প্রশ্নমালা–5(C)

1a(i). ধরি $y = \sin^{-1}\sqrt{(1-x^2)}$ এবং $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

বা $y = \sin^{-1}\sqrt{(1-\cos^2\theta)} = \sin^{-1}(\sin\theta) = \theta$

বা $y = \cos^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{-1}{\sqrt{(1-x^2)}}$.

(ii). ধরি $y = \sin^{-1}\{2ax\sqrt{(1-x^2a^2)}\}$ এবং $ax = \sin\theta \Rightarrow \theta = \sin^{-1}ax$

বা $y = \sin^{-1}\{2\sin\theta\sqrt{(1-\sin^2\theta)}\} = \sin^{-1}(2\sin\theta\cos\theta)$

বা $y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}ax$

$\therefore \dfrac{dy}{dx} = \dfrac{d}{dx}(2\sin^{-1}ax) = \dfrac{2a}{\sqrt{(1-a^2x^2)}}$.

(iii). ধরি $y = \sin(\cos^{-1}\sqrt{(1-x^2)})$ এবং $x = \sin\theta$

বা $y = \sin\{\cos^{-1}\sqrt{(1-\sin^2\theta)}\} = \sin(\cos^{-1}\cos\theta) = \sin\theta$

বা $y = x \Rightarrow \dfrac{dy}{dx} = 1$.

(iv). ধরি $y = \sin^{-1}\{-2x(1-x^2)\}$ এবং $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

বা $y = \sin^{-1}\{-2\sin\theta\sqrt{(1-\sin^2\theta)}\} = \sin^{-1}(-2\sin\theta\cos\theta)$

বা $y = \sin^{-1}(-\sin 2\theta) = -\sin^{-1}(\sin 2\theta) = -2\theta = -2\sin^{-1}x$

$\therefore \dfrac{dy}{dx} = \dfrac{d}{dx}(-2\sin^{-1}x) = \dfrac{-2}{\sqrt{(1-x^2)}}$.

(v). ধরি $y = \sin^{-1}(3x-4x^3)$ এবং $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

বা $y = \sin^{-1}(3\sin\theta - 4\sin^3\theta) = \sin^{-1}(\sin 3\theta) = 3\theta$

বা $y = 3\sin^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{3}{\sqrt{(1-x^2)}}$.

(vi). ধরি $y = \sin^{-1}\sqrt{\dfrac{x^2}{a^2+x^2}}$ এবং $x = a\tan\theta \Rightarrow \theta = \tan^{-1}\dfrac{x}{a}$

বা $y = \sin^{-1}\sqrt{\dfrac{a^2\tan^2\theta}{a^2(1+\tan^2\theta)}} = \sin^{-1}\left[\dfrac{\tan\theta}{\sec\theta}\right] = \sin^{-1}\left[\dfrac{\sin\theta/\cos\theta}{1/\cos\theta}\right]$

বা $y = \sin^{-1}(\sin\theta) = \theta = \tan^{-1}\dfrac{x}{a}$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\tan^{-1}\dfrac{x}{a}\right) = \dfrac{1}{1+x^2/a^2}\cdot\dfrac{1}{a} = \dfrac{a}{a^2+x^2}$.

(vii). $y = \sin^{-1}\dfrac{x}{\sqrt{1+x^2}} \cdots (1)$

ধরি $x = \tan\theta$ তবে $\theta = \tan^{-1}x$

$(1) \Rightarrow y = \sin^{-1}\dfrac{\tan\theta}{\sqrt{\sec^2\theta}} = \sin^{-1}\dfrac{\tan\theta}{\sec\theta} = \sin^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\cos\theta\right)$

বা $y = \sin^{-1}(\sin\theta) = \theta$

বা $y = \tan^{-1}x$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{1+x^2}.$

(viii). ধরি $y = \cos^{-1}\{2x\sqrt{(1-x^2)}$ এবং $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

বা $y = \cos^{-1}\{2\cos\theta\sqrt{(1-\cos^2\theta)}\} = \cos^{-1}(2\cos\theta.\sin\theta)$

বা $y = \cos^{-1}(\sin2\theta) = \cos^{-1}[\cos(\pi/2 - 2\theta)] = \pi/2 - 2\theta$

বা $y = \dfrac{\pi}{2} - 2\cos^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{2}{\sqrt{(1-x^2)}}.$

(ix). ধরি $y = \cos^{-1}(1-2x^2)$ এবং $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

বা $y = \cos^{-1}(1-2\sin^2\theta) = \cos^{-1}(\cos2\theta) = 2\theta = 2\sin^{-1}x$

$\therefore \dfrac{dy}{dx} = \dfrac{d}{dx}(2\sin^{-1}x) = \dfrac{2}{\sqrt{(1-x^2)}}.$

(x). ধরি $y = \tan^{-1}\dfrac{x}{\sqrt{(1-x^2)}}$ এবং $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

বা $y = \tan^{-1}\dfrac{\sin\theta}{\sqrt{(1-\sin^2\theta)}} = \tan^{-1}\dfrac{\sin\theta}{\cos\theta} = \tan^{-1}\tan\theta = \theta$

বা $y = \sin^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{(1-x^2)}}.$

(xi). ধরি $y = \tan^{-1}\left[\dfrac{x - \sqrt{(a^2-x^2)}}{x + \sqrt{(a^2-x^2)}}\right]$ এবং $x = a\cos\theta \Rightarrow \theta = \cos^{-1}\dfrac{x}{a}$

বা $y = \tan^{-1}\left[\dfrac{a\cos\theta - \sqrt{\{a^2(1-\cos^2\theta)\}}}{a\cos\theta + \sqrt{\{a^2(1-\cos^2\theta)\}}}\right]$

বা $y = \tan^{-1}\left[\dfrac{a\cos\theta - a\sin\theta}{a\cos\theta + a\sin\theta}\right]$

বা $y = \text{tna}^{-1}\left[\dfrac{a\cos\theta(1-\tan\theta)}{a\cos\theta(1+\tan\theta)}\right] = \tan^{-1}\left[\dfrac{\tan(\pi/4) - \tan\theta}{1 + \tan(\pi/4)\tan\theta}\right]$

বা $y = \tan^{-1}\tan\left(\frac{\pi}{4} - \theta\right) = \frac{\pi}{4} - \theta = \frac{\pi}{4} - \cos^{-1}\frac{x}{a}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\pi}{4} - \cos^{-1}\frac{x}{a}\right] = -\frac{(-1)}{\sqrt{(1 - x^2/a^2)}} \cdot \frac{1}{a} = \frac{1}{\sqrt{(a^2 - x^2)}}.$$

(xii). ধরি $y = \tan^{-1}\frac{1}{\sqrt{(x^2 - 1)}}$ এবং $x = \sec\theta \Rightarrow \theta = \sec^{-1}x$

বা $y = \tan^{-1}\frac{1}{\sqrt{(\sec^2\theta - 1)}} = \tan^{-1}\frac{1}{\tan\theta} = \tan^{-1}(\cot\theta)$

বা $y = \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \theta\right)\right] = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \sec^{-1}x$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left[\frac{\pi}{2} - \sec^{-1}x\right] = \frac{-1}{x\sqrt{(x^2 - 1)}}.$$

(xiii). ধরি $y = \tan^{-1}\frac{\sqrt{(1 + x^2)} - 1}{x}$ এবং $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

বা $y = \tan^{-1}\left[\frac{\sqrt{(1 + \tan^2\theta)} - 1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$

বা $y = \tan^{-1}\left[\frac{1/\cos\theta - 1}{\sin\theta/\cos\theta}\right] = \tan^{-1}\left[\frac{1 - \cos\theta}{\sin\theta}\right]$

বা $y = \tan^{-1}\left[\frac{2\sin^2\theta/2}{2\sin\theta/2.\ \cos\theta/2}\right] = \tan^{-1}\left[\tan\frac{\theta}{2}\right] = \frac{1}{2}\theta$

বা $y = \frac{1}{2}\tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{2(1 + x^2)}.$

(xiv). ধরি $y = \tan^{-1}\sqrt{\frac{1 - x}{1 + x}}$ এবং $x = \cos\theta \Rightarrow \theta = \cos^{-1}x.$

বা $y = \tan^{-1}\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \tan^{-1}\sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}} = \tan^{-1}\tan\frac{\theta}{2} = \frac{1}{2}\theta$

বা $y = \frac{1}{2}\cos^{-1}x \Rightarrow \frac{dy}{dx} = \frac{-1}{2\sqrt{(1 - x^2)}}.$

(xv). ধরি $y = \tan^{-1}\left[\frac{\sqrt{(1 + x)} - \sqrt{(1 - x)}}{\sqrt{(1 + x)} + \sqrt{(1 - x)}}\right]$

ধরি $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

$\therefore y = \tan^{-1}\left[\frac{\sqrt{(1 + \cos\theta)} - \sqrt{(1 - \cos\theta)}}{\sqrt{(1 + \cos\theta)} + \sqrt{(1 - \cos\theta)}}\right]$

বা $y = \tan^{-1}\left[\frac{\sqrt{(2\cos^2\theta/2)} - \sqrt{(2\sin^2\theta/2)}}{\sqrt{(2\cos^2\theta/2)} + \sqrt{(2\sin^2\theta/2)}}\right]$

বা $y = \tan^{-1}\left[\frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}\right] = \tan^{-1}\left[\frac{1 - \tan\theta/2}{1 + \tan\theta/2}\right]$

বা $y = \tan^{-1}\left[\frac{\tan\pi/4 - \tan\theta/2}{1 + \tan\pi/4.\ \tan\theta/2}\right] = \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

বা $y = \frac{\pi}{4} - \frac{1}{2}\theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{(1 - x^2)}}$.

(xvi). $y = \cot^{-1}(\sqrt{1 + x^2} - x) \cdots (1)$

ধরি $x = \tan\theta$ তবে $\theta = \tan^{-1}x$

এখন $\sqrt{1 + x^2} - x = \sqrt{1 + \tan^2\theta} - \tan\theta = \sqrt{\sec^2\theta} - \tan\theta$

$= \sec\theta - \tan\theta = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{1 - \sin\theta}{\cos\theta} = \frac{\cos^2(\theta/2) + \sin^2(\theta/2) - 2\sin(\theta/2)\cos(\theta/2)}{\cos^2(\theta/2) - \sin^2(\theta/2)}$

$= \frac{\{\cos(\theta/2) - \sin(\theta/2)\}^2}{\cos^2(\theta/2) - \sin^2(\theta/2)} = \frac{\cos(\theta/2) - \sin(\theta/2)}{\cos(\theta/2) + \sin(\theta/2)}$

$= \frac{1 - \tan(\theta/2)}{1 + \tan(\theta/2)} = \frac{\tan(\pi/4) - \tan(\theta/2)}{1 + \tan(\pi/4).\ \tan(\theta/2)}$

$= \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = \tan\left\{\frac{\pi}{2} - \left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right\} = \cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

$\therefore (1) \Rightarrow y = \cot^{-1}\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{\pi}{4} + \frac{\theta}{2}$

বা $y = \frac{\pi}{4} + \frac{1}{2}\tan^{-1}x$

$\therefore \frac{dy}{dx} = 0 + \frac{1}{2(1 + x^2)}$.

b(i). ধরি $y = \sin^{-1}\frac{2x}{1 + x^2}$ এবং $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

বা $y = \sin^{-1}\frac{2\tan\theta}{1 + \tan^2\theta} = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1}x) = \frac{2}{1 + x^2}$.

(ii). ধরি $y = \cos^{-1}\dfrac{1-x^2}{1+x^2}$ এবং $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

বা $y = \cos^{-1}\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore \dfrac{dy}{dx} = \dfrac{d}{dx}(2\tan^{-1}x) = \dfrac{2}{1+x^2}.$

(iii). ধরি $y = \tan^{-1}\dfrac{2x}{1-x^2} = \tan^{-1}\dfrac{x+x}{1-x.x} = \tan^{-1}x + \tan^{-1}x$

বা $y = 2\tan^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}(2\tan^{-1}x) = \dfrac{2}{1+x^2}.$

(iv). ধরি $y = \tan^{-1}\dfrac{3x-x^3}{1-3x^2}$ এবং $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

বা $y = \tan^{-1}\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \tan^{-1}(\tan 3\theta) = 3\theta$

বা $y = 3\tan^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}(3\tan^{-1}x) = \dfrac{3}{1+x^2}.$

(v). ধরি $y = \tan^{-1}\left[\dfrac{3a^2x - x^3}{a(a^2-3x^2)}\right]$ এবং $x = a\tan\theta \Rightarrow \theta = \tan^{-1}\dfrac{x}{a}$

বা $y = \tan^{-1}\left[\dfrac{3a^3\tan\theta - a^3\tan^3\theta}{a(a^2-3a^2\tan^2\theta)}\right]$

বা $y = \tan^{-1}\left[\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right] = \tan^{-1}(\tan 3\theta) = 3\theta$

বা $y = 3\tan^{-1}\dfrac{x}{a} \Rightarrow \dfrac{dy}{dx} = \dfrac{3}{1+x^2/a^2}\cdot\dfrac{1}{a} = \dfrac{3a}{a^2+x^2}.$

(vi). ধরি $y = \tan^{-1}\left[\dfrac{3\sqrt{x} - x\sqrt{x}}{1-3x}\right]$

$= \tan^{-1}\left[\dfrac{3\sqrt{x} - (\sqrt{x})^3}{[1-3(\sqrt{x})^2}\right]$

ধরি $\sqrt{x} = \tan\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$\therefore y = \tan^{-1}\left[\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right] = \tan^{-1}(\tan 3\theta) = 3\theta = 3\tan^{-1}\sqrt{x}$

$\therefore \dfrac{dy}{dx} = \dfrac{d}{dx}[3\tan^{-1}\sqrt{x}] = \dfrac{3}{1+(\sqrt{x})^2}\dfrac{d}{dx}(\sqrt{x}) = \dfrac{3}{(1+x)}\cdot\dfrac{1}{2\sqrt{x}}.$

(vii). ধরি $y = \tan^{-1}\dfrac{4\sqrt{x}}{1-4x} = \tan^{-1}\dfrac{2\sqrt{x}+2\sqrt{x}}{1-2\sqrt{x}.\,2\sqrt{x}}$

বা $y = \tan^{-1}(2\sqrt{x}) + \tan^{-1}(2\sqrt{x}) = 2\tan^{-1}(2\sqrt{x})$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}[2\tan^{-1}(2\sqrt{x})] = \frac{2}{1+(2\sqrt{x})^2}\frac{d}{dx}(2\sqrt{x})$$

$$= \frac{2}{1+4x}\cdot\frac{2}{2\sqrt{x}} = \frac{2}{\sqrt{x}(1+4x)}.$$

(viii). ধরি $y = \tan^{-1}\dfrac{a^{1/3}+x^{1/3}}{1-a^{1/3}x^{1/3}} = \tan^{-1}a^{1/3} + \tan^{-1}x^{1/3}$

$$\frac{dy}{dx} = 0 + \frac{1}{1+(x^{1/3})^2}\cdot\frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}(1+x^{2/3})}.$$

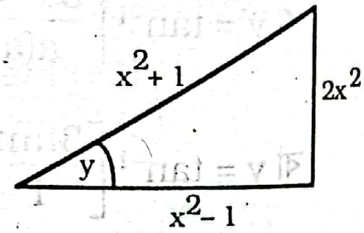
(ix). ধরি $y = \text{tna}^{-1}\dfrac{a+x}{1-ax} = \tan^{-1}a + \tan^{-1}x$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}[\tan^{-1}a + \tan^{-1}x] = 0 + \frac{1}{1+x^2} = \frac{1}{1+x^2}.$$

(x). ধরি $y = \cot^{-1}\dfrac{1-x}{1+x} = \tan^{-1}\dfrac{1+x}{1-x} = \tan^{-1}1 + \tan^{-1}x$

বা $y = \dfrac{\pi}{4} + \tan^{-1}x \Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{\pi}{4} + \tan^{-1}x\right) = \dfrac{1}{1+x^2}.$

(xi). $y = \sec^{-1}\dfrac{x^2+1}{x^2-1}$



বা $\sec y = \dfrac{x^2+1}{x^2-1}$

$\therefore \ln(\sec y) = \ln(x^2+1) - \ln(x^2-1)$

$$\therefore \frac{1}{\sec y}\sec y.\tan y\frac{dy}{dx} = \frac{2x}{x^2+1} - \frac{2x}{x^2-1}$$

$$\text{বা } \tan y\frac{dy}{dx} = \frac{2x(x^2-1-x^2-1)}{(x^2+1)(x^2-1)}$$

$$\text{বা } \frac{dy}{dx} = \cot y.\frac{(-4x)}{(x^2+1)(x^2-1)} \cdots (1)$$

কিন্তু আমাদের আছে $\sec y = \dfrac{x^2+1}{x^2-1}$

সুতরাং $\cot y = \dfrac{x^2-1}{2x}$

$$\therefore (1) \Rightarrow \frac{dy}{dx} = \frac{(x^2-1)}{2x}\cdot\frac{(-4x)}{(x^2+1)(x^2-1)} = \frac{-2}{(x^2+1)}.$$

2(i). $x = a\cos^3\theta$ এবং $y = a\sin^3\theta$

$\therefore \frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$ $\quad \therefore \frac{dy}{d\theta} = 3a\sin^2\theta.\cos\theta$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$$

(ii). $x = \sec\theta$ এবং $y = \sec\theta(1 + \sin\theta)$

$\therefore \frac{dx}{d\theta} = \sec\theta\tan\theta;$ বা $y = \sec\theta + \sec\theta.\sin\theta$

বা $y = \sec\theta + \tan\theta$

$$\frac{dy}{d\theta} = \sec\theta\tan\theta + \sec^2\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta} = \frac{\sec\theta(\tan\theta + \sec\theta)}{\sec\theta\tan\theta} = 1 + \operatorname{cosec}\theta.$$

(iii). $x = a(\theta - \sin\theta)$ এবং $y = a(1 + \cos\theta)$

$\frac{dx}{d\theta} = a(1 - \cos\theta);$ $\quad \frac{dy}{dx} = -a\sin\theta$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta} = \frac{-a\sin\theta}{a(1 - \cos\theta)} = \frac{-2\sin\theta/2.\cos\theta/2}{2\sin^2\theta/2} = -\cot\frac{\theta}{2}.$$

(iv). $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$

$$\frac{dx}{dt} = a[-\sin t + t.\cos t + 1.\sin t] = at\cos t$$

এবং $\frac{dy}{dt} = a[\cos t - 1\cos t - t(-\sin t)] = at\sin t$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{at\sin t}{at\cos t} = \tan t.$$

(v). $x = \sin^2 t$, $\quad y = \tan t$

$\frac{dx}{dt} = 2\sin t\cos t;$ $\frac{dy}{dt} = \sec^2 t$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{\sec^2 t}{2\sin t.\cos t} = \frac{\sec^2 t}{\sin 2t} = \sec^2 t.\operatorname{cosec}2t.$$

(vi). $x = a(2\cos\theta + \cos 2\theta)$, $y = a(2\sin\theta - \sin 2\theta)$

$\therefore \frac{dx}{d\theta} = a(-2\sin\theta - 2\sin 2\theta)$; $\frac{dy}{d\theta} = a(2\cos\theta - 2\cos 2\theta)$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} / \frac{dx}{d\theta} = \frac{2a(\cos\theta - \cos 2\theta)}{-2a(\sin\theta + \sin 2\theta)} = \frac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta}$$

$$= \frac{2\sin(3\theta/2)\sin(-\theta/2)}{2\sin(3\theta/2)\cos(\theta/2)} = -\tan\frac{\theta}{2}$$

(vii). $x = 2a\sin^2 t\cos 2t,\ y = 2a\sin^2 t\sin 2t$

$$\therefore \frac{dx}{dt} = 2a[2\sin t\cos t.\cos 2t + \sin^2 t.(-2\sin 2t)]$$

$$= 4a\sin t[\cos 2t\cos t - \sin 2t.\sin t]$$

$$= 4a\sin t.\cos(2t + t) = 4a\sin t\cos 3t$$

এবং $\frac{dy}{dt} = 2a[2\sin t\cos t.\sin 2t + \sin^2 t.2\cos 2t]$

$$= 4a\sin t[\sin 2t\cos t + \cos 2t.\sin t]$$

$$= 4a\sin t.\sin(2t + t) = 4a\sin t\sin 3t$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{4a\sin t\sin 3t}{4a\sin t\cos 3t} = \tan 3t.$$

(viii). $x = \frac{a\cos t}{t},\ y = \frac{a\sin t}{t}$

$$\frac{dx}{dt} = \frac{t(-a\sin t) - a\cos t}{t^2};\ \frac{dy}{dt} = \frac{t.a\cos t - a\sin t}{t^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{a(t\cos t - \sin t)/t^2}{-a(t\sin t + \cos t)/t^2} = \frac{\sin t - t\cos t}{\cos t + t\sin t}.$$

(ix). $x = \sin\left\{2\tan^{-1}\sqrt{\frac{1-t}{1+t}}\right\};\ y = ln^2 t$

এখন $t = \cos\theta$ ধরি, তবে

$$\sqrt{\frac{1-t}{1+t}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}} = \tan\frac{\theta}{2}$$

$\therefore x = \sin\{2\tan^{-1}(\tan\theta/2)\} = \sin(2.\theta/2) = \sin\theta$

বা $x = \sqrt{(1 - \cos^2\theta)} = \sqrt{(1 - t^2)}$

$$\therefore \frac{dx}{dt} = \frac{d}{dt}\sqrt{(1 - t^2)} = \frac{1(-2t)}{2\sqrt{(1 - t^2)}} = \frac{-t}{\sqrt{(-t^2)}}$$

এবং $y = (lnt)^2$

$$\therefore \frac{dy}{dt} = \frac{d}{dt}(lnt)^2 = 2(lnt)\frac{1}{t} = \frac{2lnt}{t}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{(2lnt)/t}{-t/\sqrt{(1 - t^2)}} = \frac{2lnt.\sqrt{(1 - t^2)}}{-t^2}.$$

(x). $x = \ln t + \sin t,\ y = e^t + \cos t$

$$\frac{dx}{dt} = \frac{1}{t} + \cos t;\ \frac{dy}{dt} = e^t - \sin t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{e^t - \sin t}{1/t + \cos t}.$$

(xi). $\tan y = \dfrac{2t}{1 - t^2}$ এবং $\sin x = \dfrac{2t}{1 + t^2}$

$\Rightarrow y = \tan^{-1}\dfrac{2t}{1 - t^2}$ এবং $x = \sin^{-1}\dfrac{2t}{1 + t^2}$

ধরি $t = \tan\theta$ তবে $\theta = \tan^{-1}t$

$$\therefore y = \tan^{-1}\frac{2\tan\theta}{1 - \tan^2\theta} = \tan^{-1}(\tan 2\theta) = 2\theta = 2\tan^{-1}t$$

$$\therefore \frac{dy}{dt} = \frac{d}{dt}(2\tan^{-1}t) = \frac{2}{1 + t^2}$$

এবং $x = \sin^{-1}\dfrac{2\tan\theta}{1 + \tan^2\theta} = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}t$

$$\frac{dx}{dt} = \frac{d}{dt}(2\tan^{-1}t) = \frac{2}{1 + t^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2/(1 + t^2)}{2/(1 + t^2)} = 1.$$

(xii). $x = \dfrac{2t}{1 + t^3},\ y = \dfrac{2t^2}{1 + t^3}$

$$\frac{dx}{dt} = \frac{(1 + t^3)\,2 - 2t\,3t^2}{(1 + t^3)^2} = \frac{2(1 - 2t^3)}{(1 + t^3)^2}$$

এবং $\dfrac{dy}{dt} = \dfrac{(1 + t^3)4t - 2t^2.3t^2}{(1 + t^3)^2} = \dfrac{2t(2 - t^3)}{(1 + t^3)^2}$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2t(2 - t^3)/(1 + t^3)^2}{2(1 - 2t^3)/(1 + t^3)^2} = \frac{t(2 - t^3)}{1 - 2t^3}.$$

(xiii). $x = \dfrac{3at}{1 + t^3},\quad y = \dfrac{3at^2}{1 + t^3}$

$$\therefore \frac{dx}{dt} = \frac{(1 + t^3)\,3a - 3at.\,3t^2}{(1 + t^3)^2} = \frac{3a(1 - 2t^3)}{(1 + t^3)^2}.$$

এবং $\dfrac{dy}{dt} = \dfrac{(1 + t^3)\,6at - 3at^2.\,3t^2}{(1 + t^3)^2} = \dfrac{3at(2 - t^3)}{(1 + t^3)^2}$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{3at(2 - t^3)/(1 + t^3)^2}{3a(1 - 2t^3)/(1 + t^3)^2} = \frac{t(2 - t^3)}{1 - 2t^3}.$$

(xiv). $x = t - \sqrt{(1-t^2)},\ y = e^{\sin^{-1}t}$.

$\therefore \dfrac{dx}{dt} = 1 - \dfrac{(-2t)}{2\sqrt{(1-t^2)}};\ \dfrac{dy}{dt} = \dfrac{e^{\sin^{-1}t}}{\sqrt{(1-t^2)}}$

বা $\dfrac{dx}{dt} = \dfrac{\sqrt{(1-t^2)} + t}{\sqrt{(1-t^2)}}$

$\therefore \dfrac{dy}{dx} = \dfrac{dy}{dt} \Big/ \dfrac{dx}{dt} = \dfrac{(e^{\sin^{-1}t})/\sqrt{(1-t^2)}}{\{t + \sqrt{(1-t^2)}\}/\sqrt{(1-t^2)}} = \dfrac{e^{\sin^{-1}t}}{t + \sqrt{(1-t^2)}}$.

(xv). $x = 2\sin^{-1}\dfrac{t}{\sqrt{(1+t^2)}};\ \ y = \cos^{-1}\dfrac{1}{\sqrt{(1+t^2)}}$

ধরি $t = \tan\theta \Rightarrow \theta = \tan^{-1}t$

$\therefore x = 2\sin^{-1}\left[\dfrac{\tan\theta}{\sqrt{(1+\tan^2\theta)}}\right] = 2\sin^{-1}\left[\dfrac{\tan\theta}{\sec\theta}\right] = 2\sin^{-1}(\sin\theta) = 2\theta$

বা $x = 2\tan^{-1}t \Rightarrow \dfrac{dx}{dt} = \dfrac{2}{1+t^2}$

এবং $y = \cos^{-1}\left[\dfrac{1}{\sqrt{(1+\tan^2\theta)}}\right] = \cos^{-1}\left[\dfrac{1}{\sec\theta}\right] = \cos^{-1}(\cos\theta) = \theta$

বা $y = \tan^{-1}t \Rightarrow \dfrac{dy}{dt} = \dfrac{1}{1+t^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{dy}{dt} \Big/ \dfrac{dx}{dt} = \dfrac{1/(1+t^2)}{2/(1+t^2)} = \dfrac{1}{2}$.

(xvi). $x = 3\sin^{-1}\dfrac{2t}{1-t^2},\ y = 2\sin^{-1}\dfrac{2t}{1+t^2}$

$$\frac{dx}{dt} = \frac{3}{\sqrt{1 - 4t^2/(1-t^2)^2}}\frac{d}{dt}\left(\frac{2t}{1-t^2}\right)$$

$$= \frac{3(1-t^2)}{\sqrt{(1-t^2)^2 - 4t^2}}\cdot\frac{(1-t^2)2 - 2t(-2t)}{(1-t^2)^2}$$

$$= \frac{3}{\sqrt{1 - 6t^2 + t^4}}\cdot\frac{2(1+t^2)}{1-t^2}$$

$y = 2\sin^{-1}\dfrac{2\tan\theta}{1+\tan^2\theta}$; যখন $t = \tan\theta$

বা $y = 2\sin^{-1}(\sin 2\theta) = 2\theta$

বা $y = 2\tan^{-1}t$

$\therefore \dfrac{dy}{dt} = \dfrac{2}{1+t^2}$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2/(1+t^2)}{6(1+t^2)/(1-t^2)\sqrt{1-6t^2+t^4}}$$

$$= \frac{(1-t^2)\sqrt{1-6t^2+t^4}}{3(1+t^2)^2}.$$

(xvii). $x = \sin^{-1}\dfrac{2t}{1+t^2}$, $y = \tan^{-1}\dfrac{2t}{1-t^2}$

ধরি $t = \tan\theta$ তবে $\theta = \tan^{-1}t$

$$\therefore x = \sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta} = \sin^{-1}(\sin 2\theta) = 2\theta$$

বা $x = 2\tan^{-1}t \quad \therefore \dfrac{dx}{dt} = \dfrac{2}{1+t^2}$

এবং $y = \tan^{-1}\dfrac{2\tan\theta}{1-\tan^2\theta} = \tan^{-1}(\tan 2\theta) = 2\theta$

বা $y = 2\tan^{-1}t \quad \therefore \dfrac{dy}{dt} = \dfrac{2}{1+t^2}$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2/(1+t^2)}{2/(1+t^2)} = 1.$$

(xviii). $x = 3\tan^{-1}\dfrac{2t}{1-t^2}$, $y = 2\sin^{-1}\dfrac{2t}{1+t^2}$

ধরি $t = \tan\theta$, তবে $\theta = \tan^{-1}t$

$$\therefore x = 3\tan^{-1}\frac{2\tan\theta}{1-\tan^2\theta} = 3\tan^{-1}(\tan 2\theta) = 3.2\theta$$

বা $x = 6\tan^{-1}t \Rightarrow \dfrac{dx}{dt} = \dfrac{6}{1+t^2}$

এবং $y = 2\sin^{-1}\dfrac{2\tan\theta}{1+\tan^2\theta} = 2\sin^{-1}(\sin 2\theta) = 2.2\theta$

বা $y = 4\tan^{-1}t \Rightarrow \dfrac{dy}{dt} = \dfrac{4}{1+t^2}$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{4/(1+t^2)}{6/(1+t^2)} = \frac{2}{3}.$$

3(i). ধরি $y = \sin x^0$ এবং $z = x^3$, কাজেই $\dfrac{dy}{dz}$ নির্ণয় করা আবশ্যক।

বা $y = \sin(\pi x/180)$ | $\dfrac{dz}{dx} = 3x^2$

$$\therefore \frac{dy}{dx} = \cos(\pi x/180).\frac{\pi}{180}$$

$$= (\pi/180)\cos x^0$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{(\pi/180)\cos x^0}{3x^2} = \frac{\pi\cos x^0}{540x^2}.$$

(ii). ধরি $y = \log_{10}x$ এবং $z = x^3$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

বা $y = lnx.\ \log_{10}e$ | $\frac{dz}{dx} = 3x^2$

$$\therefore \frac{dy}{dx} = \frac{1}{x}\log_{10}e$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{(1/x)\log_{10}e}{3x^2} = \frac{\log_{10}e}{3x^3}.$$

(iii). ধরি $y = \log_a x$ এবং $z = x^3$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

বা $y = lnx.\ \log_a e$ | $\frac{dz}{dx} = 3x^2$

$$\therefore \frac{dy}{dx} = \frac{1}{x}\log_a e$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{(1/x)\log_a e}{3x^2} = \frac{\log_a e}{3x^3}.$$

(iv). ধরি $y = e^x$ এবং $z = \sqrt{x}$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$\therefore \frac{dy}{dx} = e^x$; | $\frac{dz}{dx} = \frac{1}{2\sqrt{x}}$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{e^x}{1/2\sqrt{x}} = 2\sqrt{x}.\ e^x.$$

(v). ধরি $y = \sec^{-1}\left[\frac{1}{2x^2 - 1}\right]$ এবং $z = \sqrt{(1 - x^2)}$, কাজেই $\frac{dy}{dz}$ নির্ণয় ক
আবশ্যক।

ধরি $x = \cos\theta$, তবে $\theta = \cos^{-1}x$

$$y = \sec^{-1}\left[\frac{1}{2\cos^2\theta - 1}\right] = \sec^{-1}\left[\frac{1}{\cos 2\theta}\right] = \sec^{-1}(\sec 2\theta) = 2\theta$$

বা $y = 2\cos^{-1}x \Rightarrow \frac{dy}{dx} = \frac{-2}{\sqrt{(1 - x^2)}}$

এবং $z = \sqrt{(1 - x^2)} \Rightarrow \frac{dz}{dx} = \frac{(-2x)}{2\sqrt{(1 - x^2)}} = \frac{-x}{\sqrt{(1 - x^2)}}$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{-2/\sqrt{(1 - x^2)}}{-x/\sqrt{(1 - x^2)}} = \frac{2}{x}.$$

(vi). ধরি $y = ln\left(\frac{1 + \sqrt{x}}{1 - \sqrt{x}}\right)$ এবং $z = x^{3/2}$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$\therefore y = ln(1 + \sqrt{x}) - ln(1 - \sqrt{x})$

$$\therefore \frac{dy}{dx} = \frac{1}{(1 + \sqrt{x})} \cdot \frac{d}{dx}(1 + \sqrt{x}) - \frac{1}{(1 - \sqrt{x})} \frac{d}{dx}(1 - \sqrt{x})$$

$$= \frac{1}{(1 + \sqrt{x})} \frac{1}{2\sqrt{x}} - \frac{1}{1 - \sqrt{x}} \cdot \frac{(-1)}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}(1 + \sqrt{x})} + \frac{1}{2\sqrt{x}(1 - \sqrt{x})} = \frac{1 - \sqrt{x} + 1 + \sqrt{x}}{2\sqrt{x}(1 + \sqrt{x})(1 - \sqrt{x})}$$

$$= \frac{1}{\sqrt{x}(1 - x)}$$

এবং $\frac{dz}{dx} = \frac{d}{dx}(x^{3/2}) = \frac{3}{2}\sqrt{x}$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{1/\sqrt{(x)}\,(1 - x)}{3\sqrt{x}/2} = \frac{2}{3x(1 - x)}.$$

(vii). ধরি $y = \tan^{-1}x$ এবং $z = x^2$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2} \text{ এবং } \frac{dz}{dx} = 2x$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{1/(1 + x^2)}{2x} = \frac{1}{2x(1 + x^2)}.$$

(viii). যদি $y = (\sin x)^x$ এবং $z = x^{\sin x}$ ধরা হয়, তবে $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$\therefore lny = x ln(\sin x)$

$$\therefore \frac{1}{y}\frac{dy}{dx} = x \frac{d}{dx} ln(\sin x) + ln(\sin x) \cdot \frac{dx}{dx}$$

$$\text{বা } \frac{dy}{dx} = y\left[x \cdot \frac{\cos x}{\sin x} + ln(\sin x)\right]$$

$lnz = \sin x \cdot lnx$

$$\therefore \frac{1}{z}\frac{dz}{dx} = \sin x \frac{d}{dx}(lnx) + lnx \frac{d}{dx}(\sin x)$$

$$\text{বা } \frac{dz}{dx} = z\left[\frac{\sin x}{x} + lnx \cdot \cos x\right]$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{(\sin x)^x\,[x\cot x + ln(\sin x)]}{x^{\sin x}[(\sin x)/x + lnx \cdot \cos x]}.$$

(ix). ধরি $y = a^x$ এবং $z = e^{bx}$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$$\therefore \frac{dy}{dx} = a^x \log a; \quad \frac{dz}{dx} = b e^{bx}$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{a^x \log a}{be^{bx}}.$$

(x). ধরি $y = \ln x$ এবং $z = \tan x$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$$\therefore \frac{dy}{dx} = \frac{1}{x}; \quad \frac{dz}{dx} = \sec^2 x$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{1/x}{\sec^2 x} = \frac{\cos^2 x}{x}.$$

(xi). ধরি $y = \sec x$ এবং $z = \tan x$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$$\therefore \frac{dy}{dx} = \sec x \tan x; \quad \frac{dz}{dx} = \sec^2 x$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{\sec x \tan x}{\sec^2 x} = \sin x.$$

(xii). যদি $y = e^{\sin^{-1}x}$ এবং $z = \cos 3x$ ধরা হয়, তবে $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$\therefore y = e^{\sin^{-1}x}$

$$\therefore \frac{dy}{dx} = e^{\sin^{-1}x} \frac{d}{dx}(\sin^{-1}x)$$

বা $$\frac{dy}{dx} = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$$

এবং $z = \cos 3x$

$$\therefore \frac{dz}{dx} = -3\sin 3x$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} \Big/ (-3\sin 3x)$$

$$= \frac{e^{\sin^{-1}x}}{-3\sin 3x.\sqrt{1-x^2}}.$$

(xiii). ধরি $y = \ln(2 + x)$ এবং $z = \cos 2x^2$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$$\therefore \frac{dy}{dx} = \frac{1}{2+x}; \quad \frac{dz}{dx} = -\sin 2x^2.(4x) = -4x\sin 2x^2$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{1/(2+x)}{-4x\sin 2x^2} = \frac{-1}{4x(2+x)\sin 2x^2}.$$

(xiv). যদি $y = x^n\, ln(\tan^{-1}x)$ এবং $z = \frac{\sin\sqrt{x}}{x^{3/2}}$ ধরা হয় তবে $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$\therefore y = x^n\, ln(\tan^{-1}x)$

$$\frac{dy}{dx} = x^n \frac{d}{dx} ln(\tan^{-1}x) + ln(\tan^{-1}x).\frac{d}{dx}(x^n)$$

$$= \frac{x^n}{\tan^{-1}x}\cdot\frac{1}{1+x^2} + ln(\tan^{-1}x).\, nx^{n-1}$$

$$= \frac{x^{n-1}[x + n(1+x^2)\tan^{-1}x.\, ln(\tan^{-1}x)]}{(1+x^2)\tan^{-1}x}$$

এবং $z = \frac{\sin\sqrt{x}}{x^{3/2}}$

$$\therefore lnz = ln(\sin\sqrt{x}) - \frac{3}{2} lnx$$

$$\therefore \frac{1}{z}\frac{dz}{dx} = \frac{\cos\sqrt{x}}{\sin\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} - \frac{3}{2x}$$

$$\text{বা}\ \frac{dz}{dx} = z\left[\frac{\sqrt{x}\cos\sqrt{x} - 3\sin\sqrt{x}}{2x\sin\sqrt{x}}\right]$$

$$\text{বা}\ \frac{dz}{dx} = \frac{\sin\sqrt{x}}{x^{3/2}}\frac{[\sqrt{x}\cos\sqrt{x} - 3\sin\sqrt{x}]}{2x\sin\sqrt{x}} = \frac{\sqrt{x}\cos\sqrt{x} - 3\sin\sqrt{x}}{2x^{5/2}}$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx}\Big/\frac{dz}{dx} = \frac{[x + n(1+x^2)\tan^{-1}x.\, ln(\tan^{-1}x)]x^{n-1}}{(1+x^2)\tan^{-1}x} \times \frac{2x^{5/2}}{[\sqrt{x}\cos\sqrt{x} - 3\sin\sqrt{x}]}$$

$$\therefore \frac{dy}{dz} = \frac{2[x + n(1+x^2)\tan^{-1}x.\, ln(\tan^{-1}x)]x^{(2n+3)/2}}{(1+x^2)\tan^{-1}x[\sqrt{x}\cos\sqrt{x} - 3\sin\sqrt{x}]}.$$

(xv). ধরি $y = x^{\sin x}$ এবং $z = (\sin x)^x$; কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$$lny = \sin x.\, lnx$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \cos x\, lnx$$

$$\text{বা}\ \frac{dy}{dx} = y\left[\frac{\sin x}{x} + \cos x\, lnx\right] = x^{\sin x}\left[\frac{\sin x}{x} + \cos x\, lnx\right]$$

এবং $\ln z = x\ln(\sin x)$

$$\therefore \frac{1}{z}\frac{dz}{dx} = x.\frac{\cos x}{\sin x} + 1.\ln(\sin x)$$

বা $\dfrac{dz}{dx} = z[x\cot x + \ln(\sin x)] = (\sin x)^x\,[x\cot x + \ln(\sin x)]$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{x^{\sin x}[(\sin x)/x + \cos x\ \ln x]}{(\sin x)^x\,[x\cot x + \ln(\sin x)]}.$$

(xvi). ধরি $y = (\ln x)^{\tan x}$ এবং $z = \sin(m\cos^{-1}x)$, কাজেই $\dfrac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$\therefore \ln y = \tan x.\ \ln(\ln x)$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \tan x.\ \frac{1}{\ln x}.\frac{1}{x} + \sec^2 x.\ \ln(\ln x)$$

$$\therefore \frac{dy}{dx} = y\left[\frac{\tan x}{x\ln x} + \sec^2 x\ \ln(\ln x)\right]$$

এবং $z = \sin(m\cos^{-1}x)$

$$\therefore \frac{dz}{dx} = \cos(m\cos^{-1}x).\ \frac{(-m)}{\sqrt{(1-x^2)}}$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx}$$

$$= \frac{(\ln x)^{\tan x}[(\tan x)/x\ln x + \sec^2 x\ \ln(\ln x)]\ \sqrt{(1-x^2)}}{-m\cos(m\cos^{-1}x)}.$$

(xvii). ধরি $y = x^{\sin^{-1}x}$ এবং $z = \sin^{-1}x$, কাজেই $\dfrac{dy}{dz}$ নির্ণয় করা আবশ্যক।

$\Rightarrow \ln y = \sin^{-1}x.\ \ln x$

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{(1-x^2)}}$$

বা $\dfrac{dy}{dx} = y\left[\dfrac{\sin^{-1}x}{x} + \dfrac{\ln x}{\sqrt{(1-x^2)}}\right] = x^{\sin^{-1}x}\left[\dfrac{\sin^{-1}x}{x} + \dfrac{\ln x}{\sqrt{(1-x^2)}}\right]$

এবং $z = \sin^{-1}x \Rightarrow \dfrac{dz}{dx} = \dfrac{1}{\sqrt{(1-x^2)}}$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = x^{\sin^{-1}x}\left[\frac{\sin^{-1}x}{x} + \frac{\ln x}{\sqrt{(1-x^2)}}\right] \times \frac{\sqrt{(1-x^2)}}{1}$$

$$= x^{\sin^{-1}x}\left[\frac{\sqrt{(1-x^2)}\ \sin^{-1}x}{x} + \ln x\right].$$

(xviii). ধরি $y = \tan^{-1}\left[\frac{\sqrt{(1+x^2)}-1}{x}\right]$ এবং $z = \tan^{-1}x$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

ধরি $x = \tan\theta$, তবে $\theta = \tan^{-1}x$

$$\therefore y = \tan^{-1}\left[\frac{\sqrt{(1+\tan^2\theta)}-1}{\tan\theta}\right] = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$$

$$\text{বা } y = \tan^{-1}\left[\frac{1/\cos\theta - 1}{\sin\theta/\cos\theta}\right] = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$\text{বা } y = \tan^{-1}\left[\frac{2\sin^2\theta/2}{2\sin\theta/2.\ \cos\theta/2}\right] = \tan^{-1}\left[\tan\frac{\theta}{2}\right] = \frac{\theta}{2}$$

$$\text{বা } y = \frac{1}{2}\tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)} \text{ এবং } \frac{dz}{dx} = \frac{1}{1+x^2}$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{1/2(1+x^2)}{1/(1+x^2)} = \frac{1}{2}.$$

(xix). ধরি $y = \tan^{-1}\frac{2x}{1-x^2}$ এবং $z = \cos^{-1}\frac{1-x^2}{1+x^2}$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

ধরি $x = \tan\theta$, তবে $\theta = \tan^{-1}x$

$$\therefore y = \tan^{-1}\left[\frac{2\tan\theta}{1-\tan^2\theta}\right] = \tan^{-1}(\tan 2\theta) = 2\theta = 2\tan^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1}x) = \frac{2}{1+x^2}.$$

$$\text{এবং } z = \cos^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$$

$$\therefore \frac{dz}{dx} = \frac{d}{dx}(2\tan^{-1}x) = \frac{2}{1+x^2}$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \Big/ \frac{dz}{dx} = \frac{2/(1+x^2)}{2/(1+x^2)} = 1.$$

(xx). ধরি $y = \tan^{-1}\frac{x}{\sqrt{(1-x^2)}}$ এবং $z = \sec^{-1}\frac{1}{2x^2-1}$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

ধরি $x = \sin\theta$, তবে $\theta = \sin^{-1}x$

$\therefore y = \tan^{-1}\left[\frac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}\right] = \tan^{-1}\left[\frac{\sin\theta}{\cos\theta}\right] = \tan^{-1}(\tan\theta) = \theta$

বা $y = \sin^{-1} x \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{(1-x^2)}}$

আবার ধরি $x = \cos\theta$, তবে $\theta = \cos^{-1}x$.

$\therefore z = \sec^{-1}\left[\frac{1}{2\cos^2\theta - 1}\right] = \sec^{-1}\left[\frac{1}{\cos 2\theta}\right] = \sec^{-1}(\sec 2\theta) = 2\theta$

বা $z = 2\cos^{-1}x \Rightarrow \frac{dz}{dx} = \frac{-2}{\sqrt{(1-x^2)}}$

$\therefore \frac{dy}{dz} = \frac{dy}{dx} / \frac{dz}{dx} = \frac{1/(1-x^2)}{-2/(1-x^2)} = \frac{-1}{2}$.

(xxi). ধরি $y = \sin^{-1}\frac{2x}{1+x^2}$ এবং $z = \cos^{-1}\frac{1-x^2}{1+x^2}$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা বশ্যক।

ধরি $x = \tan\theta$, তবে $\theta = \tan^{-1}x$

$\therefore y = \sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right] = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1}x) = \frac{2}{1+x^2}$

এবং $z = \cos^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore \frac{dz}{dx} = \frac{d}{dx}(2\tan^{-1}x) = \frac{2}{1+x^2}$

$\therefore \frac{dy}{dz} = \frac{dy}{dx} / \frac{dz}{dx} = \frac{2/(1+x^2)}{2/(1+x^2)} = 1.$

(xxii). ধরি $y = \cos^{-1}\frac{1-x^2}{1+x^2}$ এবং $z = \tan^{-1}\frac{2x}{1-x^2}$, কাজেই $\frac{dy}{dz}$ নির্ণয় করা আবশ্যক।

ধরি $x = \tan\theta$, তবে $\theta = \tan^{-1}x$

$\therefore y = \cos^{-1}\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right] = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(2\tan^{-1}x) = \frac{2}{1+x^2}$

এবং $z = \tan^{-1}\left[\frac{2\tan\theta}{1 - \tan^2\theta}\right] = \tan^{-1}(\tan 2\theta) = 2\theta = 2\tan^{-1}x$

$\therefore \frac{dz}{dx} = \frac{d}{dx}(2\tan^{-1}x) = \frac{2}{1 + x^2}$

$\therefore \frac{dy}{dz} = \frac{dy}{dx} / \frac{dz}{dx} = \frac{2/(1 + x^2)}{2/(1 + x^2)} = 1.$

4(i). $x = y^{y^{y^{\cdots\infty}}}$

বা $x = y^x \Rightarrow lnx = xlny$

$\therefore \frac{1}{x} = x.\frac{1}{y}\frac{dy}{dx} + lny$

বা $\frac{x}{y}\frac{dy}{dx} = \frac{1}{x} - lny = \frac{1 - xlny}{x}$

$\therefore \frac{dy}{dx} = \frac{y(1 - xlny)}{x^2}.$

(ii). $f(x) = \left(\frac{a + x}{1 + x}\right)^{a+1+2x}$, কাজেই $f(0) = a^{a+1}$

$\Rightarrow lnf(x) = (a + 1 + 2x)\,[ln(a + x) - ln(1 + x)]$

$\therefore \frac{f'(x)}{f(x)} = (a + 1 + 2x)\left[\frac{1}{a + x} - \frac{1}{1 + x}\right] + 2[ln(a + x) - ln(1 + x)]$

এখন উভয় পক্ষে $x = 0$ বসাইয়া পাই

$\frac{f'(0)}{f(0)} = (a + 1)\left[\frac{1}{a} - 1\right] + 2[lna - ln1]$

বা $f'(0) = f(0)\left\{(a + 1)\frac{(1 - a)}{a} + 2lna\right\}$

$= a^{a+1}\left[\frac{1 - a^2}{a} + 2lna\right].$

(iii). $f(x) = ln\left[\frac{\sqrt{(a + bx)} - \sqrt{(a - bx)}}{\sqrt{(a + bx)} + \sqrt{(a - bx)}}\right]$

বা $f(x) = ln\left[\frac{\{\sqrt{(a + bx)} - \sqrt{(a + bx)}\}^2}{\{\sqrt{(1 + bx)}\}^2 - \{\sqrt{(a - bx)}\}^2}\right]$

বা $f(x) = ln\left[\frac{a + bx + a - bx \quad 2\sqrt{(a + bx)(a - bx)}}{(a + bx) - (a - bx)}\right]$

বা $f(x) = ln\left[\frac{2a - 2\sqrt{(a^2 - b^2x^2)}}{2bx}\right] = ln\left[\frac{a - \sqrt{(a^2 - b^2x^2)}}{bx}\right]$

বা $f(x) = ln\{a - \sqrt{(a^2 - b^2x^2)}\} - ln bx$

$\therefore f'(x) = \frac{1}{a - \sqrt{(a^2 - b^2x^2)}}\left\{\frac{2b^2x}{2\sqrt{(a^2 - b^2x^2)}}\right\} - \frac{b}{bx}$

বা $f'(x) = \frac{a + \sqrt{(a^2 - b^2x^2)}}{a^2 - (a^2 - b^2x^2)} \frac{b^2x}{\sqrt{(a^2 - b^2x^2)}} - \frac{1}{x}$

বা $f'(x) = \frac{a + \sqrt{(a^2 - b^2x^2)}}{b^2x^2} \frac{b^2x}{\sqrt{(a^2 - b^2x^2)}} - \frac{1}{x}$

বা $f'(x) = \frac{a + \sqrt{(a^2 - b^2x^2)}}{x\sqrt{(a^2 - b^2x^2)}} - \frac{1}{x} = \frac{a + \sqrt{(a^2 - b^2x^2)} - \sqrt{(a^2 - b^2x^2)}}{x\sqrt{(a^2 - b^2x^2)}}$

বা $f'(x) = \frac{a}{x\sqrt{(a^2 - b^2x^2)}}$

$\therefore \frac{1}{f'(x)} = \frac{x\sqrt{(a^2 - b^2x^2)}}{a}$

এখন $\frac{1}{f'(x)} = 0$ যখন $x\sqrt{(a^2 - b^2x^2)} = 0$

$\Rightarrow x = 0$ অথবা $a^2 - b^2x^2 = 0 \Rightarrow x = \pm\frac{a}{b}$

$\therefore x = 0, \pm\frac{a}{b}$.

(iv). $y = \sqrt{x\sqrt{x\sqrt{x \cdots \infty}}}$

বা $y = \sqrt{xy}$, বা $y^2 = xy$, বা $y = x$ $\therefore \frac{dy}{dx} = 1$.

(v). $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots \infty}}}$

বা $y = \sqrt{(\sin x + y)} \Rightarrow y^2 = \sin x + y$

$\therefore 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$

বা $(2y - 1)\frac{dy}{dx} = \cos x \Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y - 1}$

(vi). $y = x^2 + \frac{1}{x^2 +}\frac{1}{x^2 +}\cdots\infty$

বা $y = x^2 + \frac{1}{y} \Rightarrow y^2 = x^2y + 1$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$2y\frac{dy}{dx} = x^2\frac{dy}{dx} + 2xy + 0$$

বা $[2y - x^2]\frac{dy}{dx} = 2xy \Rightarrow \frac{dy}{dx} = \frac{2xy}{2y - x^2}$.

(v). $y = x + \frac{1}{x +}\frac{1}{x +}\frac{1}{x +}\cdots\infty$

বা $y = x + \frac{1}{y} \Rightarrow y^2 = xy + 1$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$2y\frac{dy}{dx} = x\frac{dy}{dx} + 1.y + 0$$

বা $(2y - x)\frac{dy}{dx} = y \Rightarrow \frac{dy}{dx} = \frac{y}{2y - x}$.

(vi). $\sqrt{(1 - x^2)} + \sqrt{(1 - y^2)} = a(x - y) \cdots (1)$

ধরি $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$ এবং $y = \sin\phi \Rightarrow \phi = \sin^{-1}y$

$\therefore (1) \Rightarrow \sqrt{(1 - \sin^2\theta)} + \sqrt{(1 - \sin^2\phi)} = a(\sin\theta - \sin\phi)$

বা $\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

বা $2\cos(\theta + \phi)/2.\ \cos(\theta - \phi)/2 = a.\ 2\cos(\theta + \phi)/2.\ \sin(\theta - \phi)/2$

বা $\cos(\theta - \phi)/2 = a.\sin(\theta - \phi)/2$, বা $a = \cot(\theta - \phi)/2$

বা $(\theta - \phi)/2 = \cot^{-1}a$, বা $\theta - \phi = 2\cot^{-1}a$

বা $\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$

$$\therefore \frac{1}{\sqrt{(1 - x^2)}} - \frac{1}{\sqrt{(1 - y^2)}}\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = \frac{\sqrt{(1 - y^2)}}{\sqrt{(1 - x^2)}}.$$

(vii). $y = \sqrt{\{4 + \sqrt{(4 - x)}\}}$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{[4 + \sqrt{(4 - x)}]}}\frac{d}{dx}[4 + \sqrt{(4 - x)}]$$

$$= \frac{1}{2\sqrt{[4 + \sqrt{(4 - x)}]}}\left[0 + \frac{(-1)}{2\sqrt{(4 - x)}}\right]$$

$$= \frac{-1}{4\sqrt{(4 - x)}\sqrt{\{4 + \sqrt{(4 - x)}\}}}.$$

(viii). $y = b\tan^{-1}\left(\dfrac{x}{a}\tan^{-1}\dfrac{x}{a}\right)$

$y = b\tan^{-1}z$, যখন $z = \dfrac{x}{a}\tan^{-1}\dfrac{x}{a}$

$$\frac{dy}{dz} = \frac{b}{1+z^2}$$

$$= \frac{b}{1+\{x\tan^{-1}(x/a)\}^2/a^2}$$

$$= \frac{a^2b}{a^2+x^2\{\tan^{-1}(x/a)\}^2}$$

এবং

$$\frac{dz}{dx} = \frac{1}{a}\tan^{-1}\frac{x}{a} + \frac{x}{a(1+x^2/a^2)}\frac{1}{a}$$

$$= \frac{1}{a}\tan^{-1}\frac{x}{a} + \frac{x}{a^2+x^2}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx}$$

$$= \frac{a^2b}{a^2+x^2\{\tan^{-1}(x/a)\}^2}\cdot\frac{1}{a}\left[\tan^{-1}\frac{x}{a} + \frac{ax}{a^2+x^2}\right]$$

$$= \frac{ab}{a^2+x^2\{\tan^{-1}(x/a)\}^2}\left[\tan^{-1}\frac{x}{a} + \frac{ax}{a^2+x^2}\right].$$

(ix). $\sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2}) = c \cdots (1)$

ধরি $x = \sin\theta$, $y = \sin\phi$ তবে $\theta = \sin^{-1}x$ এবং $\phi = \sin^{-1}y$

$\therefore (1) \Rightarrow \sin^{-1}(\sin\theta\sqrt{1-\sin^2\phi} + \sin\phi\sqrt{1-\sin^2\theta}) = c$

বা $\sin^{-1}(\sin\theta\cos\phi + \sin\phi\cos\theta) = c$

বা $\sin^{-1}\{\sin(\theta+\phi)\} = c$

বা $\theta + \phi = c$

বা $\sin^{-1}x + \sin^{-1}y = c.$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

বা $\dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{1-x^2}}$

বা $\dfrac{dy}{dx} = \dfrac{-\sqrt{1-y^2}}{\sqrt{1-x^2}}.$

(x). $x\sqrt{1+y}+y\sqrt{1+x}=0$

বা $x\sqrt{1+y}=-y\sqrt{1+x}$ বর্গ করিয়া পাই

বা $x^2(1+y)=y^2(1+x)$

বা $x^2-y^2+x^2y-y^2x=0$

বা $(x-y)(x+y)+xy(x-y)=0$

বা $(x-y)(x+y+xy)=0$

$\therefore x+y+xy=0$ যেহেতু $x\neq y$

বা $y(1+x)=-x$

বা $y=\frac{-x}{1+x}$

$\therefore \frac{dy}{dx}=-\frac{(1+x)1-x.1}{(1+x)^2}=\frac{-1}{(1+x)^2}$.

(xi). $y=\cot(\cos^{-1}x)$

বা $y=\cot\theta$ যখন $\cos^{-1}x=\theta$, বা $x=\cos\theta$

$\therefore \frac{dy}{d\theta}=-\operatorname{cosec}^2\theta$ এবং $\frac{dx}{d\theta}=-\sin\theta$

$\therefore \frac{dy}{dx}=\frac{dy}{d\theta}/\frac{dx}{d\theta}=\frac{-\operatorname{cosec}^2\theta}{-\sin\theta}=\frac{1}{\sin^3\theta}$

$=\frac{1}{(1-\cos^2\theta)^{3/2}}=\frac{1}{(1-x^2)^{3/2}}$.

(xii) (ক) $(1+x)^n=c_0+c_1x+c_2x^2+c_3x^3+\cdots+c_nx^n\cdots(1)$

ইহাকে x এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$n(1+x)^{n-1}=c_1+2c_2x+3c_3x^2+\cdots+nc_n\,x^{n-1}$

এখন উভয় পক্ষে $x=1$ বসাইয়া পাই

$n(1+1)^{n-1}=c_1+2c_2+3c_3+\cdots+\cdots nc_n$

$\therefore c_1+2c_2+3c_3+\cdots+nc_n=n.2^{n-1}\cdots(2)$

(খ) এখন (1) নং এর উভয় পক্ষে $x=1$ স্থাপন করিয়া পাই

$2^n=c_0+c_1+c_2+\cdots+c_n$

অর্থাৎ $c_0+c_1+c_2+\cdots+c_n=2.2^{n-1}\cdots(3)$

এখন (2) নং এবং (3) নং কে যোগ করিয়া পাই

$c_0+2c_1+3c_2+\cdots+(n+1)c_n=(n+2)2^{n-1}$.

(xiii). $\cos\frac{\theta}{2}\cdot\cos\frac{\theta}{2^2}\cdot\cos\frac{\theta}{2^3}\cdots\cos\frac{\theta}{2^n}=\frac{\sin\theta}{2^n\sin(\theta/2^n)}$

$$\Rightarrow \ln\cos(\theta/2)+\ln\cos(\theta/2^2)+\ln\cos(\theta/2^3)+\cdots+\ln\cos(\theta/2^n)$$
$$=\ln(\sin\theta)-\ln 2^n-\ln\sin(\theta/2^n)$$

এখন উভয় পক্ষকে θ এর সাপেক্ষে অন্তরীকরণ করিয়া পাই

$$\frac{(-1/2)\sin(\theta/2)}{\cos(\theta/2)}+\frac{(-1/2^2)\sin(\theta/2^2)}{\cos(\theta/2^2)}+\frac{(-1/2^3)\sin(\theta/2^3)}{\cos(\theta/2^3)}+\cdots+\frac{(-1/2^n)\sin(\theta/2^n)}{\cos(\theta/2^n)}$$

$$=\frac{\cos\theta}{\sin\theta}-0-\frac{(1/2^n)\cos(\theta/2^n)}{\sin(\theta/2^n)}$$

বা $-\frac{1}{2}\tan\frac{\theta}{2}-\frac{1}{2^2}\tan\frac{\theta}{2^2}-\frac{1}{2^3}\tan\frac{\theta}{2^3}\cdots-\frac{1}{2^n}\tan\frac{\theta}{2^n}=\cot\theta-\frac{1}{2^n}\cot\frac{\theta}{2^n}$

$\therefore \frac{1}{2}\tan\frac{\theta}{2}+\frac{1}{2^2}\tan\frac{\theta}{2^2}+\frac{1}{2^3}\tan\frac{\theta}{2^3}+\cdots+\frac{1}{2^n}\tan\frac{\theta}{2^n}=\frac{1}{2^n}\cot\frac{\theta}{2^n}-\cot\theta.$

(xiv). দেওয়া আছে $y=\ln(\cos 2x)^{1/2}$

বা $y=\frac{1}{2}\ln(\cos 2x)$

$$\therefore \frac{dy}{dx}=\frac{-2\sin 2x}{2\cos 2x}=-\tan 2x$$

$$\therefore \frac{dy}{dx}+1=-\tan 2x+1$$

$\therefore \frac{dy}{dx}+1=0$ হইলে $-\tan 2x+1=0$

বা $\tan 2x=1=\tan\frac{\pi}{4}$

$\Rightarrow 2x=\frac{\pi}{4}$, বা $x=\frac{\pi}{8}$.

———×———