1 3 3 10 -5 12 2

1 3 3 10 -5

Math

Differential Equation (Mamd sir)

Statistics (Belal Sir)

Methods of statistics

Book

→ Ali Azgor

→ Dr. Abdul Rashid Ahmed

Introduction

→ what is statistics

→ function (**) any one definition

→ charactistic

→ Uses of statistics (5)

Variable and Frequency distribution

MAT (Masud Sir)

Differential Equations

(i) $\frac{dy}{dx} + y = 0$

(ii) $\frac{dy}{dx} + x = 0$

(iii) $dy = x\xi \cdot dx$

Differential Equation

9-10

(i) $\frac{dy}{dx} = e^x + x + 1$ O: 1, D: 1

(ii) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + x = 0$ O: 2, D: 1

(iii) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5 = 0$ ← Degree = 1
O: 2

(iv) $\frac{dy}{dx} = \frac{x}{y}$

$\int y \cdot dy = \int x \cdot dx$

or, $\frac{y^2}{2} = \frac{x^2}{2} + c$ ← arbitrary

$y^2 = x^2 + c$

$\frac{dy}{d} = x$

$\int dy = \int x \cdot dx$

or, $y = \frac{x^2}{2} + c$

Order, → highest order deri.
degree, → power
solution, → General, Particular
Differential eq.
arbitrary constant

$\boxed{\rightarrow c, c_1, c_2, \ldots, c_n}$

Value Separation Method

$\frac{dy}{dx} = \frac{M}{N}$

or, $M\,dx = N\,dy$

$\frac{dy}{dx} = \frac{f(y)}{f(x)}$

$\frac{dy}{f(y)} = \frac{dx}{f(x)}$

(1) $\frac{dy}{dx} = \frac{e^x}{e^y}$ (2) $\frac{dy}{dx} = \frac{y^2+b^2}{x^2+a^2}$ (3) $\frac{dy}{dx} = \frac{1}{x^2+2x+5}$

\# $\int \frac{1}{\sqrt{ax^2+bx+c}} \cdot dx$, $\int \sqrt{ax^2+bx+c} \cdot dx$,

$\int \frac{px+q}{ax^2+bx+c} dx$

Math

Math Equation reducible to the form in which variables are seperable

$\frac{dy}{dx} = f(ax+by+c)$

let, $ax+by+c = v$

or, $a + b\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{b}\left(\frac{dv}{dx} - a\right)$

$\frac{dy}{dx} = f(ax+by+c)$

$\frac{1}{b}\left(\frac{dv}{dx} - a\right) = f(v)$

$\frac{dv}{dx} - a = bf(v)$

$\frac{dv}{dx} = a + bf(v)$

or, $\int \frac{dv}{a+bf(v)} = \int dx$

$\boxed{\frac{dy}{dx} = (4x+y+1)^2}$

$4x+y+1 = v$

or, $4 + \frac{dy}{dx} = \frac{dv}{dx}$

or, $\frac{dy}{dx} = \left(\frac{dv}{dx} - 4\right)$

$v^2 = \frac{dv}{dx} - 4$

or, $\frac{dv}{dx} = v^2 + 4$

or, $\frac{dv}{v^2+4} = dx$

or, $\int \frac{dv}{v^2+4} = \int dx$

or, $\frac{1}{2}\tan^{-1}\frac{4x+y+1}{2}=x$

or, $4x+y+1 = 2\tan(2x)$

chapter II

$y = x^2 + xy$

$y = x^2 + xy$

Ex 2

$(x-y)^2 \frac{dy}{dx} = a^2$

Let,

$x-y=z$

or, $1-\frac{dy}{dx}=\frac{dz}{dx}$

or, $\frac{dy}{dx}=1-\frac{dz}{dx}$

or, $\frac{a^2}{z^2}=1-\frac{dz}{dx}$

or, $\frac{dz}{dx}=\frac{z^2-a^2}{z^2}$

or, $\frac{z^2\cdot dz}{z^2-a^2}=dx$

or, $\cancel{\int\frac{dz}{1-\frac{a^2}{z^2}}}=\int dx$

void fun (int a

int n)

$$\int \frac{z^2 - a^2 + a^2}{z^2 - a^2}\, dz = \int dx$$

$$\Rightarrow \int \left(1 + \frac{a^2}{z^2 - a^2}\right) . dz = x$$

Definition

(*) Homogenius eq.

$$\frac{dy}{dx} = \frac{f_1(x, y)}{f_2(x, y)}$$

or $y = vx$

Pattern of guess!

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\Rightarrow$

$$(x^2 + y^2)\, dx + 2xy\, dy = 0$$

$$\frac{dy}{dx} = -\frac{x^2 + y^2}{2xy}$$

$$y = vx$$

$$\text{or } \frac{dy}{dx} = v + x . \frac{dv}{dx}$$

$$v + x . \frac{dv}{dx} = -\frac{x^2 + (vx)^2}{2xvx}$$

$$\Rightarrow v + x . \frac{dv}{dx} = -\frac{1 + v^2}{2v}$$

$$\Rightarrow x . \frac{dv}{dx} = -v - \frac{1 + v^2}{2v}$$

$$= \frac{-2v^2 - 1 - v^2}{2v}$$

$$\Rightarrow \frac{-3v^2 - 1}{2v} = x . \frac{dv}{dx}$$

$$\Rightarrow \frac{1}{x} . dx = -\frac{2v . dv}{3v^2 + 1}$$

$$\frac{dy}{dx} \quad xy \quad \frac{dy}{dx}$$

or, Let, or, $y^2 = (xy - x^2)\frac{dy}{dx}$

or, $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ ——— (1)

Let, $y = vx$

or, $\frac{dy}{dx} = v + x.\frac{dv}{dx}$

~~or, $\frac{dv}{dx}(1-x) = v$~~

from (1)

$$v + x\frac{dv}{dx} = \frac{v^2x^2}{vx^2 - x^2}$$

or, $v + x.\frac{dv}{dx} = \frac{v^2}{v-1}$

or, $x.\frac{dv}{dx} = \frac{v^2}{v-1} - v$

or, $x.\frac{dv}{dx} = \frac{\cancel{v^2 - v^3 - v}}{v-1} = \frac{v^2 - v(v-1)}{} = \frac{v^2 - v^2 + v}{v-1}$

or, $x.\frac{dv}{dx} = \frac{v}{v-1}$

or, $(v^2 - 1)\,dv = \frac{dx}{x}$

or, $\int (v^2 - 1).dv = \int \frac{1}{x}.dx$

Theorem

$$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'} \quad \text{——— (1)}$$

If $\frac{a}{a'} \neq \frac{b}{b'}$, then $x = X+h$, $y = Y+k$

$\frac{dy}{dx} = \frac{dY}{dX}$ We get from (1),

$$\frac{dY}{dX} = \frac{aX+bY+(ah+bk+c)}{a'X+b'Y+(a'h+b'k+c')} \qquad \begin{matrix} =0 \\ =0 \end{matrix}$$

Now, choose h and k such that

Example

$$\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$$

Here, $\frac{1}{2} \neq \frac{2}{1}$

$x = X+h$
$y = Y+k$

$$\frac{dY}{dX} = \frac{X+h+2Y+2k-3}{2X+2h+Y+k-3} = \frac{X+2Y+(h+2k-3)}{2X+Y+(2h+k-3)}$$

We choose h and k

such that

$$h+2k-3=0$$
$$2h+k-3=0$$

$$h=1, k=1$$

$$\frac{dy}{dx} = \frac{X+2Y}{2X+Y}$$

Let,
$y = vx$

or, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\frac{dy}{dx} = \frac{x+2y}{2x+y} = v + x\frac{dv}{dx}$

or, $\frac{x+2vx}{2x+vx} = v + x\frac{dv}{dx}$

$\frac{1+v+1}{(1+v)(1-v)}$

or, $\frac{1+2v}{2+v} = v + x\frac{dv}{dx}$

or, $\frac{1+2v}{2+v} - v = x\frac{dv}{dx}$

$\cancel{2v}$

$1-v^2$

$-2v+v+?$

or, $\frac{1+2v-2v-v^2}{2+v} = x\frac{dv}{dx}$

or, $\frac{1}{x}dx = \frac{2+v}{1-v^2}\cdot dv$

or, $\int \frac{1}{x}\cdot dx = \int \frac{1+v+1}{1-v^2}\cdot dv$

or, $\ln|x| = \int \frac{1}{1-v}\cdot dv + \int \frac{1}{1-v^2}\cdot dv$

~~or, $\ln|x| = \sin^{-1}\sqrt{v} + \sin^{-1}v$~~

$$\frac{dy}{dx} = \frac{3y - 7x + 7}{3x - 7y - 3}$$

$$= \frac{-7x + 3y + 7}{3x - 7y - 3}$$

Let, $X = x + h$

$Y = y + k$

$$\frac{dY}{dX} = \frac{dy}{dx} = \cancel{3y}\ \frac{-7(x+h) + 3(y+k) + 7}{3(x+h) - 7(y+k) - 3}$$

$$= \frac{-7x + 3y + 7 - 7h + 3k}{3x - 7y - 3 + 3h - 7k}$$

Here, $7 - 7h + 3k = 0$

or, $-7h + 3k = -7$

$-3 + 3h - 7k = 0$

or, $-7k + 3h = 3$

By solving, $h = 1$, $k = 0$

Here, $$\frac{dy}{dx} = \frac{dY}{dX} = \cancel{7+}\ \frac{-7x + 3y \cancel{+7}}{3x - 7y}$$

$y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} = \frac{-7x + 3(vx)}{3x - 7(vx)} = \frac{-7x + 3vx}{3x - 7vx}$$

$$= \frac{-7 + 3v}{3 - 7v}$$

Math 2.7 Linear Differential Equations

A differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where, P, Q are functions of x, or constants, is called the linear differential equation of the first order

$\therefore$ I.F. (Integrating Factor) $= e^{\int P.dx}$

Then it becomes,

$$e^{\int P dx} \frac{dy}{dx} + P y e^{\int P dx} = Q e^{\int P dx}$$

or, $\frac{d}{dx}\left[y e^{\int P dx}\right] = Q . e^{\int P.dx}$

or, $y e^{\int P dx} = \int \left[Q . e^{\int P dx}\right] . dx + c$

Ex. 1 $(1-x^2)\frac{dy}{dx} - xy = 1$

$$\frac{dy}{dx} - \frac{x}{1-x^2} y = \frac{1}{1-x^2}$$

$P = \cancel{\frac{x}{1-x}}$

Math

→ Linear differential equations:

$$\frac{dy}{dx} + Py = Q \quad \ldots\ldots (1)$$

definition

P and Q are the function of x or const.

(*) I.F. (Integrating Factor) $= e^{\int P.dx}$

Eq. solution

(1) × I.F.

or, $\underbrace{e^{\int P.dx}}_{I.F.} . \frac{dy}{dx} + Py . \underbrace{e^{\int P.dx}}_{I.F.} = Q . \underbrace{e^{\int P.dx}}_{J.F.}$

or, $\frac{d}{dx}\left[e^{\int P.dx} . y\right] = Q.e^{\int P.dx}$

or, $e^{\int P.dx} . y = \int Q.e^{\int P.dx} . dx + c$

$$I.F. y = \int Q. I.F. dx + C$$

(7) $(1-x^2)\frac{dx}{dy} - xy = 1$ $\qquad 1-x^2$

$\qquad -2x$

or, $\frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{1}{1-x^2}$

$$I.F. = e^{\int -\frac{x}{1-x^2}\cdot dx}$$

$$= e^{\frac{1}{2}\log_e(1-x^2)}$$

$$= e^{\log_e\sqrt{1-x^2}}$$

$$= \sqrt{1-x^2}$$

Sol. $y\cdot\sqrt{1-x^2} = \int \frac{1}{1-x^2}\cdot\sqrt{1-x^2}\cdot dx + c$

or, $y\cdot\sqrt{1-x^2} = \int \frac{1}{\sqrt{1-x^2}}\cdot dx + c$

or, $y\cdot\sqrt{1-x^2} = \sin^{-1}x + c.$

(8) $x\frac{dy}{dx} + 2y = x^2\log x$ | here

Math (2 page) (1) problem with theorem

Bernoulli:
$$\frac{dy}{dx} + Py = Q y^n$$

$$y^{-n} \frac{dy}{dx} + P y^{1-n} = Q$$

or, $\frac{1}{1-n} \frac{dv}{dx} + PV = Q$

or, $\frac{dv}{dx} + \underbrace{P(1-n)}_{T} v = \underbrace{(1-n) Q}_{N}$

$$I.F. = e^{\int P(1-n)\,dx}$$

$$I.F.\ v = \int (1-n) Q\ (I.F)\ dx + C$$

$y^{1-n} = V$

or, $(1-n) y^{-n} \cdot \frac{dy}{dx} = \frac{dv}{dx}$

or, $y^{-n} \frac{dy}{dx} = \frac{1}{1-n} \cdot \frac{du}{dx}$

Example $\frac{dy}{dx} = x^3 y^3 - xy$

or, $\frac{dy}{dx} + xy = x^3 y^3$

or, $y^{-3} \frac{dy}{dx} + \frac{x}{y^2} = x^3$

or, $-\frac{1}{2} \frac{dv}{dx} + xv = x^3$

or, $\frac{dv}{dx} - 2xv = -2x^3$

$y^{-2} = V$

or, $-2 y^{-3} \frac{dy}{dx} = \frac{dv}{dx}$

or, $y^{-3} \frac{dy}{dx} = -\frac{1}{2} \frac{dv}{dx}$

Equations of first order and first degree

$$Mdx + Ndy = 0$$

Sol^n: Working rule :

If $Mdx + Ndy = 0$

$\therefore$ satisfies the condition $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

then it is exact.

$$\int M\,dx + \int (N \text{ free from } x) \cdot dy = C$$

Example :

$$(y^4 + 4x^3y + 3x)\,dx + (x^4 + 4xy^3 + y + 1)\,dy = 0$$

Here, $M = y^4 + 4x^3y + 3x$, $N = x^4 + 4xy^3 + y + 1$

$\frac{\partial M}{\partial y} = 4y^3 + 4x^3$ $\qquad$ $\frac{\partial N}{\partial x} = \cancel{3x^3}\, 4x^3 + 4y^3$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, Equation is exact.

then,

$$\int (y^4 + 4x^3y + 3x) \cdot dx + \int (y+1) \cdot dy = C$$

or, $y^4x + \frac{4x^4}{4}y + \frac{3x^2}{2} + \frac{y^2}{2} + y = c$

Math

→ If $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = f(x)$, a function of x only, then $e^{\int f(x).dx}$ is an integrating factor.

→ If $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{M} = g(y)$ is a function of y, then $e^{\int g(y) dy}$ is an integrating factor.

11 page

Integrating factor? A (must)

$$(x^2+y^2+x).dx + xy\, dy = 0 \quad \text{——} (1)$$

$$\frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = y$$

$$\therefore \frac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = \frac{2y-y}{xy} = \frac{1}{x} = f(x)$$

$$\text{I.F.} = e^{\int f(x).dx} = e^{\log x} = x$$

$x(x^2+y^2+x).dx + x^2y.dy = 0$

Exact equation.

solution:

$$\int (x^3+xy^2+x^2)dx + \int 0.dy = c$$

or, $\frac{x^4}{4} + \frac{x^2}{2}.y^2 + \frac{x^3}{3} + c_1 = c$

or, $\frac{x^4}{4} + \frac{x^2 y^2}{2} + \frac{x^3}{3} = c$

~~2~~ 3 (from book) → 45 page

$(x^2+y^2).dx - 2xy.dy = 0$ .... (1)

$M = x^2+y^2$; $N = -2xy$

$\frac{\partial M}{\partial y} = 2y$; $\frac{\partial N}{\partial x} = -2y$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{-2xy} = -\frac{2}{x} = f$$

$$\text{I.F.} = e^{\int f(x).dx} = e^{-2\int \frac{1}{x}.dx} = e^{\log x^{-2}} = \frac{1}{x^2}$$

Math (continuing next class)

Linear Differential Equations with Constant Coefficients

$$\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$$

$$(D^3 + 6D^2 + 11D + 6)\,y = 0$$

Let, $y = e^{mx}$, then $\frac{dy}{dx} = me^{mx}$, $\frac{d^2y}{dx^2} = m^2e^{mx}$, $\frac{d^3y}{dx^3} = m^3 \cdot e^{mx}$

Then eq. becomes $(m^3 + 6m^2 + 11m + 6)\,e^{mx} = 0$

Auxiliary eq. is $m^3 + 6m^2 + 11m + 6 = 0$ Roots $\begin{cases} m_1 = -1 \\ m_2 = -2 \\ m_3 = -3 \end{cases}$

The complete solution is $y = c_1e^{-x} + c_2e^{-2x} + c_3e^{-3x}$