Magnetic fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers.

**Production of a Magnetic Field: Ampere's Law**

The basic law governing the production of a magnetic field by a current is Ampere's law:

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{net}$$

where **H** is the magnetic field intensity produced by the current $I_{net}$ and *dl* is a differential element of length along the path of integration.
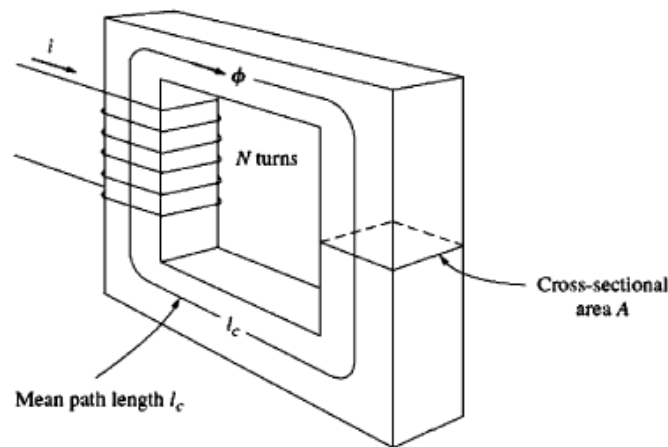


Figure shows a rectangular core with a winding of *N* turns of wire wrapped around one leg of a ferromagnetic core; essentially all the magnetic field produced by the current will remain inside the core, so the path of integration in Ampere's law is the mean path length of the core. The current passing within the path of integration $I_{net}$ is then *Ni,* since the coil of wire cuts the path of integration N times while carrying current i. Ampere's law thus becomes

$$Hl_c = Ni$$

$$H = \frac{Ni}{l_c}$$

The magnetic field intensity **H** is in a sense a measure of the "effort" that a current is putting into the establishment of a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. The relationship between the magnetic field intensity **H** and the resulting magnetic flux density **B** produced within a material is given by

$$\boldsymbol{B} = \mu\,\boldsymbol{H}$$

Now the total flux in a given area is given by

$$\emptyset = \int_A \boldsymbol{B} \cdot d\boldsymbol{A}$$

where d**A** is the differential unit of area. If the flux density vector is perpendicular to a plane of area A, and if the flux density is constant throughout the area, the equation reduces to:

$$\emptyset = BA = \frac{\mu N i A}{l_c}$$

**FARADAY'S LAW-INDUCED VOLTAGE FROM A TIME-CHANGING MAGNETIC FIELD**

Faraday's law states that if a flux passes through a turn of a coil of wire, a voltage will be induced in the turn of wire that is directly proportional to the *rate of change* in the flux with respect to time. In equation form,

$$e_{ind} = -\frac{d\emptyset}{dt}$$

If a coil has *N* turns and if the same flux passes through all of them, then the voltage induced across the whole coil is given by

$$e_{ind} = -N\frac{d\emptyset}{dt}$$

The minus sign in the equations is an expression of *Lenz's law.* Lenz's law states that the direction of the voltage buildup in the coil is such that if the coil ends were short circuited, it would produce current that would cause a flux *opposing* the original flux change.
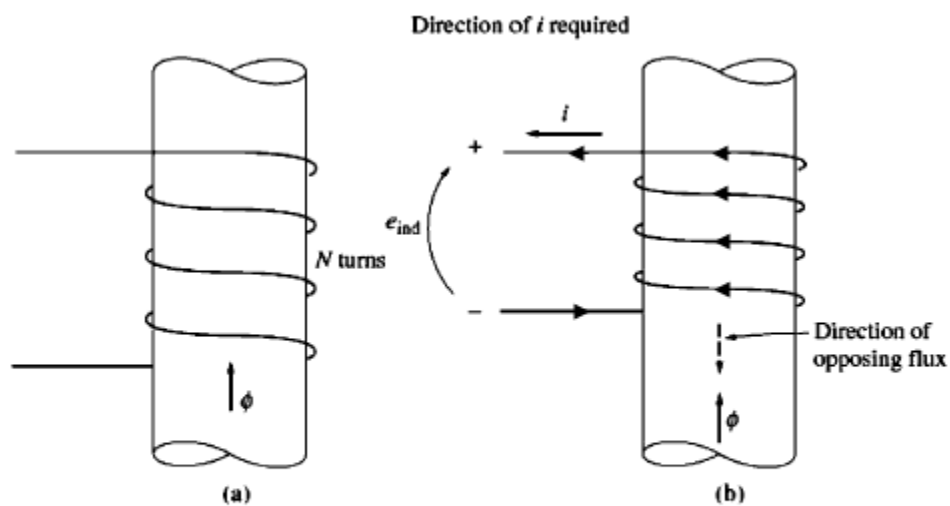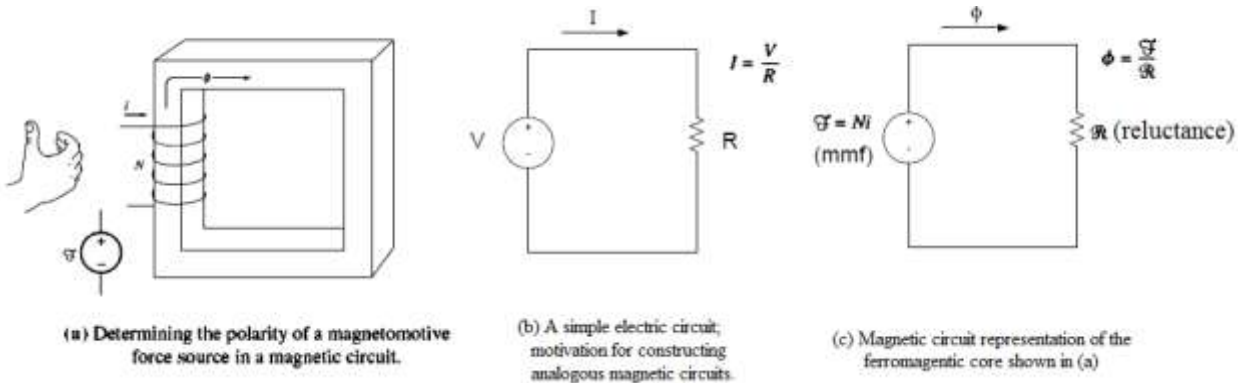


Illustration of Lenz's Law: (a) A coil enclosing a time-varying magnetic field (b) Determination of polarity of induced voltage according to Lenz's Law

**Magnetic Circuit:**

The magnetic circuit model of magnetic behavior is often used in the design of electric machines and transformers to simplify the otherwise quite complex design process.

The simplest magnetic circuit consists of a current carrying coil wrapped around a core can be considered analogous to an electrical circuit consisting a voltage source connected to a resistor.



(a) Determining the polarity of a magnetomotive force source in a magnetic circuit.

(b) A simple electric circuit; motivation for constructing analogous magnetic circuits.

(c) Magnetic circuit representation of the ferromagentic core shown in (a)

In an electric circuit, the applied voltage (electromotive force) causes a current to flow. Similarly, in a magnetic circuit, the applied magnetomotive force causes flux to be produced. The relationship between voltage and current in an electric circuit is known as Ohm's law *(I = V/R).* The relationship between magnetomotive force and flux is known as Hopkinson's law and given by a similar formula.

$$\emptyset = \frac{\mathcal{F}}{\mathcal{R}}$$

The magnetomotive force of the magnetic circuit is equal to the effective current flow applied to the core, or $\mathcal{F} = Ni$ where $\mathcal{F}$ the symbol for magnetomotive force, measured in ampere-turns.

The reluctance ($\mathcal{R}$) of a magnetic circuit is the counterpart of electrical resistance, and its units are ampere-turns per weber.

$$\mathcal{R} = \frac{l_c}{\mu A}$$

Like the voltage source in the electric circuit, the magnetomotive force in the magnetic circuit has a polarity associated with it. The *positive* end of the mmf source is the end from which the flux exits, and the *negative* end of the mmf source is the end at which the flux reenters. The polarity of the mmf from a coil of wire can be determined from a modification of the right-hand rule: If the fingers of the right hand curl in the direction of the current flow in a coil of wire, then the thumb will point in the direction of the positive mmf.

**Magnetic Behavior of Ferromagnetic Materials**

Materials which are classified as non-magnetic all show a linear relationship between the flux density B and coil current I. In other words, they have constant permeability. Thus, for example, in free space, the permeability is constant. But in iron and other ferromagnetic materials it is not constant.

For magnetic materials, a much larger value of B is produced in these materials than in free space. Therefore, the permeability of magnetic materials is much higher than $\mu_0$. However, the permeability is not linear anymore but does depend on the current over a wide range.

Thus, the **permeability is the property of a medium that determines its magnetic characteristics**. In other words, the concept of magnetic permeability corresponds to the ability of the material to permit the flow of magnetic flux through it.

In electrical machines and electromechanical devices a somewhat linear relationship between B and I is desired, which is normally approached by limiting the current.

Look at the magnetization curve and B-H curve. Note: The curve corresponds to an increase of DC current flow through a coil wrapped around the ferromagnetic core
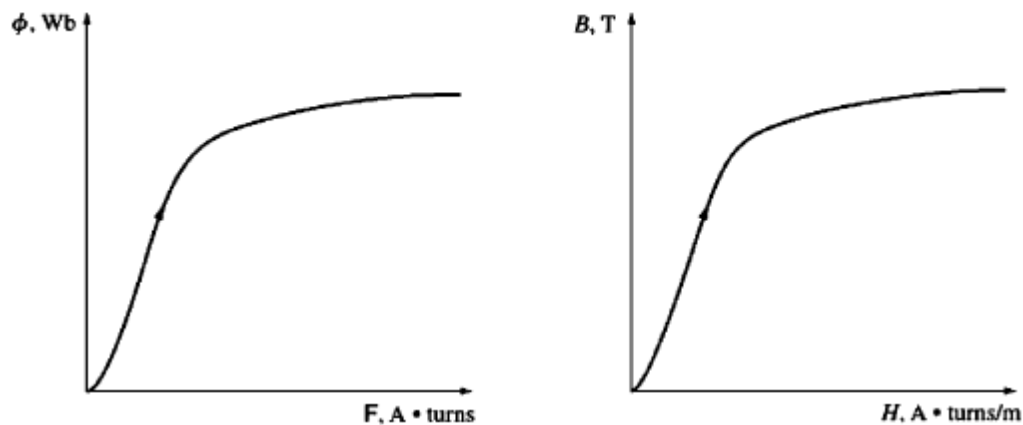
For a "perfect" ferromagnetic core magnetic flux can be expected to grow (until saturation) in positive direction during the positive half–cycle and in the opposite direction during the negative half-cycle. When the ac voltage (or current) goes through zero, magnetic flux should return to zero as well.
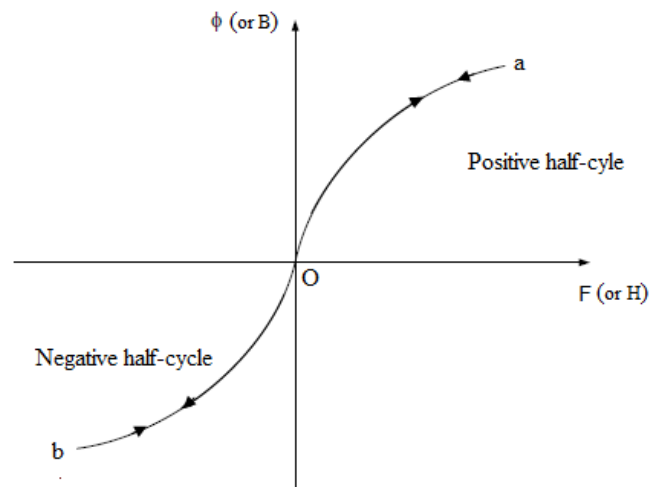
Theoretical magnetic behavior for a "loss-less" ferromagnetic core under ac supply

However, in a real ferromagnetic core, magnetic behavior deviates from that of an "ideal" ferromagnetic core. In a real ferromagnetic core, the amount of flux present in the core depends not only on the amount of current applied to the windings of the core, but also on the previous history of the flux in the core. The dependence of core flux on the preceding flux history and the resulting failure to retrace flux paths is known as hysteresis loss.

Due to hysteresis, the value of core flux is nonzero when the input current becomes zero. This is known as residual flux. To force the flux to zero, an amount of mmf (known as coercive mmf) is to be applied. This coercive mmf is required during every half-cycle and represents a power loss.
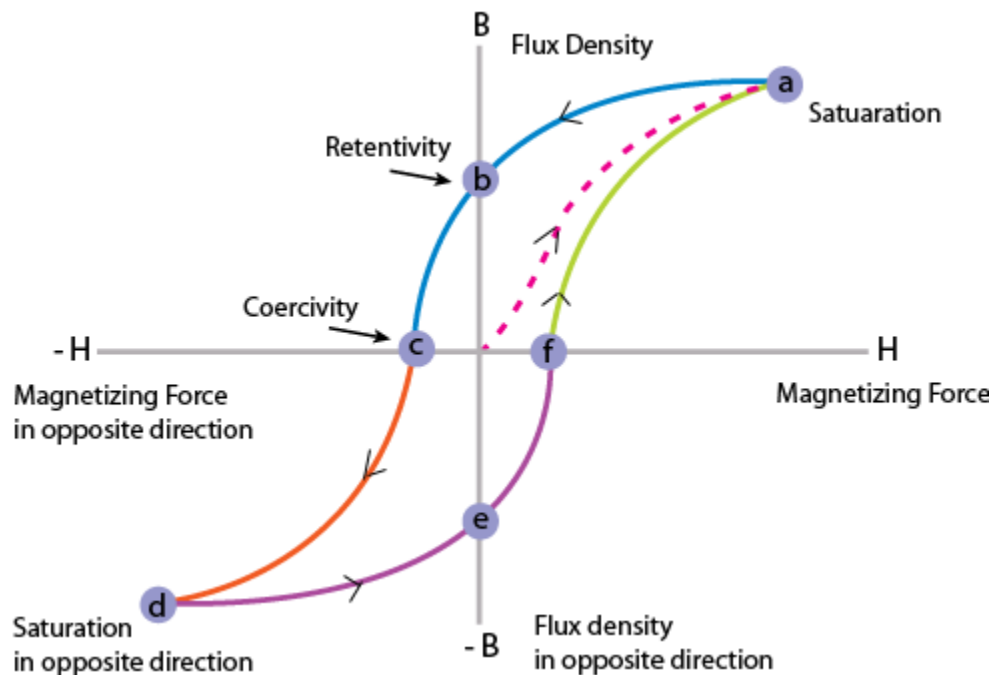
**Explanation of Hysteresis from Magnetic Domain Theory:**

Within each atom, the orbiting electrons are also spinning as they revolve around the nucleus. The atom, due to its spinning electrons, has a magnetic field associated with it. In nonmagnetic materials, the net magnetic field is effectively zero since the magnetic fields due to the atoms of the material oppose each other.

However, in magnetic materials such as iron and steel, the magnetic fields of groups of atoms numbering are aligned, forming very small bar magnets. This group of magnetically aligned atoms is called a **domain.** Each domain is a separate entity; that is, each domain is independent of the surrounding domains. For an unmagnetized sample of magnetic material, these domains appear in a random manner; therefore net magnetic field in any one direction is zero.
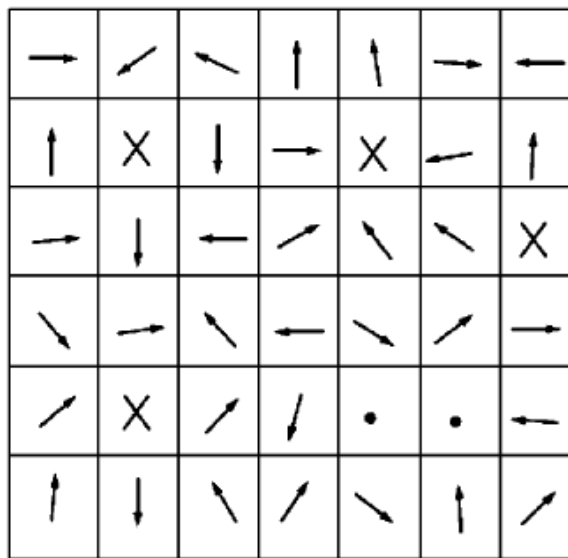


Randomly oriented magnetic domains
in a feerromagnetic material
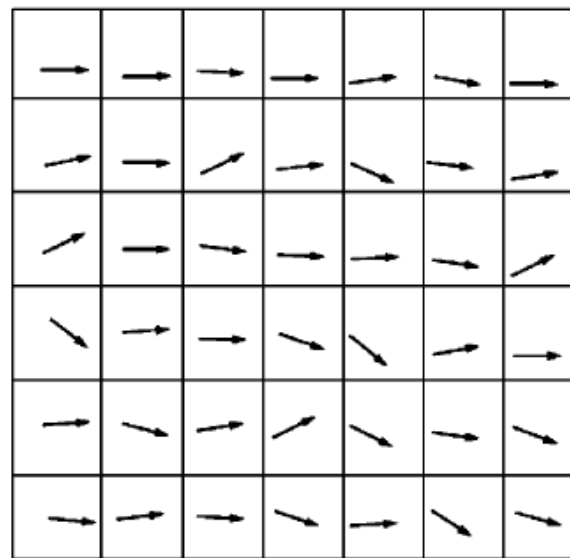
Alignment of magnetic domains in the
presence of an external field

When an external magnetizing force is applied, the domains that are nearly aligned with the applied field will grow at the expense of the less favorably oriented domains. Domains pointing in the direction of the magnetic field grow because the atoms at their boundaries physically switch orientation to align themselves with the applied magnetic field. The extra atoms aligned with the field increase the magnetic flux in the iron, which in turn causes more atoms to switch orientation, further increasing the strength of the magnetic field. It is this positive feedback effect that causes iron to have permeability much higher than air.
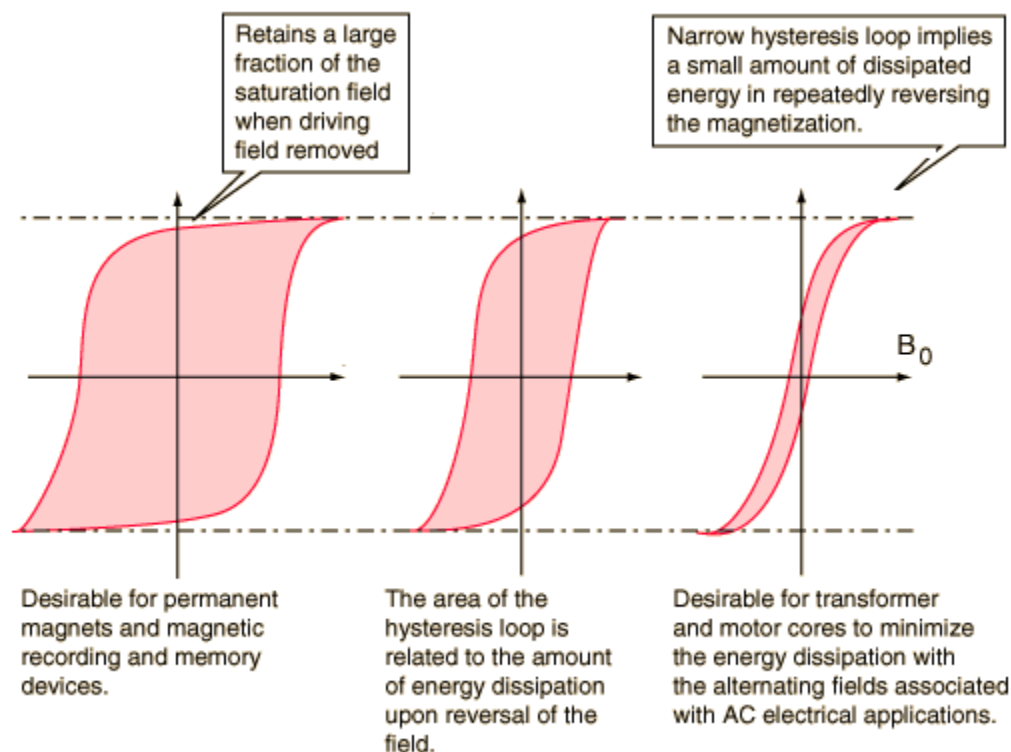
Eventually, if a sufficiently strong field is applied, all of the domains will have the orientation of the applied magnetizing force, and any further increase in external field will not increase the strength of the magnetic flux through the core—a condition referred to as *saturation.*

The key to hysteresis is that when the external magnetic field is removed, the domains do not completely randomize again.  Because turning the atoms in them requires *energy.* Originally, energy was provided by the external magnetic field to accomplish the alignment; when the field is removed, there is no source of energy to cause all the domains to rotate back. The piece of iron is now a permanent magnet.

Once the domains are aligned, some of them will remain aligned until a source of external energy is supplied to change them. Examples of sources of external energy that can change the alignment of domains are magnetomotive force applied in another direction, a large mechanical shock, and heating. Any of these events can impart energy to the domains and enable them to change alignment. (It is for this reason that a permanent magnet can lose its magnetism if it is dropped, hit with a hammer, or heated.)

The fact that turning domains in the iron requires energy leads to a common type of energy loss in all machines and transformers. **The *hysteresis loss* in an iron core is the energy required to accomplish the reorientation of domains during each cycle of the alternating current applied to the core. The area enclosed in the hysteresis loop formed by applying an alternating current to the core is directly proportional to the energy lost in a given ac cycle.**

Hysteresis losses can be effectively reduced by the injection of small amounts of silicon into the magnetic core, constituting some 2% or 3% of the total composition of the core. This must be done carefully, however, because too much silicon makes the core brittle and difficult to machine into the shape desired.



Retains a large fraction of the saturation field when driving field removed

Narrow hysteresis loop implies a small amount of dissipated energy in repeatedly reversing the magnetization.

$B_0$

Desirable for permanent magnets and magnetic recording and memory devices.

The area of the hysteresis loop is related to the amount of energy dissipation upon reversal of the field.

Desirable for transformer and motor cores to minimize the energy dissipation with the alternating fields associated with AC electrical applications.

**Eddy Current Loss:**

When an alternating current passing through a coil wrapped around a ferromagnetic core, it will develop a changing magnetic flux. Due to this time-changing flux, voltage is developed within the core due to electromagnetic induction. This voltage causes swirls of current to flow within the core known as eddy current. Energy is dissipated (in the form of heat) because these eddy currents are flowing in a resistive material (iron). The amount of energy lost to eddy currents is proportional to the **size of the paths** they follow within the core.
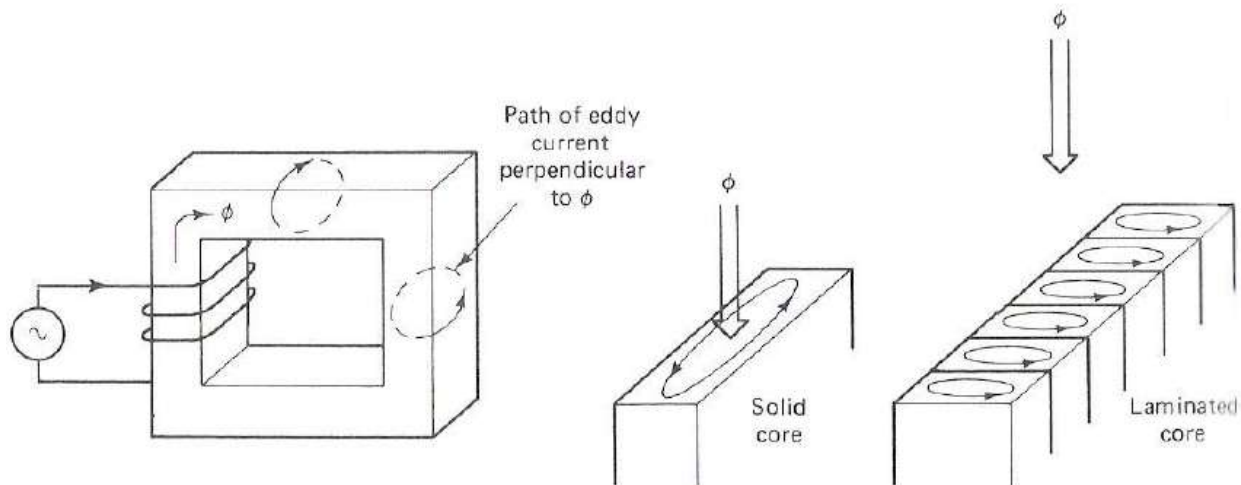


To reduce energy loss, ferromagnetic core should be broken up into small strips, or laminations, and build the core up out of these strips. An insulating oxide or resin is used between the strips, so that the current paths for eddy currents are limited to small areas.

If the core is non-ferromagnetic and has a high resistivity like air, the eddy current losses can be neglected.

Eddy current losses can be reduced if the core is constructed of thin, laminated sheets of ferromagnetic material insulated from one another and aligned parallel to the magnetic flux. Such construction reduces the magnitude of the eddy currents by placing more resistance in their path.