Solution-(i) : ধরি [we put] $I_n = \int \tan^n x\, dx$

বা $I_n = \int \tan^{n-2}x \tan^2 x\, dx = \int \tan^{n-2}x (\sec^2 x - 1)\, dx$

বা $I_n = \int \tan^{n-2}x \sec^2 x\, dx - \int \tan^{n-2}x\, dx$

বা $I_n = \int (\tan x)^{n-2}\, d(\tan x) - I_{n-2}$

বা $I_n = \dfrac{(\tan x)^{n-1}}{n-1} - I_{n-2}$

i. e. $\int \tan^n x\, dx = \dfrac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\, dx$

ইহাই নির্ণেয় লঘুকরণ সূত্র [which is the required reduction formula]

Solution-(ii) : অনুরূপভাবে $\int \cot^n x\, dx$ এর লঘুকরণ সূত্রে নির্ণয় করা যায়। [Similarly we can determine the reduction formula for $\int \cot^n x\, dx$]

Solution-(iii) : ধরি $I_n = \int \sec^n x\, dx$

বা $I_n = \int \sec^{n-2}x.\ \sec^2 x\, dx$

বা $I_n = \sec^{n-2}x \int \sec^2 x\, dx - \int \left\{\dfrac{d}{dx}(\sec^{n-2}x) \int \sec^2 x\, dx\right\} dx$

বা $I_n = \sec^{n-2}x \tan x - (n-2) \int \sec^{n-3}x.\ \sec x \tan x.\ \tan x\, dx$

বা $I_n = \sec^{n-2}x \tan x - (n-2) \int \sec^{n-2}x \tan^2 x\, dx$

বা $I_n = \sec^{n-2}x \tan x - (n-2) \int \sec^{n-2}x (\sec^2 x - 1)\, dx$

বা $I_n = \sec^{n-2}x \tan x - (n-2) \int \sec^n x\, dx + (n-2) \int \sec^{n-2}x\, dx$

বা $I_n = \sec^{n-2}x \tan x - (n-2) I_n + (n-2) I_{n-2}$

বা $(1 + n - 2) I_n = \sec^{n-2}x \tan x + (n-2) I_{n-2}$

বা $I_n = \dfrac{\sec^{n-2}x \tan x}{n-1} + \dfrac{n-2}{n-1} I_{n-2}$

অর্থাৎ [that is] $\int \sec^n x\, dx = \dfrac{\sec^{n-2}x \tan x}{n-1} + \dfrac{n-2}{n-1} \int \sec^{n-2}x\, dx.$

ইহাই নির্ণেয় লঘুকরণ সূত্র। [This is the required reduction formula]

Solution-(iv) : অনুরূপভাবে $\int \mathrm{cosec}^n x\, dx$ এর লঘুকরণ সূত্র নির্ণয় করা যায়। [Similarly we can determine the reduction formula for $\int \mathrm{cosec}^n x\, dx$]

দ্বিতীয় অংশ **[Part–II]** ঃ (3) নং সূত্রকে নিম্নরূপে লিখা যায় [Formula (3) can be written in the following form]

$$\int \cos^m x \cos nx\, dx = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n}\int \cos^{m-1}x \cos(n-1)x\, dx \cdots (4)$$

এখন (4) নং সূত্রে 0 হইতে $\pi/2$ পর্যন্ত সীমা গ্রহন করিয়া পর্যায়ক্রমে $m = 3, n = 2$ এবং $m = 2, n = 1$ স্থাপন করিয়া পাই [Now taking limits from 0 to $\pi/2$ in the formula (4) and putting $m = 3, n = 2$ and $m = 2, n = 1$ successively we get]

$$\int_0^{\pi/2} \cos^3 x \cos 2x\, dx = \left[\frac{\cos^3 x \ \sin 2x}{3+2}\right]_0^{\pi/2} + \frac{3}{3+2}\int_0^{\pi/2} \cos^2 x .\ \cos x\, dx$$

বা $\displaystyle\int_0^{\pi/2} \cos^3 x \cos 2x\, dx = 0 + \frac{3}{5}\int_0^{\pi/2} \cos^2 x \cos x\, dx \cdots (5)$

এবং $\displaystyle\int_0^{\pi/2} \cos^2 x \cos x\, dx = \left[\frac{\cos^2 x \sin x}{2+1}\right]_0^{\pi/2} + \frac{2}{2+1}\int_0^{\pi/2} \cos x\, dx$

বা $\displaystyle\int_0^{\pi/2} \cos^2 x .\ \cos x\, dx = 0 + \frac{2}{3}\int_0^{\pi/2} \cos x\, dx = \frac{2}{3}\ [\sin x]_0^{\pi/2}$

বা $\displaystyle\int_0^{\pi/2} \cos^2 x \cos x\, dx = \frac{2}{3}[\sin \pi/2 - \sin 0] = \frac{2}{3}(1-0)$

$\therefore \displaystyle\int_0^{\pi/2} \cos^2 x \cos x\, dx = \frac{2}{3} \cdots (6)$

এখন (5) নং এবং (6) নং হইতে পাই [Now from (5) and (6) we get]

$$\int_0^{\pi/2} \cos^3 x \cos 2x\, dx = \frac{3}{5}\cdot\frac{2}{3}$$

$$\therefore \int_0^{\pi/2} \cos^3 x \cos 2x\, dx = \frac{2}{5}.$$

5.8 ঃ [Find the reduction formula for]

(i). $\int \tan^n x\, dx$ (ii). $\int \cot^n x\, dx$

(iii). $\int \sec^n x\, dx$ এবং (iv). $\int \mathrm{cosec}^n x\, dx.$

এর লঘুকরণ সূত্র নির্ণয় কর।

5.7 ঃ যদি [If] $I_{m,n} = \int \cos^m x \cos nx\, dx$ হয়, তবে দেখাও যে [then show that]

$$I_{m,n} = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n} I_{m-1,\,n-1}$$

এবং ইহা হইতে [and from this find the value of]

$\int_0^{\pi/2} \cos^3 x \cos 2x\, dx$ এর মান নির্ণয় কর। [DU–82, CU–85]

Solution ঃ দেওয়া আছে [Given that]

$$I_{m,n} = \int \cos^m x \cos nx\, dx$$

বা $I_{m,n} = \cos^m x \int \cos nx\, dx - \int \left\{ \frac{d}{dx} (\cos^m x) \int \cos nx\, dx \right\} dx$

বা $I_{m,n} = \frac{\cos^m x .\ \sin nx}{n} - m \int \cos^{m-1} x\, (-\sin x) \frac{\sin nx}{n}\, dx$

বা $I_{m,n} = \frac{\cos^m x \sin nx}{n} + \frac{m}{n} \int \cos^{m-1} x \sin nx \sin x\, dx \cdots (1)$

এখন [Now] $\cos(n-1)x = \cos(nx - x)$

বা $\cos(n-1)x = \cos nx .\ \cos x + \sin nx \sin x$

বা $\cos(n-1)x - \cos nx \cos x = \sin nx \sin x \cdots (2)$

(1) নং এবং (2) নং হইতে পাই [From (1) and (2) we get]

$$I_{m,n} = \frac{\cos^m x \sin nx}{n} + \frac{m}{n} \int \cos^{m-1} x \{\cos(n-1)x - \cos nx \cos x\}\, dx$$

বা $I_{m,n} = \frac{\cos^m x .\ \sin nx}{n} + \frac{m}{n} \int \cos^{m-1} x \cos(n-1)x\, dx - \frac{m}{n} \int \cos^m x \cos nx\, dx$

বা $I_{m,n} = \frac{\cos^m x \sin nx}{n} + \frac{m}{n} I_{m-1,\,n-1} - \frac{m}{n} I_{m,n}$

বা $\left(1 + \frac{m}{n}\right) I_{m,n} = \frac{\cos^m x \sin nx}{n} + \frac{m}{n} I_{m-1,\,n-1}$

বা $\left(\frac{m+n}{n}\right) I_{m,n} = \frac{\cos^m x \sin nx}{n} + \frac{m}{n} I_{m-1,\,n-1}$

বা $I_{m,n} = \frac{\cos^m x \sin nx}{m+n} + \frac{m}{m+n} I_{m-1,\,n-1} \cdots (3)$ Proved

$$=\frac{1}{4}\int\left[\frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x\right]dx$$

$$=\frac{1}{4}\left[\frac{3x}{2}-\sin 2x+\frac{\sin 4x}{8}\right]+c\cdots(4)$$

(3) নং এবং (4) নং হইতে পাই [From (3) and (4) we get]

$$I_{4,2}=\frac{\cos x\sin^5 x}{6}+\frac{1}{24}\left[\frac{3x}{2}-\sin 2x+\frac{\sin 4x}{8}\right]+c.$$

Solution–(ii) : $U_{m,n}=\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

বা $U_{m,n}=\int_0^{\pi/2}\cos^{n-1}x.\ \sin^m x\cos x\,dx$

বা $U_{m,n}=\int_0^{\pi/2}\cos^{n-1}x\,(\sin x)^m\,d(\sin x)$

বা $U_{m,n}=\left[\frac{\cos^{n-1}x\ \sin^{m+1}x}{m+1}\right]_0^{\pi/2}$

$$-(n-1)\int_0^{\pi/2}\cos^{n-2}x\,(-\sin x)\frac{\sin^{m+1}x}{m+1}dx$$

বা $U_{m,n}=0+\frac{n-1}{m+1}\int_0^{\pi/2}\cos^{n-2}x\sin^m x\sin^2 x\,dx$

বা $U_{m,n}=\frac{n-1}{m+1}\int_0^{\pi/2}\cos^{n-2}x\sin^m x\,(1-\cos^2 x)\,dx$

বা $U_{m,n}=\frac{n-1}{m+1}\int_0^{\pi/2}\sin^m x\cos^{n-2}x\,dx-\frac{n-1}{m+1}\int_0^{\pi/2}\sin^m x\cos^n x\,dx$

বা $U_{m,n}=\frac{n-1}{m+1}U_{m,n-2}-\frac{n-1}{m+1}U_{m,n}$

বা $\left(1+\frac{n-1}{m+1}\right)U_{m,n}=\frac{n-1}{m+1}U_{m,n-2}$

বা $\left(\frac{m+n}{m+1}\right)U_{m,n}=\frac{n-1}{m+1}U_{m,n-2}$

বা $U_{m,n}=\frac{n-1}{m+n}U_{m,n-2}.$ Proved

ষষ্ঠ অধ্যায় [CHAPTER–6]
দ্বিতীয় পরিচ্ছেদ [SECTION–2]
নির্দিষ্ট ইনটিগ্র্যাল কর্তৃক ধারার সমষ্টি [Summation of series by definite integral]

6–2.1 : 6-1.1 এ প্রমাণ করা হইয়াছে [It has proved in 6-1.1]

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f(a + r/n),$$ যখন [when] $nh = b - a$

উপরের সূত্রে $a = 0$, $b = 1$ বসাইয়া পাই [Putting $a = 0$, $b = 1$ in the above formula we get]

$$\int_0^1 f(x)\,dx = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f(r/n),$$ যখন [when] $nh = 1$

i. e. $$\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f(r/n) = \int_0^1 f(x)\,dx \cdots (1)$$

যদি (1) নং এর উভয় পক্ষকে তুলনা করা হয়, তবে আমরা দেখিতে পাই যে, ধারাকে নির্দিষ্ট ইনটিগ্র্যালে প্রকাশ করিতে হইলে অবশ্যই [Comparing both sides of (1) we find that in order to express the series interms of a definite integral we must]

(i). $\frac{r}{n}$ এর পরিবর্তে x $\left[\text{Replace } \frac{r}{n} \text{ by } x\right]$

(ii). $\lim\limits_{n\to\infty} \frac{1}{n}$ এর পরিবর্তে dx $\left[\text{Replace } \lim\limits_{n\to\infty} \frac{1}{n} \text{ by } dx\right]$

(iii). $\sum$ এর পরিবর্তে $\int$ ধরিতে হয় $\left[\text{Replace } \sum \text{ by } \int\right]$

(iv). $\sum$ এর উপরে n এর সহগ যত $\int$ এর উর্ধসীমা তত এবং নিম্নসীমা সর্বদাই 0 [শুন্য] হয়।

(v). r এর নিচে সমান ঘাতের n এবং $\sum$ এর বাহিরে $\frac{1}{n}$ গঠন করিতে হইবে।

Proof : (iv). নিম্নসীমা [Lower limit]

$$= \lim_{n\to\infty} \frac{r}{n}$$

$$= \lim_{n\to\infty} \frac{1}{n}$$; যেহেতু প্রথম পদে $r = 1$ [since in the 1st term $r = 1$]

$= 0.$

উর্ধসীমা [Upper limit]

$$= \lim_{n\to\infty} \frac{r}{n}$$

$$= \lim_{n\to\infty} \frac{n}{n},$$ যেহেতু শেষপদে [since in the last term] $r = n$

$$= \lim_{n\to\infty} (1)$$

$$= 1.$$

উদাহরণ ঃ [Find the lower limit and upper limit of]

$$\lim_{n\to\infty} \frac{1}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \cdots + f(b)\right]$$

এর নিম্নসীমা এবং উর্ধসীমা নির্ণয় কর।

Solution ঃ Let $S = \lim_{n\to\infty} \frac{1}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \cdots + f\left(\frac{nb}{n}\right)\right]$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{nb} f\left(\frac{r}{n}\right)$$

এখানে নিম্নসীমা [Here lowe limit]

$$= \lim_{n\to\infty} \frac{r}{n}$$

$$= \lim_{n\to\infty} \frac{1}{n};$$ যেহেতু প্রথম পদে $r = 1$ [since in the 1st term $r= 1$]

$$= 0.$$

উর্ধসীমা [Upper limit]

$$= \lim_{n\to\infty} \frac{r}{n}$$

$$= \lim_{n\to\infty} \frac{nb}{n},$$ যেহেতু শেষ পদে [since in the last term] $r = nb$

$$= \lim_{n\to\infty} b$$

$$= b.$$

নোট-1 ঃ ধারার সমষ্টি হইতে নির্দিষ্ট ইনটিগ্র্যালে পরিবর্তিত করার সময় উপরে উল্লেখিত নিয়মে বিস্তারিত ভাবে উর্ধসীমা ও নিম্নসীমা নির্ণয় করার প্রয়োজন নাই।

এখানে $\sum\limits_{r=1}^{nb}$ এর উপরে n এর সহগ b হইল $\int$ এর উর্ধসীমা এবং r এর সসীম মানের জন্য নিম্নসীমা সর্বদাই 0 [শুন্য] হয়।

কার্য্য পদ্ধতি [working rule] : r এর নিচে সমান ঘাতের n থাকিবে এবং $\sum$ এর বাহিরে $\frac{1}{n}$ গঠন করিতে পারিলে নিম্নের 4টি সূত্র প্রয়োগ করিতে হয়।

(i). $\frac{r}{n} = x$ (ii). $\lim\limits_{n\to\infty} \frac{1}{n} = dx$ (iii). $\sum = \int$

(iv). $\sum$ এর উপরে n এর সহগ যত $\int$ এর উর্ধসীমা তত এবং নিম্নসীমা সর্বদাই 0 শুন্য হয়।

উদাহরণমালা [EXAMPLES]

Example-1 ঃ [Evaluate]

(i). $\lim\limits_{n\to\infty}\left[\frac{1}{n+m}+\frac{1}{n+2m}+\dots+\frac{1}{n+nm}\right]$ **[CU-81, DU-85]**

(ii). $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-2^2}}+\dots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right]$ এর মান নির্ণয় কর। **[RU-80, DU-82]**

Solution ঃ (i). ধরি [we put]

$$S = \lim_{n\to\infty}\left[\frac{1}{n+m}+\frac{1}{n+2m}+\dots+\frac{1}{n+nm}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+rm} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n(1+mr/n)}$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+mr/n} = \int_0^1\frac{dx}{1+mx}.$$

$$= \frac{1}{m}\int_0^1\frac{m\,dx}{1+mx} = \frac{1}{m}\ [\log(1+mx)]_0^1$$

$$= \frac{1}{m}[\ln(1+m) - \ln 1]$$

$$= \frac{\ln(1+m)}{m}.$$

Solution ঃ (ii). ধরি [We put]

$$S = \lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2-0^2}}+\frac{1}{\sqrt{n^2-1^2}}+\frac{1}{\sqrt{n^2-2^2}}+\cdots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right]$$

$$= \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{n^2-r^2}}$$

$$= \lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{n^2(1-r^2/n^2)}}$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{\sqrt{1-r^2/n^2}} = \int_0^1\frac{dx}{\sqrt{1-x^2}}$$

$$= \left[\sin^{-1}x\right]_0^1 = \sin^{-1}1-\sin^{-1}0 = \frac{\pi}{2}-0.$$

$$= \pi/2.$$

Example–2 ঃ [Evaluate] $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\cdots+\frac{1}{8n}\right]$ এর মান নির্ণয় কর। **[RU–78, DU–86, NUH–95, NU–03]**

Solution ঃ ধরি [We put]

$$S = \lim_{n\to\infty}\left[\frac{n^2}{(n+0)^3}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\cdots+\frac{n^2}{(n+n)^3}\right]$$

$$= \lim_{n\to\infty}\sum_{r=0}^{n}\frac{n^2}{(n+r)^3} = \lim_{n\to\infty}\sum_{r=0}^{n}\frac{n^2}{\{n(1+r/n)\}^3}$$

$$= \lim_{n\to\infty}\sum_{r=0}^{n}\frac{n^2}{n^3(1+r/n)^3} = \lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n}\frac{1}{(1+r/n)^3}$$

$$= \int_0^1\frac{dx}{(1+x)^3} = -\frac{1}{2}\left[\frac{1}{(1+x)^2}\right]_0^1 = -\frac{1}{2}\left(\frac{1}{4}-1\right)$$

$$= \frac{3}{8}.$$

Example–3 ঃ [Evaluate]

$\lim_{n\to\infty}\left[\frac{n+2}{n^2+1}+\frac{n+4}{n^2+4}+\frac{n+6}{n^2+9}+\cdots+\frac{n+2n}{n^2+n^2}\right]$ এর মান নির্ণয় কর।

Solution ঃ ধরি [We put]

$$S = \lim_{n\to\infty}\left[\frac{n+2.1}{n^2+1^2}+\frac{n+2.2}{n^2+2^2}+\frac{n+2.3}{n^2+3^2}+\cdots+\frac{n+2n}{n^2+n^2}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{n+2r}{n^2+r^2} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{n(1+2r/n)}{n^2(1+r^2/n^2)}$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1+2r/n}{1+r^2/n^2} = \int_0^1\frac{(1+2x)\,dx}{1+x^2}$$

$$= \int_0^1\left(\frac{1}{1+x^2}+\frac{2x}{1+x^2}\right)dx$$

$$= \left[\tan^{-1}x+\log(1+x^2)\right]_0^1$$

$$= \tan^{-1}1 + ln2 - \tan^{-1}0 - ln1$$

$$= \pi/4 + ln2.$$

Example–4 ঃ [Evaluate] $\lim\limits_{n\to\infty}\left[\frac{1^2}{n^3+1^3}+\frac{2^2}{n^3+2^3}+\cdots+\frac{n^{}}{n^3+}\right.$

এর মান নির্ণয় কর। [CU

Solution ঃ ধরি [We put]

$$S = \lim_{n\to\infty}\left[\frac{1^2}{n^3+1^3}+\frac{2^2}{n^3+2^3}+\cdots+\frac{n^2}{n^3+n^3}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^2}{n^3+r^3} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^2}{n^3(1+r^3/n^3)}$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r^2/n^2}{1+r^3/n^3} = \int_0^1\frac{x^2\,dx}{1+x^3}$$

$$= \frac{1}{3}\int_0^1\frac{3x^2\,dx}{1+x^3} = \frac{1}{3}\left[ln(1+x^3)\right]_0^1$$

$$= \frac{1}{3}[ln2 - ln1]$$

$$= \frac{1}{3}ln2.$$

Example–5 ঃ [Evaluate] $\lim_{n\to\infty}\left[\left(1+\frac{1}{2^2}\right)\left(1+\frac{n^2}{n^2}\right)\cdots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$

এর মান নির্ণয় কর। **[DU–88]**

Solution ঃ ধরি [We put]

$$S=\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\cdots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$$

উভয় পক্ষে ln গ্রহন করিয়া পাই [Taking ln on both sides we get]

$$ln\, S=\lim_{n\to\infty}\frac{1}{n}\left[ln\left(1+\frac{1^2}{n^2}\right)+ln\left(1+\frac{2^2}{n^2}\right)+\cdots+ln\left(1+\frac{n^2}{n^2}\right)\right]$$

বা $ln\, S=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}ln\left(1+\frac{r^2}{n^2}\right)=\int_0^1 ln\,(1+x^2)\,dx$

বা $ln\, S=\left[\log(1+x^2).x\right]_0^1-\int_0^1\frac{2x}{1+x^2}.x\,dx$

বা $ln\, S=ln2-0-2\int_0^1\frac{x^2\,dx}{1+x^2}=ln2-2\int_0^1\left(\frac{(1+x^2)-1}{1+x^2}\right)dx$

বা $ln\, S=ln2-2\int_0^1 dx+2\int_0^1\frac{dx}{1+x^2}$

বা $ln\, S=ln2-2\,[x]_0^1+2\left[\tan^{-1}x\right]_0^1$

বা $ln\, S=ln2-2(1-0)+2\,(\tan^{-1}1-0)$

বা $ln\, S=ln2-2+2.\pi/4=ln2-2+\pi/2$

বা $ln\, S=ln2+(\pi-4)/2=ln2+ln e^{(\pi-4)/2}$

বা $ln\, S=ln2\, e^{(\pi-4)/2}$

$\Rightarrow S=2e^{(\pi-4)/2}$.

প্রশ্নমালা [EXERCISE]–6(B)

নিম্নলিখিত সমষ্টি সমূহের সীমায়িত মান নির্ণয় কর [Find the limiting values of the following summations]

1(i). $\lim\limits_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\dots+\frac{1}{2n}\right]$

(ii). $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\dots+\frac{1}{3n}\right]$

(iii). $\lim\limits_{n\to\infty}\left[\frac{1}{3n+1}+\frac{1}{3n+2}+\frac{1}{3n+3}+\dots+\frac{1}{3n+n}\right]$

(iv). $\lim\limits_{n\to\infty}\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+\dots+\frac{1}{nb}\right]$

(v). $\lim\limits_{n\to\infty}\left[\frac{1}{\sqrt{2n-1^2}}+\frac{1}{\sqrt{4n-2^2}}+\dots+\frac{1}{n}\right]$ [DUH–87, 91, 93]

(vi). $\lim\limits_{n\to\infty}\left[\frac{1}{\sqrt{n^2-1^2}}+\frac{1}{\sqrt{n^2-2^2}}+\dots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right]$

(vii). $\lim\limits_{n\to\infty}\left[\frac{1}{\sqrt{n^2+2n}}+\frac{1}{\sqrt{n^2+4n}}+\frac{1}{\sqrt{n^2+6n}}+\dots+\frac{1}{\sqrt{n^2+2n^2}}\right]$

(viii). $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\sqrt{n^2-r^2}}$ [NUH–04]

(ix). $\lim\limits_{n\to\infty}\frac{1}{\sqrt{n}}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots+\frac{1}{\sqrt{n}}\right]$

2(i). $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\dots+\frac{n}{n^2+(n-1)^2}\right]$

(ii). $\lim\limits_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\dots+\frac{n}{n^2+n^2}\right]$

(iii). $\lim\limits_{n\to\infty}\left[\frac{n^2}{(n^2+1)^{3/2}}+\frac{n^2}{(n^2+2^2)^{3/2}}+\dots+\frac{n^2}{\{n^2+(n+1)^2\}^{3/2}}\right]$

(iv). $\lim\limits_{n\to\infty}\left[\frac{\sqrt{n}}{n^{3/2}}+\frac{\sqrt{n}}{(n+3)^{3/2}}+\dots+\frac{\sqrt{n}}{\{n+3(n-1)\}^{3/2}}\right]$

(v). $\lim\limits_{n\to\infty}\left[\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\dots+\frac{n}{(n+n)^2}\right]$

(vi). $\lim\limits_{n\to\infty} n\left[\frac{1}{n^2}+\frac{1}{n^2+2^2}+\dots+\frac{1}{n^2+(2n-2)^2}\right]$

(vii). $\lim\limits_{n\to\infty}\left[\frac{\sqrt{n}}{\sqrt{n^3}}+\frac{\sqrt{n}}{\sqrt{(n+4)^3}}+\frac{\sqrt{n}}{\sqrt{(n+8)^3}}+\dots+\frac{\sqrt{n}}{\sqrt{\{n+4(n-1)\}^3}}\right]$

(viii). $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{n^3}{(n^2+r^2)(n^2+2r^2)}$

(ix). $\lim\limits_{n\to\infty}\left[\frac{n}{(n+1)(n+2)}+\frac{n}{(n+2)(n+4)}+\frac{n}{(n+3)(n+6)}+\dots+\frac{1}{6n^2}\right]$

(x). $\lim\limits_{n\to\infty}\left[\frac{n}{(n+1)\sqrt{2n+1}}+\frac{n}{(n+2)\sqrt{2(2n+2)}}+\dots+\frac{n}{2n\sqrt{n.3n}}\right]$

(xi). $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{n}{(n+r)\sqrt{2nr+r^2}}$

(xii). $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{\sqrt{n}}{(9n+40r)^{3/2}}$

3(i). $\lim\limits_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\frac{n+3}{n^2+3^2}+\dots+\frac{1}{n}\right]$

(ii). $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{\sqrt{n^2-1^2}}{n^2}+\frac{\sqrt{n^2-2^2}}{n^2}+\dots+\frac{\sqrt{n^2-(n-1)^2}}{n^2}\right]$

(iii). $\lim\limits_{n\to\infty}\left[\frac{\sqrt{n+1}+\sqrt{n+2}+\sqrt{n+3}+\dots+\sqrt{2n}}{n\sqrt{n}}\right]$

(iv). $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n-1}\frac{\sqrt{n+r}}{n\sqrt{n-r}}$

(v). $\lim\limits_{n\to\infty}\left[\frac{(n-m)^{1/3}}{n}+\frac{(2^2n-m)^{1/3}}{2n}+\frac{(3^2n-m)^{1/3}}{3n}+\dots+\frac{(n^3-m)^{1/3}}{n^2}\right]$

4(i). $\lim\limits_{n\to\infty}\left[\frac{1+2^{10}+3^{10}+\dots+n^{10}}{n^{11}}\right]$

(ii). $\lim\limits_{n\to\infty}\left[\frac{1}{1^2+3n^2}+\frac{2}{2^2+3n^2}+\frac{3}{3^2+3n^2}+\dots+\frac{n}{n^2+3n^2}\right]$

(iii). $\lim\limits_{n\to\infty}\left[\frac{1^m+2^m+3^m+\dots+n^m}{n^{m+1}}\right]$

(iv). যদি f(x) ফাংশন অবিচ্ছিন্ন হয়, তবে [If f(x) is a continuous function, then express into definite integral]

$\lim\limits_{n\to\infty}\frac{1}{n}\left[f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\dots+f\left(\frac{n}{n}\right)\right]$ কে নির্দিষ্ট ইনটিগ্র্যালে প্রকাশ কর এবং ইহার সাহায্যে [and by using this evaluate]

$\lim\limits_{n\to\infty}\left[\frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\dots+\sqrt{n}}{n^{3/2}}\right]$ এর মান নির্ণয় কর। [DUH-90]

(v). $\lim\limits_{n\to\infty}\left[\frac{1}{n^2}\sec^2(1/n^2)+\frac{2}{n^2}\sec^2(4/n^2)+\frac{3}{n^2}\sec^2(9/n^2)+\dots+\frac{1}{n}\sec^2 1\right]$,

(vi). $\lim\limits_{n\to\infty}\frac{1}{n}\left[\sin^{2k}\frac{\pi}{2n}+\sin^{2k}\frac{2\pi}{2n}+\sin^{2k}\frac{3\pi}{2n}+\dots+\sin^{2k}\frac{\pi}{2}\right]$

5(i). $\lim\limits_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right]^{1/n}$

(ii). $\lim\limits_{n\to\infty}\left[\left\{1+\left(\frac{1}{n}\right)^4\right\}^1\left\{1+\left(\frac{2}{n}\right)^4\right\}^{1/2}\cdots\{2\}^{1/n}\right]$

(iii). $\lim\limits_{n\to\infty}\left[\left(2+\frac{1}{n^2}\right)^{1/n^2}\left(2+\frac{2^2}{n^2}\right)^{2/n^2}\cdots\left(2+\frac{n^2}{n^2}\right)^{n/n^2}\right]$. [DUH-89] (iv). $\lim\limits_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{1}\left(1+\frac{2}{n}\right)^{1/2}\left(1+\frac{2}{n}\right)^{1/3}\cdots\left(1+\frac{n}{n}\right)^{1/n}\right]$

(v). $\lim\limits_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)^{2/n^2}\left(1+\frac{2^2}{n^2}\right)^{4/n^2}\cdots\left(1+\frac{n^2}{n^2}\right)^{2n/n^2}\right]$ **[NUH–94]**

(vi). $\lim\limits_{n\to\infty}\frac{\{(n+1)(n+2)(n+3)\cdots(n+n)\}^{1/n}}{n}$

(vii). $\lim\limits_{n\to\infty}\left(\frac{n!}{n^n}\right)^{1/n}$

(viii). $\lim\limits_{n\to\infty}\left[\sin\frac{\pi}{2n}\cdot\sin\frac{2\pi}{2n}\cdot\sin\frac{3\pi}{2n}\cdots\sin\frac{n\pi}{2n}\right]^{1/n}$

(ix). $\lim\limits_{n\to\infty}\left[\tan\frac{\pi}{2n}\cdot\tan\frac{2\pi}{2n}\cdot\tan\frac{3\pi}{2n}\cdots\tan\frac{n\pi}{2n}\right]^{1/n}$

উত্তরমালা [ANSWER]

1(i). $ln2$	(ii). $ln3$	(iii). $ln(4/3)$
(iv). $ln(b/a)$	(v). $\pi/2$	(vi). $\pi/2$
(vii). $\sqrt{3}-1$	(viii). $\pi/2$	(ix). 2
2(i). $\pi/4$	(ii). $\pi/4$,	(iii). $1/\sqrt{2}$
(iv). $1/3$	(v). $1/2$	(vi). $\frac{1}{2}\tan^{-1}2$
(vii). $(5-\sqrt{5})/10$	(viii). $\sqrt{2}.\tan^{-1}\sqrt{2}-\pi/4$	(ix). $ln(3/2)$
(x). $\pi/3$	(xi). $\pi/3$	(xii). $1/105$
3(i). $\pi/4+(1/2)\,ln2$	(ii). $\frac{\pi}{4}$	(iii). $2(2\sqrt{2}-1)/3$
(iv). $\pi/2+1$	(v). $3/2$	**4**(i). $1/11$
(ii). $\frac{1}{2}ln(4/3)$	(iii). $1/(m+1)$	(iv). $\int_0^1 f(x)\,dx;\ \frac{2}{3}$
(v). $\frac{1}{2}\tan 1$	(vi). $\frac{(2k)!}{2^{2k}(k!)^2}$	**5**(i). $4/e$
(ii). $e^{\pi^2/48}$	(iii). $\sqrt{27/4e}$	(iv). $e^{\pi^2/12}$
(v). $4/e$	(vi). $4/e$	(vii). $1/e$
(viii). $1/2$	(ix). 1.	

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