+1}{x}\right)^{x^2} \Rightarrow \ln y = x^2[\ln(x+1) - \ln x]$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{y}\frac{dy}{dx} = x^2\left[\frac{1}{x+1} - \frac{1}{x}\right] + 2x[\ln(x+1) - \ln x]$$

বা $\frac{1}{y}\frac{dy}{dx} = x^2.\frac{(-1)}{x(x+1)} + 2x\ln\frac{x+1}{x}$

$$\therefore \frac{dy}{dx} = y[2x\ln(1+1/x) - x/(x+1)].$$

(x). $y = \left(\frac{a^2+ax+x^2}{a^2-ax+x^2}\right)^{1/2}.$

$$\Rightarrow \ln y = \frac{1}{2}[\ln(a^2+ax+x^2) - \ln(a^2-ax+x^2)]$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{a+2x}{a^2+ax+x^2} - \frac{-a+2x}{a^2-ax+x^2}\right]$$

$$\therefore \frac{dy}{dx} = \frac{y}{2}\left[\frac{2x+a}{a^2+ax+x^2} - \frac{2x-a}{a^2-ax+x^2}\right].$$

(xi). $y = (1+x)^x \Rightarrow \ln y = x\ln(1+x)$

$$\therefore \frac{1}{y}\frac{dy}{dx} = x.\frac{1}{1+x} + 1.\ln(1+x)$$

$$\therefore \frac{dy}{dx} = y\left[\frac{x}{1+x} + \ln(1+x)\right].$$

4(i). $y = \sqrt{(x+1)}^{\ln\sqrt{(x+1)}}$

$$\Rightarrow \ln y = \ln\sqrt{(x+1)}.\ \ln\sqrt{(x+1)}$$

$$\Rightarrow \ln y = \frac{1}{2}\ln(x+1).\ \frac{1}{2}\ln(x+1) = \frac{1}{4}\{\ln(x+1)\}^2$$

$$\Rightarrow \frac{d}{dx}(\ln y) = \frac{1}{4}\frac{d}{dx}\{\ln(x+1)\}^2$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{4}.\ 2\ln(x+1).\ \frac{1}{x+1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}.\frac{\ln(x+1)}{(x+1)} = \frac{1}{2}\sqrt{(x+1)}^{\ln\sqrt{(x+1)}}\frac{\ln(x+1)}{(x+1)}.$$

(ii). $y = x^{\cos^{-1}x} \Rightarrow \ln y = \cos^{-1}x.\ \ln x$

$$\therefore \frac{d}{dx}(\ln y) = \frac{d}{dx}(\cos^{-1}x.\ \ln x)$$

বা $\dfrac{1}{y}\dfrac{dy}{dx} = \cos^{-1}x.\ \dfrac{1}{x} + \dfrac{(-1)}{\sqrt{(1-x^2)}}.\ \ln x$

বা $\dfrac{dy}{dx} = y\left[\dfrac{\cos^{-1}x}{x} - \dfrac{\ln x}{\sqrt{(1-x^2)}}\right] = x^{\cos^{-1}x}\left[\dfrac{\cos^{-1}x}{x} - \dfrac{\ln x}{\sqrt{(1-x^2)}}\right].$

(iii). $y = x^{\tan^{-1}x} \Rightarrow \ln y = \tan^{-1}x.\ \ln x$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{y}\frac{dy}{dx} = \frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}$$

$$\therefore \frac{dy}{dx} = y\left[\frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}\right] = x^{\tan^{-1}x}\left[\frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}\right].$$

(iv). $y = x^{(\ln x)^{\ln(\ln x)}}$

$$\Rightarrow \ln y = (\ln x)^{\ln(\ln x)}.\ \ln x$$

$$\Rightarrow \ln(\ln y) = \ln(\ln x).\ \ln(\ln x) + \ln(\ln x)$$

$$\Rightarrow \ln(\ln y) = \{\ln(\ln x)\}^2 + \ln(\ln x)$$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{lny}\cdot\frac{1}{y}\frac{dy}{dx} = 2ln(lnx).\ \frac{1}{lnx}\cdot\frac{1}{x} + \frac{1}{lnx}\cdot\frac{1}{x}$$

$$\therefore \frac{dy}{dx} = ylny\left[\frac{2ln(lnx)}{xlnx} + \frac{1}{xlnx}\right]$$

$$= x^{(lnx)^{ln(lnx)}}(lnx)^{ln(lnx)}\ lnx\left[\frac{2ln(lnx)}{xlnx} + \frac{1}{xlnx}\right]$$

$$= x^{(lnx)^{ln(lnx)}}(lnx)^{ln(lnx)}\ .\frac{1}{x}[2ln(lnx) + 1].$$

(v). $y = x^{2sinx} \Rightarrow lny = 2sinx.\ lnx$

$$\therefore \frac{1}{y}\frac{dy}{dx} = 2sinx.\ \frac{1}{x} + 2cosx.\ lnx$$

$$\therefore \frac{dy}{dx} = y\left[\frac{2sinx}{x} + 2cosx.\ lnx\right].$$

(vi). $y = (x^2 + 1)^{tan^{-1}x}\ ln(1 + sin^2x)$

$\Rightarrow lny = tan^{-1}x.\ ln(1 + x^2) + ln\{ln(1 + sin^2x)\}$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{y}\frac{dy}{dx} = tan^{-1}x.\ \frac{2x}{1 + x^2} + \frac{ln(1 + x^2)}{1 + x^2} + \frac{1}{ln(1 + sin^2x)}\cdot\frac{2sinx\ cosx}{(1 + sin^2x)}$$

$$\frac{dy}{dx} = y\left[\frac{2xtan^{-1}x}{1 + x^2} + \frac{ln(1 + x^2)}{1 + x^2} + \frac{sin2x}{(1 + sin^2x)\ ln(1 + sin^2x)}\right].$$

5(i). $y = (sin^{-1}x)^{lnx}$

$\Rightarrow lny = lnx.\ ln(sin^{-1}x)$

$$\therefore \frac{1}{y}\frac{dy}{dx} = lnx.\ \frac{d}{dx}\{ln(sin^{-1}x)\} + ln(sin^{-1}x).\ \frac{d}{dx}(lnx)$$

$$\text{বা } \frac{1}{y}\frac{dy}{dx} = lnx.\ \frac{1}{sin^{-1}x}\cdot\frac{1}{\sqrt{(1 - x^2)}} + ln(sin^{-1}x).\ \frac{1}{x}$$

$$\frac{dy}{dx} = y\left[\frac{lnx}{\sqrt{(1 - x^2)}.\ sin^{-1}x} + \frac{ln(sin^{-1}x)}{x}\right].$$

(ii). $y = (sinx)^{lnx} \Rightarrow lny = lnx.\ ln(sinx)$

$$\frac{d}{dx}(lny) = \frac{d}{dx}\{lnx.\ ln(sinx)\}$$

বা $\frac{1}{y}\frac{dy}{dx} = \ln x \frac{d}{dx}\ln(\sin x) + \ln(\sin x)\frac{d}{dx}(\ln x)$

বা $\frac{dy}{dx} = y\left[\ln x.\ \cot x + \frac{\ln(\sin x)}{x}\right]$.

(iii). $y = \tan x)^{\cot x} \Rightarrow \ln y = \cot x\ \ln(\tan x)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\frac{1}{y}\frac{dy}{dx} = \cot x.\ \frac{1}{\tan x}.\ \sec^2 x - \operatorname{cosec}^2 x.\ \ln(\tan x)$

$\therefore \frac{dy}{dx} = y[\cot^2 x\ \sec^2 x - \operatorname{cosec}^2 x.\ \ln(\tan x)]$.

(iv). $y = (\sin x)^{\cos x} \Rightarrow \ln y = \cos x.\ \ln(\sin x)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\frac{1}{y}\frac{dy}{dx} = \cos x.\ \frac{1}{\sin x}\cos x - \sin x.\ \ln(\sin x)$

বা $\frac{dy}{dx} = y[\cos x.\ \cot x - \sin x\ \ln(\sin x)]$.

(v). $y = (\sin x)^{\cos^{-1}x} \Rightarrow \ln y = \cos^{-1}x\ \ln(\sin x)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\frac{1}{y}\frac{dy}{dx} = \cos^{-1}x.\ \frac{1}{\sin x}\cos x - \frac{1}{\sqrt{(1-x^2)}}\ln(\sin x)$

$\therefore \frac{dy}{dx} = y\left[\cos^{-1}x.\ \cot x - \frac{\ln(\sin x)}{\sqrt{(1-x^2)}}\right]$.

(vi). $y = (\sin x)^{\tan x} \Rightarrow \ln y = \tan x\ \ln(\sin x)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\frac{1}{y}\frac{dy}{dx} = \tan x.\ \frac{1}{\sin x}.\ \cos x + \sec^2 x\ \ln(\sin x)$

$\therefore \frac{dy}{dx} = y[1 + \sec^2 x.\ \ln(\sin x)]$.

(vii). $y = \left\{\frac{1/\cos x - (\sin x)/\cos x}{1/\cos x + (\sin x)/\cos x}\right\}^{\cos x}$

বা $y = \left(\frac{1-\sin x}{1+\sin x}\right)^{\cos x}$

$\Rightarrow lny = \cos x[ln(1-\sin x) - ln(1+\sin x)]$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \cos x\left[\frac{-\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x}\right]$$

$$-\sin x[ln(1-\sin x) - ln(1+\sin x)]$$

বা $\frac{1}{y}\frac{dy}{dx} = -\cos^2 x\left[\frac{1}{1-\sin x} + \frac{1}{1+\sin x}\right] - \sin x.\ ln\frac{1-\sin x}{1+\sin x}$

বা $\frac{1}{y}\frac{dy}{dx} = -\cos^2 x.\ \frac{2}{(1-\sin^2 x)} - \sin x.\ ln\frac{1-\sin x}{1+\sin x}$

$$\therefore \frac{dy}{dx} = y\left[-2 - \sin x.\ ln\frac{1-\sin x}{1+\sin x}\right].$$

6(i). $y = x^x + x^{1/x}$

বা $y = u + v$, যখন $u = x^x$ এবং $v = x^{1/x}$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$

এখন $u = x^x \Rightarrow lnu = xlnx$

$\frac{1}{u}\frac{du}{dx} = x.\frac{1}{x} + 1.lnx$

$\therefore \frac{du}{dx} = u[1 + lnx] = x^x[1 + lnx]$.

এবং $v = x^{1/x} \Rightarrow lnv = \frac{1}{x}\ lnx$

$\therefore \frac{1}{v}\frac{dv}{dx} = \frac{1}{x}.\frac{1}{x} - \frac{1}{x^2}\ lnx$

$\therefore \frac{dv}{dx} = v.\ \frac{1}{x^2}\ [1 - lnx]$.

$(1) \Rightarrow \frac{dy}{dx} = x^x[1 + lnx] + x^{1/x}.\ \frac{1}{x^2}\ (1 - lnx)$.

(ii). $y = x^{lnx} + x^{\cos^{-1}x}$

বা $y = u + v$, যখন $u = x^{lnx}$ এবং $v = x^{\cos^{-1}x}$

$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$

এখন $u = x^{\ln x} \Rightarrow \ln u = \ln x . \ln x \Rightarrow \ln u = (\ln x)^2$

$$\therefore \frac{1}{u}\frac{du}{dx} = 2(\ln x)^{2-1} . \frac{1}{x} = \frac{2\ln x}{x}$$

$$\therefore \frac{du}{dx} = u . \frac{2\ln x}{x} = x^{\ln x} . \frac{2\ln x}{x}$$

এবং $v = x^{\cos^{-1}x} \Rightarrow \ln v = \cos^{-1}x . \ln x$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \frac{\cos^{-1}x}{x} - \frac{\ln x}{\sqrt{(1-x^2)}}$$

$$\therefore \frac{dv}{dx} = v\left[\frac{\cos^{-1}x}{x} - \frac{\ln x}{\sqrt{(1-x^2)}}\right]$$

$$(1) \Rightarrow \frac{dy}{dx} = x^{\ln x} . \frac{2\ln x}{x} + x^{\cos^{-1}x}\left[\frac{\cos^{-1}x}{x} - \frac{\ln x}{\sqrt{(1-x^2)}}\right].$$

(iii). $y = x^{\tan^{-1}x} + (\sin x)^{\ln x}$

বা $y = u + v$, যখন $u = x^{\tan^{-1}x}$ এবং $v = (\sin x)^{\ln x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$$

এখন $u = x^{\tan^{-1}x} \Rightarrow \ln u = \tan^{-1}x . \ln x$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{u}\frac{du}{dx} = \frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}$$

$$\text{বা } \frac{du}{dx} = u\left[\frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}\right] = x^{\tan^{-1}x}\left[\frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}\right]$$

এবং $v = (\sin x)^{\ln x} \Rightarrow \ln v = \ln x . \ln(\sin x)$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \ln x . \frac{\cos x}{\sin x} + \frac{\ln(\sin x)}{x}$$

$$\therefore \frac{dv}{dx} = v\left[\ln x . \cot x + \frac{\ln(\sin x)}{x}\right]$$

$$\therefore (1) \Rightarrow \frac{dy}{dx} = x^{\tan^{-1}x}\left[\frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}\right]$$

$$+ (\sin x)^{\ln x}\left[\ln x . \cot x + \frac{\ln(\sin x)}{x}\right]$$

(iv). $y = x^x + (\sin x)^{ln x}$

বা $y = u + v$ যখন $u = x^x$ এবং $v = (\sin x)^{ln x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \cdots (1)$$

এখন $u = x^x \Rightarrow ln u = x ln x$

$$\therefore \frac{1}{u}\frac{du}{dx} = x.\frac{1}{x} + 1.ln x$$

$$\therefore \frac{du}{dx} = u[1 + ln x] = x^x[1 + ln x]$$

এবং $v = (\sin x)^{ln x} \Rightarrow ln v = ln x.\ ln(\sin x)$

$$\therefore \frac{1}{v}\frac{dv}{dx} = ln x.\frac{\cos x}{\sin x} + \frac{1}{x} ln(\sin x)$$

$$\therefore \frac{dv}{dx} = v\left[ln x.\ \cot x + \frac{ln(\sin x)}{x}\right]$$

$$(1) \Rightarrow \frac{dv}{dx} = x^x[1 + ln x] + (\sin x)^{ln x}\left[\cot x.\ ln x + \frac{ln(\sin x)}{x}\right].$$

(v). $y = (1 + x^2)^{\tan x} + (2 - \sin x)^{ln x}$

বা $y = u + v$, যখন $u = (1 + x^2)^{\tan x}$ এবং $v = (2 - \sin x)^{ln x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \cdots (1)$$

এখন $u = (1 + x^2)^{\tan x} \Rightarrow ln u = \tan x.\ ln(1 + x^2)$

$$\therefore \frac{1}{u}\frac{du}{dx} = \tan x.\ \frac{2x}{1 + x^2} + \sec^2 x.\ ln(1 + x^2)$$

$$\therefore \frac{du}{dx} = u\left[\frac{2x\tan x}{1 + x^2} + \sec^2 x.\ ln(1 + x^2)\right]$$

এবং $v = (2 - \sin x)^{ln x} \Rightarrow ln v = ln x.\ ln(2 - \sin x)$

$$\therefore \frac{1}{v}\frac{dv}{dx} = ln x.\ \frac{(-\cos x)}{2 - \sin x} + \frac{1}{x} ln(2 - \sin x)$$

$$\therefore \frac{dv}{dx} = v\left[\frac{-ln x.\ \cos x}{2 - \sin x} + \frac{ln(2 - \sin x)}{x}\right]$$

$$(1) \Rightarrow \frac{dy}{dx} = (1 + x^2)^{\tan x}\left[\frac{2x\tan x}{1 + x^2} + \sec^2 x.\ ln(1 + x^2)\right]$$

$$+ (2 - \sin x)^{ln x}\left[\frac{ln(2 - \sin x)}{x} - \frac{\cos x.\ ln x}{2 - \sin x}\right].$$

(vi). $y = (\sin x)^{\ln x} + x^{\sin x}$

বা $y = u + v$, যখন $u = (\sin x)^{\ln x}$ এবং $v = x^{\sin x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$$

এখন $u = (\sin x)^{\ln x} \Rightarrow \ln u = \ln x. \ln(\sin x)$

$$\therefore \frac{1}{u}\frac{du}{dx} = \ln x. \frac{\cos x}{\sin x} + \frac{1}{x} \ln(\sin x)$$

$$\text{বা } \frac{du}{dx} = u\left[\ln x. \cot x + \frac{\ln(\sin x)}{x}\right]$$

এবং $v = x^{\sin x} \Rightarrow \ln v = \sin x. \ln x$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \frac{\sin x}{x} + \cos x. \ln x$$

$$\therefore \frac{dv}{dx} = v\left[\frac{\sin x}{x} + \cos x. \ln x\right]$$

$$\therefore (1) \Rightarrow \frac{dy}{dx} = (\sin x)^{\ln x}\left[\ln x. \cot x + \frac{\ln(\sin x)}{x}\right] + x^{\sin x}\left[\frac{\sin x}{x} + \cos x. \ln x\right].$$

(vii). $y = x^x + \sin x \ln x$

বা $y = u + v$, যখন $u = x^x$ এবং $v = \sin x \ln x$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$$

এখন $u = x^x \Rightarrow \ln u = x \ln x$

$$\therefore \frac{1}{u}\frac{du}{dx} = x. \frac{1}{x} + 1. \ln x$$

$$\text{বা } \frac{du}{dx} = u[1 + \ln x] = x^x[1 + \ln x]$$

এবং $v = \sin x \ln x$

$$\therefore \frac{dv}{dx} = \sin x \frac{d}{dx}(\ln x) + \ln x. \frac{d}{dx}(\sin x)$$

$$\text{বা } \frac{dv}{dx} = \frac{\sin x}{x} + \ln x \cos x$$

$$(1) \Rightarrow \frac{dy}{dx} = x^x(1 + \ln x) + \frac{\sin x}{x} + \ln x \cos x.$$

(viii). $y = (\tan x)^{\cot x} + (\sin x)^{\ln x}$

বা $y = u + v$ যখন $u = (\tan x)^{\cot x}$ এবং $v = (\sin x)^{\ln x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$$

এখন $u = (\tan x)^{\cot x} \Rightarrow \ln u = \cot x \ln(\tan x)$

$$\therefore \frac{1}{u}\frac{du}{dx} = \cot x . \frac{\sec^2 x}{\tan x} - \operatorname{cosec}^2 x . \ln(\tan x)$$

$$\therefore \frac{du}{dx} = u[\cot^2 x . \sec^2 x - \operatorname{cosec}^2 x . \ln(\tan x)]$$

$$= (\tan x)^{\cot x} [\operatorname{cosec}^2 x - \operatorname{cosec}^2 x \ln(\tan x)]$$

এবং $v = (\sin x)^{\ln x} \Rightarrow \ln v = \ln x . \ln(\sin x)$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \ln x . \frac{\cos x}{\sin x} + \frac{1}{x} \ln(\sin x)$$

$$\therefore \frac{dv}{dx} = v\left[\ln x . \cot x + \frac{\ln(\sin x)}{x}\right].$$

$$(1) \Rightarrow \frac{dy}{dx} = (\tan x)^{\cot x} [\operatorname{cosec}^2 x - \operatorname{cosec}^2 x . \ln(\tan x)]$$

$$+ (\sin x)^{\ln x} [\ln x \cot x + (1/x) . \ln(\sin x)].$$

(ix). $y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$

বা $y = u + v$, যখন $u = (\tan x)^{\cot x}$ এবং $v = (\cot x)^{\tan x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$$

এখন $u = (\tan x)^{\cot x} \Rightarrow \ln u = \cot x . \ln(\tan x)$

$$\therefore \frac{1}{u}\frac{du}{dx} = \cot x . \frac{\sec^2 x}{\tan x} - \operatorname{cosec}^2 x . \ln(\tan x)$$

বা $$\frac{du}{dx} = u[\cot^2 x . \sec^2 x - \operatorname{cosec}^2 x . \ln(\tan x)]$$

$$= (\tan x)^{\cot x} [\operatorname{cosec}^2 x - \operatorname{cosec}^2 x . \ln(\tan x)]$$

এবং $v = (\cot x)^{\tan x} \Rightarrow \ln v = \tan x . \ln(\cot x)$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \tan x . \frac{1}{\cot x} (-\operatorname{cosec}^2 x) + \sec^2 x . \ln(\cot x)$$

$$\therefore \frac{dv}{dx} = v[-\sec^2 x + \sec^2 x . \ln(\cot x)]$$

$$(1) \Rightarrow \frac{dy}{dx} = (\tan x)^{\cot x} . \operatorname{cosec}^2 x [1 - \ln(\tan x)]$$

$$+ (\cot x)^{\tan x} . \sec^2 x [\ln(\cot x) - 1].$$

(x). $y = (\sin x)^{\cos x} + (\tan x)^{\sin x}$

বা $y = u + v$, যখন $u = (\sin x)^{\cos x}$ এবং $v = (\tan x)^{\sin x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$$

এখন $u = (\sin x)^{\cos x} \Rightarrow \ln u = \cos x.\ \ln(\sin x)$

$$\therefore \frac{1}{u}\frac{du}{dx} = \cos x.\frac{\cos x}{\sin x} - \sin x.\ \ln(\sin x)$$

বা $\dfrac{du}{dx} = u[\cos x.\ \cot x - \sin x.\ \ln(\sin x)]$

এবং $v = (\tan x)^{\sin x} \Rightarrow \ln v = \sin x.\ \ln(\tan x)$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \sin x.\frac{\sec^2 x}{\tan x} + \cos x.\ \ln(\tan x)$$

বা $\dfrac{dv}{dx} = v[\sec x + \cos x\ \ln(\tan x)]$

$$(1) \Rightarrow \frac{dy}{dx} = (\sin x)^{\cos x}[\cos x\ \cot x - \sin x.\ \ln(\sin x)]$$
$$+ (\tan x)^{\sin x}[\sec x + \cos x.\ \ln(\tan x)]$$

(xi). $y = (\tan x)^{\sec x} + (\cot x)^{\operatorname{cosec} x}$

বা $y = u + v$, যখন $u = (\tan x)^{\sec x}$ এবং $v = (\cot x)^{\operatorname{cosec} x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$$

এখন $u = (\tan x)^{\sec x} \Rightarrow \ln u = \sec x.\ \ln(\tan x)$

$$\therefore \frac{1}{u}\frac{du}{dx} = \sec x.\frac{\sec^2 x}{\tan x} + \sec x.\ \tan x.\ \ln(\tan x)$$

বা $\dfrac{du}{dx} = u[\cot x.\ \sec^3 x + \sec x\ \tan x.\ \ln(\tan x)]$

এবং $v = (\cot x)^{\operatorname{cosec} x} \Rightarrow \ln v = \operatorname{cosec} x.\ \ln(\cot x)$

$$\therefore \frac{1}{v}\frac{dv}{dx} = \operatorname{cosec} x.\frac{(-\operatorname{cosec}^2 x)}{\cot x} - \operatorname{cosec} x.\ \cot x.\ \ln(\cot x)$$

বা $\dfrac{dv}{dx} = v[-\tan x\ \operatorname{cosec}^3 x - \operatorname{cosec} x\ \cot x.\ \ln(\cot x)]$

$$\therefore (1) \Rightarrow \frac{dy}{dx} = (\tan x)^{\sec x}[\cot x.\ \sec^3 x + \sec x\ \tan x.\ \ln(\tan x)]$$
$$- (\cot x)^{\operatorname{cosec} x}[\tan x.\ \operatorname{cosec}^3 x + \operatorname{cosec} x.\ \cot x.\ \ln(\cot x)].$$

(xii). $y = (\sin x)^{\ln x} + \sin^2(\cos^{-1}x)$

বা $y = u + v$, যখন $u = (\sin x)^{\ln x}$ এবং $v = \sin^2(\cos^{-1}x)$

$\therefore \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx} \cdots (1)$

এখন $u = (\sin x)^{\ln x} \Rightarrow \ln u = \ln x.\ \ln(\sin x)$

$\therefore \dfrac{1}{u}\dfrac{du}{dx} = \ln x.\ \dfrac{\cos x}{\sin x} + \dfrac{1}{x}\ln(\sin x)$

বা $\dfrac{du}{dx} = u\left[\ln x.\ \cot x + \dfrac{1}{x}\ln(\sin x)\right]$

এবং $v = \sin^2(\cos^{-1}x)$

$\therefore \dfrac{dv}{dx} = 2\sin(\cos^{-1}x).\ \cos(\cos^{-1}x).\ \dfrac{(-1)}{\sqrt{(1-x^2)}}$

$= -2\sin(\cos^{-1}x).\ x/\sqrt{(1-x^2)}$

$(1) \Rightarrow \dfrac{dy}{dx} = (\sin x)^{\ln x}\left[\ln x.\ \cot x + \dfrac{\ln(\sin x)}{x}\right] - \dfrac{2x\sin(\cos^{-1}x)}{\sqrt{(1-x^2)}}.$

(xiii). $y = x^{\cot^{-1}x} + (\cos^{-1}x)^{\tan x}$

$\therefore y = u + v$ যখন $u = x^{\cot^{-1}x}$ এবং $v = (\cos^{-1}x)^{\tan x}$

$\therefore \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx} \cdots (1)$

এখন $u = x^{\cot^{-1}x} \Rightarrow \ln u = \cot^{-1}x \ln x$

$\therefore \dfrac{1}{u}\dfrac{du}{dx} = \dfrac{\cot^{-1}x}{x} - \dfrac{\ln x}{1+x^2}$

$\therefore \dfrac{du}{dx} = u\left[\dfrac{\cot^{-1}x}{x} - \dfrac{\ln x}{1+x^2}\right]$

এবং $v = (\cos^{-1}x)^{\tan x} \Rightarrow \ln v = \tan x \ln(\cos^{-1}x)$

$\therefore \dfrac{1}{v}\dfrac{dv}{dx} = \dfrac{\tan x}{\cos^{-1}x} \cdot \dfrac{(-1)}{\sqrt{1-x^2}} + \sec^2 x \cdot \ln(\cos^{-1}x)$

$\therefore \dfrac{dv}{dx} = v\left[\dfrac{-\tan x}{\sqrt{1-x^2} \cdot \cos^{-1}x} + \sec^2 x \ln(\cos^{-1}x)\right]$

$\therefore (1) \Rightarrow \dfrac{dy}{dx} = x^{\cot^{-1}x}\left[\dfrac{\cot^{-1}x}{x} - \dfrac{\ln x}{1+x^2}\right]$

$+ (\cos^{-1}x)^{\tan x}\left[\dfrac{-\tan x}{\sqrt{1-x^2} \cdot \cos^{-1}x} + \sec^2 x \ln(\cos^{-1}x)\right].$

(xiv). $y = (x^2 + 1)^{\sqrt{(1-x^2)}} + (\sin^{-1}x)^2$

বা $y = u + v$, যখন $u = (x^2 + 1)^{\sqrt{(1-x^2)}}$ এবং $v = (\sin^{-1}x)^2$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \cdots (1)$$

এখন $u = (x^2 + 1)^{\sqrt{(1-x^2)}} \Rightarrow \ln u = \sqrt{(1 - x^2)}\ \ln(x^2 + 1)$

$$\therefore \frac{1}{u}\frac{du}{dx} = \sqrt{(1 - x^2)}.\ \frac{2x}{x^2 + 1} + \frac{(-2x)}{2\sqrt{(1 - x^2)}}\ln(x^2 + 1)$$

$$\therefore \frac{du}{dx} = u\left[\frac{2x\sqrt{(1 - x^2)}}{x^2 + 1} - \frac{x\ln(x^2 + 1)}{\sqrt{(1 - x^2)}}\right]$$

এবং $v = (\sin^{-1}x)^2$

$$\therefore \frac{dv}{dx} = 2(\sin^{-1}x)^{2-1}\cdot\frac{1}{\sqrt{(1 - x^2)}} = \frac{2\sin^{-1}x}{\sqrt{(1 - x^2)}}$$

$$(1) \Rightarrow \frac{dy}{dx} = (x^2 + 1)^{\sqrt{(1-x^2)}}\left[\frac{2x\sqrt{(1 - x^2)}}{x^2 + 1} - \frac{x\ln(x^2 + 1)}{\sqrt{(1 - x^2)}}\right] + \frac{2\sin^{-1}x}{\sqrt{(1 - x^2)}}$$

7(i). $y = a^{xy} \Rightarrow \ln y = xy.\ \ln a$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করয়া পাই

$$\frac{1}{y}\frac{dy}{dx} = \ln a\left[x.\ \frac{dy}{dx} + 1.y\right]$$

$$\text{বা}\left[\frac{1}{y} - x\ln a\right]\frac{dy}{dx} = y\ln a$$

$$\text{বা}\ \frac{(1 - xy\ln a)}{y}\frac{dy}{dx} = y\ln a$$

$$\therefore \frac{dy}{dx} = \frac{y^2\ \ln a}{1 - xy\ \ln a}.$$

(ii). $x^y = e^{x-y} \Rightarrow y\ln x = x - y$

বা $y(1 + \ln x) = x$, বা $y = \dfrac{x}{1 + \ln x}$

$$\therefore \frac{dy}{dx} = \frac{(1 + \ln x).\ 1 - x.(1/x)}{(1 + \ln x)^2} = \frac{\ln x}{(1 + \ln x)^2}$$

(iii). $y = x^{y^x} \Rightarrow \ln y = y^x\ \ln x$

$\Rightarrow \ln(\ln y) = \ln y^x + \ln(\ln x)$

বা $\ln(\ln y) = x\ln y + \ln(\ln x)$

$$\frac{d}{dx}[ln(lny)] = \frac{d}{dx}(xlny) + \frac{d}{dx}\{ln(lnx)\}$$

বা $\frac{1}{lny}\cdot\frac{1}{y}\frac{dy}{dx} = x\frac{1}{y}\frac{dy}{dx} + 1.lny + \frac{1}{lnx}\cdot\frac{1}{x}$

বা $\left(\frac{1}{ylny} - \frac{x}{y}\right)\frac{dy}{dx} = lny + \frac{1}{xlnx}$

বা $\frac{(1 - xlny)}{ylny}\cdot\frac{dy}{dx} = \frac{(xlnx\, lny + 1)}{xlnx}$

$\therefore \frac{dy}{dx} = \frac{ylny(1 + xlnx.\, lny)}{xlnx(1 - xlny)}$.

(iv). $x^y y^x = 1 \Rightarrow lnx^y + lny^x = ln1$

বা $ylnx + xlny = 0$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$y.\frac{1}{x} + lnx.\frac{dy}{dx} + x.\frac{1}{y}\frac{dy}{dx} + 1.lny = 0$

বা $\left[lnx + \frac{x}{y}\right]\frac{dy}{dx} = -\left[\frac{y}{x} + lny\right]$

বা $\frac{(ylnx + x)}{y}\frac{dy}{dx} = \frac{-(y + xlny)}{x}$

$\therefore \frac{dy}{dx} = \frac{-y(y + xlny)}{x(x + ylnx)}$.

(v). $y^n = \frac{(x + y)}{(x - y)}$

$\Rightarrow nlny = ln(x + y) - ln(x - y)$

$\frac{n}{y}\frac{dy}{dx} = \frac{1}{(x + y)}\left[1 + \frac{dy}{dx}\right] - \frac{1}{(x - y)}\left[1 - \frac{dy}{dx}\right]$

বা $\left[\frac{n}{y} - \frac{1}{x + y} - \frac{1}{x - y}\right]\frac{dy}{dx} = \frac{1}{x + y} - \frac{1}{x - y}$

বা $\frac{n(x^2 - y^2) - xy + y^2 - xy - y^2}{y(x + y)(x - y)}\cdot\frac{dy}{dx} = \frac{x - y - x - y}{(x + y)(x - y)}$

বা $\frac{n(x^2 - y^2) - 2xy}{y}\cdot\frac{dy}{dx} = -2y$

$\therefore \frac{dy}{dx} = \frac{2y^2}{2xy - n(x^2 - y^2)}$.

(vi). $x^p y^q = (x+y)^{p+q} \Rightarrow p\ln x + q\ln y = (p+q)\ln(x+y)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{(p+q)}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$

বা $\left(\frac{q}{y} - \frac{p+q}{x+y}\right)\frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$

বা $\left[\frac{qx + qy - py - qy}{y(x+y)}\right]\frac{dy}{dx} = \frac{px + qx - px - py}{x(x+y)}$

বা $\frac{(qx - py)}{y}\frac{dy}{dx} = \frac{(qx - py)}{x}$

বা $\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$;

(vii). $(x+y)^{m+n} = x^m y^n \Rightarrow (m+n)\ln(x+y) = m\ln x + n\ln y$

$$\Rightarrow \frac{(m+n)}{(x+y)}\left[1 + \frac{dy}{dx}\right] = \frac{m}{x} + \frac{n}{y}\frac{dy}{dx}$$

বা $\left[\frac{(m+n)}{x+y} - \frac{n}{y}\right]\frac{dy}{dx} = \frac{m}{x} - \frac{m+n}{x+y}$

বা $\frac{my + ny - nx - ny}{y(x+y)} \cdot \frac{dy}{dx} = \frac{mx + my - mx - nx}{x(x+y)}$

বা $\frac{(my - nx)}{y} \cdot \frac{dy}{dx} = \frac{(my - nx)}{x}$

বা $\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$.

(viii). $(\cos x)^{x+y} = \sin^3 x \Rightarrow (x+y)\ln(\cos x) = 3\ln(\sin x)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$(x+y)\frac{(-\sin x)}{\cos x} + \ln(\cos x)\left[1 + \frac{dy}{dx}\right] = \frac{3\cos x}{\sin x}$$

বা $-(x+y)\tan x + \ln(\cos x) + \ln(\cos x)\frac{dy}{dx} = 3\cot x$

বা $\ln(\cos x)\frac{dy}{dx} = 3\cot x + (x+y)\tan x - \ln(\cos x)$

$$\therefore \frac{dy}{dx} = \frac{3\cot x + (x+y)\tan x - \ln(\cos x)}{\ln(\cos x)}.$$

(ix). $(\sec x)^y = (\tan y)^x \Rightarrow y\,ln(\sec x) = x\,ln(\tan y)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\therefore y.\frac{\sec x\tan x}{\sec x} + ln(\sec x)\frac{dy}{dx} = x.\frac{\sec^2 y}{\tan y}\frac{dy}{dx} + 1.ln(\tan y)$$

বা $[ln(\sec x) - x\sec^2 y.\cot y]\frac{dy}{dx} = ln(\tan y) - y\tan x$

$$\therefore \frac{dy}{dx} = \frac{ln(\tan y) - y\tan x}{ln(\sec x) - x\sec^2 y\cot y}.$$

8(i). $x^y + y^x = a^b$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{d}{dx}(x^y) + \frac{d}{dx}(y^x) = 0$$

বা $\left\{yx^{y-1}\frac{dx}{dx} + x^y\,lnx\frac{dy}{dx}\right\} + \left\{xy^{x-1}\frac{dy}{dx} + y^x\,lny\frac{dx}{dx}\right\} = 0$

বা $[x^y\,lnx + xy^{x-1}]\frac{dy}{dx} = -[yx^{y-1} + y^x\,lny]$

$$\therefore \frac{dy}{dx} = \frac{-[yx^{y-1} + y^x\,lny]}{x^y\,lnx + xy^{x-1}}.$$

(ii). $(\sin x)^{\cos y} + (\cos x)^{\sin y} = a$

$$\therefore \frac{d}{dx}(\sin x)^{\cos y} + \frac{d}{dx}(\cos x)^{\sin y} = 0 \cdots (1)$$

এখন $\frac{d}{dx}(\sin x)^{\cos y} = \cos y(\sin x)^{\cos y-1}\frac{d}{dx}(\sin x)$

$$+ (\sin x)^{\cos y}\,ln(\sin x)\frac{d}{dx}(\cos y)$$

$= \cos y.\,(\sin x)^{\cos y}(\sin x)^{-1}\cos x + (\sin x)^{\cos y}\,ln(\sin x)\,(-\sin y)\frac{dy}{dx}$

এবং $\frac{d}{dx}(\cos x)^{\sin y} = \sin y(\cos x)^{\sin y-1}\frac{d}{dx}(\cos x)$

$$(\cos x)^{\sin y}\,ln(\cos x)\frac{d}{dx}(\sin y)$$

$= (\sin y).\,(\cos x)^{\sin y}(\cos x)^{-1}(-\sin x) + (\cos x)^{\sin y}\,ln(\cos x).\,\cos y\frac{dy}{dx}$

$\therefore (1) \Rightarrow \cos y(\sin x)^{\cos y}\cot x - (\sin x)^{\cos y}\,ln(\sin x).\,\sin y\frac{dy}{dx}$

$$- \sin y(\cos x)^{\sin y}\tan x + (\cos x)^{\sin y}\,ln(\cos x)\cos y\frac{dy}{dx} = 0$$

বা $[(\cos x)^{\sin y} \ln(\cos x).\ \cos y - (\sin x)^{\cos y} \ln(\sin x).\ \sin y] \dfrac{dy}{dx}$

$= (\cos x)^{\sin y} \sin y \tan x - (\sin x)^{\cos y} \cos y \cot x$

$$\therefore \frac{dy}{dx} = \frac{(\cos x)^{\sin y} \sin y \tan x - (\sin x)^{\cos y} \cos y \cot x}{(\cos y)^{\sin y} \ln(\cos x).\ \cos y - (\sin x)^{\cos y} \ln(\sin x) \sin y}.$$

9(i). $x^2 + y^2 - 3axy = 0$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$2x + 2y\frac{dy}{dx} - 3a\left[x\frac{dy}{dx} + 1.y\right] = 0$$

বা $(2y - 3ax)\dfrac{dy}{dx} = 3ay - 2x \Rightarrow \dfrac{dy}{dx} = \dfrac{3ay - 2x}{2y - 3ax}.$

(ii). $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$2ax + 2h\left[x\frac{dy}{dx} + 1.y\right] + 2by\frac{dy}{dx} + 2g + 2f\frac{dy}{dx} = 0$$

বা $[hx + by + f]\dfrac{dy}{dx} = -[ax + hy + g]$

$$\therefore \frac{dy}{dx} = \frac{-[ax + hy + g]}{hx + by + f}.$$

(iii). $x^{2/3} + y^{2/3} = a^{2/3}$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

বা $y^{-1/3}\dfrac{dy}{dx} = -x^{-1/3}$

$$\therefore \frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}}.$$

(iv). $x^n + y^n = a^n$ কে x এর সাপেক্ষে অন্তরীকরণ করি

$$nx^{n-1} + ny^{n-1}\frac{dy}{dx} = 0$$

বা $y^{n-1}\dfrac{dy}{dx} = -x^{n-1}$

$$\therefore \frac{dy}{dx} = -\frac{x^{n-1}}{y^{n-1}} = -\left(\frac{x}{y}\right)^{n-1}.$$

(v). $x^2y + y^2x + \sqrt{(xy)} = 1$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$x^2\frac{dy}{dx} + 2xy + 2y.x\frac{dy}{dx} + y^2.1 + \frac{1}{2\sqrt{(xy)}}\left[x\frac{dy}{dx} + 1.y\right] = 0$$

বা $\left[x^2 + 2xy + \frac{x}{2\sqrt{(xy)}}\right]\frac{dy}{dx} = -\left[2xy + y^2 + \frac{y}{2\sqrt{(xy)}}\right]$

$\therefore \frac{dy}{dx} = \frac{-[2xy + y^2 + \sqrt{y}/2\sqrt{x}]}{x^2 + 2xy + \sqrt{x}/2\sqrt{y}}$.

(vi). $\frac{\sqrt{x}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}} = 1$, বা $\frac{x + y}{\sqrt{y}\,\sqrt{x}} = 1$

বা $x + y = \sqrt{x}\,\sqrt{y}$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$1 + \frac{dy}{dx} = \frac{\sqrt{x}}{2\sqrt{y}}\frac{dy}{dx} + \frac{1}{2\sqrt{x}}.\sqrt{y}$$

বা $\left[1 - \frac{\sqrt{x}}{2\sqrt{y}}\right]\frac{dy}{dx} = \frac{\sqrt{y}}{2\sqrt{x}} - 1$

বা $\left[\frac{2\sqrt{y} - \sqrt{x}}{2\sqrt{y}}\right]\frac{dy}{dx} = \frac{\sqrt{y} - 2\sqrt{x}}{2\sqrt{x}} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{y}(\sqrt{y} - 2\sqrt{x})}{\sqrt{x}(2\sqrt{y} - \sqrt{x})}$.

(vii). $y = \sin^{-1}\left(\frac{y}{x}\right)$ বা $\sin y = \frac{y}{x}$

বা $y = x\sin y$

$\therefore \frac{dy}{dx} = 1.\sin y + x\cos y.\frac{dy}{dx}$

বা $(1 - x\cos y)\frac{dy}{dx} = \sin y \Rightarrow \frac{dy}{dx} = \frac{\sin y}{1 - x\cos y}$.

(viii). $y = \sin(x + y)^2$

$\therefore \frac{dy}{dx} = \cos(x + y)^2.\frac{d}{dx}(x + y)^2$

বা $\frac{dy}{dx} = \cos(x + y)^2.\,2(x + y)\left[1 + \frac{dy}{dx}\right]$

বা $[1 - 2(x + y)\cos(x + y)^2]\frac{dy}{dx} = 2(x + y)\cos(x + y)^2$

$\therefore \frac{dy}{dx} = \frac{2(x + y)\cos(x + y)^2}{1 - 2(x + y)\cos(x + y)^2}$.

(ix). $y = x\ln\left\{\frac{y}{(a + bx)}\right\}$

বা $y = x[\ln y - \ln(a + bx)]$

$\therefore \frac{dy}{dx} = x\left[\frac{1}{y}\frac{dy}{dx} - \frac{b}{a + bx}\right] + 1.[\ln y - \ln(a + bx)]$

বা $\left[1 - \frac{x}{y}\right]\frac{dy}{dx} = \ln y - \ln(a + bx) - \frac{bx}{a + bx}$

বা $\frac{y - x}{y}\frac{dy}{dx} = \ln\frac{y}{a + bx} - \frac{bx}{a + bx}$

$\therefore \frac{dy}{dx} = \frac{y}{y - x}\left[\ln\frac{y}{a + bx} - \frac{bx}{a + bx}\right].$

(x). $x\cos y = \sin(x + y)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$1.\cos y - x\sin y\frac{dy}{dx} = \cos(x + y)\left[1 + \frac{dy}{dx}\right]$

বা $\cos y - \cos(x + y) = [x\sin y + \cos(x + y)]\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos y - \cos(x + y)}{x\sin y + \cos(x + y)}.$

(xi). $\sin y = x\sin(a + y) \cdots (1)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$\cos y\frac{dy}{dx} = x\cos(a + y)\frac{dy}{dx} + 1.\sin(a + y)$

বা $[\cos y - x\cos(a + y)]\frac{dy}{dx} = \sin(a + y)$

বা $\frac{dy}{dx} = \frac{\sin(a + y)}{\cos y - x\cos(a + y)}$

$= \frac{\sin(a + y)}{\cos y - \frac{\sin y}{\sin(a + y)}\cos(a + y)}$; (1) নং দ্বারা

$= \frac{\sin^2(a + y)}{\sin(a + y)\cos y - \cos(a + y)\sin y}$

$= \frac{\sin^2(a + y)}{\sin a}.$

(xii). $\sin(x + y) = a$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\cos(x + y)\left[1 + \frac{dy}{dx}\right] = 0$$

$\Rightarrow 1 + \frac{dy}{dx} = 0$ যেহেতু $\cos(x + y) \neq 0$

$\therefore \frac{dy}{dx} = -1.$

(xiii). $y = \sin^{-1}\left(\frac{ax}{y}\right)$, বা $\sin y = \frac{ax}{y}$

বা $ax = y \sin y \cdots (1)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$a = y\cos y \frac{dy}{dx} + \sin y . \frac{dy}{dx}$$

বা $a = [y\cos y + \sin y] \frac{dy}{dx}$

বা $\frac{dy}{dx} = \frac{a}{y\cos y + \sin y} = \frac{a}{y\sqrt{(1 - \sin^2 y)} + \sin y}$

$$= \frac{a}{y\sqrt{(1 - a^2x^2/y^2)} + ax/y} = \frac{ay}{y\sqrt{(y^2 - a^2x^2)} + ax}.$$

(xiv). $y = x\ln(xy)$

বা $y = x[\ln x + \ln y]$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{dy}{dx} = x\left[\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}\right] + 1. [\ln x + \ln y]$$

বা $\left[1 - \frac{x}{y}\right]\frac{dy}{dx} = 1 + \ln x + \ln y$

বা $\frac{(y - x)}{y}\frac{dy}{dx} = 1 + \ln x + \ln y$

$\therefore \frac{dy}{dx} = \frac{y(1 + \ln x + \ln y)}{y - x}.$

(xv). $y = x\ln y$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{dy}{dx} = x.\frac{1}{y}\frac{dy}{dx} + 1.\ln y$$

বা $\left[1 - \frac{x}{y}\right]\frac{dy}{dx} = \ln y$

বা $\frac{(y - x)}{y}\frac{dy}{dx} = \ln y \Rightarrow \frac{dy}{dx} = \frac{y\ln y}{y - x}$.

(xvi). $x^2 + y^2 = \sin(xy)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$2x + 2y\frac{dy}{dx} = \cos(xy).\left[x\frac{dy}{dx} + 1.y\right]$$

বা $[2y - x\cos(xy)]\frac{dy}{dx} = y\cos(xy) - 2x$

$\therefore \frac{dy}{dx} = \frac{y\cos(xy) - 2x}{2y - x\cos(xy)} = -\frac{y\cos(xy) - 2x}{x\cos(xy) - 2y}$.

(xvii). $\tan^{-1}y.\ \ln x + \cos y + 3x = 9$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{\tan^{-1}y}{x} + \frac{\ln x}{(1 + y^2)}\frac{dy}{dx} - \sin y\frac{dy}{dx} + 3 = 0$$

বা $\left[\frac{\ln x}{1 + y^2} - \sin y\right]\frac{dy}{dx} = -\left[\frac{\tan^{-1}y}{x} + 3\right]$

বা $\frac{\ln x - (1 + y^2)\sin y}{1 + y^2}.\frac{dy}{dx} = -\frac{\tan^{-1}y + 3x}{x}$

$\therefore \frac{dy}{dx} = \frac{-(1 + y^2)(\tan^{-1}y + 3x)}{x[\ln x - (1 + y^2)\sin y]}$.

------ ------x------------