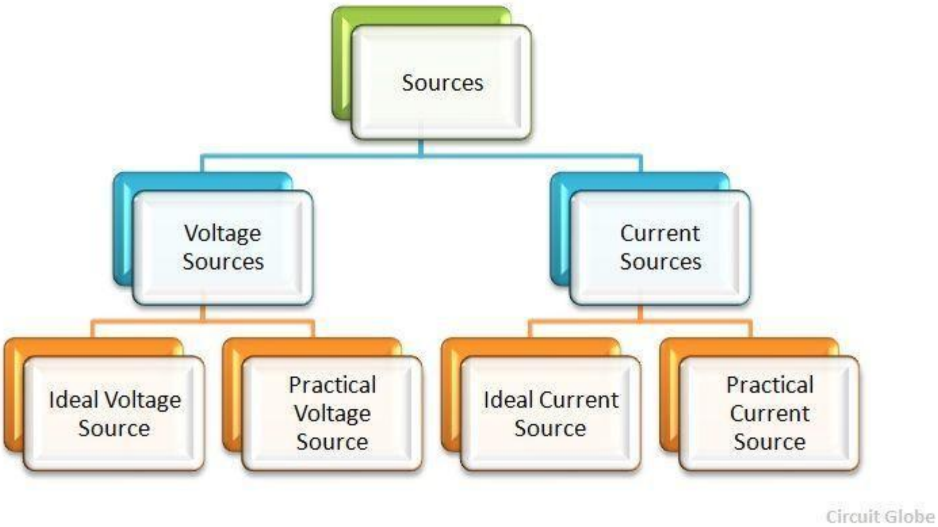
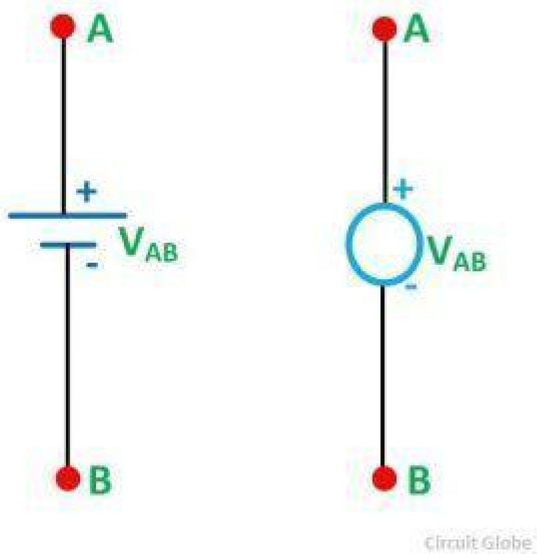
Voltage Source and Current Source

A **Source** is a device which converts mechanical, chemical, thermal or some other form of energy into electrical energy. In other words, the source is an active network element meant for generating electrical energy.





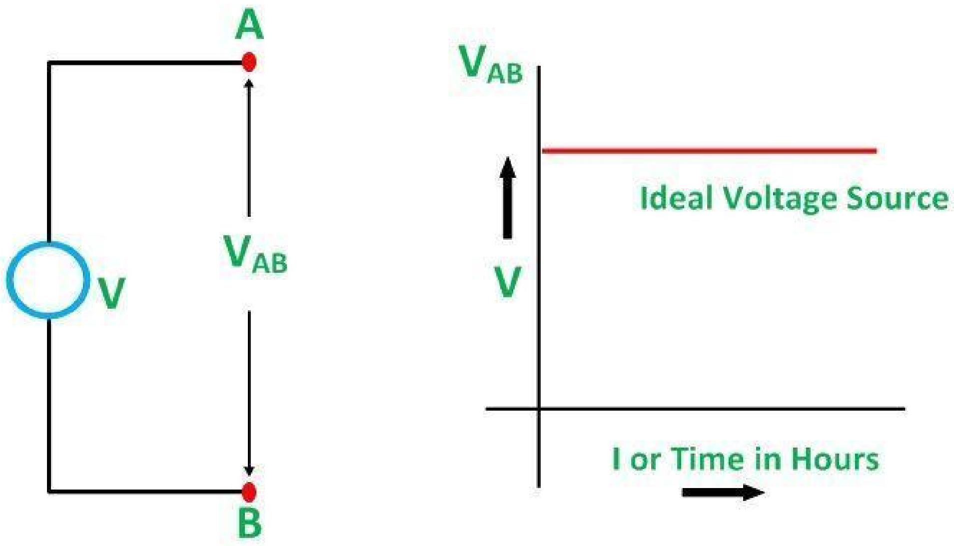


Figure A

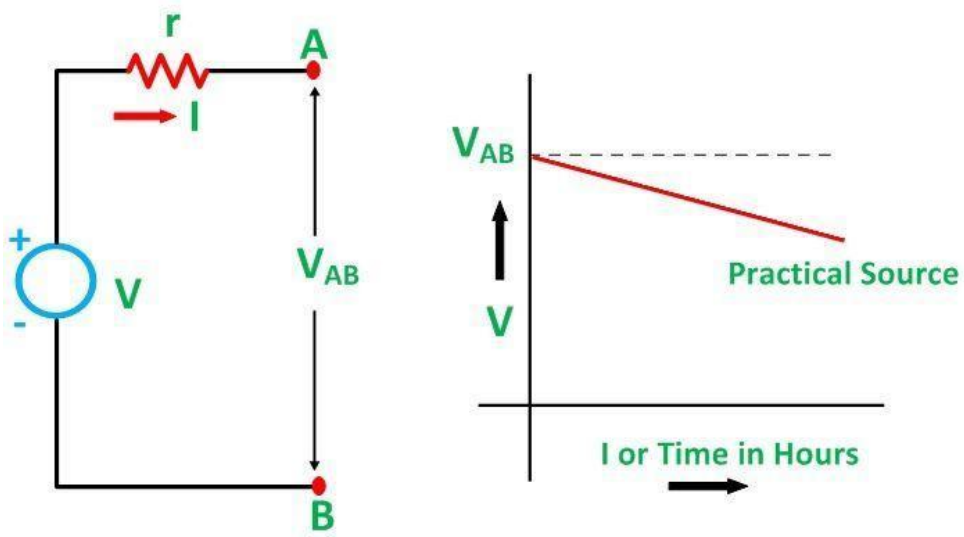
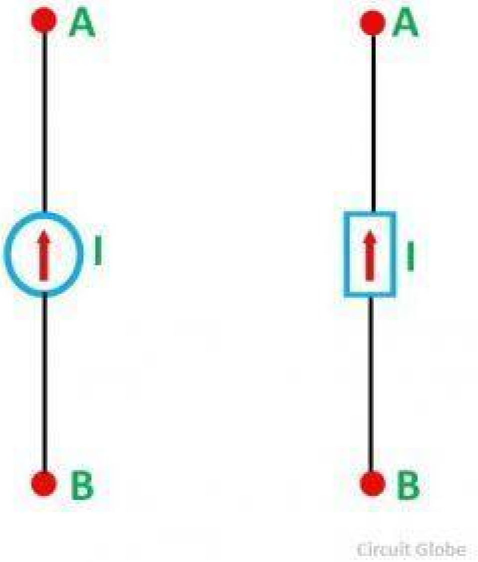


Figure B


A
I
B
A
I
B
Circuit Globe

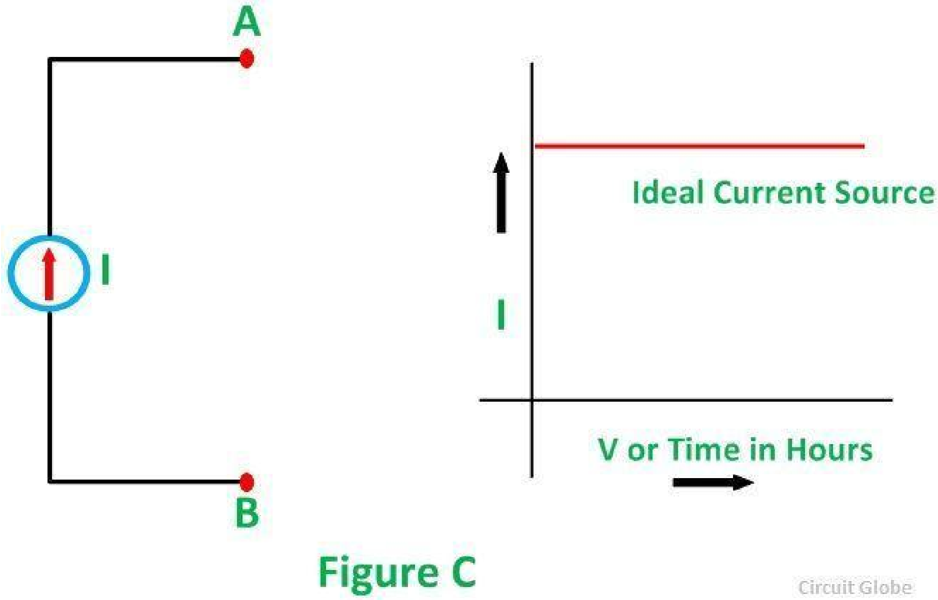


Figure C

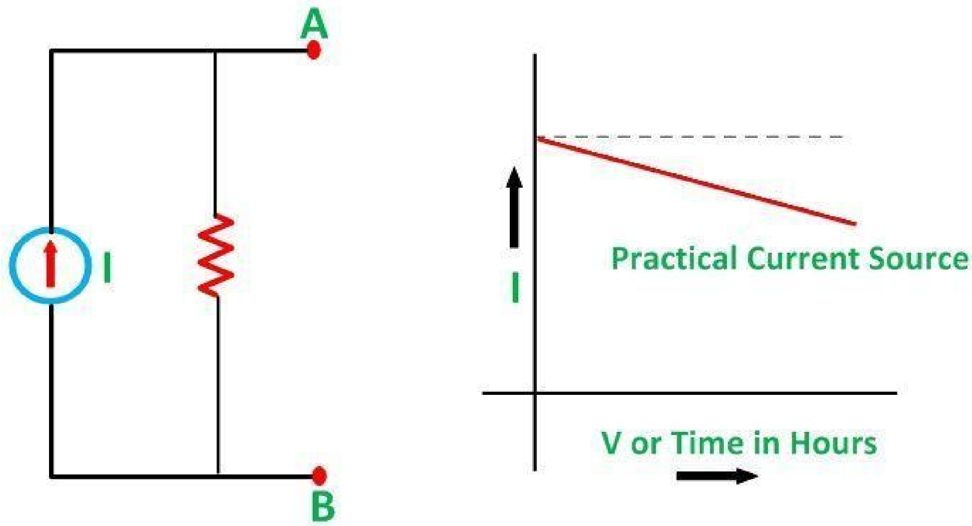


Figure D

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Kirchhoff's First & Second Laws with Solved Example

A German Physicist “Robert Kirchhoff” introduced two important electrical laws in 1847 by which, we can easily find the equivalent resistance of a complex network and flowing currents in different conductors. Both AC and DC circuits can be solved and simplified by using these simple laws which is known as Kirchhoff’s Current Law (KCL) and Kirchhoff’s Voltage Law (KVL).

Also note that KCL is derived from the charge continuity equation in electromagnetism while KVL is derived from Maxwell – Faraday equation for static magnetic field (the derivative of B with respect to time is 0)

Kirchhoff’s Current Law (KCL):
According to KCL, at any moment, the algebraic sum of flowing currents through a point (or junction) in a network is Zero (0) or in any electrical network, the algebraic sum of the currents meeting at a point (or junction) is Zero (0). This law is also known as Point Law or Current law.
In any electrical network, the algebraic sum of incoming currents to a point and outgoing currents from that point is Zero. Or the entering currents to a point are equal to the leaving currents of that point.

In other words, the sum of the currents flowing towards a point is equal to the sum of those flowing away from it. Or the algebraic sum of the currents entering a node equals the algebraic sum of the currents leaving it.

Explanation of KCL:
Suppose some conductors are meeting at a point “A” as shown in fig 1.a. in some conductors, currents are incoming to the point “A” while in other conductors, Currents are leaving or outgoing from point “A”.

Consider the incoming or entering currents as “Positive (+) towards point “A” while the leaving or outgoing currents from point “A” is “Negative (-)”.
then:

$$I_1 + (-I_2) + (-I_3) + (-I_4) + I_5 = 0$$

OR

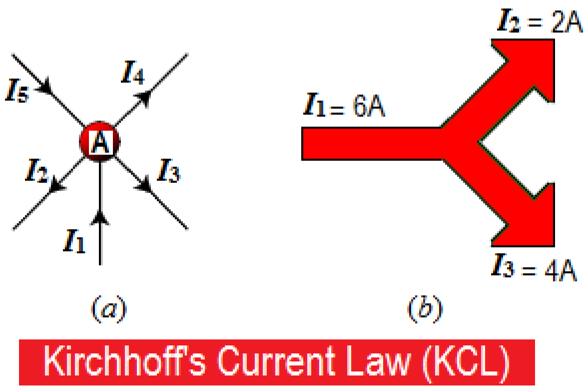
$$I_1 + I_5 - I_2 - I_3 - I_4 = 0$$

OR

$$I_1 + I_5 = I_2 + I_3 + I_4 = 0$$

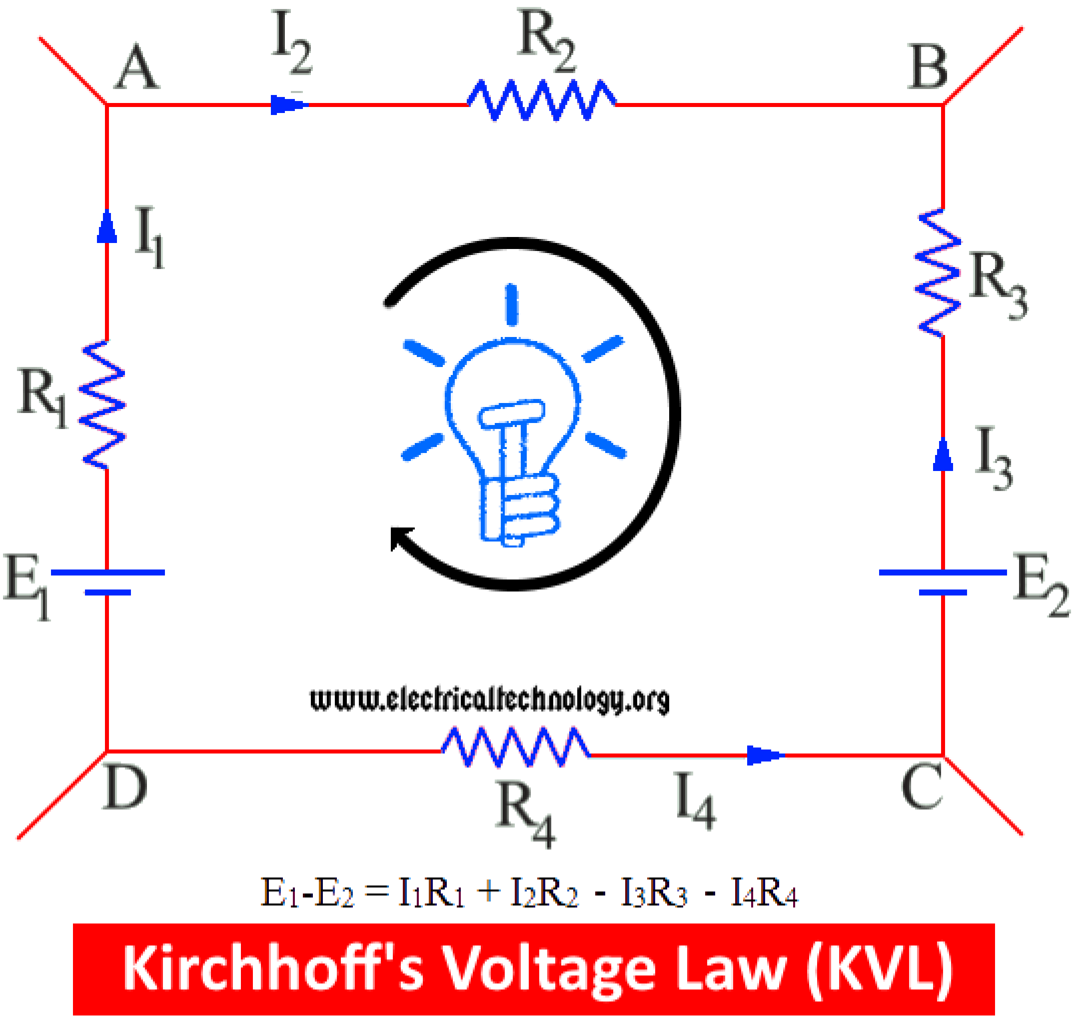
i.e.
Incoming or Entering Currents = leaving or Outgoing Currents
Or
ΣI Entering = ΣI Leaving
For instance, 8A is coming towards a point and 5A plus 3A are leaving that point in fig 1.b, therefore,

8A = 5A + 3A
8A = 8A.



Demonstrating Kirchhoff’s Current Law (KCL)

Kirchhoff’s Voltage Law (KVL):
The Kirchhoff’s second law stated that;
In any closed path (or circuit) in a network, the algebraic sum of the IR product is equal to the EMF in that path.
In other words, in any closed loop (which also known as Mesh), the algebraic sum of the EMF applied is equal to the algebraic sum of the voltage drops in the elements.
Kirchhoff’s second law is also known as Voltage Law or Mesh law.
$\Sigma IR = \Sigma E$
Explanation of KVL:
A closed circuit is shown in fig which contains on two connection of batteries E_1 and E_2. The overall sum of E.M.F’s of the batteries is indicated by E_1-E_2. The imaginary direction of current is also shown in the fig.
E_1 drive the current in such a direction which is supposed to be positive while E_2 interfere in the direction of current (i.e. it is in the opposite direction of the supposed direction of current) hence, it is taken as negative. The voltage drop in this closed circuit is depends on the product of Voltage and Current.



The voltage drop occurs in the supposed direction of current is known as Positive voltage drop while the other one is negative voltage drop.

In the above fig, I_1R_1 and I_2R_2 is positive voltage drop and I_3R_3 and I_4R_4 are negative V.D.

If we go around the closed circuit (or each mesh), and multiply the resistance of the conductor and the flowing current in it, then the sum of the IR is equal to the sum of the applied EMF sources connected to the circuit.

The overall equation for the above circuit is:

$$E_1 - E_2 = i_1R_1 + i2R_2 - {}_{i3}R_3 - {}_{i4}R_4$$

If we go in the supposed direction of the current as shown in the fig, then the product of the IR is taken as positive otherwise negative.

Good to Know:

Direction of the Current:

It is very important to determine the direction of current whenever solving circuits via Kirchhoff's laws.

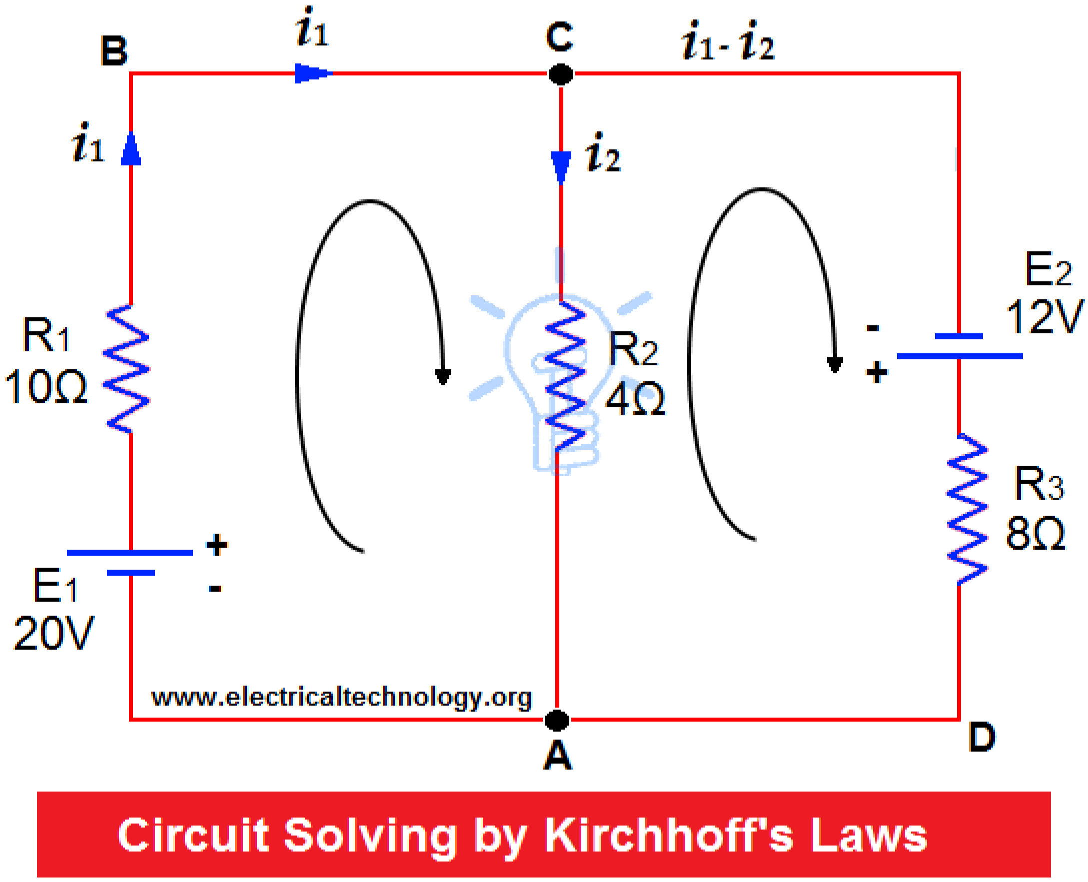
The direction of current can be supposed through clockwise or anticlockwise direction. Once you select the custom direction of the current, you will have to apply and maintain the same direction for over all circuit until the final solution of the circuit. If we got the final value as positive, it means, the supposed direction of the current were correct. In case of negative values, the current of the direction is reversal as compared to the supposed one then.

Circuit Analysis by Kirchhoff's Laws

Solved Example on KCL and KVL (Kirchhoff's Laws)

Example:

Resistors of R_1= 10Ω, R_2 = 4Ω and R_3 = 8Ω are connected up to two batteries (of negligible resistance) as shown. Find the current through each resistor.



Solution:

Assume currents to flow in directions indicated by arrows.

Apply KCL on Junctions C and A.

Therefore, current in mesh ABC = i_1

Current in Mesh CA = i_2

Then current in Mesh CDA = $i_1 - i_2$

Now, Apply KVL on Mesh ABC, 20V are acting in clockwise direction. Equating the sum of IR products, we get;

$10i_1 + 4i_2 = 20$ ……………. (1)

In mesh ACD, 12 volts are acting in clockwise direction, then:

$8(i_1–i_2) – 4i_2= 12$

$8i_1 – 8i_2 – 4i_2 = 12$
$8i_1 – 12i_2 = 12$ ……………. (2)
Multiplying equation (1) by 3;
$30i_1 + 12i_2 = 60$
Solving for i_1
$30i_1 + 12i_2 = 60$
$8i_1 – 12i_2 = 12$

$38i_1 = 72$

The above equation can be also simplified by Elimination or Cramer's Rule.
i_1 = 72/38 = **1.895 Amperes** = Current in 10 Ohms resistor
Substituting this value in (1), we get:
$10(1.895) + 4i_2 = 20$
$4i_2 = 20 – 18.95$
i_2 = **0.263 Amperes** = Current in 4 Ohms Resistors.
Now,
$i_1 – i_2$ = 1.895 – 0.263 = **1.632 Amperes**

Applications of Kirchhoff's Laws

- Kirchhoff's laws can be used to determine the values of unknown values like current, Voltage, current as well as the direction of the flowing values in the circuit.
- These laws can be applied on any circuit* (See the limitation of Kirchhoff's Laws at the end of the article), but useful to find the unknown values in complex circuits and networks.
- Also used in Nodal and Mesh analysis to find the values of current and voltage.
- Current through each independent loop is carried by applying KVL (each loop) and current in any element of a circuit by counting all the current (Applicable in Loop Current Method).
- Current through each branch is carried by applying KCL (each junction) KVL in each loop of a circuit (Applicable in Loop Current Method).
- Kirchhoff's Laws are useful in understanding the transfer of energy through an electric circuit.

Good To Know:

These rules of thumbs must be taken into account while simplifying and analyzing electric circuits by Kirchhoff's Laws:

- The Voltage Drop in a loop due to current in clockwise direction is considered as Positive (+) Voltage Drop.
- The Voltage Drop in a loop due to current in anticlockwise direction is considered as Negative (-) Voltage Drop.
- The deriving current by the battery in clockwise direction is taken as Positive (+).
- The deriving current by the battery in anticlockwise direction is taken as Positive (-).

Limitations of Kirchhoff's laws:

- KCL is applicable on the assumption that current flows only in conductors and wires. While in High Frequency circuits where, parasitic capacitance can no longer be ignored. In such cases, Current can flow in an open circuit because in these cases, conductors or wires are acting as transmission lines.
- KVL is applicable on the assumption that there is no fluctuating magnetic field linking the closed loop. While, in presence of changing magnetic field in a High Frequency but short wave length AC circuits, the electric field is not a conservative vector field. So, the electric field cannot be the gradient of any potential and the line integral of the electric field around the loop is not zero, directly contradicting KVL. That's why KVL is not applicable in such a condition.
- During the transfer of energy from the magnetic field to the electric field where fudge has to be introduced to KVL to make the P.d (potential differences) around the circuit equal to 0.

Thevenin's Theorem. Step by Step Procedure with Solved Example

Thevenin's Theorem in DC Circuit Analysis

A French engineer, **M.L Thevenin**, made one of these quantum leaps in 1893. **Thevenin's Theorem** (also known as **Helmholtz–Thévenin Theorem**) is not by itself an analysis tool, but the basis for a very useful method of simplifying active circuits and complex networks. This theorem is useful to quickly and easily solve complex linear circuits and networks, especially electric circuits and electronic networks.

Thevenin's Theorem may be stated below:

Any linear electric network or a complex circuit with current and voltage sources can be replaced by an equivalent circuit containing of a single independent voltage source $\boldsymbol{V_{TH}}$ *and a Series Resistance* $\boldsymbol{R_{TH}}$*.*

- **V_{TH} = Thevenin's Voltage**
- **R_{TH} = Thevenin's Resistance**

Steps to Analyze an Electric Circuit using Thevenin's Theorem

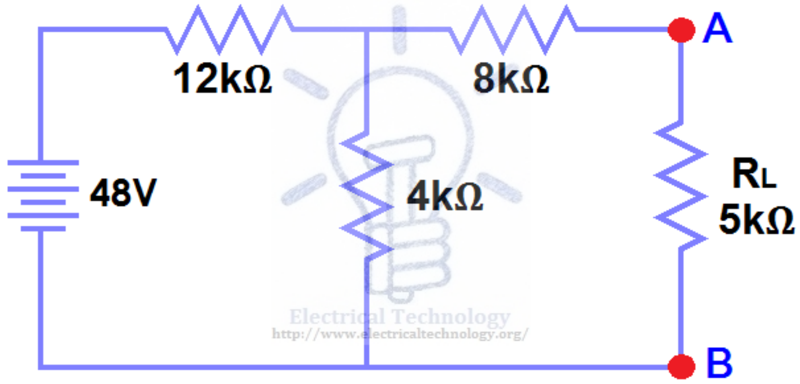
1. Open the load resistor.
2. Calculate / measure the open circuit voltage. This is the **Thevenin Voltage (V_{TH})**.
3. Open current sources and short voltage sources.
4. Calculate /measure the Open Circuit Resistance. This is the **Thevenin Resistance (R_{TH})**.

5. Now, redraw the circuit with measured **open circuit Voltage (V_{TH})** in Step (2) as voltage source and measured **open circuit resistance (R_{TH})** in step (4) as a series resistance and connect the load resistor which we had removed in Step (1). This is the **equivalent Thevenin circuit** of that **linear electric network** or **complex circuit** which had to be ***simplified and analyzed by Thevenin's Theorem***. You have done.
6. Now find the Total current flowing through load resistor by using the [Ohm's Law](): **$I_T = V_{TH} / (R_{TH} + R_L)$.**

Solved Example by Thevenin's Theorem:

Example:

Find V_{TH}, R_{TH} and the load current **I_L** flowing through and load voltage across the load resistor in fig (1) *by using Thevenin's Theorem*.

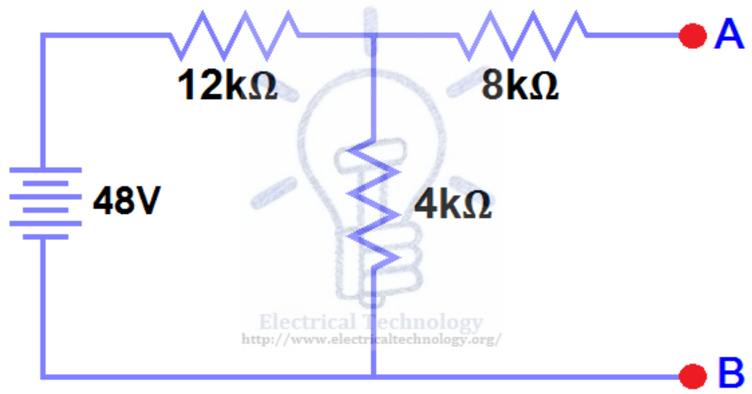


Thevenin's Theorem. Easy Step by Step Procedure with Example (Pictorial Views)

Solution:-

STEP 1.

Open the **5kΩ load resistor** (Fig 2).



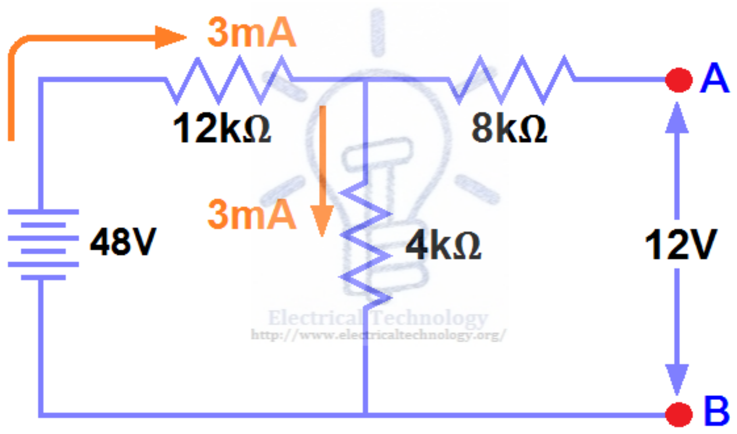
STEP 2.

Calculate / measure the open circuit voltage. This is the **Thevenin Voltage (V_{TH})**. Fig (3).

We have already removed the load resistor in figure 1, so the circuit became an **open circuit** as shown in fig 2. Now we have to calculate the Thevenin's Voltage. Since **3mA** current flows in both **12kΩ** and **4kΩ resistors** as this is a series circuit and current will not flow in the **8kΩ** resistor as it is open.

This way, **12V (3mA x 4kΩ)** will appear across the **4kΩ resistor**. We also know that current is not flowing through the 8kΩ resistor as it is an open circuit, but the 8kΩ resistor is in parallel with **4k resistor**. So the same voltage i.e. **12V will appear across the 8kΩ resistor** as well as 4kΩ resistor. Therefore 12V will appear across the AB terminals. i.e,

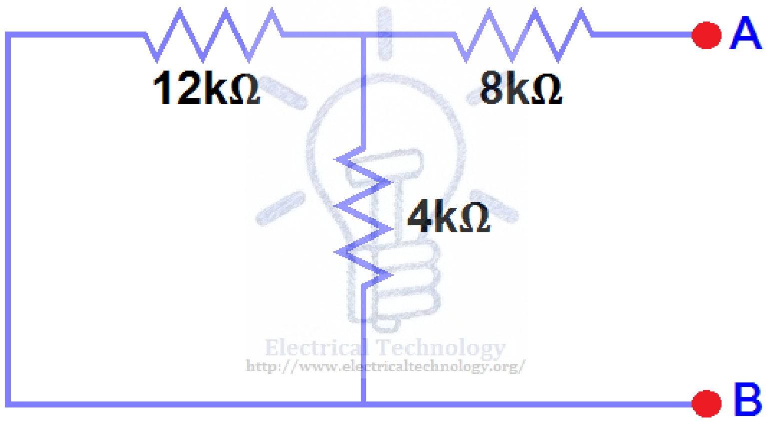
V_{TH} = 12V



3mA x 4kΩ = 12V

STEP 3.

Open current sources and **short voltage sources** as shown below. Fig (4)



STEP 4.

Calculate / **measure the open circuit resistance**. This is the Thevenin Resistance (R_{TH})

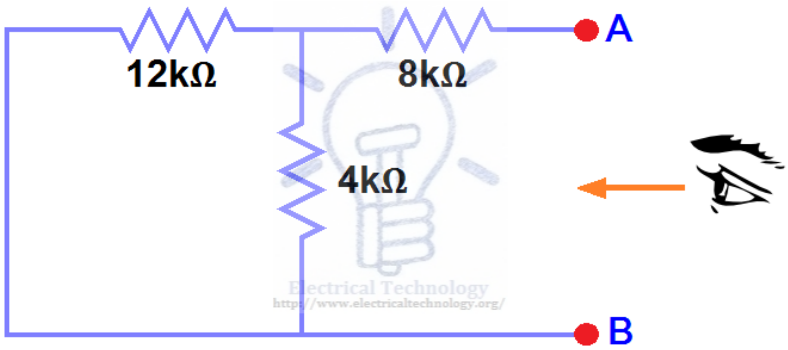
We have removed the **48V DC source** to **zero** as equivalent i.e. 48V DC source has been replaced with a short in step 3 (as shown in figure 3). We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

8kΩ + (4k Ω || 12kΩ) ….. (|| = in parallel with)

R_{TH} = 8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)]

R_{TH} = 8kΩ + 3kΩ

R_{TH} = 11kΩ

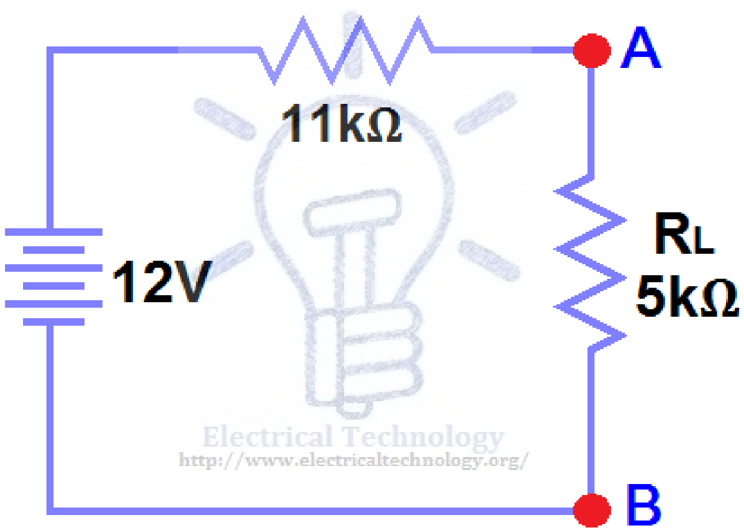


$$= 8k\Omega + (4k\ \Omega \parallel 12k\Omega) \rightarrow = 8k\Omega + 3k\Omega$$

$$\boxed{R_{TH} = 11k\Omega}$$

STEP 5.

Connect the R_{TH} in series with Voltage Source V_{TH} and re-connect the load resistor. This is shown in fig (6) i.e. Thevenin circuit with load resistor. This the **Thevenin's equivalent circuit**.



Thevenin's equivalent circuit

STEP 6.

Now apply the last step i.e Ohm's law . **Calculate the total load current and load voltage** as shown in fig 6.

$$I_L = V_{TH} / (R_{TH} + R_L)$$

I_L = 12V / (11kΩ + 5kΩ) → = 12/16kΩ

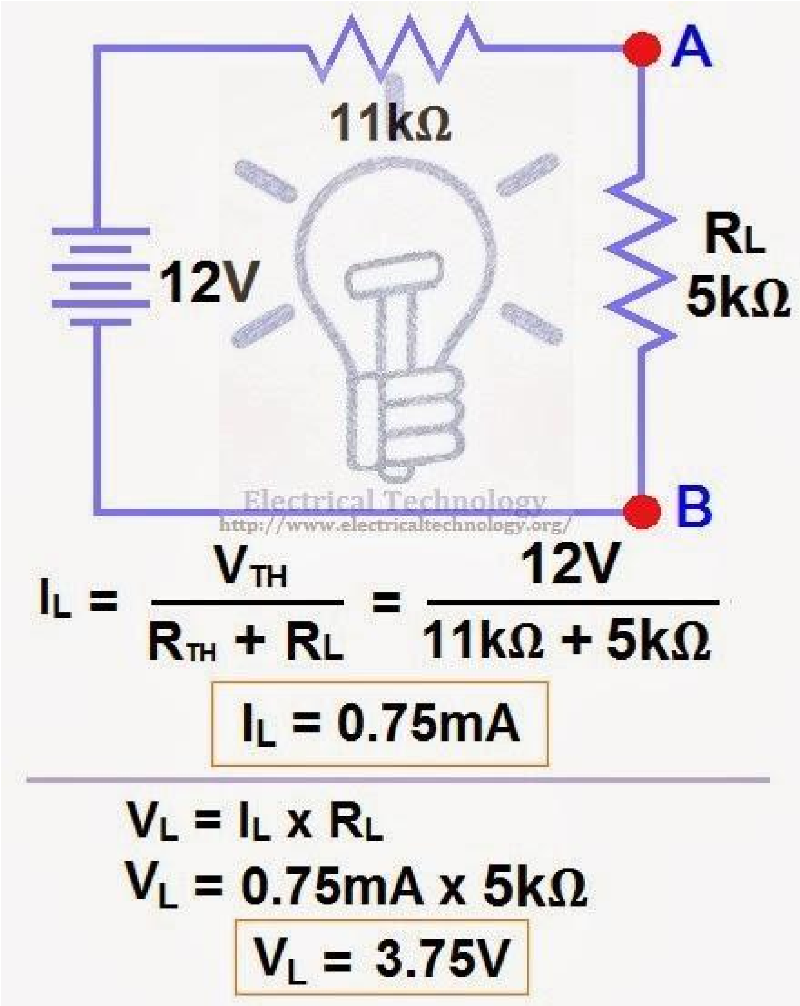
I_L = 0.75mA

And

$$V_L = I_L \times R_L$$

V_L = 0.75mA x 5kΩ

V_L= 3.75V



Norton's Theorem. Easy Step by Step Procedure with Example

Norton's Theorem in DC Circuit Analysis

Norton's theorem is another useful tool to analyze electric circuits like using the Thevenin's Theorem, which reduces linear, active circuits and complex networks into a simple equivalent circuit. The main **difference between Thevenin's theorem and Norton's theorem** is that, Thevenin's theorem provides an equivalent voltage source and an equivalent series resistance, while Norton's theorem provides an equivalent Current source and an equivalent parallel resistance.

Norton's Theorem states that:

Any linear electric network or complex circuit with current and voltage sources can be replaced by an equivalent circuit containing a single independent current source I_N and a parallel resistance R_N.

In other but simple words, Any linear circuit is equivalent to a real and independent current source in specific terminals.

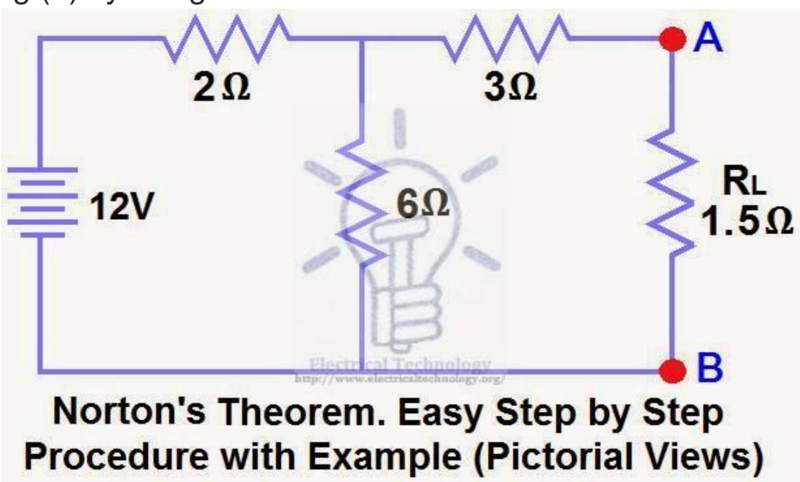
Steps to Analyze an Electric Circuit using Norton's Theorem

1. Short the load resistor.
2. Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).
3. Open Current Sources, Short Voltage Sources and Open Load Resistor.
4. Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N).
5. Now, Redraw the circuit with measured short circuit Current (I_N) in Step (2) as Current Source and measured open circuit resistance (R_N) in step (4) as a parallel resistance and connect the load resistor which we had removed in Step (3). This is the Equivalent Norton Circuit of that Linear Electric Network or Complex circuit which had to be simplified and analyzed. You have done.
6. Now find the Load current flowing through and Load Voltage across Load Resistor by using the Current divider rule. $I_L = I_N / (R_N / (R_N + R_L))$ ((For better understanding…check the solved example).

Solved Example by Norton's Theorem:

Example:

Find R_N, I_N, the current flowing through and Load Voltage across the load resistor in fig (1) by using Norton's Theorem.

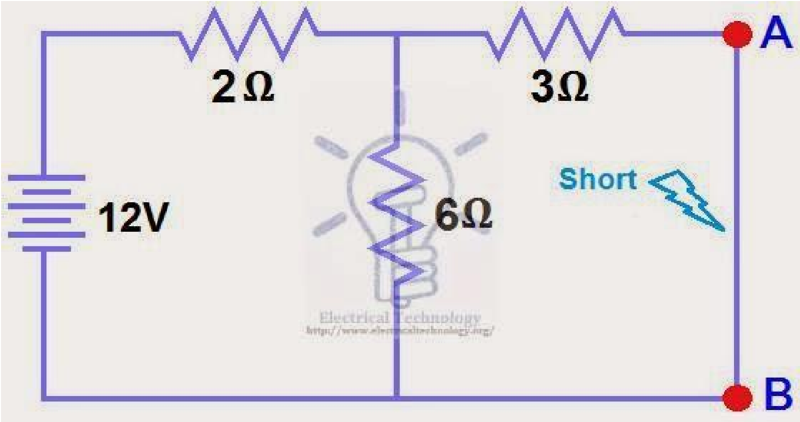


Norton's Theorem: Step by Step Procedure with Examples

Solution:-

STEP 1.

Short the 1.5Ω load resistor as shown in (Fig 2).



STEP 2.

Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).

We have shorted the AB terminals to determine the Norton current, I_N. The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω.

So the Total Resistance of the circuit to the Source is:-

2Ω + (6Ω || 3Ω) ….. (|| = in parallel with).

R_T = 2Ω + [(3Ω x 6Ω) / (3Ω + 6Ω)] → I_T= 2Ω + 2Ω = 4Ω.

R_T = 4Ω

$I_T = V / R_T$

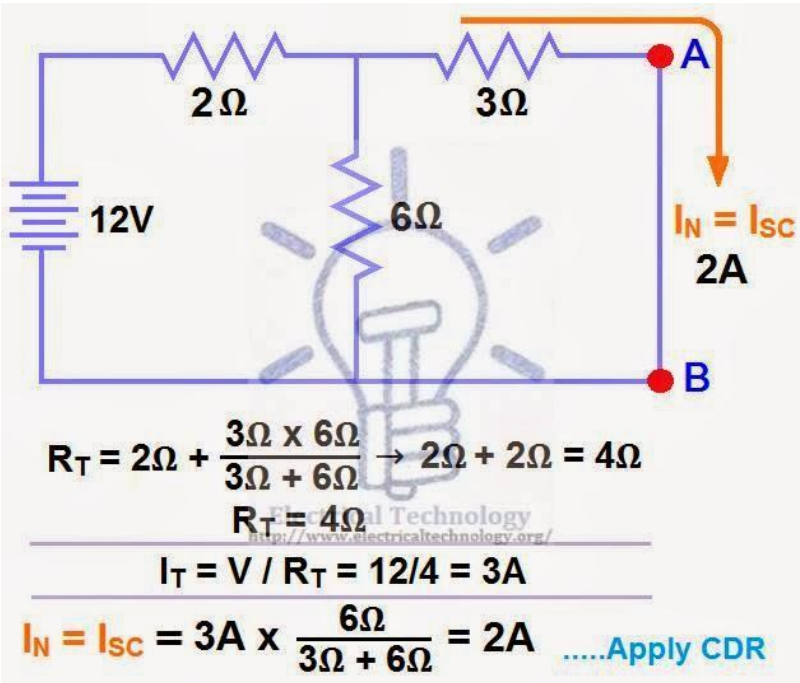
I_T = 12V / 4Ω

I_T = 3A..

Now we have to find $I_{SC} = I_N$… Apply CDR… (Current Divider Rule)…

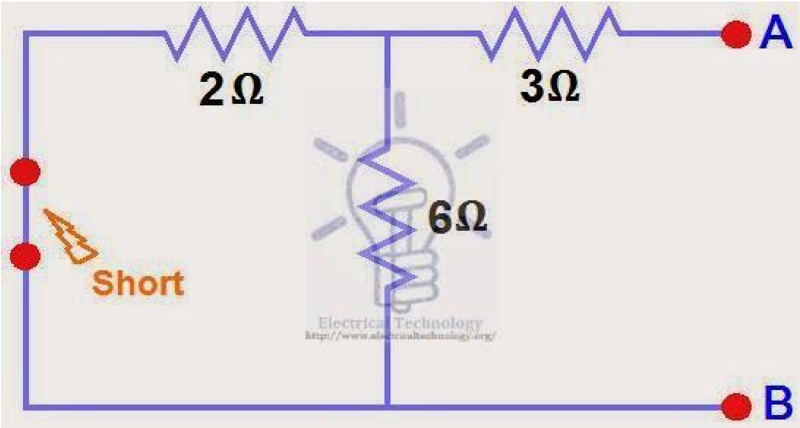
$I_{SC} = I_N$ = 3A x [(6Ω / (3Ω + 6Ω)] = 2A.

$I_{SC} = I_N$ = 2A.



STEP 3.

Open Current Sources, Short Voltage Sources and Open Load Resistor. Fig (4)



STEP 4.

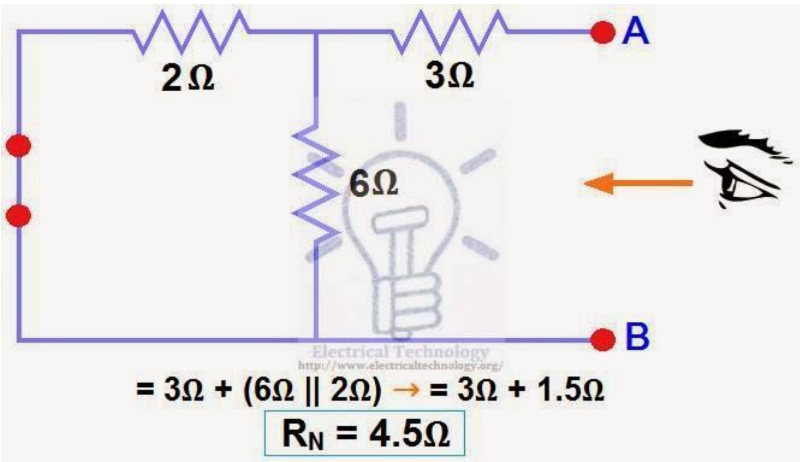
Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N) We have Reduced the 12V DC source to zero is equivalent to replace it with a short in step (3), as shown in figure (4) We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

$$3\Omega + (6\Omega \,||\, 2\Omega) \;\ldots.. \; (|| = \text{in parallel with})$$

$$R_N = 3\Omega + [(6\Omega \times 2\Omega) / (6\Omega + 2\Omega)]$$

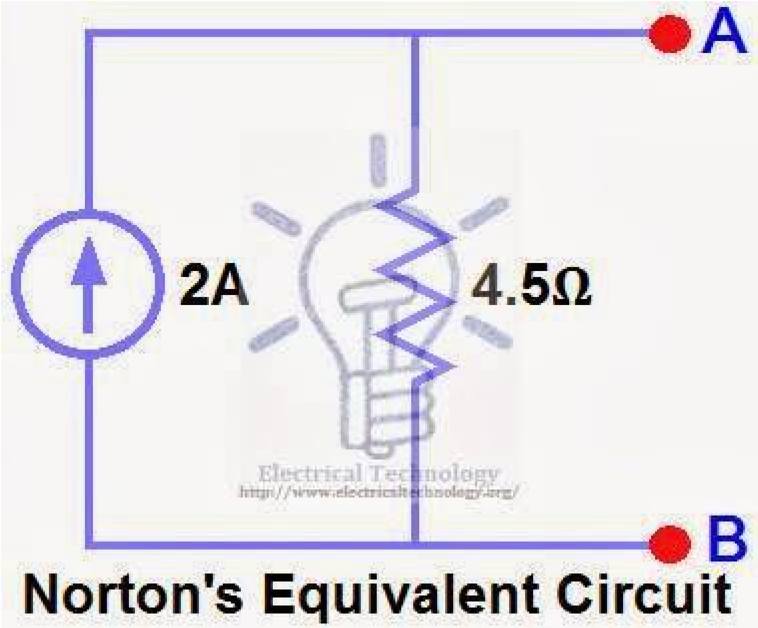
$$R_N = 3\Omega + 1.5\Omega$$

$$\mathbf{R_N = 4.5\Omega}$$



STEP 5.

Connect the R_N in Parallel with Current Source I_N and reconnect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.



Norton Equivalent Circuit

STEP 6.

Now apply the last step i.e. calculate the load current through and Load voltage across the load resistor by Ohm's Law as shown in fig 7.

Load Current through Load Resistor…

$$\mathbf{I_L = I_N \times [R_N / (R_N + R_L)]}$$

= 2A x (4.5Ω /4.5Ω +1.5Ω) → = 1.5A

I_L = 1. 5A

And

Load Voltage across Load Resistor…

$$\mathbf{V_L = I_L \times R_L}$$

V_L = 1.5A x 1.5Ω

V_L= 2.25V

Now apply the last step i.e. calculate the load current through and Load voltage across the load resistor by Ohm's Law as shown in fig 7.

Load Current through Load Resistor…

$$\mathbf{I_L = I_N \times [R_N / (R_N + R_L)]}$$

= 2A x (4.5Ω /4.5Ω +1.5Ω) → = 1.5A

I_L = 1. 5A

And

Load Voltage across Load Resistor…

$$\mathbf{V_L = I_L \times R_L}$$

V_L = 1.5A x 1.5Ω

V_L= 2.25V

Maximum Power Transfer Theorem for AC & DC Circuits

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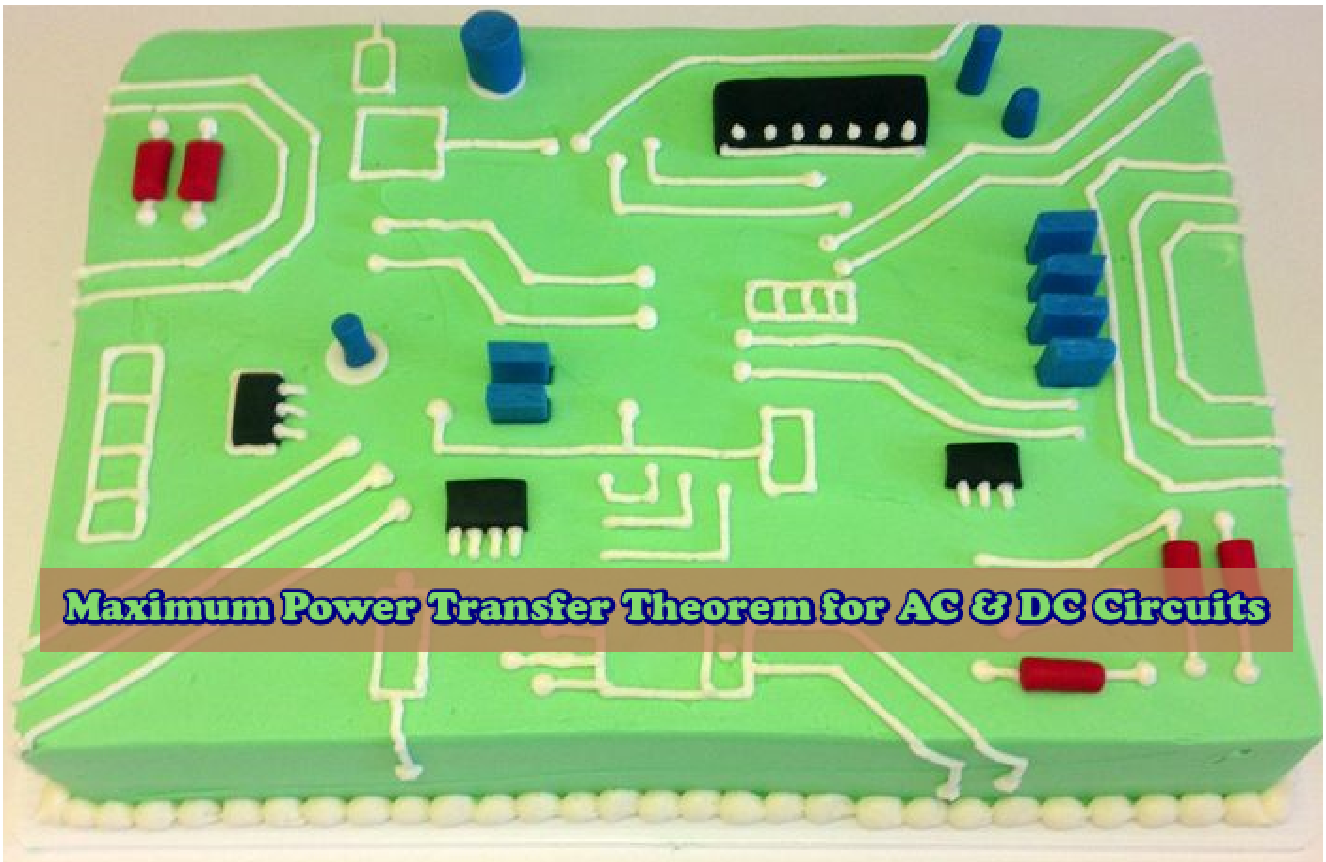
Introduction to Maximum Power Transfer Theorem

Very often we come across various real time circuits that works based on maximum power transfer theorem. For effective way of connecting source to load, an impedance matching transformer is used. In case of transmission lines, the distortion and reflections are avoided by making source and load impedances to be matched to the characteristic impedance of the line.

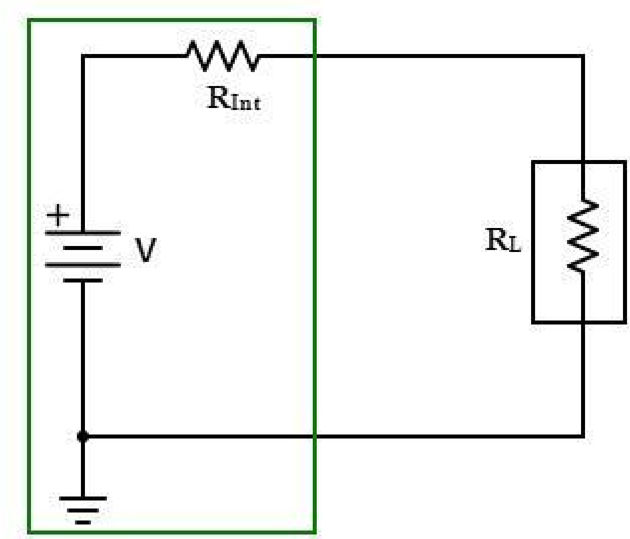


In case of solar photovoltaic (PV) systems, Maximum Power Point Tracking (MPPT) is achieved with incremental conductance method (ICM) in which the load resistance must be equal to the output resistance of the PV panel and Solar Cell

So there are several cases or applications that use maximum power transfer theorem for effectively connecting the source to a load. This theorem can be applied for both DC and AC circuits. Let us discuss this theorem for DC as well AC circuits with examples.

Maximum Power Transfer Theorem for DC circuits

This theorem describes the condition for maximum power transfer from an active network to an external load resistance. It states that ***in a linear, active, bilateral DC network, the maximum power will be transferred from source to the load when the external load resistance equals to the internal resistance of the source.***
This theorem can be developed with reference to practical current or voltage source.



If the source is a practical or independent voltage source, its internal series resistance must match with load resistance to deliver maximum power. In case of practical or independent current source, parallel internal resistance should match with load resistance.

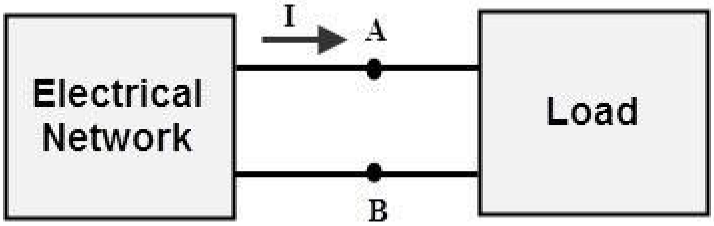
In the above circuit internal source series resistance alters the power delivered to the load and hence the maximum current delivered from the source to the load is limited.

Explanation of Maximum Power Transfer Theorem

Let us consider the electrical system with load as shown below, to which we are going to determine the value of load resistance so as to deliver the maximum power to the load.

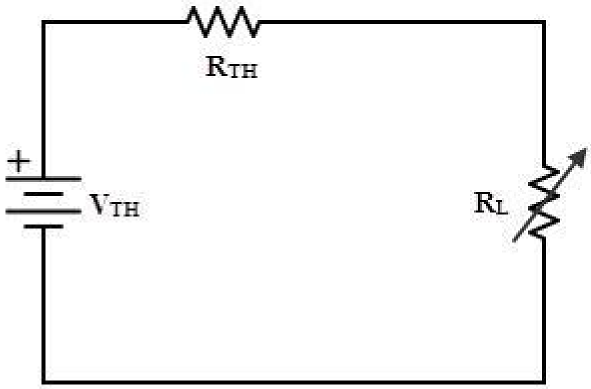
Basically, the condition at which maximum power transfer can be obtained by deriving an expression of power absorbed by the load using mesh or nodal current techniques and then finding its derivative with respect to the load resistance.

In below figure, electrical system may be a complex circuit consisting of several elements and sources. In such case finding of maximum power transfer condition



can be tedious.

Alternatively we can find the maximum power transfer with the use of Thevenin's equivalent circuit (Read Here the step by step Thevenin's Theorem with solved examples). Now we will replace the electrical system which we are considered as complex part with its Thevenin's equivalent circuit as shown in below.



From the above circuit, the current flowing through the load, 'I' is given as

$$I = \frac{V_{th}}{R_L + R_{TH}}$$

Power transferred to the load,

$$P_L = I^2 R_L = \left(\frac{V_{th}}{R_L + R_{TH}}\right)^2 \cdot R_L$$

In the above equation R_L is a variable, therefore the condition for maximum power delivered to the load is determined by differentiating load power with respect to the load resistance and equating it to zero.

$$\frac{\partial P_L}{\partial R_L} = V_{\mathrm{TH}}^2 \left[\frac{(R_L + R_{TH})^2 - 2R_L(R_{TH} + R_L)}{(R_L + R_{TH})^4} \right] = 0$$

$$\rightarrow (R_L + R_{TH})^2 = 2R_L(R_{Th} + R_L)$$

$$\rightarrow R_L + R_{TH} = 2R_L$$

$$\rightarrow R_L = R_{TH}$$

This is the condition for maximum power transfer, which states that power delivered to the load is maximum, when the load resistance R_L matches with Thevenin's resistance R_{TH} of the network.

Under this condition, power transfer to the load is

$$P_L = \left(\frac{V_{\mathrm{th}}}{R_{TH} + R_{TH}} \right)^2 \cdot R_{TH} \quad \textbf{(by substituting } \mathbf{R_L = R_{TH}}\textbf{)}$$

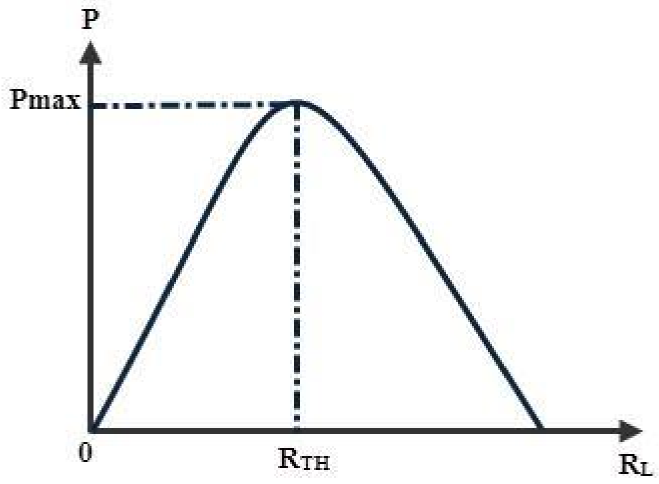
Therefore the power transfer to the load

$$P_L = \frac{V_{\mathrm{th}}^2}{4R_{TH}}$$

Input power, $P_{in} = V_{\mathrm{th}} \times I = V_{\mathrm{th}} \times \dfrac{V_{\mathrm{th}}}{2R_{TH}} = \dfrac{V_{\mathrm{th}}^2}{2R_{TH}}$

Efficiency, $\eta = \dfrac{P_L}{P_{in}} = \dfrac{V_{\mathrm{th}}^2 / 4R_{TH}}{V_{\mathrm{th}}^2 / 2R_{TH}} = 50\%$

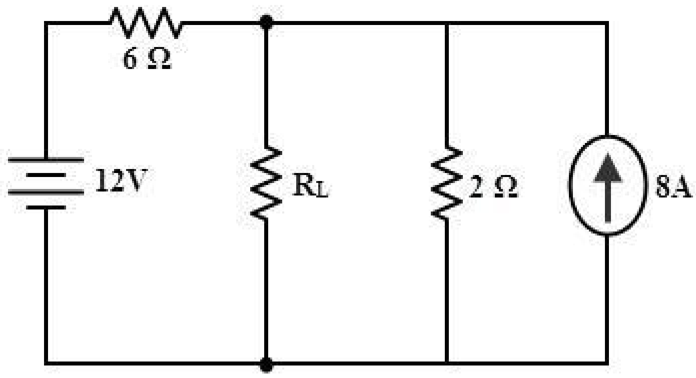
The above equation shows that the efficiency is 50% under maximum power transfer condition. Due to this 50 percent efficiency, maximum power transfer is not always desirable. For a given values the Thevenin's voltage and Thevenin's resistance, the variation of power delivered to the load with varying load resistance is shown in



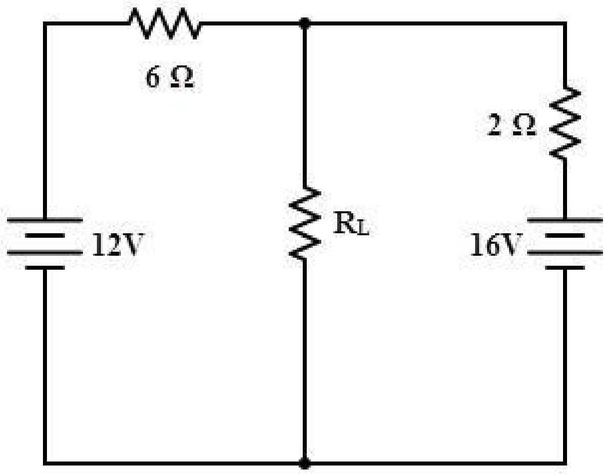
below figure.

Solved Example on Maximum Power Transfer Theorem in DC Circuits

Consider the below circuit for which we are going to determine the value of load resistance, R_L for which maximum power will transfer from source to load.

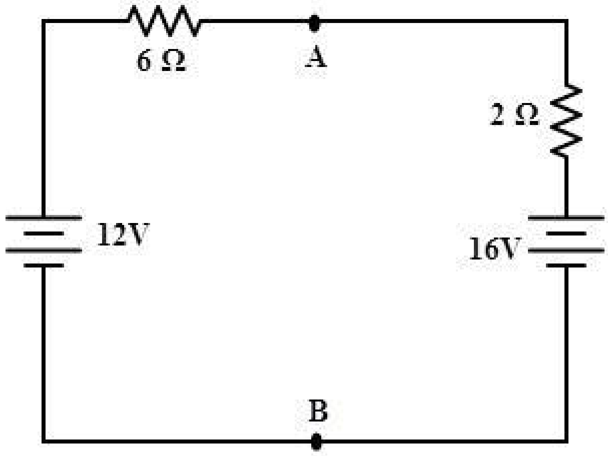


Now, the given circuit can be further simplified by converting the current source into



equivalent voltage source as follows.

We need to find the Thevenin's equivalent voltage Vth and Thevenin's equivalent resistance Rth across the load terminals in order to get the condition for maximum power transfer. By disconnecting the load resistance, the open-circuit voltage across the load terminals can be calculated as;



By applying Kirchhoff's voltage law, we get

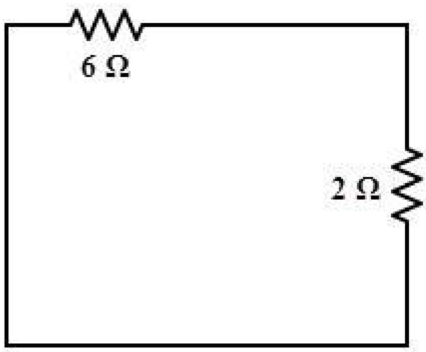
$$12 - 6I - 2I - 16 = 0$$

$$-8I = 4$$

$$I = -0.5\ A$$

The open-circuit voltage across the terminals A and B, $V_{AB} = 16 - 2 \times 0.5$
$= 15\ V$

Thevenin's equivalent resistance across the terminals A and B is obtained by short-circuiting the voltage sources as shown in the figure.



Req = (6 × 2) / (6 + 2)

= 1.5 Ω

So the maximum power will transferred to the load when R_L = 1.5 ohm.

Current through the circuit, I = 15 / (1.5 + 1.5)

= 5 A

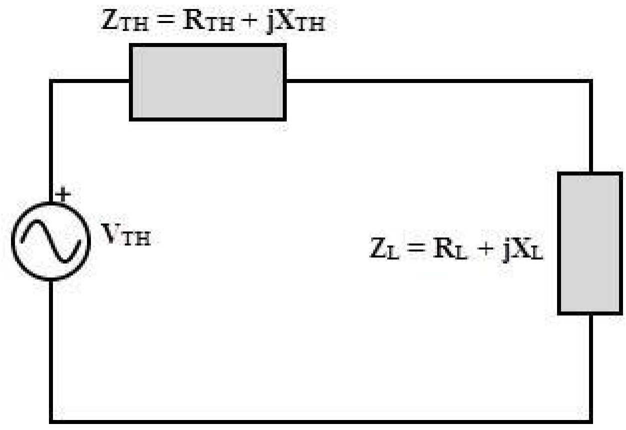
Therefore, the maximum power = $5^2 \times 1.5 = 37.5$ W

Maximum Power Transfer Theorem for AC circuits

This theorem gives the impedance conditions in AC circuit for maximum power transfer to a load. It states that in an active AC network consisting of source with internal impedance Z_S which is connected to a load Z_L, the maximum power transfer occurs from source to load when the load impedance is equal to the complex conjugate of source impedance Z_S.

Explanation & Proof of the Maximum Power Transfer Theorem

Consider the below circuit consisting of Thevenin's voltage source with series Thevenin's equivalent resistance (which are actually replacing the complex part of the circuit) connected across the complex load.



From the above figure, Let $Z_L = R_L + jX_L$ and $Z_{TH} = R_{TH} + jX_{TH}$ then the current through the circuit is given as,

$$I = \frac{V_{TH}}{Z_{TH} + Z_L}$$

By substituting above given impedances, we get

$$I = \frac{V_{TH}}{R_{TH} + jX_{TH} + R_L + jX_L}$$

$$I = \frac{V_{TH}}{(R_L + R_{TH}) + j(X_L + X_{TH})}$$

Power delivered to the load, $P_L = I^2 R_L$

$$P_L = \frac{V_{TH}^2 \times R_L}{(R_L + R_{TH})^2 + (Z_L + Z_{TH})^2}$$

For power to be maximized, the above equation must be differentiated with respect to X_L and equates it to zero.

Then we get

$$X_L + X_{TH} = 0$$

$$X_L = -X_{TH}$$

Substituting the X_L in power equation, we get

$$P_L = \frac{V_{TH}^2 \times R_L}{(R_L + R_{TH})^2}$$

Again taking derivative of the above equation and equating it to zero,

we get $$R_L + R_{TH} = 2R_L$$

$$R_L = R_{TH}$$

Again taking derivative of the above equation and equating it to zero, we get

$$R_L + R_{TH} = 2 R_L$$
$$R_L = R_{TH}$$

Therefore, in AC circuits, if $X_L = – X_{TH}$ and $R_L = R_{TH}$, maximum power transfer takes place from source to load. This implies that maximum power transfer occurs when the impedance of the load is complex conjugate of the source impedance, i.e., $Z_L = Z^*_{TH}$

The maximum power,

$$P_{Max} = \frac{V_{TH^2}}{4R_{TH}}$$

Or

$$P_{Max} = \frac{V_{TH^2}}{4R_L}$$

Superposition Theorem – Circuit Analysis with Solved Example

Superposition Theorem – Step by Step Guide with Solved Example

Table of Contents

- Superposition Theorem Experiment
- Application of Superposition theorem
    - Drawback of Superposition Theorem

What is Superposition Theorem?

The **Superposition Theorem** is used to solve complex networks with a number of energy sources. It is an important concept to determine voltage and current across the elements by calculating the effect of each source individually. And combine the effect of all sources to get the actual voltage and current of the circuit element.

Superposition theorem states that;

"In any linear bilateral network having a greater number of sources, the response (voltage and current) in any element is equal to the summation of all responses caused by individual source acting alone. While other sources are eliminated from the circuit."

In other words, we will consider only one independent source acting at a time. So, we need to remove other sources. The voltage sources are short-circuited and the current sources are open-circuited for ideal sources. If the internal resistance of sources is given, you need to consider the circuit.

The superposition theorem is only applied to the circuit which follows Ohm's law.

When to Use the Superposition Theorem?

The network must follow the below requirements to apply the superposition theorem.

- The components used in the circuit must be linear. It means, for resistors, the flow of current is proportional to the voltage; for inductors, the flux linkage is proportional to current. Therefore, the resistor, inductor, and capacitor are linear elements. But the diode, transistor is not a linear element.
- The circuit components must be bilateral elements. It means, the magnitude of the current is independent of the polarity of energy sources.
- With the help of the superposition theorem, we can find the current passes through an element, voltage-drop of resistance, and node voltage. But we cannot find the power dissipated from the element.

Steps to Follow for Superposition Theorem

Step-1 Find out a number of independent sources available in the network.

Step-2 Choose any one source and eliminate all other sources. If the network consists of any dependent source, you cannot eliminate it. It remains as it is throughout the calculation.

If you have considered all energy sources are ideal sources, you need not consider internal resistance. And directly short-circuit voltage source and open-circuit current source. But in case, if internal resistance of sources is given, you have to replace internal resistance.

Step-3 Now, in a circuit, only one independent energy source is present. You need to find a response with only one energy source in the circuit.

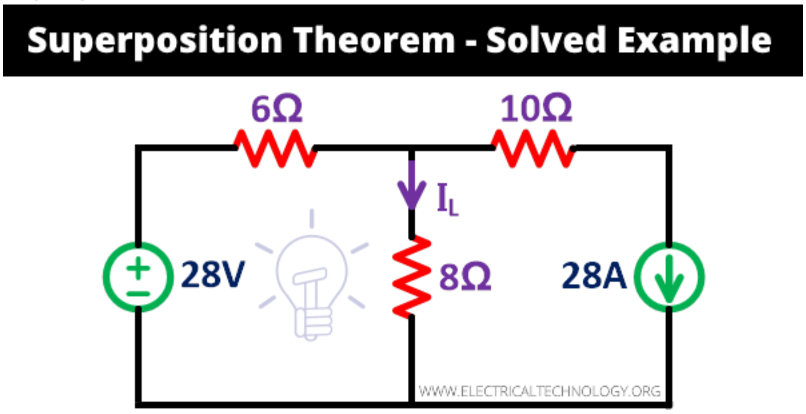
Step-4 Repeat step-2 and 3 for all energy sources available in the network. If there are three independent sources, you need to repeat these steps three times. And every time you get some value of the response.

Step-5 Now, combine all responses by algebraic summation obtained by individual sources. And you will get a final value of response for a particular element of a network. If you need to find a response for other elements, you need to follows these steps again for that element.

Superposition Theorem Solved Example

Example:

Let's understand the working of the superposition theorem by example. Find the current (I_L) passes through the 8Ω resister in the given network using the superposition theorem.

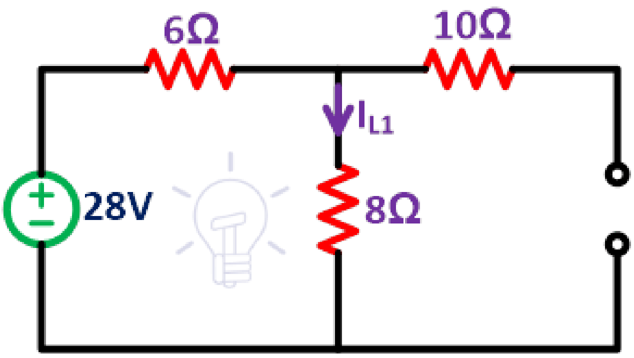


Solution:

Step-1 As shown in the above network, one voltage source, and one current source is given. Therefore, we need to repeat the procedure two times.

Step-2 First we consider 28V voltage source is present in the network. So, you need to remove the current source by open-circuited terminals. As here, we consider the current source as an ideal current source. So, we need not connect the internal resistance.

The remaining circuit is as shown in the below figure.



Step-3 Find the current (I_{L1}) passes through 8Ω resister. It gives the effect only of a voltage source.

Due to the open circuit of a current source, no current passes through the 10Ω resister. So, the network consists of only one loop.

Apply KVL to the loop;

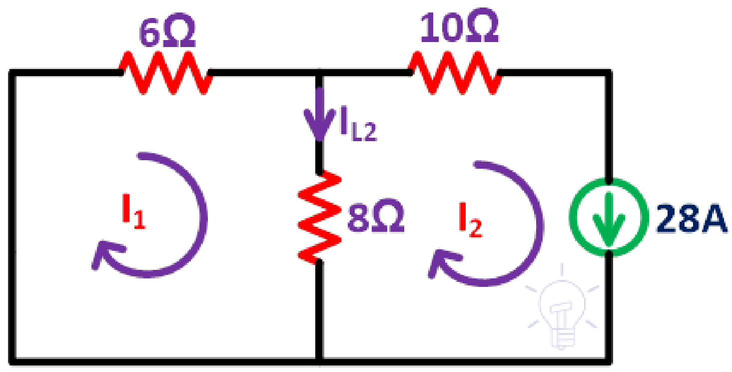
$$28 = 6I_{L1} + 8I_{L1}$$

$$28 = 14I_{L1}$$

$$I_{L1} = 28/14$$

$$I_{L1} = 2\text{A}$$

Step-4 Now, we repeat the same procedure for the current source. In this condition, we remove the voltage source by short-circuiting. A remaining circuit is as shown in the below figure.



Here, we have to consider two loops. I_1 and I_2 are loop current. And find the current I_{L2}.

Apply KVL to the loop-1;

$$0 = 6I_1 + 8I_1 - 8I_2$$

$$14I_1 - 8I_2 = 0$$

The current passes through the loop-2 is calculated from the current source. And it will be;

$$I_2 = 28\text{A}$$

Put this value in the above equation;

$$14I_1 - 8\ (28) = 0$$

$$I_1 = 16\text{A}$$

Now, the 8Ω resister branch is common in both loops. So, we need to find the resultant current (I_{L2}) passes through the 8Ω resister.

$$I_{L2} = I_1 - I_2$$

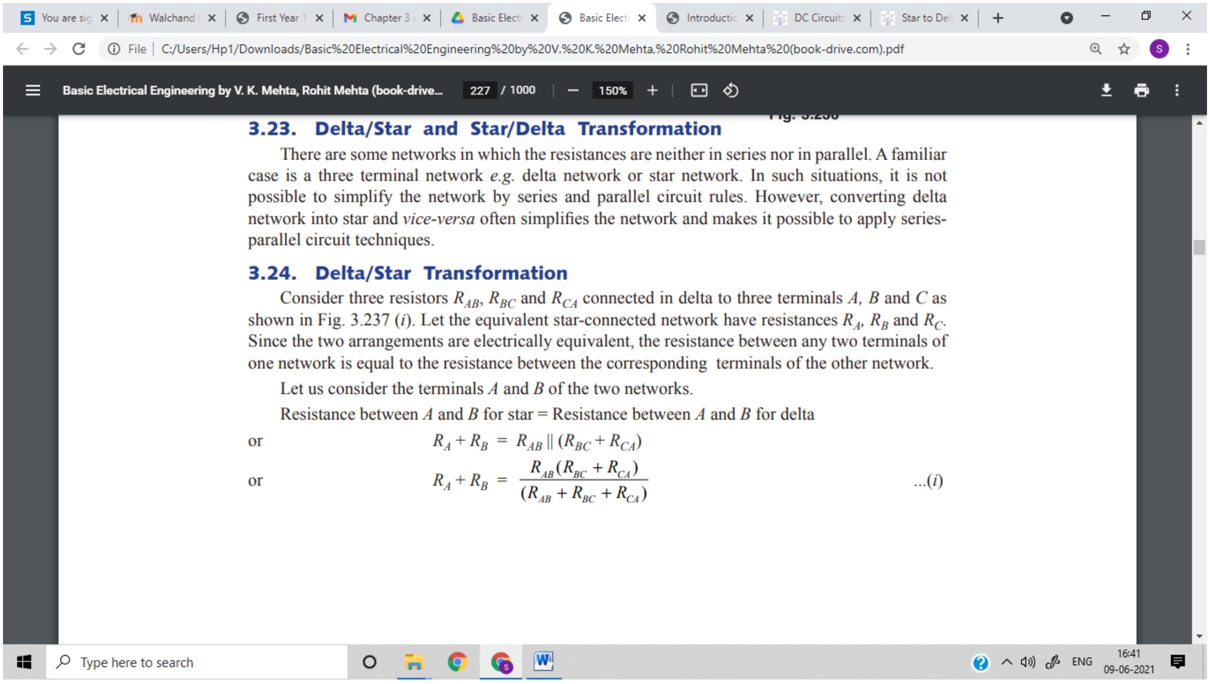
Step-5 Now, we combine the effect of both sources by algebraic summation of current. So, the total current passes through the 8Ω resister are I_L. Here, the direction of the current is most important. Current I_{L2} has a minus sign. It means during the 28A source, the current flows in opposite direction. And we cannot change the direction. That's the reason while combining all sources, we are doing algebraic summation.

$$I_L = I_{L1} - I_{L2}$$

$$I_L = 2 + (\text{-}10)$$

$$I_L = \text{-}8\text{A}$$

Here, we have assumed that the current passes through the 8Ω resister are in the direction of the arrow shown in the figure. Minus sign shows the opposite direction. And the amount of current is 8A.



3.23. Delta/Star and Star/Delta Transformation

There are some networks in which the resistances are neither in series nor in parallel. A familiar case is a three terminal network *e.g.* delta network or star network. In such situations, it is not possible to simplify the network by series and parallel circuit rules. However, converting delta network into star and *vice-versa* often simplifies the network and makes it possible to apply series-parallel circuit techniques.

3.24. Delta/Star Transformation

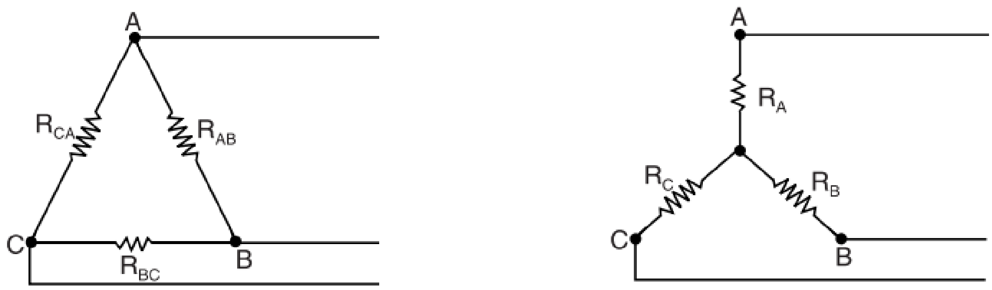
Consider three resistors R_{AB}, R_{BC} and R_{CA} connected in delta to three terminals A, B and C as shown in Fig. 3.237 (*i*). Let the equivalent star-connected network have resistances R_A, R_B and R_C. Since the two arrangements are electrically equivalent, the resistance between any two terminals of one network is equal to the resistance between the corresponding terminals of the other network.

Let us consider the terminals A and B of the two networks.

Resistance between A and B for star = Resistance between A and B for delta

or $$R_A + R_B = R_{AB} \parallel (R_{BC} + R_{CA})$$

or $$R_A + R_B = \frac{R_{AB}(R_{BC} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})} \qquad ...(i)$$



(*i*) (*ii*)

Fig. 3.237

Similarly, $$R_B + R_C = \frac{R_{BC}(R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(ii)$$

and $$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(iii)$$

Subtracting eq. (*ii*) from eq. (*i*) and adding the result to eq. (*iii*), we have,

$$R_A = \frac{R_{AB}\, R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(iv)$$

Similarly, $$R_B = \frac{R_{BC}\, R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \qquad ...(v)$$