PATUAKHALI SCIENCE AND TECHNOLOGY UNIVERSITY

Group Assignment of MAT-211

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Equation Reducible to Homogenous

An equation that is reducible to a homogeneous form is one that, through some transformations, can be converted into a form where the function can be expressed as a ratio of variables, often leading to a separable equation.

Homogeneous Differential Equations

A first-order ordinary differential equation (ODE) of the form:

$M(x,y)\, dx+N(x,y)\, dy= 0$

is homogeneous if both $M(x,y)M(x,y)$ and $N(x,y)N(x,y)$ are homogeneous functions of the same degree. A function $f(x,y)f(x,y)$ is homogeneous of degree n if:

$f(tx,ty)=t^n f(x,y)$

for any $t\in R$.

Transformations to Homogeneous Form

Sometimes, an equation that is not explicitly in homogeneous form can be transformed into one that is. One common transformation is the substitution $y=vx$ where v=y/x .

Example: Reduction to Homogeneous Form

Consider the differential equation:

$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{ax+by+c}}{dx+ey+f}$

This is not homogeneous as is. However, we can make a substitution to reduce it to a homogeneous form.

1. **Substitution**: $y=vx$

   $\frac{\mathrm{dy}}{\mathrm{dx}}=\mathrm{v}+x\frac{\mathrm{dv}}{dx}$

2. **Rewrite the Original Equation**: Substitute $y=vx$ into the original equation:

   $v+x\frac{\mathrm{dv}}{dx}=\frac{\mathrm{ax+b(vx)+c}}{dx+e(vx)+f}$

   $v+x\frac{\mathrm{dv}}{dx}=\frac{\mathrm{ax+bvx+c}}{dx+evx+f}$

3. **Simplify**:
4. $v+x\frac{\mathrm{dv}}{dx}=\frac{\mathrm{x(a+bv)+c}}{x(d+ev)+f}$

   $v+x\frac{\mathrm{dv}}{dx}=\frac{\mathrm{a+bv}+\frac{c}{x}}{d+ev+\frac{f}{x}}$

5. **Analyze the Terms**:

   If c and f are zero, the equation becomes:

   $v+x\frac{\mathrm{dv}}{dx}=\frac{\mathrm{a+bv}}{d+ev}$

This is a homogeneous differential equation in terms of vv and xx.

Solving the Homogeneous Form

1. **Separate Variables**:

   $x\frac{\mathrm{dv}}{dx}=\frac{\mathrm{a+bv}}{d+ev}-\mathrm{v}$

   $x\frac{\mathrm{dv}}{dx}=\frac{\mathrm{a+bv-vd}-v^2\mathrm{e}}{d+ev}$

$$x\frac{dv}{dx} = \frac{a-vd}{d+ev}$$

2. **Integrate**:

$$\int \frac{d+ev}{a-vd}dv = \int \frac{1}{x}dx$$

Example-1:Solve the DE dy/dx=(x+2y−1)/(x+2y+1).

Solution:

Note that *h*, *k* do not exist in this case which can reduce this DE to homogeneous form. Thus, we use the substitution

x+2y=v

⇒ 1+2 dy/dx

Thus, our DE becomes

1/2(dv/dx−1) = (v−1)/(v+1)

⇒ dv/dx=(2v−2)/(v+1)+1

= (3v−1)/(v+1)

⇒ (v+1)/(3v−1)dv=dx

⇒ 1/3(1+4/(3v−1))dv=dx

Integrating, we have

1/3(v+4/3ln(3v−1))=x+C1

Substituting v=x+2y, we have

x+2y+4/3ln(3x+6y−1)=3x+C2

⇒ y−x+2/3ln(3x+6y−1)=C

Example 2:Reducible to homogeneous differential equation.

$$\text{Solve the DE} \quad \frac{dy}{dx} = \frac{2y - x - 4}{y - 3x + 3}$$

Solution: We substitute x→X+h and y→Y+k where h, k need to be determined :

$$\frac{dy}{dx} = \frac{dY}{dX} = \frac{\{(2Y - X) + (2K - h - 4)\}}{\{(Y - 3X) + (k - 3h + 3)\}}$$

h and k must be chosen so that

$2k-h-4=0$

$k-3h+3=0$

This gives h=2 and k=3.k=3. Thus,

$x=X+2$

$y=Y+3$

Our DE now reduces to

$$\frac{dY}{dX} = \frac{2Y - X}{Y - 3X}$$

Using the substitution Y=vX, and simplifying, we have (verify),

$$\frac{v - 3}{v^2 - 5v + 1} dv = \frac{-dX}{X}$$

We now integrate this DE which is VS; the left-hand side can be integrated by the techniques described in the unit on Indefinite Integration.

Finally, we substitute $v = \frac{Y}{X}$ and

$X = x - 2$

$Y = y - 3$

to obtain the general solution.

Suppose our DE is of the form

$$\frac{dy}{dx} = f\left(\frac{ax + by + c}{dx + ey + f}\right)$$

We try to find h, k so that

$ah + bk + c = 0$

$dh + ek + f = 0$

What if this system does not yield a solution? Recall that this will happen if $\frac{a}{b} = \frac{d}{e}$. How do we reduce the DE to a homogeneous one in such a case ?

let $\frac{a}{d} = \frac{b}{e} = \lambda$ (say). Thus,

$$\frac{ax + by + c}{dx + ey + f} = \frac{\lambda(dx + ey) + c}{dx + ey + f}$$

This suggests the substitution $dx + ey = v$, which'll give

$$d + e\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e}\left(\frac{dv}{dx} - d\right)$$

Thus, our DE reduces to

$$\frac{1}{e}\left(\frac{dv}{dx} - d\right) = \frac{\lambda v + c}{v + f}$$

$$\Rightarrow \frac{dv}{dx} = \frac{\lambda ev + ec}{v + f} + d$$

$$= \frac{(\lambda e+d)v+(ec+df)}{v+f}$$

$$\Rightarrow \frac{v+f}{(\lambda e+d)v+(ec+df)}$$

which is in VS form and hence can be solved.

Example 3. Solve (3x-7y-3) $\frac{dy}{dx} = 3y - 7x + 7$

Solution: $\frac{dy}{dx} = \frac{3y=7x+7}{3x-7y-3}$

Put x= X+h, y=Y+k, where h,k are some constants. Then $\frac{dy}{dx} = \frac{dY}{dX}$

And the given eq. becomes, $\frac{dY}{dX} = \frac{3Y-7X+(3K-7h+7)}{3X-7Y+(3h-7k-3)}$

Choose h,k such that 3h-7k-3=0 and 3k-7h+7=0, which give h=1,k=0.

$\therefore \frac{dY}{dX} = \frac{3Y-7X}{3X-7Y}$

put Y= vX, $\frac{dY}{dX} = v + X\ \frac{dv}{dX}$

$\therefore\ v + X\ \frac{dv}{dX} = \frac{3v-7X}{3X-7vX} = \frac{3v-7}{3-7v}$

Or, X$\frac{dv}{dX}$= $\frac{3v-7}{3X-7vX}$ -v = $\frac{7(v2-1)}{3-7v}$

Or, $\frac{7dX}{X}$= $\frac{3-7v}{v2-1}dv = -\left(\frac{2}{v-1} + \frac{5}{v+1}\right)dv$

Integrating, 7log X = -2log(v-1)-5log(v+1)+log C

Or, $X^7(v-1)^2(v+1)^5$=C

Or, $X^7(\frac{Y}{X}=1)^2(\frac{Y}{X}+1)^6$=Cas Y=vX

Or, $(Y - X)^2(Y+X)^6$=C

Or,$(y-x + 1)^2(y+x-1)^6$=C   as x=X+1, y= Y+0

Example 4: Solve the DE $\frac{dy}{dx} = \frac{x+2y-1}{X+2y+1}$

Solution: Note that *h*, *k* do not exist in this case which can reduce this DE to homogeneous form. Thus, we use the substitution

x+2y=v

$\Rightarrow 1+2y\frac{dy}{dx} = \frac{dv}{dx}$

Thus, our DE becomes

$$\frac{1}{2}\left(\frac{dv}{dx} - 1\right) = \frac{v-1}{v+1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{2v-2}{v+1} + 1$$

$$= \frac{3v-1}{v+1}$$

$$\Rightarrow \frac{v+1}{3v-1}dv = dx$$

$$\Rightarrow \frac{1}{3}\left(1 + \frac{4}{3v-1}\right)dv = dx$$

Integrating, we have

$$\frac{1}{3}\left(v + \frac{4}{3}\ln(3\,\mathrm{v} - 1)\right) = x + C_1$$

Substituting v= x+2y, we have

$$x + 2y + \frac{4}{3}(\ln 3x + 6y - 1) = 3x + C_2$$

$$\Rightarrow y - x + \frac{2}{3}(\ln 3x + 6y - 1) = C$$

Example 5: Find the solution of the differential equation $\frac{dy}{dx} = \frac{x+y+3}{2x+2y+1}$

Solution: Here, $\frac{a}{a_1} = \frac{b}{b_1} = \frac{1}{2}$ i.e., the coefficients of x and y in the Nr and Dr of the expression for $\frac{dy}{dx}$ are proportional. Proper substitution in this case, therefore, will be to put v for x + y. Let x + y = v. Then, $1 + \frac{dy}{dx} = \frac{dv}{dx}$ with these substitutions the given equation reduces to

$$\frac{dv}{dx} - 1 = \frac{v+3}{2v+1}$$

or $\frac{dv}{dx} = \frac{v+3}{2v+1} + 1 = \frac{3v+4}{2v+1}$

Or $dx = \frac{2v+1}{3v+4} dv = [\frac{2}{3} - \frac{\frac{5}{3}}{3v+4}] dv$

$\therefore$ On integrating, $x + C = \frac{2}{3} v - \frac{5}{3} . \frac{1}{3} \log (3v + 4)$

$\Rightarrow \Rightarrow x + C = \frac{2}{3} v - \frac{5}{9} \log (3v + 4)$

Or $x + C = \frac{2}{3}(x + y) - (\frac{5}{9}) \log (3x + 3y + 4)$, here v=x + y

Which is the required solution.

Example 6. Solve (2x+y+3) $\frac{dy}{dx} = x + 2y + 3$

Solution: $\frac{dy}{dx} = \frac{x+2y+3}{2x+y+3}$

Put x= X+h, y=Y+k, where h,k are some constants. Then $\frac{dy}{dx} = \frac{dY}{dX}$

And the given eq. becomes, $\frac{dY}{dX} = \frac{X+2Y+(h+2k+3)}{2X+Y+(2h+k+3)}$

Choose h,k such that $(2h + k + 3)$=0 and $(h + 2k + 3)$=0, which give h=-1,k=-1.

$\therefore \frac{dY}{dX} = \frac{X+2Y}{2X+Y}$

put Y= vX, $\frac{dY}{dX} = v + X\,\frac{dv}{dX}$

$\therefore v + X\,\frac{dv}{dX} = \frac{X+2vX}{2X+vX}$

Or, X$\frac{dv}{dX} = \frac{1+2v}{2+v}$ -v

Or, $\frac{dX}{X} = \frac{2+v}{1-v^2}dv = \left(\frac{3/2}{1-v} + \frac{1/2}{v+1}\right)dv$

Integrating, 2log X = -3log(1-v) + log(v+1) + log C

Or, $X^2(1-v)^{3/}(v+1) = C$

Or, $X^2\,\frac{(1-Y/X)^3}{(1+Y/X)} = C$

Or, $(X-Y)^3 = C(Y+X)$; where x=X-1, y=Y-1

Or, $(X-Y)^3 = C(Y+X-2)$ is the solution.

Example 7: Solve $(3x - 3y + 4)\frac{dy}{dx} = 2x - 2y + 4$

Solution: The equation is $(3x - 3y + 6)\frac{dy}{dx} = 2x - 2y + 4$

$$\Rightarrow \frac{dy}{dx} = \frac{2x-2y+4}{(3x-3y+6)}$$

Put $x - y = v, so\ that\ 1 - \frac{dy}{dx} = \frac{dv}{dx}$

$$\text{Or } \frac{dy}{dx} = 1 - \frac{dv}{dx}.$$

∴ The equation becomes

$$1 - \frac{dv}{dx} = \frac{2v+4}{(3v+6)}$$

$$or, \frac{dv}{dx} = 1 - \frac{2v+4}{(3v+6)}$$

$$= \frac{v+2}{3v+6}$$

$$\text{Or, } dx = \frac{3v+6}{v+2}\, dv$$

∴ Integrating, $x = 3v + \log(v + 2) + C$

$\Rightarrow x = 3(\mathrm{x} - \mathrm{y}) + \log(x - y + 2) + C$; here $v = x - y$

$\Rightarrow 3y - 2x = \log(x - y + 2) + C,$

Which is the required solution.

Reducible to homogeneous differential equation

Example 1: Solve the DE $\frac{dy}{dx}=\frac{2y-x-4}{y-3x+3}$

Solution: We substitute $x \to X+h$ and $y \to Y+k$ where h, k need to be determined

$$\frac{Dy}{dx}=\frac{dY}{dY}=\frac{(2Y-X)+(2k-h-4)}{(Y-3X)+(k-3h+3)}$$

h and k must be chosen so that

$$2k-h-4=0$$
$$k-3h+3=0$$

This gives $h=2$ and $k=3$. Thus,

$$x=X+2$$
$$y=Y+3$$

Our DE now reduces to

$$\frac{dy}{dx}=\frac{2Y-X}{Y-3X}$$

Using the substitution $Y=vX$ and simplifying, we have (verify),

$$\frac{v-3}{v^2-5v+1}dv=\frac{-dX}{X}$$

We now integrate this DE which is VS; the left-hand side can be integrated by the techniques described in the unit on Indefinite Integration.

Finally, we substitute $v = YX$ and

$$X=x-2$$
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to obtain the general solution.

Suppose our DE is of the form

$$\frac{dY}{dX}=f\left(\frac{ax+by+c}{dx+ey+f}\right)$$

We try to find *h*, *k* so that

$$ah+bk+c=0$$
$$dh+ek+f=0$$

Let $\frac{a}{d}=\frac{b}{e}=\lambda$ (say). Thus,

$$\frac{ax+by+c}{dx+ey+f}=\frac{\lambda(dx+ey)+c}{dx+ey+f}$$

This suggests the substitution $dx+ey=v$ which'll give

$$d+e\frac{dy}{dx}=\frac{dv}{dx}$$

$$\frac{dy}{dx}=\frac{1}{e}\left(\frac{dv}{dx}-d\right)$$

Thus, our DE reduces to

$$\frac{1}{e}\left(\frac{dv}{dx}-d\right)=\frac{\lambda v+c}{v+f}$$

$$\Rightarrow \frac{dv}{dx}=\frac{\lambda ev+ec}{v+f}+d$$

$$=\frac{(\lambda e+d)v+(ec+df)}{v+f}$$

$$\Rightarrow \frac{(v+f)}{(\lambda e+d)\,v+ec+df}\,dv=dx$$

which is in VS form and hence can be solved.