

4.23] Isobaric and Isochoric process? Work done by an ideal gas in these process.

Isobaric: In thermodynamics, an isobaric process is a type of thermodynamic process in which the pressure of the system stays constant, $\Delta P = 0$.

Work done by an ideal gas in these process, $W = \int P \, dV = P \Delta V$

$$P \Delta V = n R \Delta T$$

Isochoric (same as isobaric...) in which the volume of the system ~~set~~ stays constant, $\Delta V = 0$.

Work done, $W = P \Delta V = 0$.

24] Account of 1st Law of thermodynamics. This law particular form of the general of conservation of energy.

1st law of thermodynamics states that the total energy entering a system in the form of energy is equal to the sum of the increase in the system's internal energy and the energy leaving the system in the form of work done by the system on its surrounding. ($ds = du + dw$)

The 1st law of thermodynamics is also known as the conservation of energy or a particular form of conservation of energy. ^{Because} The conservation of energy states that Energy can neither be created nor destroyed but it can be changed from one form to another.

4.25 prove that $c_p - c_v = R$ from 1st law of thermodynamics.

We know, $q = n c_v \Delta T$

At constant pressure, $q_p = n c_p \Delta T = dq$

At constant volume, $q_v = n c_v \Delta T = du$

We know, $dw = p \Delta v = n R \Delta T$

We know, $dq = du + dw \Rightarrow c_p = c_v + R$

$$\therefore c_p - c_v = R$$

4.26 Work done by an ideal gas in isothermal expansion.

In isothermal process the temperature of a system remain constant, $\Delta T = 0$.

The internal energy depends only & only change in temperature. So that in

this case, $du = 0$; $\therefore dq = dw$

$$\begin{aligned} W &= \int p dv = \int_{v_1}^{v_2} \frac{nRT}{v} dv = nRT \int_{v_1}^{v_2} \frac{1}{v} dv \\ &= nRT \ln(v_2 - v_1) = nRT \ln \frac{v_2}{v_1} \end{aligned}$$

4.27 Derivation of work done by an ideal gas in adiabatic process.

In thermodynamic, an adiabatic process is a thermodynamic process in which the transfer of heat of a system and its surrounding is equal to zero ($dq = 0$). $\therefore dw = -du$

Let, an ideal gas of n moles & temperature change from T_1 to T_2

$$\therefore \text{work done } W = \int p dv$$

in isothermal process we know,

$$W = \int_{V_1}^{V_2} \frac{K}{V^{\gamma}} dV$$

$$= K \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$$= \frac{nR(T_2 - T_1)}{1-\gamma}$$

$$PV^{\gamma} = K \quad \therefore P = \frac{K}{V^{\gamma}}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} = K$$

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$$= \frac{K}{-\gamma+1} (V_2^{1-\gamma} - V_1^{1-\gamma}) = \frac{1}{-\gamma+1} (P_2 V_2 - P_1 V_1)$$

4.28] Show that the slope of an adiabatic is γ times the slope of isothermal.

we know that in adiabatic process $PV^{\gamma} = K$

$$\therefore \frac{d}{dV} (PV^{\gamma}) = \frac{d}{dV} (K) \quad \left[\frac{d}{dx} (uv) = u \frac{d}{dx} v + v \frac{d}{dx} u \right]$$

$$\therefore P \gamma V^{\gamma-1} + V^{\gamma} \frac{dP}{dV} = 0$$

$$\therefore \left(\frac{dP}{dV} \right) = -\gamma \left(\frac{P}{V} \right)$$

Slope of Adiabatic process

Slope of Isothermal process

4.29] Prove that $PV^{\gamma} = \text{constant}$ in adiabatic process.

We know, in adiabatic process $dQ = 0$.

for 1 mole of ideal gas, $PV = RT$

$$P dV + V dP = R dT \quad (\text{differentiating on both side})$$

$$P dV + V dP = (C_p - C_v) dT \quad (R = C_p - C_v)$$

$$\therefore P dV + C_v dT = C_p dT - V dP \quad \text{--- (A)}$$

Again,

$$\boxed{dw = -pdv} \rightarrow (dw = -du \text{ in Adiabatic})$$
$$\text{or, } c_v dT = -pdv$$

$$\therefore c_v dT + pdv = 0 \quad \text{--- (i)}$$

$$c_p dT - v dp = 0 \quad \text{--- (ii)} \quad (\text{for A putting the value of i})$$

$$pdv = -c_v dT \quad \text{--- (iii)}$$

$$v dp = c_p dT \quad \text{--- (iv)}$$

$$\text{or, (iii) } \div \text{ (iv)}$$

$$\frac{p dv}{v dp} = \frac{-c_v}{c_p}$$

$$\text{or, } \frac{dv}{v} \left(\frac{-c_p}{c_v} \right) = \frac{dp}{p}$$

$$\text{or, } \int \frac{dv}{v} - \gamma = \int \frac{dp}{p} \quad (\text{integrating on both side})$$

$$\text{or, } -\gamma \ln v + c = \ln p$$

$$\text{or, } \ln p + \gamma \ln v = \ln e$$

$$\text{or, } \ln p + \ln v^\gamma = \ln e$$

$$\text{or, } \ln(pv^\gamma) = \ln e$$

$$\boxed{\text{or, } pv^\gamma = e = \text{constant}}$$

4.3a

(same as 4.2g)

4.31 Why does temperature of a gas drops when it is subjected to adiabatic expansion.

We know, Internal Energy = $U = n c_v \Delta T = n c_v (T_2 - T_1)$

And also, in Adiabatic process $dq=0$; $dW = p dV = p (V_2 - V_1) = - n c_v (T_2 - T_1)$

$$\Rightarrow - p dV = n c_v (T_2 - T_1)$$

"+" because of adiabatic expansion ($V_2 > V_1$)

$$\therefore (T_2 - T_1) = \Delta T = - \frac{p dV}{n c_v} = - K$$

$$\therefore T_2 < T_1 \text{ (Temperature drops)}$$

4.32 Show that the work done during an adiabatic process only upon the initial & final temperature.

In (4.27) we derive work done in adiabatic process, $W = \frac{nR}{1-\gamma} (T_2 - T_1)$

where, $\left(\frac{nR}{1-\gamma} = K\right)$ is constant $\therefore W = K \Delta T$
 $W \propto \Delta T$

Hence, we can say that work done during an adiabatic process only depends upon the initial & final temperature.

4.33 Explain why gas have two specific heat. Why $c_p > c_v$. Prove, $c_p - c_v = R$.

A solid or a liquid when heated does not undergo any change in the volume or pressure. But in case of gas, both the pressure and volume changes on heating. That why specific heat of a gas is defined for at constant volume and constant pressure.



C_v is the molar specific heat capacity. When gas is heated at a constant volume, it increases the internal energy of the system. On the other side, When gas is heated at constant pressure both volume and internal energy of the system increases. That why $C_p > C_v$.

$$C_p - C_v = R \quad (4.25)$$

$$4.34 \quad (4.24)$$

4.35] Distinguish between isothermal and adiabatic changes. Show that

$$pV^\gamma = \text{constant} \quad (4.29)$$

isothermal process

1. In thermodynamics process, an isothermal process is a type of thermodynamic process in which the temperature of a system remains constant $\Delta T = 0$.

2. Work done in this process is;

$$W = nRT \ln \frac{V_2}{V_1}$$

adiabatic process

1. In thermodynamics, an adiabatic process is a type of thermodynamic process in which the transfers of heat is equal to zero, $\Delta Q = 0$.

2. Work done in this process is;

$$W = \frac{nR}{1-\gamma} (T_2 - T_1)$$