

CHAPTER-5 Surface Tension

5.1 Introduction

A liquid flowing out slowly from the tip of a medicine dropper comes out in the form of fine drops and not as a continuous stream. A sweing needle, if placed carefully on the water surface, makes a small depression on the surcace and rests there without sinking, even though its density is higher than that of water. When a clean glass tube of small bore (a Capillary tube is dipped into water, the water rises in the tube, but if the tube is dipped in mordury, the morecury is depressed. All those phenomena, and many others of a similar nature, are associated with the existence of a boundary surface between a liquid and some other substance.

All surface phenomena indicate that the surface of a liquid can be considered to be in a state of stress such that if one considers any line lying in or bounding the surface, the material on either side of the line exerts a pull on the material on the other side. This pull lies in the plane of the surface and perpendicular to the line. The effect can be demonstrated with the simple apparatus shown in Fig. 5.1. A wire ring a few mm in diameter has attached to it a loop of thread, as shown. Then the ring and thread are dipped in a soap solution and removed. A thin film of liquid is formed in which the thread "floats" freely as shown in Fig. 5.1 (a) If the film inside the loop of thread is punctured, the thread springs out into a circular shape as in Fig. 5.1(a) as if the surface of the liquid were pulling radially outward on it, as shown by

the arrows. Presumably, the same forces were acting before the film was punctured, but the film on every portion of the thread was zero.

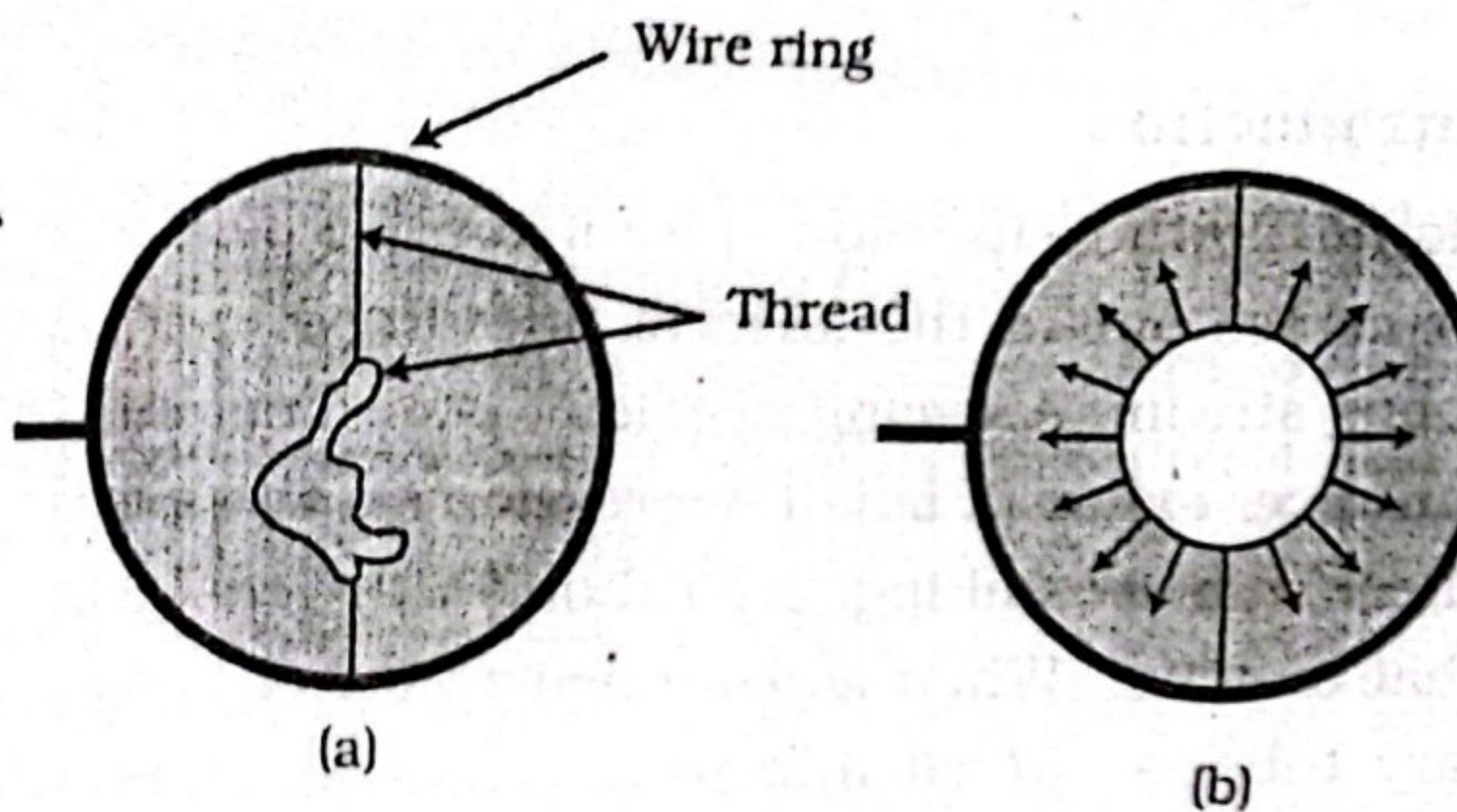


Fig. 5.1

5.2 Definition of surface tension

The surface of a liquid mass behaves somewhat like a stretched elastic membrane. The property of a liquid by virtue of which the free surface of a liquid behave as like a stretched elastic membrane is called **surface tension**. If on the surface of a liquid a straight line say, AB Fig. 5.2 is imagined to be drawn, the portions of the liquid surface on both sides of the straight line tend to draw away from each other.

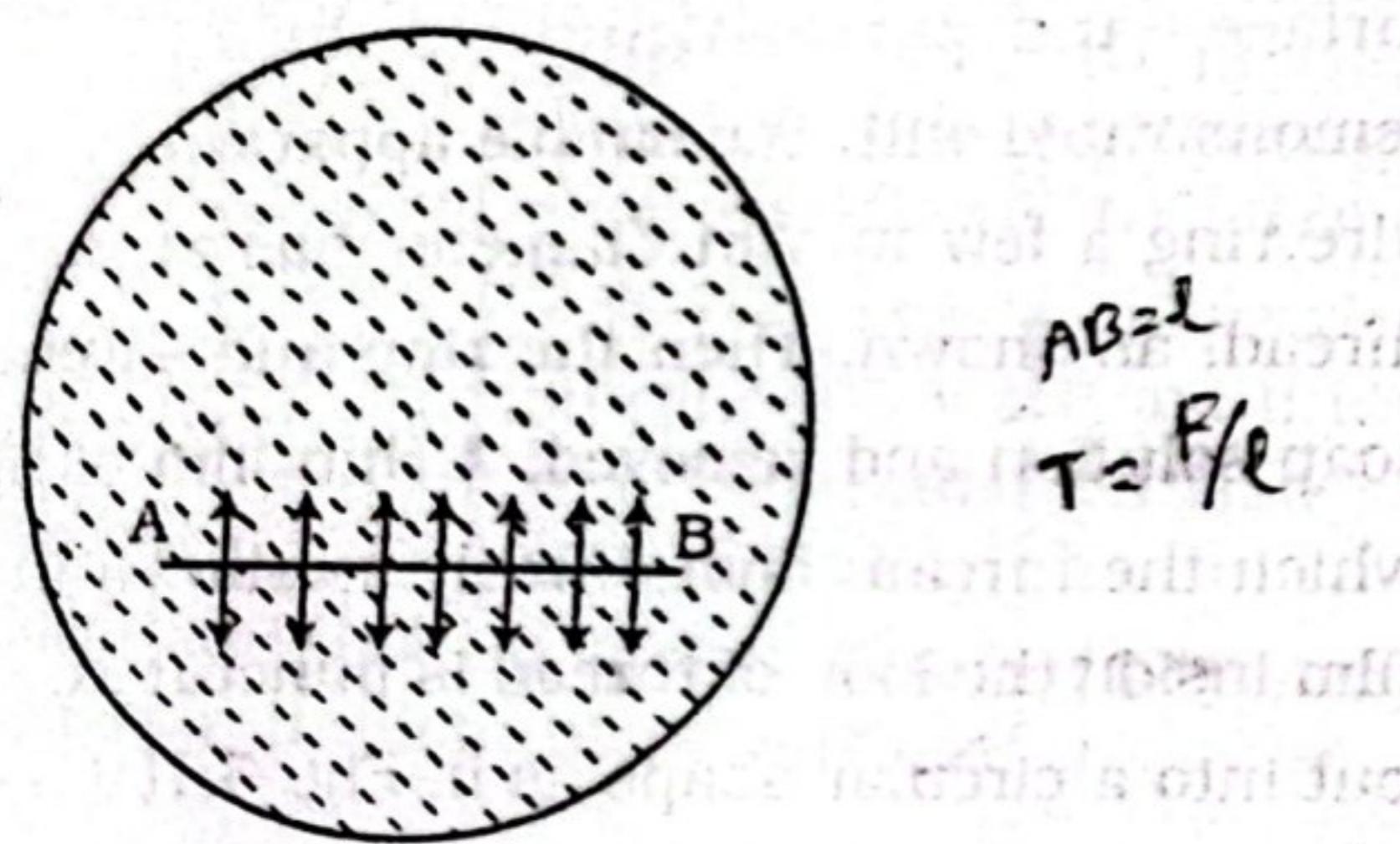


Fig. 5.2

The consequent split of the liquid surface is prevented by the force of attraction lying between the molecules on opposite side of the straight line. The force, with which each portion of the surfaces on opposite sides of the straight line of unit length is pulled away, is called the **Surface Tension** of the liquid.

It must be observed that this force acts perpendicular to the straight line and tangential to the liquid surface.

5.3 Surface Film and surface Energy

If plane CD be drawn parallel to the free surface layer AB Fig. 5.3 of a liquid at a distance equal to the molecular range from it, the layer of the liquid, lying in between the free surface and this plane, is called the **surface film**. Clearly, all the molecules in the film are being acted upon by a resultant cohesive force in the downward direction, the magnitude of which increases as the free surface of the liquid is approached.

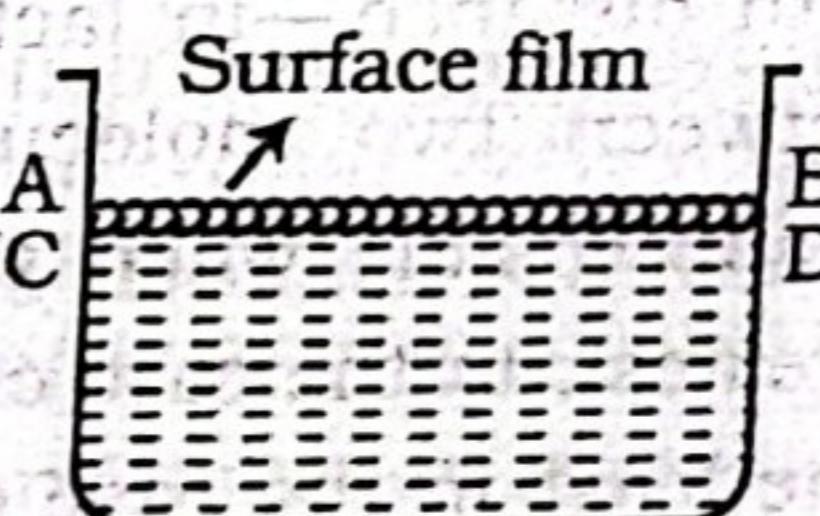


Fig. 5.3

When a molecule is brought from the interior of the liquid to the surface-film, work has to be done against the downward cohesive force acting upon it, and its potential energy is thus increased. It follows, therefore, that the potential energy of the molecules in the surface film is greater than that of those inside the liquid.

Since, however, the potential energy of a system tends towards a minimum, (i.e., a system tends to have the lowest possible potential energy), the film tends to have the least surface area, (its thickness, equal to the molecular range, being already fixed) in order that the number of molecules in it may be a minimum. The potential energy per unit area of the surface film is called its **surface energy**.

* When the area of a liquid surface is increased, work is done against surface tension. This work is stored in the surface or surface energy.

5.4 Molecular Forces

There are two types of molecular forces:

- (i) forces of adhesion, or adhesive forces, and
- (ii) forces of cohesion or cohesive forces.

(i) **Adhesion** is the force of attraction between molecules of different substances, and is different for different pairs of substances, e.g., gum has a greater adhesive force than water or alcohol.

(ii) **Cohesion**, on the other hand, is the force of attraction between molecules of the same substance. This force is different from the ordinary gravitational force and does not obey the ordinary inverse square law; the force varying inversely probably as the eighth power of the distance between two molecules and thus decreases rapidly with distance,—in fact it is appreciable when the distance between two molecules is inappreciable and becomes inappreciable when the distance is appreciable. It is the greatest, in the case of solids, less in the case of liquids and the least in the case of gases, almost negligible at ordinary temperature and pressure. When the molecules lie very much further apart for it to be appreciable. This explains at once why a solid has a definite shape, a liquid has a definite free surface and a gas has neither.

5.5 Molecular Theory of surface Tension

Laplace's investigations, on the surface tension of liquids from the molecular conception of matter, throw a great deal of light on the subject and explain many phenomena observed in the behaviour of liquids, i.e. latent heat of vaporisation of liquids, etc.

He started with the assumption that the force between two molecules of a liquid is very intense when the distance between them is small. But the force falls so rapidly with the

increase of distance between the molecules that it may be taken as vanishingly small when the distance between molecules exceeds a certain value c called the range of molecular attraction or simply molecular range as mentioned earlier. A spherical surface imagined to be drawn round a molecule with radius c is called the sphere of attraction of the molecule or sphere of influence.

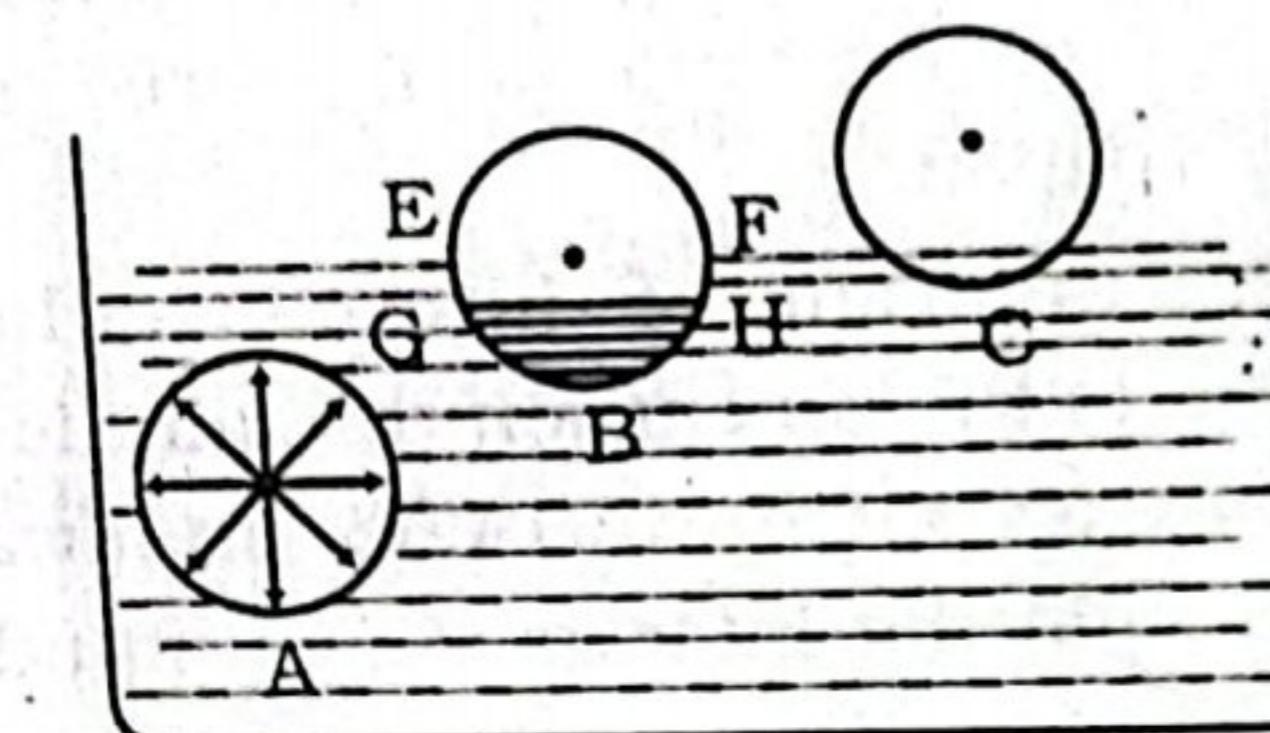


Fig. 5.4

When a molecule A Fig. 5.4 of a liquid is situated well inside the liquid, its sphere of attraction lies wholly within the liquid. This molecule exerts equal attraction simultaneously in all directions and is itself uniformly attracted by the molecules all around it and therefore no resultant force, tending to move it in any direction is exerted on it.

Near the surface, at a distance less than c (the range of molecular attraction) from the surface, a molecule behaves in a different way. The molecule in this case is subjected to a greater force on one side than on the other, the resultant force tending to drive the molecule towards the interior. Let B represents a molecule near the surface. The sphere of attraction of B will have some portion above the surface EF of the liquid. Draw a plane GH parallel to the surface as far below B as the surface is above it. The molecules of the liquid, lying within the plane and the surface, exert no resultant force on the molecule at B, as the forces exerted by the molecules in this region on the molecule at B mutually counterbalance. The liquid molecules in the portion (deeply

shaded of the sphere below the plane GH will exert a resultant force on B which is unbalanced.

Again the molecules being on the surface itself experience only the force of attraction of these molecules within the range of molecular attraction. The resultant of these forces is directed towards the interior of the liquid.

But if the molecule be at C, the downward force becomes less and less and the molecule is then free to fly up as a molecule of gas or vapour.

Thus the assumption of intermolecular forces of attraction leads us to the conclusion that all the molecules in or very near the surface must experience an inward force and tend to move towards the interior of the liquid and so the surface of the liquid tends to contract and behave like a stretched membrane. Thus the force along the surface tending it to contract is known as **surface tension**.

When we consider the molecules situated near or on the surface their potential energy is greater than when they were in the interior of the liquid, for work must be done by the molecules against the resultant downward pull towards the interior of the liquid while moving up towards the surface of the liquid and therefore the molecules gain potential energy at the expense of their kinetic energy.

We know that any system of bodies arranges itself in such a manner that the potential energy of the system is minimum. So to diminish the total surface energy the area of the surface should be diminished and thus the tendency of the surface to contract is explained. The molecules on the surface lying side by side cling to one another and endow the surface with properties somewhat resembling those of a stretched membrane. Hence the existence of surface tension. The tension in Newton exerted across unit length of any line imagined in the surface of the liquid is called the surface tension of the liquid.

The surface tension depends not only on the liquid itself but also on the medium on the other side of the surface of the liquid

The value of the surface tension of any liquid depends on the temperature and with all liquids it decreases with the rise of temperature. It vanishes at the critical temperature at which there is no definite line of separation between gas and liquid.

The presence of impurities in a liquid reduces the surface tension. This is illustrated in the movement of the few pieces of camphor when thrown in water.

Surface Tension

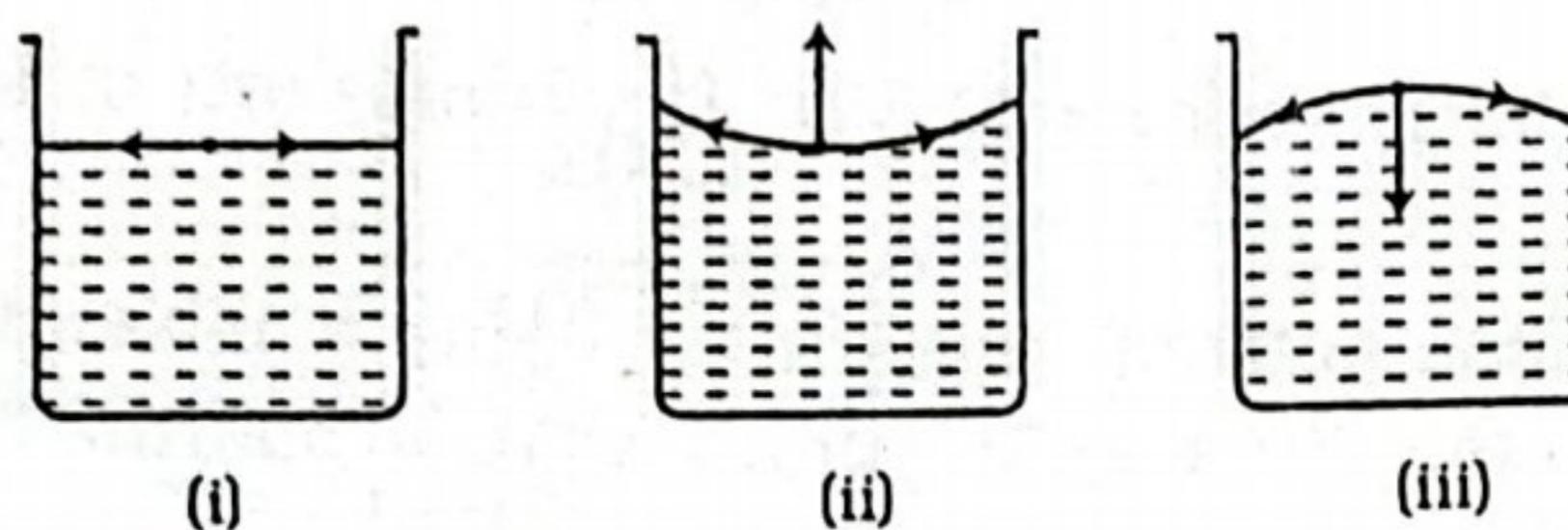


Fig. 5.7

(ii) When the free surface of the liquid is concave outward [Fig. 5.7 (ii)], the resultant force due to surface tension on a molecule on the surface is upwards.

(iii) When the free surface of the liquid is convex outside [Fig. 5.7 (iii)], the resultant force due to surface tension on a molecule on the surface is downwards.

Thus for a curved liquid surface, the pressure on the *concave side is greater than that on the convex surface*. The difference of pressure depends on surface tension of the liquid and the radius of curvature of its surface.

5.9 Excess of Pressure Inside a Spherical Liquid Drop or an Air Bubble in a Liquid.

Consider a spherical liquid drop of radius r (or an air bubble inside a liquid) (Fig. 5.8). The molecules near the surface of the drop experience a resultant pull inward due to surface tension.

Therefore, the pressure inside the drop must be greater than the pressure outside. This excess of pressure can be calculated and it depends upon the surface tension of the liquid and the radius of the drop.

Imagine the drop to be divided into two equal hemispheres by horizontal plane ABCD. Consider the equilibrium of the upper hemisphere. Let the pressures inside and outside be p_1 and p_2 respectively.

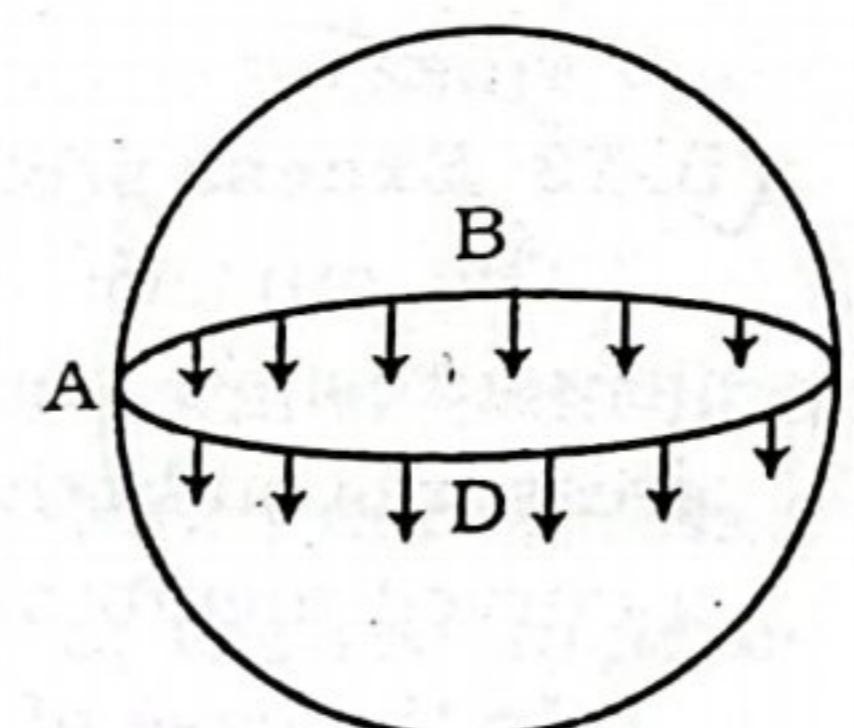


Fig. 5.8

Properties of Matter

The thrust on the hemisphere due to surface tension + the thrust on the hemisphere due to external pressure p_2 = the thrust on the hemisphere due to the internal pressure p_1 .

$$\text{The face area } ABCD = \pi r^2$$

$$\text{The thrust due to } P_2 = p_2 \times \pi r^2$$

$$\text{The thrust due to } p_1 = p_1 \times \pi r^2$$

$$\text{The thrust due to surface tension} = 2\pi r$$

The surface tension acts round the circumference of the drop which has a length = $2\pi r$

∴ For equilibrium,

$$T \times 2\pi r + p_2 \times \pi r^2 = p_1 \times \pi r^2$$

$$= \frac{T \cdot 2\pi r}{\pi r^2} + p_2 = p_1$$

$$(P_1 - P_2) \pi r^2 = \frac{T \cdot 2\pi r}{\pi r^2} = \frac{2T}{r}$$

Let the excess of pressure inside over the pressure outside is represented by p

Then we have,

$$p = p_1 - p_2$$

$$\therefore p = \frac{2T}{r} \quad \dots \dots \dots \quad (ii)$$

The unit of excess of pressure in C.G.S system is dynes/sq. cm and in M.K.S. system in newtons/sq metre.

5.10 Excess of Pressure Inside a Soap Bubble

A soap bubble has two liquid surfaces in contact with air, one inside the bubble and the other outside the bubble. Therefore the total length

$$= 2 \times 2\pi r = 4\pi r$$

For equilibrium,

$$T \times 4\pi r + p_2 \times \pi r^2 = p_1 \times \pi r^2$$

$$\text{or } (p_1 - p_2) = \frac{4\pi r T}{\pi r^2} = \frac{4T}{r} \quad \dots \dots \dots \quad (i)$$

∴ Excess of pressure inside over the pressure outside

$$p = \frac{4T}{r} \quad \dots \dots \dots \quad (ii)$$

Surface Tension

Thus, the excess of pressure is inversely proportional to the radius of the drop or the bubble. (i.e. $P \propto \frac{1}{r}$)

Thus, the smaller the bubble, the greater the excess pressure inside it.

5.5. What amount of energy will be liberated if 1000 droplets of water each 10^{-8} m in radius coalesce to form one large spherical drop? Surface tension of water = 72×10^{-3} N/m.

$$r = 1 \times 10^{-8} \text{ m}$$

$$\frac{4}{3} \pi R^3 = 1000 \times \frac{4}{3} \cdot \pi r^3$$

$$R = 10r = 1 \times 10^{-7} \text{ m}$$

Decrease in surface area

$$= 1000 \times 4\pi r^2 - 4\pi R^2$$

$$= 4\pi (1000 \times (1 \times 10^{-8})^2 - (1 \times 10^{-7})^2)$$

$$= 4\pi \times 9 \times 10^{-14} \text{ m}^2$$

Energy liberated

$$= (72 \times 10^{-3}) (4\pi \times 9 \times 10^{-14}) \text{ J}$$

$$= 8.14 \times 10^{-14} \text{ J}$$

5.6. A spherical soap bubble of radius 2 cm is blown in air. How much energy will be needed to increase the radius to 3 cm? Surface tension of soap solution is 0.04 N/m.

$$\text{Here, } r_1 = 2 \text{ cm} = 0.02 \text{ m}$$

$$r_2 = 3 \text{ cm} = 0.03 \text{ m}$$

$$\text{Increase in area} = 2 [4\pi(r_2^2 - r_1^2)]$$

$$= 8\pi[(0.03)^2 - (0.02)^2]$$

$$= 40\pi \times 10^{-4} \text{ m}^2$$

$$\text{Surface Tension, } T = 0.04 \text{ N/m}$$

$$\text{Energy required} = \text{Increase in area} \times \text{Surface Tension}$$

$$= 40\pi \times 10^{-4}$$

$$= 5.027 \times$$

5.7. A soap bubble is slowly enlarged from a radius of 0.05 m to 0.15 m. Calculate the work done in the process. Surface tension of soap solution is 30 N/m.

$$r_1 = 0.05 \text{ m}$$

$$r_2 = 0.15 \text{ m}$$

$$\frac{4}{3} \pi (a^2 + b^2) = 0$$

$$30 \times 10^{-3} \text{ N/m}$$

Surface Tension

$$\begin{aligned} \text{Increase in area} &= 2 [4\pi(r_2^2 - r_1^2)] \\ &= 8\pi[0.15^2 - (0.05)^2] \\ &= 8\pi \times 0.02 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Surface energy per unit area} \\ = 30 \times 10^{-3} \text{ J/m}^2 \end{aligned}$$

$$\begin{aligned} \text{Increase in the energy of the film} \\ = 30 \times 10^{-3} \times 8\pi \times 0.02 \\ = 15.08 \times 10^{-3} \text{ J} \end{aligned}$$

5.8 A mercury drop of radius 1.4 cm is sprayed into one million drops of the same size. Calculate the energy expended.

Surface tension of mercury = $35 \times 10^{-3} \text{ N/m}$.
Here $T = 35 \times 10^{-3} \text{ N/m}$.

$$\begin{aligned} R &= 1.4 \text{ cm} \\ &= 1.4 \times 10^{-2} \text{ m} \end{aligned}$$

Radius of each small drop = r

$$\therefore \frac{4}{3}\pi R^3 = 10^6 \times \frac{4}{3}\pi r^3$$

$$R = 100r$$

$$\begin{aligned} r &= \frac{R}{100} \\ &= \frac{1.4 \times 10^{-2}}{100} \end{aligned}$$

$$= 1.4 \times 10^{-4} \text{ m}$$

$$\begin{aligned} \text{Increase in area} &= (10^6 \times 4\pi r^2) - 4\pi R^2 \\ &= [(10^6 \times 4\pi(1.4 \times 10^{-4})^2) - 4\pi(1.4 \times 10^{-2})^2] \\ &= 4\pi \times 19.4 \times 10^{-3} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Energy expended} &= (\text{surface tension}) \text{ increase in area} \\ &= 35 \times 10^{-3} \times 4\pi \times 19.4 \times 10^{-3} \text{ J} \\ &= 8.533 \times 10^{-3} \text{ J} \end{aligned}$$

Properties of Matter

5.9. Assuming that the surface tension of rain water is 72 dynes/cm., find the difference of pressures between inside and outside of a rain drop of diameter 0.02 cm.

Since a rain drop has only one surface

$$\begin{aligned} P &= \frac{2T}{r} \\ &= \frac{2 \times 72}{0.01} \text{ 14400 dynes/cm}^2 \end{aligned}$$

5.10. An air bubble of radius 0.2 mm is situated just below the surface of water. Calculate the gauge pressure (excess of pressure) inside the air bubble. Surface tension of water = $7.2 \times 10^{-2} \text{ N/m}$.

$$\text{Here } r = 0.2 \text{ mm} = 2 \times 10^{-4} \text{ m}$$

$$T = 7.2 \times 10^{-2} \text{ N/m}$$

$$\begin{aligned} \text{Excess of pressure} &= \frac{2T}{r} \\ &= \frac{2 \times 7.2 \times 10^{-2}}{2 \times 10^{-4}} \\ &= 720 \text{ N/m}^2 \end{aligned}$$

5.11. Calculate the excess of pressure between the inside and outside of a soap bubble of radius 1 cm. Surface tension of soap solution is $3.0 \times 10^{-2} \text{ N/m}$.

$$\text{Here } r = 1 \text{ cm} = 10^{-2} \text{ m}$$

$$T = 3.2 \times 10^{-2} \text{ N/m}$$

$$\begin{aligned} \text{Excess of pressure} &= \frac{4T}{r} \\ &= \frac{4 \times 3.0 \times 10^{-2}}{10^{-2}} \\ &= 12.0 \text{ N/m}^2 \end{aligned}$$

5.12. A large soap bubble of radius a contains inside it a smaller soap bubble of radius b . The smaller bubble bursts isothermally with no leakage of air from the system as a whole, and a new bubble of radius R is formed.

$$\text{Show that } P[R^3 - a^3] + 4T[R^2 - a^2 - b^2] = 0$$