Math (mid – 19 - 20)

2/ Given matrix,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{vmatrix} = 30 \neq 0$$

Hence, A is non singular, A^{-1} exist in the system.

let A_{ij} is the cofactor of a_{ij} in the ~~matri~~ determinant A.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 4 \\ 1 & 1 \end{vmatrix} = -1 - 4 = -5$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = -(2-12) = 10$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 2+3 = 5$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -(2-3) = 1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = (1-9) = -8$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -(1-6) = 5$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} = (8+3) = 11$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = -(4-6) = 2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1-4 = -5$$

$$\text{adj} A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -5 & 1 & 11 \\ 10 & -8 & 2 \\ 5 & 5 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}A = \frac{1}{30}\begin{bmatrix} -5 & 1 & 11 \\ 10 & -8 & 2 \\ 5 & 5 & -5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{30} & \frac{11}{30} \\ \frac{1}{3} & -\frac{4}{15} & \frac{1}{15} \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \end{bmatrix}$$

Ans.