Integration Rules

(i) $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

(ii) $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left|x+\sqrt{a^2+x^2}\right| + C$

(iii) $\int \sqrt{a^2+x^2} \Rightarrow \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right| + C$

(iv) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$

(v) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

(vi) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$

(vii) $\int \sqrt{a^2-x^2} = x/2\sqrt{a^2-x^2} + a^2/2 \sin^{-1} x/a + C$

(viii) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

(ix) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$

(x) $\int \sqrt{x^2-a^2} = x/2\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

T-1 : $\int \frac{dx}{ax^2+bx+c}$

$\Rightarrow \int \frac{dx}{x^2+x+1} = \int \frac{dx}{x^2+2.x.\frac{1}{2}+1+(\frac{1}{2})^2-(\frac{1}{2})^2}$

$\Rightarrow \int \frac{dx}{(x+1/2)^2+(\sqrt{3}/2)^2}$ (x^2, a^2)

$\Rightarrow \frac{1}{\sqrt{3}/2} \tan^{-1} \frac{x+1/2}{\sqrt{3}/2}$ Ans

T-2 : $\frac{dx}{\sqrt{x^2+x+1}}$

$\Rightarrow \int \frac{dx}{\sqrt{(x+1/2)^2+(\frac{\sqrt{3}}{2})^2}}$ (x^2, a^2)

$\Rightarrow \ln\left[(x+1/2)+\sqrt{x^2+x+1}\right]+C$

T-3 : $\int \sqrt{x^2+x+1}$

$\Rightarrow \int \sqrt{(x+1/2)^2+(\sqrt{3}/2)^2}$

$\Rightarrow \frac{(x+1/2)\sqrt{x^2+x+1}}{2} + \frac{(\frac{\sqrt{3}}{2})^2}{2} \cdot \ln\left[x+\frac{1}{2}+\sqrt{x^2+x+1}\right]$

T-4 : $\int \frac{3x+4}{x^2+x+1} dx$ { লব < হর, x-এর সাপেক্ষে }

differentiation of x^2+x+1

$\Rightarrow \int \frac{\frac{3}{2}(2x+1)+4-3/2}{x^2+x+1} dx$

$\Rightarrow \frac{3}{2}\int \frac{\frac{d}{dx}(x^2+x+1)}{x^2+x+1} dx + \frac{5}{2}\int \frac{1}{x^2+x+1} dx$

$\Rightarrow \frac{3}{2}\ln(x^2+x+1) + \frac{5}{2}\int \frac{1}{(x+1/2)^2+(\frac{\sqrt{3}}{2})^2} dx$

$\Rightarrow \frac{3}{2}\ln(x^2+x+1) + \frac{5}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\frac{x+1/2}{\sqrt{3}/2}$

Rule: $\frac{f'(x)}{f(x)} = \ln f(x)$

Rule: $\frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)}$

T-5: $\int \frac{3x+4}{\sqrt{x^2+x+1}} dx$

$\Rightarrow \int \frac{\frac{3}{2}(2x+1)+(4-\frac{3}{2})}{\sqrt{x^2+x+1}} dx$

$\Rightarrow \frac{3}{2}\int \frac{\frac{d}{dx}(x^2+x+1)}{\sqrt{x^2+x+1}} + \frac{5}{2}\int \frac{dx}{\sqrt{(x+1/2)^2+(\frac{\sqrt{3}}{2})^2}}$

$\Rightarrow \frac{3}{2}\times 2\sqrt{x^2+x+1} + $ T-2

T-6: (x-এর সাপেক্ষে হর = লব হলে)

$\int \frac{2x^2+3x+4}{x^2+x+1} dx$

$\Rightarrow \int \frac{2(x^2+x+1)+\frac{1}{2}(2x+1)+(4-2-1/2)}{x^2+x+1} dx$

$\Rightarrow 2\int dx + \frac{1}{2}\ln(x^2+x+1) +$ T-1

$\Rightarrow 2x + \frac{1}{2}\ln(x^2+x+1) + \frac{1}{\sqrt{3}/2}\tan^{-1}\frac{x+1/2}{\sqrt{3}/2}$

T-7: $\int \frac{1}{(cx+d)\sqrt{ax+b}}\,dx$

$\Rightarrow ax+b = z^2$ ধরি

বা: $\int \frac{dx}{(x+1)\sqrt{x+5}}$

$\Rightarrow \int \frac{2z\,dz}{(z^2-5+1)z}$ $\quad x+5=z^2$

$\therefore dx = 2z\,dz$

$\Rightarrow 2\int \frac{dz}{z^2-2^2} = 2\times\frac{1}{2\times 2}\ln\left|\frac{z-2}{z+2}\right|$

T-8: $\int \frac{1}{(x+1)(\sqrt{x^2+5})}$

$x+1=\frac{1}{z}$ ধরি,

$dx = -\frac{1}{z^2}\,dz$

$\Rightarrow \int \frac{1}{\frac{1}{z}\sqrt{(\frac{1}{z}-1)^2+5}} \cdot -1/z^2\,dz$

$\Rightarrow \int \frac{-\frac{1}{z^2}\,dz}{\frac{1}{z}\left(\sqrt{\frac{1}{z^2}-\frac{2}{z}+1+5}\right)}$

$\Rightarrow \int \frac{-1/z^2\,dz}{\frac{1}{z^2}\left(\sqrt{1-2z+6z^2}\right)}$

$\Rightarrow -\int \frac{dz}{\sqrt{1-2z+6z^2}}$

$\Rightarrow$ T-2 rule ১৯

I-2 $\int \frac{dx}{(x^2+1)\sqrt{x^2+5}}$

1st -ই $\Rightarrow x=\frac{1}{z}$ ধরি,

$\therefore dx = -\frac{1}{z^2}\,dz$

$\Rightarrow \int \frac{dx}{(\frac{1}{z^2}+1)\sqrt{(\frac{1}{z})^2+5}}$

$\Rightarrow \int \frac{-1/z^2\,dz}{(\frac{1}{z^2}+1)\sqrt{\frac{1}{z^2}+5}}$

2nd-ই,

$\Rightarrow \int \frac{-\frac{1}{z^2}\,dz}{(\frac{1+z^2}{z^2})\sqrt{\frac{1+5z^2}{z^2}}}$

$\Rightarrow \int \frac{-1/z\,dz}{(1+z^2)\frac{1}{z}\sqrt{1+5z^2}}$

2nd-ই,

$1+5z^2 = t^2$

$\therefore 0+5\times 2z = 2t\frac{dt}{dz}$

$\therefore 10z\,dz = 2t\,dt$

$\therefore zdz = t/5\,dt$

$\Rightarrow -\int \frac{t/5\,dt}{(\frac{t^2-1}{5}+1)\sqrt{t^2}}$

$\Rightarrow -\frac{1}{8}\int \frac{5\times dt}{t^2-1+5}$

$\Rightarrow -\int \frac{dt}{t^2+4} \Rightarrow -\frac{1}{2}\tan^{-1}\frac{t}{2}+c$

T-10 : $\int \sqrt{\frac{a+x}{a-x}}\,dx$

হরের অনুবর্গী নিয়ে হবে ⇒

$\int \sqrt{\frac{5+x}{5-x}}\,dx$

$\Rightarrow \int \frac{\sqrt{5+x}\,\sqrt{5+x}}{\sqrt{5-x}\,\sqrt{5+x}}\,dx$

$\Rightarrow \int \frac{5+x}{\sqrt{25-x^2}}\,dx$

$\Rightarrow \int \frac{5}{\sqrt{25-x^2}}\,dx + \int \frac{x}{\sqrt{25-x^2}}\,dx$

$\Rightarrow 5\sin^{-1}\frac{x}{5} + \boxed{T-5}$

T-11 অপকৃত ভগ্নাংশ হবে,

$\int \frac{x-2}{x+5}\,dx$ (হর এর মতো বানাতে হবে)

$\Rightarrow \int \frac{(x+5)-7}{x+5}\,dx$

$\Rightarrow \int 1\,dx - \int \frac{7}{x+5}\,dx$

$\Rightarrow x - 7\ln(x+5)$

T-12 : $\int \frac{x^2-2}{x+5}\,dx$

$\Rightarrow \int \frac{x(x+5)-5(x+5)-2+25}{x+5}\,dx$

$\Rightarrow \int x\,dx - \int 5\,dx - \int \frac{23}{x+5}\,dx$

$\Rightarrow \ldots\ldots$ (do it)

T-13 : $\int \frac{dx}{\sqrt[m]{x}-\sqrt[n]{x}}$ বা $\int \frac{x^{1/m}\,dx}{x^{1/n}\phi - 1}$

$x = z^p$ ধরি, ($p = m, n$ এর লসাগু)

$\int \frac{dx}{\sqrt[3]{x}-\sqrt[2]{x}}$ এ $x = z^6$, $dx = 6z^5$

$\Rightarrow \frac{6z^5}{z^3-z^2}$

$\Rightarrow 6\int \frac{z^2(z-1)+z(z-1)+(z-1)+1}{z-1}\,dz$

$\Rightarrow 6\left\{\int z^2 + z + 1 + \frac{1}{z-1}\right\}dz$

$\Rightarrow \ldots\ldots$ (do it)

T-14 :

$\int uv\,dx = u\int v\,dx - \int \left[\frac{d}{dx}u \cdot \int v\,dx\right]dx$

LIATE → u

T-15 :

$\int \frac{dx}{a+be^{mx}}$ বা $\int \frac{dx}{e^{mx}+e^{-mx}}$

↓ ↓

e^{-mx} দ্বারা লব e^{mx} দ্বারা লব

T-16 :

$e^{ax}[af(x) + f'(x)]\,dx$

$\Rightarrow e^{ax}\cdot f(x)$

T-17: $\int \left[\frac{1}{x} - \frac{1}{x^2}\right] dx = \frac{1}{x}$

সমাধান: $\int \left[\frac{1}{\ln(x)} - \frac{1}{\ln(x)^2}\right] dx$

$\Rightarrow \ln x = z$

বা $e^z = x$

$e^z dz = dx$

$\Rightarrow \int \left[\frac{1}{z} - \frac{1}{z^2}\right] e^z dz = e^z/z$

T-18: $\sin ax \cos bx$ / $\sin ax \sin bx$ / $\cos ax \cos bx \, dx$

$\Rightarrow \int \sin 7x \cos 3x \, dx$

$\Rightarrow \frac{1}{2} \int 2 \sin 7x \cos 3x \, dx$

$\Rightarrow \frac{1}{2} \int (\sin 10x + \sin 4x) \, dx$

Rule:

$2 \sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$

$2 \sin ax \sin bx = \cos(a-b)x + \cos(a+b)x$

$2 \cos ax \cos bx = \cos(a+b)x + \cos(a-b)x$

T-19: $\int \sin^m x \, dx$ বা $\int \cos^m x \, dx$

m = বিজোড় হলে,

সমাধান: $\int \cos^5 x \, dx$

$\Rightarrow \int \cos^4 x \cdot \cos x \, dx$

$\sin x = z$

$\cos x \, dx = dz$

$\Rightarrow \int (1 - \sin^2 x)^2 \, dz$

$\Rightarrow \int (1 - z^2)^2 \, dz$

---(do it)

Rule:

$2 \cos^2 x = 1 + \cos 2x$

m = জোড় হলে,

$\int \cos^4 x \, dx = \frac{1}{4} (2 \cos^2 x)^2$

$\Rightarrow \frac{1}{4} (1 + \cos 2x)^2$

$\Rightarrow \frac{1}{4} (1 + 2\cos 2x + \cos^2 2x)$

$\Rightarrow \frac{1}{4} + \frac{1}{2} \cos 2x + \frac{1}{8} (1 + \cos 4x)$

$\Rightarrow \int \frac{1}{4} + \frac{1}{2} \cos 2x + \frac{1}{8} + \frac{1}{8} \cos 4x \, dx$

---(do it)

T-20: $\int \tan^m x \, dx$ বা $\int \cot^m x \, dx$

জোড়, বিজোড় fact না

$\Rightarrow \int \tan^5 x \, dx$

$\tan x = z$

$\sec^2 x \, dx = dz$

$\Rightarrow \frac{z^5}{z^2 + 1} \, dz$

---(do it)

T-21: $\int \sin^m x \cos^n x \, dx$

m = জোড়, n = বিজোড়

যার power জোড় $\theta = z$

সমাধান: $\int \sin^4 x \cos^3 x \, dx$

$\Rightarrow \int \sin^4 x \cos^2 x \cos x \, dx$

$\Rightarrow \int z^4 (1 - z^2) \, dz$

$\sin x = z$

$\cos x \, dx = dz$

---(do it)

T-22: m = n = জোড় হলে

$\int \sin^4 x \cos^2 x \, dx$

~~$\sin x = z$~~

~~$\cos x \, dx = dz$~~

~~$\int z^4 (1-z^2)$~~

⇒ formula (বিকল্পভাবে) apply করে solve করতে হবে

T-23: m = n = বিজোড়

যার power বেশি = z

$\int \sin^3 x \cos^3 x \, dx$ $\quad \sin x = z$

$\cos x \, dx = dz$

⇒ $\int \sin^3 x \cos^2 x \cdot \cos x \, dx$

⇒ $z^5 (1-z^2) \, dz$

..... (do it)

T-24: $\int \frac{dx}{a+b\cos x}$ // $\int \frac{dx}{a+b\sin x}$ + $\int \frac{dx}{a+b\cos x + \sin x}$

Full:

$\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$

$\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$

$\tan x = \frac{2\tan x/2}{1-\tan^2 x/2}$

Example: $\int \frac{dx}{3+4\sin x} = \int \frac{dx}{3+4\left(\frac{2\tan x/2}{1+\tan^2 x/2}\right)}$

⇒ $\int \frac{1+\tan^2 \frac{x}{2} \, dx}{3+3\tan^2 \frac{x}{2} + 8\tan x/2}$

⇒ $\frac{\sec^2 \frac{x}{2} \, dx}{3+3z^2+8z}$ $\quad \tan \frac{x}{2} = z$

$\frac{1}{2}\sec^2 \frac{x}{2} \, dx = dz$

⇒ $\frac{2\,dz}{3+3z^2+8z}$

..... (do it)

T-25: $\int \frac{1}{a+b\cos^2 x} dx$ or $\int \frac{dx}{a+b\sin^2 x}$

or $\int \frac{dx}{a+b\cos^2 x + c\sin^2 x}$

(Tan এর convert করতে হবে)

definite integral

T-1 $\int_a^b f(x) \, dx$

T-2: $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$

T-3: $\int_a^b f(x) = k$

or, $\int_a^b f(kx) = \frac{P}{k}$