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Assignment 01: Function, Limit, Continuous Function.

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INTRODUCTION TO FUNCTION

✔️Function :
A function is an expression or rule, or law that defines a relationship between an independent variable and another variable that is dependent.

Examples-
$f(x) = \cos x$, $f(x) = 3x + 4$; these refers to different types of functions.

↪ Domain :
The domain represents a set of all the possible inputs for a function.

↪ Range :
The range represents a set of all the possible values that the function will give when we give in the domain as input.

Examples-

1. Domain and Range of Square-Root Function:

$$y = 2 - \sqrt{-3x+2}$$

➜This is defined only when the value inside the square root is a non-negative number. So for a domain,

$$-3x + 2 \geq 0$$

$$-3x \geq -2$$

$$x \leq 2/3$$

And for the range- We already know that the square root function results in a non-negative value always.

$$\sqrt{(-3x+2)} \geq 0$$

$$-\sqrt{(-3x+2)} \leq 0$$

$$2 - \sqrt{(-3x+2)} \leq 2$$

$$y \leq 2$$

So the Corresponding Domain and Range of $y = 2 - \sqrt{-3x+2}$ is $x \leq 2/3$ and $y \leq 2$.

2. Domain and Range of Trigonometric Functions:

➜The domain and range of trigonometric functions “sine” and “cosine” are given by-

Domain = All Real Numbers $(-\infty, \infty)$

Range = $[-1, 1]$.

3. Find the domain and range of:

$$y = -|x| + 4$$

➜Here,
The range of $y = |x|$ is $y \geqslant 0$ since the absolute value function returns only positive numbers or 0 if the input is 0.

This means that the range of $y = -|x|$ is $y \leqslant 0$ since we are taking every value in the range and making it negative.

So, the Range of $y = -|x| + 4$ is $y \leqslant 4$;since we are adding 4 to every value and $|x| \geqslant 0$.

Therefore, the domain is $X \in R$ and the range is $y \leq 2$.

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↪ Inependent Variable :
An independent variable is a variable that represents a quantity that is being altered in an experiment.

↪ Dependent Variable :
The dependent variable is the one that depends on the value of some other number known as independent variables.
A dependent variable is the variable that changes as a result of the independent variable's alternation.

[Ex: **y=x+2**, is a function where the value of x is not defined. And, it can be manipulated as of wish. Thus it's called an independent variable.
Whether, the value of y on the other hand is completely dependent on the choice of the value of x. Thus it’s called a dependent variable.]

☑️Types of Functions :

There are various types of functions in mathematics which are explained below in detail. The different function types covered here are,

🔸 Based on Elements:

▪ One One Function (Injective function)
▪ Many One Function
▪ Onto Function (Surjective Function)
▪ One One and Onto Function
▪ Into Function
▪ Constant Function

🔸 Based on Equation:

▪ Identity Function
▪ Linear Function
▪ Quadratic Function
▪ Cubic Function
▪ Polynomial Functions

🔸 Based on the Range:

▪ Modulus Function
▪ Rational Function
▪ Signum Function
▪ Even and Odd Functions
▪ Periodic Functions
▪ Greatest Integer Function
▪ Inverse Function
▪ Composite Functions

🔸 Based on the Domain:

▪ Algebraic Functions
▪ Trigonometric Functions
▪ Logarithmic Functions

▪ Fractional Part Function (Decimal Part Function)

INTRODUCTION TO LIMIT

✔️Limit of a Function:
The limit of a function is a value of the function, towards which- the input of the function gets closer or approaches near.

[Limits are used to define integrals, derivatives, and continuity. The limit of a function is always concerned with the behavior of the function at a particular point.]

☑️Types of Limits :

The theory of limits of functions is the at the foundation of calculus because upon these, the notion of a derivative depends.

A function may approach two different limits. One where the variable approaches its limit through values larger than the limit and the other where the variable approaches its limit through values smaller than the limit. In such a case, the limit is not defined but the right and left-hand limit exist.

↪ Right-hand Limit:
The right-hand limit of a function is the value of the function approaches when the variable approaches its limit from the right.

If $\lim\limits_{x \longrightarrow a^+} f(x)$ is the expected value of f at x = a given the values of 'f' close to x to the right of a. This value is known as the right-hand limit of f(x) at a.

↪ Left-hand Limit
The left-hand limit of a function is the value of the function approaches when the variable approaches its limit from the left.

If $\lim\limits_{x \longrightarrow a^-} f(x)$ is the expected value of f at x = a given the values of 'f' close to x to the left of a. This value is known as the left-hand limit of 'f' at a.

The limit of a function exists if and only if the left-hand limit is equal to the right-hand limit.

Finding out if a Function has Limit or not:

Examples-

1. Does the limit of following function exists?

$$f(x) = \frac{|x|}{x}$$

➜ When,

Left-hand limit x→

$$\lim_{x \to 0^-} \frac{-x}{x} = \lim_{x \to 0^-} -1 = -1$$

Right-hand limit x→

$$\lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$$

Here, $\lim_{x \to 0^+} \frac{x}{x} \neq \lim_{x \to 0^-} \frac{-x}{x}$.

Therefore, $\lim_{x \to 0} \frac{|x|}{x}$ does not exists;meaning the limit of $f(x) = \frac{|x|}{x}$ does not exists.

2. At x=2, check the limit of $f(x) = x + 2$.

➜ At the point of x = 2, the limit exists if,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$$

$$\lim_{x \to 2^+} (x+2) = \lim_{x \to 2^-} (x+2)$$

Now, **h** is a sufficiently small number that, $h \longrightarrow 0$.

So,

$$x \longrightarrow 2^+$$

x = 2 + h

For the right-hand limit,

$$\lim_{x \longrightarrow 2^+} (x+2) \; = \; \lim_{h \longrightarrow 0} (2+h+2) \; = 4$$

And,

$$x \longrightarrow 2^-$$

x = 2 - h

For the left-hand limit,

$$\lim_{x \longrightarrow 2^-} (x+2) \; = \; \lim_{h \longrightarrow 0} (2-h+2) \; = \; 4$$

As $\lim_{x \longrightarrow 2^+} (x+2) = \lim_{x \longrightarrow 2^-} (x+2)$ =4.

Therefore the limit of $f(x) \; = x+2$ exists at the point x=2.

INTRODUCTION TO CONTINUOUS FUNCTION

✔️Continuous Function:
A continuous function is a function that does not have any discontinuities.

If, $f(x)$ is a real function on a subset of Real Numbers and let c be a point in the domain of f. Then f is continuous at c if,

$$\lim_{x \longrightarrow c^+} f(x) = \lim_{x \longrightarrow c^-} f(x) = f(c)$$

Here the left-hand limit, right-hand limit, and the function's value at x = c exist and are equal to each other, the function f is continuous at x = c.

In other words, if f is not continuous at c, then f is discontinuous at c; thus c is called a point of discontinuity of the given function f.

Examples-

1. Find out if the below function is continuous at x=2 or not.

f(x) = 3x + 7.

➜ f(x) will be continuous at x=1, if and only if

$$\lim_{x \longrightarrow 2^+} f(x) = \lim_{x \longrightarrow 2^-} f(x) = f(2)$$

Now, **h** is a sufficiently small number that, $h \longrightarrow 0$.
So,

$$x \longrightarrow 2^-$$

x = 2 - h

For the left-hand limit,

$$\lim_{x \longrightarrow 2^-} (3x+7) \;=\; \lim_{h \longrightarrow 0} (3(2-h)+7) \;=\; \lim_{h \longrightarrow 0} (6-3h+7) \;=\; 13$$

And,

$$x \longrightarrow 2^+$$

x = 2 + h

For the right-hand limit,

$$\lim_{x \longrightarrow 2^+} (3x+7) \;=\; \lim_{h \longrightarrow 0} (3(2+h)+7) \;=\; \lim_{h \longrightarrow 0} (6+3h+7) \;=\; 13$$

Therefore the limit of the function of f(x) exists. And, f(x) = 3x + 7 is continuous at x=2.

2. Check the continuity of the below function at x = 1.

$$f(x) = \begin{cases} x+1 & x > 1 \\ 2 & x = 1 \\ x^2+1 & x < 1 \end{cases}$$

➜ f(x) will be Continuous if and only if,

$$\lim_{x \longrightarrow 1^+} f(x) \;=\; \lim_{x \longrightarrow 1^-} f(x) \;=\; f(1)$$

Let, **h** be a sufficiently small number that,

$$h \longrightarrow 0$$

For the left-hand limit,

$$x \longrightarrow 1^-$$

x = 1 - h;

$$\lim_{x\longrightarrow 1^-} f(x) = \lim_{x\longrightarrow 1^-}(x+1) = \lim_{h\longrightarrow 0}\left((1-h)^2+1\right) = \lim_{h\longrightarrow 0}\left(1^2+2h+h^2+1\right) = 2$$

For the right-hand limit,

$$x \longrightarrow 1^+$$

x = 1 + h;

$$\lim_{x\longrightarrow 1^+} f(x) = \lim_{x\longrightarrow 1^+}(x+1) = \lim_{h\longrightarrow 0}(1+h+1) = 2$$

And given that,
When, x = 1 then, f(x)=2.

So,

$$\lim_{x\longrightarrow 1^+} f(x) = \lim_{x\longrightarrow 1^-} f(x) = f(1)$$

Therefore, f(x) is continuous at x=1.

And, $\lim_{x\to 1} f(x)$ exists; when $f(x) = \begin{cases} x+1 & x>1 \\ 2 & x=1 \\ x^2+1 & x<1 \end{cases}$.