Transformer all day efficiency

A distribution transformer cannot be run with constant load throughout 24 hours. At day peak time it's loading is high, whereas in night lean time its loading may be negligible. So selecting a transformer depending upon its conventional efficiency is not practical and economical, too. As a solution of these problems, the concept of all day efficiency of distribution transformer came into the picture.

In this concept, we use the ratio of total energy delivered by the transformer to the total energy fed to the transformer, during a 24 hrs span of time instead of ratio of power output and input of the transformer. Hence, all day efficiency is determined as, total KWh at the secondary of the total KWh at the primary of the transformer for a long specific period preferably 24 hrs. i.e,

$$All\ day\ efficiency = \frac{Output\ energy\ (kWh)\ in\ a\ fixed\ time\ interval\ of\ 24\ hours}{Input\ energy\ (kWh) over\ the\ same\ interval}$$

$$All\ day\ efficiency = \frac{\sum Output\ power \times Hour}{\sum Output\ power \times Hour + \sum Copper\ loss\ \times Hour + Core\ loss\ \times 24\ hour}$$

This is very much useful to judge the performance of a distribution transformer, whose primary is connected to the system forever, but secondary load varies tremendously throughout the day. Since the load is continually varying, conventional efficiency would not be an accurate reflection of the transformer's capability

Transformer maximum efficiency

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{out}}{P_{out} + P_{core} + P_{Cu}} = \frac{V_s I_s cos\theta_s}{V_s I_s cos\theta_s + P_{core} + I_s^2 R_{eq,s}}$$

$$\frac{1}{\eta} = \frac{V_s I_s cos\theta_s + I_s^2 R_{eq,s} + P_{core}}{V_s I_s cos\theta_s} = 1 + \frac{I_s^2 R_{eq,s} + P_{core}}{V_s I_s cos\theta_s}$$

For maximum efficiency condition,

$$\frac{d}{dI_s}\left(\frac{1}{\eta}\right) = 0$$

$$\frac{R_{eq,s}}{V_s cos\theta_s} - \frac{P_{core}}{V_s I_s^2 cos\theta_s} = 0$$

$$P_{core} = I_s^2 R_{eq,s} = P_{Cu}$$

$$P_{fixed} = P_{variable}$$

Hence, the transformer will operate at the maximum efficiency if variable loss (copper loss) equals the fixed loss (core loss).

Per-Unit System

In the per-unit system, the voltages, currents, powers, impedances, and other electrical quantities are not measured in their usual SI units (volts, amperes, watts, ohms, etc.). Instead, *each electrical quantity is measured as a decimal fraction* of some base level. Any quantity can be expressed on a per-unit basis by the equation where "actual value" is a value in volts, amperes, ohms, etc.

$$Quantity\ per\ unit = \frac{Actual\ value}{Base\ value\ of\ quantity}$$

There is another approach to solving circuits containing transformers which eliminates the need for explicit voltage-level conversions at every transformer in the system. Instead, the required conversions are handled automatically by the method itself, without ever requiring the user to worry about impedance transformations. Because such impedance transformations can be avoided, circuits containing many transformers can be solved easily with less chance of error. This method of calculation is known as the *per-unit (pu) system* of measurements.

There is yet another advantage to the per-unit system that is quite significant for electric machinery and transformers. *As* the size of a machine or transformer varies, its internal impedances vary widely. However, it turns out that in a per-unit system related to the device's ratings, *machine and transformer impedances fall within fairly narrow ranges* for each type and construction of device. This fact can serve as a useful check in problem solutions. Information regarding the impedance of transformer windings is generally available from the manufacturer or from the transformer nameplate as per-unit or percent impedance.

It is customary to select two base quantities to define a given per-unit system. The ones usually selected are voltage and apparent power. Once these base quantities have been selected, all the other base values are related to them by the usual electrical laws. In a single-phase system, these relationships are

$$S_{base} = V_{base}\ I_{base}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

In a power system, a base values for apparent power and voltage are selected at a specific point in the system. A transformer has no effect on the base apparent power of the system, since the apparent power into a transformer equals the apparent power out of the transformer. On the other hand, voltage changes when it goes through a transformer, so the value of V_{base} changes at every transformer in the system according to its turns ratio. Because the *base quantities* change in passing through a transformer, the process of referring quantities to a common voltage level is automatically taken care of during per-unit conversion.

For a simple system with a single transformer, the rated apparent power and rated voltages are taken as base values.

$$S_{base} = S_{rated}$$

$$V_{base,LS} = V_{rated,LS} \text{ and } V_{base,HS} = V_{rated,HS}$$

$$Z_{base,LS} = \frac{V_{rated,LS}^2}{S_{rated}} \text{ and } Z_{base,HS} = \frac{V_{rated,HS}^2}{S_{rated}}$$

$$\left(Z_{pu}\right)_{LS} = \frac{Z_{eq,LS}}{Z_{base,LS}}$$

$$\left(Z_{pu}\right)_{HS} = \frac{Z_{eq,HS}}{Z_{base,HS}} = \frac{a^2 Z_{eq,LS}}{V_{rated,HS}^2/S_{rated}} = \frac{a^2 Z_{eq,LS}}{\left(aV_{rated,LS}\right)^2/S_{rated}} = \frac{Z_{eq,LS}}{Z_{base,LS}}$$

$$\therefore \left(Z_{pu}\right)_{LS} = \left(Z_{pu}\right)_{HS}$$

Thus per-unit impedance will be same whether calculated using all high-side or all low-side values.

If required, base currents can be found using the base voltages and base apparent power.

$$I_{base,HS} = \frac{S_{rated}}{V_{rated,HS}} = I_{rated,HS} \text{ and } I_{base,LS} = \frac{S_{rated}}{V_{rated,LS}} = I_{rated,LS}$$

it is to be noted that base impedance serves as the base for both resistance and reactance.

$$R_{pu} = \frac{R_{eq}}{Z_{base}} = \frac{I_{rated}\ R_{eq}}{V_{rated}}$$

$$X_{pu} = \frac{X_{eq}}{Z_{base}} = \frac{I_{rated}\ X_{eq}}{V_{rated}}$$

Here, $I_{rated}, V_{rated}, R_{eq}, X_{eq}$ must be all high-side or all low-side values.

The per-unit impedance in terms of its components, $\boldsymbol{Z}_{pu} = R_{pu} + jX_{pu}$

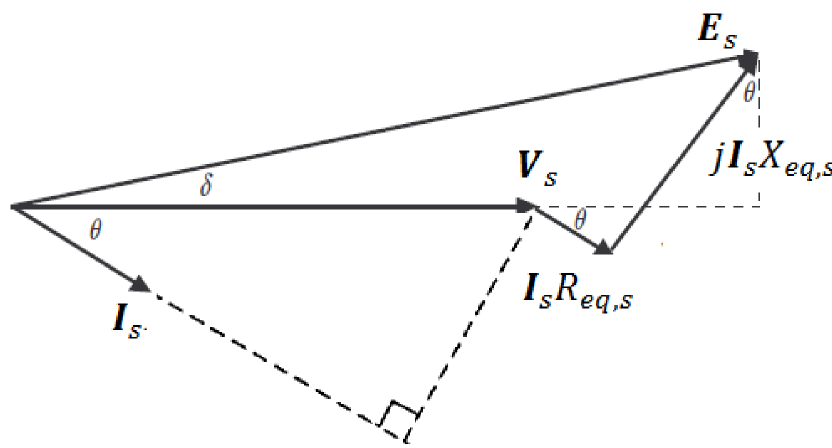
Transformers rated above 100 kVA have conductors of such large cross-sectional area that $X_{pu} >> R_{pu}$

Thus for very large transformers, $\boldsymbol{Z}_{pu} \approx jX_{pu}$

If percent values are given, per-unit values can be found simply dividing by 100.

$$Z_{pu} = \frac{\%Z}{100}$$

Calculating regulation from per-unit values



From phasor diagram, $\boldsymbol{E}_s = \boldsymbol{V}_s + \boldsymbol{I}_s R_{eq,s} + j\boldsymbol{I}_s X_{eq,s}$

$\boldsymbol{V}_s$ can be decomposed into components along $\boldsymbol{I}_s$ and in the perpendicular direction of $\boldsymbol{I}_s$.

$$E_s = \sqrt{\left(I_s R_{eq,s} + V_s cos\theta\right)^2 + \left(I_s X_{eq,s} + V_s sin\theta\right)^2}$$

$$reg = \frac{V_{nl} - V_{fl}}{V_{fl}} = \frac{E_s - V_s}{V_s}$$

$$reg = \frac{\sqrt{\left(I_s R_{eq,s} + V_s cos\theta\right)^2 + \left(I_s X_{eq,s} + V_s sin\theta\right)^2} - V_s}{V_s}$$

$$reg = \sqrt{\left(\frac{I_s R_{eq,s}}{V_s} + cos\theta\right)^2 + \left(\frac{I_s X_{eq,s}}{V_s} + sin\theta\right)^2} - 1$$

$$reg = \sqrt{\left(\frac{I_s}{I_{rated\,,s}} \frac{I_{rated\,,s}\, R_{eq,s}}{V_s} + cos\theta\right)^2 + \left(\frac{I_s}{I_{rated\,,s}} \frac{I_{rated\,,s}\, X_{eq,s}}{V_s} + sin\theta\right)^2} - 1$$

$$reg = \sqrt{\left(\frac{I_s}{I_{rated\,,s}} \frac{I_{rated\,,s}\, R_{eq,s}}{V_{rated\,,s}} + cos\theta\right)^2 + \left(\frac{I_s}{I_{rated\,,s}} \frac{I_{rated\,,s}\, X_{eq,s}}{V_{rated\,,s}} + sin\theta\right)^2} - 1$$

$$reg = \sqrt{\left(\frac{I_s}{I_{rated\,,s}} \frac{R_{eq,s}}{Z_{base\,,s}} + cos\theta\right)^2 + \left(\frac{I_s}{I_{rated\,,s}} \frac{X_{eq,s}}{Z_{base\,,s}} + sin\theta\right)^2} - 1$$

$$reg = \sqrt{\left(I_{pu}\, R_{pu} + cos\theta\right)^2 + \left(I_{pu}\, X_{pu} + sin\theta\right)^2} - 1$$

$$\boxed{I_{pu} = \frac{I_s}{I_{rated\,,s}} = \frac{I_s\, V_{rated\,,s}}{I_{rated\,,s} V_{rated\,,s}} = \frac{S}{S_{rated}} = S_{pu}}$$

$$reg = \sqrt{\left(S_{pu}\, R_{pu} + cos\theta\right)^2 + \left(S_{pu}\, X_{pu} + sin\theta\right)^2} - 1$$

Here, $\theta = \cos^{-1} pf$ for lagging power-factor loads and $\theta = -\cos^{-1} pf$ for leading power-factor loads

If the transformer operates at rated load, then S_{pu} =1 and the expression reduces to

$$reg = \sqrt{\left(R_{pu} + cos\theta\right)^2 + \left(X_{pu} + sin\theta\right)^2} - 1$$

Calculating efficiency from per-unit values

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{out}}{P_{out} + P_{core} + P_{Cu}} = \frac{S \times pf}{S \times pf + P_{core} + I_s^2 R_{eq,s}}$$

$$\eta = \frac{(S/S_{rated}) \times pf}{(S/S_{rated}) \times pf + (P_{core}/S_{rated}) + \left(I_s^2 R_{eq,s}/S_{rated}\right)}$$

$$\eta = \frac{(S/S_{rated}) \times pf}{(S/S_{rated}) \times pf + (P_{core}/S_{rated}) + \left(I_s^2 R_{eq,s}/V_{rated} I_{rated}\right)}$$

$$\eta = \frac{(S/S_{rated}) \times pf}{(S/S_{rated}) \times pf + (P_{core}/S_{rated}) + \left(I_s^2/{I_{rated}}^2\right) \times \left(I_{rated} R_{eq,s}/V_{rated}\right)}$$

$$\eta = \frac{S_{pu} \times pf}{S_{pu} \times pf + P_{core,pu} + I_{pu}^2 \times R_{pu}}$$

$$\boxed{I_{pu} = \frac{I_s}{I_{rated,s}} = \frac{I_s V_{rated,s}}{I_{rated,s} V_{rated,s}} = \frac{S}{S_{rated}} = S_{pu}}$$

$$\eta = \frac{S_{pu} \times pf}{S_{pu} \times pf + P_{core,pu} + S_{pu}^2 \times R_{pu}}$$

If the transformer operates at rated load, then $S_{pu} = 1$ and the expression reduces to

$$\eta = \frac{pf}{pf + P_{core,pu} + R_{pu}}$$

Conversion of per-unit values from one base to another base

Nameplate impedances are given in per-unit values respective to their own base. However, analyzing a power system with numerous motors, generators and transformers requires to select a base apparent power and a base voltage at a specific point of the system. If the chosen base is different from the actual rating, then per-unit values to be converted into the new base.

$$Z_{pu,new} = \frac{Z_{eq}}{Z_{base,new}}$$

$$Z_{pu,old} = \frac{Z_{eq}}{Z_{base,old}}$$

$$\frac{Z_{pu,new}}{Z_{pu,old}} = \frac{Z_{base,old}}{Z_{base,new}}$$

$$\frac{Z_{pu,new}}{Z_{pu,old}} = \frac{V_{old}^2/S_{old}}{V_{new}^2/S_{new}}$$

$$Z_{pu,new} = \frac{S_{new}}{S_{old}} \times \frac{V_{old}^2}{V_{new}^2} \times Z_{pu,old}$$

Parallel Operation of Transformers:

https://electricalnotes.wordpress.com/2012/07/17/parallel-operation-of-transformers/

For supplying a load in excess of the rating of an existing transformer, two or more transformers may be connected in parallel with the existing transformer. It is usually economical to install another transformer in parallel instead of replacing the existing transformer by a single larger unit. In addition, at least half the load can be supplied with one transformer out of service and total power outage can be avoided.

To maximize electrical power system flexibility: There is always a chance of increasing or decreasing future demand of power system. If it is predicted that power demand will be increased in future, there must be a provision of connecting transformers in system in parallel to fulfill the extra demand because, it is not economical from business point of view to install a bigger rated single transformer by forecasting the increased future demand as it is unnecessary investment of money. Again if future demand is decreased, transformers running in parallel can be removed from system to balance the capital investment and its return.

To maximize electrical power system availability: If numbers of transformers run in parallel, we can shutdown any one of them for maintenance purpose. Other parallel transformers in system will serve the load without total interruption of power.

To maximize power system reliability: if any one of the parallel transformers is tripped due to fault, other transformers will share the load. Hence power supply may not be interrupted if the shared loads do not make other transformers over loaded.

To maximize electrical power system efficiency: Generally electrical power transformer gives the maximum efficiency at full load. If we run numbers of transformers in parallel, we can switch on only those transformers which will give the total demand by running nearer to its full load rating for that time. When load increases, we can switch none by one other transformer connected in parallel to fulfill the total demand. In this way we can run the system with maximum efficiency.

Conditions for Parallel Operation of Transformers

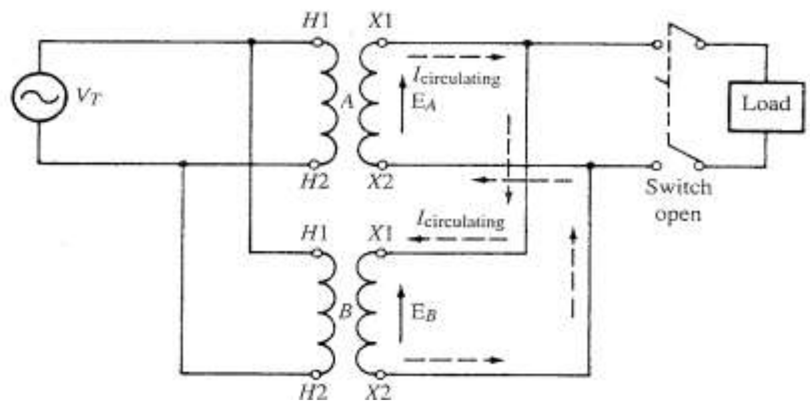
When two or more transformers run in parallel, they must satisfy the following conditions for satisfactory performance. These are the conditions for parallel operation of transformers.

1. Same voltage ratio of transformer
2. Same percentage impedance
3. Same polarity
4. Same phase sequence (for 3-phase transformers)

Unmatched impedances result in unequal load sharing between the paralleled transformers, whereas huge circulating current flows if the other conditions are not maintained.

Effect of unequal voltage ratio- Circulating Current:

If two transformers of different voltage ratio are connected in parallel with same primary supply voltage, there will be a difference in secondary voltages. Now say the secondary of these transformers are connected to same bus, there will be a circulating current between secondaries. As the internal impedance of transformer is small, a small voltage difference may cause sufficiently high circulating current causing unnecessary extra I^2R loss.



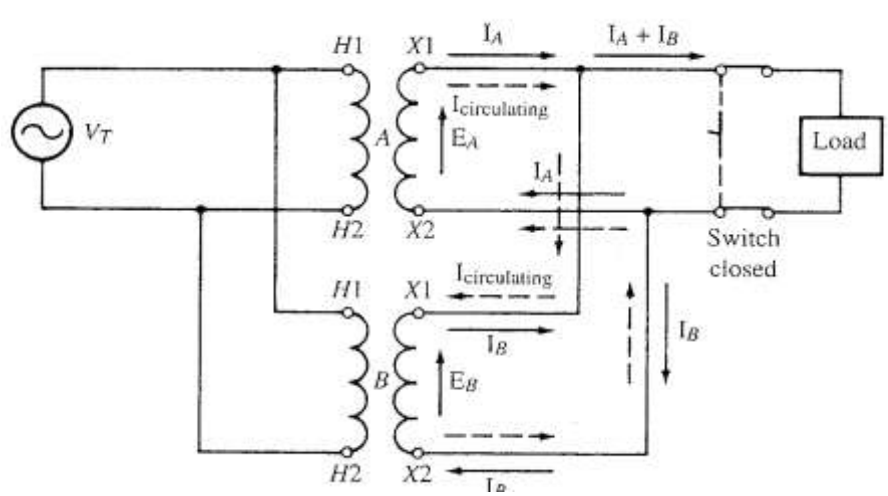
The circulating flows in the closed loop formed by the two secondaries even when no-load is connected.

Considering transformer A has a higher voltage ratio compared to transformer B, the circulating current can be expressed as:

$$\boldsymbol{I_{cir}} = \frac{\boldsymbol{E_A} - \boldsymbol{E_B}}{\boldsymbol{Z_A} + \boldsymbol{Z_B}}$$

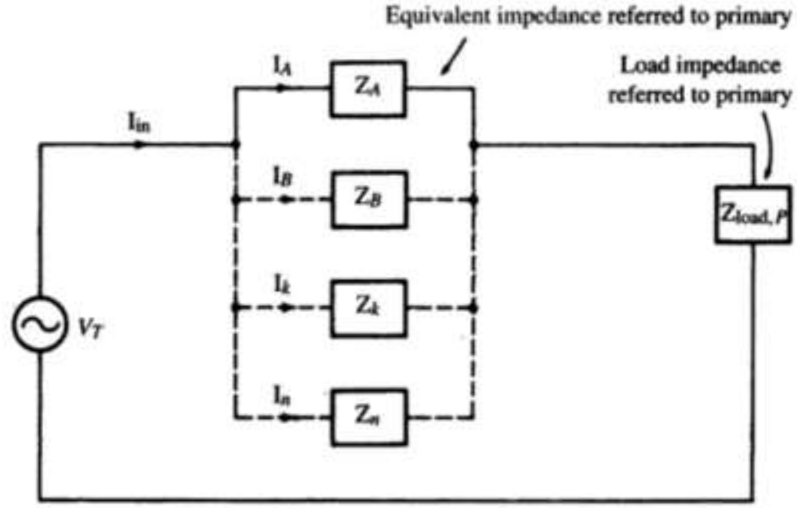
where Z_A and Z_B are the equivalent impedance of both transformers respective to their secondaries.

When the load switch is closed, the circulating current adds to the load current in one transformer and subtracts from the load current in the other transformer. Thus, if the transformer is operating at rated load, the transformer with the higher secondary voltage will be overloaded, and the other transformer will be underloaded. Overloading has adverse effects on transformer service life, because resultant overheating damages the winding insulation.



Effect of unmatched impedances – unequal load distribution

The current shared by two transformers running in parallel should be proportional to their MVA ratings. Again, current carried by these transformers are inversely proportional to their internal impedance. From these two statements it can be said that, impedance of transformers running in parallel are inversely proportional to their MVA ratings. In other words, percentage impedance or per unit values of impedance should be identical for all the transformers that run in parallel.



Equivalent circuit for paralleled transformers.

If all the transformers have the same turns ratio,, they may be represented by paralleled impedances. According to current-divider rule, current through the k-th transformer can be expressed as:

$$\boldsymbol{I_k} = \frac{1/\boldsymbol{Z_k}}{1/\boldsymbol{Z_1} + 1/\boldsymbol{Z_2} + \cdots + 1/\boldsymbol{Z_k} + \cdots + 1/\boldsymbol{Z_n}} \boldsymbol{I_{bank}} = \frac{\boldsymbol{Y_k}}{\boldsymbol{Y_p}} \boldsymbol{I_{bank}}$$

where $\boldsymbol{Y_k}$ is the equivalent impedance of transformer k and $\boldsymbol{Y_p}$ is the equivalent impedance of all paralleled transformers.

If all the transformers have identical impedances (both in magnitude and angle), current will be equally divided among all the paralleled transformers i.e. load will be shared equally. If the transformer impedances differ either in magnitude or phase (or both), loads will be unequally shared among them. Due to this, the transformer with the lower impedance may get overheated at rated load.

Note that, if the transformer impedances are given in percent or per-unit and they have the same base impedance (identical voltage rating and KVA rating), percent or per-unit values may be used in place of the equivalent ohmic impedance to calculate the current drawn by each transformer.

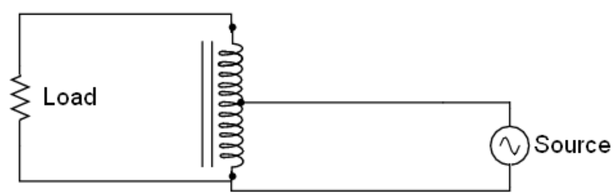
For two transformers connected in parallel, load shared by each transformer can be found from

$$\frac{S_1}{S_2} = \frac{Z_{eq,2}}{Z_{eq,1}}$$

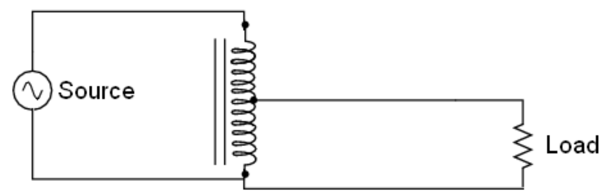
Autotransformer

An autotransformer is a single coil transformer with one or more taps to provide transformer action. Unlike conventional two-winding transformers, there is a direct physical connection between the primary and the secondary circuits. Therefore autotransformers should only be used in applications where lack of electrical isolation does not present a safety hazard.

It is common practice in power systems to use autotransformers whenever two voltages fairly close to each other in level need to be transformed, In those applications, where continuous non-interruptible adjustment of voltage is required, autotransformers are used. They are also used as variable transformers, where the low-voltage tap moves up and down the entire winding.



Autotransformer *(As a Step-Up Transformer)*



Autotransformer *(As a Step-Down Transformer)*

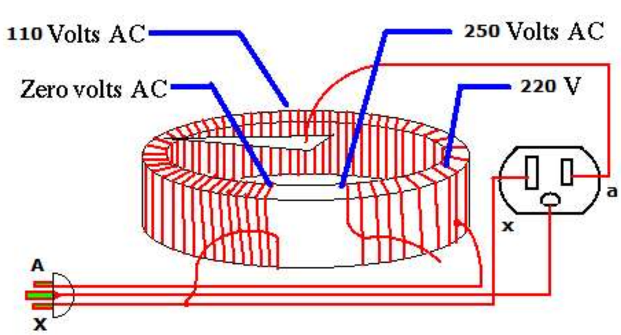
Comparison of autotransformers and conventional transformers

Advantages: Autotransformers provide a great deal of advantages compared to their two winding counterparts due to their single-coil construction.

- less leakage flux,
- less copper, less iron,
- less weight,
- takes up less space
- more efficient
- cost less

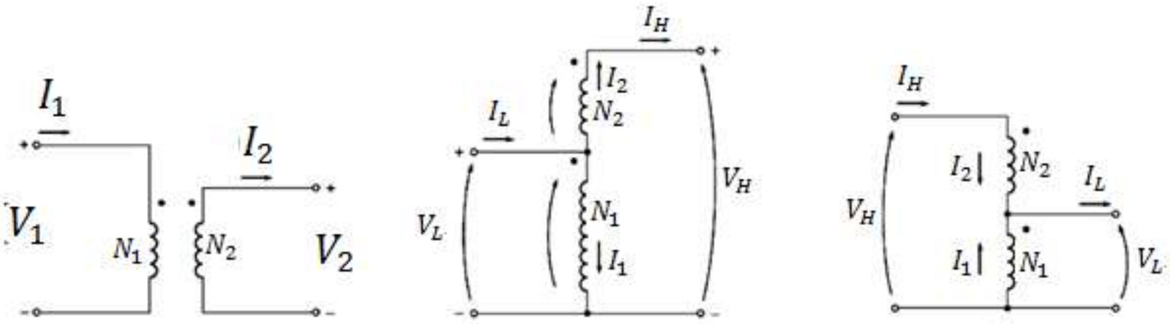
Disadvantages: No electrical isolation between the primary and the secondary.

Variac: Variacs, which are AC power supplies, are the most popular variable auto transformers on the market because they are cheaper, lighter, and smaller than dual-winding transformers. These adjustable power supplies can be used for a number of hobbies and are very versatile.



Input between A-X, output between a-x

The Apparent Power Rating Advantage of Autotransformers



(a) A transformer with its windings connected in the conventional manner

(b) Primary and secondary windings reconnected as a step-up autotransformer

(c) Primary and secondary windings reconnected as a step-down autotransformer

Apparent power rating as a 2-winding transformer,

$$S_{2w} = V_1 I_1 = V_2 I_2$$

Considering the transformer as ideal $\quad \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a$

Apparent power rating as an autotransformer,

$$S_{at,HS} = V_H I_H = (V_1 + V_2) I_2 = \left(\frac{V_1}{V_2} + 1\right) V_2 I_2 = (a+1) S_{2w}$$

$$S_{at,LS} = V_L I_L = V_1 (I_1 + I_2) = V_1 I_1 \left(1 + \frac{I_2}{I_1}\right) = (a+1) S_{2w}$$

Apparent power rating advantage of an autotransformer over a conventional two-winding transformer,

$$\frac{S_{at}}{S_{2w}} = (a+1)$$

Note: If the primary *current* of the transformer flows *into* the dotted end of the primary winding, the secondary current will flow *out* of the dotted end of the secondary winding.

Internal Impedance of an Autotransformer

$$Z_{eq} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1}$$

$$Z_{eq}^{'} = \frac{\boldsymbol{V}_L}{\boldsymbol{I}_L} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1 + \boldsymbol{I}_2} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1} \frac{1}{1 + \boldsymbol{I}_2/\boldsymbol{I}_1} = \frac{1}{a+1} Z_{eq}$$

Effective impedance of an autotransformer is smaller by a factor equal to the reciprocal of the power advantage of the autotransformer connection.