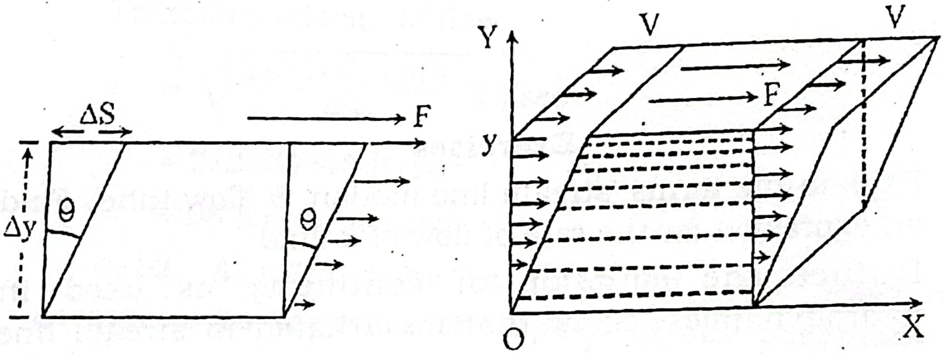
read ⇒ theory

CHAPTER - 7
VISCOSITY

7.1 Introduction (self)

We defined the shear modulus as the ratio of the Shearing stress to shearing strain. Mathematically we can write

$$n = \frac{\frac{F_1}{A}}{\frac{\Delta s}{\Delta y}}$$



(a) (b)

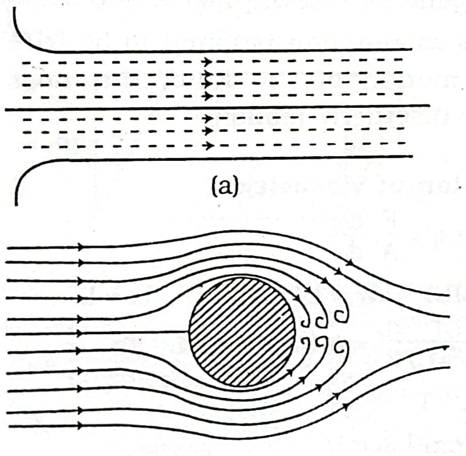
Fig. 7.1

The ratio $\frac{\Delta s}{\Delta y}$, being small, is identified with the angle θ. (Fig : 7.1). If the figure drawn in 7.1 (a) were a side of a cube of liquid as in Fig. 7.1 (b) the tangential force F would not produce a finite displacement ΔS; it would, in fact, cause the horizontal layers of liquid to flow with varying velocities, If the bottom of the cube is in contact with a stationary wall, observation shows that the lowest layer would be at rest and the higher ones would have uniformly increasing velocities as shown at the left. By virtue of its viscous behavior an upper layer drags the lower ones along, but the lower layer flows with a somewhat smaller speed.

So whenever there is a relative motion among the different layers of a fluid tangential forces are called into play within the fluid tending to destroy the relative motion. This property of a fluid which opposes a relative motion among its different parts is called **viscosity**. The opposing force which comes into play inside a fluid when its different layers are moving relative to one another is the friction exerted by the molecules of the moving layers and hence viscosity is called the **Internal Friction**. In a perfect fluid this internal friction is supposed to be absent and hence a perfect fluid is defined as a fluid which does not possess viscosity.

7.2 Orderly and Turbulent Motion : Critical Velocity

A fluid possesses two characteristic types of motion. In one the path of a moving particle coincides with the line of motion of the fluid. This motion is called the **orderly** or streamline flow. In stream line flow which is illustrated in fig. 7.2a, every particle of liquid passing a particular point follows the same path (called a streamline) as the particles that passed the point previously. In the other type the path of a particle does not coincide with the line of motion of the fluid. This motion which is characterized by the presence whirls and eddies like the closed of a cigarette smoke is celled the **turbulent or disorderly motion** [Fig : 7.2 (b)].


(a)

(b)

Fig.7.2

7.3 Reynold's number

Toward the end of the last century Sir Osborne Reynolds investigated the change from laminar to turbulent flow in liquid cylindrical tubes as the velocity of flow was increased. He found that for any liquid there was a critical velocity at which there was a sudden change from the laminar to the turbulent type of motion. His experiment consisted in inserting a narrow thread of colored liquid into the main body of liquid flowing through the tube. With laminar flow the narrow thread followed a straight line, but once the critical velocity was passed, the thread was broken up and the colored liquid ultimately diffused into the main body of the liquid. To determine the point at which turbulence sets in, we must ask ourselves on what quantities we might expect this critical velocity V_c to depend.

There is a critical velocity of the flow of a liquid, below which the motion of a fluid is orderly but above which the motion is turbulent. It is inversely proportional to the radius of the tube and the density of the liquid and directly proportional to the viscosity of the liquid. i. e.

$$V_c = \frac{k\eta}{\rho r}$$

where $\frac{\eta}{\rho}$ is frequently denoted by γ and termed kinematic viscosity, k is termed Reynold's number according to the name of Osborne Reynolds and generally has a value about 1000 for liquid like water.

Deduction of the relation : $V_c = \frac{k\eta}{\rho r}$

The above relation may easily be established by the method of dimension Since V_c is found to depend on the factors η, ρ and r, we may express V_c as flows :

Let $V_c = k\eta^a\rho^b r^c$. [k = a.dimensionless constant]

Inserting dimensions of different quantities in the above equation.

$$[LT^{-1}] = [ML^{-1}T^{-1}]^a\,[ML^{-3}]^b\,[L]^c$$

or, $$[LT^{-1}] = [M^{a+b}.L^{-a-3b+c}.T^{-a}]$$

Thus $a + b = 0$; $-a - 3b + c = 1$ and $-a = -1$

Solving we get $a = 1$, $b = -1$ and $c = -1$

$$\therefore \quad V_c = k\eta^1\rho^{-1}r^{-1} = \frac{k\eta}{\rho r}$$

Thus (i) $V_c \propto \eta$ (ii) $V_c \propto \frac{1}{\rho}$ and (ii) (i) $V_c \propto \frac{1}{r}$

7.4 Coefficient of Viscosity

Consider a stream of fluid flowing over a fixed horizontal plane AB from left to right (fig. 7.3). The layer of the fluid in contact with AB is at rest while the velocities of different layers above AB are increasing uniformly according to their heights above AB. If we consider an imaginary horizontal plane CD within the fluid, the fluid molecules below CD exerts a force on it from right to left as shown by the arrow-head which tends to destroy the relative motion of the fluid. If V be the velocity of the fluid-layer CD at a height x above AB.

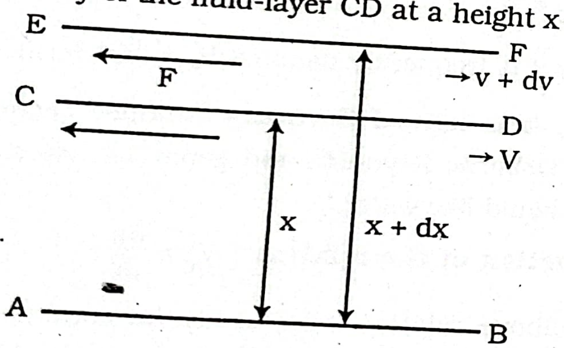


Fig. 7.3

Let us consider another layer EF moving with velocity V + dv at a distance x + dx from the fixed plane. The velocity gradient i.e. the change of velocity per unit distance is dv/dx.

Newton assumed that the tangential stress opposing the relative motion is directly proportional to the velocity e.g.

$$\frac{F}{A} \propto \frac{dv}{dx} \qquad [F = \text{tangential force, } A = \text{area of CD}]$$

or, $$\frac{F}{A} = \frac{\eta dv}{dx}$$

or, $$F = \eta\, A\frac{dv}{dx} \qquad \ldots \qquad \ldots \qquad \text{(i)}$$

$$\therefore \quad \eta = \frac{F}{A}\frac{dx}{dv}$$

where η is a constant depending on the nature of the fluid. The constant η is called the **coefficient of viscosity** of the fluid. Equation (i) is known as **Newton's formula** of viscous flow for stream-line-flow.

If the velocity gradient $\frac{dv}{dx}$ is unity, $\eta = \frac{F}{A}$. Thus the coefficient of viscosity may be defined as the tangential force per unit area per unit velocity gradient.

If $A = 1$ and $\frac{dv}{dx} = 1$ in equation (i), There $\eta = F$

Thus coefficient of viscosity of a fluid may, be defined as the tangential force required per unit area to maintain unit velocity gradient.

Newton's assumption is found to be true for orderly or streamline motion of the fluid, but does not hold for turbulent or disorderly motion.

7.5 Dimension of viscosity

Since $\eta = \frac{F}{A}.\frac{dv}{dx}$

$F = MLT^{-2}$, $dv = LT^{-1}$, $A = L^2$, $dx = L$

$$\therefore \quad \eta = \frac{MLT^{-2}\times L}{L^2\times LT^{-1}} = MLT^{-2}.\,L.L^{-2}\,L^{-1}.T$$

$$= ML^{-1}\,T^{-1}$$

$$= \text{kg cm}^{-1}\ \text{sec}^{-1}$$

Units : If $A = 1$ sq. cm, $\frac{dv}{dx} = \frac{1\text{cm/s}}{1\text{cm}}$

From a similarity between the definitions of rigidity for solids and viscosity for fluids, Maxwell pointed out that fluids possess a certain amount of rigidity. This rigidity is short-lived. If appears for a short time and then disappears and appears again. For this reason the viscosity of fluids is also called **fugitive elasticity**.

7.7 Flow of a liquid through a capillary tube : Poiseuille's equation

Rate of flow of a liquid through capillary tube depends upon

(a) length of the capillary tube (l)

(b) radius of the capillary tube (r)

(c) pressure difference between two ends of the capillary tube (P)

(d) coefficient of viscosity of the liquid (η)

When a liquid flows through a capillary tube of length l and radius r in a stream-line motion under a difference of pressure P between the ends, a relation can be established between V, the volume of the liquid that flows out per second; η the coefficient of viscosity, P, l and r.

Poiseuille to whom we owe for this deduction assumed that the flow of the liquid is stream line, (ii) the liquid in contact with the walls of the tube is at rest, (iii) the tube is kept horizontal and (iv) the pressure at any given cross section is constant and there is no radial flow of the liquid.

If a cylinder (Fig. 7.5) is imagined to be drawn within the capillary tube co-axial with it, the viscous drag on the surface of this cylinder will retard the motion of the liquid cylinder whereas the force on the cylinder due to the pressure difference between the ends will accelerate the liquid cylinder. When these two forces are equal, the flow of the liquid through the tube will became laminar or stream-line and there will be no acceleration in the motion of the liquid.

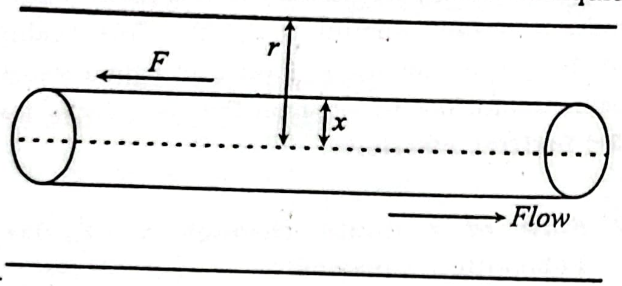


Fig. 7.5

If x be the radius of the cylinder and v, the velocity of the liquid on its surface, the velocity gradient is $\frac{dv}{dx}$ and the viscous drag on the liquid cylinder = $2\pi x\, l\eta \frac{dv}{dx}$. The force on the liquid cylinder due to the external pressure difference = $\pi x^2 P$

Thus for steady flow of the liquid, we have

$$-2\pi x\, l\eta \frac{dv}{dx} = \pi x^2 P$$

— sign because v decreases with the increase of x and $\frac{dv}{dx}$ is negative.

$$\text{or,} \quad -\frac{2\, l\eta}{P} dv = x dx$$

Integrating this we have

$$-\frac{2l\eta}{P} v = \frac{x^2}{2} + c \qquad \dots \qquad \dots \qquad \text{(i)}$$

where c is an integration constant.
To evalute c, we see that at x = r, v =0

$$\therefore \quad 0 = \frac{r^2}{2} + c$$

$$\text{or,} \quad c = -\frac{r^2}{2}$$

Replacing c in equation ((i)

$$\frac{2\, l\eta}{P} v = \frac{r^2 - x^2}{2}$$

$$\text{or,} \quad v = \frac{P}{4l\eta}(r^2 - x^2) \qquad \dots \qquad \dots \qquad \text{(ii)}$$

This equation gives us the magnitude of the velocity of the liquid layer at a distance x from the axis of the tube.

To determine the volume of the liquid flowing out of the tube per second, let us first of all determine the volume of the liquid flowing out through an annular space lying between two co-axial cylinders of radii x and x + dx, where dx is supposed to be so small that the velocity of flow of a liquid layer, within the annular space is the same and is equal to v. In calculating this, we easily see that in one second the liquid contained in the annular space bounded by radii x and x + dx and length v flows out. Therefore if dV is the volume of the liquid flowing per sec out of the annulus between the coaxial cylinders of radii x and x + dx

$$dV = 2\pi x\, dx\, v$$

Substituting for v from equation (ii).

$$dV = \frac{\pi P}{2l\eta}(r^2 - x^2)\, x dx$$

Integrating this expression between the limit. x = o to x = r, the volume V of the liquid flowing out of the tube per second is obtained.

$$\therefore \quad V = \int_0^r \frac{\pi P}{2l\eta}(r^2 - x^2)\, x dx = \frac{\pi P}{2l\eta}\left\{r^2 \int_0^r x dx - \int_0^r x^3 dx\right\}$$

$$= \frac{\pi P}{2l\eta}\left\{\frac{r^4}{2} - \frac{r^4}{4}\right\} = \frac{\pi P r^4}{8l\eta} \qquad \dots \qquad \dots \qquad \text{(iii)}$$

The assumptions made in deducing Poiseuille's equation for the rate of flow through a horizontal capillary tube (Fig. 7.5) are more or less valid *if the velocity of the flow is small* and *the tube is narrow*, for in that case only a streamline motion of the liquid occurs. For tubes of wider bores, the value of critical velocity is small and is soon reached ($V_c \propto I/r$) so that turbulence sets in and the relation breaks down. This may easily be seen by plotting a graph between the pressure difference P across the two ends of the tube and the *rate of flow* V of the liquid through the tube, as shown in Fig. 7.6.

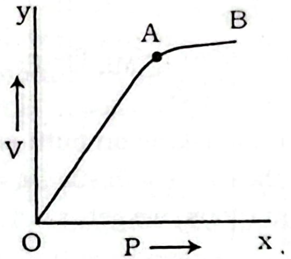


Fig. 7.6

The straight part OA of the graph corresponds to velocities below the cirtical value and clearly indicates that within the range the rate of flow V is proportional to the pressure difference P and thus depends chiefly on the viscosity (η) of the liquid. The curved part AB, on the other hand, which corresponds to velocities above the cirtical value, signifies that the rate of flow is no longer proportional to the pressure difference which is now almost wholly used up in combating the trubulence set up in the liquid and in imparting kinetic energy to it. The rate of flow thus no longer depends upon η, and is, in fact found to depend mainly on the density (ρ) of the liquid and to be approximetely proportional to square root of presoure (P).

Then, again, the assumption regarding the layer of the liquid in contact with the wall of the tube remaining stationary is also found to be valid for really small velocities of flow.

It would thus appear that if the tube is really narrow and the velocity of flow of the liquid as to take the form of almost a trickle of drops, Poiseuille's equation will be fully applicable.

There are, however, two factors which have not been taken account of in deducing this equation : (i) part of the thrust due to the pressure difference across the capillary tube imparts kinetic energy to the liquid and only the rest of it is

available for overcoming the viscous resistance of the liquid: the effective value of the pressure difference is thus less than P. (ii) the motion of the liquid where it enters the capillary tube is some what accelerated with the result that the velocity of flow is not uniform over the first short length of the tube. Proper correction have, therefore, to be incorporated.

7.8 Correction to Poiseuille's Equation

In the Poiseuille's equation two important corrections are to be applied.

(i) In deriving the equation, it has been assumed that the liquid does not possess any kinetic energy at the outlet end of the tube. However the liquid flowing out of the tube has a certain velocity and hence possesses kinetic energy. The effective pressure P_1 is given by

$$P_1 = P - P' \quad \ldots \quad \ldots \quad \ldots \quad \text{(i)}$$

Kinetic energy given to the liquid per second

$$E' = \int_0^r \frac{1}{2} mu^2 = \int_0^r \frac{1}{2} [(2\pi x dx)u\rho]u^2$$

$$E' = \pi\rho \int_0^r u^3 x dx.$$

$$\text{But } u = \frac{P(r^2-x^2)}{4\eta l}$$

$$\therefore \quad E' = \pi\rho \int_0^r \left(\frac{F}{4\eta l}\right)^3 (r^2-x^2)^3 x dx$$

$$E' = \pi\rho \left(\frac{P}{4\eta l}\right)^3 \int_0^r (r^6x-3r^4x^3+3r^2x^5-x^7)dx$$

$$E' = \pi\rho \left[\frac{P}{4\eta l}\right]^3 \left(\frac{r^8}{8}\right) = \left(\frac{\pi P r^4}{8\eta l}\right)^3 \frac{\rho}{\pi^2 r^4}$$

$$E' = \frac{V^3\rho}{\pi^2 r^4}$$

$\therefore \quad P'V = E'$

$$P' = \frac{E'}{V} = \frac{V^2\rho}{\pi^2 r^4}$$

$$\therefore \quad P_1 = P - \left(\frac{V^2\rho}{\pi^2 r^4}\right) \qquad \dots \qquad \dots \qquad \text{(ii)}$$

If V is negligibly small, $P_1 = P$

(ii) At the inlet end of the tube, the flow of the liquid is not stream line for some distance. Consequently the liquid is accelerated. The effective length of the tube is given by $(l + 1.64r)$

$$\therefore \quad \eta = \frac{\pi P r^4}{8V(l+1.64r)} - \left(\frac{V^2 P}{\pi^2 r^4}\right) \frac{\pi r^4}{8V(1+1.6r)}$$

$$\eta = \frac{\pi P r^4}{8V(l+1.64r)} - \frac{V^2\rho}{8\pi(l+1.64r)} \qquad \dots \qquad \dots \qquad \text{(iii)}$$

7.11 Stoke's law

Since the method he used is beyond the scope of this book, we shall give only a partial solution by dimensional analysis. In applying dimensional analysis to this problem, we must first attempt to ascertain the physical quantities that determine the force exerted on a sphere moving with constant velocity in a viscous liquid. This must be done by intelligent guesswork or intuition.

A resisting force that varies directly with the speed is found frequently in nature. Any small spherical body of radius r, like a raindrop, an oil droplet or a steal fluid (liquid or gas) is subjected to a force F, where

$$F = 6\pi\eta rv$$

and η is the viscosity. This relation is known as **Stoke's law** because Sir George Stokes found it in 1845.

The experiments show that the resisting force F to the motion of a sphere which falls through a viscous fluid depends on **the radius of the sphere** r, **the coefficient of**

viscosity of the fluid η and **the terminal velocity of the sphere v.**

Let $F = Kv^a\eta^b r^c$ [K = dimensionless constant]

$$[MLT^{-2}] = [LT^{-1}]^a [ML^{-1}T^{-1}]^b [L]^c$$

Thus, $a - b + c = 1; - a - b = 2; b = 1$

Solving we get, $b = 1; a = 1; c = 1$

$\therefore$ $F = K\pi\eta vr$

Stoke determined the value of K to be equal to 6π; therefore

$$F = 6\pi\eta vr$$

Letting $b = 6\pi\eta r$

We may write Stoke's law

$$F = + bv$$

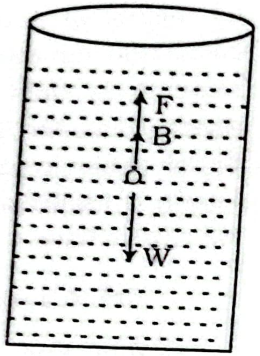


Fig: 7.9

A small sphere falling through a viscous fluid is subjected to three vertical forces as shown: the weight W, the bouyant force B and the resisting force F.

Let u's suppose that the sphere starts from rest and that the positive y-direction is downward. Then from Newton's second law

$$W - B - bv = ma$$

or, $$W - B - bv = m\frac{dv}{dt}$$

At first when v = o, the resisting force is zero, and the initial acceleration a_0 is positive.

$$a_o = \frac{W-B}{m} \quad ..... \quad ... \quad (1)$$

So long as $W>(B + bv)$, $\frac{dv}{dt}$ will be +ve and will increase. Finally, however, $\frac{dv}{dt}$ will be zero and the drop will fall with constant velocity. This so-called terminal velocity is attained when $bv = W- B$. Thus

$$W- B - bv_m = 0$$

or, $$v_m = \frac{W-B}{b} \quad ... \quad ... \quad (2)$$

Thus a sphere falling in a viscous fluid reaches a terminal velocity v_T at which the viscous retarding force plus the buoyant force equals the weight of the sphere. Let ρ be the density of the sphere and ρ' the density of the liquid. The weight of the sphere is then $\frac{4}{3}\pi r^3\rho g$, the bouyant force is $\frac{4}{3}\pi r^3\rho' g$ and when the terminal velocity is reached

$$\frac{4}{3}\pi r^3 r\rho' g + 6\pi\eta r v_T = \frac{4}{3}\pi r^3\rho g \text{ or, } 6\pi\eta r v_T = \frac{4}{3}\pi r^3(\rho - \rho')g$$

or, $$V_1 = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - \rho') \quad ... \quad ... \quad (3)$$

or, $$V_T = \frac{2r^2 g\left[1 - \frac{\rho'}{\rho}\right]\rho}{9\eta} \quad ... \quad ... \quad (4)$$

Thus the terminal velocity of a body, (of course, of a small size) falling through a viscous medium, is (i) directly proportional to the square of its radius (ρ^2), (ii) directly proportional to the difference in the densities of the body and the medium, $(\rho - \rho')$ and (ii) inversely proportional to the coefficient of viscosity of the medium (η).

By measuring the terminal velocity of a sphere of known radius and density, the viscosity of the fluid in which it is falling can be found from the equation above. This equation was also used by Millikan to calculate the radius of the tiny sub-microscopic electrically charged oil drops by means of which he determained the charge on an individual electron. In this case, the terminal velocity of the drops was measured as they fell in air of known viscosity.

From equation (4) one can experimentally determine the viscosity of a liquid since all the remaining quantities in Eq. (4) can be easily measured. However, one must be careful to see that the motion is stream line or laminar. This type of motion depends very much on the viscosity of the liquid and the shape of the body. A marble falling through glycerine does so with stream-line flow, and the motion is in a straight line with constant speed. If however, the same marble is dropped through water, then the motion may be irregular because turbulence has been created in the liquid.

very lower pressures, where the mean free path becomes so long that a molecule can travel across a container without making numerous collisions on the way and at very high pressures, where the molecules are so close together that inter-molecular forces become significant.

WORKED EXAMPLES

7.1 : Water jiows through a horizontal capillary tube of 1 mm internal diameter and length 30 cm under pressure of a column of water 70 cm in height. Find the rate of flow of water through the capillary tube. Viscosity of water = 10^{-3} N-s/m^2.

Rate of flow,

$$V = \frac{\pi P r^4}{8\eta l}$$

Here $P = 0.7$ m of water. $= 0.7\times10^3\times0.8$ N/m^2.

$r = 0.5$ mm $= 5\times10^{-4}$ m

$l = 0.3$ m

$\eta = 10^{-3}$ N-s/m^2

$$\therefore \quad V = \frac{\pi\times0.7\times10^3\times9.8\times[5\times10^{-4}]^4}{8\times10^{-3}\times0.3}$$

$$V = 1.43\times10^{-6} \text{ m}^3/\text{s}$$

7.2 In an experiment with Poiseulle's apparatus the following data were obtained.

Volume of water issuing per minute-7.08. c.c.

Head of water = 34.1 cm.

Length of tube = 56.45 cm.

Radius of tube = 0.0514 cm.

Find the coefficient of viscosity.

$$\eta = \frac{\pi P r^4}{8Vl} = \frac{\pi\times34.1\times981\times(0.0514)^4\times60}{8\times7.08\times56.45}$$

$= 0.0138$ C.G.S. Unit (or poise)

7.3. A plate of metal 2m² area rests on a layer of castor oil 2×10⁻³ m thick, whose coefficient of viscosity is 2N-s/m². Calculate the horizontal force requred to move the plate with a uniform speed of 2×10⁻² m/s.

$$F = \eta A\frac{dv}{dx}$$

Here $\eta = 2$ N-s/m²
$A = 10^{-2}$ m²
$dv = 2\times10^{-2}$ m/s
$dx = 2\times10^{-3}$ m

$$\therefore\ F = \frac{2\times2\times2\times10^{-2}}{2\times10^{-3}} - 40 \text{ N}$$

7.4. A flat plate of area 20 sq. cm. is separated from a large plate by a layer of glycerine 2 mm. thick. If the coefficient of viscosity of glycerine is 20 poise, what force is required to keep the plate moving with a velocity of 1.5 cm/sec.?

$$F = \eta A\frac{dv}{dx} = \frac{20\times20\times1.5}{0.2} = 3000 \text{ dynes}$$

7.5. Caluctate the mass of water flowing in 20 seconds through a horizontal capillatry tube of circular cross section of radius 2×10⁻³ m. The tube is fitted at the bottom of a constant level tank at a depth of 1.5 m. Length of the tube is 0.35 m. Given, $\eta = 10^{-3}$ N-s/m²

$$V = \frac{\pi P r^4}{8\eta l}$$

Mass of water flowing in t seconds

$$M = V\rho t = \frac{\pi P r^4 \rho t}{8\eta l}$$

Here, $P = 1.5\times10^3\times9.8$ N/m²
$r = 2\times10^{-3}$ m
$\rho = 10^3$ kg/m³
$t = 20$ s
$\eta = 10^{-3}$ N-s/m²
$l = 0.35$ m

$$M = \frac{3.142\times1.5\times10^3\times9.8\times(2\times10^{-3})^4\times10^3\times20}{8\times10^{-23}\times0.35}$$

$M = 5.2752$ kg

7.6. A spherical glass ball of mass 1.35×10⁻⁴ kg and diameter 4.6×10⁻³ m takes 7 s to fall steadily through a height of 0.42 m inside a large volume of oil of specific gravity 0.94 Calculate the coeffifient of viscosity of the oil.

According to Stoke's law

$$F = 6\pi\eta r v_T = mg - \frac{4}{3}\pi r^3\rho g$$

$$\eta = \frac{Mg - \frac{4}{3}\pi r^3\rho g}{6\pi\eta r v_T}$$

Here $V = \frac{0.42}{7} = 0.06$ m/s

$m = 1.35\times10^{-4}$ Kg,
$r = 2.3\times10^{-3}$ m
$\rho = 0.94\times10^3$ Kg/m³
$g = 9.8$ m/s²

$$\therefore\ \eta = \frac{1.35\times10^{-4}\times9.8\times\frac{4}{3}\pi(2.3\times10^{-3})^3\times0.94\times10^3\times9.8}{6\pi\times0.94\times10^{-3}(0.06)}$$

$$\text{or,}\quad \eta = \frac{1.323\times10^{-3} - 0.2\times10^{-3}}{2.6\times10^{-3}}$$

$$= \frac{1.123}{2.6}$$

$$= 0.473 \text{ N/m}^2$$

7.7 : A lead shot of radius 0.05 cm falls through glycerine with a terminal velocity of 0.648 cm/sec. If the spcific gravities of lead and glycerine be 11.36 and 1.26 respectively, calculate the coefficient of viscosity of glycerine.

$$\eta = \frac{2}{9}\frac{r^2(\rho - \rho')g}{v} = \frac{2}{9}\frac{(0.05)^2(11.36 - 1.26)\times980}{0.648}$$

$$= 8.486 \text{ poise.}$$

7.8 : Water is conveyed through a horizontal tube 8 cm. in diameter and 4 kilometers in length at the rate of 20 liters per sec. Assuming only viscous resistence, calculate pressure required to maintain the flow. (η for water : 0.91 C.G.S. unit).

We have, $\eta = \pi P r^4 / 8 Vl$ Where P is the pressure,

r = radius of the tube, l = length of the tube,

V = volume flowing per second

$\therefore\ P = 8Vl\eta/\pi r^4 = (8\times2\times10^4\times4\times10^5\times0.01/3.14\times(4)^4$

$= 8\times10^5$ dynes/cm²

7.9 : A large bottle is fitted with a siphon made of capillary glass tubing. Compare the coefficients of viscosity of water and petrol if the time taken to empty the bottle in the two cases is in the ratio 3 : 7. Specific gravity of petrol = 0.8

$$V_1 = \frac{\pi P_1 r^4}{8\eta_1 l}$$

$$V_2 = \frac{\pi P_2 r^4}{8\eta_2 l}$$

$$V_1 = \frac{V}{t_1}$$

$$V_2 = \frac{V}{t_2}$$

$$\frac{V_1}{V_2} = \frac{t_2}{t_1} = \frac{7}{3}$$

Also, $\dfrac{V_1}{V_2} = \dfrac{P_1\eta_2}{P_2\eta_1}$

Again, $\dfrac{P_1}{P_2} = \dfrac{\rho_1}{\rho_2} = \dfrac{10^3}{0.8\times10^3} = 1.25$

$$\therefore\ \frac{7}{3} = 1.25 \times \frac{\eta_2}{\eta_1}$$

$$\frac{\eta_1}{\eta_2} = \frac{1.25\times4}{7} = 0.7$$

$$\eta_1 : \eta_2 :: 5 : 7$$

7.10 : A capillary tube 2×10^{-3} m in diameter and 0.4 in length is fitted horizontally to a vessel kept full of alcohol of density 0.8×10^3 Kg/m³. The depth of the centre of the capillary tube below the surface of alcohol is 0.4 m. Viscosity of alchol is 0.0012 N-s/m². Calculate the volume of alcohol that flows in 10 minutes.

$$V = \frac{\pi P r^4}{8\eta l}$$

Here $P = h\rho g = 0.4\times0.8\times10^3\times9.8$ N/m²

$r = 10^{-3}$ m

$\eta = 0.0012$ N-s/m²

$l = 0.4$ m

$t = 10$ minutes $= 600$ s

$$\text{Total volume} = V\times t = \frac{\pi P r^4 \times t}{8\eta l}$$

$$= \frac{\pi\times0.4\times0.8\times10^3\times9.8\times10^3\times600}{8\times0.0012\times0.4}$$

$$= 1.54\times10^{14}\ \text{m}^3$$

7.11 : A cylindrical vessel of diameter 10 cm has at its bottom a horizontal capillary tube of length 20 cm and internal radius 0.5 mm. If the vessel is filled with water, find the time in which the water level becomes half the initial height. Veiscosity of water = 10^{-3} N-s/m²

Let the rate of fall of height with time be $-\dfrac{dh}{dt}$.

Rate of flow of water = V

Area of cross section of the cylinder = A

$$\therefore\ V = -A\left(\frac{dh}{dt}\right)$$

But $V = \dfrac{\pi P r^4}{8\eta l}$