Magnetic fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers.

Production of a Magnetic Field: Ampere's Law

The basic law governing the production of a magnetic field by a current is Ampere's law:

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{net}$$

where **H** is the magnetic field intensity produced by the current I_{net} and ***dl*** is a differential element of length along the path of integration.

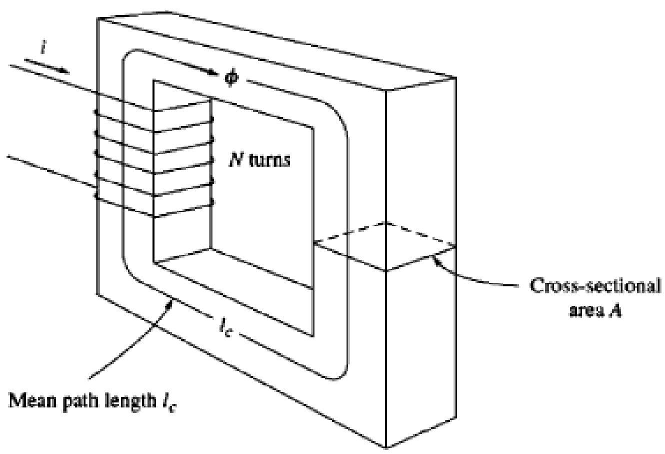


Figure shows a rectangular core with a winding of *N* turns of wire wrapped around one leg of a ferromagnetic core; essentially all the magnetic field produced by the current will remain inside the core, so the path of integration in Ampere's law is the mean path length of the core. The current passing within the path of integration I_{net} is then *Ni,* since the coil of wire cuts the path of integration N times while carrying current i. Ampere's law thus becomes

$$Hl_c = Ni$$

$$H = \frac{Ni}{l_c}$$

The magnetic field intensity **H** is in a sense a measure of the "effort" that a current is putting into the establishment of a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. The relationship between the magnetic field intensity **H** and the resulting magnetic flux density **B** produced within a material is given by

$$\boldsymbol{B} = \mu\, \boldsymbol{H}$$

Now the total flux in a given area is given by

$$\emptyset = \int_A \boldsymbol{B} \cdot d\boldsymbol{A}$$

where d**A** is the differential unit of area. If the flux density vector is perpendicular to a plane of area A, and if the flux density is constant throughout the area, the equation reduces to:

$$\emptyset = BA = \frac{\mu N i A}{l_c}$$

FARADAY'S LAW-INDUCED VOLTAGE FROM A TIME-CHANGING MAGNETIC FIELD

Faraday's law states that if a flux passes through a turn of a coil of wire, a voltage will be induced in the turn of wire that is directly proportional to the *rate of change* in the flux with respect to time. In equation form,

$$e_{ind} = -\frac{d\emptyset}{dt}$$

If a coil has *N* turns and if the same flux passes through all of them, then the voltage induced across the whole coil is given by

$$e_{ind} = -N\,\frac{d\emptyset}{dt}$$

The minus sign in the equations is an expression of *Lenz's law.* Lenz's law states that the direction of the voltage buildup in the coil is such that if the coil ends were short circuited, it would produce current that would cause a flux *opposing* the original flux change.

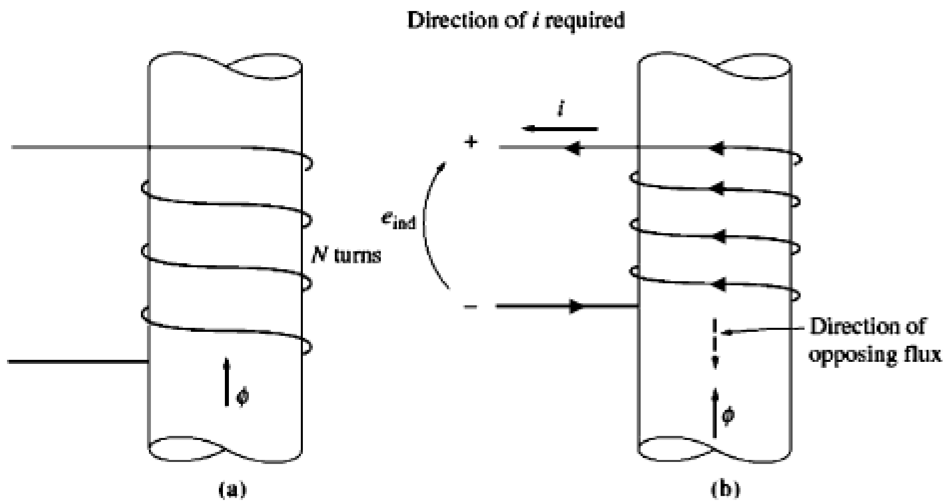
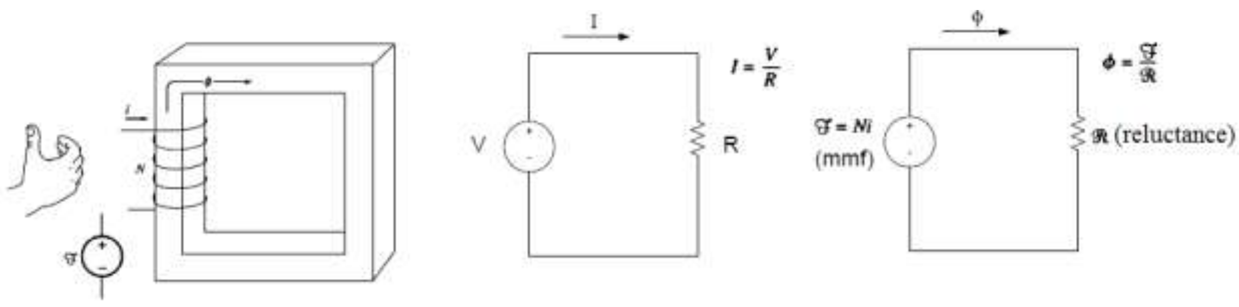


Illustration of Lenz's Law: (a) A coil enclosing a time-varying magnetic field
(b) Determination of polarity of induced voltage according to Lenz's Law

Magnetic Circuit:

The magnetic circuit model of magnetic behavior is often used in the design of electric machines and transformers to simplify the otherwise quite complex design process.

The simplest magnetic circuit consists of a current carrying coil wrapped around a core can be considered analogous to an electrical circuit consisting a voltage source connected to a resistor.



(a) Determining the polarity of a magnetomotive force source in a magnetic circuit.

(b) A simple electric circuit; motivation for constructing analogous magnetic circuits.

(c) Magnetic circuit representation of the ferromagentic core shown in (a)

In an electric circuit, the applied voltage (electromotive force) causes a current to flow. Similarly, in a magnetic circuit, the applied magnetomotive force causes flux to be produced. The relationship between voltage and current in an electric circuit is known as Ohm's law *(I = V/R).* The relationship between magnetomotive force and flux is known as Hopkinson's law and given by a similar formula.

$$\emptyset = \frac{\mathcal{F}}{\mathcal{R}}$$

The magnetomotive force of the magnetic circuit is equal to the effective current flow applied to the core, or $\mathcal{F} = Ni$ where $\mathcal{F}$ the symbol for magnetomotive force, measured in ampere-turns.

The reluctance ($\mathcal{R}$) of a magnetic circuit is the counterpart of electrical resistance, and its units are ampere-turns per weber.

$$\mathcal{R} = \frac{l_c}{\mu A}$$

Like the voltage source in the electric circuit, the magnetomotive force in the magnetic circuit has a polarity associated with it. The *positive* end of the mmf source is the end from which the flux exits, and the *negative* end of the mmf source is the end at which the flux reenters. The polarity of the mmf from a coil of wire can be determined from a modification of the right-hand rule: If the fingers of the right hand curl in the direction of the current flow in a coil of wire, then the thumb will point in the direction of the positive mmf.

Magnetic Behavior of Ferromagnetic Materials

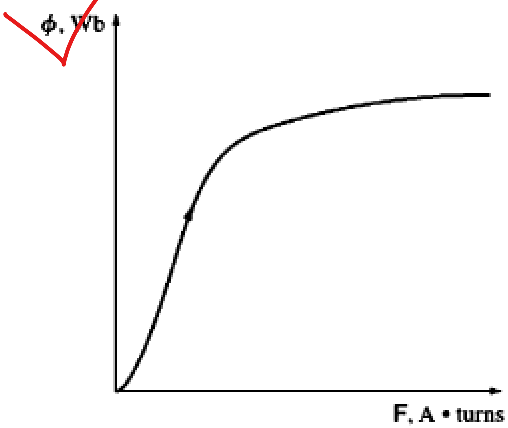
Materials which are classified as non-magnetic all show a linear relationship between the flux density B and coil current I. In other words, they have constant permeability. Thus, for example, in free space, the permeability is constant. But in iron and other ferromagnetic materials it is not constant.

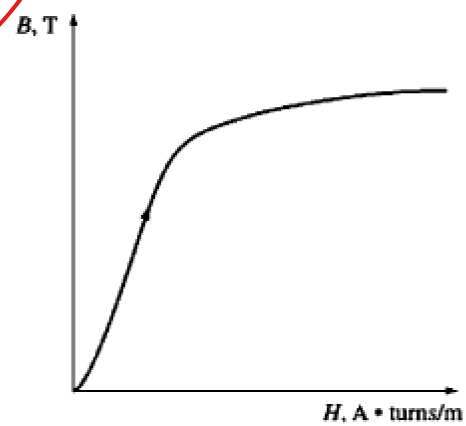
For magnetic materials, a much larger value of B is produced in these materials than in free space. Therefore, the permeability of magnetic materials is much higher than μ_0. However, the permeability is not linear anymore but does depend on the current over a wide range.

Thus, the **permeability is the property of a medium that determines its magnetic characteristics**. In other words, the concept of magnetic permeability corresponds to the ability of the material to permit the flow of magnetic flux through it.

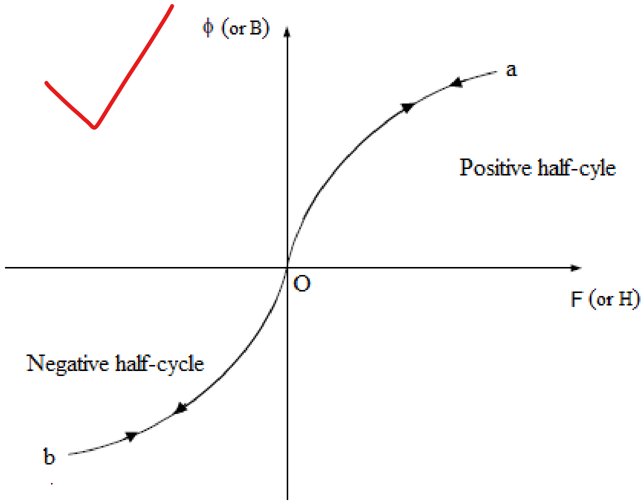
In electrical machines and electromechanical devices a somewhat linear relationship between B and I is desired, which is normally approached by limiting the current.

Look at the magnetization curve and B-H curve. Note: The curve corresponds to an increase of DC current flow through a coil wrapped around the ferromagnetic core





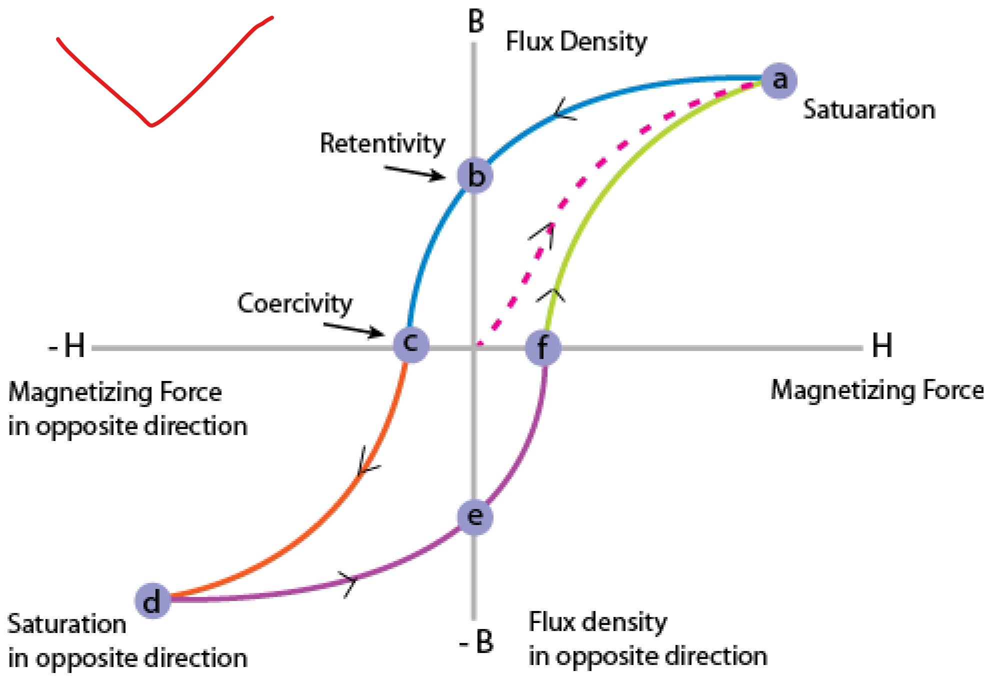
For a “perfect” ferromagnetic core magnetic flux can be expected to grow (until saturation) in positive direction during the positive half–cycle and in the opposite direction during the negative half-cycle. When the ac voltage (or current) goes through zero, magnetic flux should return to zero as well.



Theoretical magnetic behavior for a "loss-less" ferromagnetic core under ac supply

However, in a real ferromagnetic core, magnetic behavior deviates from that of an "ideal" ferromagnetic core. In a real ferromagnetic core, the amount of flux present in the core depends not only on the amount of current applied to the windings of the core, but also on the previous history of the flux in the core. The dependence of core flux on the preceding flux history and the resulting failure to retrace flux paths is known as hysteresis loss.

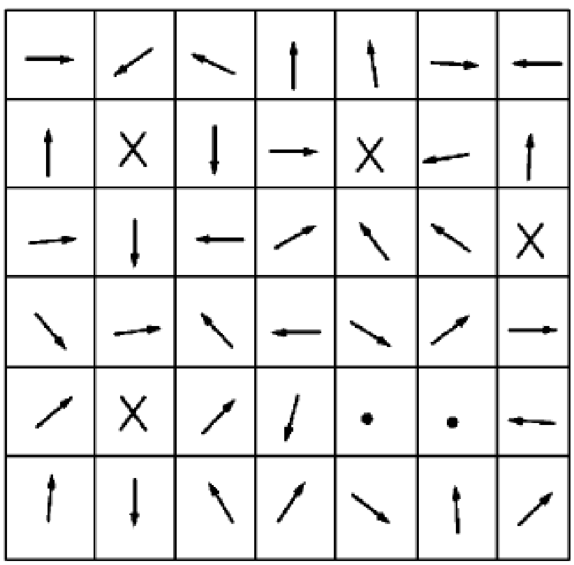
Due to hysteresis, the value of core flux is nonzero when the input current becomes zero. This is known as residual flux. To force the flux to zero, an amount of mmf (known as coercive mmf) is to be applied. This coercive mmf is required during every half-cycle and represents a power loss.



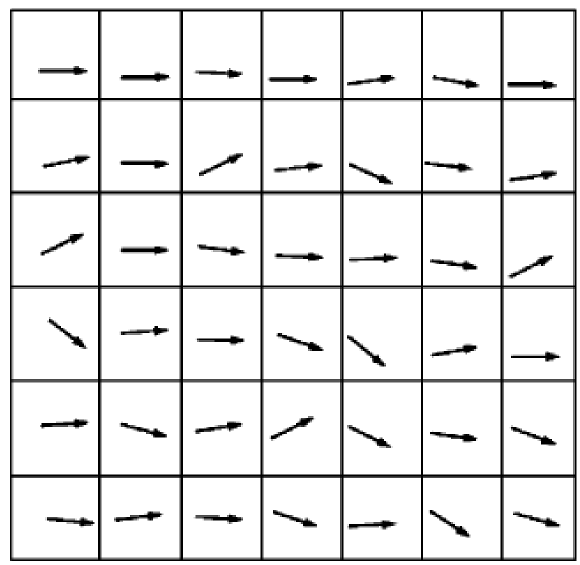
Explanation of Hysteresis from Magnetic Domain Theory:

Within each atom, the orbiting electrons are also spinning as they revolve around the nucleus. The atom, due to its spinning electrons, has a magnetic field associated with it. In nonmagnetic materials, the net magnetic field is effectively zero since the magnetic fields due to the atoms of the material oppose each other.

However, in magnetic materials such as iron and steel, the magnetic fields of groups of atoms numbering are aligned, forming very small bar magnets. This group of magnetically aligned atoms is called a **domain.** Each domain is a separate entity; that is, each domain is independent of the surrounding domains. For an unmagnetized sample of magnetic material, these domains appear in a random manner; therefore net magnetic field in any one direction is zero.



Randomly oriented magnetic domains in a feerromagnetic material



Alignment of magnetic domains in the presence of an external field

When an external magnetizing force is applied, the domains that are nearly aligned with the applied field will grow at the expense of the less favorably oriented domains. Domains pointing in the direction of the magnetic field grow because the atoms at their boundaries physically switch orientation to align themselves with the applied magnetic field. The extra atoms aligned with the field increase the magnetic flux in the iron, which in turn causes more atoms to switch orientation, further increasing the strength of the magnetic field. It is this positive feedback effect that causes iron to have permeability much higher than air.

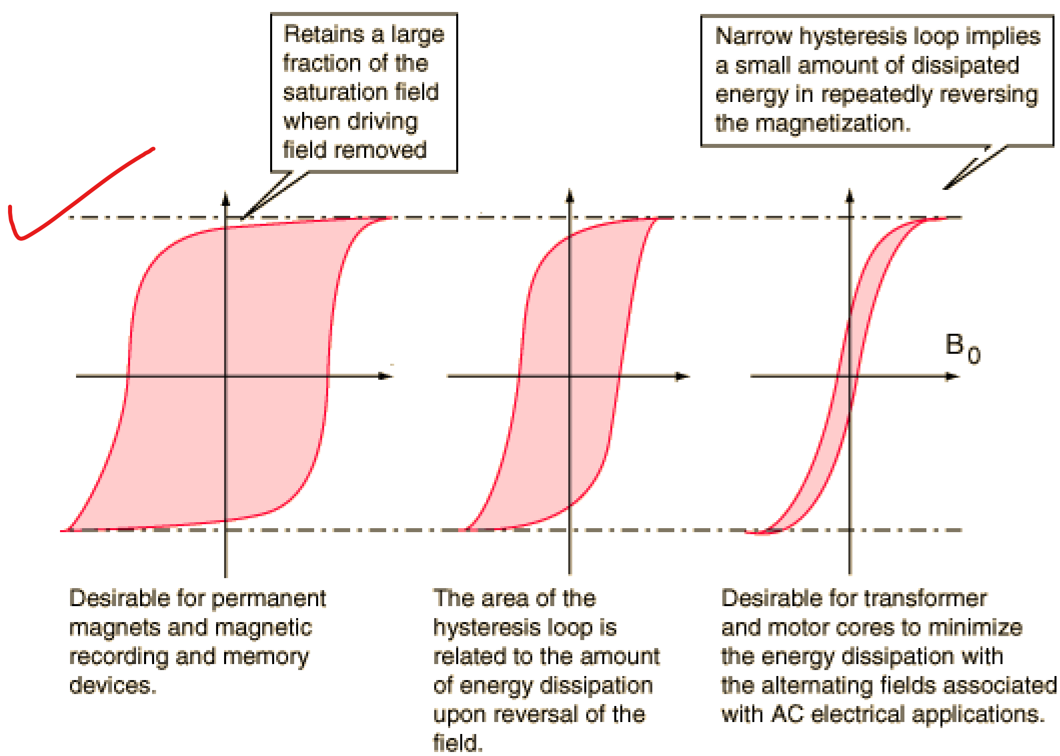
Eventually, if a sufficiently strong field is applied, all of the domains will have the orientation of the applied magnetizing force, and any further increase in external field will not increase the strength of the magnetic flux through the core—a condition referred to as *saturation.*

The key to hysteresis is that when the external magnetic field is removed, the domains do not completely randomize again. Because turning the atoms in them requires *energy.* Originally, energy was provided by the external magnetic field to accomplish the alignment; when the field is removed, there is no source of energy to cause all the domains to rotate back. The piece of iron is now a permanent magnet.

Once the domains are aligned, some of them will remain aligned until a source of external energy is supplied to change them. Examples of sources of external energy that can change the alignment of domains are magnetomotive force applied in another direction, a large mechanical shock, and heating. Any of these events can impart energy to the domains and enable them to change alignment. (It is for this reason that a permanent magnet can lose its magnetism if it is dropped, hit with a hammer, or heated.)

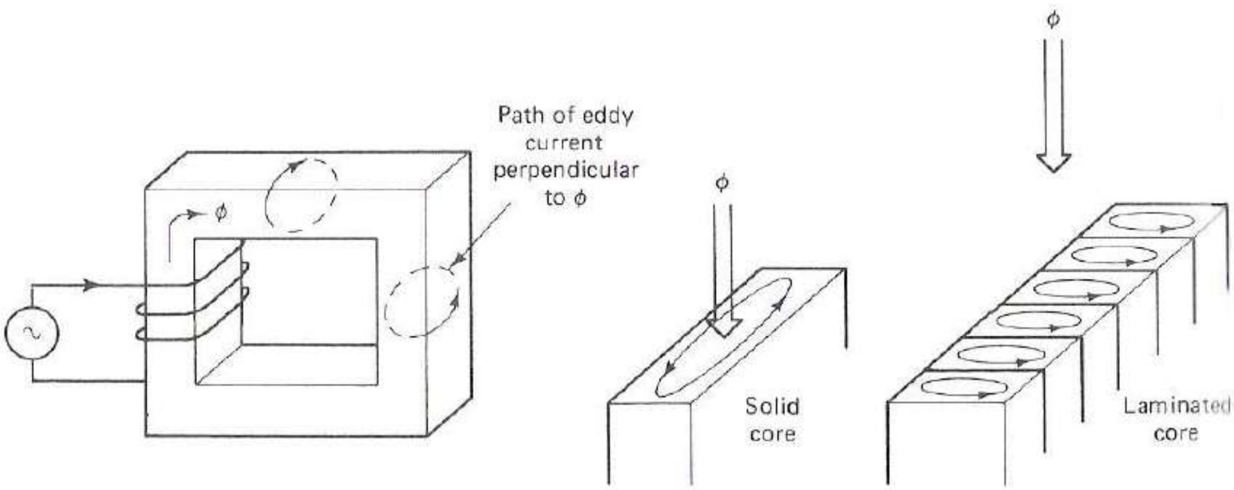
The fact that turning domains in the iron requires energy leads to a common type of energy loss in all machines and transformers. **The *hysteresis loss* in an iron core is the energy required to accomplish the reorientation of domains during each cycle of the alternating current applied to the core. The area enclosed in the hysteresis loop formed by applying an alternating current to the core is directly proportional to the energy lost in a given ac cycle.**

Hysteresis losses can be effectively reduced by the injection of small amounts of silicon into the magnetic core, constituting some 2% or 3% of the total composition of the core. This must be done carefully, however, because too much silicon makes the core brittle and difficult to machine into the shape desired.



Eddy Current Loss:

When an alternating current passing through a coil wrapped around a ferromagnetic core, it will develop a changing magnetic flux. Due to this time-changing flux, voltage is developed within the core due to electromagnetic induction. This voltage causes swirls of current to flow within the core known as eddy current. Energy is dissipated (in the form of heat) because these eddy currents are flowing in a resistive material (iron). The amount of energy lost to eddy currents is proportional to the **size of the paths** they follow within the core.



To reduce energy loss, ferromagnetic core should be broken up into small strips, or laminations, and build the core up out of these strips. An insulating oxide or resin is used between the strips, so that the current paths for eddy currents are limited to small areas.

If the core is non-ferromagnetic and has a high resistivity like air, the eddy current losses can be neglected.

Eddy current losses can be reduced if the core is constructed of thin, laminated sheets of ferromagnetic material insulated from one another and aligned parallel to the magnetic flux. Such construction reduces the magnitude of the eddy currents by placing more resistance in their path.

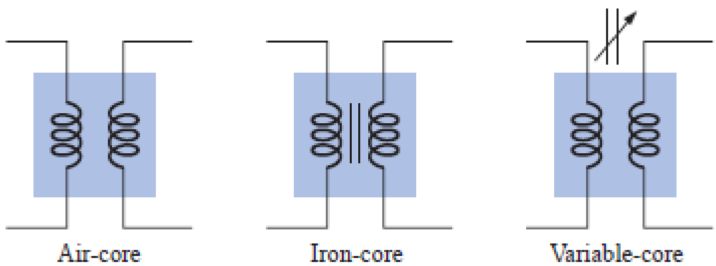
Transformer:

A *transformer* is a device that changes the voltage level of ac electric power through the action of a magnetic field. It consists of two or more coils of wire wrapped around a common ferromagnetic core. These coils are (usually) not directly connected or conductively coupled. The only connection between the coils is the common magnetic flux present within the core (magnetically coupled).

One of the transformer windings is connected to a source of ac electric power, and the remaining winding supplies electric power to loads. The transformer winding connected to the power source is called the *primary winding* or *input winding,* and the winding connected to the loads is called the *secondary winding* or *output winding.* If the secondary voltage is higher, it is known as step-up operation and if the secondary voltage is lower, it is known as step-down operation. Sometimes there is a third winding on the transformer, it is called the *tertiary winding.*

Construction of Transformers

Transformers are available in many different shapes and sizes. Some of the more common types include the power transformer, audio transformer, IF (intermediate-frequency) transformer, and RF (radio frequency) transformer. Each is designed to fulfill a particular requirement in a specific area of application. Size of a transformer decreases with the increase in operating frequency.



Transformer symbols.

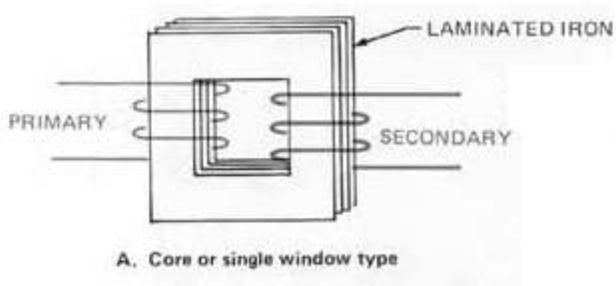
Power transformer: In case of power transformers, the core is made of laminated sheets of ferromagnetic material separated by an insulator (varnish or oxide coating) to reduce the eddy current losses. The sheets themselves will also contain a small percentage of silicon to increase the electrical resistivity of the material and further reduce the eddy current losses.

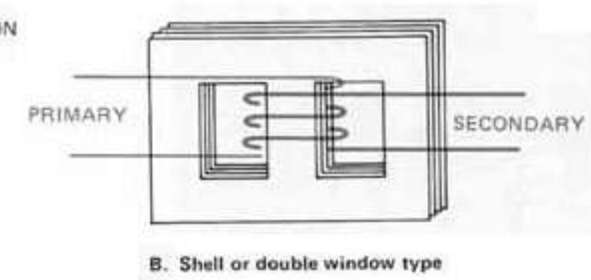
Types of cores for power transformer:

i) **Core Form:** A simple rectangular laminated piece of steel with the transformer windings wrapped around two sides of the rectangle.

ii) **Shell Form:** A three legged laminated core with the windings wrapped around the centre leg. The primary and secondary windings are wrapped one on top of the other with the low-voltage winding innermost. It serves two purposes:

- It simplifies the problem of insulating the high-voltage winding from the core.
- It results in much less leakage flux





Cooling: Heat generation in transformer is mainly due to its copper loss and partially to its core loss. It is essential to control the temperature within permissible limit to ensure the long life of transformer by reducing thermal degradation of its insulation system. Cooling is provided by air convection, forced air, insulating liquids, or gas (SF_6, C_2F_6, etc.)

Power transformers are given a variety of different names, depending on their use in power systems.

- Unit transformers – Usually located at the output of a generator. Its function is to step up the voltage level so that long distance transmission of power is possible.
- Substation transformers – Located at main distribution or secondary level transmission substations. Its function is to lower the voltage levels for 1^{st} level distribution purposes.
- Distribution Transformers – Located at small distribution substation. It lowers the voltage levels for 2^{nd} level distribution purposes.

Air-core Transformer: Due to non-linearity in ferromagnetic core (due to core saturation, power loss in the form of eddy current and hysteresis) electromagnetic field does not change uniformly and output signal gets distorted. To maintain the quality of signal in high frequency application like signal transmission, air core transformer is introduced. Here iron core of transformer is absent and the flux is linked with the windings through air. In addition to the noise-free operation, an air core transformer is quite light weight due to absence of heavy weight iron core. That is why this type of transformer is most suitable for portable, light weight electronic devices and high frequency devices. Air core transformers are generally used in radio transmitter and communication devices etc.

Principle of Operation:

A current-carrying wire produces a magnetic field in the area around it. If the current is time-varying, the resulting magnetic field will be time-varying as well. A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil; this is known as electromagnetic induction.

The magnitude of the flux is directly proportional to the current. Therefore for a sinusoidal input, the magnitude of the flux will vary as a sinusoid as well.

The sinusoidal time-varying flux that links both coils can be expressed as

$$\emptyset_M(t) = \Phi_{\max} \sin\omega t$$

The induced voltage across the primary due to a sinusoidal input can be determined by Faraday's law:

$$e_p(t) = N_p \frac{d\emptyset_p(t)}{dt} = N_p \frac{d\emptyset_M(t)}{dt} = N_p \frac{d}{dt}\Phi_{\max} \sin\omega t = \omega N_p \Phi_{\max} \cos\omega t$$

Effective value of e_p is given by,

$$E_p = \frac{\omega N_p \Phi_m}{\sqrt{2}} = \frac{2\pi f N_p \Phi_m}{\sqrt{2}} = 4.44 f N_p \Phi_{\max}$$

Similarly, effective value of the induced voltage across the secondary

$$E_s = 4.44 f N_s \Phi_{\max}$$

Ideal Transformer:

An *ideal transformer* is a lossless device with an input winding and an output winding.

$$v_p(t) i_p(t) = v_s(t) i_s(t)$$

The transformer has N_p turns of wire on its primary side and N_s turns of wire on its secondary sides. The relationship between the primary and secondary voltage is as follows:

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

where *a* is the turns ratio of the transformer.

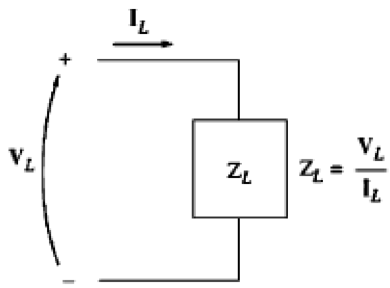
The relationship between primary and secondary current is:

$$\frac{i_p(t)}{i_s(t)} = \frac{N_s}{N_p} = \frac{1}{a}$$

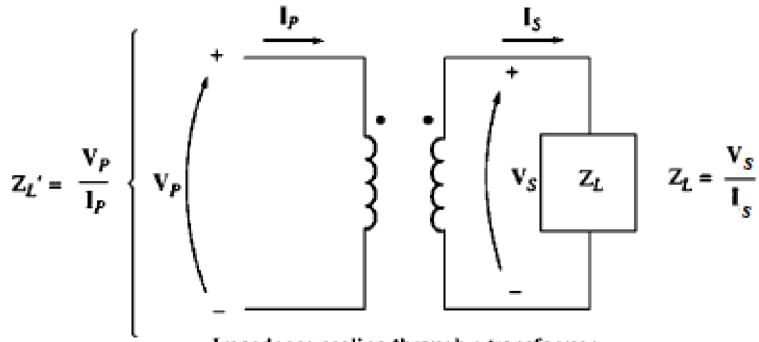
In terms of phasor quantities:

$$\frac{\boldsymbol{V}_p}{\boldsymbol{V}_s} = \frac{\boldsymbol{I}_s}{\boldsymbol{I}_p} = a$$

Impedance Transformation:



Definition of impedance.



Impedance scaling through a transformer.

$$Z_L = \frac{\boldsymbol{V}_s}{\boldsymbol{I}_s}$$

$$Z_L^{'} = \frac{\boldsymbol{V}_p}{\boldsymbol{I}_p} = \frac{a\,\boldsymbol{V}_s}{\boldsymbol{I}_s/a} = a^2\frac{\boldsymbol{V}_s}{\boldsymbol{I}_s}$$

$$Z_L^{'} = a^2\,Z_L$$

DOT Convention:

In real transformers, to determine the polarity of the secondary terminals, it is required to open the transformer casing and examine the orientation of its windings. To avoid this transformers utilize the *dot convention.* The dots appearing at one end of each winding tell the polarity of the voltage and current on the secondary side of the transformer.

- If the primary *voltage* is positive at the dotted end of the winding with respect to the undotted end, then the secondary voltage will be positive at the dotted end also.
- If the primary *current* of the transformer flows *into* the dotted end of the primary winding, the secondary current will flow *out* of the dotted end of the secondary winding.

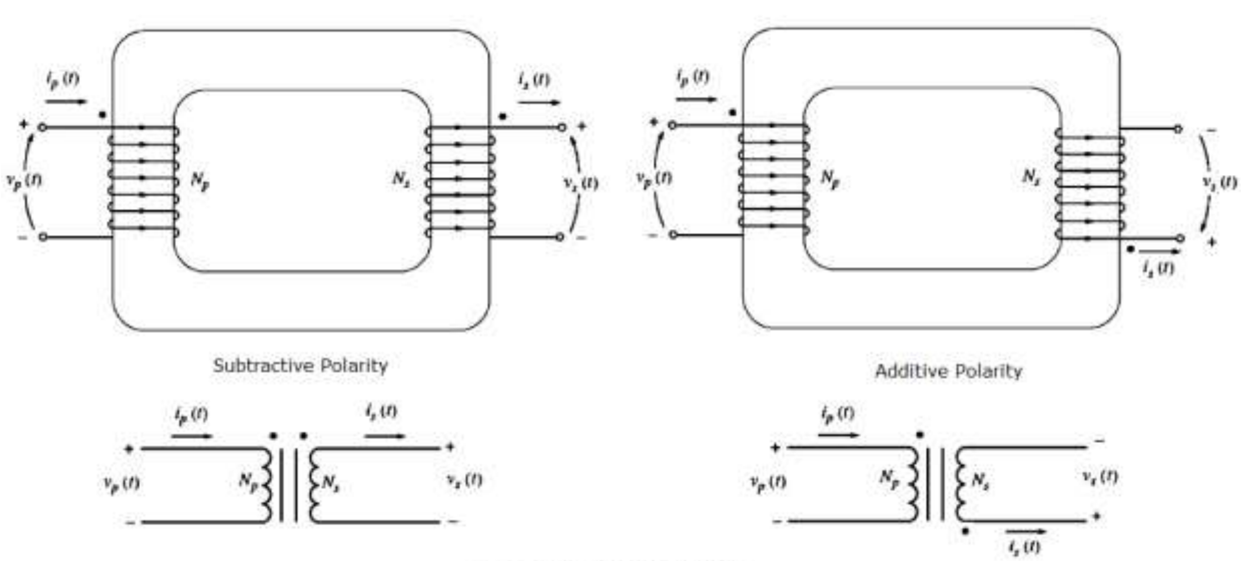


Illustration of Dot Convention

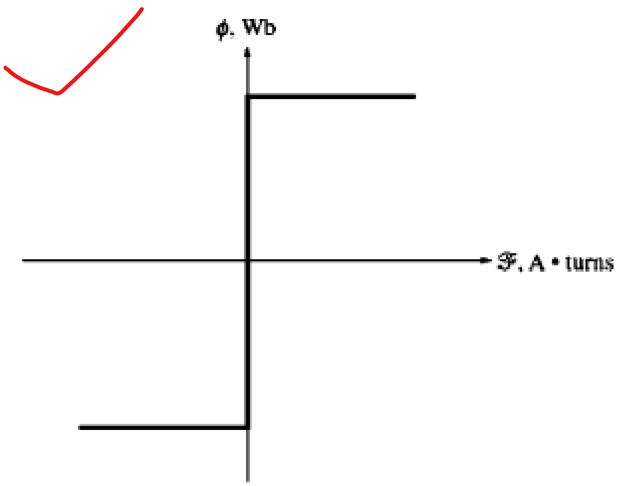
Subtractive Polarity: In this connection, the terminals with the same instantaneous polarity are opposite to each other. If an accidental contact between any two adjacent terminals occurs, the voltage across the other terminals will be the difference between the high and low voltages

Additive Polarity: Accidental contact between adjacent terminals of opposite windings will result in a voltage across the other ends equal to the sum of high and low voltages.

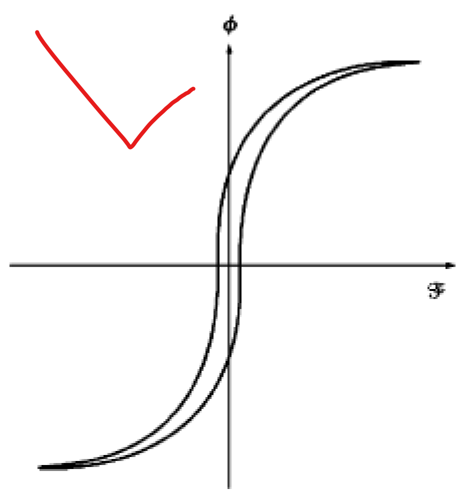
Real Transformer:

Ideal transformers may never exist due to the fact that there are losses associated to the operation of transformers. The differences between a real transformer and an ideal transformer are:

- In an ideal transformer, the resistance of the transformer windings is assumed to be zero. Whereas, real transformers have nonzero winding resistance that results in power loss (known as Cu loss).
- In an ideal transformer, the leakage flux in the core is assumed to be zero, implying that all the flux in the core couples both windings. In a real transformer leakage flux is present in both primary and secondary coils.
- When secondary terminals are open, there is no current flow in the secondary coil. If the transformer is ideal, current in the primary coil is zero as well. However, in a real transformer there is a current flow in the primary coil, when no load in the secondary. This is known as exciting current.
- In a real transformer, power losses occur in the ferromagnetic core in the form hysteresis loss and eddy current loss. Whereas, in an ideal transformer, the core must have no hysteresis or eddy currents.
- The permeability of the core is infinite; it requires no exciting current to maintain the flux. In an ideal transformer, the net magnetomotive force is zero. $F_{net} = N_p i_p - N_s i_s = 0$. The resulting magnetization curve must have the shape shown below.



The magnetization curve of an ideal transformer.

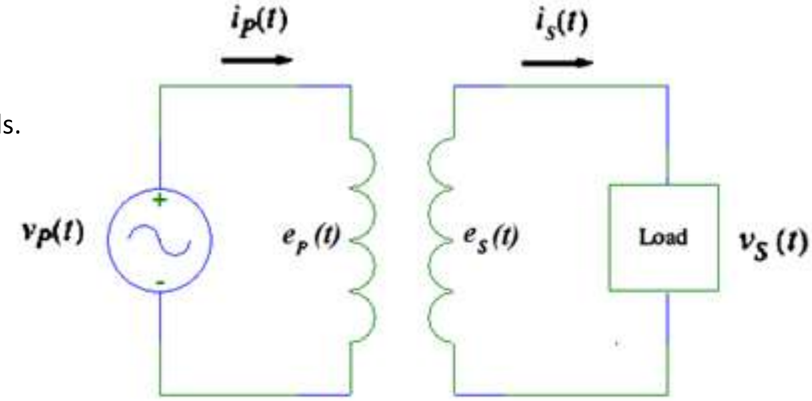


The hysteresis curve of the transformer.

Equivalent Circuit of a Real Transformer:

To analyze the performance of a real transformer, the non-ideal factors are to be considered. The equivalent circuit will take into account all the major imperfections in a real transformer.

Leakage Flux:



For an ideal transformer, same flux links both coils.

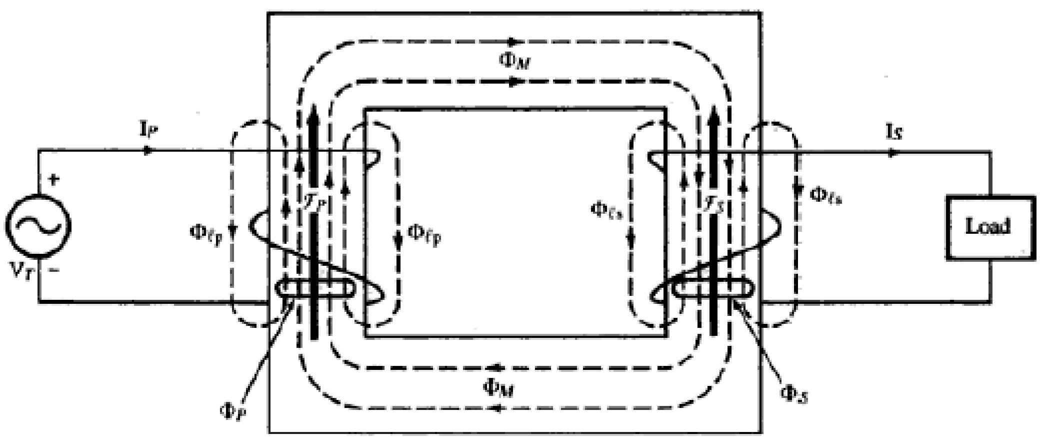
$$v_p(t) = e_p(t)$$

$$v_s(t) = e_s(t)$$

$$\frac{v_p(t)}{v_s(t)} = \frac{e_p(t)}{e_s(t)} = a$$

However, in a real transformer due to leakage flux, there will be some voltage drops and voltage ratio, $\frac{v_p(t)}{v_s(t)}$ will be different than turns ratio, a.

The portion of the flux that goes through one of the transformer coils but not the other one is called leakage flux. The following figure depicts various flux components in a loaded transformer.



$$\emptyset_p = \emptyset_M + \emptyset_{lp} \qquad \emptyset_s = \emptyset_M - \emptyset_{ls}$$

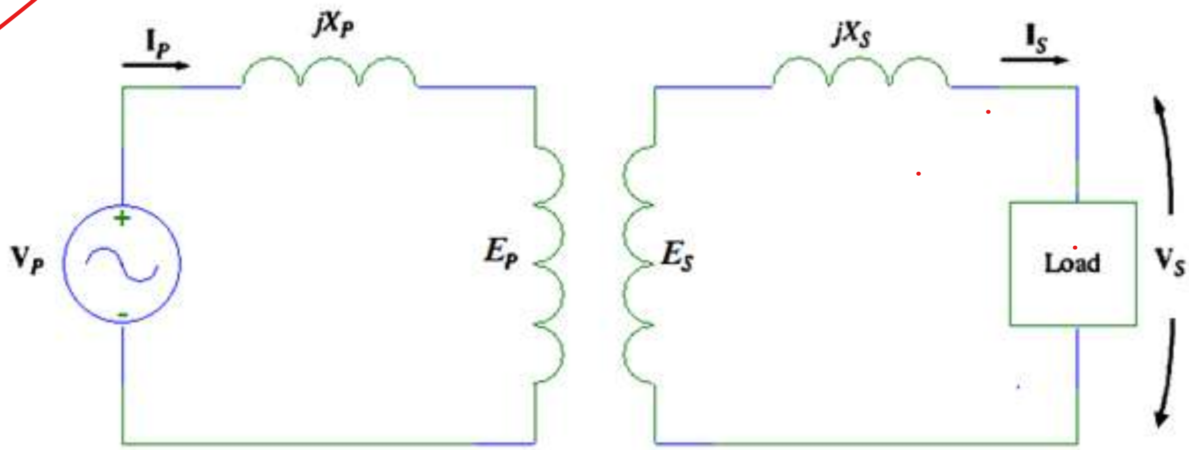
$$v_p(t) = N_p \frac{d\emptyset_p}{dt} \qquad v_s(t) = N_s \frac{d\emptyset_s}{dt}$$

$$v_p(t) = N_p \frac{d\emptyset_M}{dt} + N_p \frac{d\emptyset_{lp}}{dt} \qquad v_s(t) = N_s \frac{d\emptyset_M}{dt} - N_s \frac{d\emptyset_{ls}}{dt}$$

$$v_p(t) = e_p(t) + e_{lp}(t) \qquad v_s(t) = e_s(t) - e_{ls}(t)$$

$$e_{lp}(t) = N_p \frac{d\emptyset_{lp}}{dt} = N_p \frac{d\emptyset_{lp}}{di_p}\frac{di_p}{dt} = L_p \frac{di_p}{dt} \qquad e_{ls}(t) = N_s \frac{d\emptyset_{ls}}{dt} = N_s \frac{d\emptyset_{ls}}{di_s}\frac{di_s}{dt} = L_s \frac{di_s}{dt}$$

Therefore the **leakage element may be modeled as an inductance** connected together in series with the primary and secondary circuit respectively.



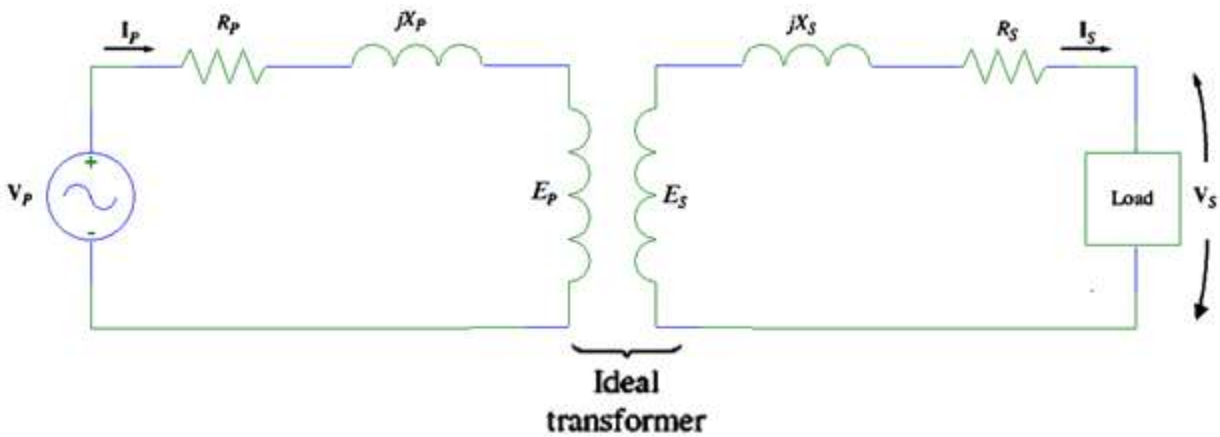
In terms of phasor quantities,

$$\boldsymbol{V}_p = \boldsymbol{E}_p + j\boldsymbol{I}_p X_p \qquad \boldsymbol{V}_s = \boldsymbol{E}_s - j\boldsymbol{I}_s X_s$$

Where X_p and X_s are leakage reactances of primary and secondary coil.

Copper Loss:

Copper losses are resistive power losses due to nonzero winding resistance of the coils. These losses are modeled by placing a resistor R_P in the primary circuit and a resistor R_S in the secondary circuit.



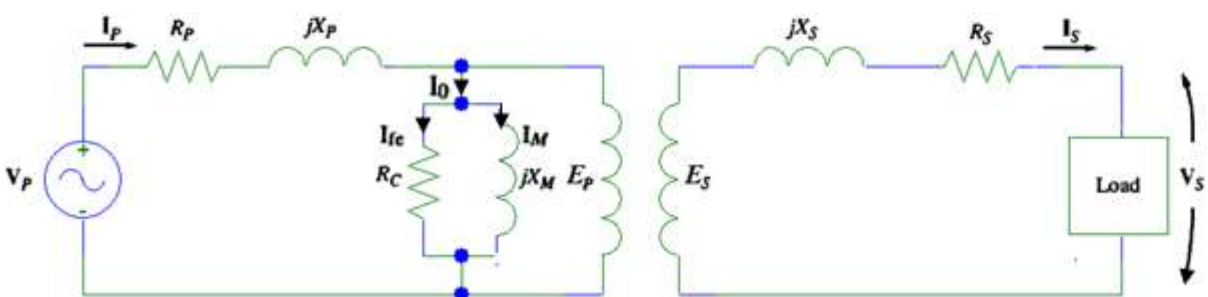
$$\boldsymbol{V}_p = \boldsymbol{E}_p + \boldsymbol{I}_p R_p + j\boldsymbol{I}_p X_p \qquad \boldsymbol{V}_s = \boldsymbol{E}_s - j\boldsymbol{I}_s X_s - \boldsymbol{I}_s R_s$$

Where R_p and R_s are winding resistances of primary and secondary coil

Exciting Current:

Exciting current flows in the primary circuit, even when *the secondary circuit is open circuited.* To accommodate this current, a parallel branch (no-load branch) is to be included in the equivalent circuit. Exciting current consists of two components:

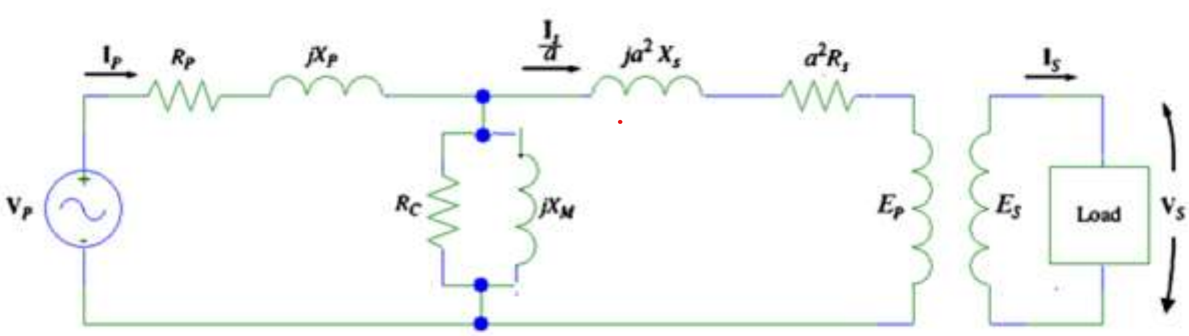
- The magnetization current *I_m*, which is the current required to produce the flux in the transformer core. The magnetization current I_m is proportional (in the unsaturated region) to the voltage applied to the core and lagging the applied voltage by 90°. Therefore, the magnetizing current can be modeled as reactance X_m across the primary voltage source.
- The *core-loss current I_{fe}* which is the current required to make up for hysteresis and eddy current losses. The core loss current I_{fe} is proportional to the voltage applied to the core and also in phase with the applied voltage. Therefore, the core-loss current can be modeled as a resistance R_C across the primary voltage source.



The no-load branch consists of a resistance, R_C (core-loss resistance) in parallel with an inductive reactance, X_M (magnetizing reactance).

To analyze practical circuits containing transformers, it is normally necessary to convert the entire circuit to an equivalent circuit at a single voltage level.

Impedances of the secondary side can be reflected to the primary side (muliplying by square of the turns ratio) and thus equivalent circuit referred to the primary circuit can be obtained.

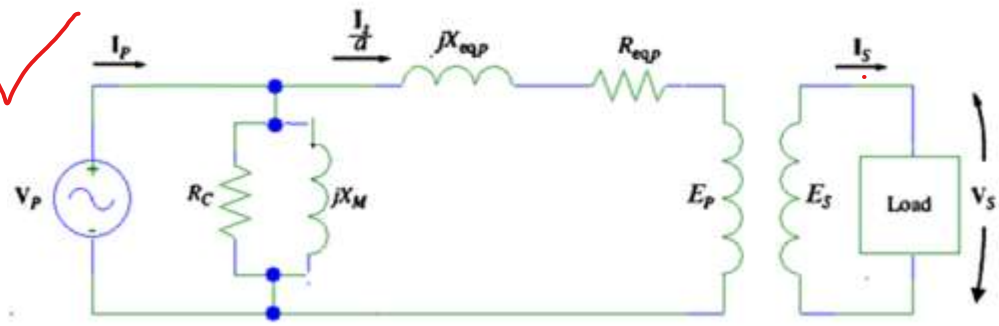


Equivalent circuit referred to primary side

The circuit can be further simplified by combining the impedance of the primary side and the reflected impedance of the secondary side.

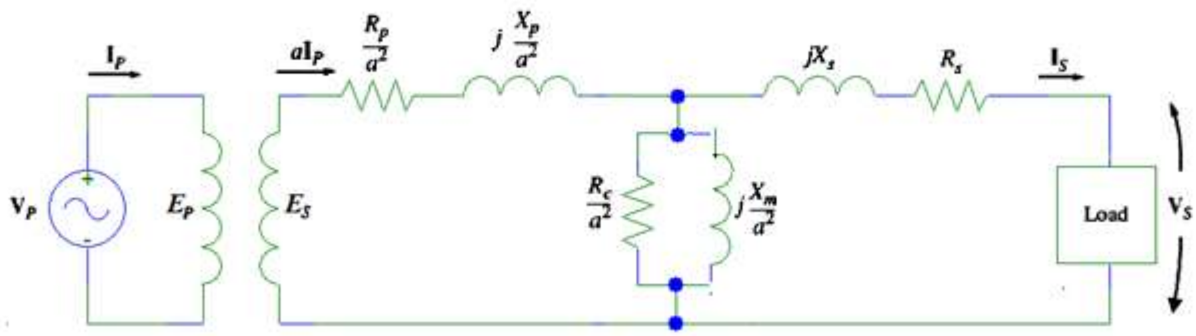
$$R_{eq,p} = R_p + a^2 R_s$$

$$X_{eq,p} = X_p + a^2 X_s$$

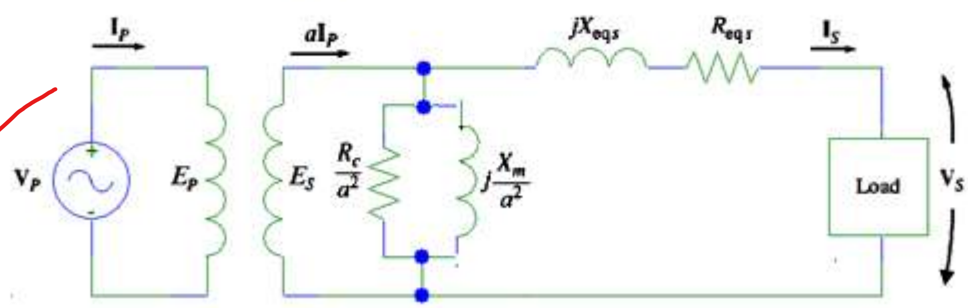


Approximate equivalent circuit referred to primary side

Alternatively, impedances of the primary side can be reflected to the secondary side (dividing by square of the turns ratio) and hence combined to form a equivalent resistance and reactance.



Equivalent circuit referred to secondary side



Approximate equivalent circuit referred to secondary side

$$R_{eq,s} = \frac{R_p}{a^2} + R_s$$

$$X_{eq,s} = \frac{X_p}{a^2} + X_s$$

Voltage Regulation:

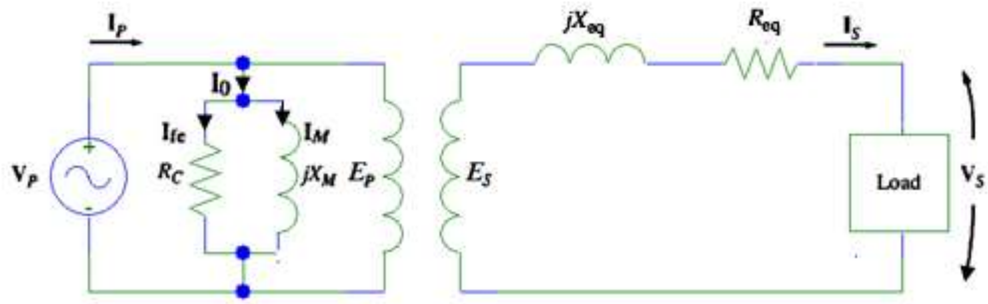
Because a real transformer has series impedances within it, the output voltage of a transformer varies with the load even if the input voltage remains constant. To conveniently compare transformers in this respect, it is customary to define a quantity called *voltage regulation* (VR). *Full-load voltage regulation* is a quantity that compares the output voltage of the transformer at no load with the output voltage at full load.

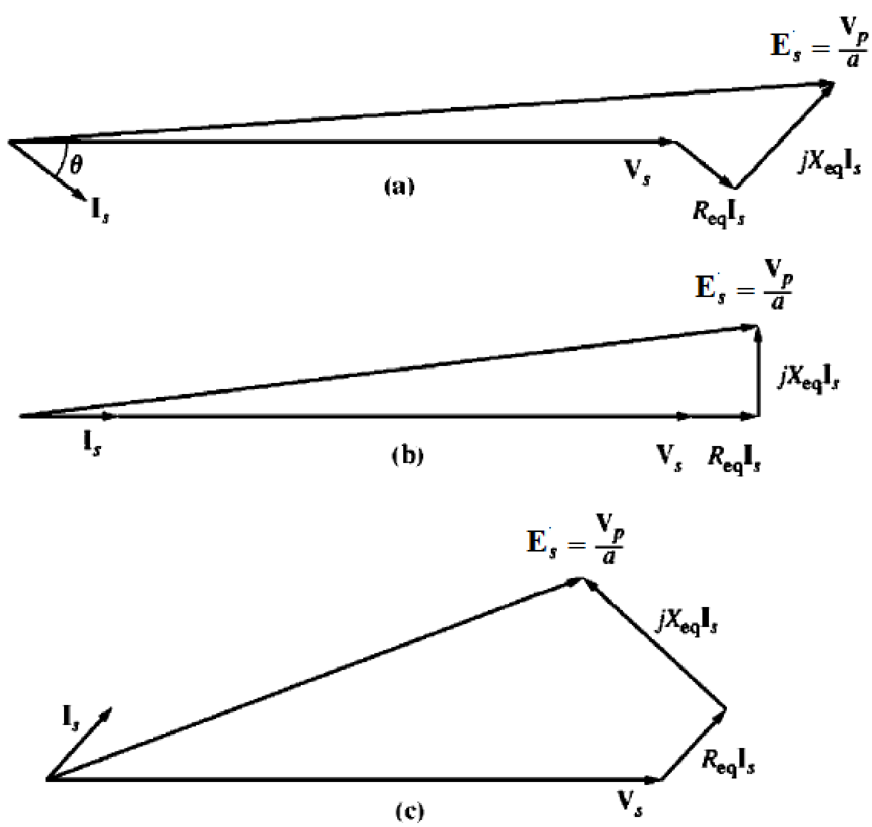
$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}}$$

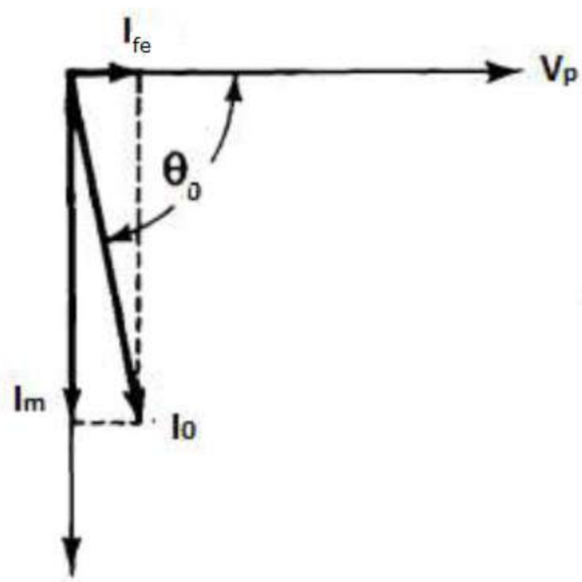
Usually it is a good practice to have as small a voltage regulation as possible. For an ideal transformer, VR = 0 percent. It is not always a good idea to have a low-voltage regulation, though-sometimes high-impedance and high-voltage regulation transformers are deliberately used to reduce the fault currents in a circuit.

Transformer Phasor Diagram:

$$\boldsymbol{E}_s = \boldsymbol{V}_s + \boldsymbol{I}_s R_{eq} + j\boldsymbol{I}_s X_{eq}$$







Phasor Diagram of the transformer while operating at no load

Phasor diagram of a transformer operating at a (a) lagging, (b) unity and (c) leading power factor

From the phasor diagrams, it is evident that voltage regulation is positive for lagging and unity power factor, whereas negative for leading power factor.

Final

Efficiency:

Transformers are also compared and judged on their efficiencies. The efficiency of a device is defined by the equation

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{out}}{P_{out} + P_{core} + P_{Cu}} = \frac{V_s I_s cos\theta_s}{V_s I_s cos\theta_s + P_{core} + P_{Cu}}$$

An ideal transformer has zero copper loss and zero core loss. Therefore, efficiency of an ideal transformer is unity (100%). In case of real transformers, efficiency is commonly above 95% and in some cases can be as high as 99%.

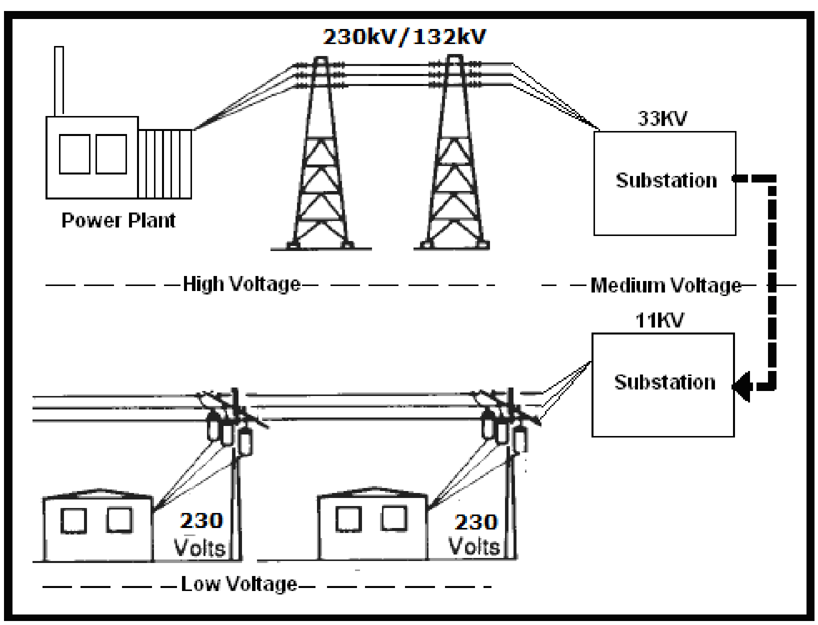
Applications of Transformer: Final

1. **Power Transformer:** Transformer plays an integral part in power distribution systems. The main purpose of using transformers is to reduce power loss and voltage drop in transmission line. Since the power loss is proportional to the square of line current, by rise of line voltage at the transmission end can significantly lower the power loss. Besides, the smaller current reduces the required conductor size, producing considerable cost savings. Without the transformer, the majority of the power generated would be lost on the transmission line and long-distance transmission would not be possible.

In a modern power system, electric power is generated at voltages of 12 to 25 kV. Transformers step up the voltage to between 110 kV and nearly 1000 kV for transmission over long distances at very low losses. Transformers then step down the voltage to the 12- to 34.5-kV range for local distribution and finally permit the power to be used safely in homes, offices, and factories at voltages as low as 120 V.

In Bangladesh, electric power is generated at voltages around 13 kV. It is then stepped up to 132 kV or 230 kV and transmitted through national grid. It is then stepped down in successive stages to 33 kV, 11 KV and finally to 400 V.

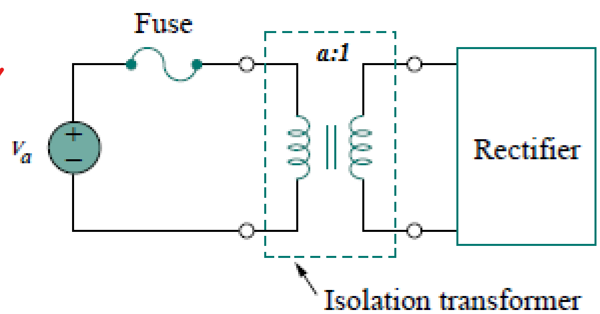
Residential users are given connections at 230V (1-phase) whereas for industrial consumers 3-phase supply is provided at 11 kV or 33 kV.



2. Isolation Transformer:

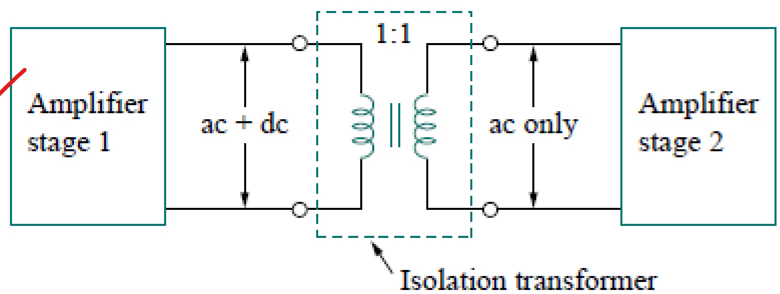
Electrical isolation is said to exist between two devices when there is no physical connection between them. In a transformer, energy is transferred by magnetic coupling, without electrical connection between the primary circuit and secondary circuit.

First, consider the circuit shown below. A rectifier is an electronic circuit that converts an ac supply to a dc supply. A transformer is often used to couple the ac supply to the rectifier. The transformer serves two purposes. First, it steps up or steps down the voltage. Second, it provides electrical isolation between the ac power supply and the rectifier, thereby reducing the risk of shock hazard in handling the electronic device.



A transformer used to isolate an ac supply from a rectifier.

A transformer is often used to couple two stages of an amplifier, to prevent any dc voltage in one stage from affecting the dc bias of the next stage. Biasing is the application of a dc voltage to a transistor amplifier or any other electronic device in order to produce a desired mode of operation. Each amplifier stage is biased separately to operate in a particular mode; the desired mode of operation will be compromised without a transformer providing dc isolation.

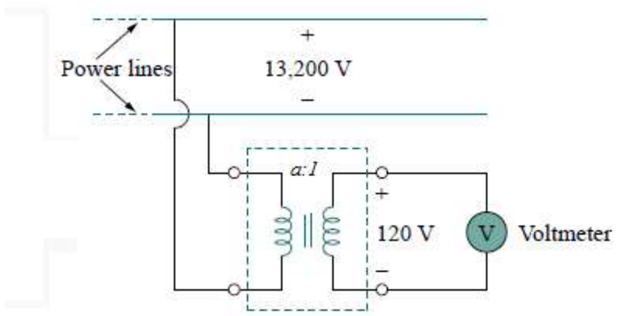


A transformer providing dc isolation between two amplifier stages.

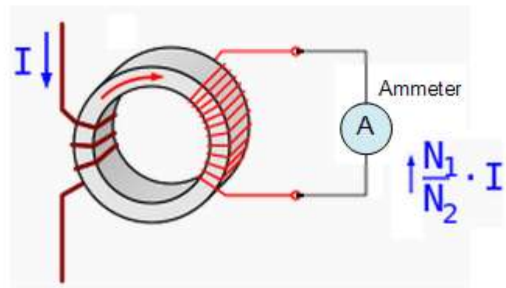
As shown in the figure above, only the ac signal is coupled through the transformer from one stage to the next. Transformers are used in radio and TV receivers to couple stages of high-frequency amplifiers. When the sole purpose of a transformer is to provide isolation, its turns ratio *a* is made unity. Thus, an isolation transformer has $a = 1$.

3. **Instrument Transformer:**

Generators in the power plant produce voltage in the range of 13.2 kV and then this voltage is raised to 230 kV or 132 kV for long distance transmission. It is obviously not safe to connect a voltmeter directly to such high-voltage lines. A transformer can be used both to electrically isolate the line power from the voltmeter and to step down the voltage to a safe level, as shown in the figure below. This transformer is called potential transformer or voltage transformer. Similarly, current transformers can be used to measure high line currents.



A potential transformer being used to sample and measure a high voltage

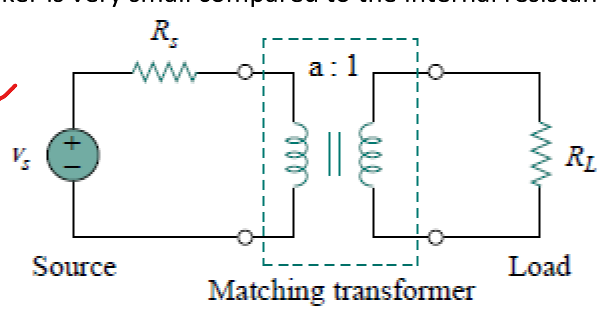


Current transformer being used to sample and measure a high current

4. **Impedance Matching Transformer:**

For maximum power transfer, the load resistance R_L must be matched with the source resistance R_S . In most cases, the two resistances are not matched; both are fixed and cannot be altered. However, an iron core transformer can be used to match the load resistance to the source resistance. This is called *impedance matching*.

For example, to connect a loudspeaker to an audio power amplifier requires a transformer, because the resistance of the speaker is very small compared to the internal resistance of the amplifier.



Transformer being used as a matching device.

The ideal transformer reflects its load back to the primary with a scaling factor of a^2.

Turns ratio of the desired matching transformer can be determined using the relation: $R_S = a^2 R_L$

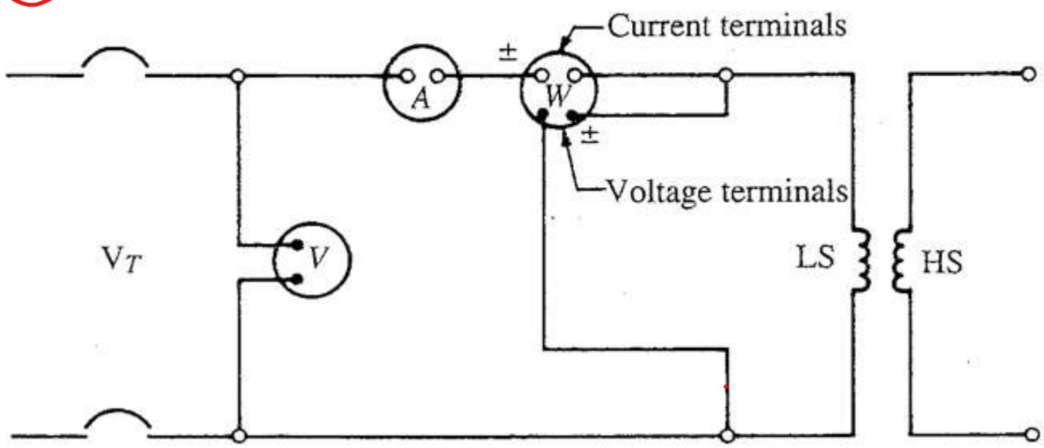
Therefore a step-down transformer ($a > 1$) is needed as the matching device when $R_S > R_L$, and a step-up ($a < 1$) is required when $R_S < R_L$.

Transformer Test

If transformer parameters are not readily available from the nameplate or from the manufacturer, they can be approximated from an open-circuit test (also called a no-load test) and a short circuit test.

Open Circuit Test

In the *open-circuit test,* a transformer's secondary winding is open-circuited, and its primary winding is connected to the full-rated line voltage at the rated frequency. The input voltage, input current, and input power to the transformer are measured using a voltmeter, an ammeter and a wattmeter respectively. For safety in testing and instrumentation, the open circuit test is generally made on the low-voltage side. To prevent accidental contact, the high side terminals should be covered with insulating material.



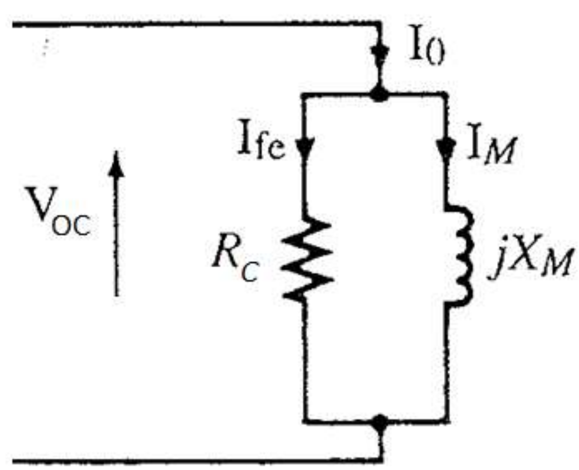
Connection for transformer open-circuit test.

Under the conditions described, all the input current must be flowing through the excitation branch of the transformer. The series elements R_p and X_p are too small in comparison to R_c and X_M to cause a significant voltage drop, so essentially all the input voltage is dropped across the excitation branch. Since the secondary is carrying no load, the copper losses in the secondary are zero, and the copper losses in the primary are negligible. Thus, the wattmeter reading for the open circuit test can be considered as the approximate core loss at rated condition.

$$R_c = \frac{V_{oc}^2}{P_{oc}}$$

$$X_m = \frac{V_{oc}^2}{Q_{oc}} = \frac{V_{oc}^2}{\sqrt{(V_{oc}I_{oc})^2 - P_{oc}^2}}$$

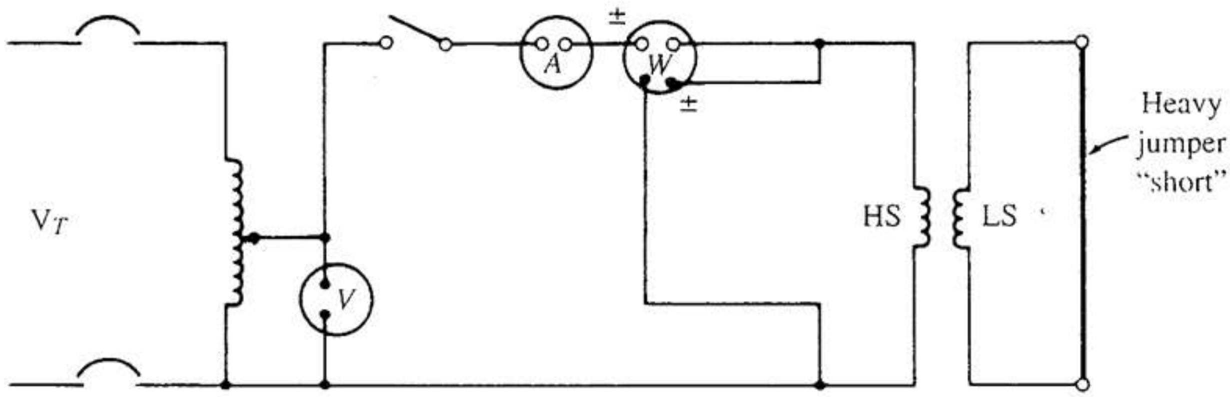
$$P_{core} \approx P_{oc}$$



Equivalent Circuit under open-circuit condition

Short Circuit Test:

In the *short-circuit test,* the secondary terminals of the transformer are short circuited, and the primary terminals are connected to the supply line through an adjustable-voltage autotransformer. The input voltage is adjusted until the current in the short circuited windings is equal to its rated value. Short circuit test can be performed using either winding. For reasons of lower current input and meter sizing, the high voltage winding is preferred.



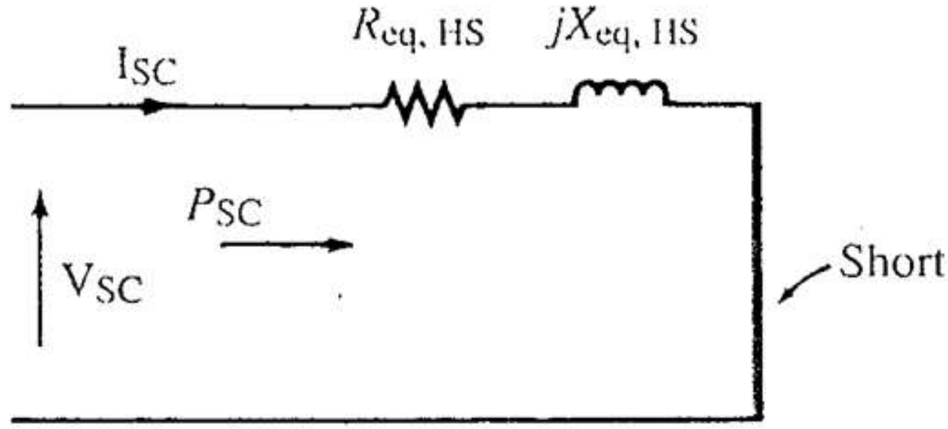
Connection for transformer short-circuit test.

Since the input voltage is so low during the short-circuit test, negligible current flows through the excitation branch. If the excitation current is ignored, then the entire voltage drop in the transformer can be attributed to the series elements in the circuit. Short circuiting the secondary causes the flux density to be reduced to a very low value, making the core losses insignificant. Thus the wattmeter reading for the short circuit test can be considered as an approximate value of copper loss at rated condition.

$$R_{eq} = \frac{P_{sc}}{I_{sc}^2}$$

$$X_{eq} = \frac{Q_{sc}}{I_{sc}^2} = \frac{\sqrt{(V_{sc}I_{sc})^2 - P_{sc}^2}}{I_{sc}^2}$$

$$P_{Cu} \approx P_{sc}$$



Equivalent Circuit of the transformer under short circuit condition

Transformer all day efficiency

A distribution transformer cannot be run with constant load throughout 24 hours. At day peak time it's loading is high, whereas in night lean time its loading may be negligible. So selecting a transformer depending upon its conventional efficiency is not practical and economical, too. As a solution of these problems, the concept of all day efficiency of distribution transformer came into the picture.

In this concept, we use the ratio of total energy delivered by the transformer to the total energy fed to the transformer, during a 24 hrs span of time instead of ratio of power output and input of the transformer. Hence, all day efficiency is determined as, total KWh at the secondary of the total KWh at the primary of the transformer for a long specific period preferably 24 hrs. i.e,

$$All\ day\ efficiency = \frac{Output\ energy\ (kWh)\ in\ a\ fixed\ time\ interval\ of\ 24\ hours}{Input\ energy\ (kWh) over\ the\ same\ interval}$$

$$All\ day\ efficiency = \frac{\sum Output\ power \times Hour}{\sum Output\ power \times Hour + \sum Copper\ loss\ \times Hour + Core\ loss\ \times 24\ hour}$$

This is very much useful to judge the performance of a distribution transformer, whose primary is connected to the system forever, but secondary load varies tremendously throughout the day. Since the load is continually varying, conventional efficiency would not be an accurate reflection of the transformer's capability

Transformer maximum efficiency

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{out}}{P_{out} + P_{core} + P_{Cu}} = \frac{V_s I_s cos\theta_s}{V_s I_s cos\theta_s + P_{core} + I_s^2 R_{eq,s}}$$

$$\frac{1}{\eta} = \frac{V_s I_s cos\theta_s + I_s^2 R_{eq,s} + P_{core}}{V_s I_s cos\theta_s} = 1 + \frac{I_s^2 R_{eq,s} + P_{core}}{V_s I_s cos\theta_s}$$

For maximum efficiency condition,

$$\frac{d}{dI_s}\left(\frac{1}{\eta}\right) = 0$$

$$\frac{R_{eq,s}}{V_s cos\theta_s} - \frac{P_{core}}{V_s I_s^2 cos\theta_s} = 0$$

$$P_{core} = I_s^2 R_{eq,s} = P_{Cu}$$

$$P_{fixed} = P_{variable}$$

Hence, the transformer will operate at the maximum efficiency if variable loss (copper loss) equals the fixed loss (core loss).

Per-Unit System

In the per-unit system, the voltages, currents, powers, impedances, and other electrical quantities are not measured in their usual SI units (volts, amperes, watts, ohms, etc.). Instead, *each electrical quantity is measured as a decimal fraction* of some base level. Any quantity can be expressed on a per-unit basis by the equation where "actual value" is a value in volts, amperes, ohms, etc.

$$Quantity\ per\ unit = \frac{Actual\ value}{Base\ value\ of\ quantity}$$

There is another approach to solving circuits containing transformers which eliminates the need for explicit voltage-level conversions at every transformer in the system. Instead, the required conversions are handled automatically by the method itself, without ever requiring the user to worry about impedance transformations. Because such impedance transformations can be avoided, circuits containing many transformers can be solved easily with less chance of error. This method of calculation is known as the *per-unit (pu) system* of measurements.

There is yet another advantage to the per-unit system that is quite significant for electric machinery and transformers. *As* the size of a machine or transformer varies, its internal impedances vary widely. However, it turns out that in a per-unit system related to the device's ratings, *machine and transformer impedances fall within fairly narrow ranges* for each type and construction of device. This fact can serve as a useful check in problem solutions. Information regarding the impedance of transformer windings is generally available from the manufacturer or from the transformer nameplate as per-unit or percent impedance.

It is customary to select two base quantities to define a given per-unit system. The ones usually selected are voltage and apparent power. Once these base quantities have been selected, all the other base values are related to them by the usual electrical laws. In a single-phase system, these relationships are

$$S_{base} = V_{base}\ I_{base}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}$$

In a power system, a base values for apparent power and voltage are selected at a specific point in the system. A transformer has no effect on the base apparent power of the system, since the apparent power into a transformer equals the apparent power out of the transformer. On the other hand, voltage changes when it goes through a transformer, so the value of V_{base} changes at every transformer in the system according to its turns ratio. Because the *base quantities* change in passing through a transformer, the process of referring quantities to a common voltage level is automatically taken care of during per-unit conversion.

For a simple system with a single transformer, the rated apparent power and rated voltages are taken as base values.

$$S_{base} = S_{rated}$$

$$V_{base,LS} = V_{rated,LS} \text{ and } V_{base,HS} = V_{rated,HS}$$

$$Z_{base,LS} = \frac{V^2_{rated,LS}}{S_{rated}} \text{ and } Z_{base,HS} = \frac{V^2_{rated,HS}}{S_{rated}}$$

$$\left(Z_{pu}\right)_{LS} = \frac{Z_{eq,LS}}{Z_{base,LS}}$$

$$\left(Z_{pu}\right)_{HS} = \frac{Z_{eq,HS}}{Z_{base,HS}} = \frac{a^2 Z_{eq,LS}}{V^2_{rated,HS}/S_{rated}} = \frac{a^2 Z_{eq,LS}}{\left(aV_{rated,LS}\right)^2/S_{rated}} = \frac{Z_{eq,LS}}{Z_{base,LS}}$$

$$\therefore \left(Z_{pu}\right)_{LS} = \left(Z_{pu}\right)_{HS}$$

Thus per-unit impedance will be same whether calculated using all high-side or all low-side values.

If required, base currents can be found using the base voltages and base apparent power.

$$I_{base,HS} = \frac{S_{rated}}{V_{rated,HS}} = I_{rated,HS} \text{ and } I_{base,LS} = \frac{S_{rated}}{V_{rated,LS}} = I_{rated,LS}$$

it is to be noted that base impedance serves as the base for both resistance and reactance.

$$R_{pu} = \frac{R_{eq}}{Z_{base}} = \frac{I_{rated}\ R_{eq}}{V_{rated}}$$

$$X_{pu} = \frac{X_{eq}}{Z_{base}} = \frac{I_{rated}\ X_{eq}}{V_{rated}}$$

Here, $I_{rated}, V_{rated}, R_{eq}, X_{eq}$ must be all high-side or all low-side values.

The per-unit impedance in terms of its components, $\boldsymbol{Z}_{pu} = R_{pu} + jX_{pu}$

Transformers rated above 100 kVA have conductors of such large cross-sectional area that $X_{pu} >> R_{pu}$

Thus for very large transformers, $\boldsymbol{Z}_{pu} \approx jX_{pu}$

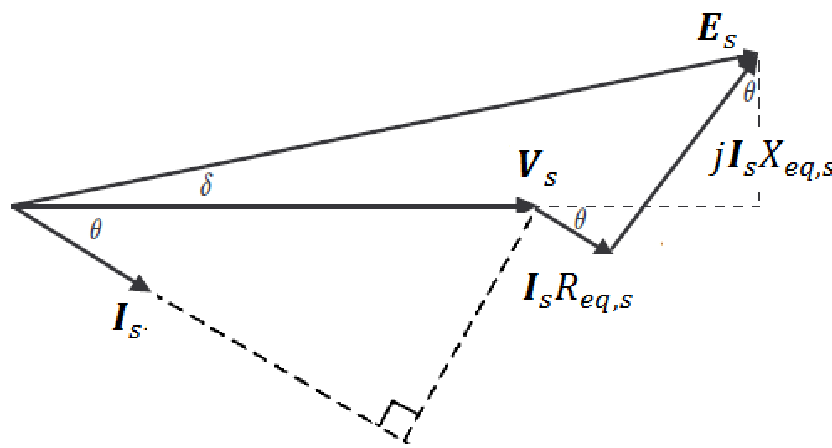
If percent values are given, per-unit values can be found simply dividing by 100.

$$Z_{pu} = \frac{\%Z}{100}$$

Calculating regulation from per-unit values

From phasor diagram, $\boldsymbol{E}_s = \boldsymbol{V}_s + \boldsymbol{I}_s R_{eq,s} + j\boldsymbol{I}_s X_{eq,s}$

$\boldsymbol{V}_s$ can be decomposed into components along $\boldsymbol{I}_s$ and in the perpendicular direction of $\boldsymbol{I}_s$.



$$E_s = \sqrt{\left(I_s R_{eq,s} + V_s cos\theta\right)^2 + \left(I_s X_{eq,s} + V_s sin\theta\right)^2}$$

$$reg = \frac{V_{nl} - V_{fl}}{V_{fl}} = \frac{E_s - V_s}{V_s}$$

$$reg = \frac{\sqrt{\left(I_s R_{eq,s} + V_s cos\theta\right)^2 + \left(I_s X_{eq,s} + V_s sin\theta\right)^2} - V_s}{V_s}$$

$$reg = \sqrt{\left(\frac{I_s R_{eq,s}}{V_s} + cos\theta\right)^2 + \left(\frac{I_s X_{eq,s}}{V_s} + sin\theta\right)^2} - 1$$

$$reg = \sqrt{\left(\frac{I_s}{I_{rated,s}}\frac{I_{rated,s} R_{eq,s}}{V_s} + cos\theta\right)^2 + \left(\frac{I_s}{I_{rated,s}}\frac{I_{rated,s} X_{eq,s}}{V_s} + sin\theta\right)^2} - 1$$

$$reg = \sqrt{\left(\frac{I_s}{I_{rated,s}}\frac{I_{rated,s} R_{eq,s}}{V_{rated,s}} + cos\theta\right)^2 + \left(\frac{I_s}{I_{rated,s}}\frac{I_{rated,s} X_{eq,s}}{V_{rated,s}} + sin\theta\right)^2} - 1$$

$$reg = \sqrt{\left(\frac{I_s}{I_{rated,s}}\frac{R_{eq,s}}{Z_{base,s}} + cos\theta\right)^2 + \left(\frac{I_s}{I_{rated,s}}\frac{X_{eq,s}}{Z_{base,s}} + sin\theta\right)^2} - 1$$

$$reg = \sqrt{\left(I_{pu} R_{pu} + cos\theta\right)^2 + \left(I_{pu} X_{pu} + sin\theta\right)^2} - 1$$

$$\boxed{I_{pu} = \frac{I_s}{I_{rated,s}} = \frac{I_s V_{rated,s}}{I_{rated,s} V_{rated,s}} = \frac{S}{S_{rated}} = S_{pu}}$$

$$reg = \sqrt{\left(S_{pu} R_{pu} + cos\theta\right)^2 + \left(S_{pu} X_{pu} + sin\theta\right)^2} - 1$$

Here, $\theta = \cos^{-1} pf$ for lagging power-factor loads and $\theta = -\cos^{-1} pf$ for leading power-factor loads

If the transformer operates at rated load, then S_{pu} =1 and the expression reduces to

$$reg = \sqrt{\left(R_{pu} + cos\theta\right)^2 + \left(X_{pu} + sin\theta\right)^2} - 1$$

Calculating efficiency from per-unit values

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{out}}{P_{out} + P_{core} + P_{Cu}} = \frac{S \times pf}{S \times pf + P_{core} + I_s^2 R_{eq,s}}$$

$$\eta = \frac{(S/S_{rated}) \times pf}{(S/S_{rated}) \times pf + (P_{core}/S_{rated}) + \left(I_s^2 R_{eq,s}/S_{rated}\right)}$$

$$\eta = \frac{(S/S_{rated}) \times pf}{(S/S_{rated}) \times pf + (P_{core}/S_{rated}) + \left(I_s^2 R_{eq,s}/V_{rated} I_{rated}\right)}$$

$$\eta = \frac{(S/S_{rated}) \times pf}{(S/S_{rated}) \times pf + (P_{core}/S_{rated}) + \left(I_s^2/{I_{rated}}^2\right) \times \left(I_{rated} R_{eq,s}/V_{rated}\right)}$$

$$\eta = \frac{S_{pu} \times pf}{S_{pu} \times pf + P_{core,pu} + I_{pu}^2 \times R_{pu}}$$

$$\boxed{I_{pu} = \frac{I_s}{I_{rated,s}} = \frac{I_s V_{rated,s}}{I_{rated,s} V_{rated,s}} = \frac{S}{S_{rated}} = S_{pu}}$$

$$\eta = \frac{S_{pu} \times pf}{S_{pu} \times pf + P_{core,pu} + S_{pu}^2 \times R_{pu}}$$

If the transformer operates at rated load, then $S_{pu} = 1$ and the expression reduces to

$$\eta = \frac{pf}{pf + P_{core,pu} + R_{pu}}$$

Conversion of per-unit values from one base to another base

Nameplate impedances are given in per-unit values respective to their own base. However, analyzing a power system with numerous motors, generators and transformers requires to select a base apparent power and a base voltage at a specific point of the system. If the chosen base is different from the actual rating, then per-unit values to be converted into the new base.

$$Z_{pu,new} = \frac{Z_{eq}}{Z_{base,new}}$$

$$Z_{pu,old} = \frac{Z_{eq}}{Z_{base,old}}$$

$$\frac{Z_{pu,new}}{Z_{pu,old}} = \frac{Z_{base,old}}{Z_{base,new}}$$

$$\frac{Z_{pu,new}}{Z_{pu,old}} = \frac{V_{old}^2/S_{old}}{V_{new}^2/S_{new}}$$

$$Z_{pu,new} = \frac{S_{new}}{S_{old}} \times \frac{V_{old}^2}{V_{new}^2} \times Z_{pu,old}$$

Parallel Operation of Transformers:

https://electricalnotes.wordpress.com/2012/07/17/parallel-operation-of-transformers/

For supplying a load in excess of the rating of an existing transformer, two or more transformers may be connected in parallel with the existing transformer. It is usually economical to install another transformer in parallel instead of replacing the existing transformer by a single larger unit. In addition, at least half the load can be supplied with one transformer out of service and total power outage can be avoided.

To maximize electrical power system flexibility: There is always a chance of increasing or decreasing future demand of power system. If it is predicted that power demand will be increased in future, there must be a provision of connecting transformers in system in parallel to fulfill the extra demand because, it is not economical from business point of view to install a bigger rated single transformer by forecasting the increased future demand as it is unnecessary investment of money. Again if future demand is decreased, transformers running in parallel can be removed from system to balance the capital investment and its return.

To maximize electrical power system availability: If numbers of transformers run in parallel, we can shutdown any one of them for maintenance purpose. Other parallel transformers in system will serve the load without total interruption of power.

To maximize power system reliability: if any one of the parallel transformers is tripped due to fault, other transformers will share the load. Hence power supply may not be interrupted if the shared loads do not make other transformers over loaded.

To maximize electrical power system efficiency: Generally electrical power transformer gives the maximum efficiency at full load. If we run numbers of transformers in parallel, we can switch on only those transformers which will give the total demand by running nearer to its full load rating for that time. When load increases, we can switch none by one other transformer connected in parallel to fulfill the total demand. In this way we can run the system with maximum efficiency.

Conditions for Parallel Operation of Transformers

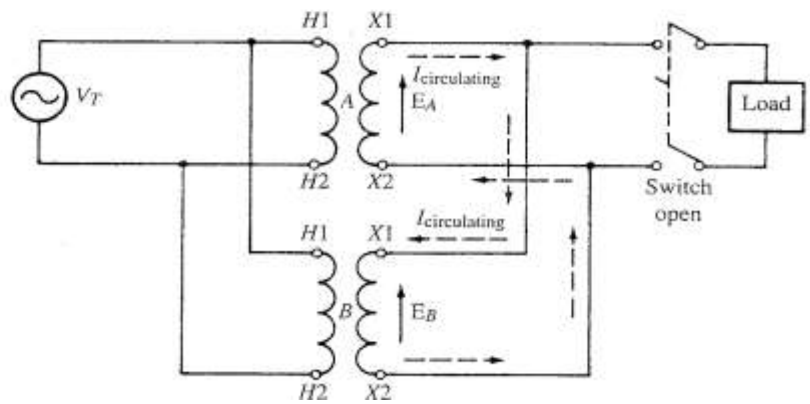
When two or more transformers run in parallel, they must satisfy the following conditions for satisfactory performance. These are the conditions for parallel operation of transformers.

1. Same voltage ratio of transformer
2. Same percentage impedance
3. Same polarity
4. Same phase sequence (for 3-phase transformers)

Unmatched impedances result in unequal load sharing between the paralleled transformers, whereas huge circulating current flows if the other conditions are not maintained.

Effect of unequal voltage ratio- Circulating Current:

If two transformers of different voltage ratio are connected in parallel with same primary supply voltage, there will be a difference in secondary voltages. Now say the secondary of these transformers are connected to same bus, there will be a circulating current between secondaries. As the internal impedance of transformer is small, a small voltage difference may cause sufficiently high circulating current causing unnecessary extra I^2R loss.



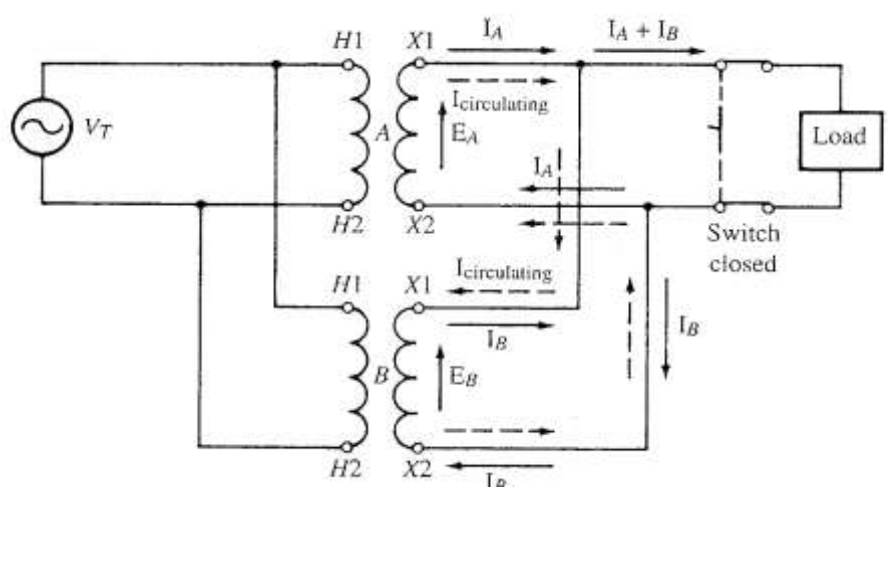
The circulating flows in the closed loop formed by the two secondaries even when no-load is connected.

Considering transformer A has a higher voltage ratio compared to transformer B, the circulating current can be expressed as:

$$\boldsymbol{I_{cir}} = \frac{\boldsymbol{E_A} - \boldsymbol{E_B}}{\boldsymbol{Z_A} + \boldsymbol{Z_B}}$$

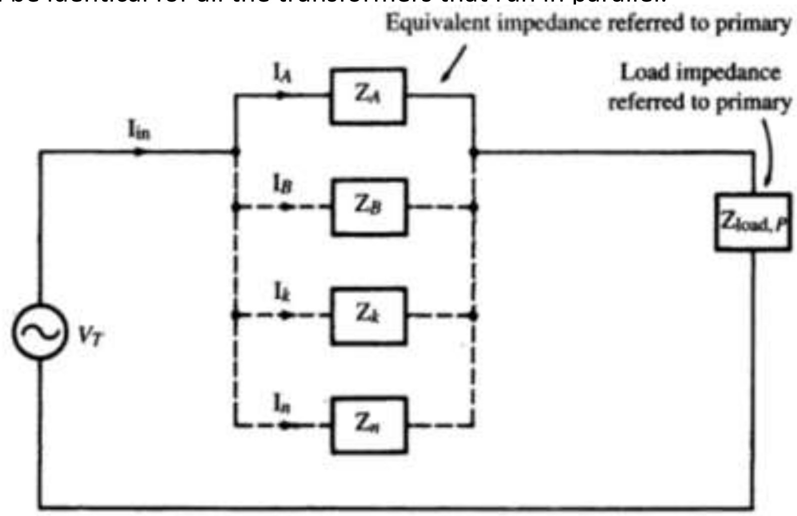
where Z_A and Z_B are the equivalent impedance of both transformers respective to their secondaries.

When the load switch is closed, the circulating current adds to the load current in one transformer and subtracts from the load current in the other transformer. Thus, if the transformer is operating at rated load, the transformer with the higher secondary voltage will be overloaded, and the other transformer will be underloaded. Overloading has adverse effects on transformer service life, because resultant overheating damages the winding insulation.



Effect of unmatched impedances – unequal load distribution

The current shared by two transformers running in parallel should be proportional to their MVA ratings. Again, current carried by these transformers are inversely proportional to their internal impedance. From these two statements it can be said that, impedance of transformers running in parallel are inversely proportional to their MVA ratings. In other words, percentage impedance or per unit values of impedance should be identical for all the transformers that run in parallel.



Equivalent circuit for paralleled transformers.

If all the transformers have the same turns ratio,, they may be represented by paralleled impedances. According to current-divider rule, current through the k-th transformer can be expressed as:

$$\boldsymbol{I_k} = \frac{1/\boldsymbol{Z_k}}{1/\boldsymbol{Z_1} + 1/\boldsymbol{Z_2} + \cdots + 1/\boldsymbol{Z_k} + \cdots + 1/\boldsymbol{Z_n}} \boldsymbol{I_{bank}} = \frac{\boldsymbol{Y_k}}{\boldsymbol{Y_p}} \boldsymbol{I_{bank}}$$

where $\boldsymbol{Y_k}$ is the equivalent impedance of transformer k and $\boldsymbol{Y_p}$ is the equivalent impedance of all paralleled transformers.

If all the transformers have identical impedances (both in magnitude and angle), current will be equally divided among all the paralleled transformers i.e. load will be shared equally. If the transformer impedances differ either in magnitude or phase (or both), loads will be unequally shared among them. Due to this, the transformer with the lower impedance may get overheated at rated load.

Note that, if the transformer impedances are given in percent or per-unit and they have the same base impedance (identical voltage rating and KVA rating), percent or per-unit values may be used in place of the equivalent ohmic impedance to calculate the current drawn by each transformer.

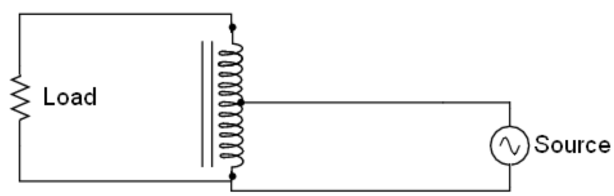
For two transformers connected in parallel, load shared by each transformer can be found from

$$\frac{S_1}{S_2} = \frac{Z_{eq,2}}{Z_{eq,1}}$$

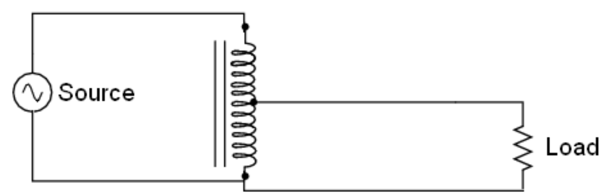
Autotransformer

An autotransformer is a single coil transformer with one or more taps to provide transformer action. Unlike conventional two-winding transformers, there is a direct physical connection between the primary and the secondary circuits. Therefore autotransformers should only be used in applications where lack of electrical isolation does not present a safety hazard.

It is common practice in power systems to use autotransformers whenever two voltages fairly close to each other in level need to be transformed, In those applications, where continuous non-interruptible adjustment of voltage is required, autotransformers are used. They are also used as variable transformers, where the low-voltage tap moves up and down the entire winding.



Autotransformer *(As a Step-Up Transformer)*



Autotransformer *(As a Step-Down Transformer)*

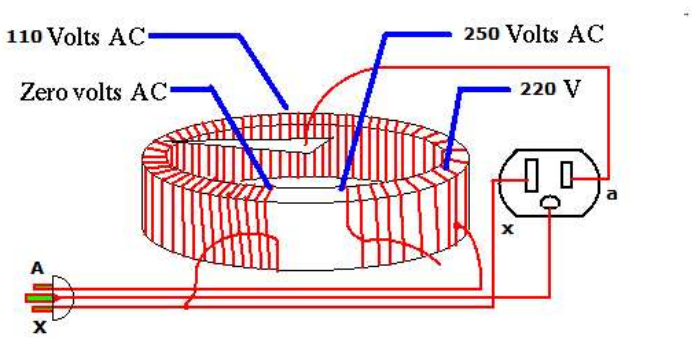
Comparison of autotransformers and conventional transformers

Advantages: Autotransformers provide a great deal of advantages compared to their two winding counterparts due to their single-coil construction.

- less leakage flux,
- less copper, less iron,
- less weight,
- takes up less space
- more efficient
- cost less

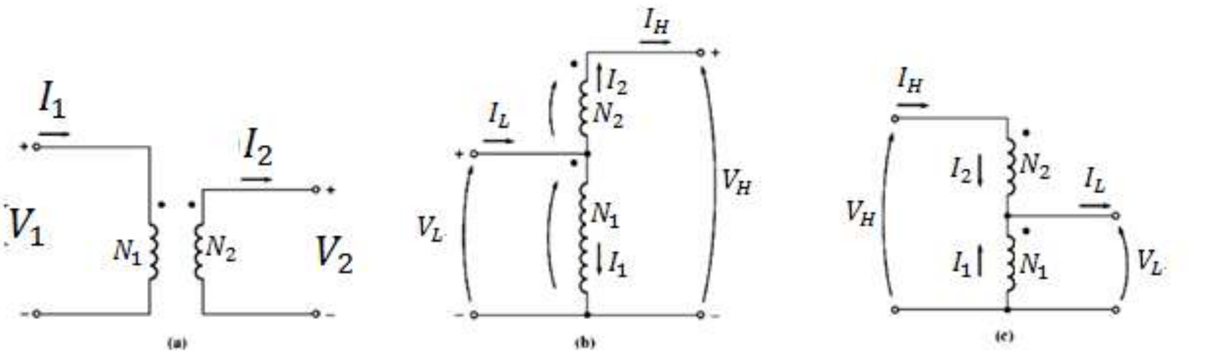
Disadvantages: No electrical isolation between the primary and the secondary.

Variac: Variacs, which are AC power supplies, are the most popular variable auto transformers on the market because they are cheaper, lighter, and smaller than dual-winding transformers. These adjustable power supplies can be used for a number of hobbies and are very versatile.



Input between A-X, output between a-x

The Apparent Power Rating Advantage of Autotransformers



A transformer with its windings connected in the conventional manner

Primary and secondary windings reconnected as a step-up autotransformer

Primary and secondary windings reconnected as a step-down autotransformer

Apparent power rating as a 2-winding transformer,

$$S_{2w} = V_1 I_1 = V_2 I_2$$

Considering the transformer as ideal $\quad \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a$

Apparent power rating as an autotransformer,

$$S_{at,HS} = V_H I_H = (V_1 + V_2) I_2 = \left(\frac{V_1}{V_2} + 1\right) V_2 I_2 = (a+1) S_{2w}$$

$$S_{at,LS} = V_L I_L = V_1 (I_1 + I_2) = V_1 I_1 \left(1 + \frac{I_2}{I_1}\right) = (a+1) S_{2w}$$

Apparent power rating advantage of an autotransformer over a conventional two-winding transformer,

$$\frac{S_{at}}{S_{2w}} = (a+1)$$

Note: If the primary *current* of the transformer flows *into* the dotted end of the primary winding, the secondary current will flow *out* of the dotted end of the secondary winding.

Internal Impedance of an Autotransformer

$$Z_{eq} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1}$$

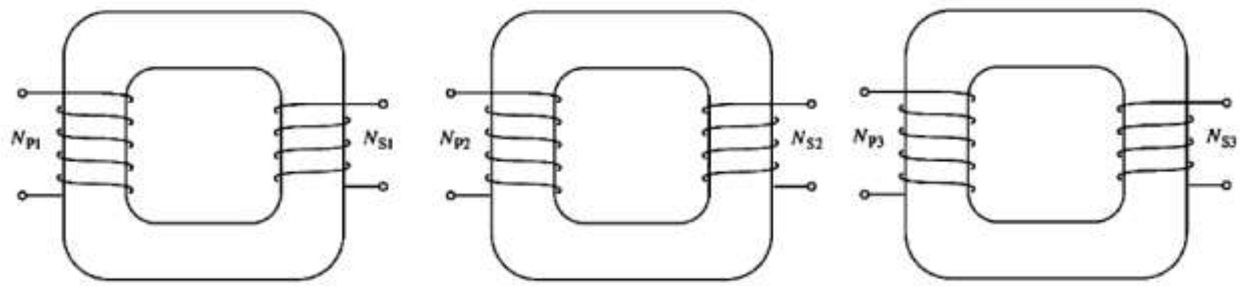
$$Z_{eq}^{'} = \frac{\boldsymbol{V}_L}{\boldsymbol{I}_L} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1 + \boldsymbol{I}_2} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1} \frac{1}{1 + \boldsymbol{I}_2/\boldsymbol{I}_1} = \frac{1}{a+1} Z_{eq}$$

Effective impedance of an autotransformer is smaller by a factor equal to the reciprocal of the power advantage of the autotransformer connection.

Three-phase Transformers:

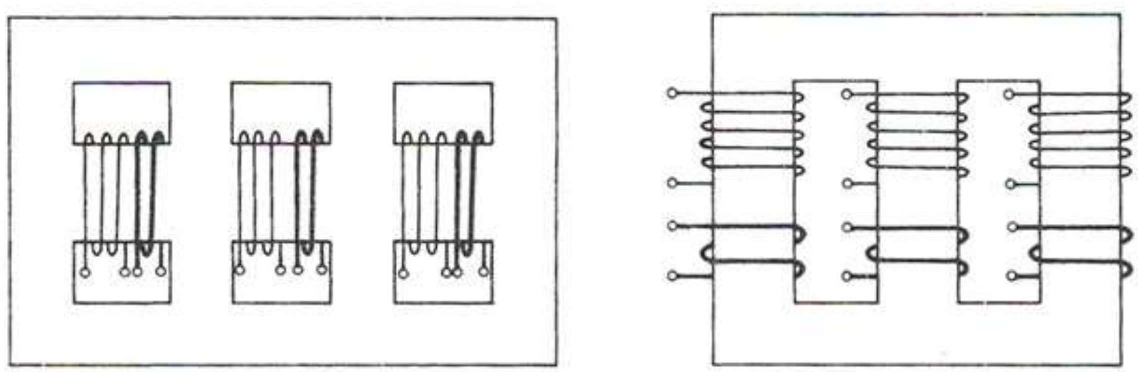
Almost all the major power generation and distribution systems in the world today are three-phase ac systems. Since three-phase systems play such an important role in modern life, it is necessary to understand how transformers are used in them. Transformers for three-phase circuits can be constructed in one of two ways.

One approach is simply to take three single-phase transformers and connect them in a three-phase bank.



A three-phase transformer bank composed of independent transformers.

An alternative approach is to make a three-phase transformer consisting of three sets of windings wrapped on a common core. Two possible types of transformer construction are shown below.



(a) (b)

Basic construction of three-phase transformers: (a) shell type; (b) core type

The construction of a single three-phase transformer is the preferred practice today, since it is lighter, smaller, cheaper, and slightly more efficient. Furthermore, since all three phases are in one tank, the wye or delta connections can be made internally, reducing the number of external high-voltage connections from six to three.

The older construction approach was to use three separate transformers. That approach had the advantage that each unit in the bank could be replaced individually in the event of trouble, but that does not outweigh the advantages of a combined three phase unit for most applications. However, there are still a great many installations consisting of three single-phase units in service.

Initial cost, cost of operation, cost of spares, cost of repairs, cost of downtime, space requirement, need for uninterrupted operation in case of fault occurrence - all these are to be considered to make a choice between these two.

The Per-Unit System for Three-Phase Transformers

The per-unit system of measurements applies just as well to three-phase transformers as to single-phase transformers.

If the total base voltampere value of the transformer bank is called S_{base}, then the base voltampere value of one of the transformers $S_{1\emptyset,base}$ is given by

$$S_{1\emptyset,base} = \frac{S_{base}}{3}$$

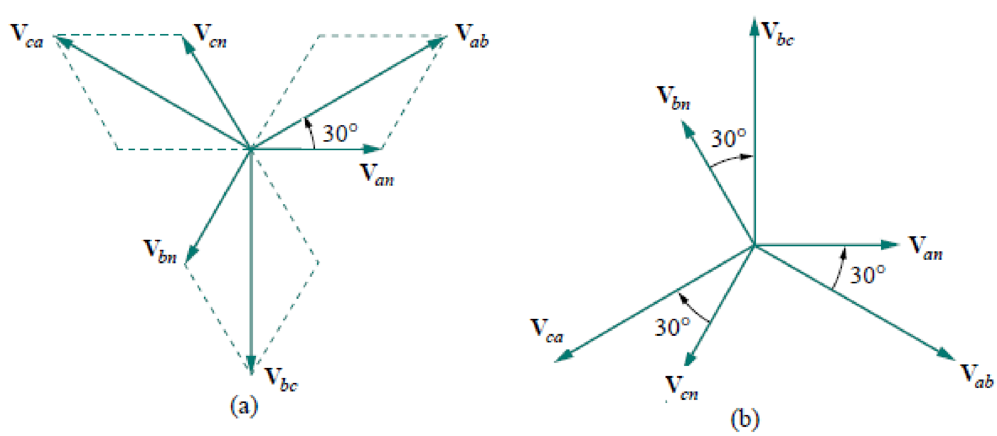
The base phase current and impedance of the transformer are expressed as:

$$I_{\emptyset,base} = \frac{S_{1\emptyset,base}}{V_{\emptyset,base}} = \frac{S_{base}}{3\,V_{\emptyset,base}}$$

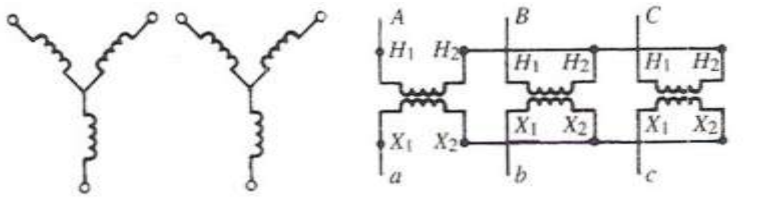
$$Z_{base} = \frac{\left(V_{\emptyset,base}\right)^2}{S_{1\emptyset,base}} = \frac{3\left(V_{\emptyset,base}\right)^2}{S_{base}}$$

Paralleling three-phase transformer banks:

In a Y-connected system, the line voltage leads corresponding phase voltage by 30° for a-b-c sequence**,** and lags by 30° for a-c-b sequence. Due to this, there is a phase shift between primary and secondary line voltages in case of Y-Δ and Δ-Y banks. In the United States, it is customary to make the secondary voltage lag the primary voltage by 30°. Although this is the standard, it has not always been observed, and older installations must be checked very carefully before a new transformer is paralleled with them, to make sure that their phase angles match.

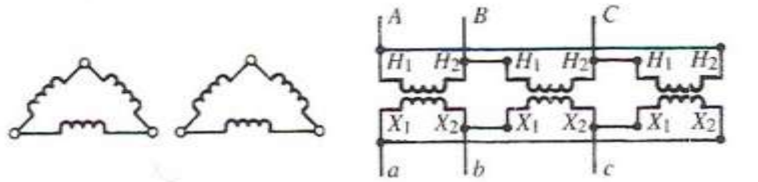


Phase relationship between phase voltage and line voltage in a balanced Y connected network for (a) a-b-c sequence and (b) a-c-b sequence



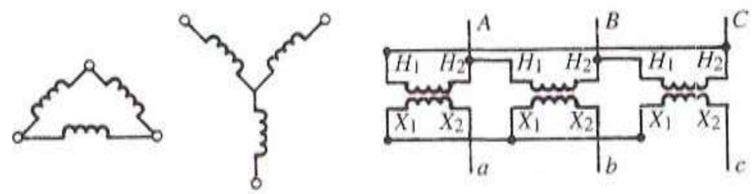
For Y-Y bank,

$$\boldsymbol{V_{an}} = \frac{1}{a}\boldsymbol{V_{AN}} \ ; \ \boldsymbol{V_{ab}} = \frac{1}{a}\boldsymbol{V_{AB}}$$



For Δ-Δ bank,

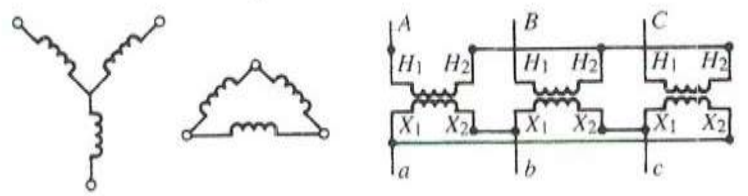
$$\boldsymbol{V_{ab}} = \frac{1}{a}\boldsymbol{V_{AB}}$$



For Δ-Y bank,

$$\boldsymbol{V_{an}} = \frac{1}{\sqrt{3}a}\boldsymbol{V_{AB}}$$

$$\boldsymbol{V_{ab}} = \sqrt{3}\boldsymbol{V_{an}}\angle 30° = \frac{1}{a}\boldsymbol{V_{AB}}\angle 30°$$



For Y-Δ bank,

$$\boldsymbol{V_{AB}} = \sqrt{3}\boldsymbol{V_{AN}}\angle 30°$$

$$\boldsymbol{V_{ab}} = \frac{\sqrt{3}}{a}\boldsymbol{V_{AN}} = \frac{1}{a}\boldsymbol{V_{AB}}\angle - 30°$$

There is an angular displacement, called phase shift, between the corresponding primary and secondary line voltages in the Y-Δ bank and also in the Δ-Y bank. There is no phase shift between corresponding primary and secondary line voltages in a Y-Y, Δ-Δ or V-V bank. Because of the phase shift inherent in Y-Δ and Δ-Y banks, they must not be paralleled with Y-Y, Δ-Δ or V-V banks. Doing so would cause large circulating currents and severe overheating of the windings.

The Y-Y connection has two very serious problems and this is why, seldom used.

- If loads on the transformer secondaries are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.
- Third-harmonic voltages can be very large, even larger than the fundamental voltage.

The Y-Δ and Δ-Y connection has no problem with third-harmonic components in its voltages, since they are consumed in a circulating current on the Δ side. This connection is also more stable with respect to unbalanced loads, since the Δ partially redistributes any imbalance that occurs. A phase shift of 30° between the primary and secondary line voltages must be taken into consideration before paralleling. The Δ-Δ connection has no phase shift associated with it and no problems with unbalanced loads or harmonics. And this bank will continue to operate if one of the phases is removed for maintenance.

The simple rule which normally dominates the selection of a three-phase transformer is the network where it is to be installed. The preferable connection for the unit transformers are YNd i.e low voltage side (generator side) is connected in Delta and high voltage side (transmission side) is connected as Wye)

From transmission network to the distribution network, the preferable connection is Dyn. (transmission side in Delta and Distribution Side in Wye). Even in the distribution networks, the transformers are Dyn. But here besides the above benefits, it has an extra advantage that power can be supplied to both three-phase and single phase loads by the distributors.

Three-phase Transformation using two Transformers

In addition to the standard three-phase transformer connections, there are ways to perform three-phase transformation with only two transformers. One such technique is ***open-delta connection*** or ***V-V connection***. It provides a convenient means for inspection, maintenance, testing and replacing of transformers one at a time, with only a brief power interruption. Transformers selected for delta-delta or open-delta connection must have the same turns ratio and same per unit impedance in order to share the load equally.

Suppose a transformer bank composed of separate transformers has a damaged phase that must be removed for repair. Open delta bank will still remain able to operate successfully, although at a lower capacity.

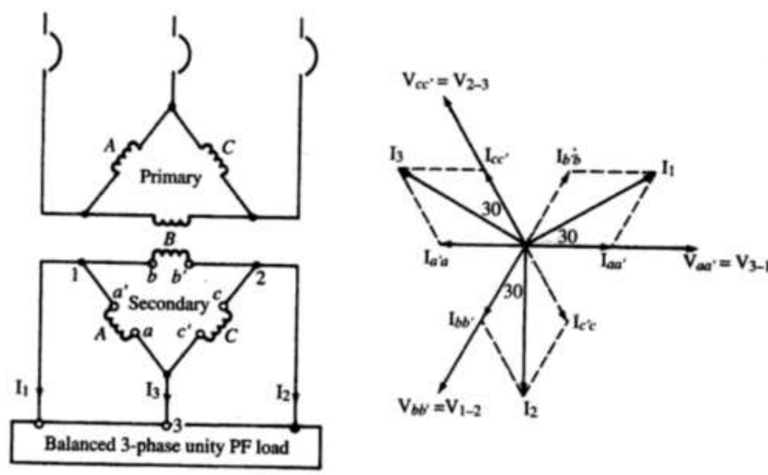
The open-delta connection is also used to provide three-phase service in applications where a future increase in load is expected. The increase may be accommodated by adding the third transformer to the bank at a later date.

Considering balanced three-phase, the phase voltages will be same in magnitude and $120°$ angle apart in phase.
Considering a-b-c phase sequence

$$\boldsymbol{V_{3-1}} = \boldsymbol{V_{aa'}} = V_p\angle 0°$$

$$\boldsymbol{V_{1-2}} = \boldsymbol{V_{bb'}} = V_p\angle -120°$$

$$\boldsymbol{V_{2-3}} = \boldsymbol{V_{cc'}} = V_p\angle{-240°} = V_p\angle 120°$$



Delta-Delta bank and corresponding phasor diagram

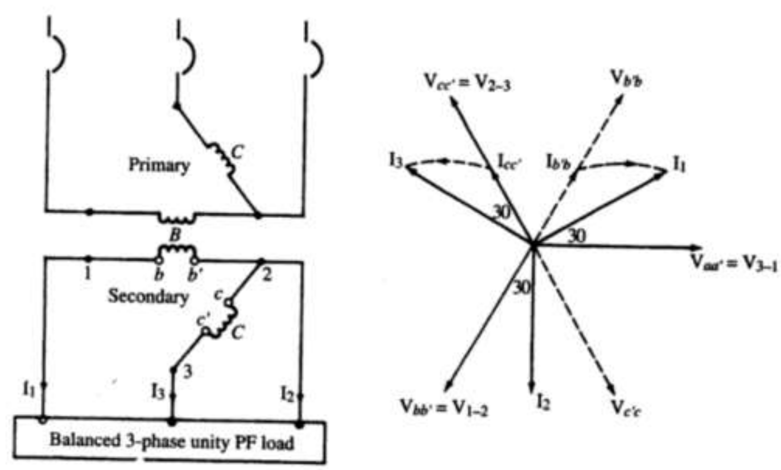
Now if transformer A is removed,

$$\boldsymbol{V_{1-3}} = \boldsymbol{V_{1-2}} + \boldsymbol{V_{2-3}}$$

$$\boldsymbol{V_{1-3}} = V_p\angle -120° + V_p\angle 120°$$

$$\boldsymbol{V_{1-3}} = V_p\angle 180°$$

$$\therefore \boldsymbol{V_{3-1}} = V_p\angle 0°$$

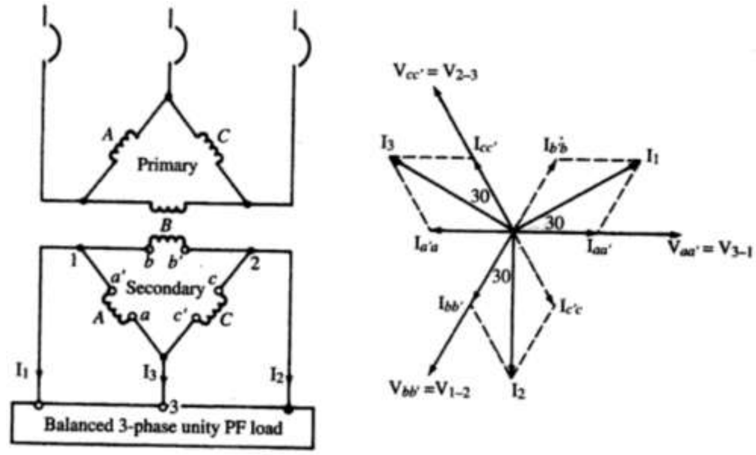


V-V bank and corresponding phasor diagram

This is exactly the same voltage that would be present if the third transformer were still there. Therefore the removed phase (phase A in this case) is termed as ***ghost phase***.

For $\Delta - \Delta$ operation, the line currents can be expressed in terms of phase currents.

$$\boldsymbol{I_1 = I_{aa'} + I_{b'b}}$$
$$\boldsymbol{I_2 = I_{bb'} + I_{c'c}}$$
$$\boldsymbol{I_3 = I_{cc'} + I_{a'a}}$$

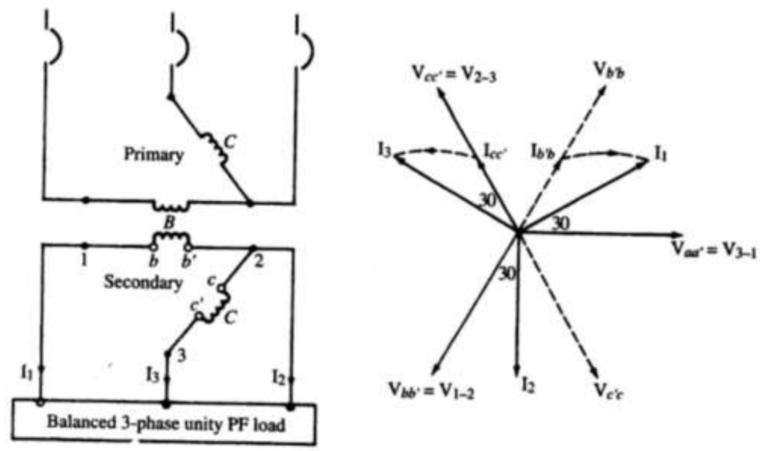


Delta-Delta bank and corresponding phasor diagram

Since the three secondary line voltages in V-V operation remain same as in $\Delta - \Delta$, and the load impedance has not changed, line currents must remain same as well. To keep the line currents unchanged, the coil currents in the remaining two transformers increase in magnitude by $\sqrt{3}$ and also shift in phase by 30° to match the corresponding line currents. This may lead to overheating of transformer windings. Therefore bank current and bank apparent power must be rerated.

For $\Delta - \Delta$ operation, the line currents can be expressed in terms of phase currents.

$$\boldsymbol{I_1 = I_{b'b}}$$
$$\boldsymbol{I_2 = I_{bb'} + I_{c'c}}$$
$$\boldsymbol{I_3 = I_{cc'}}$$



V-V bank and corresponding phasor diagram

For V-V operation,

$$P_B = V_{bb'} I_{bb'} \cos(30°) = \frac{\sqrt{3}}{2} V_\varphi I_\varphi \qquad P_C = V_{cc'} I_{cc'} \cos(-30°) = \frac{\sqrt{3}}{2} V_\varphi I_\varphi$$
$$Q_B = V_{bb'} I_{bb'} \sin(30°) = \frac{1}{2} V_\varphi I_\varphi \qquad Q_C = V_{cc'} I_{cc'} \sin(-30°) = -\frac{1}{2} V_\varphi I_\varphi$$
$$S_B = \sqrt{(P_B)^2 + (Q_B)^2} = V_\varphi I_\varphi \qquad S_C = \sqrt{(P_C)^2 + (Q_C)^2} = V_\varphi I_\varphi$$

$$P_{T,(V-V)} = P_B + P_c = \sqrt{3}\, V_\varphi I_\varphi$$
$$Q_{T,(V-V)} = Q_B + Q_c = 0$$

$$S_{T,(V-V)} = \sqrt{(P_{T,(V-V)})^2 + (Q_{T,(V-V)})^2} = \sqrt{3}\, V_\varphi I_\varphi$$

In case of $\Delta - \Delta$ operation, the phase voltage and phase currents are in phase.
Therefore, $P_A = P_B = P_C = V_\varphi I_\varphi$ and $Q_A = Q_B = Q_c = 0$. Hence, $S_{T,(\Delta-\Delta)} = P_{(T,\Delta-\Delta)} = 3V_\varphi I_\varphi$

$$S_{V-V,rated} = \frac{S_{\Delta-\Delta,rated}}{\sqrt{3}}$$

Although resistive load has been considered to reduce complicacy in analysis, the relation in terms of apparent power (as written above) holds for loads with any power factor.

Transformer Ratings and Related Problems

Transformers have four major ratings: apparent power, voltage, current, and frequency.

The voltage rating of a transformer serves two functions. One is to protect the winding insulation from breakdown due to an excessive voltage applied to it. The second function is related to the magnetization curve and magnetization current of the transformer.

The principal purpose of the apparent power rating of a transformer is that, together with the voltage rating, it sets the current flow through the transformer windings. The current flow is important because it controls the heating of the transformer coils. Overheating the coils of a transformer drastically shortens the life of its insulation. The actual voltampere rating of a transformer may be more than a single value. In real transformers, there may be a voltampere rating for the transformer by itself, and another (higher) rating for the transformer with forced cooling. The key idea behind the power rating is that the hot-spot temperature in the transformer windings must be limited to protect the life of the transformer.

If a steady state sinusoidal voltage $v(t)$ is applied to the transformer's primary winding, the flux established in the core can be found from Faraday's Law.

$$v(t) = \mathrm{V_m}\ \sin\omega\mathrm{t}$$

$$\emptyset(t) = \frac{1}{N_p}\int v(t)\, dt = \frac{1}{N_p}\int \mathrm{V_m}\ \sin\omega\mathrm{t}\, dt = -\frac{\mathrm{V_m}}{\omega N_p}\cos\omega\mathrm{t}$$

Hence, the maximum core flux depends on both voltage and frequency.

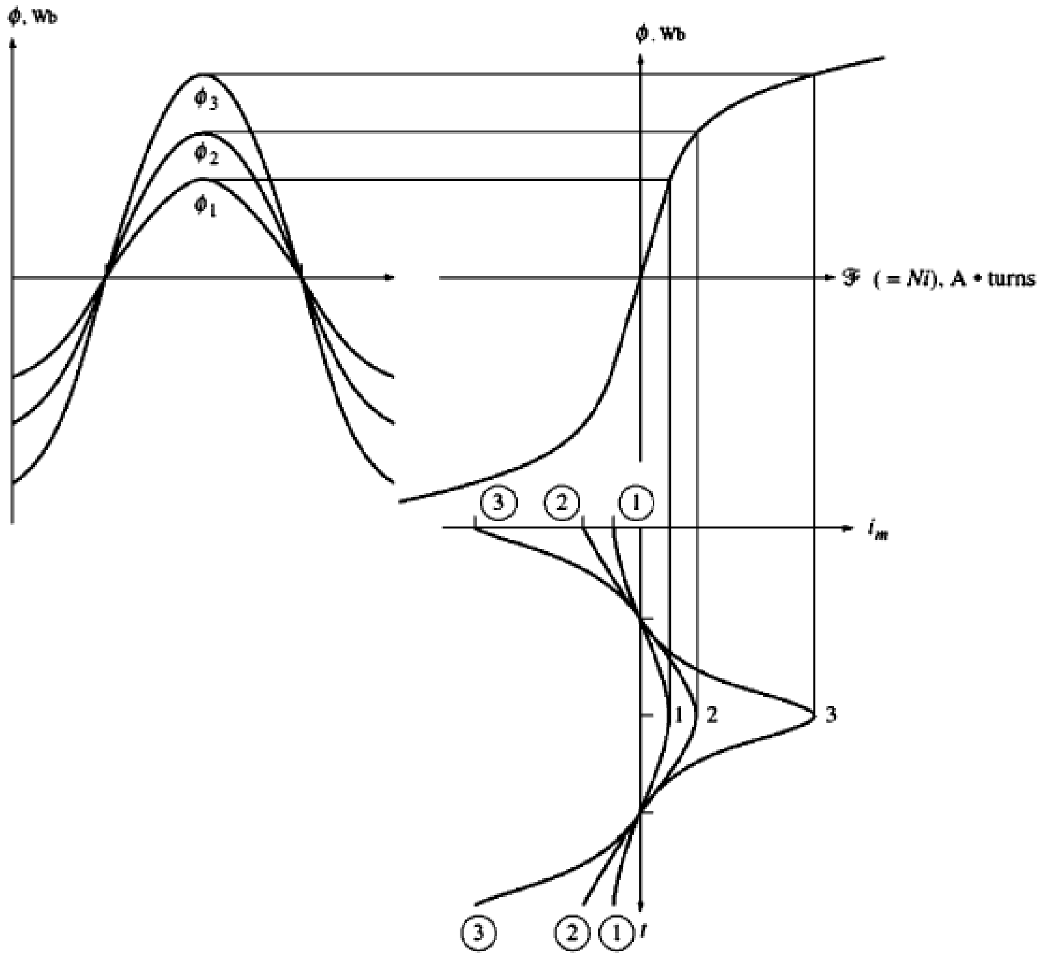
$$\Phi_{\max} = \frac{\mathrm{V_m}}{\omega N_p} = \frac{\mathrm{V_m}}{2\pi f N_p} \Rightarrow \quad \Phi_{\max} \propto \frac{\mathrm{V_m}}{f}$$

We know, magnetization curve for ferromagnetic material shows saturation when all of the magnetic domains of the material are oriented in the direction of the applied magnetomotive force. Saturation begins at the start of the knee region and is essentially complete when the curve starts to flatten. Transformers and AC machines are operated in the linear region and lower end of the knee.

If the applied voltage *v(t)* is increased, the resulting maximum flux in the core will also increase. However, a small increase in flux will require an even greater increase in magnetization current *due to saturation*. As the voltage increases, the high-magnetization currents soon become unacceptable due to overheating of the primary coils. The maximum applied voltage (and therefore the rated voltage) is set by the maximum acceptable magnetization current in the core.

Thus, if a 60-Hz transformer is to be operated on 50 Hz, its applied voltage must also be reduced by one-sixth or the peak flux in the core will be too high. This reduction in applied voltage with frequency is called **derating**. Similarly, a 50-Hz transformer may be operated at a 20 percent higher voltage on 60 Hz if this action does not cause insulation problems. Mathematically,

$$\frac{V_{new}}{V_{old}} = \frac{f_{new}}{f_{old}}$$



The effect of the peak flux in a transformer core upon the required magnetization current.

If a transformer's voltage is reduced for any reason (e.g., if it is operated at a lower frequency than normal), then the transformer's voltampere rating must be reduced by an equal amount. If this is not done, then the current in the transformer's windings will exceed the maximum permissible level and cause overheating. So while operating a 60 Hz transformer on a 50 Hz system, its kVA rating should be adjusted according to the following equation.

$$\frac{S_{new}}{S_{old}} = \frac{V_{new}}{V_{old}} \times \frac{I_{new}}{i_{old}} = \frac{f_{new}}{f_{old}} \times 1 = \frac{f_{new}}{f_{old}}$$

Harmonics in Transformer Exciting Current

The nonlinear characteristics of ferromagnetic cores used in transformers cause the magnetizing current to be nonsinusoidal even though the mutual flux is sinusoidal.

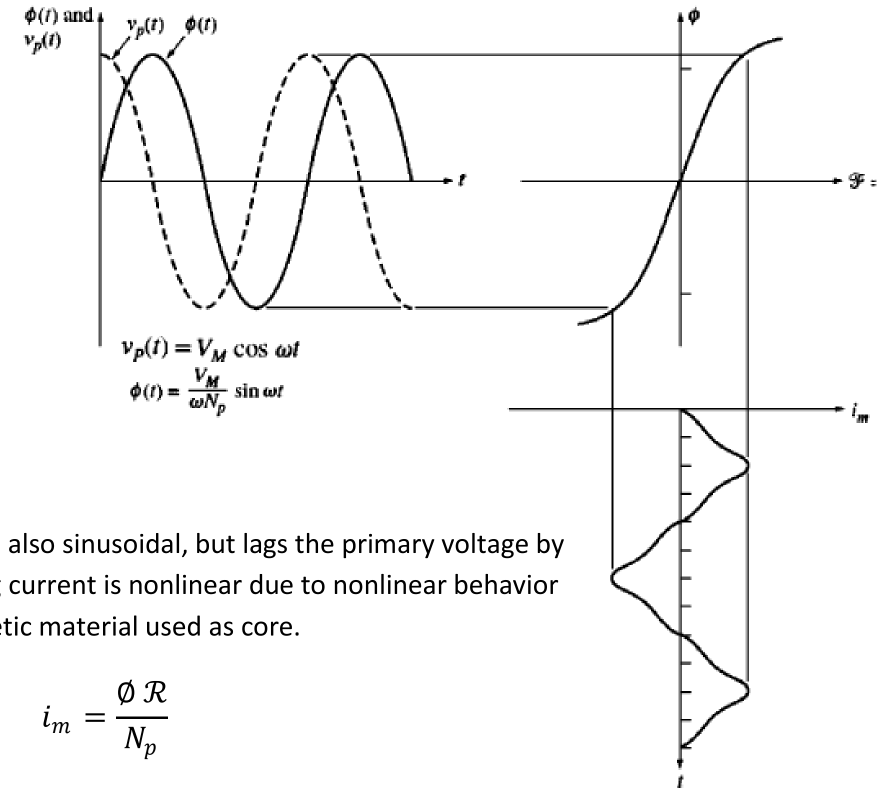
If the primary voltage is given by,

$$v_p(t) = \mathrm{V_M}\ \cos\omega t$$

The resulting flux is given by

$$\emptyset(t) = \frac{1}{N_p}\int v_p(t)\ dt$$

$$= \frac{\mathrm{V_m}}{\omega N_p}\sin\omega t$$

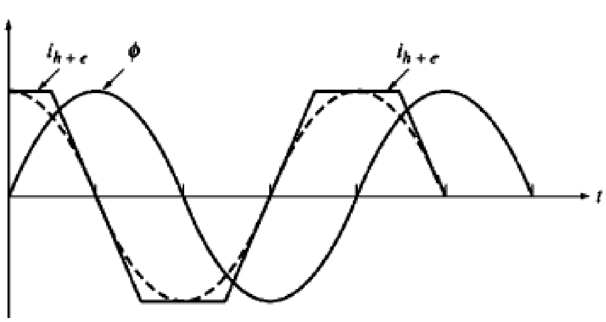


This means the resulting flux is also sinusoidal, but lags the primary voltage by 90°. However, the magnetizing current is nonlinear due to nonlinear behavior (reluctance) of the ferromagnetic material used as core.

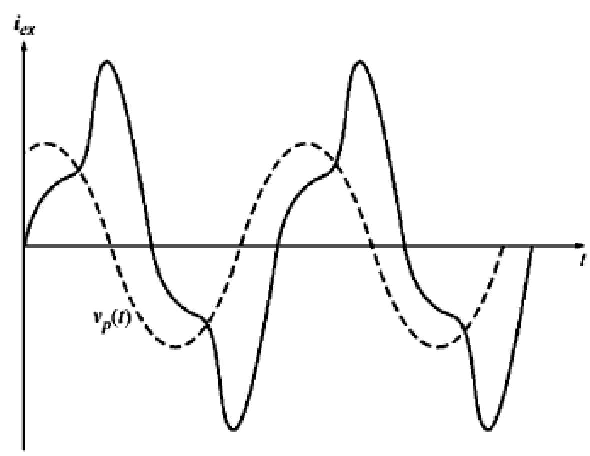
$$i_m = \frac{\emptyset\ \mathcal{R}}{N_p}$$

The nonsinusoidal magnetizing current caused by the nonlinear characteristics of ferromagnetic core

Core-loss current is nonlinear because of the nonlinear effects of hysteresis and its fundamental component is in-phase with the voltage applied to the core. Excitation current (no-load current) is just the sum of magnetization current and the core-loss current. Magnetizing current is much higher than the core-loss current; therefore, for all practical purposes, exciting current and magnetizing current may be used interchangeably.



The core-loss current in a transformer.



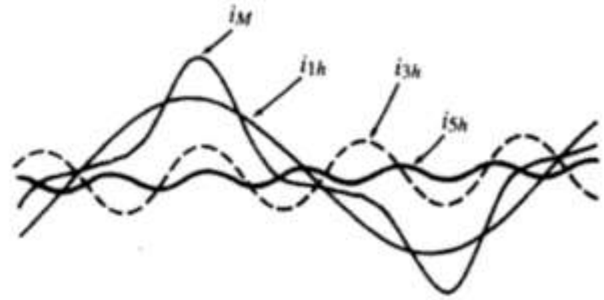
The total excitation current in a transformer.

Any periodic non-sinusoidal wave can be expressed as a sum of weighted and shifted sinusoids according to Fourier series.

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

For a wave with zero average value, $a_0 = 0$ For a half-wave odd symmetric wave, only odd harmonics are present.

The figure shows fundamental and first three harmonics present in the magnetizing current. The other harmonics are of small magnitudes and not dominant.

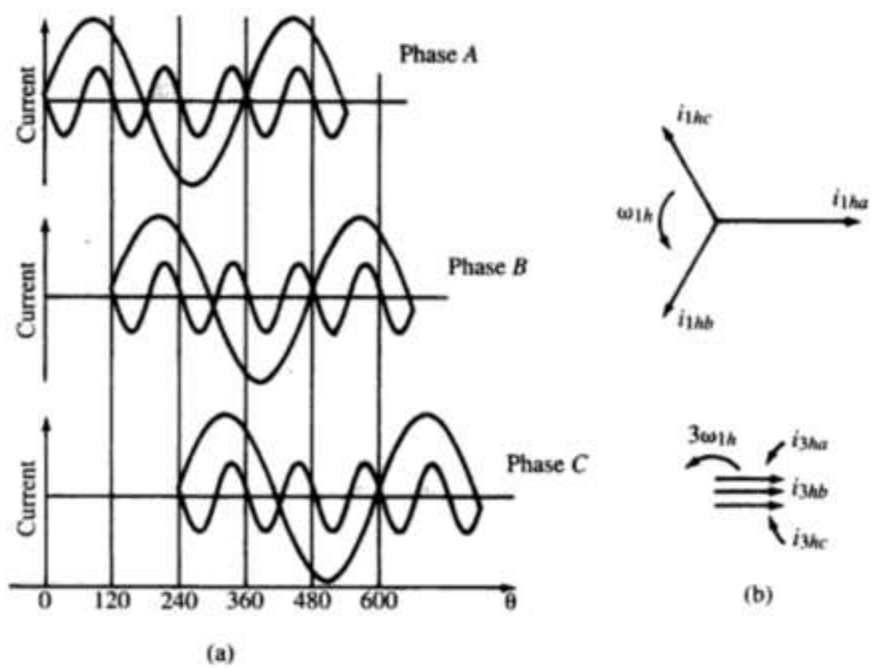


Magnetizing current and its first three harmonics.

Harmonic Suppression in Three-phase Connections

Magnetizing current that produces a sinusoidal flux and hence a sinusoidal output voltage is itself nonsinusoidal, containing many odd harmonic components. Suppressing any one of the harmonic components will result in a nonsinusoidal flux and hence a nonsinusoidal secondary voltage.

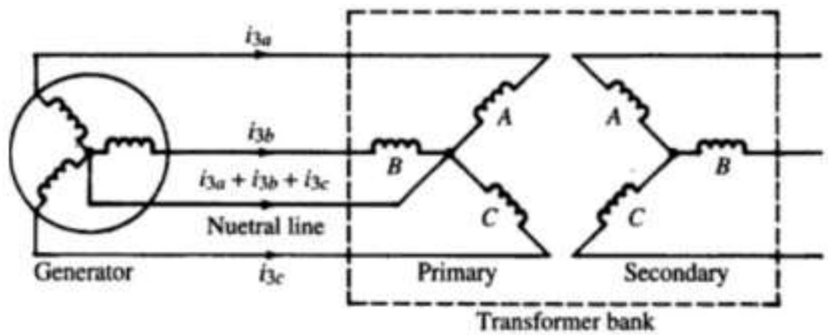
Third-harmonic currents and their multiples, called triplen harmonics, have zero phase sequence and thus are not three-phase quantities. The three third harmonics (one from each transformer) are in phase with each other. Although other triplen harmonics, such as the ninth and the fifteenth are present; their magnitudes are quite small compared to the magnitude of the third harmonic. All these components add up, and as a result, a very large third-harmonic component is observed in the secondary.



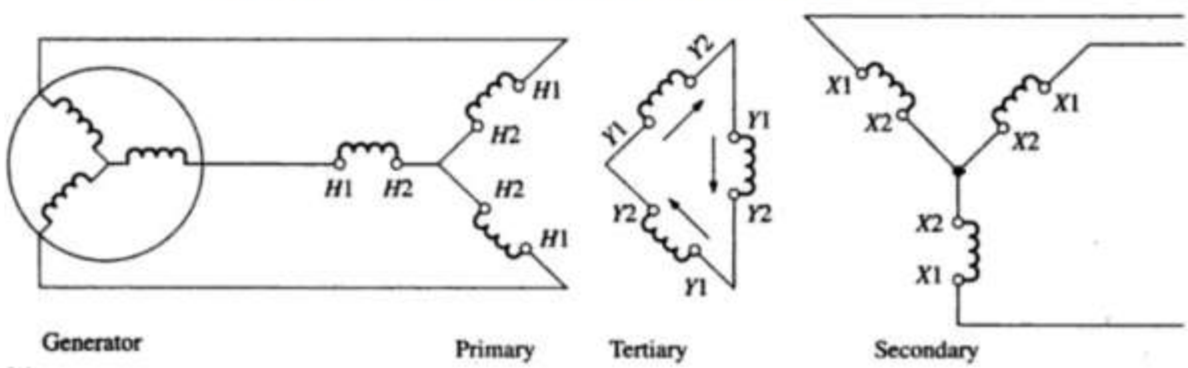
(a) Waves of the fundamental and the third harmonic
(b) Phasors of the fundamental and the third harmonic

Y-Y bank: Triplen harmonic currents of each phase of a three-phase wye-connected transformer bank are all in phase, all going in or all going out. Third harmonic voltages can be large, even larger than the fundamental voltage itself. The problem can be solved using one of two techniques.

Solidly ground the neutrals of the transformers, especially the neutral of the primary winding. This connection permits the additive third-harmonic components to cause a current flow in the neutral instead of building up large voltages.



Wye-wye bank with a neutral connection to primary; triplen harmonic components will flow in the neutral

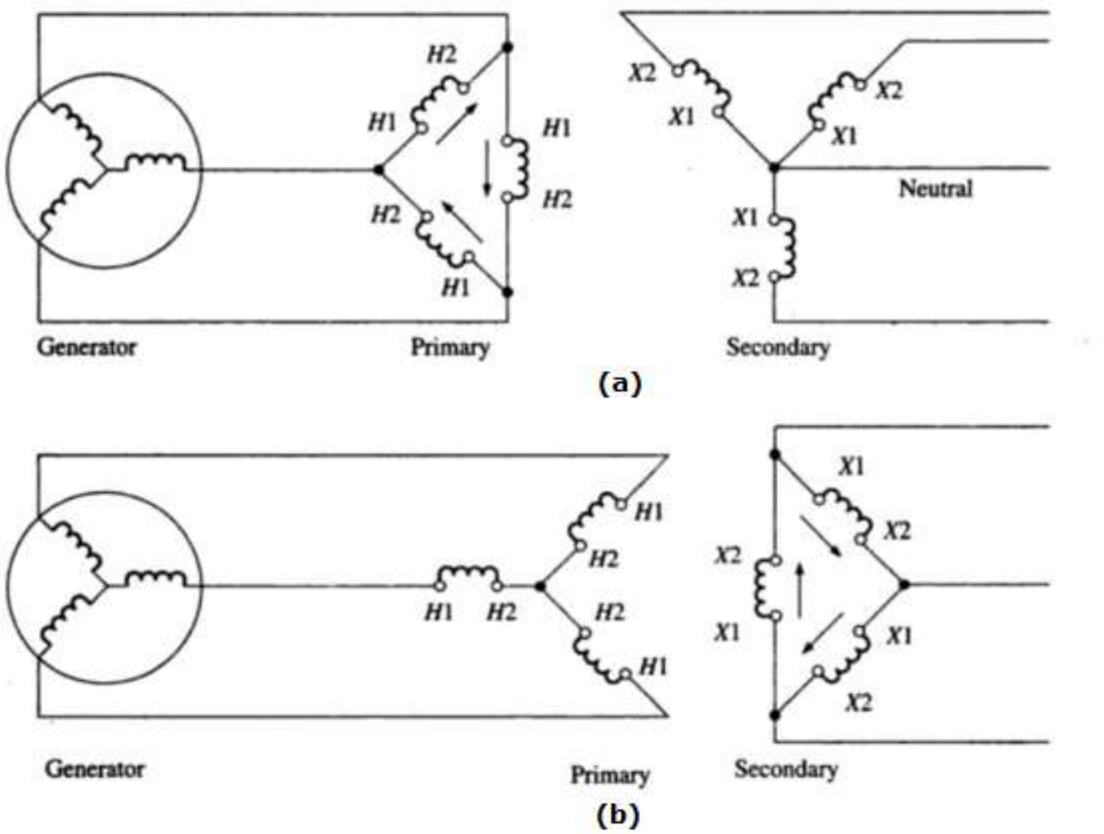


Wye-wye bank with no neutral connection; a delta connected tertiary coil is used to provide a path for triplen harmonic components

Add a third (tertiary) winding connected in Δ to the transformer bank. If a third Δ-connected winding is added to the transformer, then the third- harmonic components of voltage in the Δ will add up, causing a circulating current flow within the winding. This suppresses the third-harmonic components of voltage in the same manner as grounding the transformer neutrals.

The Δ-connected tertiary windings need not even be brought out of the transformer case, but they often are used to supply lights and auxiliary power within the substation where it is located. The tertiary windings must be large enough to handle the circulating currents, so they are usually made about one-third the power rating of the two main windings.

Delta-Wye Bank: Figure shows a Δ-Y transformer bank with Y-connected source. There can be no neutral connection to the primary. However, it has no problem with third-harmonic components because circulating triplen-harmonic currents in the delta enables a sinusoidal output voltage.



Third-harmonic currents circulating in the closed loop formed by the delta
(a) Delta-Wye bank (b) Wye-Delta bank with no neutral to the generator

Wye-Delta Bank: Figure shows a Y-Δ bank with no neutral connection to the generator. In this case, the path for triplen harmonic currents is the delta-connected secondary.

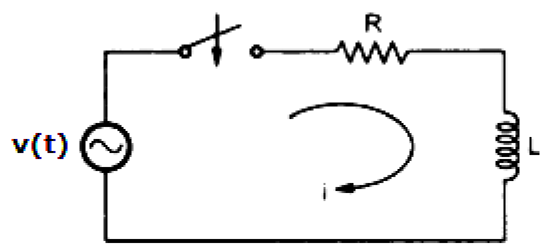
Delta-Delta Bank: Delta-delta banks have no problems with harmonics due to close loop in both primaries and secondaries.

Harmonic currents in power lines interfere with telephone communications by introducing an objectionable hum. Also, the power system is subject to overvoltages caused by possible series resonance between the capacitive reactances of the lines and the leakage reactances of the transformer at a harmonic frequency. The third harmonic is the principal trouble maker and that’s why needs to treated with caution.

Inrush Current

When a transformer is initially connected to a source of AC voltage, there may be a substantial surge of current through the primary winding called *inrush current*. The magnitude of the in-rush current depends on the magnitude and phase angle of the voltage wave at the instant the switch is closed, the magnitude and direction of the residual flux in the ferromagnetic core, and the type and magnitude of the load connected to the secondary. The high in-rush current must be taken into consideration when selecting fuses and/or circuit breakers.

When a switch is closed connecting an ac source to an R-L series circuit, the current will have a source-free response (transient component), and a forced response (steady-state component).



$$L\frac{di}{dt} + iR = V_m sin(\omega t + \phi)$$

$$\frac{di}{dt} + i\frac{R}{L} = \frac{V_m}{L} sin(\omega t + \phi)$$

$$i_{tr} = Ae^{-\frac{R}{L}t}$$

$$i_{ss} = I_m sin(\omega t + \phi - \theta)$$

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \quad = tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i(t) = i_{tr} + i_{ss} = Ae^{-\frac{R}{L}t} + I_m sin(\omega t + \phi - \theta)$$

$$i(0) = 0 = A + I_m sin(\phi - \theta) => A = -I_m sin(\phi - \theta)$$

$$i(t) = I_m \left[sin(\omega t + \phi - \theta) - sin(\phi - \theta)\, e^{-\frac{R}{L}t}\right]$$

The transient term has a damping factor associated with it and hence, decays exponentially within a few cycles. However, during first few cycles current may vary from 0 to $2I_m$ depending on the switching time.

The transient term will be zero when $\phi - \theta = 0, \pi, 2\pi,$ etc. If the R-L branch is highly inductive, the ratio of ωL to R is large, thereby causing θ to approach $\pi/2$ as an upper limit. In such cases, the transient term is zero when ϕ is approximately equal to $\pi/2, 3\pi/2, 5\pi/2,$ etc. Physically this means that, zero transient effect takes place in highly inductive circuits when the circuit is energized at points of approximately maximum voltage on the voltage wave. On the contrary, the transient term is maximum when ϕ is approximately equal to $0, \pi, 2\pi,$ etc.

We know that the rate of change of instantaneous flux in a transformer core is proportional to the instantaneous voltage drop across the primary winding. Alternatively, the voltage waveform is the derivative of the flux waveform, and the flux waveform is the integral of the voltage waveform. In a continuously-operating transformer, these two waveforms are phase-shifted by 90^o (Fig. 1). Since flux (Φ) is proportional to the magnetomotive force in the core, and the mmf is proportional to winding current, the current waveform will be in-phase with the flux waveform, and both will be lagging the voltage waveform by 90^o.

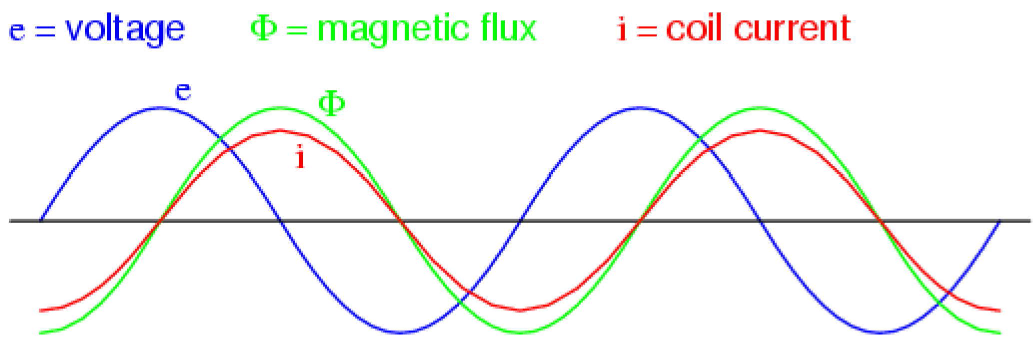


Fig 1: Continuous steady-state operation: Magnetic flux, like current, lags applied voltage by 90^o.

Let us suppose that the primary winding of a transformer is suddenly connected to an AC voltage source at the exact moment in time when the instantaneous voltage is at its positive peak value. In order for the transformer to create an opposing voltage drop to balance against this applied source voltage, a magnetic flux of rapidly increasing value must be generated. The result is that winding current increases rapidly, but actually no more rapidly than under normal conditions (Figure 2).

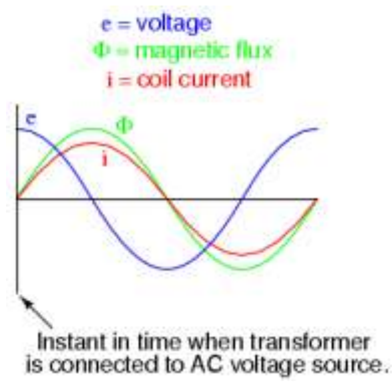


Fig 2. ***Connecting transformer to line at AC volt peak: Flux increases rapidly from zero, same as steady-state operation.***

Both core flux and coil current start from zero and build up to the same peak values experienced during continuous operation. Thus, there is no "surge" or "inrush" or current in this scenario (Fig. 2).

Alternatively, let us consider what happens if the transformer's connection to the AC voltage source occurs at the exact moment in time when the instantaneous voltage is at zero. During continuous operation (when the transformer has been powered for quite some time), this is the point in time where both flux and winding current are at their negative peaks, experiencing zero rate-of-change ($d\Phi/dt = 0$ and $di/dt = 0$). As the voltage builds to its positive peak, the flux and current waveforms build to their maximum positive rates-of-change, and on upward to their positive peaks as the voltage descends to a level of zero.

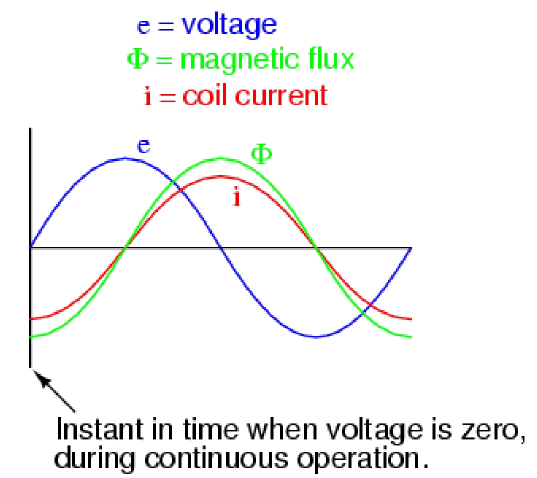


Fig 3. Starting at e=0 V is not the same as running continuously shown in Fig. 1. These expected waveforms are incorrect. Φ and i should start at zero.

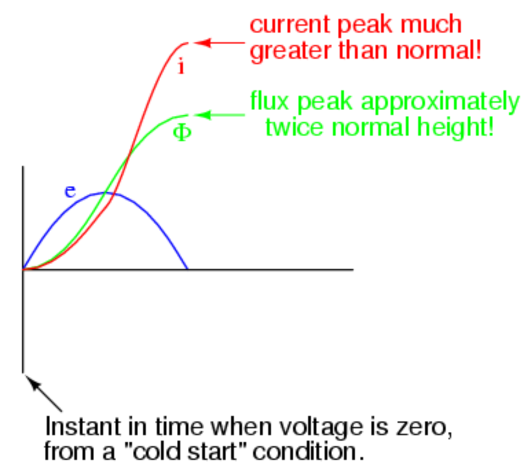


Fig 4. Starting at e=0 V, both Φ and i starts from zero. Φ increases to twice the normal value. Current also increases to twice the normal value for an unsaturated core, or considerably higher in case of saturation.

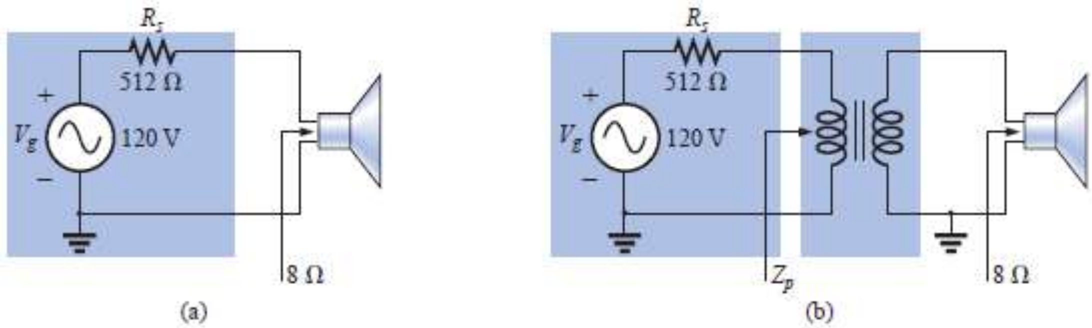
A significant difference exists, however, between continuous-mode operation and the sudden starting condition assumed in this scenario. During continuous operation, the flux and current levels were at their negative peaks when voltage was at its zero point; in a transformer that has been sitting idle, however, both magnetic flux and winding current should start at *zero*. When the magnetic flux increases in response to a rising voltage, it will increase from zero upward, not from a previously negative (magnetized) condition as we would normally have in a transformer that's been powered for awhile. Thus, in a transformer that's just "starting," the flux will reach approximately twice its normal peak magnitude as it "integrates" the area under the voltage waveform's first half-cycle (Fig. 4).

In an ideal transformer, the magnetizing current would rise to approximately twice its normal peak value as well, generating the necessary mmf to create this higher-than-normal flux. However, most transformers aren't designed with enough of a margin between normal flux peaks and the saturation limits to avoid saturating in a condition like this, and so the core will almost certainly saturate during this first half-cycle of voltage. During saturation, disproportionate amounts of mmf are needed to generate magnetic flux. This means that winding current, which creates the mmf to cause flux in the core, will disproportionately rise to a value *easily exceeding* twice its normal peak (Fig. 4).

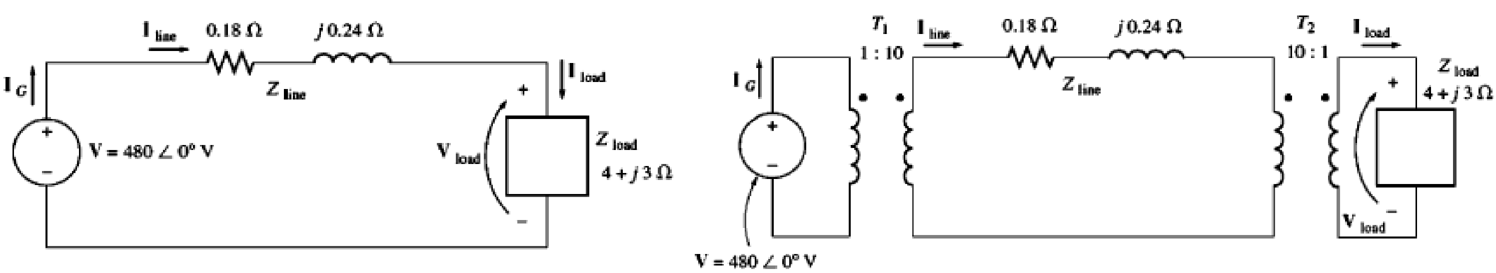
This is the mechanism causing inrush current in a transformer's primary winding when connected to an AC voltage source. The magnitude of the inrush current strongly depends on the exact time that electrical connection to the source is made. If the transformer happens to have some residual magnetism in its core at the moment of connection to the source, the inrush could be even more severe. Inductive loads increase the in-rush, whereas the resistive loads and capacitive loads decrease the in-rush.

Mathematical Problems

1. The source impedance in the following figure is 512 Ω, while the input impedance of the speaker is 8-Ω.
   a) Determine the power delivered to the speaker.
   b) Determine the turns–ratio of the audio impedance matching transformer introduced between the speaker and the source to ensure maximum power transfer. Determine the power delivered to the speaker.
   c) Compare the power delivered to the speaker under the conditions of parts (a) and (b).



2. A single-phase power system consists of a 480-V, 60-Hz generator supplying a load Z_{load} = 4 + j3 Ω through a transmission line of impedance Z_{line} = 0.18 + *j0.24* Ω. Answer the following questions about this system.
   a) If the power system is exactly as shown in fig (a), what will the voltage at the load be? What will the transmission line losses be?
   b) Suppose a 1:10 step-up transformer is placed at the generator end (sending end) of the transmission line and a 10:1 step-down transformer is placed at the load end (receiving end) of the line as shown in fig (b). What will the load voltage be now? What will the transmission line losses be now?



3. A 100 kVA, 60 Hz, 7200-480V, single phase, step-down transformer has the following parameters in ohms:

R_p=3.06	R_S=0.014	X_m =17809
X_p=6.05	X_S=0.027	R_{fe} =71400

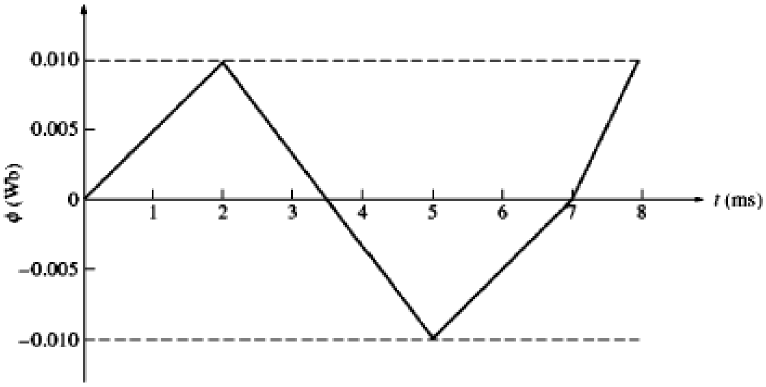
The transformer is supplying a load that draws rated current at 480V and 75 percent lagging power factor. Determine

   a) Equivalent resistance and equivalent reactance referred to the secondary side
   b) Input voltage at the primary side
   c) Voltage Regulation and Efficiency

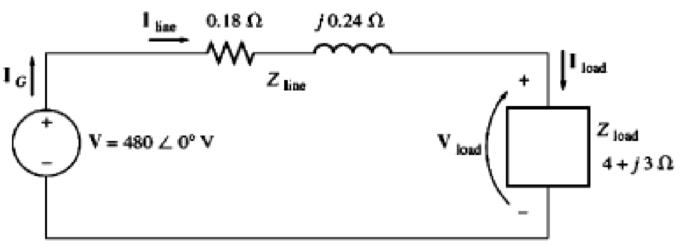
Mathematical Problems on Transformer

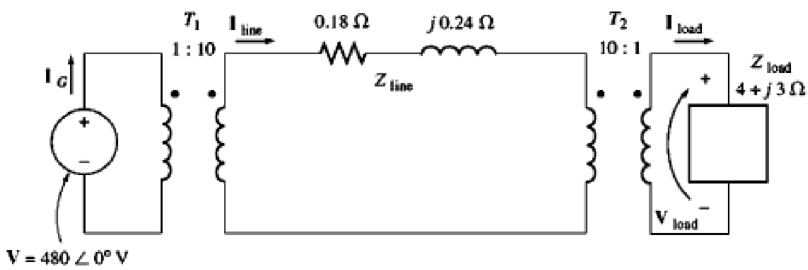
1. The flux in the transformer core is given by the equation, $\varphi = 0.05 \sin 377t\ Wb$. If there are 1000 turns on the primary and 500 turns on the secondary, determine the induced primary voltage and secondary voltage.

2. The following figure depicts the magnetic flux in a ferromagnetic core. If there are 100 turns on the primary coil, sketch the voltage present at the coil terminals.



3. A single-phase power system consists of a 480-V 60-Hz generator supplying a load, Z_{load} = 4 + j3 Ω through a transmission line of impedance, Z_{line} = 0.18 + j0.24 Ω.
   ~~a)~~ If the power system is exactly as described above calculate the loss in the transmission line.
   ~~b)~~ If a 1: 10 step-up transformer is placed at the generator end of the transmission line and a 10: 1 step-down transformer is placed at the load end of the line what will the transmission line losses be now?





4. A 15 kVA, 2400-240V, 60 Hz transformer has a magnetic core with a cross-sectional area of 50 cm^2 and a mean effective length of 66.7 cm. The application of 2400V causes a magnetic field intensity of 450 A-t/m (rms), and a maximum flux density of 1.5T. Determine the no. of turns in each winding and also the magnetizing current.

5. The hysteresis and eddy current losses for a 75 kVA, 480-120V, 60 Hz transformers are 215W and 115W respectively. The magnetizing current is 2.5 percent of rated current, and the transformer is operating in the step-up mode. Determine
   a) exciting current
   b) no-load power factor
   c) reactive power input at no load

6. A 100 kVA, 60 Hz, 7200-480V, single phase transformer has the following parameters in ohms:

R_{HS}=3.06,	R_{LS}=0.014	$X_{m,HS}$=17809
X_{HS}=6.05,	X_{LS}=0.027	$R_{fe,HS}$=71400

The transformer is supplying a load that draws rated current at 480V and 75 percent lagging power factor. Determine

a) Equivalent resistance and equivalent reactance referred to the high side
b) Input impedance of the combined transformer and load
c) Input voltage at the transformer
d) Input impedance at no load
e) Exciting current and its components (at no-load)

7. A 500 kVA, 7200-240V, 60Hz transformer with 2.2 percent impedance was severely damaged as a result of a dead short across the secondary terminals. Determine

a) short circuit current
b) required percent impedance of a replacement transformer that will limit the low-side short circuit current to 60000 A.

Note that, for large transformers (rated above 100kVA) $X_{pu} \gg R_{pu}$ and $Z_{pu} \approx R_{pu}$ X_{pu}

8. The percent resistance and percent reactance of a 50 kVA, 2400-600V, 60 Hz transformer are 0.8 and 1.2 respectively. If an accidental short circuit of 0.01Ω (resistive) occurs at the secondary (when 2500V is impressed across the primary), what will be the high-side fault current?

9. The equivalent low-side parameters of a 250 kVA, 4160-480V, 60 Hz transformer are $R_{eq,LS}$ = 0.0092 Ω and $X_{eq,LS}$ = 0.0433 Ω. The transformer is operating in the step-down mode and is delivering rated current at rated voltage to a 0.84 power factor lagging load. Determine

a) No load voltage
b) Actual input voltage at the high side
c) Voltage regulation

10. The following test data are obtained from short-circuit and open-circuit tests of a 50 kVA, 2400-600V, 60Hz transformer:

V_{oc}=600V	V_{sc}=76.4V
I_{oc}=3.34A	I_{sc}=20.8A
P_{oc}=484W	P_{sc}=754W

a) Determine all the transformer model parameters
b) What will be the regulation and efficiency if the transformer operates at rated load and 0.9 power factor lagging?
c) What will be the regulation and efficiency if the transformer operates at 85% rated load and 0.9 power factor leading?

11. A 150 kVA, 7200-600V, 60 Hz, single phase transformer operating at rated conditions has a hysteresis loss of 527 W, an eddy current loss of 373W, and a conductor loss of 2000W. The transformer is to be used on a 50 Hz system, with the restriction that it must maintain the same maximum core flux and the same total losses. Determine
    a) new voltage rating and new kVA rating
    b) conductor loss, hysteresis loss and eddy current loss at new operating condition

12. A 150 kVA, 7200-600V, 60 Hz, single phase transformer is to be used on a 50 Hz system, with the restriction that it must maintain the same maximum core flux. Determine new voltage rating and new kVA rating.

13. An autotransformer with a total of 600 turns is connected to a 60Hz, 2400V driving voltage. A load connected to the secondary draws 4.8kVA at 0.6 power factor lagging. If the secondary embraces 200 turns, neglecting losses and leakage effects, determine
    a) secondary voltage, secondary current, primary current
    b) apparent power conducted, apparent power transformed

14. A 120V, 60Hz air conditioner is to be operated in a remote area where the voltage drop in the long transmission line results in a utilization voltage of 102V. Determine
    a) required voltage ratio for satisfactory performance
    b) voltage ratio of a standard buck-boost transformer that most closely meets the requirements of the load
    c) voltage at the load with the buck-boost transformer installed
    d) Sketch the appropriate connection diagram showing the input and output terminals

15. An electric boiler rated at 50 kW, 240V, and 60 Hz is to be operated from a 60 Hz system whose utilization voltage is 269.5 V. Determine
    a) required voltage ratio for satisfactory performance
    b) voltage ratio of a standard buck-boost transformer that most closely meets the requirements of the load
    c) voltage at the load with the buck-boost transformer installed
    d) Sketch the appropriate connection diagram showing the input and output terminals

16. Three Single Phase transformers are used to supply a total of 750 kVA at 450V to a balanced three phase load. The three-phase input to the bank is 2400V. Determine
    a) Bank ratio and transformer ratio if connected as Wye-Delta
    b) Bank ratio and transformer ratio if connected as Delta-Wye
    c) Bank ratio and transformer ratio if connected as Delta-Delta

    If each transformer is rated at 400 kVA and the bank is delta-delta connected, will the bank be able to carry the load if one transformer is disconnected?

17. Two 4160-450V transformers are to be purchased to supply a 450V, 90kW, 0.75 lagging power factor three-phase load. The transformers are to be connected in open delta. Specify the minimum power rating required for each transformer.

18. Two 100 kVA, single phase, 60 Hz transformers are to be operated in parallel. Determine the circulating current in the paralleled secondaries when rated voltage is applied across the paralleled primaries.

Transformer	Voltage Ratio	$R_{eq,LS}$	$X_{eq,LS}$
A	2300-460	0.0288 Ω	0.07 Ω
B	2300-450	0.0284 Ω	0.06 Ω

19. Three 2400-480V transformers are operated in parallel. Determine the percent of bank current carried by each transformer. Calculate the maximum apparent power the bank can supply without overloading any of the transformers.

Transformer	kVA	Nameplate Impedance
A	100	3.68%
B	167	4.02%
C	250	4.25%

20. Two 7200-240V, 75 kVA transformers are to be operated in parallel. The per-unit impedances are $Z_{A,pu} = 0.01 + j\,0.055$ and $Z_{B,pu} = 0.02 + j0.038$. Determine the contribution in bank current from each transformer. Calculate the maximum apparent power the bank can supply without overloading any of the transformers.