Probability

Page No. | Date

★ Probability distribution -

Outcomes -

Sample space -

event -

if $n(s) = p$,

possible events $= 2^p$

• Mutually exclusive events (disjoint)

$$A \cap B = \phi$$

• Independent events -

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A') = 1 - P(A)$$

$$P(A \cap B') = P(A) - P(A \cap B)$$

$$P(B \cap A') = P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$P(\phi) = 0$$

★ Random variables

1) Discrete 2) Continuous

• Discrete

$S = \{HH, HT, TH, TT\}$

X - no. of tails obtained

$X(HH) = 0$ $X(HT) = 1$

$X(TH) = 1$ $X(TT) = 2$

Probability mass function :- (PMF)

$$x \rightarrow \overset{\text{Discrete}}{r.v.} \rightarrow S$$

$$\{x_1, x_2, x_3, \ldots, x_n\}$$
$$p_1, p_2, p_3, \ldots, p_n$$

i) $p_i \geq 0 \quad i = 0, 1, 2, 3, \ldots$

ii) $\sum p_i = 1$

Probability density function (PDF) -

$$x \rightarrow \text{continuous } r.v. \rightarrow S$$

$f(x) \rightarrow$ i) $f(x) \geq 0 \quad \forall x \in R$

ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

Q.1 The pmf of r.v. x is 0 except at $x = 0, 1, 2$. At these points $P(0) = 3c^3$, $P(1) = 4c - 10c^2$, $P(2) = 5c - 1$

i) Find c ii) Find $P(0 < x \leq 2)$

$\rightarrow$ $\Sigma P = 1$

$\therefore 3c^3 + 4c - 10c^2 + 5c - 1 = 1$

$3c^3 - 10c^2 + 9c - 2 = 0$

2: ± 1, ± 2

for $c = 1$, LHS = RHS

By ~~syn~~ synthetic division,

1	3	-10	9	-2
		3	-7	2
	3	-7	2	0

Page No.
Date

$\therefore \ (c-1)(3c^2-7c+2)=0$

$(c-1)(3c^2-6c-c+2)=0$

$(c-1)(c-2)(3c-1)=0$

$\therefore \ c=1 \ , \ c=2 \ , \ c=1/3$

if $c=1$ $\quad P(0)=3$ $\quad$ not possible

if $c=2$ $\quad P(0)=24$ $\quad$ not possible

$\therefore \ c=1/3$

x	0	1	2
$P(x)$	$1/9$	$2/9$	$2/3$

$$P(0<x\le 2) = P(1)+P(2) = \frac{8}{9}$$

Q.2 Verify whether the following func is pdf

1) $f(x) = \frac{2}{9}\, x\left(2-x/2\right) \quad \ldots,\ 0<x\le 3$

2) $f(x) = \frac{1}{2}\, e^{-|x|} \quad \ldots,\ -\infty<x<\infty$

1) $f(x) = \frac{2}{9}\, x\left(2-x/2\right) \quad \ldots\ 0<x\le 3$

$f(x)\ge 0 \quad \forall \quad 0<x\le 3$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx$$

$$\int_{0}^{3} \frac{2}{9}\left(2x-\frac{x^2}{2}\right)dx = \frac{2}{9}\left[x^2-\frac{x^3}{6}\right]_0^3$$

$$=1$$

$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore$ Given $f(x)$ is pdf

2) $f(x) = \frac{1}{2} e^{-|x|} \quad \ldots \quad -\infty < x < \infty$

$f(x) \geq 0 \quad \forall \quad -\infty < x < \infty$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{1}{2} e^{-|x|}\,dx$$

$$= \frac{1}{2}\left\{\int_{-\infty}^{0} e^{-|x|}\,dx + \int_{0}^{\infty} e^{-|x|}\,dx\right\}$$

$$\because |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{2}\left\{\int_{-\infty}^{0} e^{x}\,dx + \int_{0}^{\infty} e^{-x}\,dx\right\}$$

$$= \frac{1}{2}\left\{\left[e^{x}\right]_{-\infty}^{0} - \left[e^{-x}\right]_{0}^{\infty}\right\}$$

$$= \frac{1}{2}\left\{(1-0) - (0-1)\right\}$$

$$= \frac{1}{2}(2)$$

$$= 1$$

$\therefore$ Given $f(x)$ is pdf

Page No.
Date

Q.3 Find the value of k if each of the foll func is pdf

1) $f(x) = kx^4 e^{-x/2} \qquad 0 \le x < \infty$

$\because f(x)$ is pdf

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{0} kx^4 e^{-x/2}\, dx + \int_{0}^{\infty} kx^4 e^{-x/2}\, dx = 1$$

$$k \frac{\sqrt{5}}{\left(\frac{1}{2}\right)^5} = 1$$

$$k = 1/768$$

2) $f(x) = \frac{k}{1+x^2}, \quad -\infty < x < \infty$

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$k \int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx = 1$$

$$k \left[\tan^{-1}(x)\right]_{-\infty}^{\infty} = 1$$

$$k \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = 1$$

$$\pi k = 1$$

$$k = \frac{1}{\pi}$$

Page No
Date

* Binomial Distribution & Poission distribution -

1) Binomial -

result - either succes or failure

$p \rightarrow$ prob. of succes $q \rightarrow$ prob of failure

if experiment is repeated n times, then

prob of success $= {}^nC_x \, p^x q^{n-x}$

e.g. tossing of coin, result of exam, etc.

mean $= \bar{x} = np$

variance $= \sigma^2 = npq$

if experiment, each n trials is repeated N times then expected freq^n = N F(x)

2) Poission -

Here the no. of succes only observed not number of failure.

$p \rightarrow$ prob. of success

* $n \rightarrow \infty$ & $p \rightarrow 0$

$$p(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \qquad \lambda = np$$

mean $= \bar{x} = np = \lambda$

variance $= \sigma^2 = np = \lambda$

$P(x > a) = 1 - P(x \leq a)$

$P(x \geq a) = 1 - P(x < a)$

1 If 10% bolts are defective, calculate the probability that out of sample selected at random of 10 bolts, not more than 1 bolt are defective.

success — defective bolt

$$p = 10\% = \frac{1}{10} \qquad q = \frac{9}{10}$$

$$P(x \leq 1) = P(x=0) + P(x=1)$$
$$= {}^{n}C_{x}\, p^{x} q^{n-x} + {}^{n}C_{x}\, p^{x} q^{n-x}$$
$$= {}^{10}C_{0}\, p^{0} q^{10-0} + {}^{10}C_{1}\, p\, q^{10-1}$$
$$= \left(\frac{9}{10}\right)^{10} + 10 \cdot \frac{1}{10}\left(\frac{9}{10}\right)^{9}$$
$$= 0.7361$$

2 Each of the five questions of mcq has four choices. Only one of which is correct. A student is attempting to guess the answer. What is the probability that student will get

1) Exactly 3 ans correct.
2) At most 3 ans correct.
3) At least 1 correct ans.

Here we use binomial distribution.
success — ans is correct

$$p = \frac{1}{4} \qquad q = \frac{3}{4}$$

i) $P(x=3) = {}^nC_x \, p^x q^{n-x}$

$= {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2$

$\frac{5!}{3!\,2!}$ $\frac{5\times4}{2}$ $= 10 \; \frac{3^2}{4^5}$

$= 0.0879$

ii) ~~$P(x\leq 3) = P(0)+P(1)+P(2)+P(3)$~~

$P(x\leq 3) = 1 - P(x>3)$

$= 1 - P(x=4) - P(x=5)$

$= 1 - {}^5C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^1 - {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0$

$= 1 - \frac{5\times 3}{4^5} - \frac{1}{4^5}$

$= 1 - \frac{16}{4^5}$

$= 0.9844$

iii) $P(x\geq 1) = 1 - P(x<1)$

$= 1 - P(0)$

$= 1 - {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5$

$= 0.7627$

Q.3) The probability that the person who undergoes the kidney operation will recover is 0.7. Find the probability that of 6 patients undergo similar operation

1) None will recover

2) all will recover

3) half will recover
4) atleast half will recover

success → recovery
$p = 0.7$ $q = 0.3$

$P(x)$ → no. of patients recovered
$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

i) $P(0) = {}^{6}C_{0}\,(0.7)^{0}(0.3)^{6}$
$= 0.000729$

ii) $P(6) = {}^{6}C_{6}\,(0.7)^{6}(0.3)^{0}$
$= 0.1176$

iii) $P(x=3) = {}^{6}C_{3}\,(0.7)^{3}(0.3)^{3}$
$= 0.1852$

iv) $P(x \geq 3) = P(x=3) + P(x=4) + P(x=5) + P(x=6)$
$= 0.9295$

Q.4) Betn 2 & 3 pm, the avg no. of phonecalls per min coming into a switchboard of company is 2.5. Find the probability during a minute there will be
1) no phone call 2) exactly 3 calls

Use poisson distribution.
$np = \lambda = 2.5$

$$P(x) = \frac{e^{-\lambda}\,\lambda^{x}}{x!}$$

i) no phone calls

$$P(x=0) = \frac{e^{-2.5}\lambda^0}{0!}$$

$$= 0.0821$$

ii) $$p(x=3) = \frac{e^{-2.5}(2.5)^3}{3!}$$

$$= 0.2138$$

Q.5) If the probability that an indivisual suffers bad reaⁿ from injection is 0. Determine prob. that out of 2000 indivisuals
1) exactly 3 2) more than 2
suffer bad reaction.

→ Use poisson distribution $\because n >> p$

$n = 2000$ $p = 0.001$ $\lambda = np = 2$

$p(x) = p$(no. of indivisuals suffered bad reaⁿ)

$$= \frac{e^{-\lambda}\lambda^x}{x!}$$

i) $$p(x=3) = \frac{e^{-2}2^3}{3!} = 0.1804$$

ii) $$p(x>2) = 1 - p(x \le 2)$$

$$= 1 - p(0) - p(1) - p(2)$$

$$= 1 - \frac{e^{-2}2^0}{0!} - \frac{e^{-2}2^1}{1!} - \frac{e^{-2}2^2}{2!}$$

$= \cancel{0.1879}$ $= 0.3233$

$\cancel{= 0.180}$

$\cancel{= 0.18}$

Q.6 Six dice are thrown 729 times. How many times do you expect atleast 3 dice to show 5 or 6

Use binomial distribution.

$n = 6 \qquad N = 729$

success → getting a number 5 or 6.

$p = \frac{1}{3} \qquad q = \frac{2}{3}$

$P(x) = {}^nC_x\, p^x\, q^{n-x}$

$P(x \geq 3) = 1 - P(x<3)$

$= 1 - P(x=0) - P(x=1) - P(x=2)$

$= 1 - {}^6C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6 - {}^6C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5 - {}^6C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$

$= 0.3196$

no. of times atleast 3 dice shows 5 or 6

$= N\, P(x)$

$= 729 \times 0.3196$

$= 232.9884$

≈ 233

Q.7 A fair coin is tossed 8 times. Find the probability that i) it shows head exactly 5 times.
ii) larger number of times than tail
iii) atleast one.

$p = \frac{1}{2} \qquad q = \frac{1}{2}$

i) $P(x=5) = 0.2188$

ii) $P(x \geq 5)$ ·

Q.8 A machine has 14 identical components that functions independently. It will stop working if 3 or more fails. If probability that component fails is 0.1. Find the probability that machine will be working.

→ We will solve this by binomial distribution.

$$P(x = \text{machine works}) = {}^{n}C_{x} (0.9)^{x} (0.1)^{n-x}$$

$$P(x > 11) = P(x=12) + P(x=13) + P(x=14)$$

$$= {}^{14}C_{12} (0.9)^{12} (0.1)^{2} + {}^{14}C_{13} (0.9)^{13} (0.1)^{1} + {}^{14}C_{14} (0.9)^{14} (0.1)^{0}$$

$$= 91 (0.9)^{12} (0.1)^{2} + 14 (0.9)^{13} (0.1)^{1} + (0.9)^{14} (0.1)^{0}$$

$$= 0.2570 + 0.3559 + 0.2288$$

$$= 0.8417$$

Q.9 During a war, one ship out of 9 sunk on an average in making a certain journey. What is the probability that exactly 3 out of 6 ships will arrive safely.

→ $q = \frac{1}{9}$ $p = \frac{8}{9}$ $n = 6$

$$P(x=3) = {}^6C_3 \left(\frac{8}{9}\right)^3 \left(\frac{1}{9}\right)^3$$

$$= 0.0193$$

Q.10 In a sampling, the mean no of defective bolts manufactured by machine in a sample of 20 is 2. Determine the expected no. of sample out of such 500 samples to contain at least 2 defective bolts.

→ $p = \frac{1}{10}$, $q = \frac{9}{10}$, $n = 20$, $N = 500$

At least 2 bolts defective.

$$\therefore P(x \geq 2) = 1 - P(x < 2)$$

$$P(x=0) = {}^{20}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20}$$

$$= 0.1216$$

$$P(x=1) = {}^{20}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{19}$$

$$= \frac{20}{10} \left(\frac{9}{10}\right)^{19}$$

$$= 0.2702$$

$$P(x<2) = 1 - (0.1216 + 0.2702)$$

$$= 1 - 0.3918$$

$$= 0.6082$$

$$500 \times 0.6082 = 304.100$$

$$\approx 300$$

11) In a certain factory, producing cycle tyres, there is a small chance one in 500 for any tyre to be defective, the tyres are supplied in lots of 20. Calculate the approximate no. of lots containing no defective, one defective, 2 defective tyres, in a thousands of 20,000 tyres.

→ We will use poison de distribution.

$p = \frac{1}{500} = 0.002$, $n = 20$, $N = 20{,}000$

$\lambda = np = 20 \times 0.002 = 0.04$

$P(x)$ = probability of no. of defective tyres.

$= \frac{e^{-\lambda} \lambda^{x}}{x!}$

i) No defective:

$P(x=0) = 0.9608$

ii) One defective:

$P(x=1) = 0.0384$

iii) Two defective:

$P(x=2) = 0.0008$

$\text{No. of lots} = \frac{20000}{20} = 1000$

No. of lots containing no defective tyres

$= 1000 \times 0.9608$

$= 960.8$

$= 961$

No. of lots containing two defective tyres

$= 1000 \times 0.0008$

$= 0.8$

≈ 1

Q.12 A company has two cars which it hires out day by day. The number of demands for a car at each day is distributed as poisson variate with mean 1.5 calculate the number of days in year for which i) neither car is in demand.
2) Demand is refused

→ $\lambda = np = 1.5$

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-1.5}(1.5)^{x}}{x!}$$

i) $$P(x=0) = \frac{e^{-1.5}(1.5)^0}{0!} = 0.2231$$

~~ii)~~ No. of days for year =

$365 \times 0.2231 = 81$ or

$366 \times 0.2231 = 82$

ii) $P(x>2) = 1 - P(x \le 2)$

$= 1 - (P(x=0) + P(x=1) + P(x=2))$

$= 1 - (0.2231 + 0.3347 + 0.2510)$

$= 0.1912$

No. of days $= 365 \times 0.1912 \cong 70$ days OR

$= 366 \times 0.1912 \cong 70$ days

Q.13 7 coins are tossed & the number of heads obtained are noted. The exp. is repeated 128 times. Following distribution is obtained.

1> No. of hands	Freq^n	Expected $= N\ ^nC_x (\frac{1}{2})^x (\frac{1}{2})^{n-x}$	
0	7	1	
1	6	7	
2	19	21	
3	35	35	Ans. (i)
4	30	35	
5	23	21	
6	7	7	
7	1	1	

i> if coin is unbiased

ii> Nature of coin is not known

→ i> $n=7$, $p=\frac{1}{2}$, $q=\frac{1}{2}$

$P(x) = {}^nC_x (p)^x (q)^{n-x}$

$P(x=0) = {}^7C_0 (\frac{1}{2})^0 (\frac{1}{2})^7$

$= 0.0078$

$0.0078 \times 128 = 1$

$P(x=1) = {}^7C_1 (\frac{1}{2})^1 (\frac{1}{2})^6 = 0.0547$

$0.0547 \times 128 = 7$

$P(x=2) = 0.1641$

$128 \times 0.1641 = 21$

$P(x=3) = 0.2734$

$0.2734 \times 128 = 35$

Page No.
Date

$P(x=4) = 0.2734$

$0.2734 \times 128 = 35$

$P(x=5) = 0.1641$

$0.1641 \times 128 = 21$

$P(x=6) = 0.0547$

$128 \times 0.0547 = 7$

$P(x=7) = 0.0078$

$0.0078 \times 128 = 1$

ii) $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

No. of heads	Freq	$x_i f_i$
0	7	0
1	6	6
2	19	38
3	35	105
4	30	120
5	23	115
6	7	42
7	1	7
	$\Sigma f_i = 128$	$\Sigma f_i x_i = 433$

$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{433}{128} = 3.3828$

mean $= np = 3.38$ $\quad 7p = 3.38$

$\therefore p = 0.48 = 12/25$

Page No.
Date

Now the nature of coin is unknown
we have $p = 0.48$, $q = 0.52$, $n = 7$

$$P(x=0) = {}^7C_0 \left(\frac{12}{25}\right)^0 \left(\frac{13}{25}\right)^7$$

$$\approx 0.0103$$

$N \times 0.0103 = 1.3159$

$N \times P(0) \cong 1$

$N \times P(x=1) = 128 \times 0.0664 \cong 9$

$N \times P(x=2) = 128 \times 0.1840 \cong 24$

$N \times P(x=3) = 128 \times 0.283 \cong 36$

$P(x=4) \times N = 0.283 \times 128 \cong 36$

$N \times P(x=5) = 128 \times 0.1447 = 18.52 \cong 19$

$N \times P(x=6) = 128 \times 0.0445 = 5.6985 \cong 6$

$N \times P(x=7) = 0.0059 \times 128 \cong 1$

Heads	Expected
0	1
1	9
2	24
3	36
4	36
5	19
6	6
7	1

* Normal distribution
Probability curve -
Area under the curve bet^n $x = \alpha$ & $x = \beta$ is $P(\alpha < x < \beta)$ & total area is 1.

$$P(\alpha < x < \beta) = \int_{\alpha}^{\beta} f(x)\, dx = 1$$

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

$m \rightarrow$ mean $\qquad \sigma \rightarrow$ std deviation

Probability of continuous random variable lies bet^n x_1 & x_2 is area under the curve bet^n x_1 & x_2

$$P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} y\, dx = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$$

$$z = \frac{x-m}{\sigma}$$

$$dz = \frac{dx}{\sigma}$$

$$\therefore dx = \sigma\, dz$$

$x = x_1 \qquad z_1 = \dfrac{x_1 - m}{\sigma}$

std normal curve :-

$\cdot m = 0, \sigma = 1$

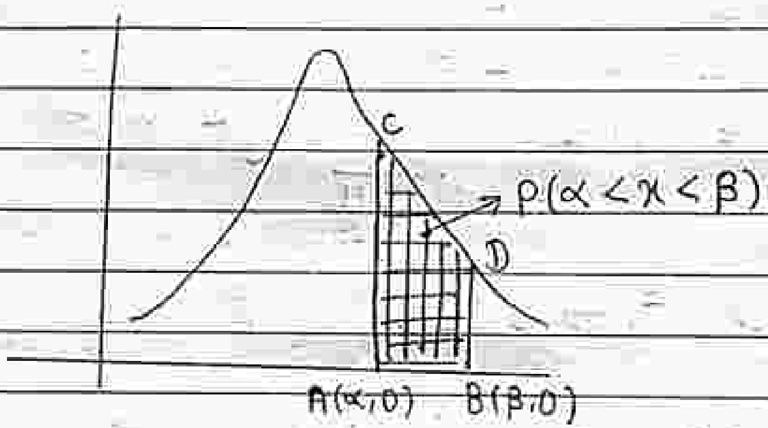
$x = z$

$x = x_2 \quad z_2 = x_2 - m/\sigma$

$$\therefore P(Z_1 \le Z \le Z_2) = \int_{Z_1}^{Z_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \sigma \, dz$$

$$= \int_{Z_1}^{Z_2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

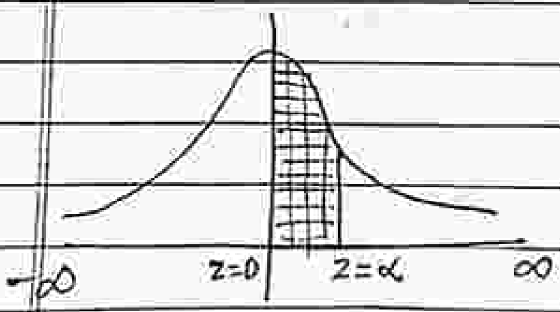
IMP * Normal curve is bell shaped, symmetric about highest ordinate.



Total area under curve = 1

* $P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$

*



$P(-\infty < Z < \infty) = 1$

$P(-\infty < Z < 0) = P(0 < x < \infty) = \frac{1}{2}$

$P(0 < z < \alpha)$ = shaded region

Page No.
Date

* Standard normal curve -

Area from $z=0$ to $z=2$ is 0.4772

$z=0$ to $z=1$ is 0.3413

$z=0$ to $z=0.525$ is 0.2

$z=0$ to $z=1.28$ is 0.4

$z=0$ to $z=0.25$ is 0.1

$z=0$ to $z=0.52$ is 0.2

$z=0$ to $z=1.645$ is 0.45

$z=0$ to $z=0.5$ is 0.19

$z=0$ to $z=1.4$ is 0.42

Q.1 Students of the class give an aptitude test. Their marks are normally distributed with mean 60, S.D.-5. What is % of students who scored more than 60.

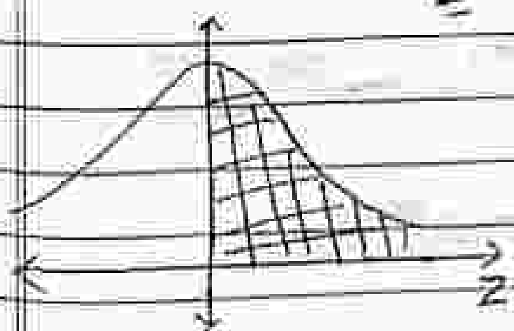
→ $m=60$, $\sigma=5$

$x \to$ marks

$$P(x>60) = P\left(\frac{x-m}{\sigma} > \frac{60-m}{\sigma}\right)$$

$$= P(z>0)$$

$$= 0.5$$



∴ 50% students got more than 60 marks

* Standard normal curve –

Area from $z=0$ to $z=2$ is 0.4772

$z=0$ to $z=1$ is 0.3413

$z=0$ to $z=0.525$ is 0.2

$z=0$ to $z=1.28$ is 0.4

$z=0$ to $z=0.25$ is 0.1

$z=0$ to $z=0.52$ is 0.2

$z=0$ to $z=1.645$ is 0.45

$z=0$ to $z=0.5$ is 0.19

$z=0$ to $z=1.4$ is 0.42

Q.1 Students of the class give an aptitude test. Their marks are normally distributed with mean 60, S.D. 5. What is % of students who scored more than 60.

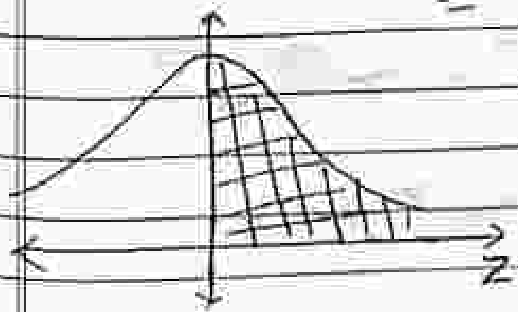
→ $m = 60$, $\sigma = 5$

$x \to$ marks

$$P(x > 60) = P\left(\frac{x-m}{\sigma} > \frac{60-m}{\sigma}\right)$$

$$= P(z > 0)$$

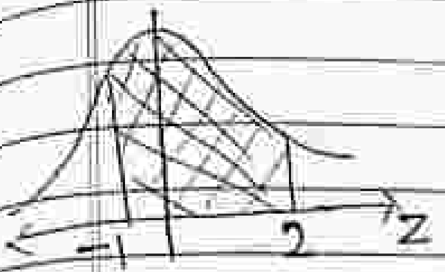
$$= 0.5$$



∴ 50% students got more than 60 marks.

ii) $P(45 \leq x \leq 60) = P\left(\frac{45-50}{5} \leq z \leq \frac{60-50}{5}\right)$

$= P(-1 \leq z \leq 2)$



$= P(-1 \leq z \leq 0) + P(0 \leq z \leq 2)$

$= 0.3413 + 0.4772$

$= 0.8185$

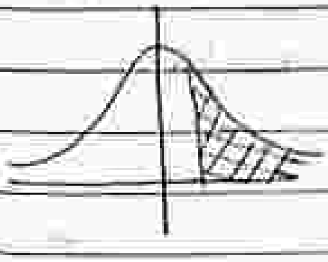
$\therefore$ No. of students $= 4000 \times 0.8185$
$= 3274$

Q.4 In an exam, given by 500 candidates if avg & SD of marks obtained are 40 & 10 respectively. Assuming the marks are distributed normally Find approximately
1) How many will pass if 50 is fixed as minimum
2) What should be min if 350 candiates are to pass

$\rightarrow$ $x \rightarrow$ marks
$m = 40$, $\sigma = 10$, $N = 500$

1) $P(x \geq 50) = P(z \geq 1)$

$= 0.5 - (0 < z < 1)$



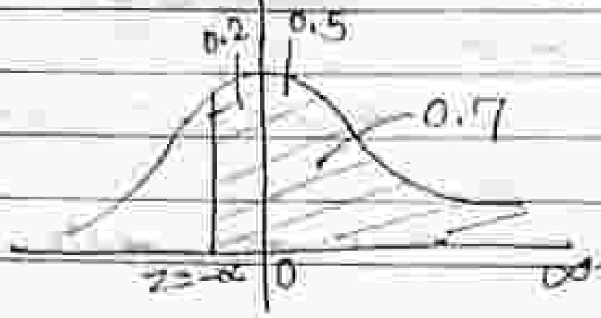
$= 0.5 - 0.3413$

$= 0.1587$

No. of students $= 500 \times 0.1587 = 79.35 \approx 79$

ii) let x be minimum marks

$P(\text{passing}) = \frac{350}{500} = 0.7$



Area from $z = -\alpha$ to $z = \infty$ is 0.7

We know that,

Area from $z = 0$ to $z = \infty$ is 0.5

$z = -\alpha$ to $z = 0$ is 0.2

$\therefore$ From given values,

$\alpha = 0.525$

$z = -0.525 = \frac{x - 40}{10}$

$x = -5.25 + 40$

$x = 34.75$

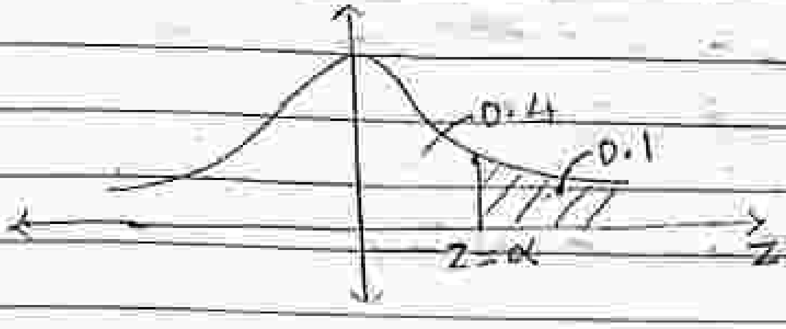
Q.5 Determine the min marks of students in order to get A grade if 10% students in order awarded A grade in an exam where mean & S.D. are 72 & 9 resp.

$\rightarrow$ $x \rightarrow$ min marks

$m = 72 \qquad \sigma = 9$

10% students awarded A grade

$P(\text{A grade}) = \frac{10}{100} = 0.1$



$P(\alpha < Z < \infty) = 0.1$
$P(0 < Z < \alpha) = 0.4$

from given values $\alpha = 1.28$

$z = \frac{x - m}{\sigma}$

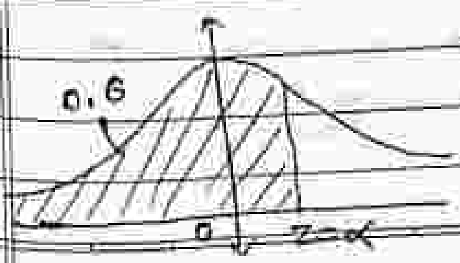
$\therefore x = 83.52$

$\therefore$ students need min 83.52 marks for A grade.

Q.6 When the mean of marks was 50% & S.D. 5%, the 60% students failed in an exam. Determine the grace marks to be avoided in order to show that 70% of the students passed, consider marks ~~due~~ are normally distributed.

→ $m = 50/100 = 0.5$, $\sigma = 5/100 = 0.05$
let x be marks obtained by student.

Before grace marks were awarded, 60% students failed.



$P(0 < z < \alpha) = 0.1$
$\therefore \alpha = 0.25$
$\therefore \alpha = \frac{x - m}{\sigma}$

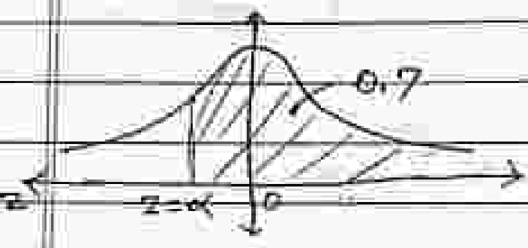
$0.25 = x - 0.5/0.05$

$x = 0.5125$

$\therefore$ minimum passing marks before grace is

$0.5125 \times 100 = 51.25$

After grace, 70% students passed.



$P(\alpha < z < 0) = 0.2$

$\alpha = -0.525$

$\alpha = \frac{x - m}{\sigma}$

$-0.525 = x - 0.5/0.05$

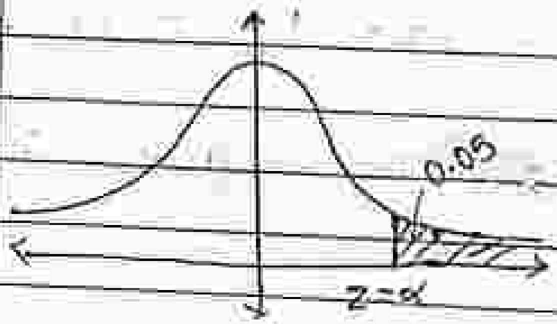
$x = 0.4740$

min marks after the grace $= 47.40$

$\therefore$ Grace marks $= 51.25 - 47.40 = 3.85$

Q.7 The income of 10,000 rupees persons is normally distributed with mean 520 Rs & σ 60 Rs. Find
1) lowest income of richest 500
2) highest income of poorest 500.

→ a) richest 500 means 5% of 10,000 = 0.05



area from
$z = 0$ to $z = \alpha$ is 0.45

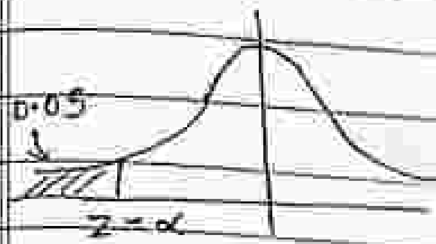
$\therefore \alpha = 1.645$

$\alpha = x - m/\sigma$

$1.645 = x - 520/60$

$x = 618.7$

b) for poorest 500



$P(0 < z < \alpha) = 0.45$

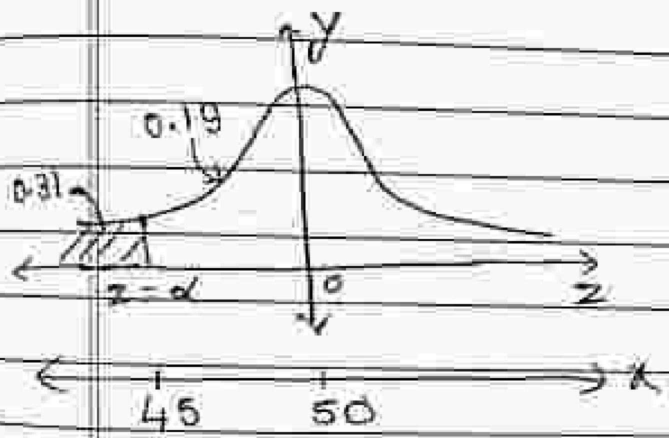
$\alpha = -1.645$

$x = 421.3$

Q.8 In a normal distribution, 31% items are under 45 & 8% are over 64 find mean & S.D.

→ 31% are under 45

0.31 are under 45



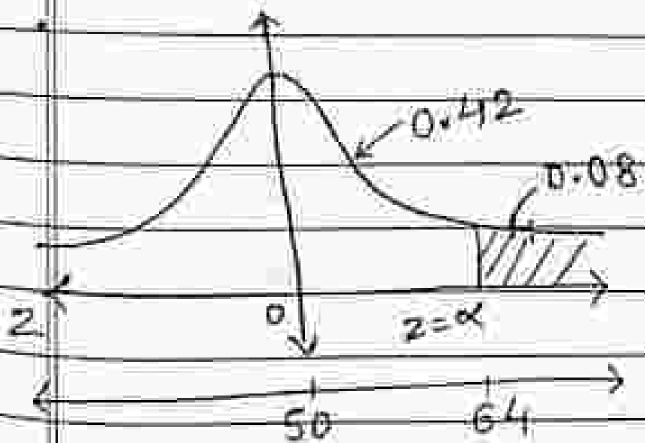
$P(\alpha < z < 0) = 0.19$

$\therefore \alpha = -0.5$

$x = \alpha\sigma + m$

$45 = -0.5 \times \sigma + m$

$m = 45 + 0.5\sigma$



$P(0 < z < \alpha) = 0.42$

$\alpha = 1.4$

$\alpha = \dfrac{x - m}{\sigma}$

$m = -1.4\sigma + 64$

$m = 45 + 0.5\sigma$

$m = 64 - 1.4\sigma$

$\sigma = 10$

$m = 50$

* Conditional Probability -

To obtain the probability of B when A has occured already.

$P(B/A)$

$P(A) \rightarrow$ proportion of area of whole sample space taken up by A

$P(B/A) \rightarrow$ proportion of area of A taken up by B.

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \qquad P(A) \neq 0$$

* Multiplication rule -

$$P(A \cap B) = P(A) \cdot P(B/A) = P(B) \; P(A/B)$$

* Partition of the sample space -

A collection of mutually exclusive & $(A \cap B) = \phi$ exhaustive events is called partition of S. $(A \cup B = S)$

ex. $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$ $\qquad B = \{2, 4, 6\}$

i.e. $A = B^c$

$A \cap B = \phi$

$A \cup B = S$

* Baye's theorem –
Suppose events $A_1, A_2, A_3 \ldots A_n$ forms partition of a sample space of random experiment and B be any another event with $P(B) > 0$ defined on same sample space. Then probability ~~of~~

$$P(A_i/B) = \frac{P(A_i) \; P(B/A_i)}{\sum_{j=1}^{n} P(A_j) \; P(B/A_j)}$$

..... (for $i = 1, 2, 3, \ldots n$)

Q.1 Four cards are drawn at random from well shuffled pack of 52 cards. Find prob.
i) Two cards are red & 2 are black
ii) All cards of the different suit
iii) all cards from same suit.
iv) one is king

→ 1) $n(S) = {}^{54}C_4$

A → two red two black

$$P(A) = \frac{{}^{26}C_2 \; {}^{26}C_2}{{}^{54}C_4}$$

$= 0.3902$

2) B → all cards diff. suit

$n(B) = {}^{13}C_1 \; {}^{13}C_1 \; {}^{13}C_1 \; {}^{13}C_1$

$$P(B) = \frac{{}^{13}C_1 \; {}^{13}C_1 \; {}^{13}C_1 \; {}^{13}C_1}{{}^{54}C_4}$$

$= \cancel{0.0106}$

$= 0.1055$

3) C → all cards from same suit

$$P(C) = \frac{n(C)}{n(S)} = \frac{{}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4}{{}^{54}C_4}$$

$$= 0.0106$$

4) D → one is king

$$P(D) = \frac{n(D)}{n(S)} = \frac{{}^{48}C_3 \; {}^{4}C_1}{{}^{52}C_4} = 0.2556$$

Q2. If a pair of unbiased coins are used. Obtain the probability of occurance of
a) both heads b) single head
c) atleast one head.

→ $S = \{HH, HT, TH, TT\}$ $n(S) = 4$

a) A → getting both heads

$A = \{HH\}$ $n(A) = 1$

$P(A) = 1/4$

b) B → getting single head

$B = \{HT, TH\}$ $n(B) = 2$

$P(B) = 2/4 = 1/2$

c) C → getting atleast one head

$C \rightarrow \{HH, TH, HT\}$

$n(C) = 3$

$$P(C) = \frac{3}{4}$$

Date

Q.3 A machine consists of two parts P_1 & P_2. Probability of deffect in P_1 is 0.08 & that in P_2 is 0.05. What is the probability that the assembled machine will not have any deffect.

→ Event A → P_1 is free from deffect
Event B → P_2 is free from deffect

$$P(A) = 1 - 0.08 = 0.92$$
$$P(B) = 1 - 0.05 = 0.95$$

Event C → machine not have any deffect

$$P(C) = P(A \cap B)$$
$$= P(A) \cdot P(B)$$
$$= 0.92 \times 0.92$$
$$P(C) = 0.874$$

Q.4 The odds that A speaks truth are 5 : 3 & that of B are 8 : 3. In what % of cases are A & B likely to contradict each other on an identical point?

→ E_1 → A speaks truth
E_2 → B speaks truth

$$P(E_1) = \frac{5}{5+3} = \frac{5}{8}, \quad P(E_1') = \frac{3}{8}$$
$$P(E_2) = \frac{8}{8+3} = \frac{8}{11}, \quad P(E_2') = \frac{3}{11}$$

Date

They will contradict in the following cases

i) A speaks truth, B tells lie $= P(E_1 \cap E_2')$

$\cancel{P(E_1 \cap E_2') = -P(E_1 \cap E_2) + P(E_1)}$

$\cancel{= \frac{5}{8} -}$

ii) A speaks lie & B speaks truth $= P(E_1' \cap E_2)$

Required probability $= P(E_1 \cap E_2') + P(E_1' \cap E_2)$

$= P(E_1)\,P(E_2') + P(E_1')\,P(E_2)$

$= \frac{5}{8} \cdot \frac{3}{11} + \frac{3}{8} \cdot \frac{8}{11}$

$= \frac{39}{88}$

$= 0.4432$

$\% = 44.32$

Q.5 A problem is given to 4 students A, B & C whose chances of solving the same are $1/2$, $1/3$ & $1/4$ resp. If all 3 students solve the problem independently, What is the prob. that the problem will be solved?

→ Req. prob. $= 1 - (P(A') \cdot P(B')\,P(C'))$

$= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$

$= \frac{3}{4}$

$= 0.75$

OR

$P(A \cup B \cup C) = \frac{3}{4}$

Page No.
Date

Q.6 Pair of fair dice is rolled. If the sum of 8 has appeared. Find the probability that one of the dice shows 3.

$A \to$ 1 dice shows 3
$B \to$ sum of 8 has appeared

$A = \{(1,3), (2,3), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,3), (5,3), (6,3)\}$
$B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$n(A \cap B) = 2$
$P(A/B) = n(A \cap B) / n(B)$
$= \frac{2}{5}$
$= 0.4$

Q.7 A random experiment results in an integer outcome bet^n 1 & 10 (both including). All numbers are equally likely. let A be event that odd no. occurs & B be event that no. is divisible by 3.
Obtain (1) $P(A/B)$ (2) $P(B/A)$ (3) $P(A'/B)$
(4) $P(A/B')$ (5) $P(A'/B')$

$\to$ $n(S) = 10$
$A \to$ odd no. occurs
$A = \{1, 3, 5, 7, 9\}$ $n(A) = 5$
$B \to$ no. divisible by 3
$B = \{3, 6, 9\}$
$n(A \cap B) = 2$ $n(B) = 3$
$n(A' \cap B) = 1$ $n(A \cap B') = 3$
$P(A \cap B) = \frac{2}{10} = \cancel{\frac{1}{5}}$

1) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/10}{3/10} = \frac{2}{3}$

2) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{2/10}{5/10} = \frac{2}{5}$

3) $P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{1/10}{3/10} = \frac{1}{3}$

4) $P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{3/10}{7/10} = \frac{3}{7}$

5) $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{4/10}{7/10} = \frac{4}{7}$

Q.8 A card is drawn from an ordinary pack of 52 playing cards. What is prob. that it is a king given that it is face card.

→ $A \rightarrow$ card is king $\quad n(A) = 4/52$
$B \rightarrow$ card is face $\quad n(B) = 12/52$

$$P(A/B) = \frac{n(A \cap B)}{n(B)} = \frac{4}{12} = \frac{1}{3}$$

Q.9 The personnel development of company has 100 computers whose distribution is

Age (year)	B.E.	M.E.	Total
20 - 30	20	5	25
30 - 40	25	10	35
≥ 40	10	30	40
Total	55	45	100

Page No.
Date

Q.10 If an engineer is selected at random, find
a) prob that he is only B.E.
b) prob that he has M.E. given that his age is beyond 40.
c) The prob that he is under 30 given that he has M.E. degree.

→ A - only BE   B → only ME
C → age beyond 40   D → age below 30

$n(A) = 55$ $n(B) = 45$
$n(C) = 40$ $n(D) = 25$

a) $P(A) = 55/100 = 0.55$

b) $P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{30}{40} = \frac{3}{4}$

c) $P(D/B) = \frac{P(B \cap D)}{P(B)} = \frac{5}{45} = \frac{1}{9}$