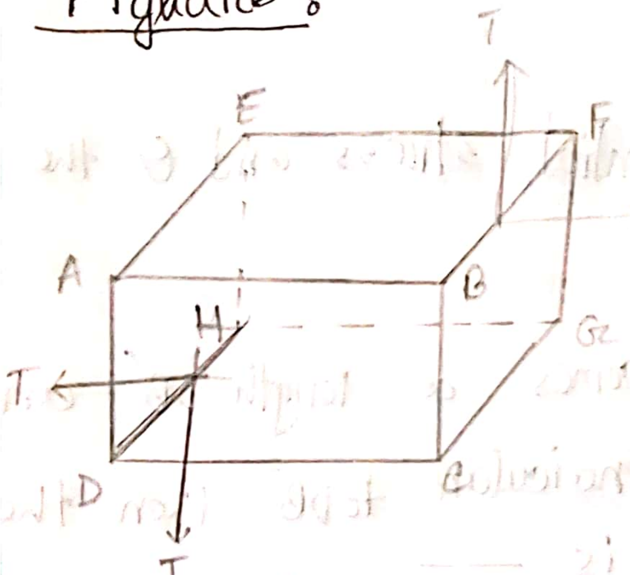
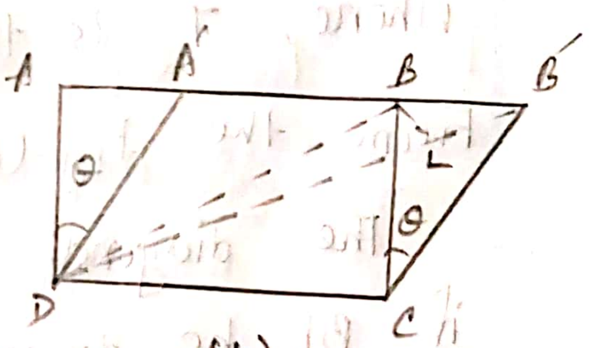
TOPIC NAME : 2102026 DAY : TIME : DATE : / /

Assignment name: Relation between Young's modulus (Y), Bulk modulus (K), Rigidity modulus (η) and poisson's ration (σ).

Figrare:



(a)



(b)

Describe:

Let a unit cube of a sustance be under the action of tangential stresses in fig (a).

The result will be distort the cube so that the faces ABCD becomes a rhombus A'B'CD fig(b). There is only a change of shape but the size remains the same since the area ABCD is equal to that of A'B'CD. The body is unchanged in dimension in a direction perpendicular to ABCD.

Here, the forces applied to the cube must be in equilibrium among them. That denoted by T.

Now, According to Hook's law —

$$\eta = \frac{T}{\theta}$$

where, T is the tangential stress and θ, the strain

From the Fig-(b),

The diagonal DB becomes a length DB' and if BL be drawn perpendicular to DB' then the extension strain along DB is —

$$\frac{DB' - DB}{DB} = \frac{LB'}{DB} = \frac{BB' \cos 45^\circ}{\sqrt{2}\, BC} = \frac{BB'}{\sqrt{2}} \times \frac{1}{\sqrt{2}\, BC}$$

$$= \frac{BB'}{2BC} = \frac{\theta}{2} = \left(\frac{1}{2}\right)\left(\frac{T}{\eta}\right) \quad \text{[where } \theta \text{ is small]}$$

Now,

According to Young's law ——

$$\text{Young's modulus } Y = \frac{\text{Longitudinal Stress, } T}{\text{Longitudinal Strain}}$$

$$Y = \frac{T}{\text{Long - strain}}$$

$$\therefore \text{Long. strain} = \frac{T}{Y}$$

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Again,

$$\text{Poisson's Ratio } \sigma = \frac{\text{lateral contraction strain}}{\text{Longitudinal strain}}$$

$\therefore$ Lateral Contraction Strain $= \sigma \left(\frac{T}{Y}\right)$

$$\therefore \frac{1}{2}\frac{T}{\eta} = \frac{T}{Y} + \sigma \frac{T}{Y} \Rightarrow \frac{T}{Y}(1+\sigma).$$

$$\therefore Y = 2\eta(1+\sigma) \quad \text{———} (i)$$

If the unit cube be subjected to a uniform pressure P over each face,

$$P = \frac{dv}{V} \quad \left[\begin{array}{l} dv = \text{diminution in volume} \\ V = \text{The original volume} \end{array} \right]$$

But by Hooke's Law,

$$\frac{\text{Stress}}{\text{Strain}} = \text{Bulk modulus}, K$$

$$\therefore \frac{dV}{V} = \frac{P}{K} = 3\alpha \quad [\alpha \text{ is the contraction}]$$

$$\therefore \alpha = \frac{P}{3K}$$

Again, For the direction perpendicular to one pair of faces the pressure P on those faces, produces a compression $\frac{P}{Y}$ and the pressures on the

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other two pair of faces produce stretches each to $\sigma(P/Y)$ ——

$$\therefore \frac{P}{3K} = \left(\frac{P}{Y}\right) - \left(\frac{2\sigma P}{Y}\right) = \frac{P}{Y}(1-2\sigma).$$

$$\therefore Y = 3K(1-2\sigma) \quad \text{——} (ii)$$

From the eqn (i) and (ii)

$$\sigma = \frac{3K-2\eta}{6K+2\eta}$$

$$Y = \frac{9K\eta}{3K+\eta}$$

we have,

$$Y = 2\eta(1+\sigma)$$

$$Y = 3K(1-2\sigma)$$

$$\therefore 2\eta(1+\sigma) = 3K(1-2\sigma). \quad \text{——} (iii)$$

$$\Rightarrow \frac{3K}{2\eta} = \frac{1+\sigma}{1-2\sigma} \quad \text{——} (iv)$$

Limiting Values of σ ——

(a) If the Poisson's ratio is a possitive quantity (K, η) ——

$(1-2\sigma) > 0$

$\therefore 2\sigma < 1$ or $\sigma < 0.5$

(b) If the Poisson's ratio is negative quantity (η, K) ———
positive

$(1+\sigma) > 0$

$\therefore \sigma > -1$

If means the value σ lies between -1 and $+0.5$

However,

In actual practice the value of σ can't be negative. when $\sigma = 0.5$ (volueme no chang) & Body is completely incompressible.

Therefore,

The value of σ for most of the isotropic substances. Range is 0.2 to 0.4

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