EEE 205 Energy Conversion II

Electromechanical Energy Conversion

A H Chowdhury, PhD
Professor
EEE, BUET

July 2018

Topic Content

Construction

- Armature (stator)
- Rotating field (exciter)
- Excitation system with brushes, brushless excitation system
- Cooling
- Distributed short pitched armature winding
    - Generated voltage equation
    - Armature winding connections and harmonic cancellation

Topic Content

Equivalent circuit

- Synchronous impedance
- Generated voltage and terminal voltage, phasor diagram, voltage regulation with different power factor type loads
- Determination of synchronous impedance by tests
- Salient pole generator d-q axes parameters, equivalent circuit, generator equations, determination of d-q axes parameters by tests
- Equation of developed power and torque of synchronous machines (salient and non salient pole motor and generator)

Topic Content

Parallel operation of generators

- Requirement of parallel operation
- Conditions of parallel operation
- Synchronizing, effect of synchronizing current, hunting and oscillation, synchronoscope, phase sequence indicator
- Load distribution of alternators in parallel
  - Droop setting
  - Frequency control
  - Voltage control
  - House diagrams

Synchronous Generator

- An electromechanical converter system has three essential parts:
  1) an electric system
  2) a mechanical system
  3) a coupling field

Synchronous Generator

Ashuganj steam-turbine generator

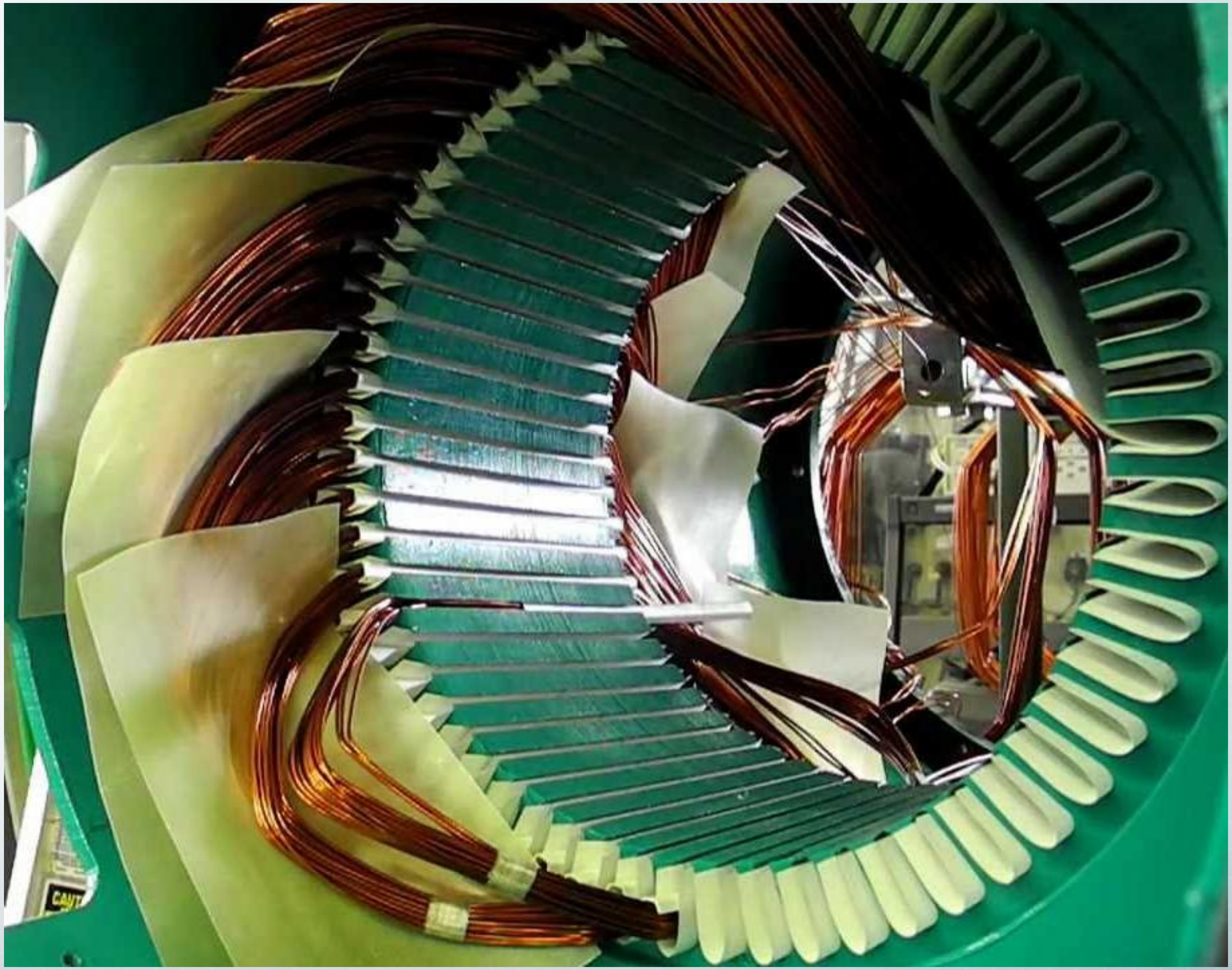
Synchronous Generator

Synchronous Generator Construction

Stator

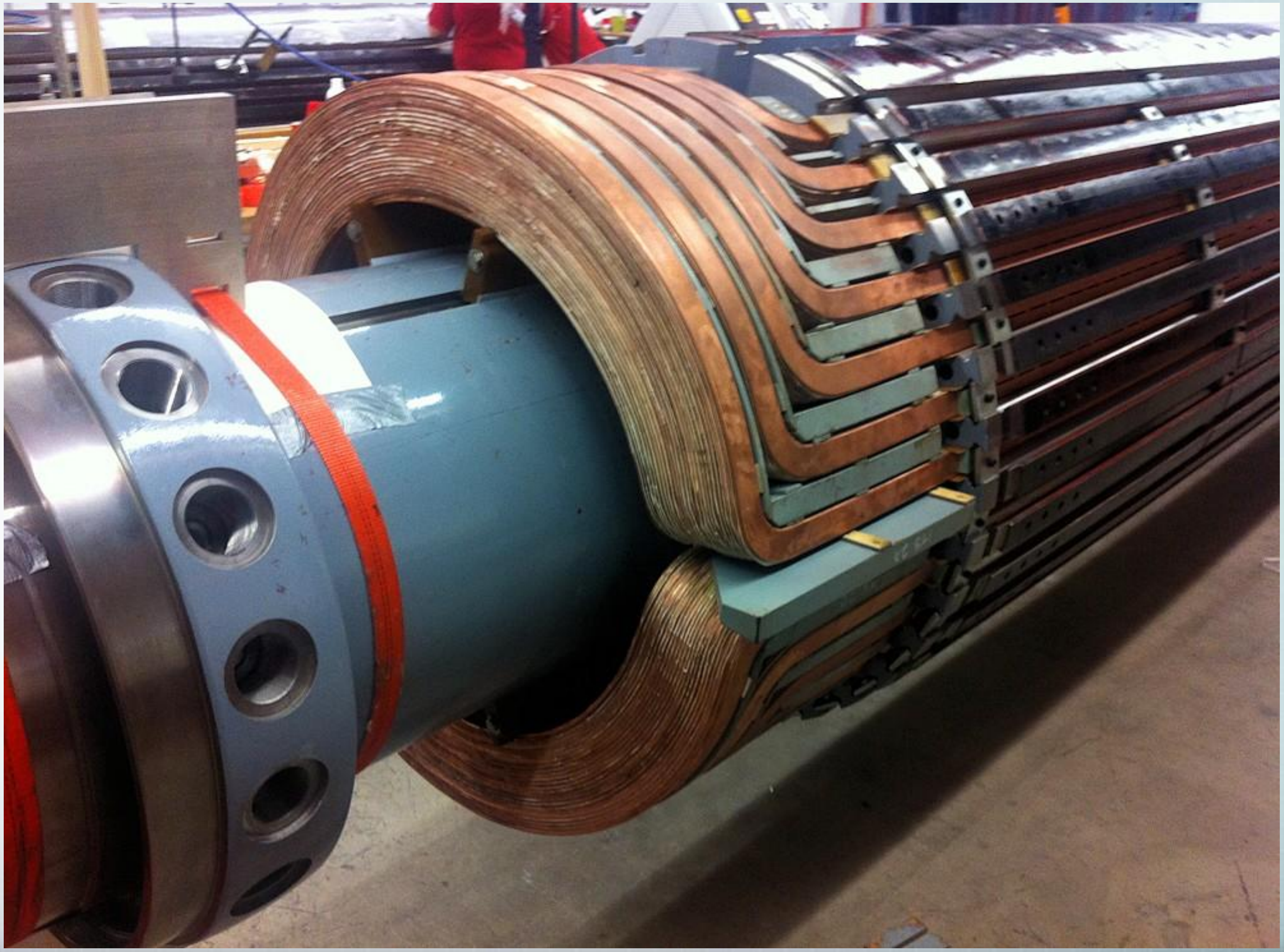
Synchronous Generator Construction

Stator



Synchronous Generator Construction

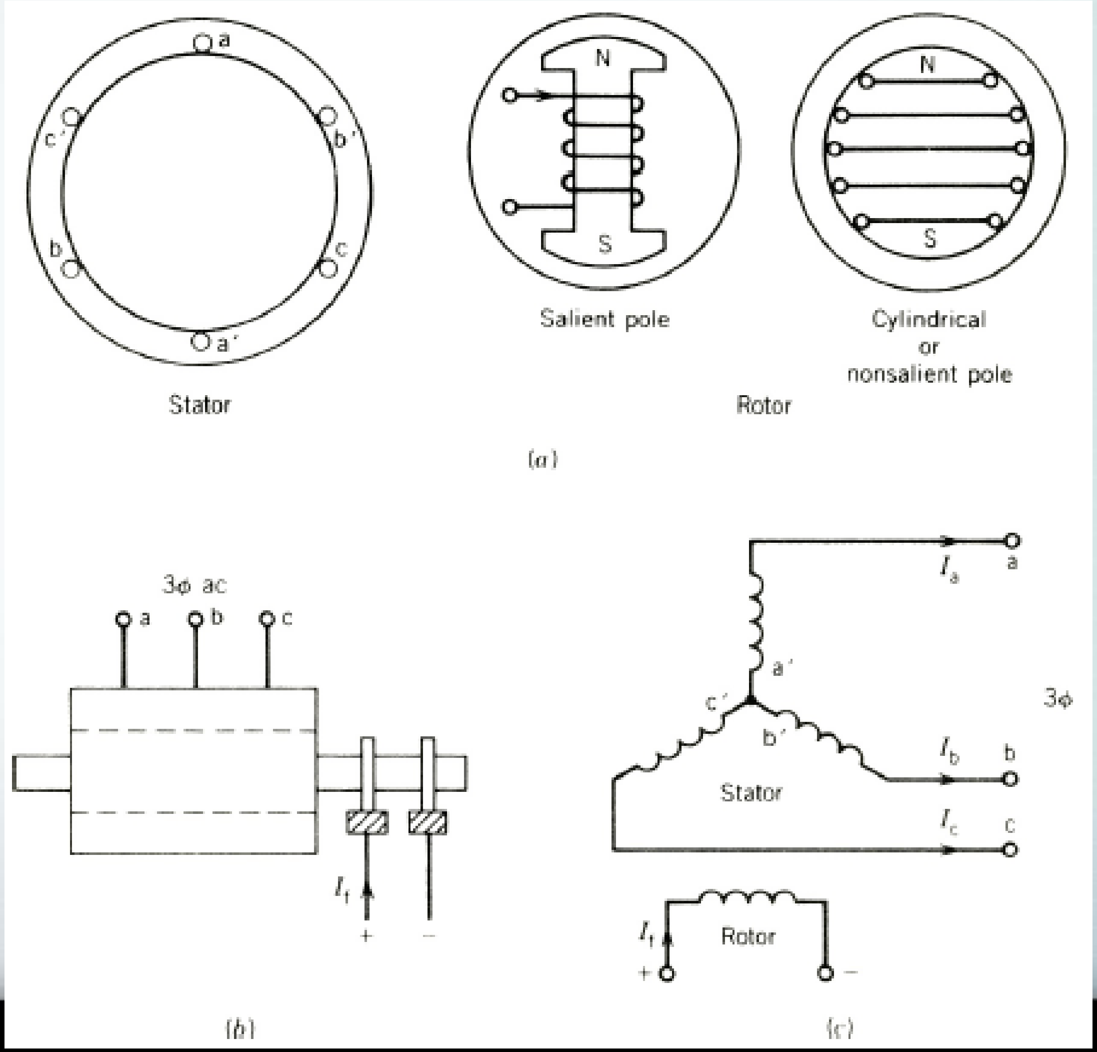
Rotor



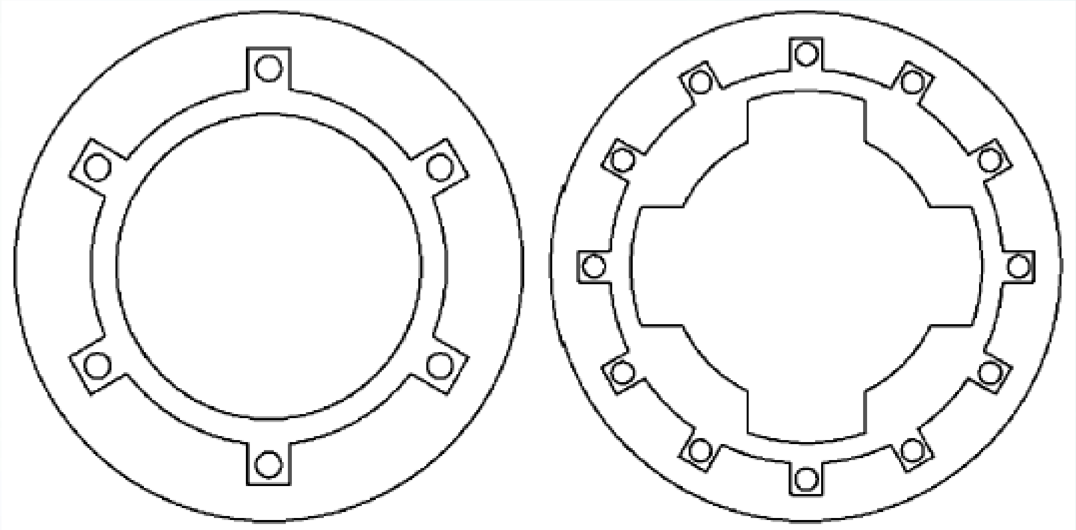
Synchronous Generator Construction

Rotor

Synchronous Generator Construction



Synchronous Generator Construction

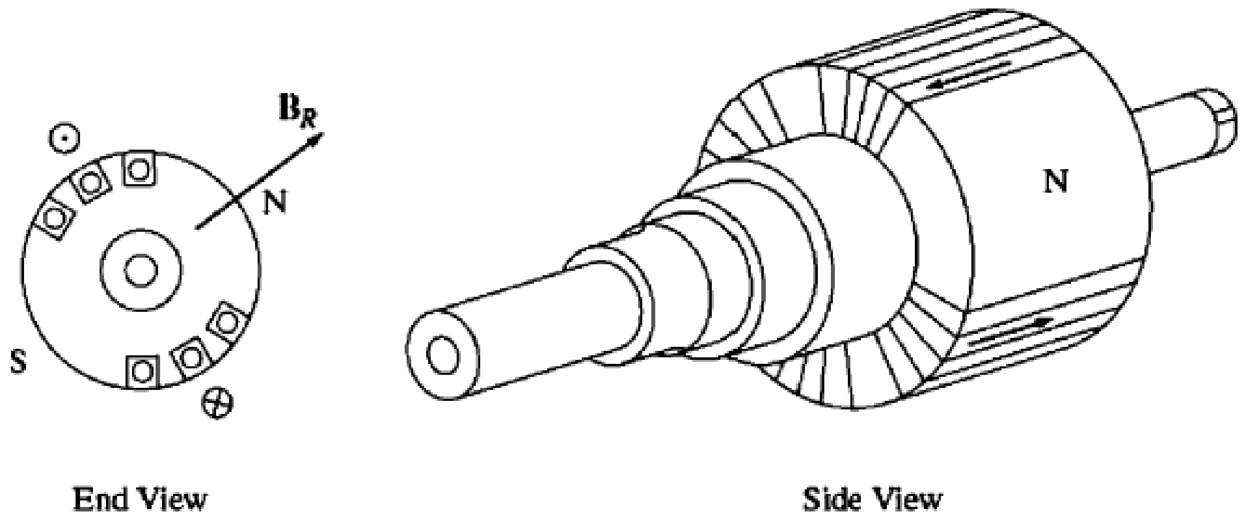


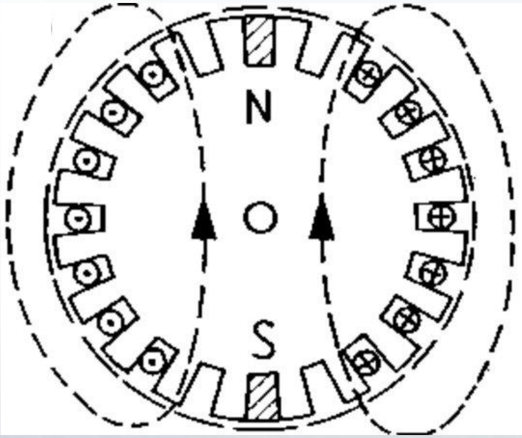
Nonsalient-pole rotor **Salient-pole rotor**

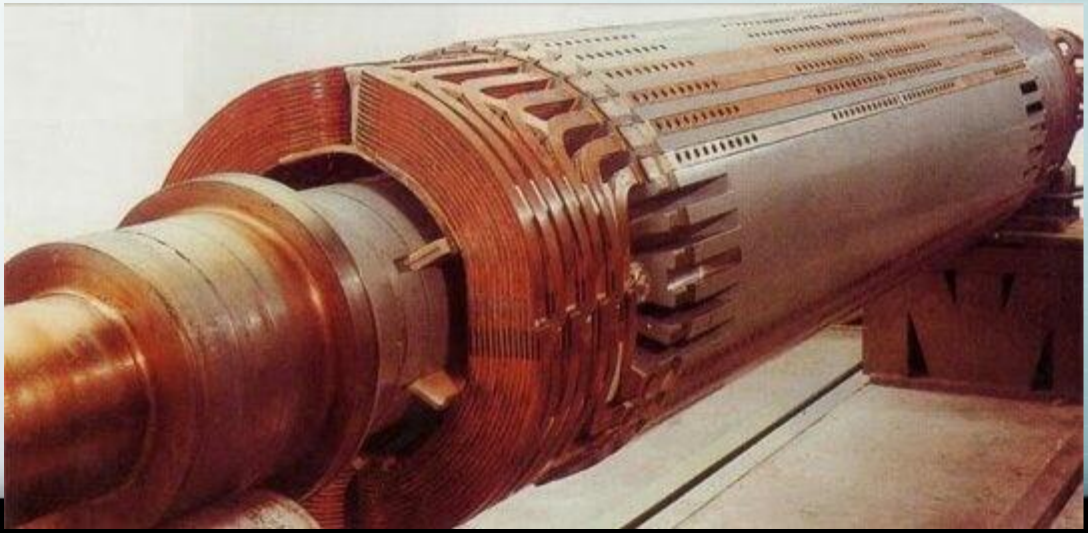
- **Nonsalient-pole rotors -** normally used for 2- and 4-pole rotors
- **Salient-pole rotors -** normally used for rotors with four or more poles
- Rotor constructed of thin laminations to reduce eddy current losses

Synchronous Generator

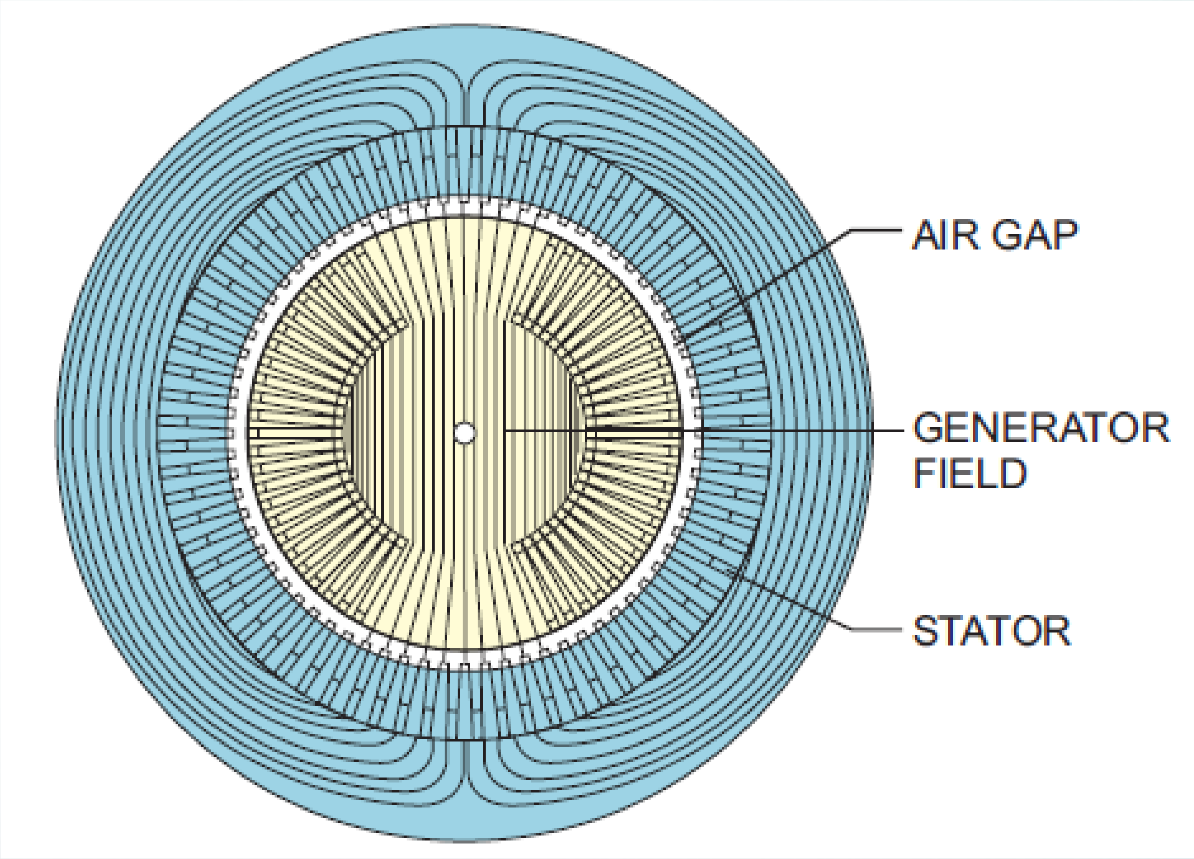
A non salient two-pole rotor for a synchronous machine







Synchronous Generator



Rotor magnetic flux linking rotor and stator

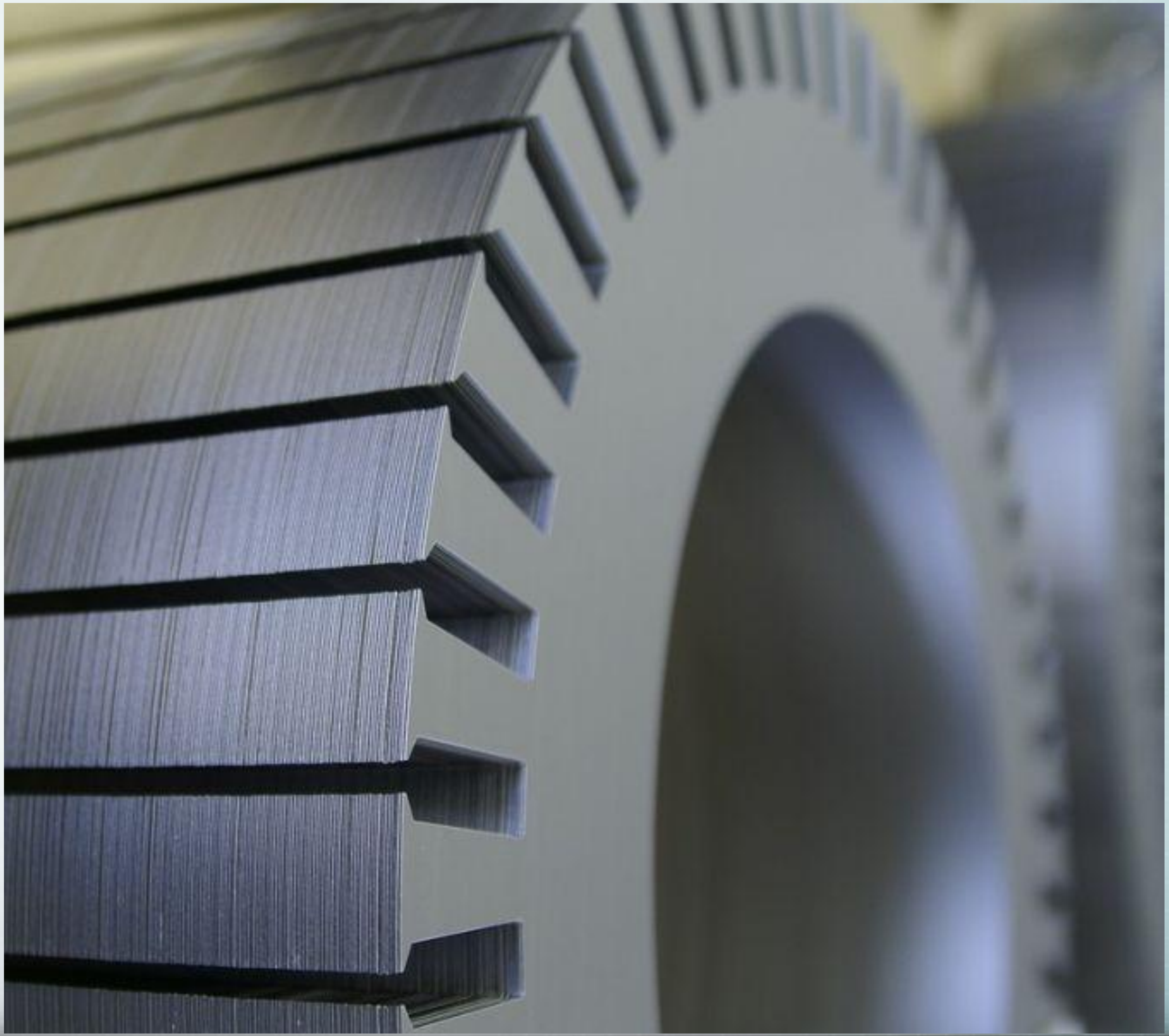
Synchronous Generator

Slip ring

A non salient two-pole rotor for a synchronous machine

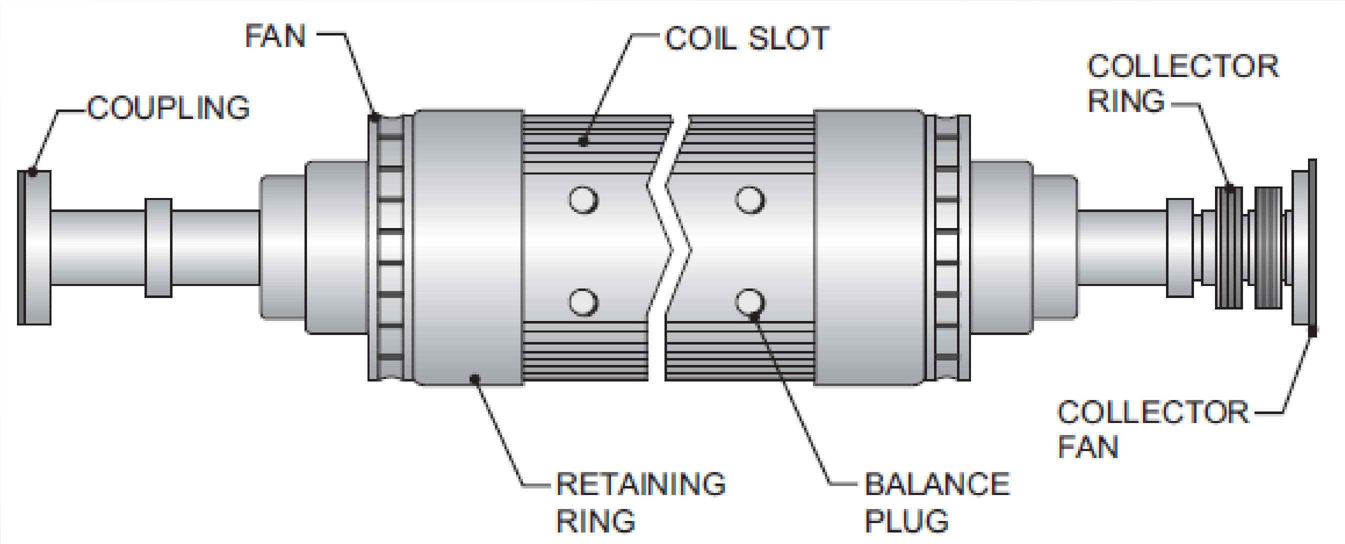
Synchronous Generator

Laminated rotor



Synchronous Generator

Mechanical outline for a typical generator field



Major components:

- Turbine coupling
- Main cooling fans
- Retaining rings
- Coil slot
- Balance plug
- Collector rings
- Collector fans

Synchronous Generator

- A dc current supplied to field circuit on the rotor
- Special arrangement required to get dc power to the field windings
- Two common approaches

   1. Supply power from external dc source to the rotor by means of *slip rings and brushes*
   2. Supply power from a special dc power source mounted directly on shaft of synchronous generator

Speed of Rotation

- Electrical frequency produced is locked in or synchronized with mechanical rate of rotation of generator
- Rate of rotation of magnetic fields related to stator electrical frequency by

$$\boxed{f_e = \frac{n_m P}{120}}$$

f_e = electrical frequency, in Hz

n_m = mechanical speed of magnetic field, in r/min (equals speed of rotor for synchronous machines)

P = number of poles

Speed of Rotation

- What will be the speed of a 4-pole rotor synchronous generator designed for a 50 Hz system?
- How many poles required for a hydro-generator rotating at 107 rpm in a 50 Hz system?

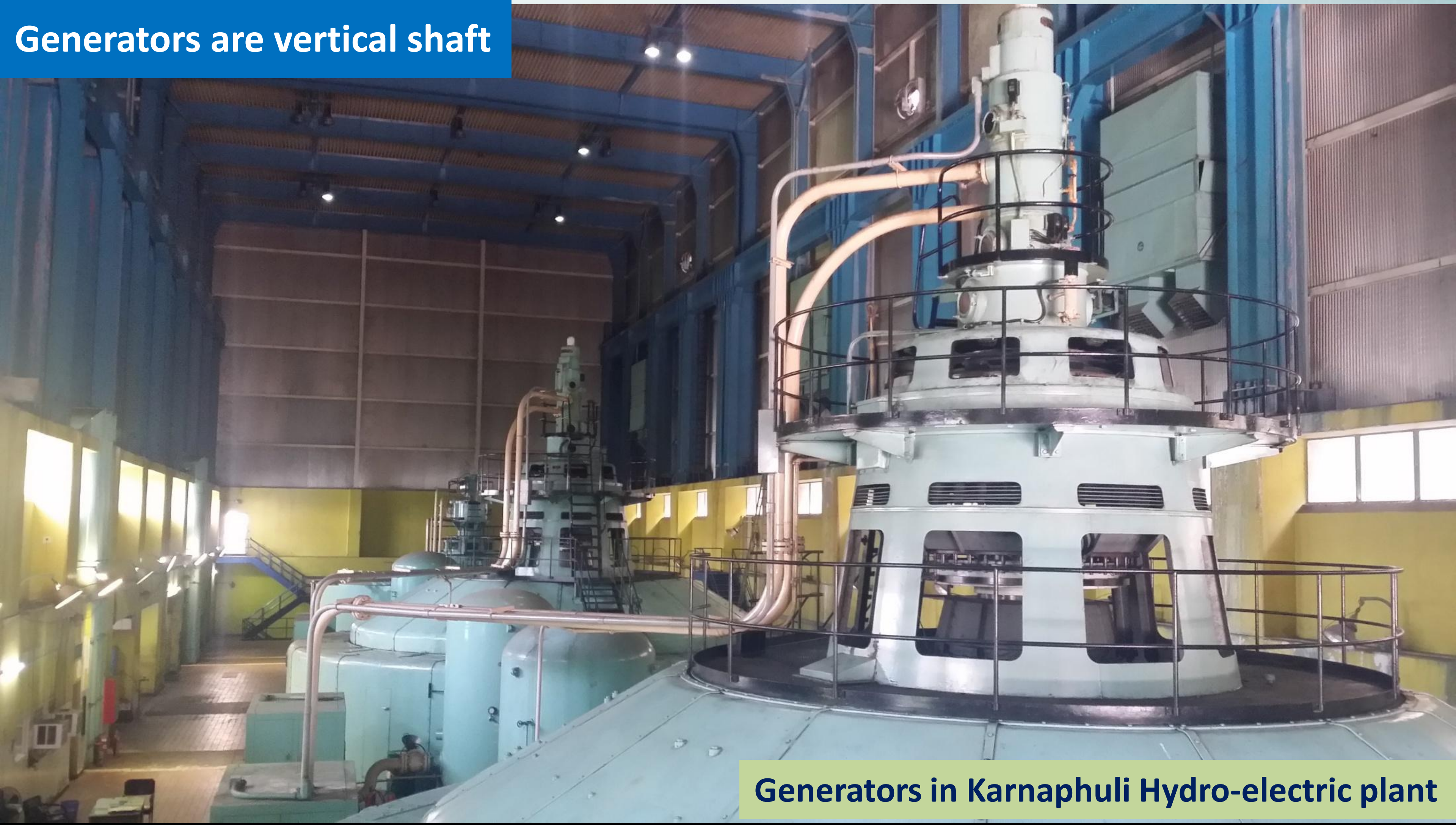
Kaptai hydro-generator name plate

Speed of Rotation

Why the low rpm? Look at the turbine blades!

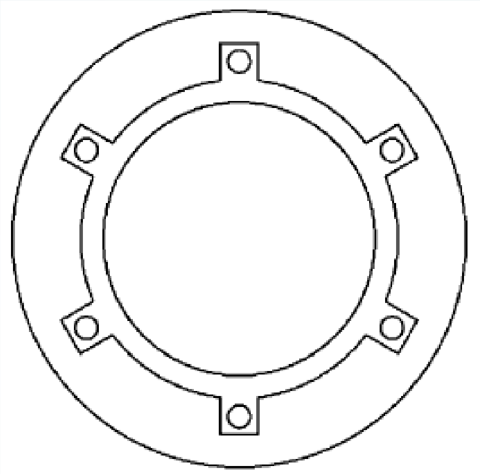
Karnaphuli water-turbine blades

Speed of Rotation



Generators in Karnaphuli Hydro-electric plant

Stator Winding

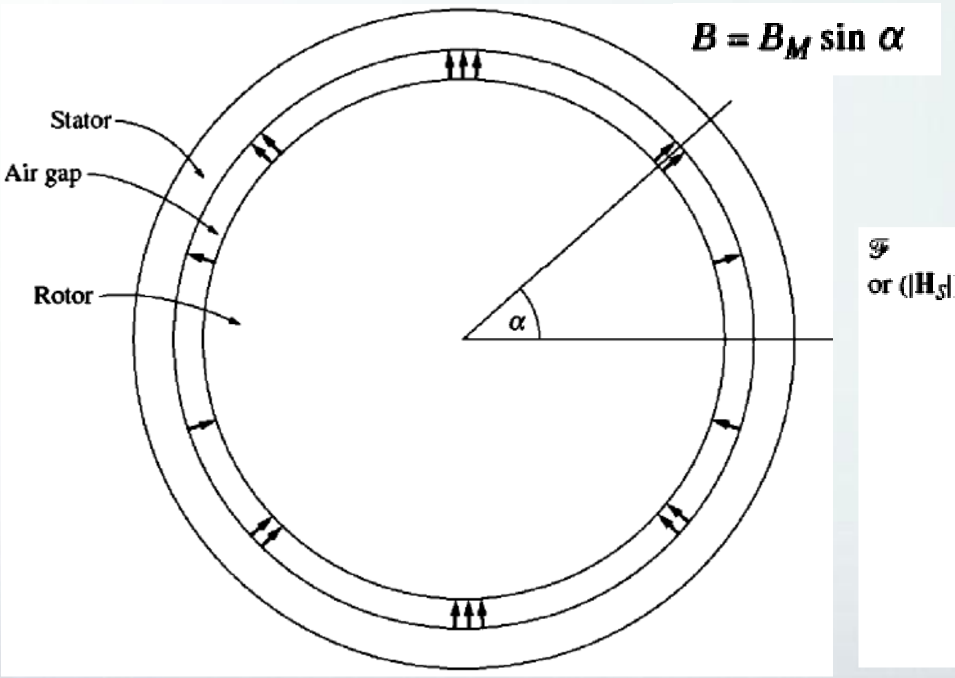


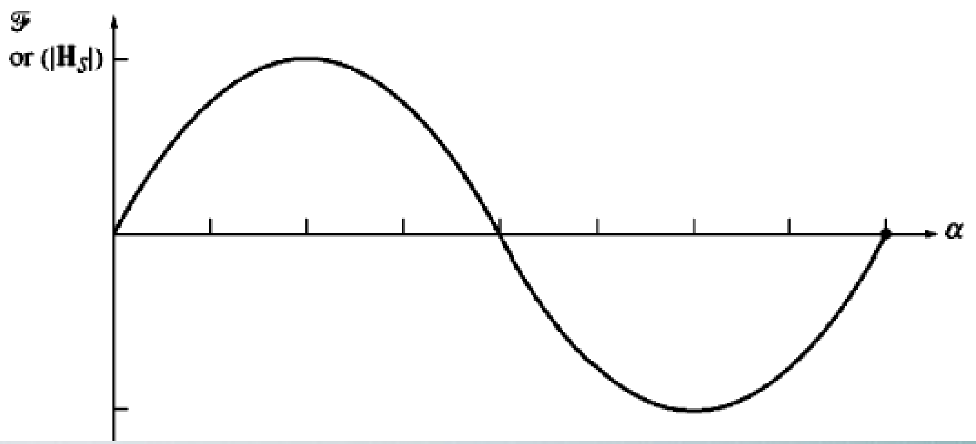
- Reluctance of air gap much higher than reluctances of rotor or stator
- Flux density vector B takes shortest possible path across air gap and jumps perpendicularly between rotor and stator

- To produce a sinusoidal voltage, magnitude of flux density vector B must vary in a sinusoidal manner along surface of air gap

Stator Winding

- Flux density will vary sinusoidally only if mmf varies in a sinusoidal manner along surface of air gap



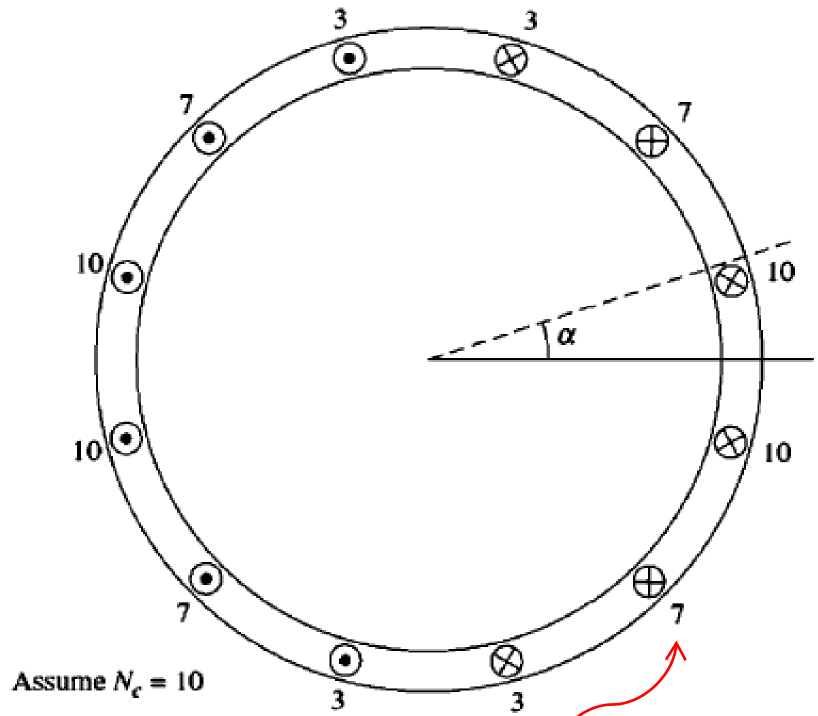


Stator Winding

- To achieve a sinusoidal variation of mmf along surface of air gap
  - Distribute turns of winding that produces mmf in closely spaced slots around surface of machine, and
  - Vary number of conductors in each slot in a sinusoidal manner
- Number of conductors in each slot

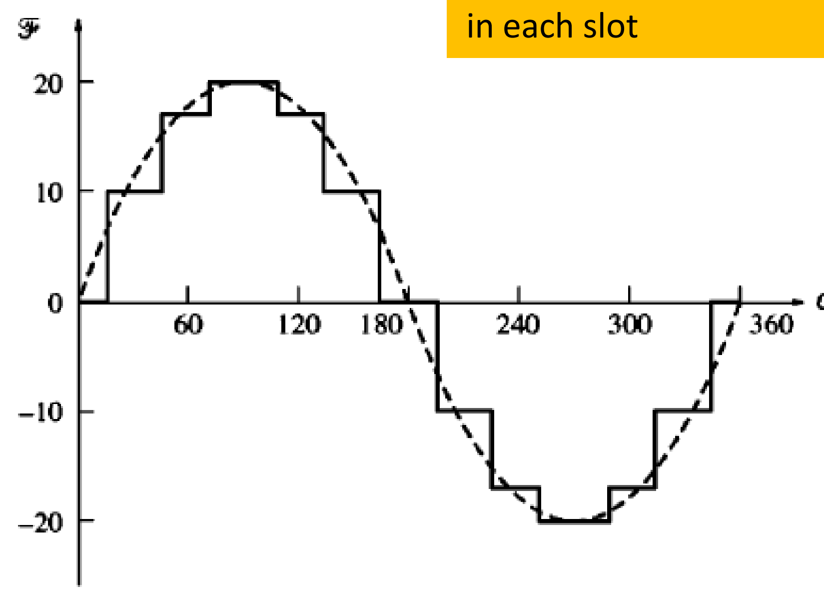
$$\boxed{n_C = N_C \cos \alpha}$$

More slots and more closely spaced, better sinusoidal



(a)

number of conductors in each slot



(b)

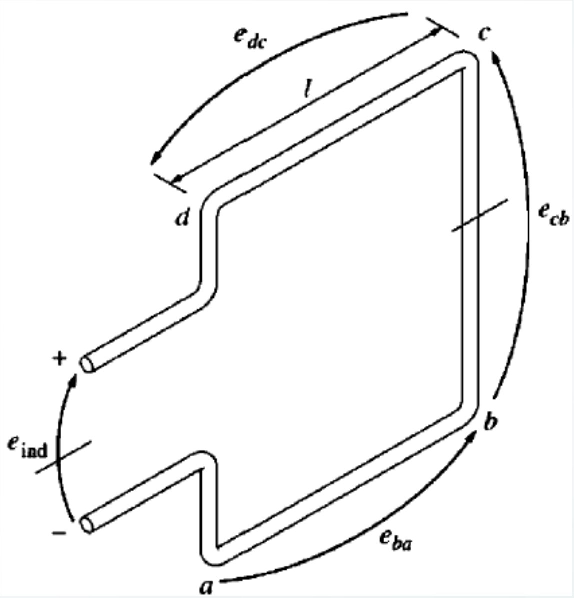
Stator Winding

- Not possible to distribute windings exactly according to $\boxed{n_C = N_C \cos \alpha}$
  - only a finite number of slots in a real machine
  - only integral numbers of conductors can be included in each slot
- Resulting mmf distribution only approximately sinusoidal → higher-order harmonic components present
- Fractional-pitch windings can suppress unwanted harmonic components

Induced Voltage in AC Machines

Induced Voltage in a Coil on a 2-Pole Stator

A rotating rotor with a sinusoidally distributed magnetic field in the center of a stationary coil



- Ideal flux distribution
  - Magnitude of flux density B in air gap between rotor and stator varies sinusoidally with mechanical angle
  - Direction of B always radially outward*
- Flux density vector B at a point around rotor

$$\boxed{B = B_M \cos\alpha}$$

α - angle measured from direction of peak rotor flux density

- Rotor rotating within stator at an angular velocity ωm , magnitude of flux density vector B at any angle α around stator $\boxed{B = B_M \cos(\omega t - \alpha)}$

* Note: at some locations around air gap, flux density vector will point in toward rotor

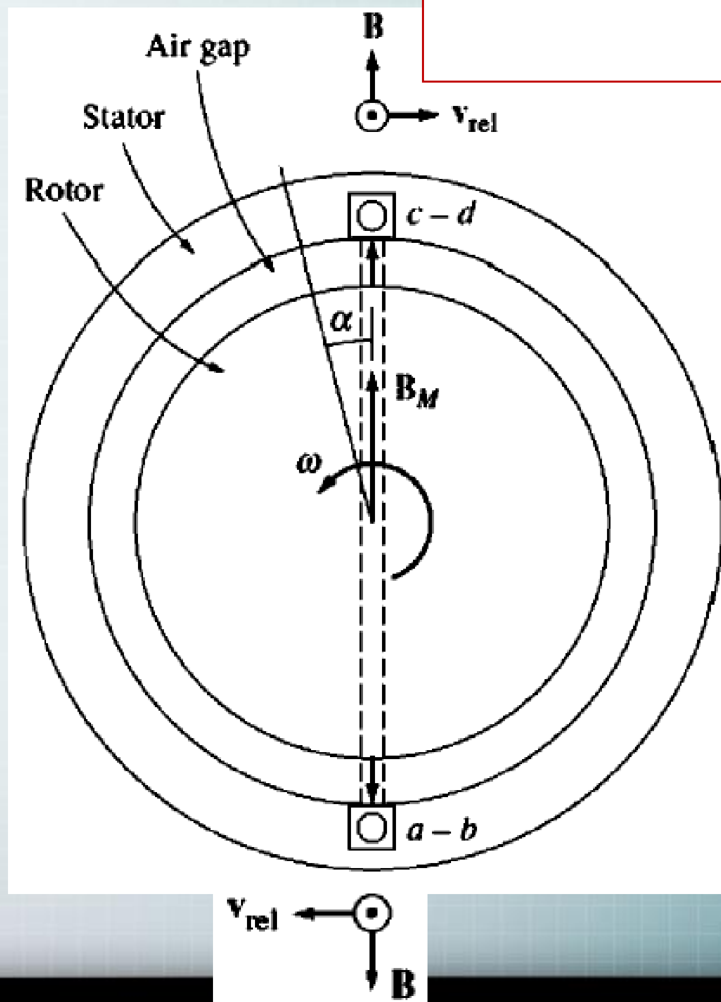
Induced Voltage in a Coil on a 2-Pole Stator

- Induced voltage in a moving wire in stationary magnetic field $e = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$

$\mathbf{v}$ = velocity of the wire *relative to the magnetic field*

- For stationary wire, moving magnetic field this equation does not directly apply
  – Need frame of reference where magnetic field appears to be stationary
- If observer 'sits on magnetic field' field appears to be stationary, coil sides appear to go by at an apparent velocity v_{rel}

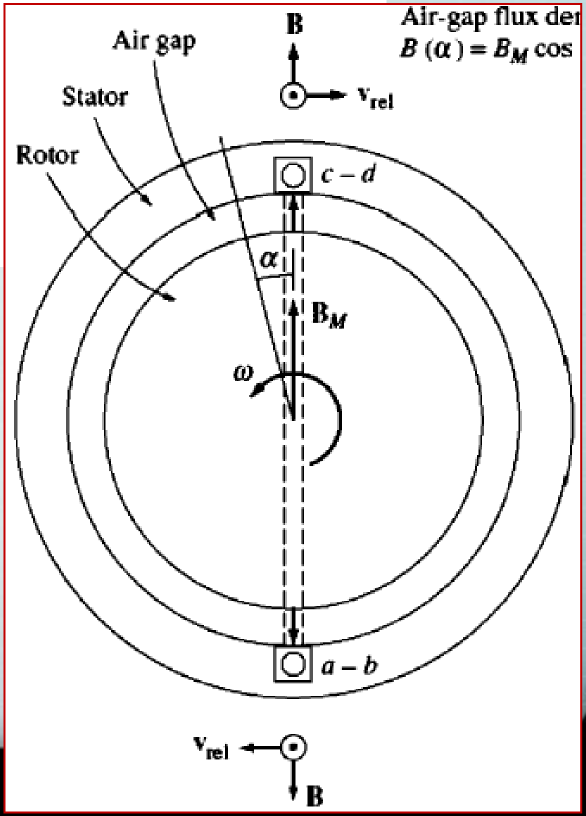
Air-gap flux density:
$B(\alpha) = B_M \cos(\omega_m t - \alpha)$



Induced Voltage in a Coil on a 2-Pole Stator

1. *Segment ab*. For segment *ab*, $\alpha = 180°$. Assuming that **B** is directed radially outward from the rotor, the angle between **v** and **B** in segment *ab* is 90°, while the quantity $\mathbf{v} \times \mathbf{B}$ is in the direction of **l**, so

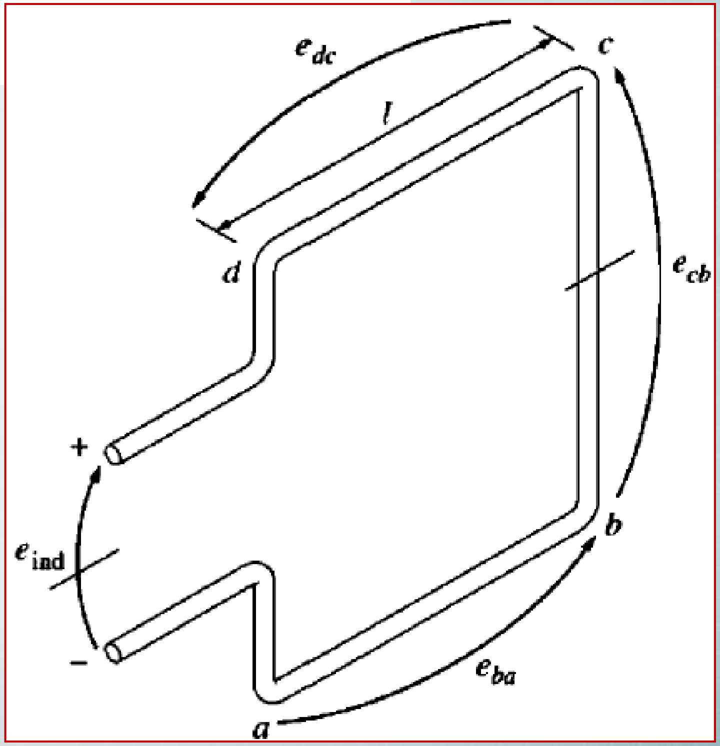
$$
\begin{aligned}
e_{ba} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\
&= vBl \qquad \text{directed out of the page} \\
&= -v[B_M \cos(\omega_m t - 180°)]l \\
&= -vB_M l \cos(\omega_m t - 180°)
\end{aligned}
$$



Induced Voltage in a Coil on a 2-Pole Stator

2. Segment *bc*. The voltage on segment *bc* is zero, since the vector quantity $\mathbf{v} \times \mathbf{B}$ is perpendicular to $\mathbf{l}$, so

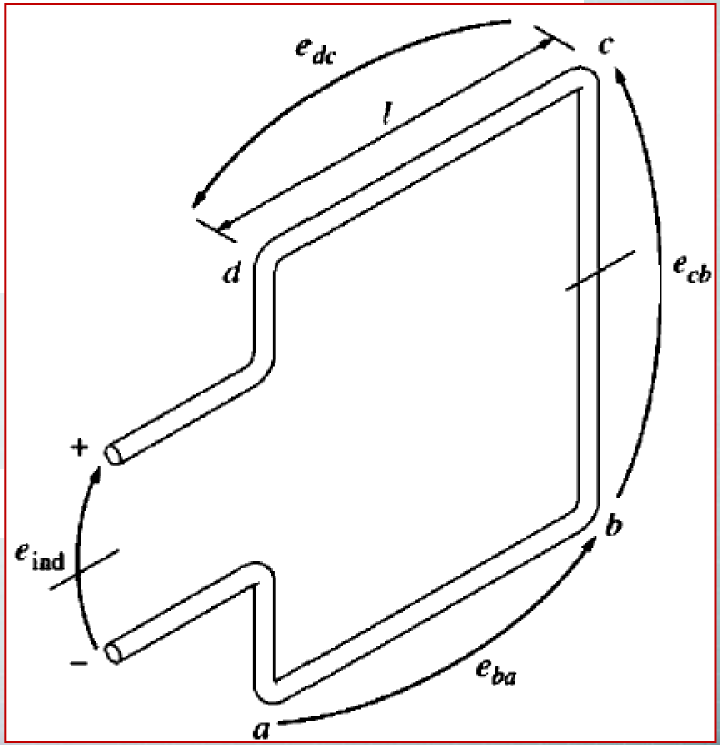
$$e_{cb} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = 0$$



Induced Voltage in a Coil on a 2-Pole Stator

3. Segment *cd*. For segment *cd*, the angle $\alpha = 0°$. Assuming that **B** is directed radially outward from the rotor, the angle between **v** and **B** in segment *cd* is 90°, while the quantity $\mathbf{v} \times \mathbf{B}$ is in the direction of **l**, so

$$\begin{aligned} e_{dc} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\ &= vBl \qquad \text{directed out of the page} \\ &= v(B_M \cos \omega_m t)l \\ &= vB_M l \cos \omega_m t \end{aligned}$$



4. Segment *da*. The voltage on segment *da* is zero,

$$e_{ad} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = 0$$

Induced Voltage in a Coil on a 2-Pole Stator

total voltage on the coil will be

$$e_{\text{ind}} = e_{ba} + e_{dc}$$
$$= -vB_M l \cos(\omega_m t - 180°) + vB_M l \cos \omega_m t$$
$$= vB_M l \cos \omega_m t + vB_M l \cos \omega_m t$$
$$= 2vB_M l \cos \omega_m t$$

Since $\cos \theta = -\cos(\theta - 180°)$,

Induced Voltage in a Coil on a 2-Pole Stator

velocity of the end conductors is given by $v = r\omega_m$,

$$e_{\text{ind}} = 2(r\omega_m)B_M l \cos \omega_m t$$
$$= 2rlB_M\omega_m \cos \omega_m t$$

flux passing through the coil can be expressed as $\phi = 2rlB_m$

$$\boxed{e_{\text{ind}} = \phi\omega \cos \omega t}$$

Induced Voltage in a Coil on a 2-Pole Stator

Voltage induced in a single-turn coil

$$\boxed{e_{\text{ind}} = \phi\omega \cos \omega t}$$

If coil in stator has N_c turns of wire, then total induced voltage of coil

$$\boxed{e_{\text{ind}} = N_C\phi\omega \cos \omega t}$$

- Voltage is sinusoidal
- Amplitude depends on
  - flux φ
  - Angular velocity ω of rotor
  - N_c – number of turns [a constant depending on construction of machine]

Induced Voltage in AC Machines

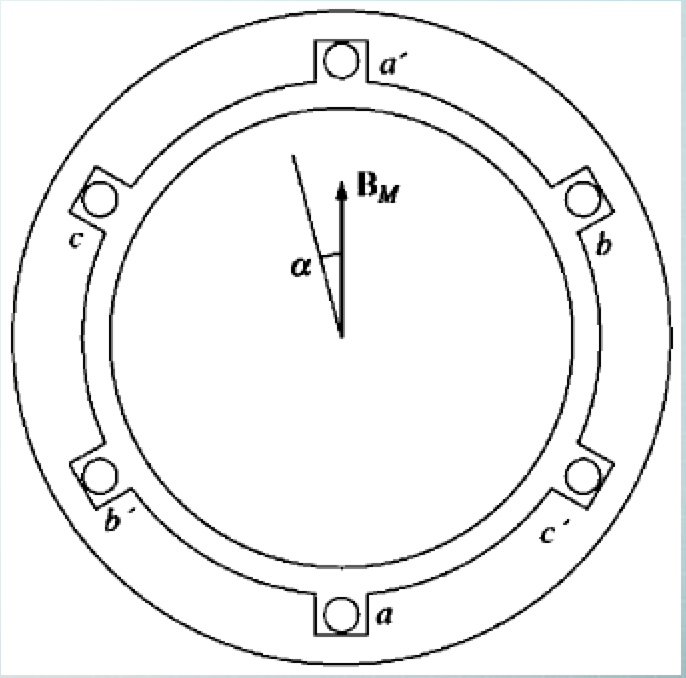
Induced Voltage in a Three-Phase Set of Coils

- Voltages induced in each coil will be the same in magnitude but differ in phase by 120°

$$e_{aa'}(t) = N_C \phi\omega \sin \omega t \qquad \text{V}$$
$$e_{bb'}(t) = N_C \phi\omega \sin (\omega t - 120^\circ) \qquad \text{V}$$
$$e_{cc'}(t) = N_C \phi\omega \sin (\omega t - 240^\circ) \qquad \text{V}$$

A 3-phase set of currents can generate a uniform rotating magnetic field in a machine stator, and a uniform rotating magnetic field can generate a 3-phase set of voltages in such a stator

3 coils, each of N_c turns, placed around rotor magnetic field



RMS Voltage in a Three-Phase Stator

- Peak voltage in any phase

$$E_{\max} = N_C \phi \omega = 2\pi N_C \phi f$$

- RMS voltage of any phase

$$E_A = \frac{2\pi}{\sqrt{2}} N_C \phi f = \sqrt{2}\pi N_C \phi f$$

RMS voltage at *terminal* depend on whether stator is Y- or Δ-connected

- Y-connected stator - terminal voltage $\sqrt{3}\, E_A$
- Δ -connected stator - terminal voltage just E_A

Distributed Fractional Pitched Armature Winding

Harmonics in Induced Voltage

- Induced voltage in an ac machine is sinusoidal only if harmonic components of air-gap flux density are suppressed
  - Air-gap flux density distribution not sinusoidal → output voltages in stator will not be sinusoidal
- Actual flux distribution consists of a fundamental sinusoidal component plus harmonics
  - generate harmonic components in stator's voltages and currents
- **Two techniques used in machine design to suppress harmonics**
  i. Fractional-pitch coil
  ii. Distributed winding

Coil Pitch

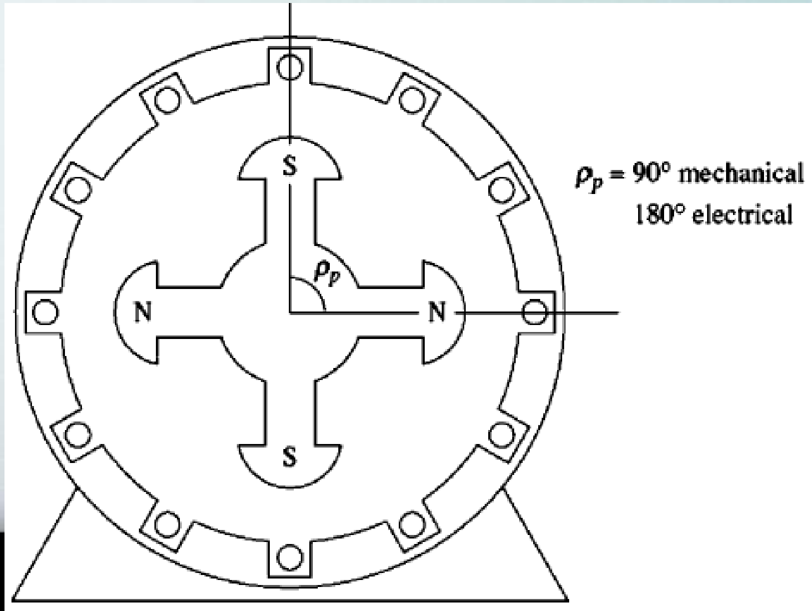
- Pole pitch is the angular distance between two adjacent poles on a machine
- Pole pitch of a machine in mechanical degrees → $\boxed{\rho_p = \frac{360^\circ}{P}}$

  ρ_p - pole pitch in mechanical degrees

  P - number of poles on the machine

- Regard less of number of poles on machine,

  a pole pitch is always 180 *electrical degrees*



Coil Pitch

- **Full-pitch coil** - stator coil stretches across same angle as pole pitch
- **Fractional-pitch coil** - stator coil stretches across an angle smaller than a pole pitch
- Pitch of a fractional-pitch coil expressed as a fraction indicating portion of pole pitch it spans
  - e.g. a 5/6-pitch coil spans five-sixths of distance between two adjacent poles
- Pitch of a fractional-pitch coil in electrical degrees →

$$\boxed{\rho = \frac{\theta_m}{\rho_p} \times 180^\circ}$$

θ_m – mechanical angle covered by coil in degrees

ρ_p – machine's pole pitch in mechanical degrees

Coil Pitch

- Pitch of a fractional-pitch coil in electrical degrees → $$\boxed{\rho = \frac{\theta_m}{\rho_p} \times 180^\circ}$$

  θ_m – mechanical angle covered by coil in degrees

  ρ_p – machine's pole pitch in mechanical degrees

  Or → $$\boxed{\rho = \frac{\theta_m P}{2} \times 180^\circ}$$

  θ_m – mechanical angle covered by coil in degrees

  P – number of poles in the machine

- Windings employing fractional-pitch coils known as **chorded windings**

Induced Voltage of a Fractional-Pitch Coil

- Pole pitch – 180°, coil pitch – ρ
- Total voltage induced in coil

  = $\sum$voltages on individual coil sides

- Assuming, Ideal flux density vector ***B***

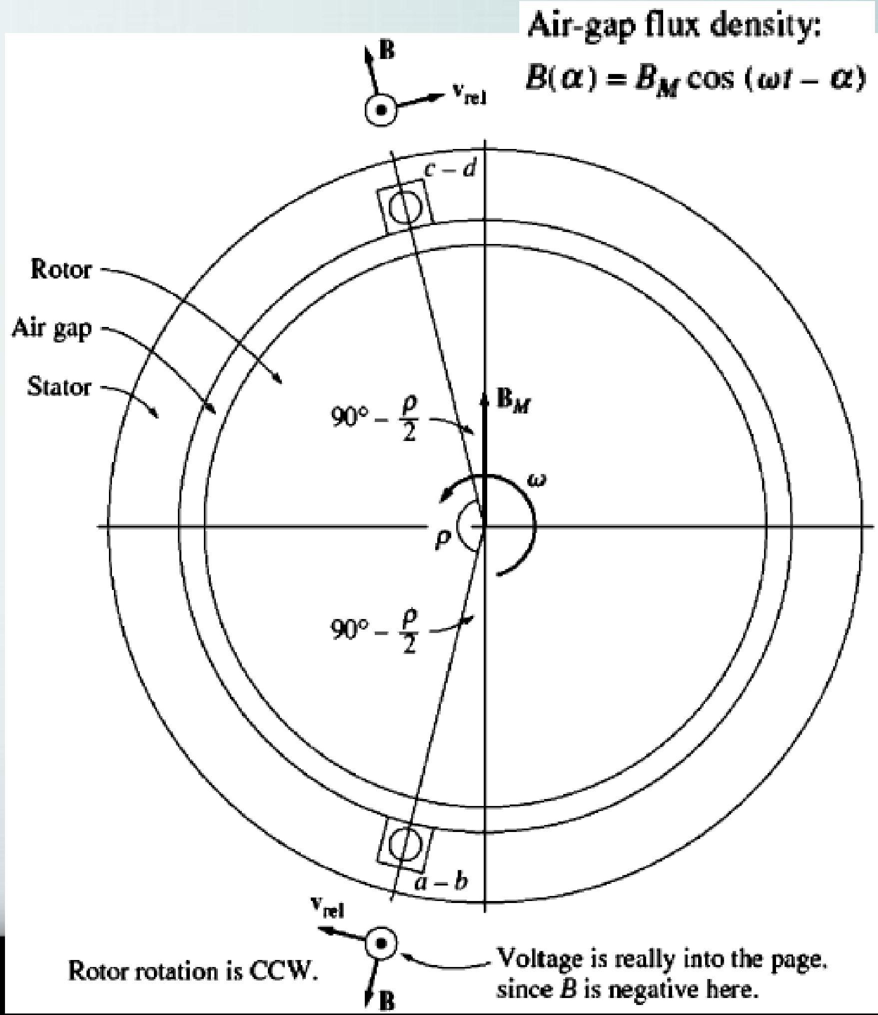
and, $\boldsymbol{\alpha}$ – angle measured from direction of peak rotor flux density

$\boldsymbol{\omega}_m$ – angular velocity of rotor

Flux density vector ***B*** around stator

$$\boxed{B = B_M \cos(\omega t - \alpha)}$$

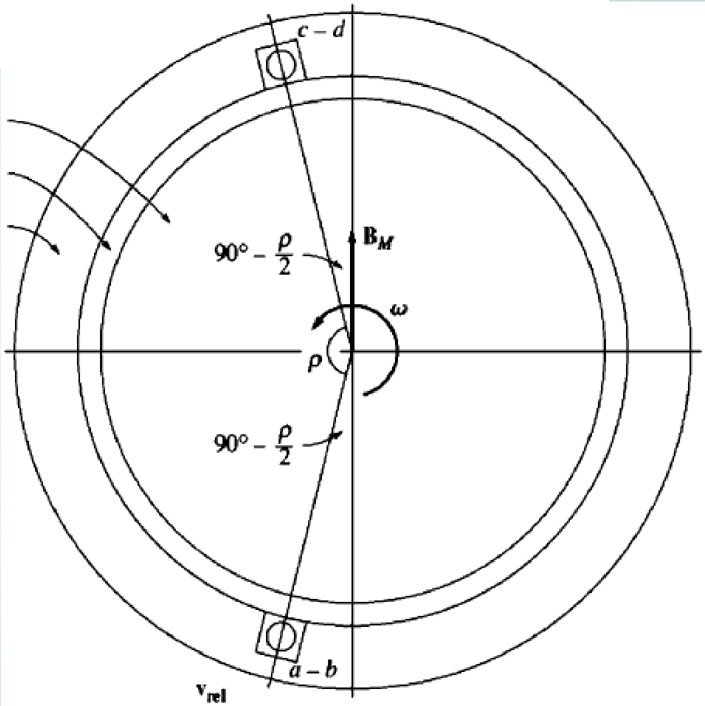
Simple two-pole machine with a fractional-pitch winding



Induced Voltage of a Fractional-Pitch Coil

1. *Segment ab*. For segment *ab* of the fractional-pitch coil, $\boxed{\alpha = 90° + \rho/2.}$ Assuming that **B** is directed radially outward from the rotor, the angle between **v** and **B** in segment *ab* is 90°, while the quantity $\mathbf{v} \times \mathbf{B}$ is in the direction of **l**, so

$$
\begin{aligned}
e_{ba} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\
&= vBl \quad \text{directed out of the page} \\
&= -vB_M \cos\left[\omega_m t - \left(90° + \frac{\rho}{2}\right)\right] l \\
&= -vB_M l \cos\left(\omega_m t - 90° - \frac{\rho}{2}\right)
\end{aligned}
$$



2. *Segment bc*. The voltage on segment *bc* is zero, since the vector quantity $\mathbf{v} \times \mathbf{B}$ is perpendicular to **l**, so

$$e_{cb} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = 0$$

Induced Voltage of a Fractional-Pitch Coil

3. *Segment cd*. For segment *cd*, the angle $\alpha = 90° - \rho/2$. Assuming that $\mathbf{B}$ is directed radially outward from the rotor, the angle between $\mathbf{v}$ and $\mathbf{B}$ in segment *cd* is 90°, while the quantity $\mathbf{v} \times \mathbf{B}$ is in the direction of $\mathbf{l}$, so

$$e_{dc} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$
$$= vBl \quad \text{directed out of the page}$$

$$e_{ba} = -vB_M \cos\left[\omega_m t - \left(90° - \frac{\rho}{2}\right)\right] l$$

$$= -vB_M l \cos\left(\omega_m t - 90° + \frac{\rho}{2}\right)$$

4. *Segment da*. The voltage on segment *da* is zero, since the vector quantity $\mathbf{v} \times \mathbf{B}$ is perpendicular to $\mathbf{l}$, so

$$e_{ad} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = 0$$

Induced Voltage of a Fractional-Pitch Coil

Total resulting voltage

$$e_{\text{ind}} = e_{ba} + e_{dc} = -vB_Ml\cos\left(\omega_m t - 90^\circ - \frac{\rho}{2}\right) + vB_Ml\cos\left(\omega_m t - 90^\circ + \frac{\rho}{2}\right)$$

$$\cos\left(\omega_m t - 90^\circ - \frac{\rho}{2}\right) = \cos(\omega_m t - 90^\circ)\cos\frac{\rho}{2} + \sin(\omega_m t - 90^\circ)\sin\frac{\rho}{2}$$

$$\cos\left(\omega_m t - 90^\circ + \frac{\rho}{2}\right) = \cos(\omega_m t - 90^\circ)\cos\frac{\rho}{2} - \sin(\omega_m t - 90^\circ)\sin\frac{\rho}{2}$$

$$\sin(\omega_m t - 90^\circ) = -\cos\omega_m t$$

$$\begin{aligned} e_{\text{ind}} = vB_Ml\Big[&-\cos(\omega_m t - 90^\circ)\cos\frac{\rho}{2} - \sin(\omega_m t - 90^\circ)\sin\frac{\rho}{2} \\ &+ \cos(\omega_m t - 90^\circ)\cos\frac{\rho}{2} - \sin(\omega_m t - 90^\circ)\sin\frac{\rho}{2}\Big] \\ = -2vB_Ml&\sin\frac{\rho}{2}\sin(\omega_m t - 90^\circ) = 2vB_Ml\sin\frac{\rho}{2}\cos\omega_m t \end{aligned}$$

Since $2vB_Ml$ is equal to $\phi\omega$,

$$\boxed{e_{\text{ind}} = \phi\omega\sin\frac{\rho}{2}\cos\omega_m t}$$

Induced Voltage of a Fractional-Pitch Coil

$$e_{ind} = \phi\omega \sin\frac{\rho}{2} \cos\omega_m t$$

Pitch factor, $k_p = \sin\frac{\rho}{2}$

Total voltage in an N_C -turn fractional-pitch coil, $e_{ind} = N_C k_p \phi\omega \cos\omega_m t$

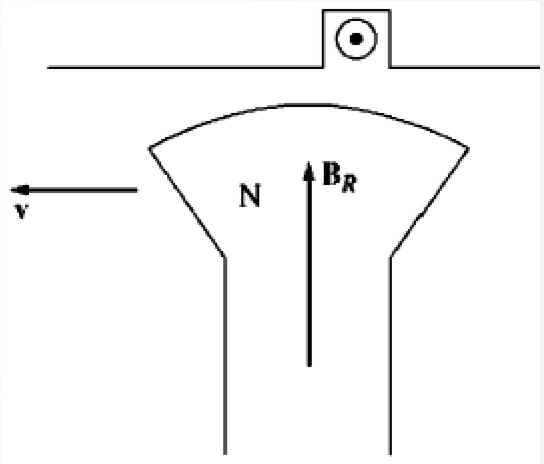
Peak voltage, $E_{max} = N_C k_p \phi\omega = 2\pi N_C k_p \phi f$

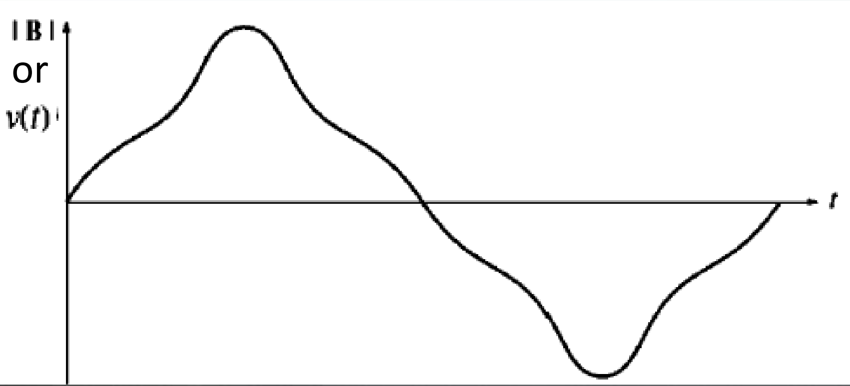
RMS voltage of any phase, $E_A = \frac{2\pi}{\sqrt{2}} N_C k_p \phi f = \sqrt{2}\pi N_C k_p \phi f$

If coil pitch is given in mechanical degrees, then pitch factor, $k_p = \sin\frac{\theta_m P}{2}$

Harmonic Problems & Fractional-Pitch Windings

A salient-pole synchronous machine; rotor sweeping across stator surface





- Reluctance of magnetic field path much lower directly under center of the rotor → flux strongly concentrated at that point
- Induced voltage not sinusoidal → contains harmonics
- No even harmonics in phase voltage → because voltage waveform is symmetric about center of rotor flux

Harmonic Problems & Fractional-Pitch Windings

- Odd harmonics (3rd, 5th, 7th, 9th etc.) present in phase voltage
- Higher the harmonic number, the lower its magnitude → beyond ~9th harmonic effects of higher harmonics may be ignored
- When phases are Y or Δ connected, some harmonics disappear from output

Fundamental component of voltage

$$e_a(t) = E_{M1} \sin \omega t \quad \text{V}$$
$$e_b(t) = E_{M1} \sin (\omega t - 120°)$$
$$e_c(t) = E_{M1} \sin (\omega t - 240°)$$

3rd harmonic component of voltage

$$e_{a3}(t) = E_{M3} \sin 3\omega t \quad \text{V}$$
$$e_{b3}(t) = E_{M3} \sin (3\omega t - 360°)$$
$$e_{c3}(t) = E_{M3} \sin (3\omega t - 720°)$$

3rd harmonic components of voltage are identical in each phase

Harmonic Problems & Fractional-Pitch Windings

- **Y-connected stator windings** → 3rd harmonic voltage between any terminals will be zero
- **Δ-connected stator windings** → 3rd harmonic components add and drive 3rd harmonic current inside Δ-winding
  - Since 3rd harmonic voltages dropped across stator impedances, no significant third-harmonic component of voltage at terminals
- **This result also applies to any multiple of a third-harmonic component**
  - These are called ***triplen harmonics***

Induced Voltage of a Fractional-Pitch Coil

- Remaining harmonic frequencies are 5^{th}, 7^{th}, 11^{th}, 13^{th} etc.
- Actual distortion in sinusoidal output of a synchronous machine caused by 5^{th} and 7^{th} harmonics
  - Also called the ***belt harmonics***
- **Solutions**
  1. Design rotor to distribute flux in an approximately sinusoidal shape
  2. Design machine with fractional-pitch windings

Induced Voltage of a Fractional-Pitch Coil

- Effect of fractional-pitch windings on voltage → electrical angle of n^{th} harmonic is n times the electrical angle of fundamental frequency
- If a coil spans 150 electrical degrees at its fundamental frequency, it will span
  - 300 electrical degrees at its 2^{nd} harmonic
  - 450 electrical degrees at its 3^{rd} harmonic
- Let, $\boldsymbol{\rho}$ - electrical angle spanned by coil at its fundamental frequency

  $\boldsymbol{v}$ **-** number of the harmonic being examined

Then, pitch factor of coil at harmonic frequency → $\boxed{k_p = \sin \frac{v\rho}{2}}$

By a proper choice of coil pitch it is possible to almost eliminate harmonic frequency components in output of machine

Induced Voltage of a Fractional-Pitch Coil

Example

A 3-phase, 2-pole stator has coils with a 5/6 pitch. What are the pitch factors for the harmonics present in this machine's coils? Does this pitch help suppress harmonic content of generated voltage?

Solution

The pole pitch in mechanical degrees of this machine is

$$\rho_p = \frac{360^\circ}{P} = 180^\circ$$

mechanical pitch angle of these coils is five-sixths of 180°, or 150°

resulting pitch in electrical degrees is

$$\rho = \frac{\theta_m}{\rho_p} \times 180^\circ = \frac{150^\circ}{180^\circ} \times 180^\circ = 150^\circ$$

Induced Voltage of a Fractional-Pitch Coil

Employing fractional-pitch windings drastically reduces harmonic content of output voltage

Fundamental: $k_p = \sin \frac{150°}{2} = 0.966$

Third harmonic: $k_p = \sin \frac{3(150°)}{2} = -0.707$

Fifth harmonic: $k_p = \sin \frac{5(150°)}{2} = 0.259$

Seventh harmonic: $k_p = \sin \frac{7(150°)}{2} = 0.259$

Ninth harmonic: $k_p = \sin \frac{9(150°)}{2} = -0.707$

Fundamental frequency component slightly suppressed

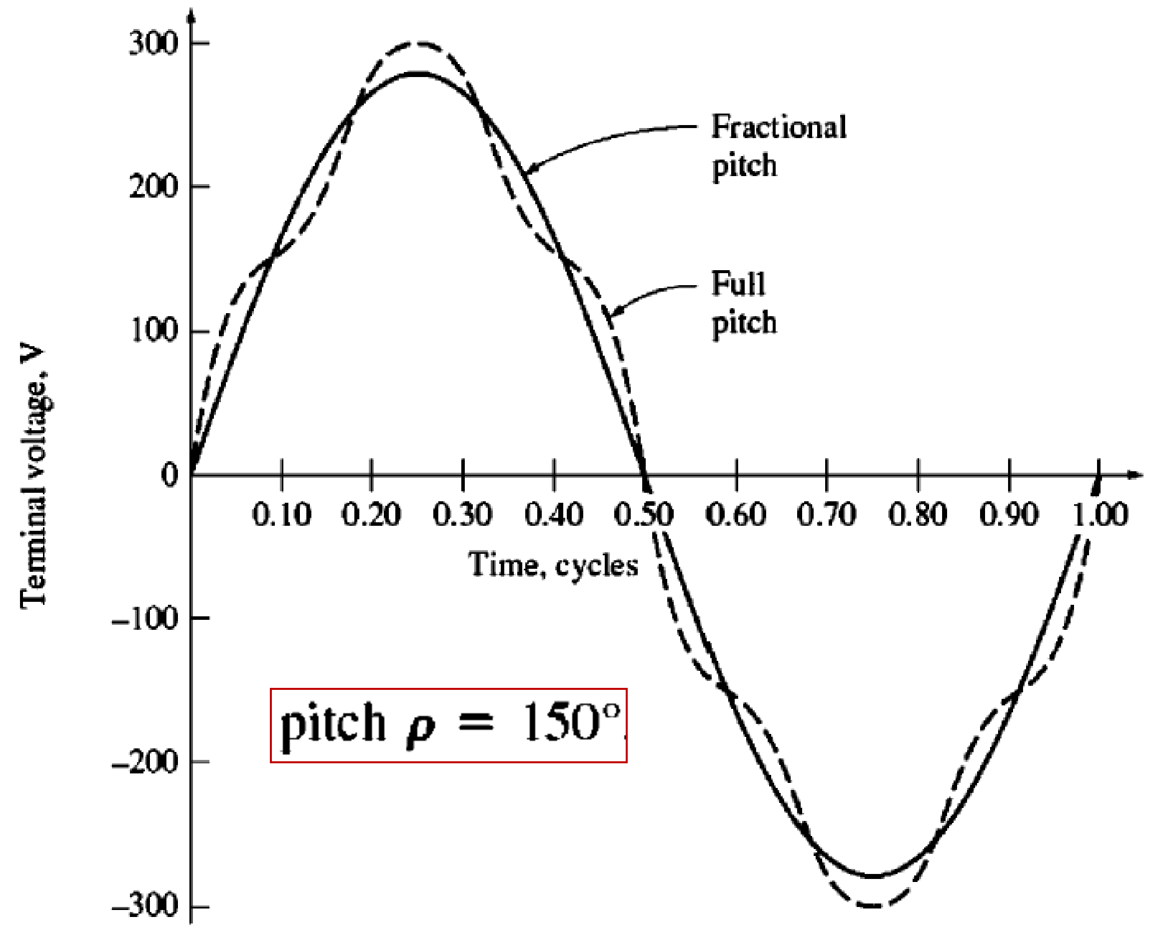
(This is a triplen harmonic not present in the three-phase output.)

3rd, 9th harmonic suppressed slightly by this coil pitch

5th, 7th harmonics suppressed

Induced Voltage of a Fractional-Pitch Coil

Line voltage of a 3-phase generator with full-pitch and fractional-pitch windings

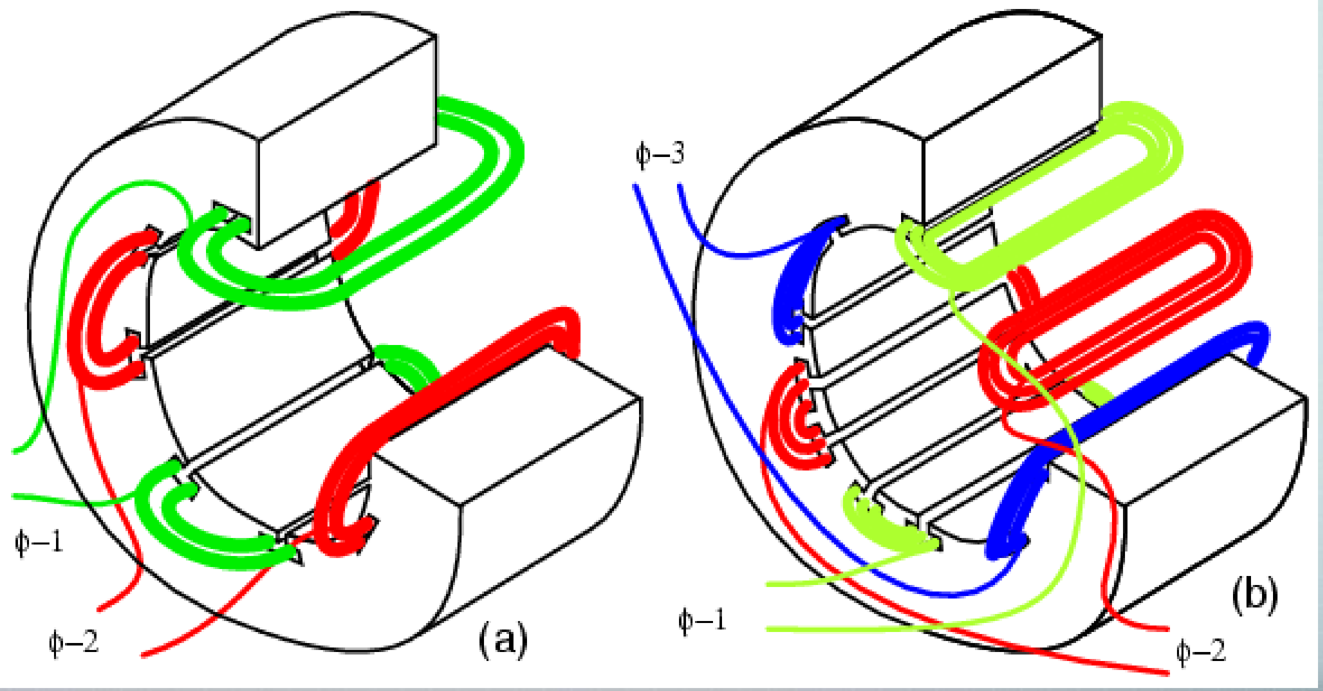


- Large visible improvement in waveform quality
- There are certain types of higher-frequency harmonics - *tooth or slot harmonics*- which cannot be suppressed by varying pitch of stator coils

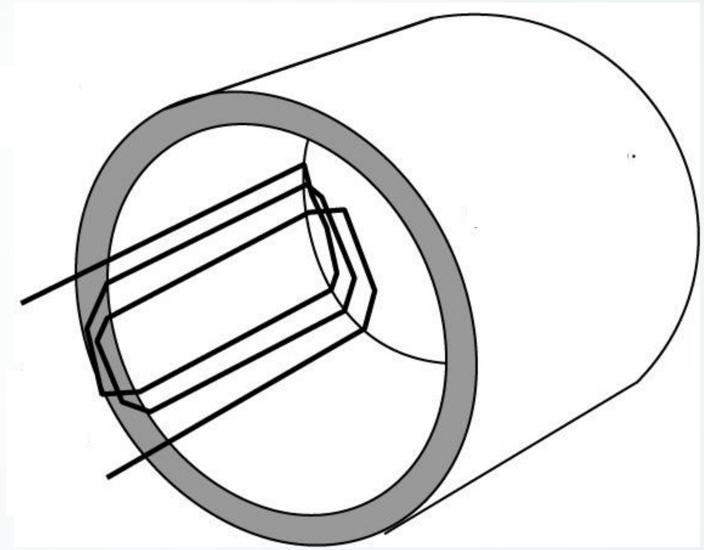
Distributed Winding

Windings associated with each phase distributed among several adjacent pairs of slots → because it is impossible to put all conductors into a single slot

- Stators consist of several coils in each phase, distributed in slots around inner surface of stator



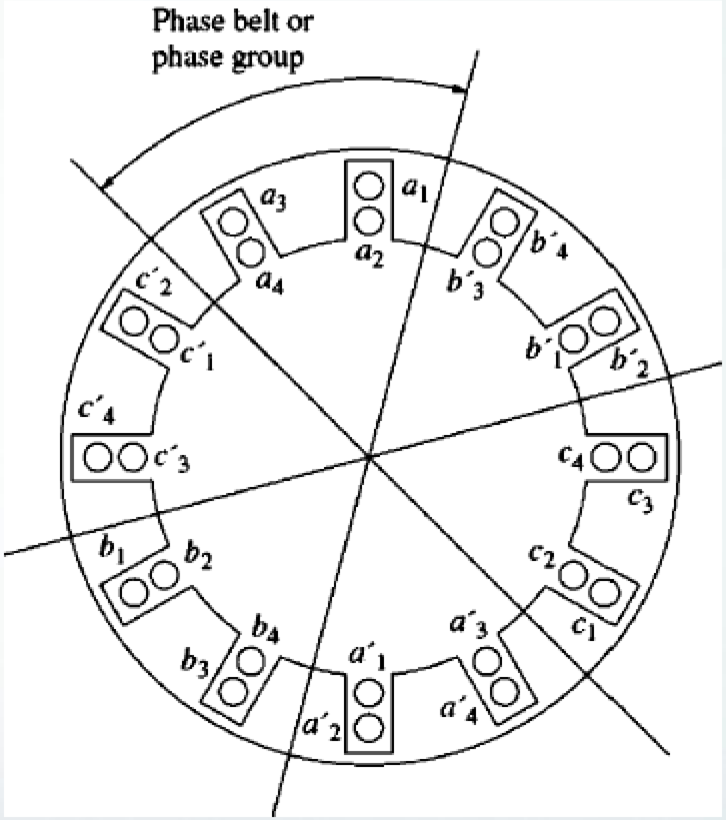
Distributed Winding



Stator structure – windings in slots

- Voltage in a single turn very small → placing many turns in series produce reasonable voltages
- Large number of turns physically divided among several coils, and coils are placed in slots equally spaced along surface of stator
- In larger machines, each coil is a preformed unit consisting of a number of turns, each turn insulated from others and from side of stator itself

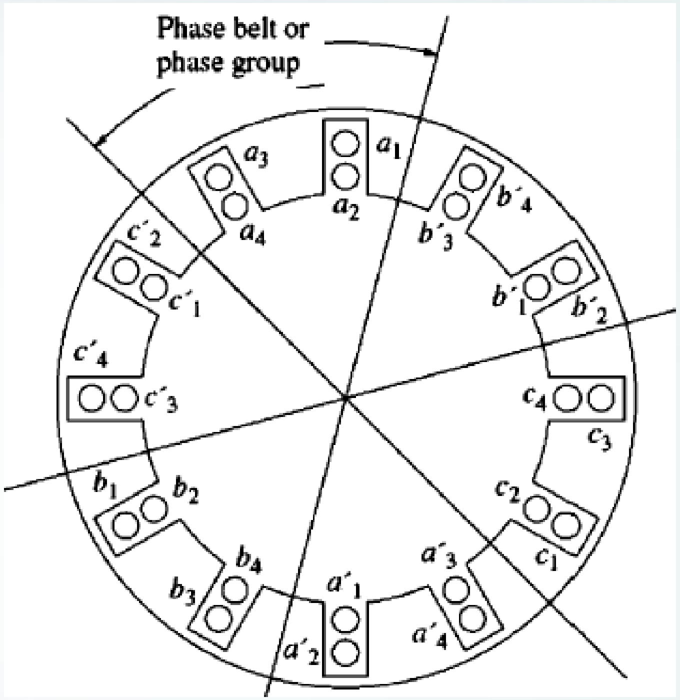
Distributed Winding

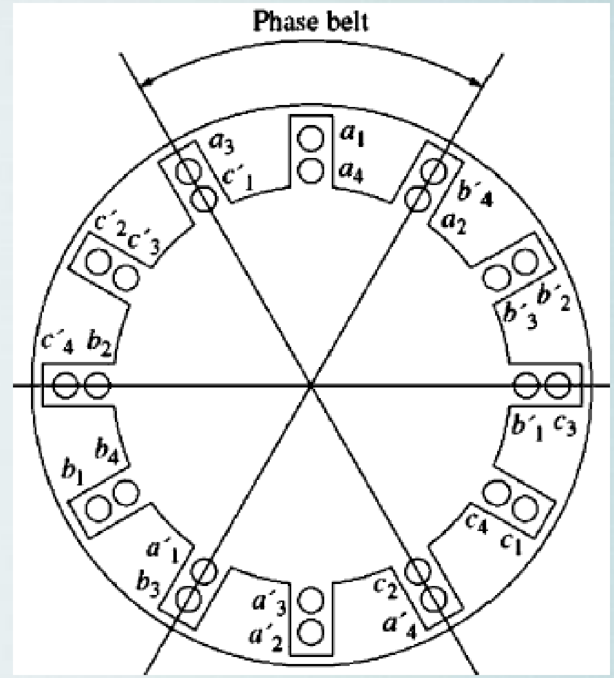


Double-layer full-pitch distributed winding for a two-pole ac machine

- **Slot pitch γ** – spacing in degrees between adjacent slots on a stator
- Stator coils normally formed into double-layer windings
- Double- layer windings usually easier to manufacture (fewer slots for a given number of coils) and have simpler end connections than single-layer windings

Distributed Winding





- 4 coils associated with each phase
- All coil sides of a given phase are placed in adjacent slots → these sides are known as phase belt or phase group → 6 phase belts on this stator
- In general, *3P* phase belts on a *P*-pole stator, *P* of them in each phase

- Distributed winding using fractional-pitch coils
- This winding has phase belts, but phases of coils within an individual slot may be mixed
- Pitch of coils 5/6 or 150 electrical degrees

Distributed Winding

- Dividing total required number of turns into separate coils permits more efficient use of inner surface of stator
  - Stator slots can be smaller → it provides greater structural strength
- Turns composing a given phase lie at different angles → their voltages will be somewhat smaller than expected

Breadth or Distribution Factor

Let initial voltage of central coil of phase *a*

$$\mathbf{E}_{a2} = E \angle 0^\circ \text{ V}$$

Voltages in other two coils in phase *a*

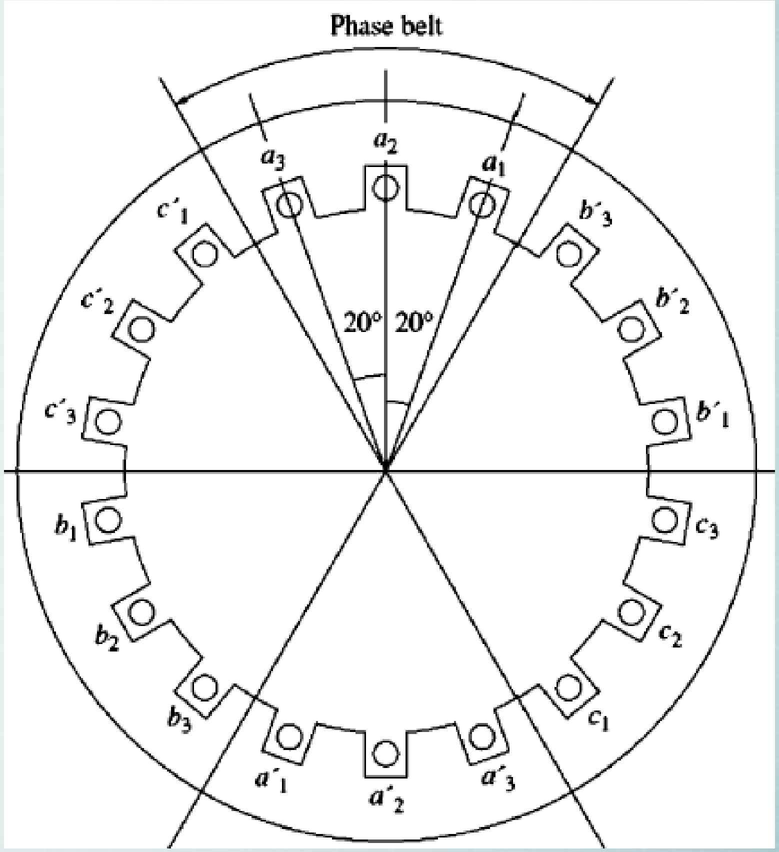
$$\mathbf{E}_{a1} = E \angle -20^\circ \text{ V}$$

$$\mathbf{E}_{a3} = E \angle 20^\circ \text{ V}$$

Total voltage in phase *a*

$$\begin{aligned}\mathbf{E}_a &= \mathbf{E}_{a1} + \mathbf{E}_{a2} + \mathbf{E}_{a3} \\ &= E \angle -20^\circ + E \angle 0^\circ + E \angle 20^\circ \\ &= E \cos(-20^\circ) + jE \sin(-20^\circ) + E \\ &\qquad + E \cos 20^\circ + jE \sin 20^\circ \\ &= E + 2E \cos 20^\circ = 2.879\,E\end{aligned}$$

2.879E* instead of *3E



2-pole stator with a single-layer winding consisting of 3 coils per phase, each separated by 20^0

Distributed Winding

- **Breadth factor or distribution factor of winding** - ratio of actual voltage in a phase of a distributed winding to its expected value in a concentrated winding with same number of turns → $$k_d = \frac{V_\phi \text{ actual}}{V_\phi \text{ expected with no distribution}}$$

- Distribution factor for the machine in last slide, $$k_d = \frac{2.879E}{3E} = 0.960$$

- For a winding with *n* slots per phase belt spaced γ degrees, $$k_d = \frac{\sin(n\gamma/2)}{n \sin(\gamma/2)}$$

- In previous example $n = 3$ and $\gamma = 20°$

$$k_d = \frac{\sin(n\gamma/2)}{n \sin(\gamma/2)} = \frac{\sin[(3)(20°)/2]}{3 \sin(20°/2)} = 0.960$$

Generated Voltage Including Distribution Effects

- RMS voltage in a single coil of N_c turns and pitch factor k_p

$$E_A = \sqrt{2}\pi N_C k_p \phi f$$

- If a stator phase consists of i coils, each containing N_c turns, then a total turns present in the phase, $\boldsymbol{N_p = iN_c}$
- Total phase voltage $\boxed{E_A = \sqrt{2}\pi N_P k_p k_d \phi f}$
- Winding factor $\boxed{k_w = k_p k_d}$
- So, $\boxed{E_A = \sqrt{2}\pi N_P k_w \phi f}$

Generated Voltage Including Distribution Effects

Example B–2. A simple two-pole, three-phase, Y-connected synchronous machine stator is used to make a generator. It has a double-layer coil construction, with four stator coils per phase distributed as shown in Figure B–8. Each coil consists of 10 turns. The windings have an electrical pitch of 150°, as shown. The rotor (and the magnetic field) is rotating at 3000 r/min, and the flux per pole in this machine is 0.019 Wb.

(*a*) What is the slot pitch of this stator in mechanical degrees? In electrical degrees?
(*b*) How many slots do the coils of this stator span?
(*c*) What is the magnitude of the phase voltage of one phase of this machine's stator?
(*d*) What is the machine's terminal voltage?
(*e*) How much suppression does the fractional-pitch winding give for the fifth-harmonic component of the voltage relative to the decrease in its fundamental component?

Generated Voltage Including Distribution Effects

Solution

(*a*) This stator has 6 phase belts with 2 slots per belt, so it has a total of 12 slots. Since the entire stator spans 360°, the slot pitch of this stator is

$$\gamma = \frac{360^\circ}{12} = 30^\circ$$

This is both its electrical and mechanical pitch, since this is a two-pole machine.

(*b*) Since there are 12 slots and 2 poles on this stator, there are 6 slots per pole. A coil pitch of 150 electrical degrees is 150°/180° = 5/6, so the coils must span 5 stator slots.

(*c*) The frequency of this machine is

$$f = \frac{n_m P}{120} = \frac{(3000\text{ r/min})(2\text{ poles})}{120} = 50\text{ Hz}$$

From Equation (B–19), the pitch factor for the fundamental component of voltage is

$$k_p = \sin\frac{\nu\rho}{2} = \sin\frac{(1)(150^\circ)}{2} = 0.966$$

Generated Voltage Including Distribution Effects

Although the windings in a given phase belt are in three slots, the two outer slots have only one coil each from the phase. Therefore, the winding essentially occupies two complete slots. The winding distribution factor is

$$k_d = \frac{\sin(n\gamma/2)}{n\sin(\gamma/2)} = \frac{\sin[(2)(30°)/2]}{2\sin(30°/2)} = 0.966$$

Therefore, the voltage in a single phase of this stator is

$$\begin{aligned} E_A &= \sqrt{2}\ \pi N_p k_p k_d \phi f \\ &= \sqrt{2}\ \pi(40 \text{ turns})(0.966)(0.966)(0.019 \text{ Wb})(50 \text{ Hz}) \\ &= 157 \text{ V} \end{aligned}$$

(d) This machine's terminal voltage is

$$V_T = \sqrt{3}E_A = \sqrt{3}(157 \text{ V}) = 272 \text{ V}$$

Generated Voltage Including Distribution Effects

(*d*) This machine's terminal voltage is

$$V_T = \sqrt{3}E_A = \sqrt{3}(157 \text{ V}) = 272 \text{ V}$$

(*e*) The pitch factor for the fifth-harmonic component is

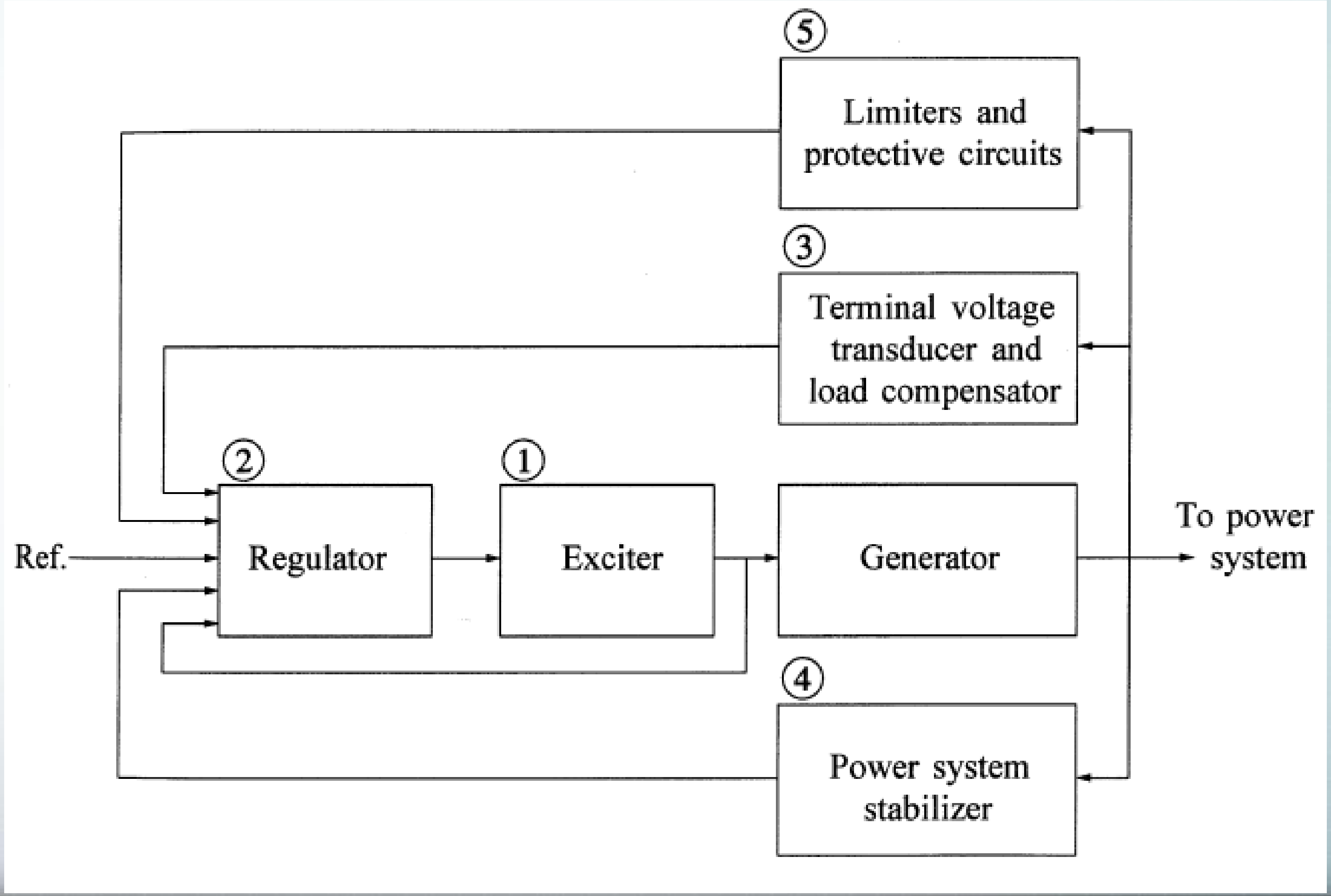
$$k_p = \sin\frac{\nu\rho}{2} = \sin\frac{(5)(150°)}{2} = 0.259$$

Since the pitch factor of the fundamental component of the voltage was 0.966 and the pitch factor of the fifth-harmonic component of voltage is 0.259, the fundamental component was decreased 3.4 percent, while the fifth-harmonic component was decreased 74.1 percent. Therefore, the fifth-harmonic component of the voltage is decreased 70.7 percent more than the fundamental component is.

Excitation System

Excitation System

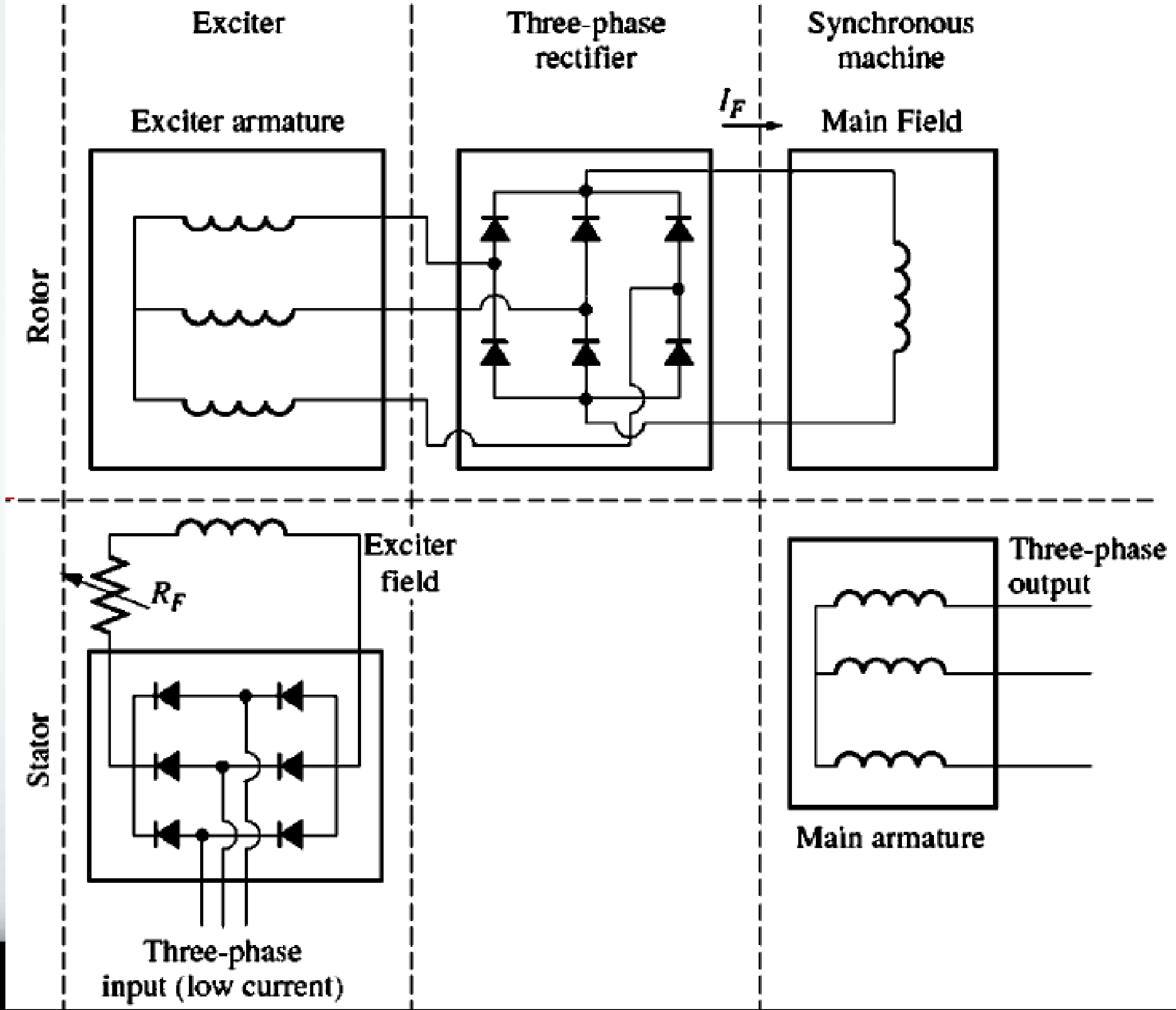
Functional block diagram of a synchronous generator excitation control system



Brushless Excitation System

Output of armature circuit of exciter rectified and used to supply field current of main machine

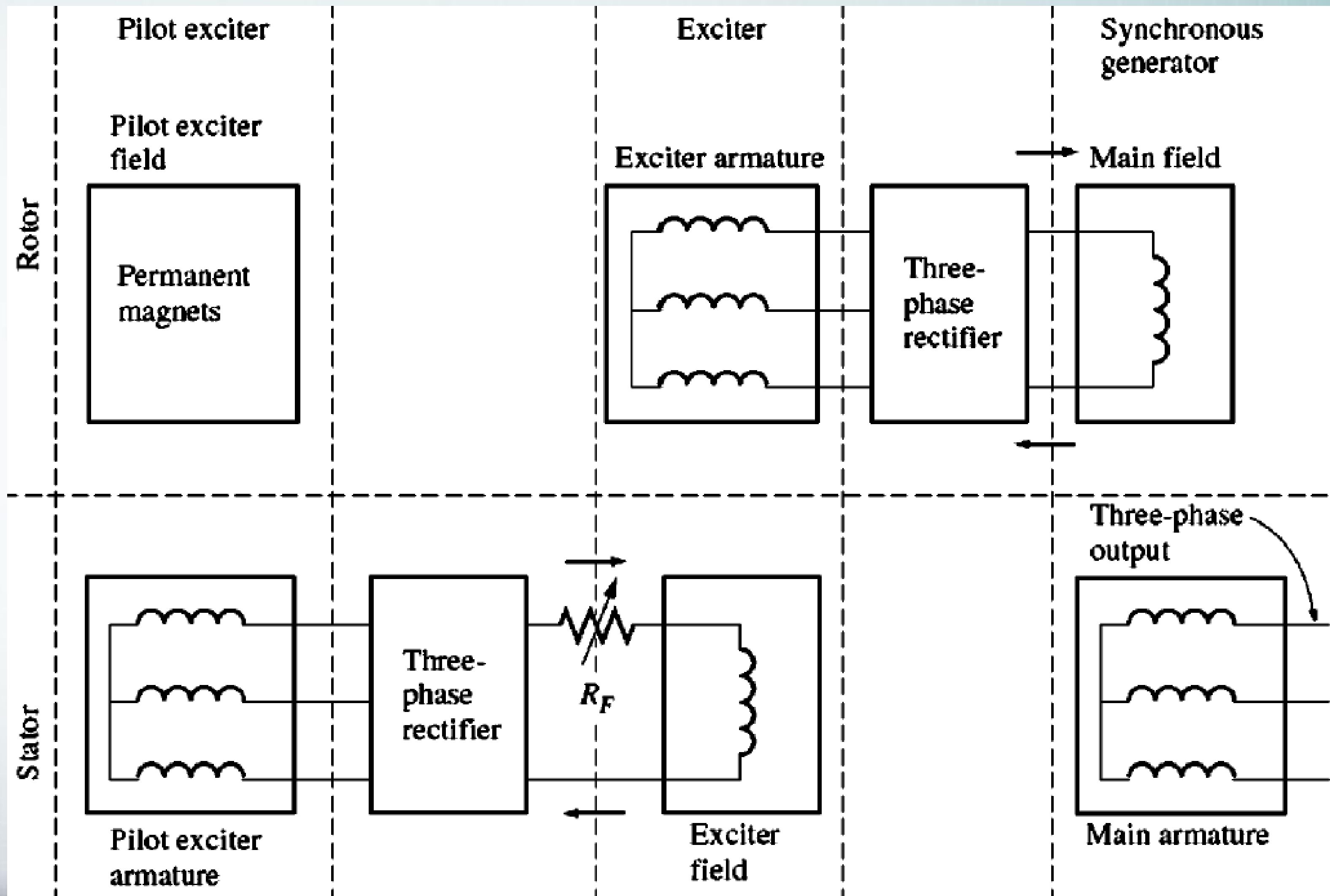
Small 3-phase current rectified and used to supply field circuit of exciter



Brushless Excitation System

A brushless excitation scheme with pilot exciter

Permanent magnets of pilot exciter produce field current of exciter, which in turn produces field current of main machine



Equivalent Circuit of non-Salient Pole Synchronous Generator

Internal Generated Voltage

- Magnitude of voltage induced in a given stator phase

$$\boxed{E_A = \sqrt{2}\pi N_P k_w \phi f}$$

- In simpler form $\boxed{E_A = K\phi\omega}$

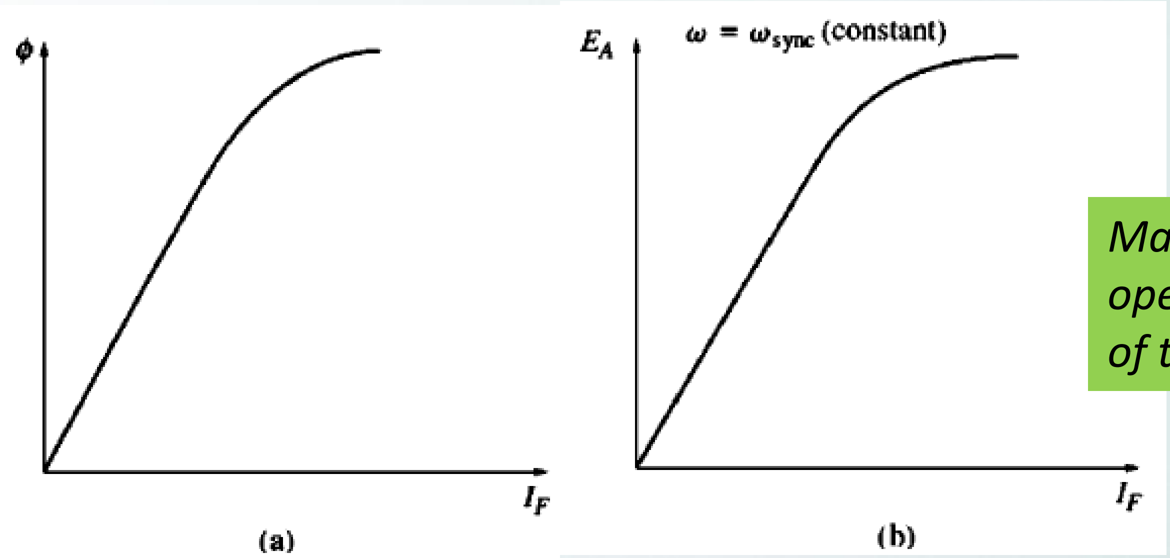
K – a constant representing construction of machine

Observations

- Internal generated voltage E_A directly proportional to flux and to speed
- Flux depends on current flowing in rotor field circuit

Internal Generated Voltage

- Field circuit I_F related to flux φ [Fig. (a)]
- Since E_A directly proportional to flux, E_A is related to field current [Fig. (b)]

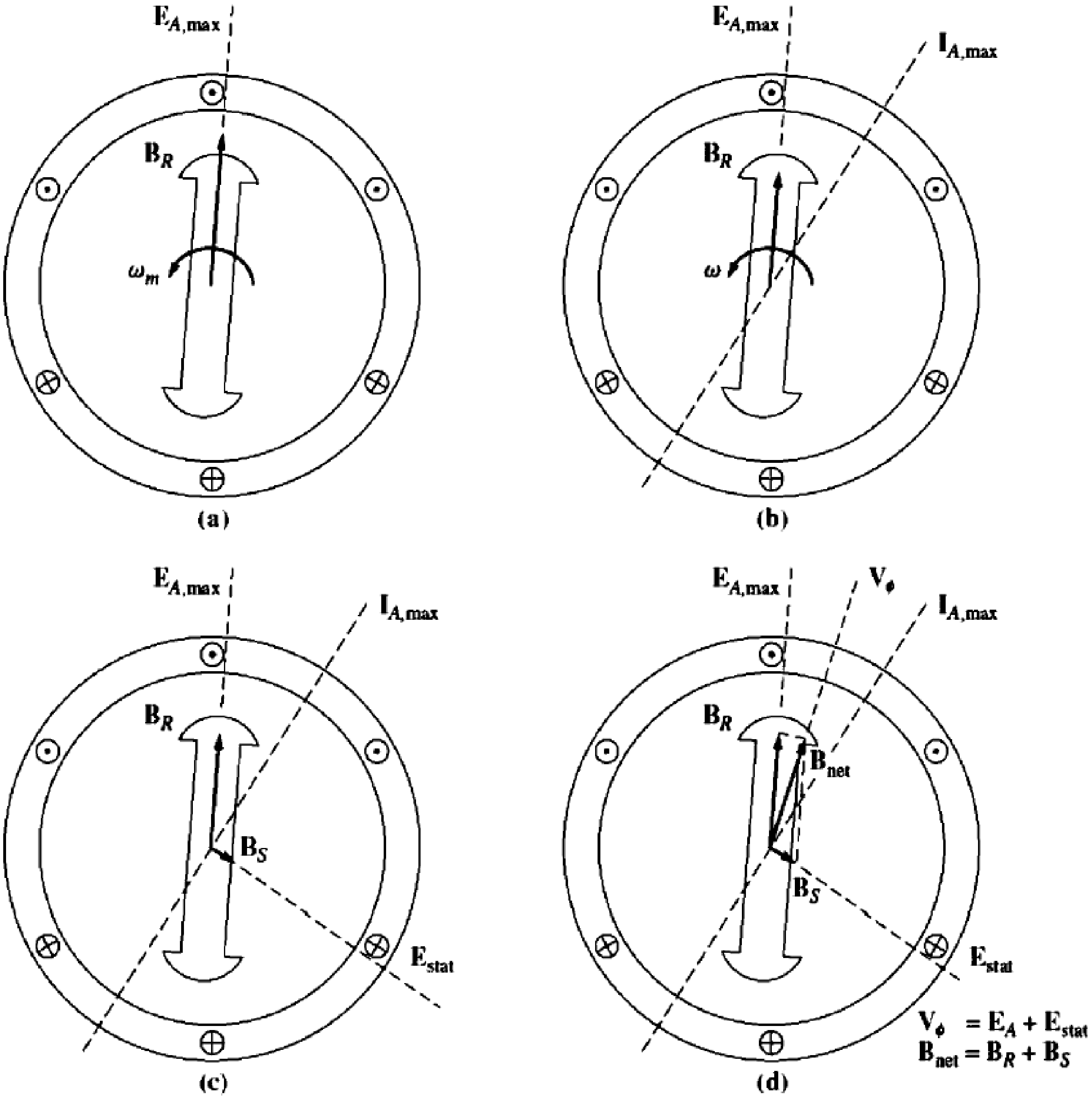


Magnetization curve or open-circuit characteristic of the generator

Saturation puts a practical limit on the maximum magnetic fields achievable in ferromagnetic-core electromagnets and transformers of around 2 T, which puts a limit on the minimum size of their cores. This is one reason why high power motors, generators, and transformers are

Equivalent Circuit of Synchronous Generator

- Factors that cause difference between **internal voltage E_A** and **phase voltage V_φ**
  1. **Armature reaction** – distortion of air-gap magnetic field by current flowing in stator
  2. Self-inductance of armature coils
  3. Resistance of armature coils
  4. Effect of salient-pole rotor shapes



(a) Rotating field B_R produces internal generated voltage $E_{A'}$

(b) $E_{A'}$ produces lagging current flow when connected to a lagging load

(c) Stator current produces its own magnetic field B_S → produces voltage E_{stat} in stator windings

(d) B_S adds to B_R → distorting it into B_{net}

Voltage E_{stat} adds to E_A producing V_φ

Angles of E_A, B_R same and angles of E_{stat}, B_S same → resulting magnetic field B_{net} coincide with net voltage V_φ

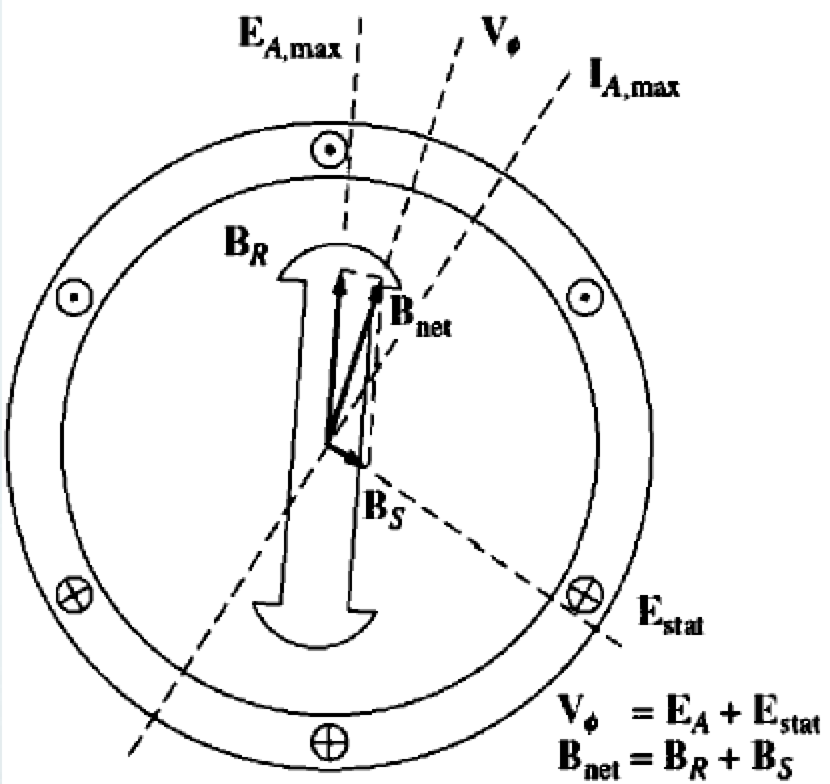
Equivalent Circuit of Synchronous Generator

- E_{stat} lies 90° behind plane of maximum current I_A
- E_{stat} directly proportional to I_A
- Let X be a constant of proportionality

→ armature reaction voltage can be expressed as $\boxed{\mathbf{E}_{\text{stat}} = -jX\mathbf{I}_A}$

- Voltage on a phase is thus

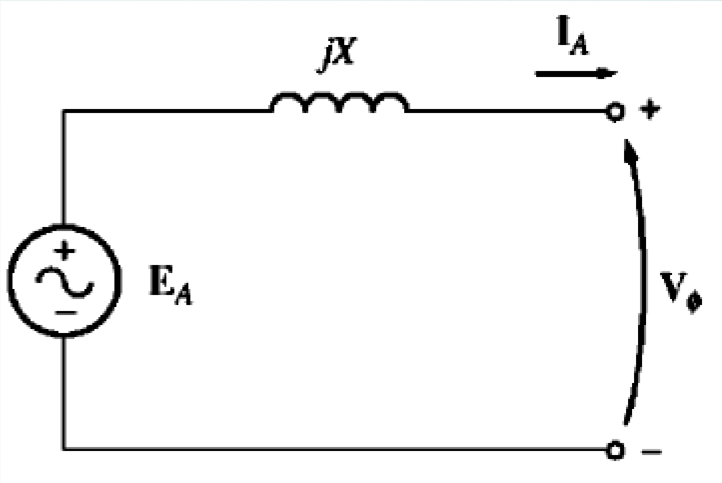
$$\boxed{\mathbf{V}_\phi = \mathbf{E}_A - jX\mathbf{I}_A}$$



(d)

Equivalent Circuit of Synchronous Generator

$$\boxed{\mathbf{V}_\phi = \mathbf{E}_A - jX\mathbf{I}_A}$$



Adding stator coil self inductance and resistance →

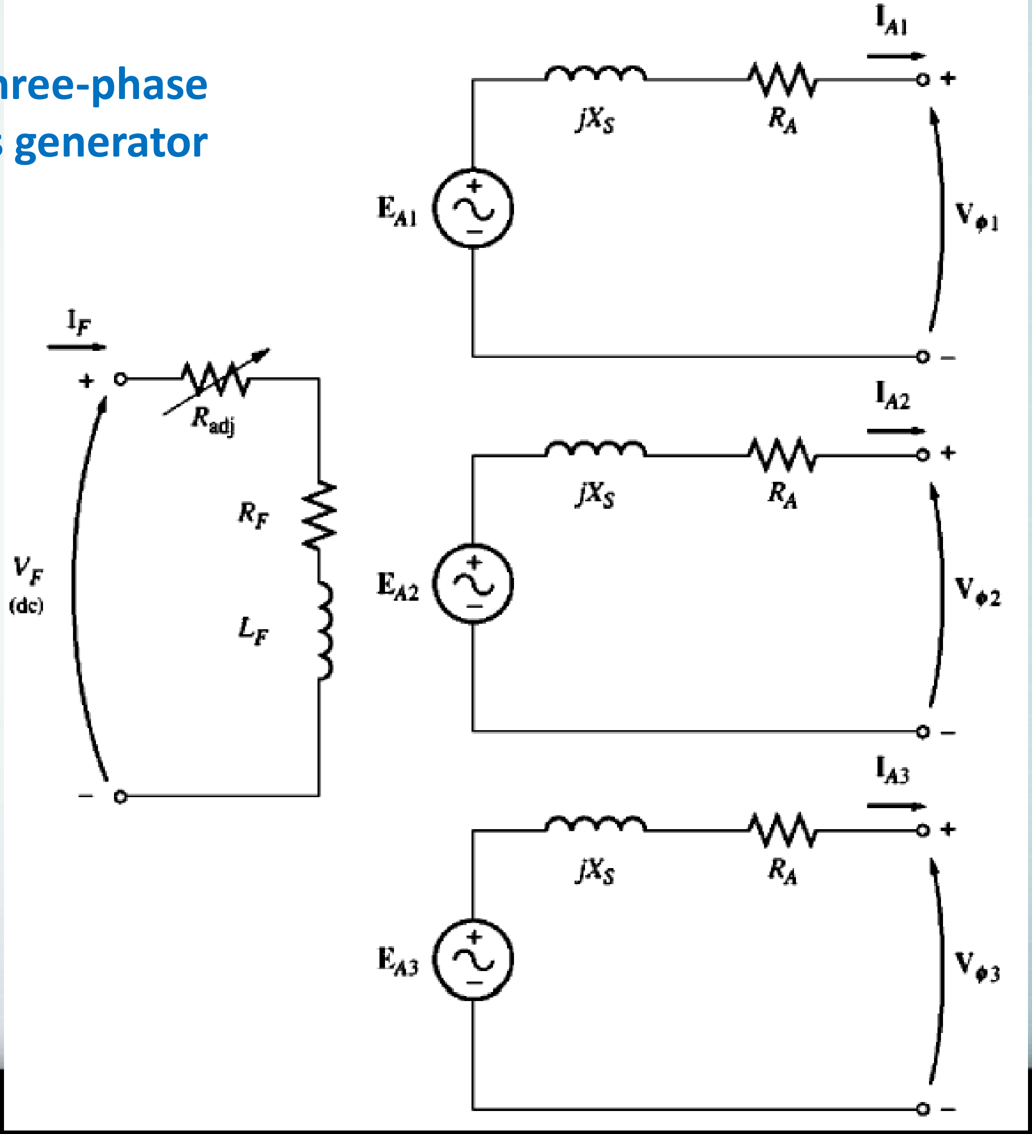
$$\boxed{\mathbf{V}_\phi = \mathbf{E}_A - jX\mathbf{I}_A - jX_A\mathbf{I}_A - R_A\mathbf{I}_A}$$

$$X_S = X + X_A$$ Synchronous reactance

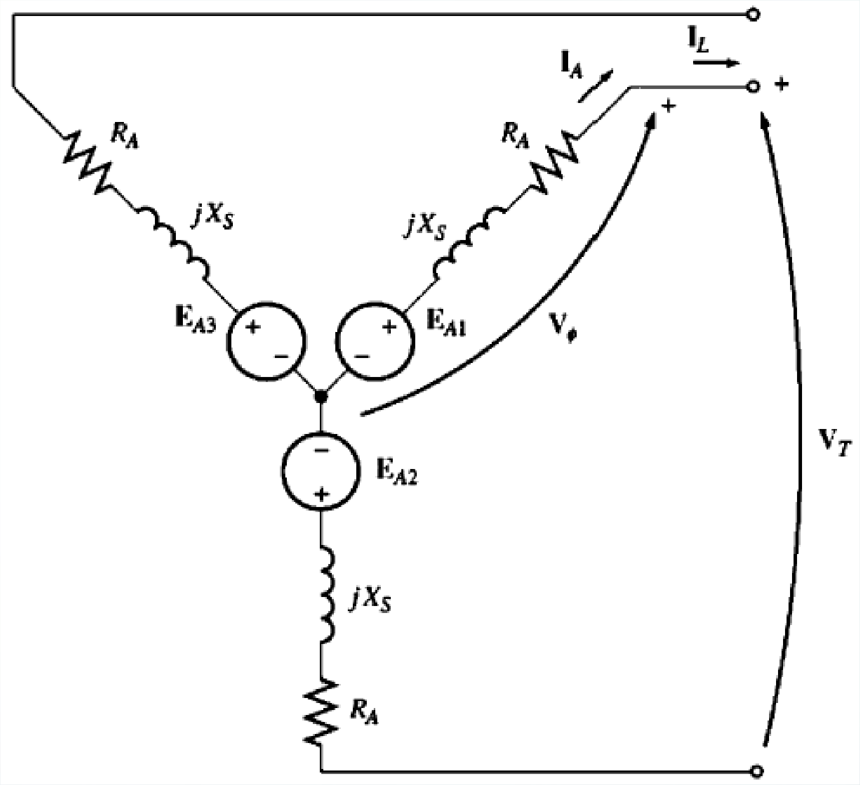
$$\boxed{\mathbf{V}_\phi = \mathbf{E}_A - jX_S\mathbf{I}_A - R_A\mathbf{I}_A}$$

Equivalent Circuit of Synchronous Generator

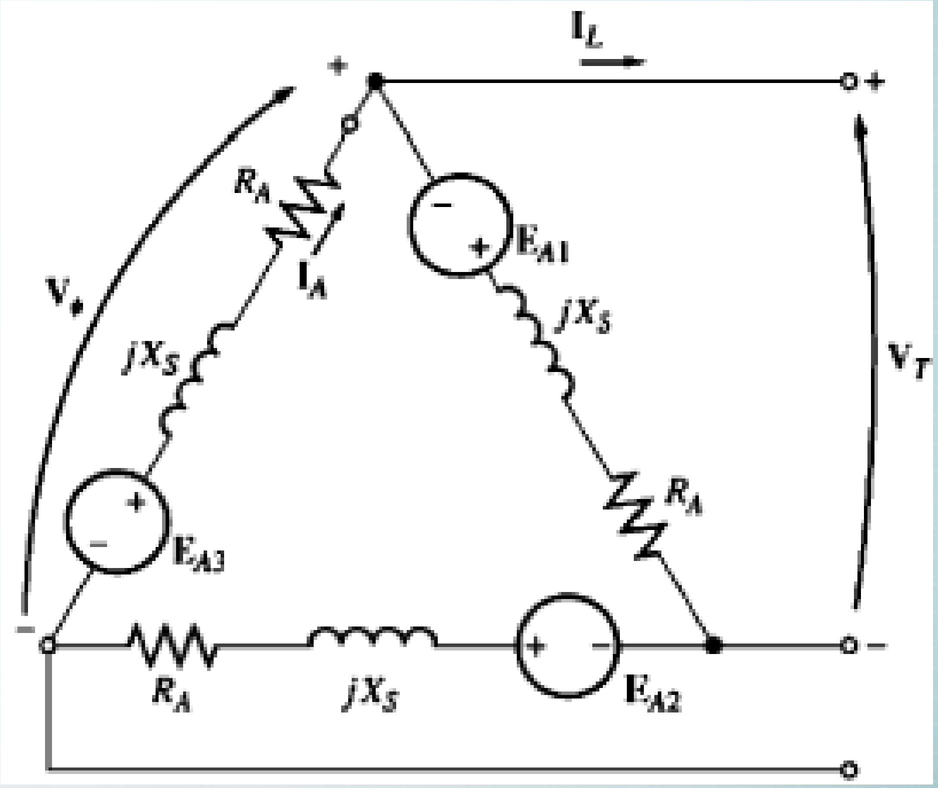
Full equivalent circuit of a three-phase synchronous generator



Equivalent Circuit of Synchronous Generator

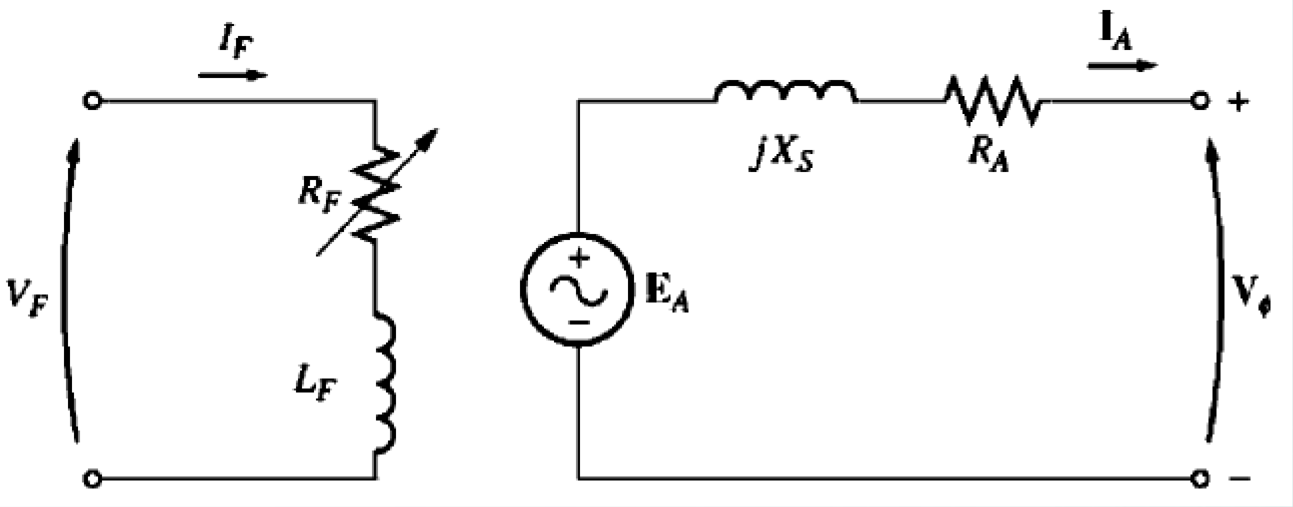


Generator equivalent circuit connected in Y



Generator equivalent circuit connected in Δ

Equivalent Circuit of Synchronous Generator

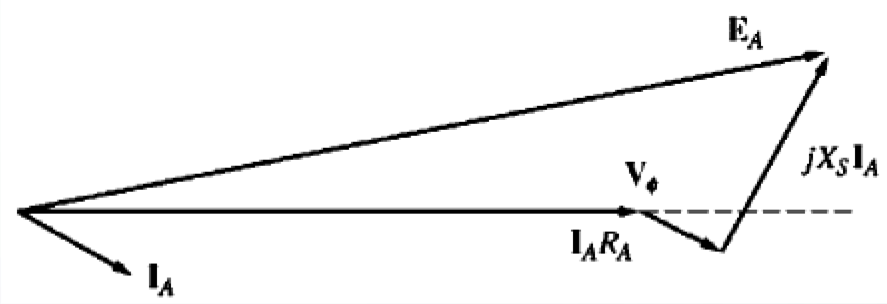


Per-phase equivalent circuit of a synchronous generator

- Internal field circuit resistance and external variable resistance combined into a single resistor R_F

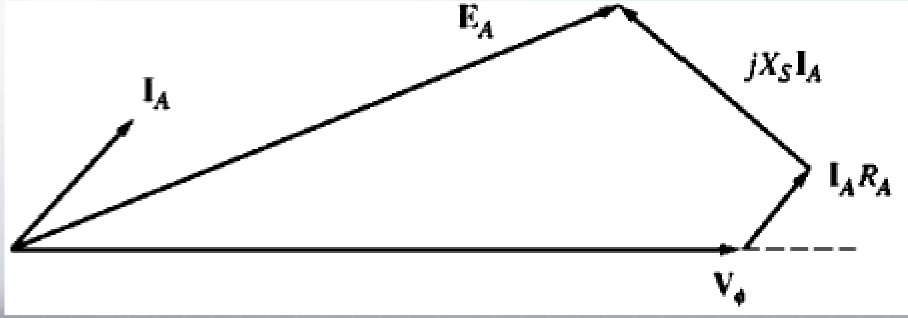
Phasor Diagram

Phasor diagram of a synchronous generator at **unity power factor**



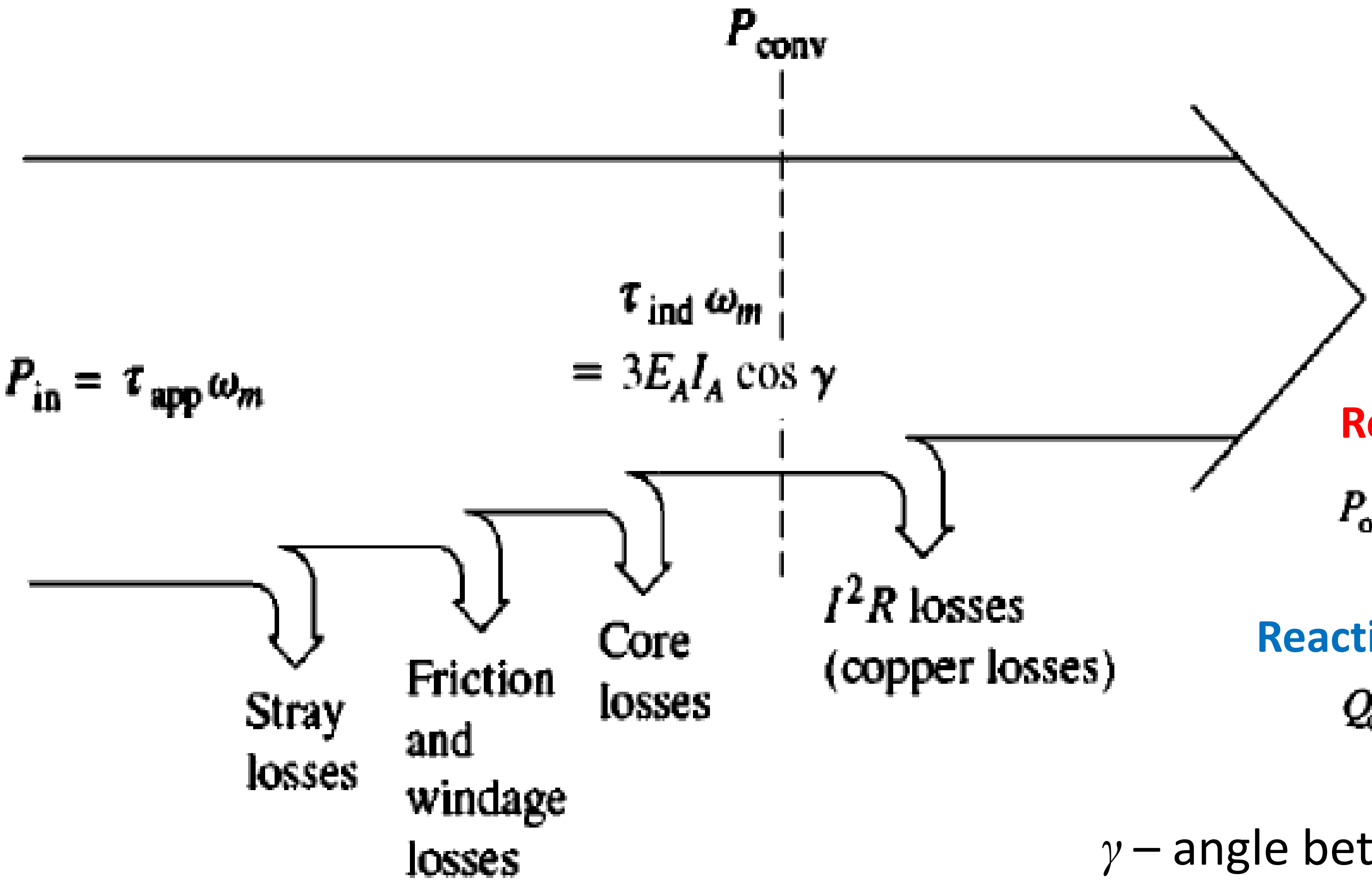
Phasor diagram of a synchronous generator at **lagging power factor**

? Comment on voltage regulation with different power factor type loads?



Phasor diagram of a synchronous generator at **leading power factor**

Power and Torque



Real power output

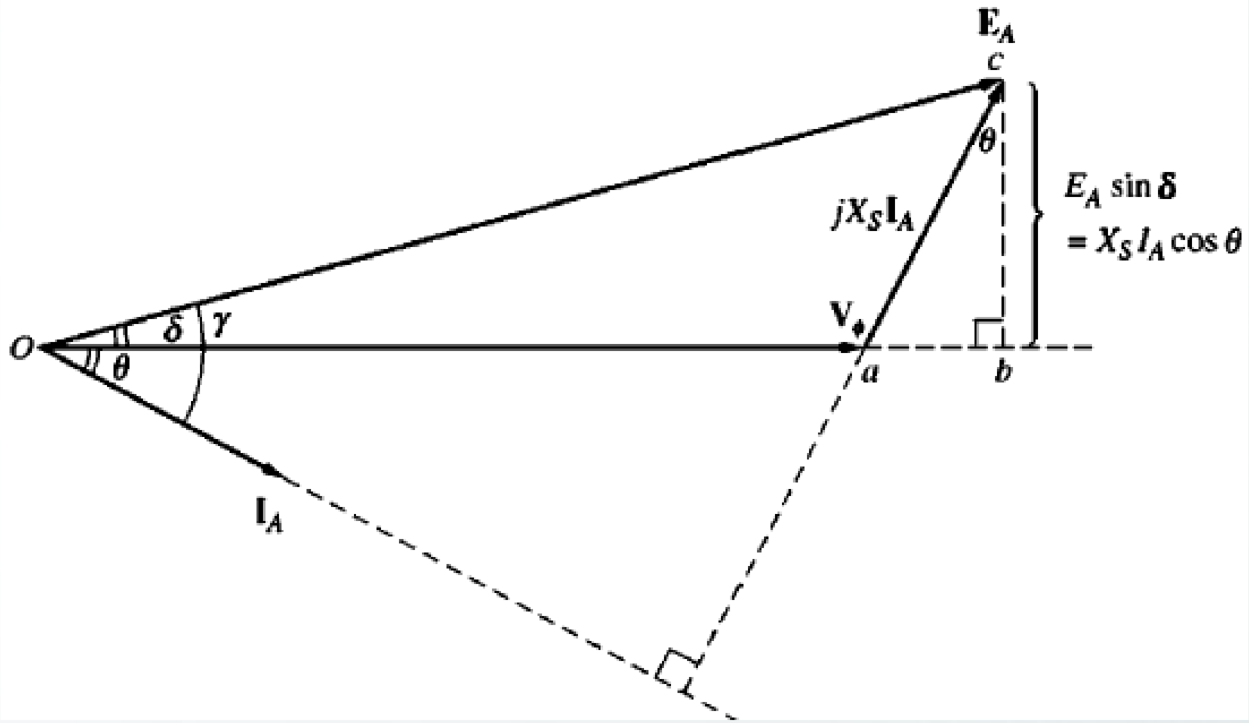
$$P_{out} = \sqrt{3}V_T I_L \cos\theta$$
$$= 3V_\phi I_A \cos\theta$$

Reactive power output

$$Q_{out} = \sqrt{3}V_T I_L \sin\theta$$

γ – angle between E_A and I_A

Power and Torque



$$E_A \sin\delta = X_S I_A \cos\theta$$

$$I_A \cos\theta = \frac{E_A \sin\delta}{X_S}$$

$$P_{out} = 3V_\phi I_A \cos\theta$$

$$= \frac{3V_\phi E_A \sin\delta}{X_S}$$

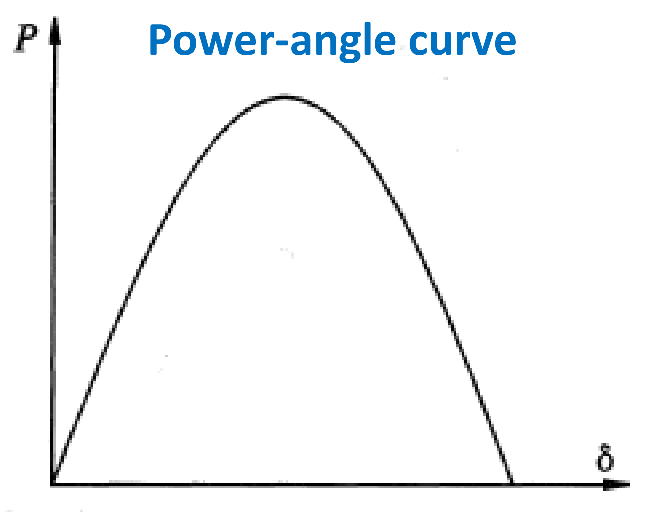
Simplified phasor diagram with armature resistance ignored

[Since X_S>> R_A, armature resistance neglected]

Power and Torque

$$P = \frac{3V_\phi E_A \sin\delta}{X_S} = P_{\max}\sin\delta$$

- Power produced by a synchronous generator depends on angle δ between V_φ and E_A
- δ is known as **torque angle of the machine**



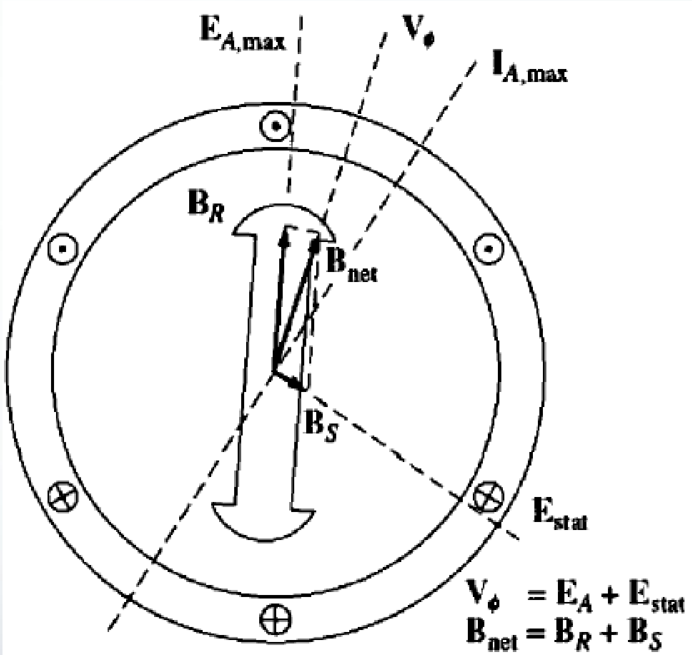
If V_φ is assumed constant

→real power output directly proportional to $I_A \cos\theta$ and $E_A \sin\delta$

→reactive power output directly proportional to $I_A \sin\theta$

Power and Torque

Induced torque $$\tau_{ind} = k\mathbf{B}_R \times \mathbf{B}_{net} = kB_R B_{net} \sin\delta$$



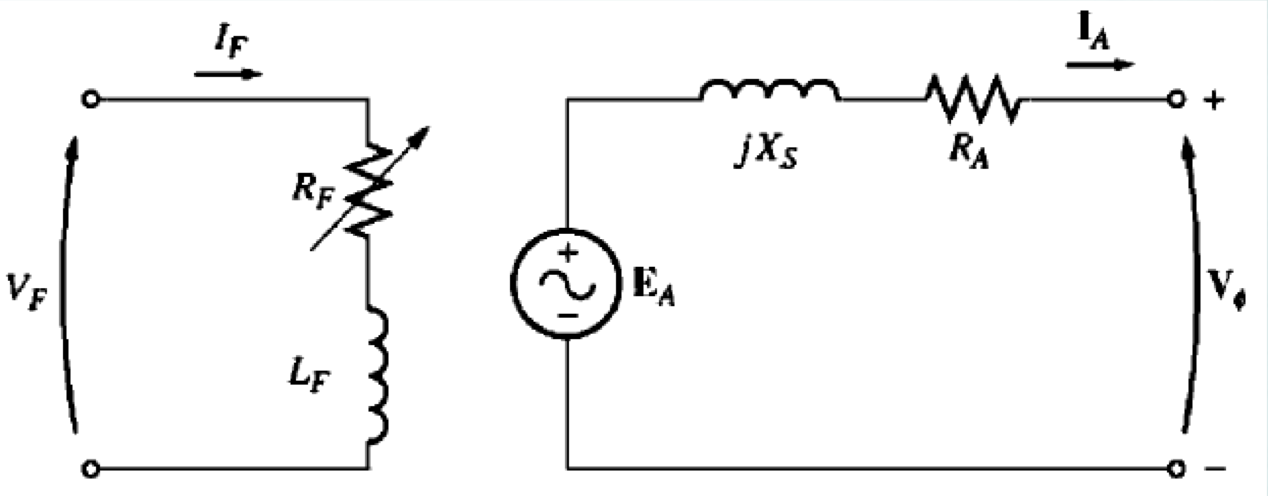
(d)

Since B_R produces E_A and B_{net} produces V_φ, angle δ between E_A and V_φ the same as the angle δ between B_R and B_{net}

Again since, $$P_{conv} = \tau_{ind}\omega_m$$

$$\tau_{ind} = \frac{3V_\phi E_A \sin\delta}{\omega_m X_S}$$

Measuring Generator Model Parameters

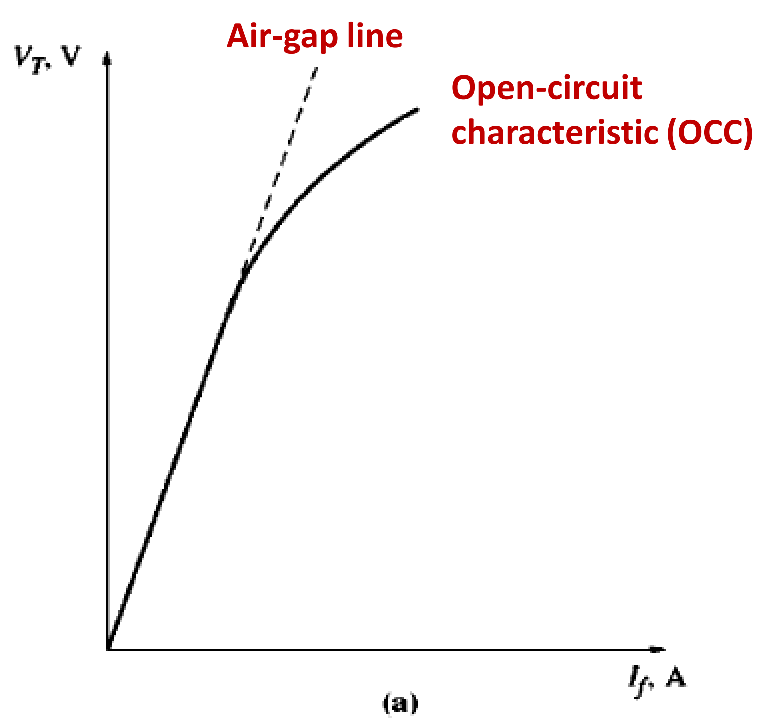


1. Open-circuit characteristic → relationship between I_F and flux (i.e. between I_F and E_A)
2. Synchronous reactance, X_S
3. Armature resistance, R_A

Measuring Generator Model Parameters

Open-circuit test

- Terminals open
- Field current gradually increased in steps, and terminal voltage at each step

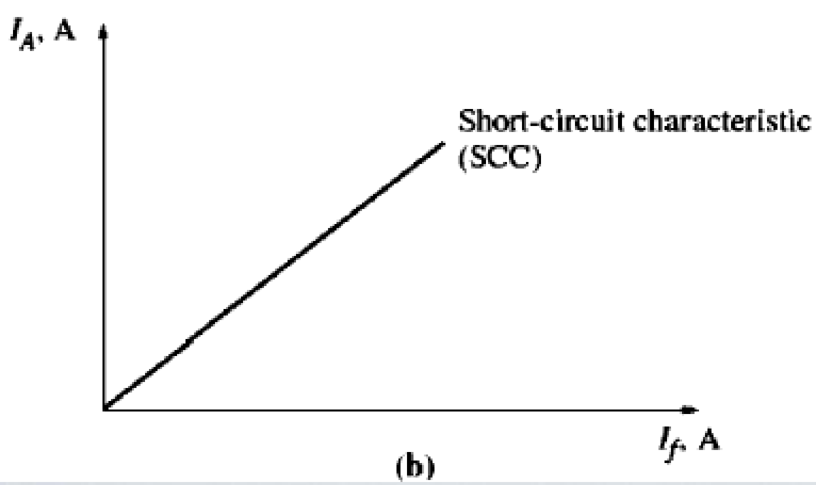


- Reluctance of unsaturated iron in machine frame several thousand times lower than air-gap reluctance
- Initially almost all mmf is across air gap → resulting flux increase linear
- When iron saturates, reluctance of iron increases dramatically → flux increases much more slowly with an increase in mmf

Measuring Generator Model Parameters

Short-circuit test

- Generator terminal short-circuited
- Armature current I_A measured as I_F is increased

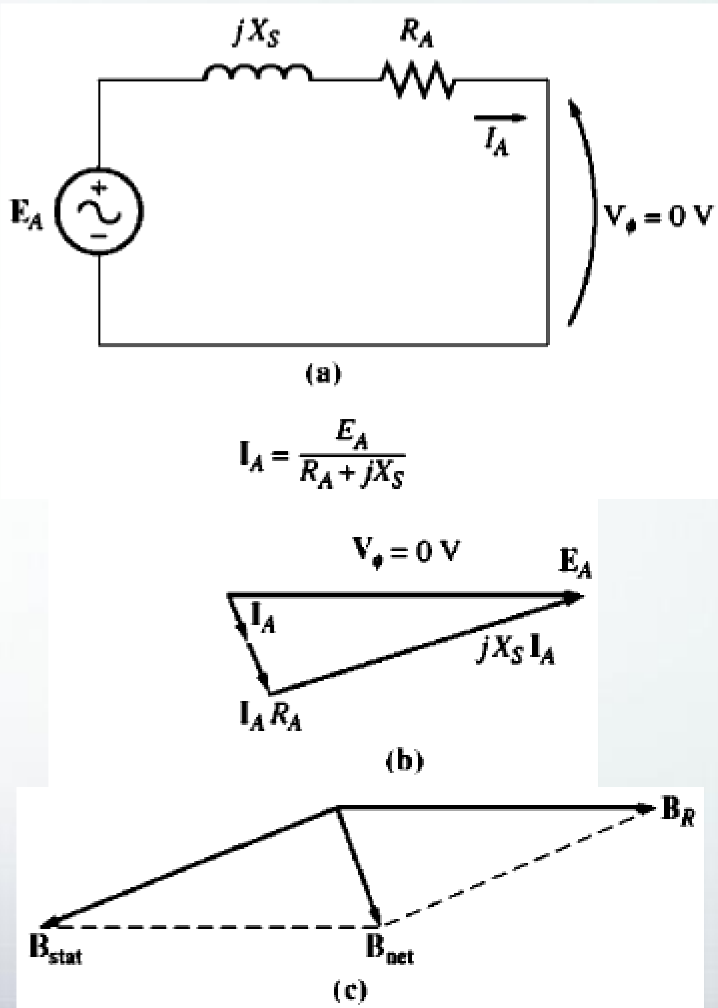


- When terminals are short-circuited, armature current

$$\mathbf{I}_A = \frac{\mathbf{E}_A}{R_A + jX_S}$$

$$|I_A| = \frac{E_A}{\sqrt{R_A^2 + X_S^2}}$$

Measuring Generator Model Parameters



- B_S almost cancels B_R → B_{net} very small (corresponding to internal resistive and inductive drops only)
- Machine is unsaturated → SCC is linear

Measuring Generator Model Parameters

- From OCC

$$Z_S = \sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_A}$$

Since $X_S >> R_A$,

$$X_S \approx \frac{E_A}{I_A} = \frac{V_{\phi,oc}}{I_A}$$

- Winding resistance can be measured by applying a dc voltage to windings while machine is stationary and measuring resulting current flow
- Not perfectly accurate → ac resistance will be slightly larger than dc resistance (due to skin effect at higher frequencies)

Approximate method for determining X_S at a given field current:

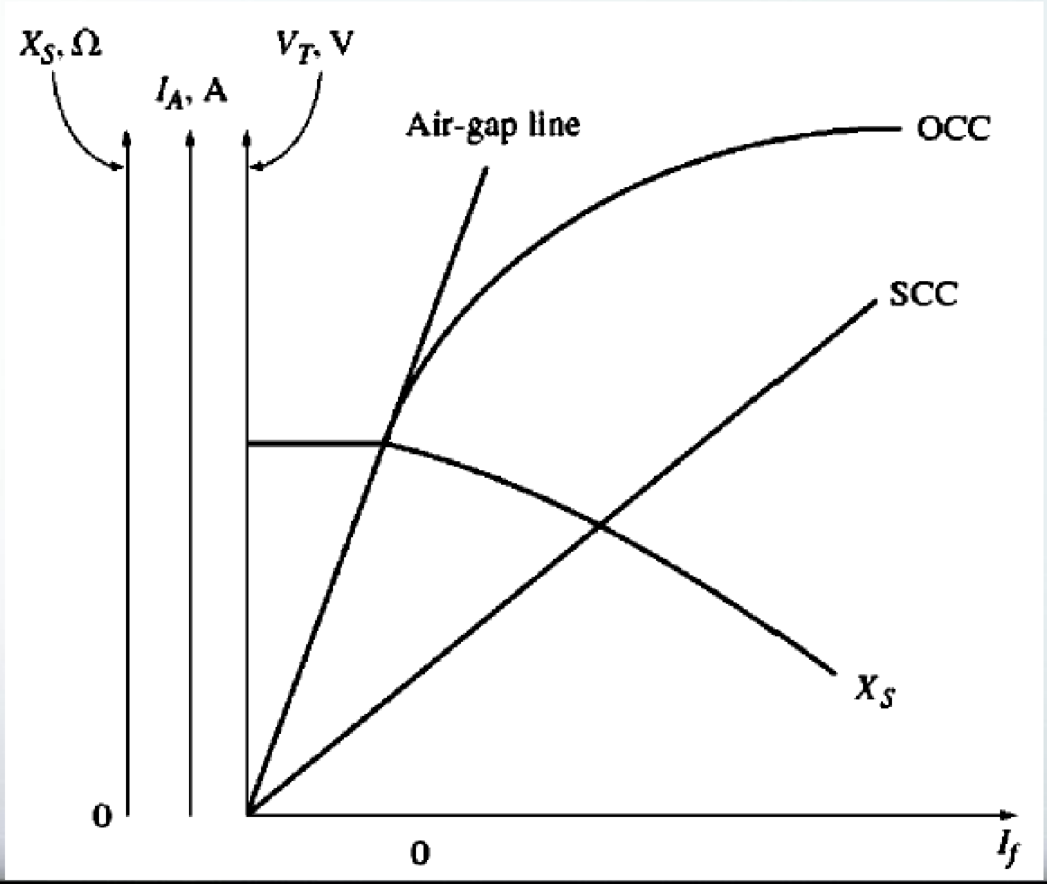
1. Get internal generated voltage E_A from OCC at that field current
2. Get short-circuit current flow $I_{A,SC}$ at that field current from SCC
3. Find Xs by applying $X_S \approx \frac{E_A}{I_A}$

Measuring Generator Model Parameters

- **Problem with approximate approach**
  - E_A taken from OCC, where machine is partially saturated for large I_F
  - I_A taken from SCC, where machine is unsaturated at all I_F
  - So, at higher field currents, E_A taken from OCC at a given I_F is not the same as E_A at the same I_F under short-circuit conditions
  - Resulting value of X_S only approximate
- This approach is accurate up to the point of saturation, i.e. unsaturated synchronous reactance X_S

Measuring Generator Model Parameters

A plot of approximate synchronous reactance as a function of field current



- Approximate value of X_S varies with degree of saturation of OCC

→ so value of X_S to be used in a given problem should be one calculated at the approximate load on machine

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Short-Circuit Ratio – a parameter used to describe synchronous generators

- It is the ratio of I_F required for rated voltage at open circuit to the field current required for rated I_A at short circuit

→ reciprocal of per-unit value of approximate saturated synchronous reactance calculated by $X_S \approx \frac{E_A}{I_A}$

→ this term is occasionally encountered in industry