Patuakhali Science and Technology University.

Group Assignment of MAT-111

Submitted By:

Nayema Ferodushi -2102026

Md. Afridi Alom Pranto -2102027

Shawan Mahamud Abdullah -2102028

Sayed Saiful Islam Shuvo -2102029

Yasin Arafat -2102030

Submitted To:

Muhammad Masudur Rahman.

Associate professor of Mathematics Department.

Faculty of Computer Science and Engineering.

Date: 11 April,2023.

Partial Differentiation:

Given a function two or more variables, ƒ (x, y), the derivative with respect to x only (treating y as a constant) is called the partial derivative of ƒ with respect to x and is denoted by either ∂ƒ / ∂ x or ƒ x. Similarly, the derivative of ƒ with respect to y only (treating x as a constant) is called the partial derivative of ƒ with respect to y and is denoted by either ∂ƒ / ∂ y or ƒ y.

The second partial dervatives of f come in four types:

Notations:

- Differentiate ƒ with respect to x twice. (That is, differentiate ƒ with respect to x; then differentiate the result with respect to x again.)

  $\frac{\partial^2 f}{\partial x^2}$ $or\ f_{xx}$

- Differentiate ƒ with respect to y twice. (That is, differentiate ƒ with respect to y; then differentiate the result with respect to y again.)

  $\frac{\partial^2 f}{\partial y^2}$ $or\ f_{yy}$

Mixed partials:

- First differentiate ƒ with respect to x; then differentiate the result with respect to y.

  $\frac{\partial^2 f}{\partial y\, \partial x}$ or f_{xx}

- First differentiate ƒ with respect to y; then differentiate the result with respect to x.

  $\frac{\partial^2 f}{\partial x\, \partial y}$ or f_{yx}

For virtually all functions ƒ (x, y) commonly encountered in practice, ƒ vx ; that is, the order in which the derivatives are taken in the mixed partials is immaterial.

Example 1: If ƒ (x, y) = 3 x 2 y + 5 x − 2 y 2 + 1, find ƒ x , ƒ y , ƒ xx , ƒ yy , ƒ xy 1, and ƒ yx .

First, differentiating ƒ with respect to x (while treating y as a constant) yields

$$f_x = 6xy + 5$$

Next, differentiating ƒ with respect to y (while treating x as a constant) yields

$$f_y = 3x^2 - 4y$$

The second partial derivative ƒ xx means the partial derivative of ƒ x with respect to x; therefore,

$$f_{xx} = (f_x)_x = \frac{\partial}{\partial x}(f_x) = \frac{\partial}{\partial x}(6xy + 5) = 6y$$

The second partial derivative ƒ yy means the partial derivative of ƒ y with respect to y; therefore,

$$f_{yy} = (f_y)_y = \frac{\partial}{\partial y}(f_y) = \frac{\partial}{\partial x}(3x^2 - 4y) = -4$$

The mixed partial ƒ xy means the partial derivative of ƒ x with respect to y; therefore,

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial x}(6xy + 5) = 6x$$

The mixed partial ƒ yx means the partial derivative of ƒ y with respect to x; therefore,

$$f_{yx} = (f_y)_x = \frac{\partial}{\partial x}(f_y) = \frac{\partial}{\partial x}(3x^2 - 4y) = 6x$$

Note that ƒ yx = ƒ xy, as expected.

Exercise Question:

- **Find fxx,fxy,fyx,fyy for the following functions.**
  1) x cos y + y cos x

  Solution:
  Let,
  f = x cos y + y cos x.
  $\therefore$ 1.cos y – y sin x $\left[\frac{d}{dx}\ (uv) = uv` + vu`\right]$
  $\therefore f_{xx} = 0 - y \cos x$
  $\therefore f_{xy} = -(\sin y + \sin x)$

  Again,
  $\therefore f_y = -x \sin y + \cos x$
  $\therefore f_{yx} = -(\sin y + \sin x)$
  $\therefore f_{yy} = -x \cos y$

- **Show that** $\boldsymbol{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0\ , if\ v = tan^{-1}\frac{y}{x}\ , then\ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0}$ **.**

  Solution:
  Now,
  $v = tan^{-1}\frac{y}{x}$

  $\frac{d}{dx}(v) = \frac{-yz}{x^2+y^2}$
  $\therefore \frac{d^2}{d\,x^2}(v) = \frac{2xyz}{(x^2+y^2)}$ -------(1)
  $\therefore \frac{d^2}{d\,y^2}(v) = \frac{-2xyz}{(x^2+y^2)}$ ---------(2)

$$\therefore \frac{d^2}{d\,z^2}(v) = 0$$ -----------(3)

$\therefore$ 1+2+3,

$$\frac{2xyz}{(x^2 + y^2)} - \frac{2xyz}{(x^2+y^2)} + 0 = 0$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$