SL-07; PHY-122 06 tk

5. *What is the physical significance of the moment inertia?*
Moment of inertia plays the same part in rotating bodies as mass plays when bodies move in straight line.
6. *What is the unit of moment of inertia?*
In C.G.S. system it is gm. cm ^{2}

EXPT. 13. TO DETERMINE THE VALUE OF g, ACCELERATION DUE TO GRAVITY, BY MEANS OF A COMPOUND PENDULUM

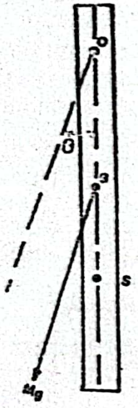

Fig.2.22a

Theory : Compound pendulum is a rigid body of any shape free to turn about a horizontal axis. In Fig. 2.22a, G is the centre of gravity of the pendulum of mass M, which performs oscillations about a horizontal axis through O. When the pendulum is at an angle θ to the vertical, the equation of motion of the pendulum is $I\omega = Mgl\sin\theta$ where ω is the angular acceleration produced, l is the distance OG and I is the moment of inertia of the pendulum about the axis of oscillations. For small amplitude of vibrations, $\sin\theta = \theta$, so that

$$I\omega = Mgl\theta$$

Hence the motion is simple harmonic, with period of vibrations.

$$T=2\pi\sqrt{\frac{I}{Mgl}}$$

If K is the radius of gyration of the pendulum about an axis through G parallel to the axis of oscillation through O, from the Parallel Axes Theorem,

$I = M(K^2+l^2)$, and so

$$T = 2\pi\sqrt{\frac{k^2+l^2}{gl}} = 2\pi\sqrt{\frac{\frac{k^2+l^2}{l}}{g}} \qquad (1)$$

Since the periodic time of a simple pendulum is given by

$T = 2\pi\sqrt{\frac{L}{g}}$, the period of the rigid body (compound pendulum) is the same as that of a simple pendulum of length

$$L = \frac{k^2+l^2}{l} \qquad (2)$$

This length L is known as the length of the simple equivalent pendulum. The expression for L can be written as a quadratic in (l). Thus from (2)

$$l^2 - lL + k^2 = 0 \qquad (3)$$

This gives two values of l (l_1 and l_2) for which the body has equal times of vibration. From the theory of quadratic equations,

$$l_1 + l_2 = L \text{ and } l_1 l_2 = k^2$$

As the sum and products of two roots are positive, the two roots are both positiue. This means that there are two positions of the centre of suspension on the same side of C.G. about which the periods (T) would be same. Similarly there will be two more points of suspension on the other side of the C. G., about which the time periods (T) will again be the same. Thus, there are altogether *four* points, *two* on either side of the C.G., about which the time periods of the pendulum are the same (T). The distance between two such points, assymetrically situated on either side of the C. G., will be the length (L) of the simple equivalent pendulum. If the length OG in Fig. 2.22a is l_1 and we measure the length $GS = \frac{k^2}{l_1}$ along OG produced, then obviously $\frac{k^2}{l_1} = l_2$ Or, $OS = OG + GS = l_1 + l_2 = L$. The period of oscillation about either O or S is the same.

The point S is called the centre of oscillation. The points O and S are interchangeable *i.e.*, when the body oscillates about O or S, the time period is the same. If this period of oscillation is T, then from the expression $T = 2\pi\sqrt{\frac{L}{g}}$ we get

$$g = 4\pi^2 . \frac{L}{T^2}$$

By finding L graphically, and determining the value of the period T, the acceleration due to gravity (g) at the place of the experiment can be determined.

Apparatus : A bar pendulum, a small metal wedge, a beam compass, a spirit level, a telescope with cross-wires in the eye-piece, stop-watch and a wooden prism with metal edge.

Description of the apparatus : The apparatus ordinarily used in the laboratory is a rectangular bar AB of brass about 1 meter long. A series of holes is drilled along the bar at intervals of 2-3 cm (Fig.2.22b). By inserting the metal wedge S in one of the holes and placing the wedge on the support S_1S_2, the bar may be made to oscillate.

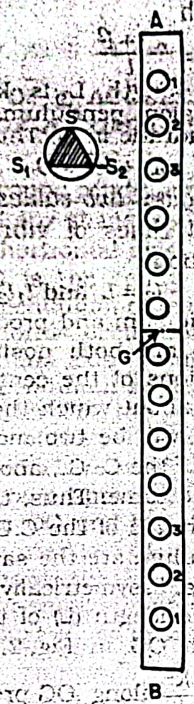

Fig.2.22b.

Procedure : (i) Find out the centre of gravity G of the bar by balancing it on the wooden prism.

(ii) Put a chalk mark on the line AB of the bar. Insert the metal wedge in the first hole in the bar towards A and place the wedge on the support S_1S_2 so that the bar can turn round S.

(iii) Place a telescope at a distance of about a metre from the bar and focus the cross-wires and rotate the collar of the tube till the cross-wires form a distinct cross. Next focus the telescope on the bar and see that the point of inter-section of the cross-wires coincides with the chalk mark along the line AB of the bar.

(iv) Set the bar to oscillate taking care to see that the amplitude of oscillations is not more than 5°. Note the time for 50 oscillations by counting the oscillations when the line AB passes the inter-section of the cross-wires in the same direction.

(v) Measure the length from the end A of the bar to the top of the first hole i.e., upto the point of suspension of the pendulum.

(vi) In the same way, suspend the bar at holes 2,3.............and each time note times for 50 oscillations. Also measure distances from the end A for each hole.

(vii) When the middle point of the bar is passed, it will turn round so that the end B is now on the top. But continue measuring distances from the point of suspension to the end A.

(viii) Now calculate the time-period T from the time recorded for 50 oscillations.

(ix) On a nice and large graph paper, plot a curve with length as abscissa and period T as ordinate with the origin at the middle of the paper along the abscissa. (Fig.2.22c).

(x) Through the point on the graph paper corresponding to the centre of gravity of the bar, draw a vertical line. Draw a second line ABCD along the abscissa. AC or BD is the length of the equivalent simple pendulum i.e., $L = l_1 + \frac{k^2}{l_1}$. AG $= l_1$ and GC $= \frac{k^2}{l_1} = l_2$, C being the centre of oscillation.

Similarly GD $= l_1$ and GB $= \frac{k^2}{l_1} = l_2$, B being the centre of oscillation. From this, $g = 4\pi^2 \frac{L}{T^2}$ can be calculated.

(xi) By drawing another line A'B'C'D' calculate another value of g

Alternate method of measuring the length of the pendulum.

Instead of measuring length from the end A to the point of suspension, length can also be measured from the point of suspension to the centre of gravity G of the bar (see Fig. 2.22b). In that case also there will be two sets of readings-one with the end A at the top and again with the end B at the top. Calculate the period T with 50 oscillations at each suspension. Now draw a graph with the centre of gravity of the bar at the origin which is put at the middle of the paper along the abscissa. Put the length measured towards the end A to the left and that measured towards the end B to the right of the origin (see Fig.2.22c). A line ABCD drawn parallel to the abscissa intersects the two curves at A B C and D.

Here also the length AC or BD is the length of the equivalent simple pendulum.

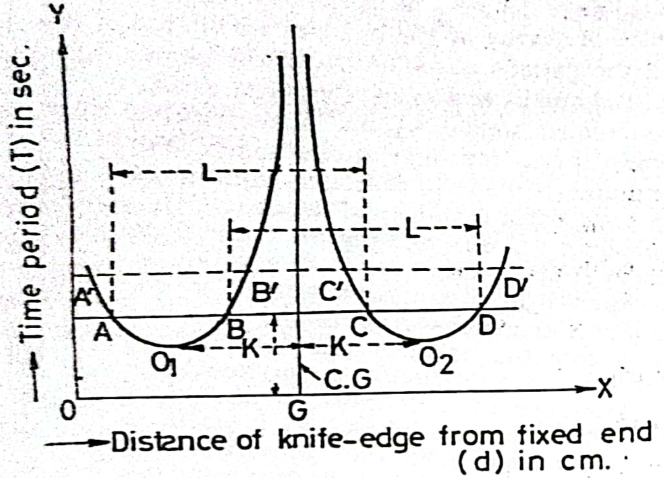

Fig. 2.22c

Results:

(A) Observation for the time period T and the distance of the point of suspension from the end A.

At the top	Hole no.	Distance from A	Time for 50 oscillations	Mean Time	Mean Period T
End A	1	=c m	(i) ....sec (ii) ...sec (iii) ...sec		
	2	= ... cm	(i) ....sec (ii) ...sec (iii) ...sec		
	3	= ... cm	(i) (ii) (iii) etc		
End B	1 2 3		etc	etc	

(B) Alternate method of measuring length.

Use the above table only changing the third column by "Distance from G", the centre of gravity.

(From graph)

Length AC=.....cm. Length BD =...cm.

Mean length $L = \frac{AC+BD}{2}$ =...cm

Corresponding time-period from the graph.

T =...sec. $g = \frac{4\pi^2 L}{T^2}$ =......cm. per sec^2

Discussions: (i) Distances are to be measured from the end A or the point G, preferably from A.

(ii) In measuring time an accurate stop-watch should be used.

(iii) Oscillations should be counted whenever the line of the bar crosses the intersecting point of the cross- wires, in the same direction.

(iv) Graph paper used should have sharp lines and accurate squares and should be sufficiently large to draw smooth and large curves.

(v) Amplitude of oscillations must not be more than 5°

(vi) Error due to the yielding of support, air resistance, and irregular knife-edge should be avoided.

(vii) Determination of the position of G only helps us to understand that $AG=l_1$ and $GC = \frac{K^2}{l_1} = l_2$ and is not necessary for determining the value of 'g'.

(viii) For the lengths corresponding to the points A, B, C and D the period is the same.

(ix) At the lowest points of the curves P_1 and P_2 the centre of suspension and the centre of oscillation coincide.

It is really difficult to locate the points P_1 and P_2 in the graph and so K is calculated from the relation

$$K=\sqrt{GA.GB}=\sqrt{GB.GC}.$$

EXPT .14. TO DETERMINE THE VALUE OF 'g' BY KATER'S REVERSIBLE PENDULUM.

Theory : In a Kater's pendulum if l_1 and l_2 be distances of two points from the centre of gravity of the bar and on opposite direction from it such that the periods of oscillations about these points are exactly equal, then period T is given by

$$T= 2\pi \sqrt{\frac{l_1+l_2}{g}} \quad \text{or } g = 4\pi^2 \frac{l_1+l_2}{T^2} \quad \ldots \quad \ldots \ldots(1)$$

But it is extremely difficult to make the periods exactly equal. It can, however, be shown in the following way that the time-periods T_1 and T_2 about these two points need not be exactly equal.

$$T_1 = 2\pi \sqrt{\frac{l_1^2+K^2}{l_1 g}}\ .\ T_2 = 2\pi \sqrt{\frac{l_2^2+K^2}{l_2 g}}$$

or $T_1^2.l_1 g = (4\pi^2\ l_1^2+K^2)$, $T_2^2.l_2 g=4\pi^2\ (l_2^2+K^2)$.

Subtracting, $(T_1^2 l_1-T_2^2\ l_2)g = 4\pi^2(l_1^2-l_2^2)$

$$\text{or, } \frac{4\pi^2}{g} = \frac{l_1T_1^2-l_2T_2^2}{l_1^2-l_2^2} = \frac{1}{2}\left[\frac{T_1^2+T_2^2}{l_1+l_2}+\frac{T_1^2+T_2^2}{l_1+l_2}\right]$$

$$\text{or, } \frac{8\pi^2}{g} = \frac{T_1^2+T_2^2}{l_1+l_2}+\frac{T_1^2-T_2^2}{l_1-l_2}.$$

From the above relation, g can be calculated.

Apparatus : Kater's pendulum, stop-watch, telescope, etc.

Description of the apparatus : The Kater's pendulum consists of a metal rod about one metre in length having a heavy mass W fixed at one end (Fig.2.23). Two steel knife-edges k_1 and k_2 are fixed to this rod with their edges turned towards each other, from which the pendulum can be suspended. Two other small weights w_1 and w_2 can slide along the rod and can be screwed anywhere on it. With the help of these two weights, centre of gravity of the rod can be altered and the periods of oscillation of the pendulum about k_1 and k_2 can be made equal. The smaller weight w_2 has a micrometer arrangement for fine adjustment. The pendulum is made to oscillate about one of the knife-edges from a rigid support.

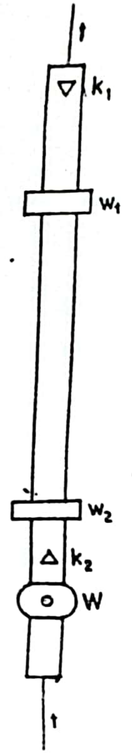

Fig. 2.23

Procedure : (i) Suspend the pendulum from a rigid support about the knife-edge k_1, so that the weight W is in downward position.

(ii) Focus the cross-wires of the telescope and rotate the collar of the tube till the cross-wires form a distinct cross. Next place the telescope at a distance of about one metre from the pendulum and focus it on the lower tail *t* of the pendulum (or alternately on a chalk line marked along the length of the pendulum) so that the vertical line of the cross-wire or the point of intersection of the cross-wires (when none of them is vertical) coincides with the tail *t* or the chalk mark.

(iii) Displace the pendulum slightly and release it. The pendulum will begin to oscillate. Note the time for 10 complete oscillations (the amplitude of oscillations should be small) with an accurate stop-watch. Repeat the same for the knife-edge k_2. The two times will generally differ.

(iv) Slide the heavier weight w_1 in one direction and note the time for 10 oscillations about k_1 and k_2. If the difference the other fingers at the top. The faces of the prism should be cleaned with a clean piece of fine linen, if necessary.

(vii) If an asbestos ring is used, it must be held in the non-lumimous part of the bunsen flame; should be supplied with fresh solution of common salt from time to time.

(viii) The width of the slit image should be as narrow as possible.

(ix) In taking reading care should be taken to ascertain whether the zero of the main circular scale has been crossed in going from one position to the other

EXPT 45. TO DETERMINE THE REFRACTIVE INDEX OF THE MATERIAL OF A PRISM.

Theory : If A be the angle of the prism and δ_m that of minimum deviation which light of a given colour undergoes by refraction through the prism in a principal section, then the refractive index of the material of the prism for light of the given colour *i.e.*, wavelength is given by the relation

$$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

The expression for μ can be deduced in the following manner.

Let a ray PQ (Fig.5.33) be incident on the first face of a prism and after passsing through the principal plane of the prism, finally emerge out through the other face in the direction RS. Let θ and φ be the respective angles of incidence and refraction at the first face of the prism and φ' and θ' the corresponding quantities for the second face. Now the deviation of the ray, given by the angle SOT, is equal to $(\theta-\varphi) + (\theta'-\varphi')$. But in the position of minimum deviation, the ray passes symmetrically through the prism so that $\theta=\theta'$ and $\varphi=\varphi'$.

Therefore the angle of minimum deviation,

$\delta_m = 2(\theta - \varphi)$ ... ... (1)

From the figure, it can be shown that the angle LMR (between the two normals at the two faces) is equal to the angle

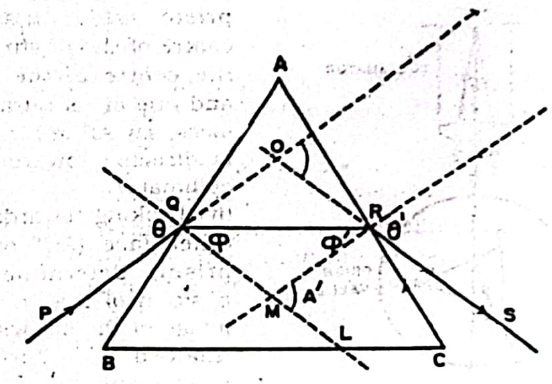

Fig. 5.33

(A) of the prism. But $\angle LMR$ is also equal to $(\varphi+\varphi')$. Therefore,

$$\angle LMR = \angle A = \varphi+\varphi' = 2\varphi \text{ ... ... (2)}$$

$$(i.e., \ \varphi = \frac{A}{2})$$

From (1) and (2) $\theta = \frac{A+\delta_m}{2}$ ... ... (3)

$$\text{Hence } \mu = \frac{\sin\theta}{\sin\varphi} = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

Apparatus : Spectrometer, sodium lamp, prism, spirit level, reading lens, etc.

Description of the apparatus : See spectrometer (Art.5.4).

Procedure : (i) Make all the necessary adjustments of the spectrometer including focussing for parallel rays by Schuster's method in the usual manner as described in Art. 5.4. Determine the vernier constants of both the verniers. Now place the prism on the prism table in such a way that the vertex of the prism coincides with the centre of the prism table.

(ii) Determine the angle of the prism in the manner described in expt 44.

(iii) To determine the angle of minimum deviation, so