Unit - II

Chapter

2

Single Phase AC Circuits

Syllabus :

Cycle, Frequency, Periodic time, Amplitude, Angular velocity, RMS value, Average value, Form factor, Peak factor, Impedance, Phase angle, and Power factor, Mathematical and phasor representation of alternating emf and current, AC in resistors, Inductors and capacitors, AC in RL series, RC series, RLC series and parallel circuits, Power in AC circuits, Power triangle.

Chapter Contents

2.1 Difference between AC and DC Quantities : W-05, S-16

MSBTE Questions

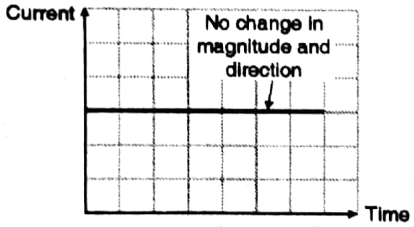
Q. 1 Define the following term related to AC fundamentals : Alternation **(W-05, 1 Mark)**

Q. 2 Define AC and DC current. **(S-16, 2 Marks)**

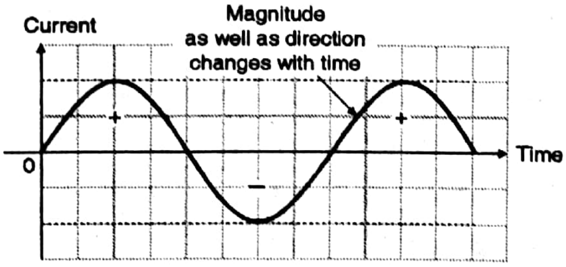
- The circuits can be either DC or AC types. In this chapter we are going to discuss the "ac circuits".
- The long form of ac is alternating current. What is alternating in the ac ? The answer is both the magnitude and direction of the alternating current changes as shown in Fig. 2.1.1(b). No such changes take place for the direct current (dc) as shown in Fig. 2.1.1(a).

Pure dc quantity :

- We can define a pure dc quantity (voltage, current or power) as the one which does not change its value or polarity with respect to time.



(a) Waveform of direct current (dc)



(b) Waveform of alternating current (ac)

(A-1539) **Fig. 2.1.1**

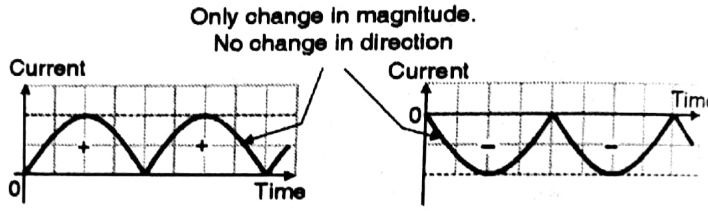
- As shown in Fig. 2.1.1(b) the alternating current changes its value with respect to time. It's value can be positive, zero, negative, maximum or in between zero and maximum. This is called as change in magnitude.
- Look at Fig. 2.1.1(b), the alternating current can be positive, negative or zero. The current that changes its polarity is called as alternating current.

Definition of an ac quantity :

- An alternating (ac) quantity (voltage, current or power) is defined as the one which changes its value as well as direction (polarity) with respect to time.
- Thus it is important to note that a quantity is called as an ac quantity if and only if both its values as well as polarity changes with respect to time.

Pulsating DC :

- In practice we may not always use the pure dc quantities. The practical dc quantities may change their magnitude (value) with respect to time. But the polarity does not change.
- Such dc quantities are called as pulsating dc quantities.
- Fig. 2.1.2 shows the waveforms for pulsating dc currents.



(a) A pulsating direct current **(b) Pulsating direct current**

(A-1540) **Fig. 2.1.2**

2.2 AC Waveforms :

2.2.1 Definition of Waveform :

- A waveform is a graph of magnitude of a quantity with respect to time.
- The quantity plotted on the X-axis is time and the quantity plotted on the Y-axis will be voltage, current, power etc.

2.2.2 Types of AC Waveforms :

- The shape of an ac quantity such as current, voltage or power need not always be a sinewave.
- The ac voltage or current can have other shapes also such as a triangular wave, square wave or a trapezoidal waveform.
- That means any waveform irrespective of its shape will be called as an alternating waveform if its magnitude and direction changes with time.

2.2.3 Waveform of Sinusoidal AC :

- One of the alternating waveforms is the sinusoidal AC waveform. The term sinusoidal is used for two types of waveforms :

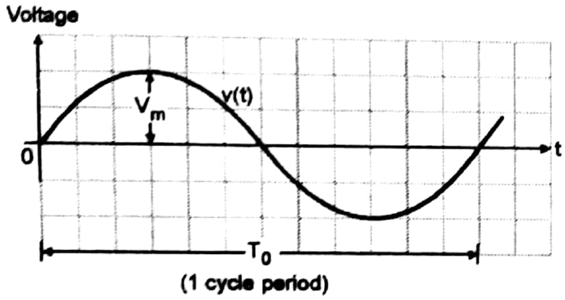
1. Sine waves 2. Cosine waves

- In this chapter we are going to deal with the sine waves only.
- The sinewaves can be represented in two different ways :
  1. Graphical representation (Waveform)
  2. Mathematical representation.

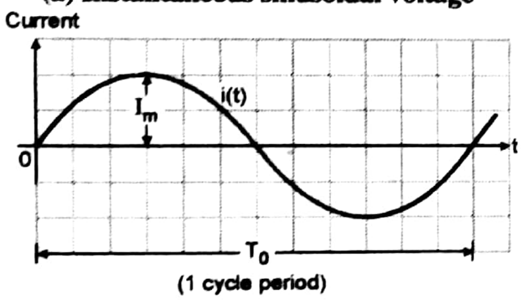
2.2.4 Graphical and Mathematical Representation of Sinusoidal AC Quantities :

Graphical Representation :

- The graphical representation (waveforms) of sinusoidal voltage and current is shown in Fig. 2.2.1.



(a) Instantaneous sinusoidal voltage



(b) Instantaneous sinusoidal current

(A-558)**Fig. 2.2.1**

- For plotting such a waveform we plot time (t) on x-axis and corresponding voltage v (t) or current i (t) on the y-axis.
- That means the graphical representation of sinusoidal voltage or current shows the instantaneous variation in the voltage or current with respect to time.

Mathematical representation :

- The voltage waveform of Fig. 2.2.1(a) is mathematically represented as,

$$v(t) = V_m \sin(2\pi f_0 t) \qquad \text{...(2.2.1)}$$

Where, $v(t)$ = Instantaneous voltage,

V_m = Peak value (or maximum value)

f_0 = Frequency in Hz. ($f_0 = 1 / T_0$)

Variable is time (A-559)

v(t)

Instantaneous voltage

and "sin" represents the shape of the waveform.

- It can also be represented as,

$$v(t) = V_m \sin(\omega_0 t) \quad \text{or} \quad V_m \sin\theta$$

where $\theta = \omega_0 t = 2\pi f_0 t$

- Similarly the current waveform of Fig. 2.2.2(b) is mathematically represented as,

$$i(t) = I_m \sin(2\pi f_0 t) \qquad \text{...(2.2.2)}$$

where $i(t)$ = Instantaneous voltage,

I_m = Peak value

f_0 = Frequency in Hz.

Variable is time (A-560)

i(t)

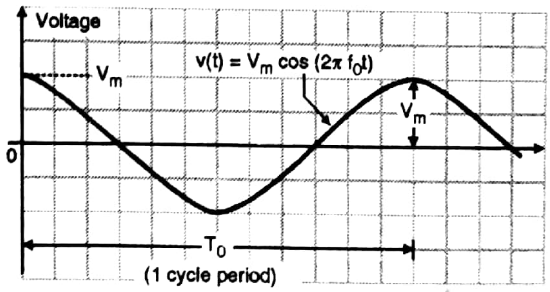
Instantaneous current

- The alternating current can also be represented mathematically as,

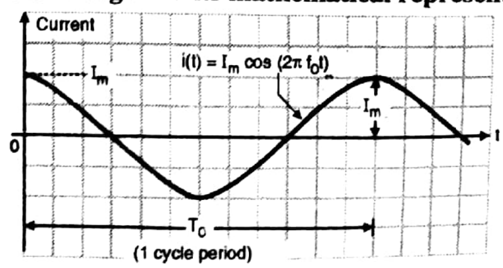
$$i(t) = I_m \sin(\omega_0 t) = I_m \sin\theta \qquad \text{...(2.2.3)}$$

where $\theta = \omega_0 t = 2\pi f_0 t$

- The sinusoidal waveform can be sinewaves or cosine waves. The cosine voltage and current waveforms and their mathematical expressions are as shown in Figs. 2.2.2(a) and (b).



(a) Cosine voltage and its mathematical representation



(b) Cosine current and its mathematical representation

(A-561)**Fig. 2.2.2**

2.3 Definitions :

In this section we are going to define some of the very basic but important terms related with the ac quantities. Remember that the AC quantity can be current, voltage, power ... anything.

Some of the important terms are :

1. Waveforms
2. Instantaneous value
3. Cycle period
4. Time
5. Frequency
6. Amplitude
7. Angular velocity
8. Electrical and mechanical angle

2.3.1 Waveform : S-05. W-06. S-11

MSBTE Questions

Q. 1 Define the following term related to ac : Waveform **(S-05, W-06, 1 Mark, S-11, 4 Marks)**

- The waveform is a graph of magnitude of an AC quantity against time. The waveform tells us about instantaneous (instant to instant) change in the magnitude (value) of an AC waveform.

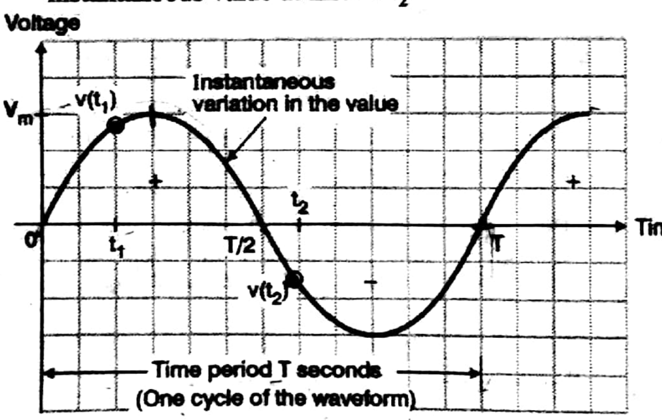
2.3.2 Instantaneous Value : S-09. S-11

MSBTE Questions

Q. 1 Define - instantaneous value and frequency of alternating quantities. **(S-09, 2 Marks)**

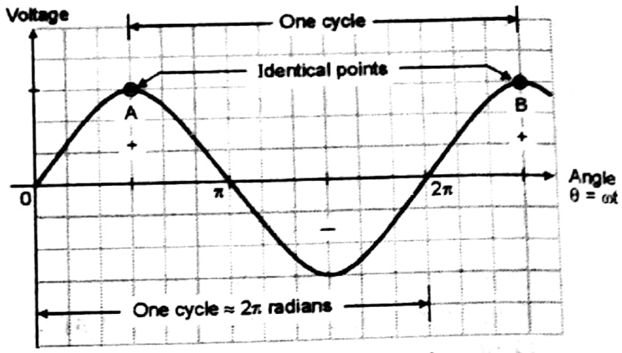
Q. 2 Define the following term related to AC : Instantaneous value. **(S-11, 4 Marks)**

- The instantaneous value of an ac quantity is defined as the value of that quantity at a particular instant of time.
- For example in Fig. 2.3.1, $v(t_1)$ is the instantaneous value of the ac voltage v (t) at instant t_1 or v (t_2) is its instantaneous value at instant t_2.



(a) Waveform and instantaneous value of an ac voltage

(A-565) **Fig. 2.3.1**



(b) Definition of cycle

(A-566) **Fig. 2.3.1**

2.3.3 Cycle :

S-05. W-05. S-08. W-08. S-11. S-12. W-12. S-15. W-16

MSBTE Questions

Q. 1 Define the following term related to ac : Cycle. **(S-05, W-05, S-08, W-08, S-11, S-12, W-12, S-15, W-16, 1 Mark)**

- In an ac waveform, a particular portion consisting of one positive and negative part repeats many times. Each repetition consisting of one positive and one identical negative part is called as **one cycle of the waveform.** Refer Fig. 2.3.1(b).
- A cycle can start at any point on the waveform (point A) and it ends when an identical point is obtained (point B) as shown in Fig. 2.3.1(b).
- If the waveform is plotted by plotting angle on the X-axis in place of time, then **cycle** is that portion of the waveform corresponding to an angle span of 2π radians as shown in Fig. 2.3.1(b).

$$\textbf{1 cycle} \equiv 2\pi \textbf{ radians} = 360^\circ$$

2.3.4 Time Period or Periodic Time (T) :

S-05. W-05. W-06. W-08. S-11. S-13. W-14. W-16. I-Scheme : W-18

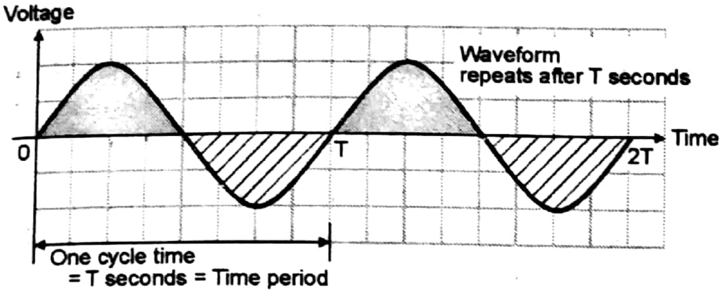
MSBTE Questions

Q. 1 Define the following term related to ac : Time period. **(S-05, W-05, W-06, W-08, S-11, S-13, W-14, W-16, 1 Mark)**

Definition :

- Time period (T) is defined as the time taken in seconds by the waveform of an ac quantity to complete one cycle. After every T seconds, the cycle repeats itself as shown in Fig. 2.3.2.

∴ **Time period T = Time corresponding to one cycle**



(A-567) **Fig. 2.3.2 : Concept of time period (T)**

2.3.5 Frequency :

S-05, W-05, W-06, S-08, S-09, W-09, W-12, S-13, W-14, S-15, W-16, S-17, I-Scheme : W-18

MSBTE Questions

Q. 1 Define the following term related to ac : Frequency.

(S-05, W-05, W-06, S-08, S-09, W-09, W-12, S-13, W-14, S-15, W-16, S-17, 1 Mark)

Definition :

– Frequency is defined as the number of cycles completed by an alternating quantity in one second. It is denoted by "f" and its units are cycles/second or Hertz (Hz).

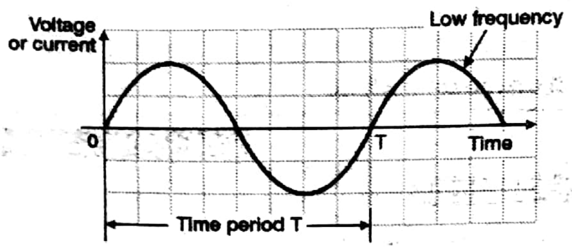
– As the time period (T) is the time in seconds per cycle denoted in seconds/cycle, hence relation between frequency and time period is as follows :

$$\text{Frequency (f)} = \frac{\text{Cycles}}{\text{Second}} = \frac{1}{\text{second / cycle}} = \frac{1}{T}$$

$$\therefore \quad f = (1 / T)\ \text{Hz}$$

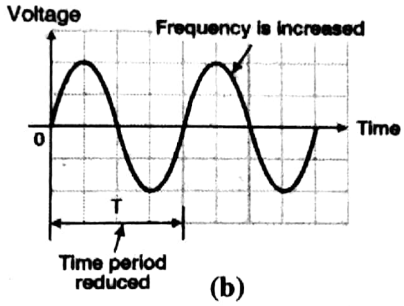
– Therefore as the time period increases, the frequency decreases and vice-versa as shown in Fig. 2.3.3.

– Thus as we decrease the time period (T) the frequency increases as there are more number of cycles per second.



(a)

Fig. 2.3.3



(b)

(A-568) **Fig. 2.3.3 : Effect of change in time period (T) on the value of frequency**

2.3.6 Amplitude :

W-06, S-08, W-08, W-09, S-13, W-16

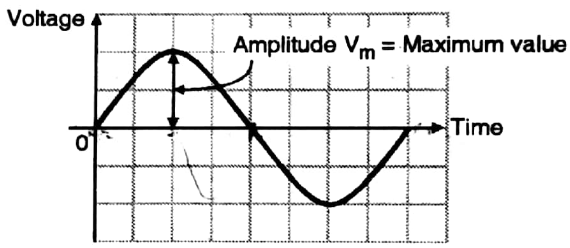
MSBTE Questions

Q. 1 Define : Maximum value

(W-06, S-08, W-08, W-09, S-13, W-16, 1 Mark)

Definition :

– The maximum value or peak value of an ac quantity is called as its amplitude. This is shown in Fig. 2.3.4. The amplitude is denoted by V_m for voltage, I_m for current waveform etc.



(A-569) **Fig. 2.3.4 : Concept of amplitude**

2.3.7 Angular Velocity (ω) :

Definition :

– The angular velocity (ω) is the rate of change of angle ωt with respect to time.

$$\therefore \quad \omega = \frac{d\theta}{dt} \qquad ...(2.3.1)$$

Where dθ is the change in angle in time dt.

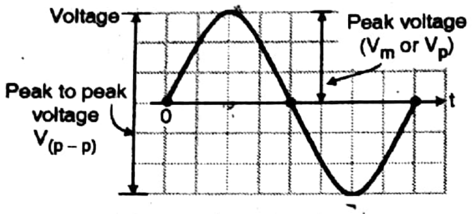
If dt = T i.e. time period, (one cycle) then the corresponding change in θ is 2π radians.

$$\therefore \quad d\theta = 2\pi$$

$$\therefore \quad \omega = \frac{2\pi}{T} \qquad ...(2.3.2)$$

But $1 / T = f$. $\quad \therefore \quad \omega = 2\pi f$

2.4 Peak and Peak to Peak Voltage :



(A-570) **Fig. 2.4.1 : Peak and peak to peak value**

Definition :

- Peak voltage is the voltage measured from the baseline of an ac waveform to its maximum or peak level. It is also called as amplitude.
- Peak voltage is denoted by V_m or V_p.
- For a typical sinewave, the positive peak voltage is equal to the negative peak voltage.
- Peak to peak voltage is the voltage measured from the maximum positive level to maximum negative level.
- Peak to peak values are most often used when measuring the magnitude on the cathode ray oscilloscope (CRO) which is a measuring equipment.
- For a typical sinewave, the peak to peak value is equal to 2 times the peak value.
- Peak to peak voltage is denoted by V_{p-p}

$$\therefore \quad V_{p-p} = 2\,V_m.$$

Ex. 2.4.1 : What is the peak-to-peak value of a sinusoidal waveform that has a peak value of 10 V ?

Soln. :

Peak-to-peak value = $2 \times V_p = 2 \times 10 = 20\ V_{p-p}$ **...Ans.**

2.5 Effective or R.M.S. Value :

S-06, S-08, S-09, W-09, S-10, W-10, S-11, S-12, S-13, W-14, S-15, W-15, W-16

MSBTE Questions

Q. 1 Write the formula for : r.m.s value of current for sinusoidal waveform. **(S-06, 1 Mark)**

Q. 2 Define RMS value. **(S-08, S-09, W-09, S-10, W-10, S-11, S-12, S-13, W-14, S-15, W-16, 1 Mark)**

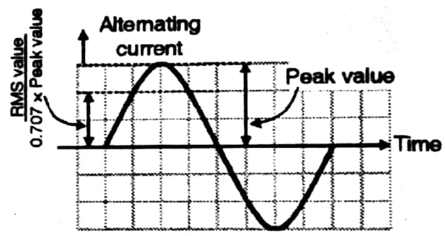
Q. 3 Define RMS value of an AC quantity. Explain its practical significance. **(W-15, 4 Marks)**

- The ac current value changes continuously with time, whereas the direct current remains constant with respect to time.
- In order to relate these currents we consider the effect which is common to both the types of currents.
- The common effect is the production of heat due to current flowing through a resistor. Hence this effect is used for comparing the ac and dc currents.
- The definition and other important points related to the effective or R.M.S. (root mean square) value are as follows.

Note : RMS value exists for any AC quantity such as voltage, current etc.

Definition :

The effective or RMS value of an ac current is equal to the steady state or DC current that is required to produce the same amount of heat as produced by the ac current provided that the resistance and time for which these currents flow are identical.



(A-571) **Fig. 2.5.1 : RMS value**

- All the ac currents or voltages are expressed as RMS values unless clearly specified otherwise.
- RMS value of ac current is denoted by I_{rms} and RMS voltage is denoted by V_{rms}.
- RMS value of a sinusoidal waveform (sine or cosine) is equal to 0.707 times its peak value.

$$I_{rms} = 0.707\, I_m$$

- RMS value is called as the heat producing component of ac current.
- Amount of light produced by a lamp or the amount of heat produced by an iron is proportional to the square of rms value of current flowing through them.

2.6 Average Value :

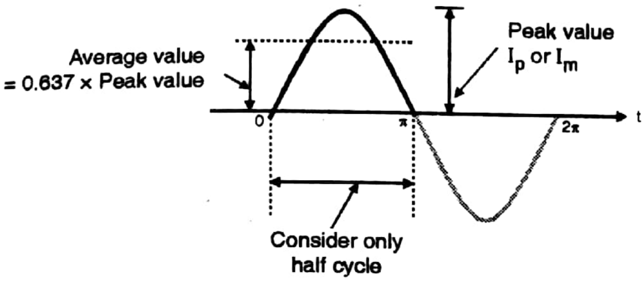
S-08, W-09, S-13, S-15

MSBTE Questions

Q. 1 Define : Average value. **(S-08, W-09, S-13, S-15, 1 Mark)**

Definition :

– The average value of an alternating quantity is equal to the average of all, the instantaneous values over a period of half cycle.



(A-574) **Fig. 2.6.1 : Average value**

– The average value of ac current denoted by I_{av} or I_{dc}.

– Average value is expressed without a positive or negative sign.

– The average value of a sinusoidal waveform is equal to 0.637 times its peak value.

$$\therefore \quad I_{av} = I_{dc} = 0.637\ I_m$$

– Similarly the average value of the sinusoidal voltage is 0.637 V_m.

– The dc ammeters or voltmeters indicate the average value.

– Average value of a full cycle of a symmetrical ac waveform is zero.

2.7 Form Factor :

S-17; I-Scheme : W-18

MSBTE Questions

Q. 1 Define :

1. Frequency 2. Form factor. **(S-17, 2 Marks)**

– The form factor of an alternating quantity is defined as the ratio of its RMS value to its average value.

$$\therefore \quad \text{Form factor } K_f = \frac{\text{RMS value}}{\text{Average value}} \qquad ...(2.7.1)$$

– Form factor is a dimensionless quantity and its value is always **higher than one.** That mean the RMS value will always be higher than the average value.

– Form factor of a sinusoidal alternating current is given by,

$$K_f = \frac{I_{rms}}{I_{av}} = \frac{0.707\ I_m}{0.637\ I_m} = \underline{1.11}$$

– The value of K_f = 1.11 is valid only for the sinusoidal (sine or cosine) ac quantities. For all the other shapes of ac quantities the form factor is different from 1.11 and should be obtained by using the general definition given in Equation (2.7.1).

2.8 Crest Factor or Peak Factor (K_p) :

Definition :

– The maximum value (peak value or amplitude) of an alternating quantity is called as the **crest value** of the quantity. The **crest factor** is defined as the ratio of the crest (peak) value to the rms value of the quantity.

$$\therefore \quad K_p = \frac{\text{Peak value}}{\text{RMS value}} \qquad ...(2.8.1)$$

– As the peak value is always higher than the rms value, crest factor is always higher than one.

– For a sinusoidal alternating quantity the crest factor is given by,

$$K_p = \frac{\sqrt{2} \times \text{RMS value}}{\text{RMS value}}$$

$$\therefore \quad K_p = \sqrt{2} \text{ or } 1.414 \text{ .... for a sinewave.}$$

2.9 Solved Examples :

Ex. 2.9.1 : An alternating voltage is represented by the following expression : v = 25 sin (200 πt)

Calculate the following :

1. Amplitude 2. Time period
3. Frequency 4. Angular velocity.
5. Form factor 6. Crest factor.

W-11, 4 Marks

Soln. :

The standard expression for a sinewave is

$$v = V_m \sin (2\pi\ ft)$$

Compare this equation with the given equation to write

1. Amplitude or peak value V_m = 25 Volts.
2. Frequency f = 100 cycles/second or Hz.
3. Time period T = 1 / f = (1 / 100) = 0.01 sec or 10 msec.
4. Angular velocity ω = 2 πf = 2π × 100 = 200 π

   = 628.31 rad/sec.

5. Form factor $K_f = \dfrac{\text{RMS value}}{\text{Average value}}$ ...(1)

$$V_{av} = 0.637\ V_m = 0.637 \times 25 = 15.925$$

$$V_{rms} = 0.707\ V_m = 0.707 \times 25 = 17.675$$

Substituting values of V_{av} and V_{rms} in Equation (1),

$$\therefore \quad K_f = 1.1098$$

6. Crest factor $K_p = \frac{\text{Peak value}}{\text{RMS value}} = 1.414$

Ex. 2.9.2 : For a cosine voltage waveform given by the following expression : v = 325 cos $(100\pi t)$. Calculate the following :

1. Peak voltage
2. Peak-to-peak voltage
3. Instantaneous voltage at $\theta = \pi$ rad.
4. RMS value
5. Average value.

Soln. :

All the expressions which we have derived for the sinewave alternating quantities are applicable to the cosine shaped alternating quantities as well.

The standard expression for a cosine wave is

$$v = V_m \cos(2\pi ft)$$

Comparing this equation with the given equation we write

1. Peak voltage $V_m = 325$ V ...Ans.
2. Peak-to-peak voltage $V_{p-p} = 2 \times V_m = 650$ V ...Ans.
3. Instantaneous value at $\theta = \pi$ rad can be obtained by expressing the given equation in the following manner :

$$v = 325 \cos\theta$$

Now substitute $\theta = \pi$ radians to obtain the instantaneous value as

$$V(\theta = \pi) = 325 \cos\pi \quad \text{...(calculator in the rad mode)}$$

$$= 325 \times -1 = -325 \text{ V} \quad \text{...Ans.}$$

4. RMS value $V_{rms} = 0.707\, V_m = 0.707 \times 325$

$$= 229.775 \text{ V} \approx 230 \text{ V} \quad \text{...Ans.}$$

5. Average value $V_{av} = 0.637 \times V_m = 207$ V ...Ans.

Ex. 2.9.3 : The current flowing through the circuit is $i = 14.14 \sin\left(314\, t - \frac{\pi}{6}\right)$. Calculate :

1. Frequency
2. I_{rms}
3. Phase difference
4. Amplitude

W-07. S-15. 4 Marks

Soln. :

Given : $i = 14.14 \sin\left(314\, t - \frac{\pi}{6}\right)$ ...(1)

The standard expression for instantaneous current is

$$i = I_m \sin(2\pi ft + \phi) \quad \text{...(2)}$$

Comparing Equations (1) and (2) we get,

1. **Frequency :** $2\pi f = 314$

$$\therefore f = \frac{314}{2\pi} = 50 \text{ Hz} \quad \text{...Ans.}$$

2. **I_{rms} :** $I_m = 14.14$ Amp.

$$\therefore I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{14.14}{\sqrt{2}} = 10 \text{ A} \quad \text{...Ans.}$$

3. **Phase difference :**

$$\phi = -\frac{\pi}{6} \text{ rad. or } -30^\circ \quad \text{...Ans.}$$

4. **Amplitude :** $I_m = 14.14$ Amp ...Ans.

Ex. 2.9.4 : An alternating voltage is mathematically expressed as $v = 141.42 \sin\left(157.08t + \frac{\pi}{2}\right)$ volts. Find its r.m.s. value frequency, time period and peak value.

W-10. 4 Marks

Soln. :

Compare the given expression with standard expression.

$$v = V_m \sin(2\pi ft)$$

1. **r.m.s. value :** $= V_{rms} = \frac{V_m}{\sqrt{2}}$

$$V_m = 141.42$$

$$\therefore V_{rms} = 141.42/\sqrt{2}$$

$$= 99.99 \approx 100 \text{ V} \quad \text{...Ans.}$$

2. **Peak value :** $V_m = 141.42$ V ...Ans.

3. **Frequency :**

Given : $2\pi f = 157.08$

$$\therefore f = \frac{157.08}{2\pi} = 25 \text{ Hz.} \quad \text{...Ans.}$$

4. **Time period :**

$$T = \frac{1}{f} = \frac{1}{25} = 0.04 \text{ sec. or } 40 \text{ mS} \quad \text{...Ans.}$$

Ex. 2.9.5 : An alternating current is represented by i = 70.7 sin 520 t. Determine :

1. Frequency
2. The current at 0.0015 second after passing through zero increasing positively
3. r.m.s. value
4. Average value.

S-12. 4 Marks

Soln. :

The standard expression for instantaneous current is given by,

$$i = I_m \sin\omega t = I_m \sin[2\pi ft] \quad \text{...(1)}$$

Compare this equation with the given equation to get,

1. **Frequency f :**

$$\omega = 2\pi f = 520 \therefore f = \frac{520}{2\pi} = 82.76 \text{ Hz} \quad \text{...Ans.}$$

2. **Current I :**

Find i for t = 0.0015 sec.

$$i = 70.7 \sin 520\, t$$

$$\therefore \quad i = 70.7 \sin (520 \times 0.0015) = 70.7 \sin (0.78)$$

$$\therefore \quad i = 49.72 \text{ A} \quad \text{...Ans.}$$

3. **Rms value I_{rms} :**

$$I_m = 70.7 \text{ A}$$

$$\therefore I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{70.7}{\sqrt{2}}$$

$$= 49.99 \approx 50 \text{ A} \quad \text{...Ans.}$$

4. **Average value I_{av} :**

$$I_{av} = 0.637 \times I_m = 0.637 \times 70.7$$

$$= 45.04 \text{ A} \quad \text{...Ans.}$$

Ex. 2.9.6 : An alternating voltage is represented by the following expression ;
V = 25 sin (200 πt). Calculate the following :
1. Amplitude 2. Time period
3. Frequency 4. Angular velocity
S-12. 4 Marks

Soln. :

The standard expression for a sinewave is,

$$v = V_m \sin (2\pi ft)$$

Compare this equation with the given equation to write

1. Amplitude or peak value V_m = 25 Volts.
2. Frequency f = 100 cycles/second or Hz.
3. Time period T = 1 / f = (1 / 100) = 0.01 sec or 10 msec.
4. Angular velocity $\omega = 2\pi f = 2\pi \times 100 = 200\pi$ = 628.31 rad/sec.

Ex. 2.9.7 : An a.c. voltage is represented by v = 141.48 sin 377t. Determine maximum value, RMS value, angular velocity and frequency.
W-12. 4 Marks

Soln. :

Given : V = 141.48 sin 377t

1. **Maximum value :** V_m = 141.48 Volts ...Ans.
2. **RMS value :** $V_{rms} = \frac{V_m}{\sqrt{2}} = 100$ Volts ...Ans.
3. **Angular velocity :** $\omega = 2\pi f = 377$ rad/sec. ...Ans.
4. **Frequency :** $f = \frac{377}{2\pi} = 60$ Hz ...Ans.

Ex. 2.9.8 : A voltage equation is expressed as V = 70.7 sin 314 t.
Determine :
1. Maximum value of voltage
2. rms value of voltage
3. Frequency and time period of waveform.
S-14, 4 Marks

Soln. :

Given : V = 70.7 sin 314 t

To find :

1. Maximum value of voltage
2. RMS value of voltage
3. Frequency and time period of waveform.

Step 1 : Find maximum rms value frequency and time period :

Compare given equation with standard equation $v = V_m \sin \omega t$

1. $\therefore \quad V_m = 70.7$ Volts ...Ans.
2. RMS value $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{70.7}{\sqrt{2}} = 50$ Volts ...Ans.
3. $\omega = 314 = 2\pi f$

   $\therefore \quad f = 49.97$ Hz ...Ans.
4. $T = \frac{1}{f} = \frac{1}{49.97} = 0.02$ seconds ...Ans.

Ex. 2.9.9 : An alternating current is given by i = 141.4 sin 314 t.
Calculate the maximum value, frequency, time period and instantaneous value when t is 3 mS.
W-14. 4 Marks

Soln. :

$$i = 141.4 \sin 314t$$

Standard expression for instantaneous current is,

$$i = I_m \sin (2\pi ft + \phi)$$

Comparing these two equations we get,

$$\therefore \quad I_m = 141.4\text{A}$$

.....Maximum value...Ans.

$$\text{Frequency, } 2\pi f = 314$$

$\therefore\ f = \frac{314}{2\pi} = 50$ Hz ...Ans.

Time period,

$$T = \frac{1}{f} = \frac{1}{50}$$

$= 0.02$ sec or 20 mS ...Ans.

Instantaneous value when t = 3 mS or 0.003 mS

$$i = 141.4 \sin 314 \times 0.003$$

$= 2.32$ A ...Ans.

Ex. 2.9.10 : An alternating voltage is mathematically expressed as,

$$v = 141.42 \sin\left(157.08\, t + \frac{\pi}{12}\right) \text{ volt}$$

Find maximum value, RMS value, frequency and periodic time. **W-15, 4 Marks**

Soln. :

Compare the given expression with standard expression.

$$V = V_m \sin(2\pi ft + \phi)$$

1. **Peak value :** $V_m = 141.42$ V ...Ans.

2. **r.m.s. value :** $= V_{rms} = \frac{V_m}{\sqrt{2}}$

$V_m = 141.42$

$\therefore\ V_{rms} = 141.42/\sqrt{2}$

$= 99.99 \approx 100$ V ...Ans.

3. **Frequency :**

Given : $2\pi f = 157.08$

$\therefore\ f = \frac{157.08}{2\pi} = 25$ Hz. ...Ans.

4. **Time period :**

$$T = \frac{1}{f} = \frac{1}{25}$$

$= 0.04$ sec. or 40 mS ...Ans.

Ex. 2.9.11 : An alternating current is represented by $i = 50.5 \sin(314\, t + \pi/2)$ calculate : 1. Amplitude 2. Frequency 3. I_{rms} 4. Phase difference **S-16, 4 Marks**

Soln. :

Given : $I = 50.5 \sin\left(314\, t + \frac{\pi}{2}\right)$ ...(1)

The standard expression for instantaneous current is

$I = I_m \sin(2\pi ft + \phi)$ ...(2)

Comparing Equations (1) and (2) we get,

1. **Frequency :** $2\pi f = 314$

$\therefore\ f = \frac{314}{2\pi} = 50$ Hz ...Ans.

2. **I_{rms} :** $I_m = 50.5$ Amp.

$\therefore\ I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{50.5}{\sqrt{2}} = 35.7$ A ...Ans.

3. **Phase difference :** $\phi = \frac{\pi}{2}$ rad. or 90°...Ans.

4. **Amplitude :** $I_m = 50.5$ Amp ...Ans.

Ex. 2.9.12 : The current flowing in a circuit is $I = 28.28 \sin\left(314t - \frac{\pi}{6}\right)$

Calculate :

1. Amplitude 2. Rms current

3. Frequency 4. Phase difference **W-16, 4 Marks**

Soln. :

Given : $i = 28.28 \sin\left(314\, t - \frac{\pi}{6}\right)$

1. **Amplitude :** From the given expression of i, we get,

$I_m = 28.28$ Amp. ...Ans.

2. **RMS current :**

$I_{RMS} = \frac{I_m}{\sqrt{2}} = \frac{28.28}{\sqrt{2}} = 20$ Amp ...Ans.

3. **Frequency :**

$2\pi f = 314$

$\therefore\ f = \frac{314}{2\pi} = 50$ Hz ...Ans.

4. **Phase angle :**

From the given expression of i, we get,

$\phi = -\pi/6$ rad or $-30°$ ...Ans.

Ex. 2.9.13 : An alternating current is given by equation $I = 10\sqrt{2} \sin 314\, t$.

Calculate :

1. Average value
2. Instantaneous value of i at t = 5 millisec.

W-16, 4 Marks

Soln. :

Given : $i = 10\sqrt{2} \sin 314\, t$

1. **Average value :**

$I_{av} = 0.637\, I_m$

But $I_m = 10\sqrt{2}$ Amp.

$\therefore\ I_{av} = 0.637 \times 10\sqrt{2} = 9$ Amp ...Ans.

2. **Instantaneous value of I at t = 5 mS :**

$t = 5\ mS = 5 \times 10^{-3}$ sec.

$$\therefore\ i = 10\sqrt{2} \sin(314 \times 5 \times 10^{-3})$$

$$= 10\sqrt{2} \sin(1.57 \text{ rad})$$

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ}$$

$$\therefore\ 1.57 \text{ rad} = \frac{180}{\pi} \times 1.57 = 90^{\circ}$$

$$\therefore\ i = 10\sqrt{2} \sin(90^{\circ}) = 10\sqrt{2}$$

$$= 14.14 \text{ Amp} \qquad \text{...Ans.}$$

Ex. 2.9.14 : Current flowing through the circuit is $i = 141.4 \sin\left(314t - \frac{\pi}{2}\right)$ Amp.

Calculate :

1. Frequency
2. RMS value
3. Phase difference
4. Amplitude

S-17, 4 Marks

Soln. :

Given : $i = 141.4 \sin\left(314t - \frac{\pi}{2}\right)$

To find : 1. Frequency 2. RMS value 3. Phase difference 4. Amplitude

Comparing the given expression with standard expression,

$$i = I_m \sin(\omega t - \phi)$$

1. **Frequency :**

$$\omega t = 314\,t$$

$$\therefore\ 2\pi f t = 314\,t$$

$$\therefore\ f = 50 \text{ Hz} \qquad \text{...Ans.}$$

2. **RMS value :**

$$I_{rms} = I_m \times 0.707 = 141.4 \times 0.707$$

$$= 99.96 \text{ Amp} \qquad \text{...Ans.}$$

3. **Phase difference :**

$$\phi = \frac{\pi}{2} \text{ or } 90^{\circ} \qquad \text{...Ans.}$$

4. **Amplitude :**

Maximum value $= I_m = 141.4$ amp ...Ans.

Ex. 2.9.15 : An A.C. voltage of v(t) = 230 sin 314 t Volts is applied to a circuit. Calculate :

1. Angular frequency
2. Frequency
3. RMS value
4. Average value

W-17, 4 Marks

Soln. :

Given : 230 sin 314 t

– The standard expression for a sinewave is,

$$v = V_m \sin(2\pi f t)$$

– Compare this equation with the given equation to get,

1. **Frequency :**

$$\omega = 2\pi f = 314$$

$$\therefore\ f = \frac{314}{2\pi} = 50 \text{ Hz} \qquad \text{...Ans.}$$

2. **Angular frequency :**

$$\omega = 314 \text{ rad/sec} \qquad \text{...Ans.}$$

3. **RMS value :**

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{230}{\sqrt{2}} = 162.63 \text{ V} \qquad \text{...Ans.}$$

4. **Average value :**

$$V_{avg} = 0.637 \times V_m$$

$$= 0.637 \times 230 = 146.51 \text{ V} \qquad \text{...Ans.}$$

2.10 Phasor Representation of an Alternative Quantity :

– We can represent an alternating quantity such as a sine or cosine wave with the help of a phasor.

– The graphical method of representing a sine wave needs drawing of sinewave which is a cumbersome job.

– So "phasor" representation which is a very simple way to represent a sinusoidal quantity is being used.

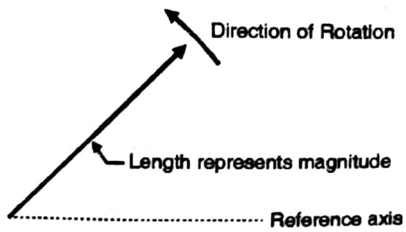
What is a phasor ? How does it look like ?

– A phasor is a straight line with an arrow marked on one side.

– The length of this straight line represents the magnitude of the sinusoidal quantity being represented and the arrow represents its direction.

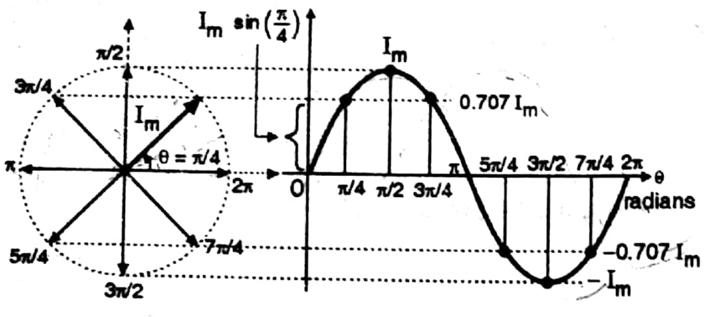
– Thus phasor representation is similar to the vector representation. However the phasors rotate in the anticlockwise direction as shown in Fig. 2.10.1.

– The length of the phasor represents the rms value of the sinusodial quantity. Sometimes the length also represents the peak value.



(A-584) **Fig. 2.10.1 : Phasor representation of a sinusoidal quantities**

- Speed of rotation of the phasor is equal to ω radians/sec, where $\omega = 2\pi f$.
- One rotation of the phasor corresponds to one cycle of the alternating waveform as shown in Fig. 2.10.2.



(A-585)**Fig. 2.10.2 : Relation between an alternating quantity and phasor**

Conclusion :

If the length of the phasor is equal to the peak value of the sinusoidal ac quantity, its angular velocity $\omega = 2\pi f$ and it rotates in the anticlockwise direction in space then at any given angle θ, its projection on the Y-axis gives you the instantaneous value of the sinusoidal ac quantity at that angle.

Can we represent two or more sinusoidal quantities simultaneously using phasors ?

Yes, we can represent two or more sinusoidal quantities simultaneously on the same phasor diagram **if and only if their frequencies are same.** If their frequencies are not same then they cannot be represented on the same phasor diagram.

2.11 Phase of an Alternating Quantity :

2.11.1 Phase Angle :

S-13

MSBTE Questions

Q. 1 Define : Phase (S-13, 4 Marks)

- Phase is a particular point in the cycle of waveform measured as an angle.

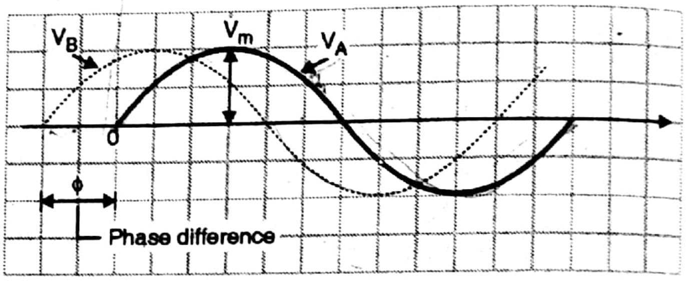
The equation of the induced emf in the conductor is

$$v = V_m \sin \omega t = V_m \sin \theta \quad ...(2.11.1)$$

In Equation (2.11.1), θ is the angle made by the conductor with the reference axis and it is called as the **Phase Angle.** Thus the phase angle can take any value between 0 and 2π radians.

2.11.2 Phase Difference :

- It is not necessary that two voltages or current waves originate at the same instant of time. Refer Fig. 2.11.1 to understand this point.
- The voltage V_A and V_B do not have the same zero crossover point so we say that there is **a phase difference** between them.
- Both V_A and V_B have the same frequency and same peak voltage V_m.



(A-586)**Fig. 2.11.1 : Concept of phase difference**

- We can represent the two voltages mathematically as follows :

$$\text{Voltage} : V_A = V_m \sin \theta$$

$$\text{Voltage} : V_B = V_m \sin (\theta + \phi) \quad ...(2.11.2)$$

- The angle ϕ is known as the **Phase difference** between V_A and V_B. It is measured in "radians" or "degrees".
- If measured in radians the phase difference can take any value between 0 and 2π, but if expressed in degrees it can take any value between 0 and 360°.

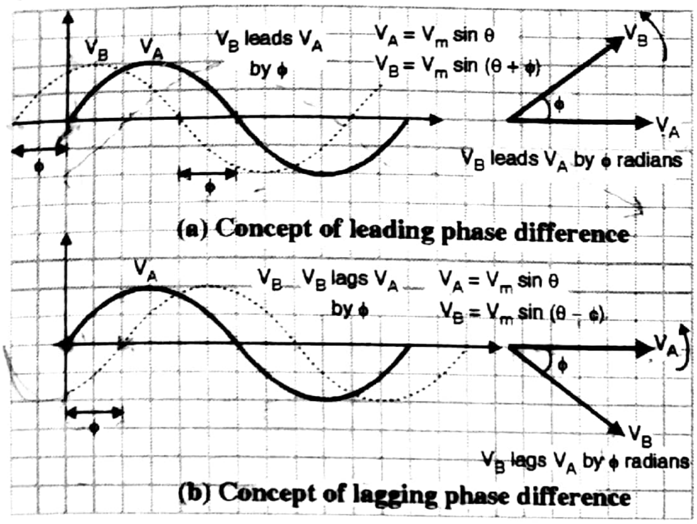
2.11.3 Leading and Lagging Phase Difference :

Leading phase difference :

- If the phase angle ϕ in Equation (2.11.2) is positive then the phase difference ϕ is said to be a leading phase difference. In other words we say that voltage V_B leads the voltage V_A.
- Thus if $V_A = V_m \sin \theta$ and $V_B = V_m \sin (\theta + \phi)$ then V_B leads V_A by an angle ϕ. This is as shown in Fig. 2.11.2(a).

Lagging phase difference :

- If the phase angle ϕ in Equation (2.11.2) is negative, then the phase difference is said to be a lagging phase difference. That means V_B lags behind V_A by ϕ.
- That means if $V_A = V_m \sin \theta$ and $V_B = V_m \sin (\theta - \phi)$ then V_B lags behind V_A by ϕ degrees and graphically it is shown in Fig. 2.11.2(b).



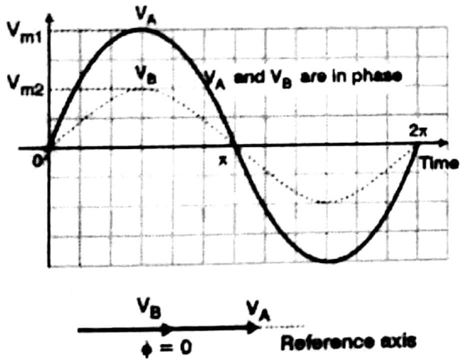
(A-589)**Fig. 2.11.2**

What is meant by "in phase" voltages ?

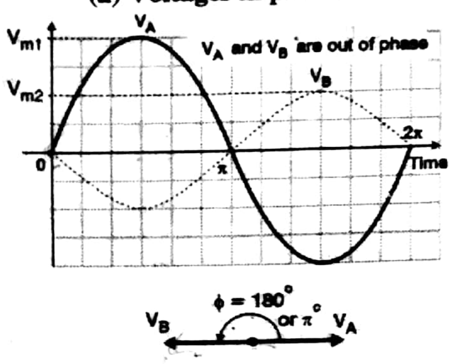
The two ac voltages (or any two ac quantities) are said to be "in phase" if the phase difference between them is equal to zero as shown in Fig. 2.11.3(a).

What is meant by voltages "out of phase" ?

The two ac voltages V_A and V_B are said to "out-of-phase" of each other if the phase difference between them is 180° or π radians (leading or lagging) as shown in Fig. 2.11.3(b).



(a) Voltages in phase



(b) Voltages out of phase

(A-590)**Fig. 2.11.3**

Practical use of phasors :

- The phase difference between any two ac quantities can be represented easily by their sinewaves.
- But when a large number of ac quantities are to be represented simultaneously, the representation using waveforms becomes extremely complicated.
- Hence a shortcut to represent all these quantities is to use the phasor representation.

Important points to be noted about the phasor diagrams :

- Phasor diagram is the diagram in which different sinusoidal alternating quantities of the same frequency are represented by phasors with their correct phase relationship.
- X and Y are the axes assumed to be fixed in space. Hence it is not necessary to draw them in the phasor diagram.
- One phasor is chosen as a reference phasor and it is drawn in the horizontal position. (coinciding with the X-axis).
- Till now the phasors were drawn to represent the peak value. But phasors are normally drawn to represent the effective or RMS value.
- The phasors are always assumed to be rotating in the counter clockwise direction. Hence the phasor which is ahead in this direction from the reference phasor is said to lead the reference phasor.
- The angle between two phasors represents the phase difference between the two sinusoidal quantities.
- Different types of arrowheads are used for differentiating phasors drawn for different types of ac quantities such as current, voltage etc.

Ex. 2.11.1 : Different phasors are shown in Fig. P. 2.11.1, identify the lead-lag relation between them.

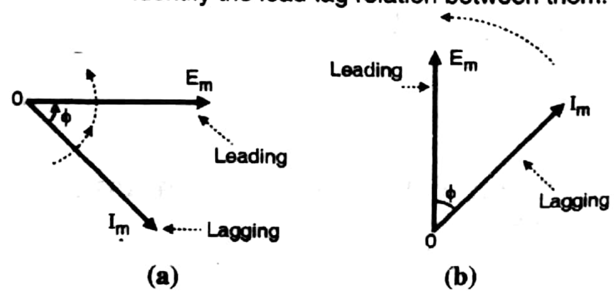
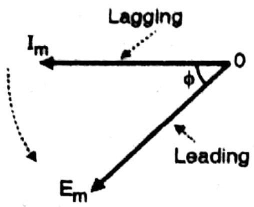


Fig. P. 2.11.1 (Contd...)



(c)

(A-592) **Fig. P. 2.11.1**

Soln. :

- Phasors are always assumed to be rotating in the counterclockwise direction. Hence the phasor which is ahead in the anticlockwise direction is said to be the leading phasor.
- Accordingly the given phasors of Fig. P. 2.11.1 have been marked as leading or lagging phasors.

Ex. 2.11.2 : The current flowing in a circuit is given by $i = 20 \sin\left(314\,t - \frac{\pi}{6}\right)$ Calculate :

1. Frequency 2. Phase difference
3. RMS value of current
4. Peak value of current

S-11, 4 Marks

Soln. :

Given : $i = 20 \sin\left(314t - \frac{\pi}{6}\right)$ ...(1)

The standard expression for instantaneous current is,

$$i = I_m \sin(2\pi ft + \phi) \quad ...(2)$$

Comparing Equations (1) and (2), we get,

1. Frequency :

$$2\pi f = 314$$

$$\therefore \quad f = \frac{314}{2\pi} = 50 \text{ Hz} \quad \textbf{...Ans.}$$

2. Phase difference :

$$\phi = -\frac{\pi}{6} \text{ rad or } -30° \quad \textbf{...Ans.}$$

3. Peak value of current :

$$I_m = 20 \text{ Amp.} \quad \textbf{...Ans.}$$

4. RMS value of current :

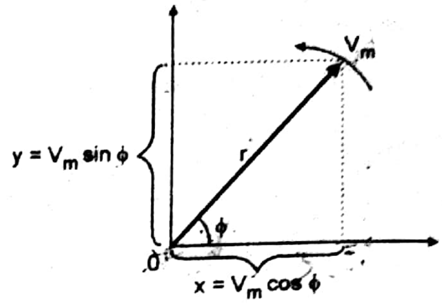
$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.14 \text{ Amp.} \quad \textbf{...Ans.}$$

2.12 Mathematical Representation of Phasor :

- A phasor can be presented in two different ways :

1. Rectangular form 2. Polar form.

- The instantaneous voltage $v(t) = V_m \sin(\omega t + \phi)$ is represented using a phasor as shown in Fig. 2.12.1.
- From Fig. 2.12.1 we can obtain the expressions for the polar and rectangular forms.



(A-608) **Fig. 2.12.1**

2.12.1 Polar Representation :

- The instantaneous voltage $v(t) = V_m \sin(\omega t + \phi)$ can be represented in the polar form as follows :

$$v(t) = r \angle \phi \quad ...(2.12.1)$$

where $r = V_m$

- That means the length of the phasor (r) represents the peak value of the ac quantity.
- For example if,

$$v(t) = 20 \sin(100\,\pi t + 60°)$$

then it is represented in the polar form as,

$$v(t) = 20 \angle 60° \text{ volts}$$

- The polar form is suitable for multiplication and addition of phasors.

2.12.2 Rectangular Representation :

- The same waveform can be represented in the rectangular form as follows :

$$v(t) = x + jy \quad ...(2.12.2)$$

- Refer Fig. 2.12.1 to write,

$$x = \text{x component of the phasor} = V_m \cos \phi$$

and $$y = \text{y component of the phasor} = V_m \sin \phi$$

- Substituting the values of x and y components into Equation (2.12.2) we get,

$$v(t) = V_m \cos \phi + j\,V_m \sin \phi \quad ...(2.12.3)$$

- The rectangular form is suitable for addition and subtraction of phasors.
- For example if $v(t) = 20 \sin(100\,\pi t + 60°)$ can be represented in the rectangular form as follows :

$$v(t) = (20 \cos 60° + j\, 20 \sin 60°)$$
$$= (10 + j\, 17.32) \text{ Volts.}$$

2.13 Single Phase AC Circuits :

- We have discussed about all the basic concepts of single phase AC circuits. So now let us discuss some practical ac single phase circuits.
- The three basic elements of any ac circuit are Resistance (R), Inductance (L) and Capacitance (C).
- We are going to discuss the following three ac circuits first and then their combinations. The three basic ac circuits are :
  1. Purely resistive AC circuit.
  2. Purely inductive AC circuit.
  3. Purely capacitive AC circuit.

2.13.1 Reactance and Impedance :

In the dc circuits we have defined "resistance" as opposition to the flow of current. Similarly for ac circuits we define two terms namely 'reactance (X)' and 'impedance (Z)'.

2.13.2 Reactance :

Reactance can be of two types :

1. Inductive reactance X_L
2. Capacitive reactance X_C

2.13.3 Inductive Reactance (X_L) :

W-07, S-11, W-11, S-14

MSBTE Questions

Q. 1 Define inductive reactance and capacitive reactance with respect to AC circuit. **(W-07, S-11, W-11, S-14, 2 Marks)**

- We define the inductive reactance X_L as, $X_L = \omega L = 2 \pi f L$ and the unit of inductive reactance is ohm (Ω).
- We can define the inductive reactance as the opposition to the flow of an alternating current, offered by an inductance.

2.13.4 Capacitive Reactance (X_C) :

S-11, S-14

MSBTE Questions

Q. 1 Define inductive and capacitive reactance. **(S-11, 2 Marks)**

Q. 2 Define inductive reactance and capacitive reactance with respect to AC circuit. **(S-14, 4 Marks)**

- We define the capacitive reactance X_C as,

$$X_C = \frac{1}{\omega C} = \frac{1}{2 \pi f C} \qquad \ldots (2.13.1)$$

- And the unit of capacitive reactance is ohms (Ω).
- Thus capacitive reactance X_C is defined as the opposition offered by a pure capacitor to the flow of alternating current.
- Equation (2.13.1) shows that the capacitive reactance is inversely proportional to the frequency of the applied voltage if C is constant. X_C decreases with increase in frequency f.

2.13.5 Impedance (Z) :

- The ac circuit may not always be purely resistive, capacitive or inductive. It will contain the combination of these elements.
- So defining resistance, and reactance is not enough.
- Hence a combination of R, X_L and X_C is defined and it is called as impedance.
- Impedance is denoted by Z and has unit Ω.
- Impedance can be expressed in the polar form as follows :

$$Z = |Z| \angle \phi \quad \Omega$$

→ Phase angle (A-1551)

→ Magnitude of Z

- And it is expressed in the rectangular form as,

$$Z = R + j X$$

→ Reactive part (A-1552)

→ Resistive part

where $|Z| = \sqrt{(R^2 + X^2)}$ and $\phi = \tan^{-1} [X / R]$

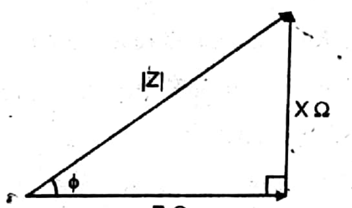
2.14 Concept of Impedance Triangle :

- Impedance triangle is the graphical way of relating the Resistance (R), reactance (X) and impedance (Z) of the given circuit.
- Impedance triangle is a right angle triangle.
- The two sides of the impedance triangle correspond to resistance (R) and reactance (X) with an angle of 90°

between them as shown in Fig. 2.14.1 and the third side corresponds to the magnitude of impedance i.e. $|Z|$ with $|Z|^2 = R^2 + X^2$.

- Also note that the angle between the sides representing $|Z|$ and R is equal to the phase angle ϕ where $\phi = \tan^{-1}\frac{X}{R}$
- Hence $R = |Z| \cos\phi$ and $X = |Z| \sin\phi$.
- Impedance triangles for different types of circuits will be different. We will discuss the impedance triangles for RL series, RC series and RLC series circuits in this chapter. It is not possible to draw impedance triangles for purely resistive, purely inductive or purely capacitive circuits because for such circuits either the resistive part or the reactive part is equal to zero.

$$|Z|^2 = R^2 + X^2$$
$$R = |Z| \cos\phi$$
$$X = |Z| \sin\phi$$



(A-3360) **Fig. 2.14.1 : Concept of an impedance triangle**

2.15 Important Terms Related to Power :

Some of the important terms related to power are as follows :

1. Apparent power (S)
2. True or Real power (P)
3. Power factor
4. Reactive power or imaginary power (Q)

2.15.1 Apparent Power (S) :

- Apparent power is defined as the product of the rms values of voltage (V) and current (I). It is denoted by S and measured in Volt-Amperes (VA) or kilovolt-ampere (kVA).

$\therefore$ Apparent power $S = V \times I$ ...(2.15.1)

- Apparent power represents the total power supplied by the source.
- It can be proved that apparent power is equal to the vector sum of active and reactive powers.

Active power + reactive power = P + Q

$$= V_1 \cos\phi + V_1 \sin\phi$$
$$= \sqrt{V^2 I^2 \cos^2\phi + V^2 I^2 \sin^2\phi} = V \times I$$
$$= \text{Apparent power S.}$$

Thus $S = \bar{P} + \bar{Q}$

- The transformers are specified in terms of the VA of kVA rating.
- If the power factor is high (close to 1) then the apparent power required to deliver the same amount of active power to the load is reduced.

2.15.2 Real Power or True Power or Active Power (P) :

- The true power or real power (P) is defined as the average power P_{av} taken by or consumed by the ac circuit. It is given by,

$$P = P_{av} = VI \cos\phi \quad \text{Watts} \quad ...(2.15.2)$$

where ϕ = Phase angle between V and I

- The true power is measured in Watts (W) or kilowatts (kW).
- V represents the rms value of the supply voltage and I is rms value of circuit current.
- Active power is the average power consumed by the resistor R in the given circuit hence it is also called as the useful power.

2.15.3 Reactive Power or Imaginary Power (Q) :

- The reactive power (Q) is defined as the product of V, I and sine of angle between V and I i.e.ϕ.

$\therefore$ Reactive Power $Q = VI \sin\phi$ VAR ...(2.15.3)

- The reactive power is also called as imaginary power. and it is expressed in **VAR** i.e. volt-ampere reactive or **kVAR** i.e. kilovolt-ampere reactive.
- Reactive power is a part of total power which is not consumed by the load. Reactive power simply travels from source to load and back from load to source. It is therefore the useless power and hence should be minimized.

2.15.4 Power Factor :

- Power factor is defined as the factor by which the apparent power (S) must be multiplied so as to obtain the true power (P). OR in other words, it is the ratio of the true power and the apparent power.
- Ideally the power factor should be 1 and practically it should be as high as possible.

$$\text{Power factor} = \frac{\text{True Power (P)}}{\text{Apparent Power (S)}} \quad ...(2.15.4)$$

- Substituting the expressions for P and S we get,

$$\text{Power factor} = \frac{VI \cos\phi}{VI} = \cos\phi \quad ...(2.15.5)$$

- Thus the power factor is equal to the cosine of the phase difference ϕ between the applied voltage (V) and current (I) flowing through the L-R series circuit.
- The power factor can also be defined as ratio of impedances as follows. From the impedance triangle the resistance R is given by,

$$R = Z\cos\phi \quad ...(2.15.6)$$

$$\therefore \text{ Power factor} = \cos\phi = \frac{R}{Z} \quad ...(2.15.7)$$

- The value of power factor will be confined between ± 1 because it is basically the cosine of the phase difference ϕ.

$$\therefore \quad -1 \le PF \le 1 \quad ...(2.15.8)$$

- Depending on the polarity of PF, it can be classified as lagging power factor or leading power factor. But the sign of ϕ depends on polarity of ϕ which ultimately depends on type of circuit. (R, R-L, R-C etc.)
- The power factor can be either leading or lagging or simply unity. This depends on whether the current I leads, lags or is in phase with the applied voltage (V).
- The current lags behind voltage in the inductive circuits hence ϕ is negative and power factor is lagging.
- But in the resistance circuit $\phi = 0$ hence power factor is unity, simply unity, it neither leads nor lags.
- For a capacitive circuit current leads the voltage. Hence ϕ is positive and power factor is leading.
- Since the power factor is equal to the ratio of R and $|Z|$, it improves with increase in the resistive part of the impedance.
- The nature of power factor is dependent on the type of circuit as illustrated in Table 2.15.1.

Table 2.15.1

Sr. No.	Type of circuit	Value of ϕ	Nature of power factor	
1.	Purely resistive	$\phi = 0$	$\cos\phi = 1$	Neither leading nor lagging.
2.	Purely inductive	$\phi = -90°$	$\cos\phi = 0$	The PF is said to be zero lagging due to negative sign of ϕ.
3.	Purely capacitive	$\phi = +90°$	$\cos\phi = 0$	The PF is said to be zero leading due to positive sign of ϕ.
4.	L-R series circuit	$0 \le \phi \le -90°$	$0 \le PF \le 1$	PF is lagging due to negative sign of ϕ.

2.15.5 Importance (Significance) of Power Factor :

- As per definition,

$$P.F. = \frac{\text{True power (P)}}{\text{Apparent or total power (S)}}$$

- Thus P.F. of a circuit represents the percentage of total input power that is being actually consumed by that circuit.
- For example, if P.F. of a circuit is 0.9 then,

$$P = 0.9 \times S \quad \text{or} \quad P = 90\% \text{ of } S.$$

- That means the given circuit consumes 90% of the total power (S) that is being supplied to it.
- If P.F. of a circuit is say 0.2, then it is able to consume or use only 20% of the total power supplied to it. The remaining power goes back to the source (it is not used). This is not desirable.
- Hence the P.F. of a circuit should be as high as possible.

Ex. 2.15.1 : Define power factor in 3 different ways and give its significance.

Soln. :

$$PF = \frac{\text{True power (P)}}{\text{Apparent power (S)}}$$

$$\text{P.F. } \cos\phi = \frac{R}{|Z|}$$

$$\text{PF } \cos\phi = \frac{\text{Fundamental component of current (RMS value)}}{\text{RMS value of total current}}$$

Refer section 2.15.5 for significance of P.F.

2.15.6 Causes of Low Power Factor :

- Low power factor indicates that a very small percentage of total power is being actually utilized. The remaining power simply travels to and from.
- So if the P.F. is low, then a large power is required to be generated to delivered the required power to the load.
- The low power factor is a result of large inductive loads such as motors. The current drawn by large machines lags behind the supply voltage by a large angle. So the power factor is low.
- If PF is low then a large power needs to be supplied in order to consume the required amount of power in the load.
- Large power means large current. To carry this large current, conductors of larger diameter are required to be used. This will increase the cost.
- Larger current increases the copper loss and reduces the efficiency.
- A large current increases the voltage drop (I_Z) in the transmission line. To compensate for this voltage drop, higher voltage needs to be generated.

2.15.7 Power Triangle :

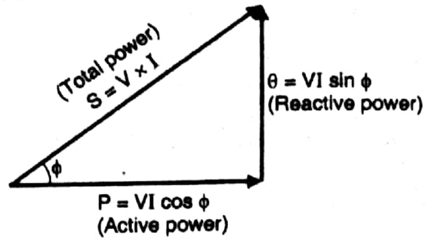
- Power triangle graphically relates the apparent power (S), true power (P) and imaginary power (Q) for any given circuit.
- Power triangle is a right angle triangle. It graphically represents the following relationship,

$$S^2 = P^2 + Q^2$$

- The two sides of a power triangle correspond to the real power (P) and the imaginary power (Q) with an angle of 90° between them as shown in Fig. 2.15.1 and the hypotenuse corresponds to the apparent of total power.
- Also note that the angle between the sides representing P and S is the phase angle ϕ where,

$$\phi = \tan^{-1}\left(\frac{X}{R}\right)$$

- We will discuss the power triangles for the RL series, RC series and RLC series circuits in this chapter.
- It is not possible to draw the power triangle for purely resistive, purely inductive and purely capacitive circuits, because for these circuits either the active power (P) or reactive power Q is equal to zero.



(A-3361) **Fig. 2.15.1 : Power triangle**

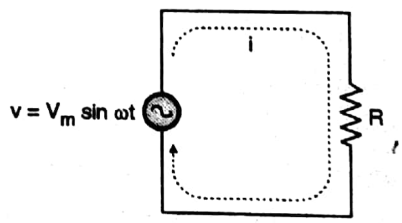
2.16 Purely Resistive AC Circuit : S-08

MSBTE Questions

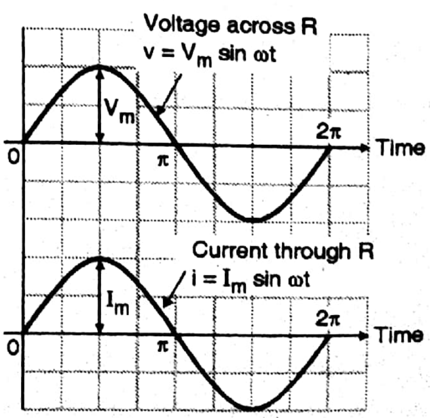
Q. 1 Draw waveform and phasor diagram of a simple resistive circuit when AC is applied across it. **(S-08, 4 Marks)**

Circuit Diagram and Waveforms :

- The purely resistive ac circuit is as shown in Fig. 2.16.1(a). It consists of an ac voltage source $v = V_m \sin \omega t$, and a resistor R connected across it.
- The instantaneous current flowing through the circuit is i.



(a) Purely resistive circuit



(b) Voltage and current waveforms

(A-622)**Fig. 2.16.1 : Purely resistive AC circuit**

2.16.1 Voltage and Current Waveforms and Equations :

- Referring to Fig. 2.16.1(a), the instantaneous voltage across the resistor (v_R) is same as the source voltage.

$$\therefore \quad v_R = v = V_m \sin \omega t \qquad ...(2.16.1)$$

- Applying the ohm's law the expression for the instantaneous current flowing through the resistor is given by,

$$i = \frac{v_R}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m \angle 0^\circ}{R \angle 0^\circ}$$

$$\text{Let } I_m = \frac{V_m}{R}, \quad i = I_m \angle 0^\circ = I_m \sin(\omega t) \qquad ...(2.16.2)$$

- From the current Equation (2.14.2) we conclude that :

1. The current flowing through a purely resistive ac circuit is sinusoidal.
2. The current through the resistive circuit and the applied voltage are in phase with each other.

- The waveforms of voltage and current are as shown in Fig. 2.16.1(b).

Note : Note that current and voltage in a purely resistive circuit are in phase, that means the phase angle between them is zero. If the voltage has a phase angle ϕ, then the current also will have the same phase angle ϕ.

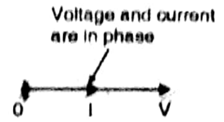
2.16.2 Phasor Diagram : S-08, S-09

MSBTE Questions

Q. 1 Draw waveform and phasor diagram of a simple resistive circuit when AC is applied across it. **(S-08, 4 Marks)**

Q. 2 Explain with phasor diagram the power consumed by a pure resistance, pure inductance and pure capacitances if connected across an ac supply. **(S-09, 4 Marks)**

- The phasor diagram for a purely resistive ac circuit is as shown in Fig. 2.16.1(c).
- The phase difference between the voltage and current phasor is $\phi = 0$.
- Also note that the phasors represent the RMS values V and I of the voltage and current respectively.



(A-623) **Fig. 2.16.1(c) : Phasor diagram**

2.16.3 Impedance of the Purely Resistive Circuit :

- The impedance Z is expressed in the rectangular form as :

$$Z = R + jX$$

Where R is the resistive part while X is the reactive part.

- When the load is purely resistive, the reactive part is zero i.e. X = 0. Hence the impedance of a resistive circuit is given by,

$$Z = R\,\Omega = (R + j0)\,\Omega$$

- In the polar form it is given by,

$$Z = R \angle 0^\circ\,\Omega$$

2.16.4 Average Power (P_{av}) : S-09

MSBTE Questions

Q. 1 Explain with phasor diagram the power consumed by a pure resistance, pure inductance and pure capacitances if connected across an ac supply. **(S-09, 4 Marks)**

- The average power supplied by the source and consumed by the pure resistor R connected in an AC circuit is given by,

$$P_{av} = V_{rms}\, I_{rms} \qquad ...(2.16.3)$$

- Thus the average power in a resistive circuit is equal to the product of rms values of voltage and current.

2.16.5 Energy in Purely Resistive Circuit :

$$\text{Energy } E = p \times t$$

- In the pure resistive circuits, the energy flow is unidirectional. The energy flows in only one direction i.e. from the source to the load.

- The resistance can not store any energy. So all the energy gets dissipated in the form of heat, in the resistance.
- This fact is utilized in the electric heaters, water heaters and electric irons. All these are examples of resistive loads.

Ex. 2.16.1 : Find the current drawn by 100 Ω resistive load when it is connected across 230 V, 50 Hz ac supply. W-06, 2 Marks

Soln. :

Given : $R = 100\ \Omega$, $V = 230\ V$, $f = 50\ Hz$.

$$I = \frac{V}{R} = \frac{230}{100} = 2.3\ \text{Amp.} \qquad \text{...Ans.}$$

Ex. 2.16.2 : A 60 Hz voltage of 115 V (r.m.s.) is supplied to 100 Ω resistance. Write time equation of voltage and current. S-10, 4 Marks

Soln. :

$$V(t) = V_m \sin(2\pi f_o t)$$
$$= \sqrt{2}\ V_{rms} \sin(2\pi f_o t)$$
$$= \sqrt{2} \times 115 \sin(2\pi \times 60 \times t)$$
$$= 162.63 \sin(377\ t)$$
$$i(t) = I_m \sin(2\pi f_o t)$$
$$= \frac{V_m}{R} \sin(2\pi \times 60 \times t)$$
$$= \frac{\sqrt{2}\ V_{rms}}{100} \sin(2\pi \times 60 \times t)$$
$$= 1.63 \sin(377\ t)$$

2.17 AC Circuit Containing Inductance Only :

W-10, W-14, S-16

MSBTE Questions

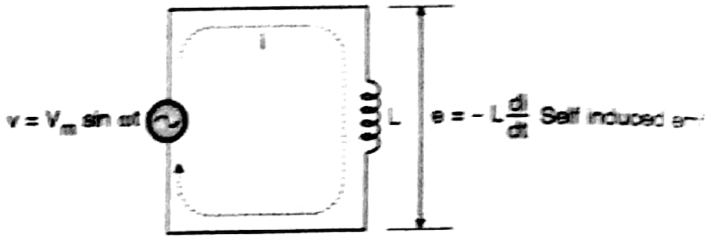
Q. 1 Draw circuit diagram and with the help of waveforms and phasor diagrams comment on the phase relationship between voltage and current in pure inductive circuit. **(W-10, 4 Marks)**

Q. 2 Draw and explain the labelled circuit and phasor diagram for purely inductive circuit. What is the power factor of the circuit ? **(W-14, 4 Marks)**

Q. 3 Draw a circuit diagram of pure inductive circuit and phasor diagram. **(S-16, 4 Marks)**

Circuit diagram :

- Fig. 2.17.1 shows a purely inductive ac circuit. It consists of an alternating voltage source of instantaneous voltage $v = V_m \sin \omega t$ and a pure inductance L.
- The pure inductance has zero ohmic resistance. It is a coil with only pure inductance of L Henries (H).



(A-626) **Fig. 2.17.1 : A purely inductive ac circuit**

2.17.1 Equations for Current i and Voltage v :

W-09, S-10, W-10

MSBTE Questions

Q. 1 For pure inductive circuit derive the relation between voltage and current. **(W-09, 4 Marks)**

Q. 2 For a purely inductive circuit, prove that :
$i(t) = I_m \sin(\omega t - \pi/2)$ **(S-10, 4 Marks)**

Q. 3 Draw circuit diagram and with the help of waveforms and phasor diagrams comment on the phase relationship between voltage and current in pure inductive circuit. **(W-10, 4 Marks)**

- Let the instantaneous voltage applied to the purely inductive ac circuit be given by,

$$v = V_m \sin(2\pi ft) \qquad \text{...(2.17.1)}$$

- Let the instantaneous current through the circuit be i. Then it can be proved that the equation for instantaneous current is given by,

$$i = I_m \sin\left(2\pi ft - \frac{\pi}{2}\right) \qquad \text{...(2.17.2)}$$

where I_m is the peak value of current.

- The expression for current can be explained as follows :

(A-627)

$i = I_m \sin(2\pi ft - \frac{\pi}{2})$

- i → Instantaneous current
- I_m → Peak value
- sin → Current wave is a sinewave
- f → Frequency of the current waveform is f
- – → Current lags behind voltage
- $\frac{\pi}{2}$ → Phase angle is $\frac{\pi}{2}$ rad. or 90°

...(2.17.3)

Peak value I_m :

The peak value of the current is given by,

$$I_m = \frac{V_m}{X_L}$$

where X_L is the reactance of the inductor.

Conclusion from Equation (2.17.3) :

- The current i is a sinusoidal current with a peak value $I_m = (V_m / \omega L)$.
- The phase angle of i is $(-\pi/2)$ radians or $-90°$. Thus the current lags behind the applied voltage by 90°.
- If we assume the current to be reference, the voltage across the inductance leads the current through it by 90°.

2.17.2 Waveforms and Phasor Diagram :

S-09, W-10, W-14, S-16

MSBTE Questions

Q. 1 Explain with phasor diagram power consumed by a pure resistance, pure inductance and pure capacitance if connected across an ac supply. **(S-09, 4 Marks)**

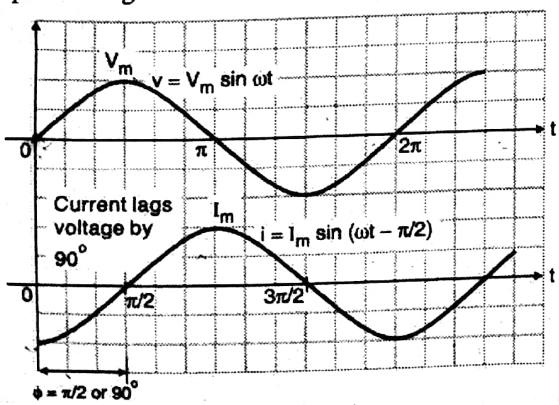
Q. 2 Draw circuit diagram and with the help of waveforms and phasor diagrams comment on the phase relationship between voltage and current in pure inductive circuit. **(W-10, 4 Marks)**

Q. 3 Draw and explain the labelled circuit and phasor diagram for purely inductive circuit. What is the power factor of the circuit ? **(W-14, 4 Marks)**

Q. 4 Draw a circuit diagram of pure inductive circuit and phasor diagram. **(S-16, 4 Marks)**

Waveforms :

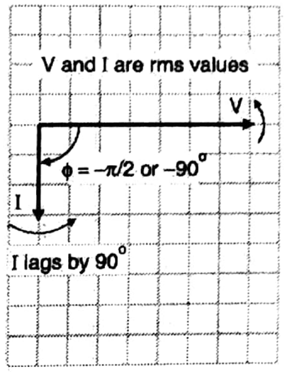
- The waveform for voltage across and current through a pure inductance are as shown in Fig. 2.17.2(a) and the phasor diagram is as shown in Fig. 2.17.2(b).



(a) Waveform of voltage and current

(A-628)Fig. 2.17.2

Phasor diagram :



(b) Phasor diagram

(A-628)Fig. 2.17.2 : Purely inductive ac circuit

2.17.3 Power in a Purely Inductive Circuit :

S-09

MSBTE Questions

Q. 1 Explain with phasor diagram power consumed by a pure resistance, pure inductance and pure capacitance if connected across an ac supply. **(S-09, 4 Marks)**

Instantaneous power (P) :

- The instantaneous power is given by the product of the instantaneous voltage across the inductance and the instantaneous current through it.

$\therefore$ Instantaneous power, $p = v \times i$

- It can be proved that the instantaneous power in a purely inductive circuit is given by,

$$p = \frac{-V_m I_m}{2} \times \sin(2\omega t) \qquad \text{...(2.17.4)}$$

- This is expression for instantaneous power, which shows that the instantaneous power is a sinusoidal waveform.

Average power :

- The average power supplied to or consumed by a pure inductor connected in an ac circuit is zero.

$$\therefore \quad P_{av} = 0 \qquad \text{...(2.17.5)}$$

2.17.4 Impedance of a Purely Inductive Circuit :

- The impedance Z is expressed in the rectangular form as,

$$Z = R + jX$$

- When the circuit is purely inductive, the resistive part is zero i.e. $R = 0$ and $X = X_L$.
- Hence the impedance of a purely inductive circuit in the rectangular form is given by,

$$Z = j X_L \, \Omega$$

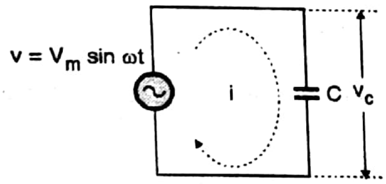
- In the polar form, it is given by,

$$Z = X_L \angle 90° \, \Omega$$

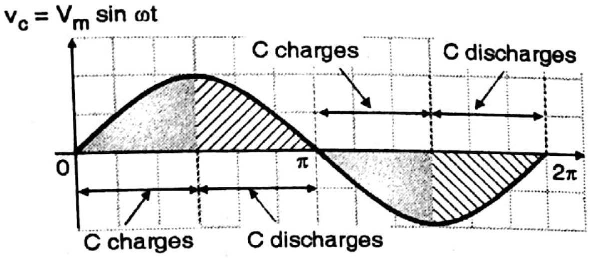
2.18 AC Circuit Containing Capacitance Only :

Circuit Diagram and Waveforms :

- The purely capacitive AC circuit is shown in Fig. 2.18.1(a). It consists of a pure capacitor C and a voltage source $v = V_m \sin \omega t$. A pure capacitor has its leakage resistance equal to infinity.
- The capacitor charges when the applied voltage is increasing and discharges when the applied voltage is decreasing. This gives rise to an alternating current i in the circuit. This is shown in Fig. 2.18.1(b).



(a) Purely capacitive circuit



(b) Capacitor charges and discharges in the alternate quarter cycles

(A-631)**Fig. 2.18.1**

2.18.1 Equation for Voltage and Current :

- Let the instantaneous applied voltage be denoted by v and be mathematically given by,

$$v = V_m \sin (2\pi ft) \qquad ...(2.18.1)$$

- Then it can be proved that the instantaneous current equation is given by,

$$i = I_m \sin\left(2\pi ft + \frac{\pi}{2}\right) \qquad ...(2.18.2)$$

where I_m is the peak value of current.

- The expression for current "i" can be explained as follows :

$$i = I_m \sin \left(2\pi ft + \frac{\pi}{2}\right) \qquad ...(2.18.3)$$

Instantaneous current ← i

I_m → Peak value

$\sin$ → Current wave is a sinewave

f → Frequency of the current waveform is f Hz

$+$ → Current leads the voltage

$\frac{\pi}{2}$ → Phase angle is $\frac{\pi}{2}$ rad. or 90°

(A-632)

Peak value I_m :

- The peak value of current is given by,

$$I_m = \frac{V_m}{X_C}$$

where X_C = The reactance of capacitor.

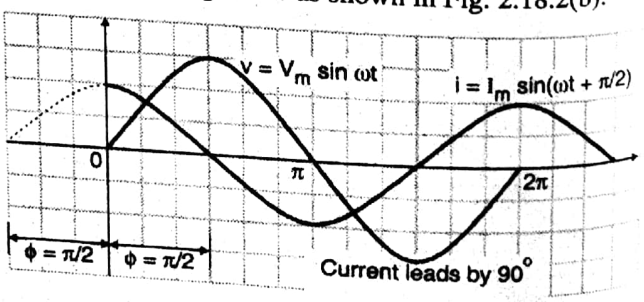
Conclusions from Equation (2.18.3) :

- Equation (2.18.3) is the expression for the instantaneous current through a purely capacitive ac circuit which indicates that,
  - The current i is purely sinusoidal current, with the same frequency as that of the voltage.
  - It has a peak value of $I_m = \frac{V_m}{X_C}$ where $X_C = 1 / 2 \pi fC$.
  - The current has a phase angle $\phi = + \frac{\pi}{2}$ or 90° which shows that it leads the voltage across the capacitor by 90°. In other words voltage across the capacitor lags behind the current by $\pi / 2$ radians or 90°.

2.18.2 Current and Voltage Waveforms and Phasor Diagram :

Waveforms :

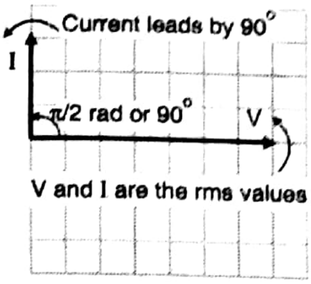
- The voltage and current waveforms for a purely capacitive ac circuit are as shown in Fig. 2.18.2(a) and the phasor diagram is as shown in Fig. 2.18.2(b).



(a) Voltage and current waveforms

(A-633)**Fig. 2.18.2 (Contd...)**

Phasor Diagram :



(b) Phasor diagram

(A-633)**Fig. 2.18.2 : A purely capacitive circuit**

2.18.3 Power in Purely a Capacitive AC Circuit :

W-08, S-09, W-09

MSBTE Questions

Q. 1 State the equation of power in case of purely capacitive circuit when the voltage $V = V_m \sin \omega t$ is applied. **(W-08, 4 Marks)**

Q. 2 Explain with phasor diagram power consumed by a pure resistance, pure inductance and pure capacitance if connected across an ac supply. **(S-09, 4 Marks)**

Q. 3 State the equation of power in case of purely capacitive circuit when the voltage is $Q = V_m \sin \omega t$ is applied. **(W-09, 4 Marks)**

Instantaneous Power :

- The instantaneous power, $p = v \times i$.
- It can be proved that the instantaneous power in a purely capacitive ac circuit is given by,

$$p = \frac{V_m I_m}{2} \sin (2\omega t) \quad ...(2.18.4)$$

Average power in purely capacitive ac circuit :

- The average value of power supplied to and consumed by a pure capacitor connected in an AC circuit is zero.

2.18.4 Impedance of a Purely Capacitive Circuit :

- The impedance Z is expressed in the rectangular form as

$$Z = R + jX$$

- When the circuit is purely capacitive, the resistive part is zero i.e. $R = 0$ and $X = X_C$.
- Hence the impedance of a purely capacitive circuit is given by,

$$Z = -j X_C$$

- In the polar form, it is given by,

$$Z = X_C \angle -90° \ \Omega$$

Ex. 2.18.1 : Find the current through a purely capacitive circuit containing a voltage source V = 230 Volts, 50 Hz and a capacitor of 1000 μF.

Soln. :

Given : $V_{rms} = 230$, $f = 50$ Hz, $C = 1000$ μF

1. Capacitive reactance $X_C = \frac{1}{2\pi fC}$

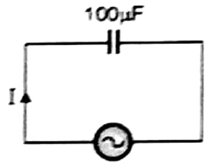
$$= \frac{1}{2\pi \times 50 \times 1000 \times 10^{-6}}$$

$$= 3.183 \ \Omega$$

2. RMS value of current $I_{rms} = \frac{V_{rms}}{X_C} = \frac{230}{3.183}$

$$= 72.26 \text{ A} \quad \text{...Ans.}$$

Ex. 2.18.2 : Refer Fig. P. 2.18.2. Find the current taken by capacitor.



(A-2646)**Fig. P. 2.18.2**

Soln. :

Reactance of capacitor $X_C = \frac{1}{2\pi fC}$

$$= \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$$= 31.83 \ \Omega$$

$\therefore$ Current $I = \frac{V}{X_C} = \frac{230}{31.83} = 7.23$ Amp. **...Ans.**

Ex. 2.18.3 : Find the current taken by a 100 μF capacitor when it is connected across 230 V, 50 Hz supply. **W-10, 4 Marks**

Soln. :

Given : $C = 100$ μF, $V_{rms} = 230$ V, $f = 50$ Hz

$$i = I_m \sin (2 \pi ft + \pi/2)$$

$$I_m = \frac{V_m}{X_C}$$

$\therefore$ Capacitance reactance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 50 \times 100 \times 10^{-6}}$$

$$= 31.83 \ \Omega$$

Peak current, $I_m = \frac{V_m}{X_C} = \frac{\sqrt{2} \times V_m}{31.83} = \frac{\sqrt{2} \times 230}{31.83}$

$$= 10.22 \text{ A}$$

$\therefore \quad I_m = 10.21 \text{ A}$

Therefore the expression for the capacitor current is

$\therefore$ Current $i = 10.21 \sin (2\pi \times 50\, t + \pi/2)$...**Ans.**

And the rms value of capacitor current is ,

$$I = \frac{10.21}{\sqrt{2}} = 7.22 \text{ Amp.} \qquad \text{...Ans.}$$

2.19 AC Circuits with Series Elements :

- We have discussed the operation of AC circuits using the basic three pure elements i.e. resistance, inductance, capacitor.
- Now we are going to interconnect by considering two elements at a time (RL, RC, LC) or interconnecting all the three elements in series (RLC).

2.20 The Series R-L Circuit :

S-16

MSBTE Questions

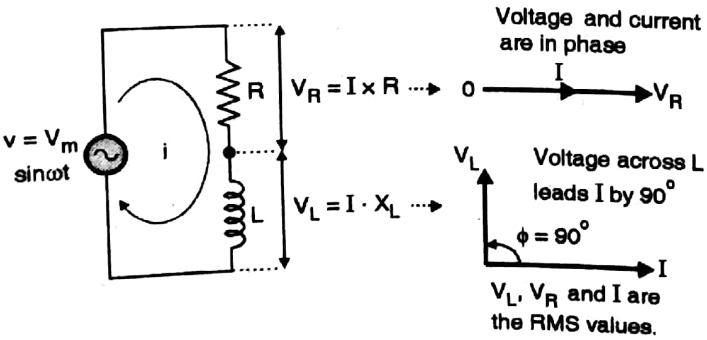
Q. 1 Draw the circuit diagram and waveforms of voltage and current in RL series circuit. **(S-16, 4 Marks)**

Circuit diagram :

- The series R-L circuit is as shown in Fig. 2.20.1. The ac voltage source of instantaneous voltage $v = V_m \sin \omega t$ is connected across the series combination of L and R.
- Assume the resistance to be of value R ohms and inductance to be a pure inductance of value L henries (H).
- Assume that the current flowing through L and R is I amperes, where I is the rms value of the instantaneous current i.
- Due to this current the voltage drops across the L and R are given by :

  Voltage drop across R, $V_R = I \cdot R$ (V_R is in phase with I)

  Voltage drop across L, $V_L = I \cdot X_L$ (V_L leads I by 90°)



(A-637) **Fig. 2.20.1 : R-L series circuit**

2.20.1 Phasor Diagram :

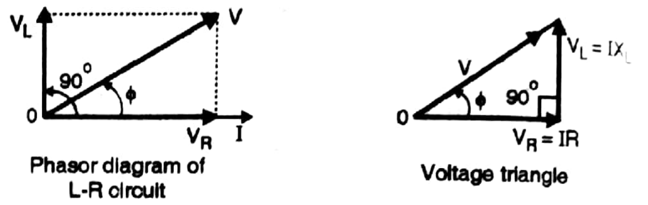
- The applied voltage v is equal to the phasor addition of V_R and V_L.

$$\bar{V} = \bar{V}_R + \bar{V}_L \qquad \text{...(phasor addition)} \qquad \text{...(2.20.1)}$$

Substituting, $\bar{V}_R = \overline{IR}$ and $\bar{V}_L = \overline{IX_L}$ we get,

$$\bar{V} = \overline{IR} + \overline{IX_L} \qquad \text{...(2.20.2)}$$

- This addition is shown as phasor diagram Fig. 2.20.2(a) and the voltage triangle is as shown Fig. 2.20.2(b).



All the voltages and currents are RMS values

(A-638(a)) **(a)** **(b)**

Fig. 2.20.2 : Phasor diagram and voltage triangle for the LR series circuit

2.20.2 Impedance of L-R Series Circuit :

- The impedance of the R-L series circuit is expressed in the rectangular form is expressed as,

$$Z = R + jX_L \qquad \text{...(2.20.3(a))}$$

- And it is expressed in the polar form as,

$$Z = |Z| \angle \phi \qquad \text{...(2.20.3(b))}$$

Where $|Z| = \sqrt{R^2 + X_L^2}$ and $\phi = \tan^{-1} [X_L / R]$

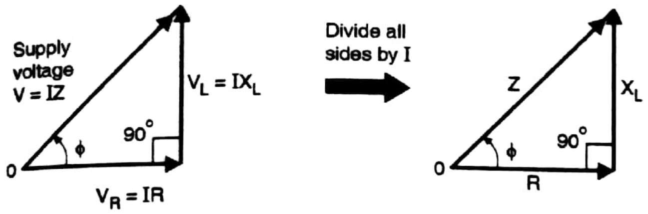
2.20.3 Impedance Triangle :

- Consider the voltage triangle of Fig. 2.20.3(a). We can obtain the values of sin ϕ, cos ϕ and tan ϕ from the voltage triangle as follows :

$$\sin\phi = \frac{V_L}{V} = \frac{X_L}{Z}, \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z},$$

$$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}$$

- Now divide each side of the voltage triangle by I to obtain another triangle called as the impedance triangle as shown in Fig. 2.20.3(b). The three sides of this triangle are R, X_L and Z.



(a) Voltage triangle **(b) Impedance triangle**

(A-639) **Fig. 2.20.3**

2.20.4 Voltage and Current Waveforms :

S-16

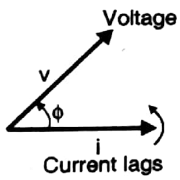
MSBTE Questions

Q. 1 Draw the circuit diagram and waveforms of voltage and current in RL series circuit. **(S-16, 4 Marks)**

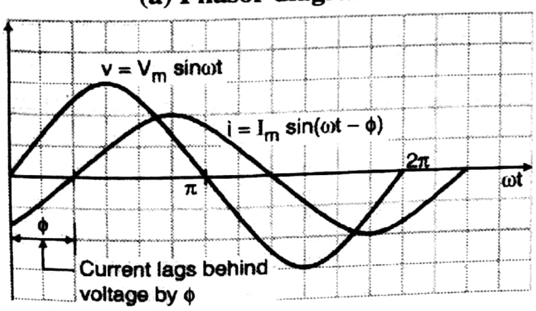
- From the phasor diagram of Fig. 2.20.4(a) it is evident that the supply voltage v leads current i by a phase angle ϕ or current lags behind voltage by ϕ.
- Hence the expressions for the voltage and current are as follows,

$$i = I_m \sin(\omega t - \phi), \quad \text{and} \quad v = V_m \sin(\omega t)$$

- The voltage and current are graphically shown in Fig. 2.20.4(b).



(a) Phasor diagram



(b) Voltage and current waveforms

(A-640)**Fig. 2.20.4**

Expression for current :

- The current through an R-L circuit is given by,

$$i(t) = \frac{v(t)}{Z} = \frac{V\angle 0^\circ}{|Z|\angle\phi} = \frac{V\angle -\phi}{|Z|}$$

Let $\frac{V}{|Z|} = I_m \quad \therefore\ i(t) = I_m \angle -\phi$ Amp.

- Let $v(t) = V_m \sin \omega t$. Hence the expression for the instantaneous current is,

$$i(t) = I_m \sin(\omega t - \phi)$$

- This equation mathematically shows that current in the R-L series circuit lags behind the supply voltage by an angle ϕ as shown in Fig. 2.20.4(b).

2.20.5 Average Power in Series L-R Circuit :

- If we represent the rms voltage and current by V and I then, the average power supplied to a series RL circuit is given by,

$$P_{av} = VI \cos\phi \text{ Watts} \quad ...(2.20.4)$$

- The average power supplied to the L-R circuit is,

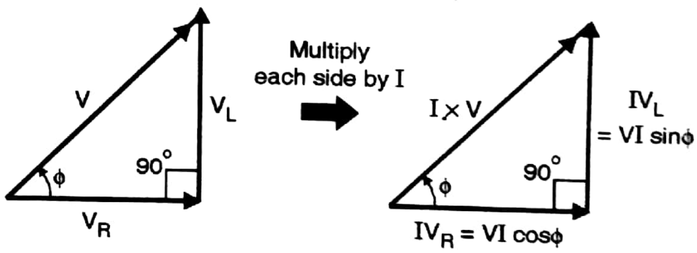
$$P_{av} = \text{(Average power consumed by R)} + \text{(Average power consumed by L)}$$

- But the average power consumed by the pure inductance is zero.

$$\therefore P_{av} = \text{Average power consumed by R}$$

2.20.6 Power Triangle for L-R Series Circuit :

- Earlier we have drawn the voltage triangle, then the impedance triangle. Now let us draw one more, called as the power triangle for the L-R series circuit.
- The power triangle is obtained from the voltage triangle by multiplying its each side by I as shown in Fig. 2.20.5.



(a) Voltage triangle **(b) Power triangle**

(A-641) **Fig. 2.20.5**

2.20.7 Apparent Power (S) :

- Apparent power is defined as the product of the rms values of voltage (V) and current (I).
- For the RL series circuit the apparent power is given by,

$$\therefore \text{Apparent power } S = V \times I \text{ Volt-ampere} \quad ...(2.20.5)$$

2.20.8 Real Power or True Power or Active Power (P) :

The true power or real power (P) is equal to the power consumed by the RL series circuit. It is given by,

$$P = P_{av} = VI \cos \phi \text{ Watts} \quad ...(2.20.6)$$

where ϕ = Phase angle between V and I.

2.20.9 Reactive Power or Imaginary Power (Q) :

For RL series circuit,

$$\therefore \text{ Reactive power } Q = VI \sin \phi \text{ VAR} \quad ...(2.20.7)$$

2.20.10 Power Factor :

For an RL series circuit, the supply current I lags behind the supply voltage V by angle ϕ. Therefore the power factor of RL series circuit is a **lagging** power factor and its value lies between 0 and 1.

$$0 \leq \cos \phi \leq 1 \text{ ...for RL series circuit}$$

2.20.11 Q-Factor :

- A resistor cannot store energy but inductor and capacitor are capable of storing energy.
- The quality of a reactive circuit is measured in terms of how efficient of L and C components are to store energy.
- And the efficiency of L and C to store energy is measured in terms of a factor called Quality factor or Q factor.
- The Q-factor thus represents the efficiency of a reactive circuit to store energy. Hence it is also called as figure of merit of the reactive circuit.

Definition :

- The Q factor is defined as the ratio of energy stored per cycle to the energy lost (or dissipated) per cycle.
- Mathematically Q can be expressed as,

$$Q = 2\pi \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}}$$

Q factor of RL series circuit :

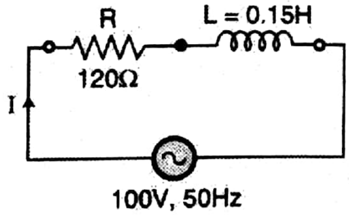
Q factor of an RL series circuit is given by,

$$Q = \frac{X_L}{R} = \frac{2\pi f L}{R}$$

Ex. 2.20.1 : A RL series circuit consist of a 120 Ω resistance and a 0.15 Henry inductance. The series circuit is connected across a 100 V, 50 Hz AC supply calculate the voltage across individual elements.

Soln. :

Step 1 : Draw the circuit, calculate reactance and impedance :



(A-1555)**Fig. P. 2.20.1**

Reactance of inductor

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.12\ \Omega$$

$$\text{Impedance } |Z| = \sqrt{R^2 + X_L^2} = \sqrt{(120)^2 + (47.12)^2}$$

$$= 128.92\ \Omega$$

Step 2 : Calculate current :

$$I = \frac{V}{|Z|} = \frac{100}{128.92} = 0.7757 \text{ A}$$

Step 3 : Calculate individual voltages :

Voltage across resistance

$$V_R = I \times R = 0.7757 \times 120$$

$$= 93 \text{ Volts} \quad \text{...Ans.}$$

Voltage across inductance $V_L = 0.7757 \times 47.12$

$$= 36.55 \text{ Volts.} \quad \text{...Ans.}$$

Ex. 2.20.2 : A coil having a resistance of 4.5 Ω and inductance of 0.03 henry is connected across 230 V, 50 Hz, l-phase supply mains, find current taken by the coil.

Soln. :

Given : $R = 4.5\ \Omega$ $L = 0.03$ H $V = 230$ V $f = 50$ Hz

Step 1 : Calculate the impedance :

Reactance $X_L = 2\pi f L = 2\pi \times 50 \times 0.03 = 9.42\ \Omega$

$$\therefore \text{ Impedance } Z = R + j X_L = 4.5 + j\, 9.42$$

$$= \sqrt{(4.5)^2 + (9.42)^2} \angle \tan^{-1} \frac{9.42}{4.5}$$

$$= 10.44 \angle 64.47^\circ\ \Omega$$

Step 2 : Calculate current :

$$I = \frac{V}{Z} = \frac{230 \angle 0^\circ}{10.44 \angle 64.47^\circ}$$

$$= 22.03 \angle -64.47^\circ \text{ Amp.} \quad \text{...Ans.}$$

Ex. 2.20.3 : A coil has an inductance of 0.02 H and resistance of 5 ohm, it is supplied with 100 V, 50 Hz a.c. supply. Determine the current taken by the coil. Also find the power consumed by it.

Soln. :

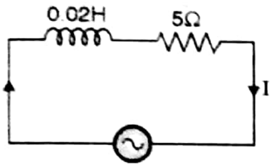
Given : $R = 5\ \Omega$, $L = 0.02$ H, $V = 100$ V, $f = 50$ Hz

Step 1 : Impedance :

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$\therefore\ |Z| = \sqrt{R^2 + X_L^2} = \sqrt{(5)^2 + (6.28)^2}$$

$$\therefore\ |Z| = 8.03\ \Omega$$



(A-1557) **Fig. P. 2.20.3**

Step 2 : Current :

$$I = \frac{V}{|Z|} = \frac{100}{8.03} = 12.45\ \text{A} \quad \text{...Ans.}$$

Step 3 : Power consumed :

$$P = I^2 R = (12.45)^2 \times 5$$

$$= 775\ \text{W} \quad \text{...Ans.}$$

Ex. 2.20.4 : A coil having resistance 10 Ω and an inductance 0.2 H is connected across 100 V, 50 Hz, supply calculate :

1. Reactance (X_L). 2. Impedance (Z).
3. Current. **W-08, S-12, S-13, 4 Marks**

Soln. :

Given : $R = 10\ \Omega$, $L = 0.2$ H, $V_S = 100$ V, $f = 50$ Hz.

1. Reactance $X_L = 2\pi fL = 2\pi \times 50 \times 0.2$

$$= 62.83\ \Omega \quad \text{...Ans.}$$

2. Impedance $|Z| = \sqrt{R^2 + X_L^2} = \sqrt{(10)^2 + (62.83)^2}$

$$= 63.62\ \Omega \quad \text{...Ans.}$$

3. Current $I = \frac{V_S}{|Z|} = \frac{100}{63.62} = 1.57$ Amp ...Ans.

Ex. 2.20.5 : A coil consist of 20 ohm resistance and 0.2 H inductance is connected across 230 V, 50 Hz. supply. Calculate :

1. Impedance of coil 2. Power factor
3. Current 4. Active power.

S-14, 4 Marks

Soln. :

Given : $R = 20$ ohm, $L = 0.2$ H, $V_S = 230$ V, $f = 50$ Hz

To find : 1. Impedance 2. Power factor 3. Current 4. Active power

Step 1 : Calculate impedance and current :

$$\text{Reactance } X_L = 2\pi f_L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$$

$$\text{Impedance } Z = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + (62.83)^2}$$

$$= 65.93\ \Omega \quad \text{...Ans.}$$

$$\text{Current } I = \frac{V_S}{Z} = \frac{230}{65.93} = 3.488\ \text{Amp.} \quad \text{...Ans.}$$

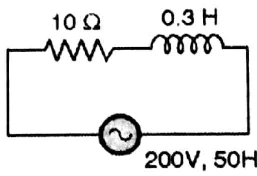
Step 2 : Calculate power factor and active power :

$$\cos\phi = \frac{R}{Z} = \frac{20}{65.93} = 0.3033 \quad \text{...Ans.}$$

$$\text{Active power} = I^2 R = (3.488)^2 \times 20$$

$$= 243.32\ \text{W} \quad \text{...Ans.}$$

Ex. 2.20.6 : Find : 1. Impedance, 2. Phase angle, 3. Current, 4. Total power for the circuit shown in Fig. P. 2.20.6. **W-14, 4 Marks**



(A-4202) **Fig. P. 2.20.6**

Soln. :

1. **Impedance :**

$$X_L = 2\pi f_L = 2\pi \times 50 \times 0.3 = 94.24\ \Omega$$

$$|Z| = \sqrt{R_2^2 + X_L^2} = \sqrt{10^2 + (94.24)^2}$$

$$= 94.76\ \Omega \quad \text{...Ans.}$$

2. **Phase angle :**

$$\phi = \tan^{-1}\left[\frac{X_L}{R}\right] = \tan^{-1}\left[\frac{94.24}{10}\right]$$

$$\therefore\ \phi = 83.94° \quad \text{...Ans.}$$

3. **Current :** $I = \frac{V_s}{|Z|} = \frac{230}{94.76} = 2.43$ Amp....Ans.

4. **Total power :**

$$P = I^2R = (2.43)^2 \times 10 = 59\ \text{W} \quad \text{...Ans.}$$

Ex. 2.20.7 : A coil having resistance 10 ohm and an inductance 0.2 H is connected across 100 Volts, 50 Hz, supply. Calculate :

1. Reactance 2. Impedance

3. Current 4. Power consumed

W-15, 4 Marks

Soln. :

Please Refer Ex. 2.20.4 for reactance, impedance and current.

4. Power consumed :

$$P = I^2R = (1.57)^2 \times 10 = 24.65 \text{ Watts} \quad \text{...Ans.}$$

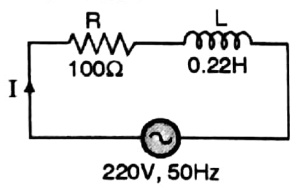
Ex. 2.20.8 : An RL series circuit consists of 100 Ω resistance and 0.22 H inductance connected across 220 V, 50 Hz AC supply. Calculate : 1. Impedance 2. Current 3. Voltage across resistor 4. Voltage across inductor

S-16, 4 Marks

Soln. :

Given : f = 50 Hz, R = 100 Ω, L = 0.22 H, V = 220 Volts.

Step 1 : Draw the circuit :



(A-4868) **Fig. P. 2.20.8**

Step 2 : Calculate reactance of inductor :

$$X_L = 2\pi fL = 2 \times \pi \times 50 \times 0.22$$

$$= 69.11\ \Omega$$

Step 3 : Calculate impedance :

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (69.11)^2}$$

$$= 121.55\ \Omega \quad \text{...Ans.}$$

Step 4 : Calculate current :

$$I = \frac{V}{|Z|} = \frac{220}{121.55} = 1.809 \text{ A} \quad \text{...Ans.}$$

Step 5 : Calculate individual voltages :

Voltage across resistor

$$V_R = I \times R = 1.809 \times 100 = 180.9 \text{ Volts}$$

Voltage across inductor

$$V_L = I X_L = 1.809 \times 69.11$$

$$= 125 \text{ Volts.} \quad \text{...Ans.}$$

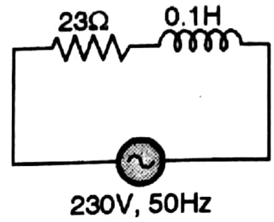
Ex. 2.20.9 : Calculate :

1. Reactance 2. Impedance

3. Current 4. Phase angle

for the circuit shown in Fig. P. 2.20.9.

W-16, 4 Marks



(A-5181) **Fig. P. 2.20.9**

Soln. :

Given : R = 23 Ω, L = 0.1 H, f = 50 Hz, V = 230 Volts.

1. Reactance :

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.42\ \Omega \quad \text{...Ans.}$$

2. Impedance :

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{(23)^2 + (31.42)^2}$$

$$= 38.94\ \Omega \quad \text{...Ans.}$$

3. Current :

$$I = \frac{V}{|Z|} = \frac{230}{38.94} = 5.9 \text{ Amp} \quad \text{...Ans.}$$

4. Phase angle :

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\left(\frac{31.42}{23}\right)$$

$$= 53.8° \quad \text{...Ans.}$$

Ex. 2.20.10 : A coil having resistance 6 Ω and reactance 8 Ω is connected across 230 V, 50 Hz, a.c. supply. Calculate :

1. Inductance 2. Impedance

3. Current 4. Active power

W-16, 4 Marks

Soln. :

Given : $R = 6\ \Omega$, $X_L = 8\ \Omega$, V = 230 Volts, f = 50 Hz.

1. Inductance :

$$L = \frac{X_L}{2\pi f} = \frac{8}{2\pi \times 50}$$

$$= 0.02546 \text{ H or } 25.46 \text{ mH} \quad \text{...Ans.}$$

2. Impedance :

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{(6)^2 + (8)^2}$$

$$= 10\ \Omega \quad \text{...Ans.}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°$$

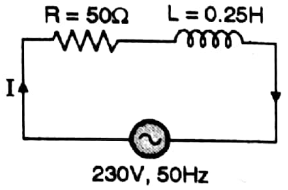
3. **Current :**

$$I = \frac{V}{|Z|} = \frac{230}{10} = 23 \text{ Amp} \quad \text{...Ans.}$$

4. **Active power :**

$$P_A = V \times I \times \cos\phi = 230 \times 23 \times \cos(53.13)$$
$$= 3174 \text{ W} \quad \text{...Ans.}$$

Ex. 2.20.11 : Calculate voltage across individual element for the circuit shown in Fig. P. 2.20.11. S-17, 4 Marks



(A-5262) **Fig. P. 2.20.11**

Soln. :

Given :

RL series circuit, R = 50 Ω, L = 0.25 H, V = 230 V, f = 50 Hz

To Find : 1. Voltage across R 2. Voltage across L

Step 1 : Calculate reactance of inductor :

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.25 = 78.54\ \Omega$$

Step 2 : Calculate impedance :

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{(50)^2 + (78.54)^2}$$
$$= 93.10\ \Omega$$

Step 3 : Calculate current :

$$I = \frac{V}{|Z|} = \frac{230}{93.10} = 2.47 \text{ Amp.}$$

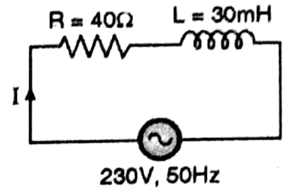
Step 4 : Calculate individual voltages :

$$V_R = I \times R = 2.47 \times 50$$
$$= 23.51 \text{ Volts} \quad \text{...Ans.}$$
$$V_L = I \times X_L = 2.47 \times 78.54$$
$$= 94 \text{ Volts} \quad \text{...Ans.}$$

Ex. 2.20.12 : A series circuit consisting of resistance 40 Ω and inductance 30 mH is supplied by 230 V, 50 Hz, a.c. supply. Calculate impedance and current taken by the circuit. W-17, 4 Marks

Soln. :

Step 1 : Draw the circuit, calculate reactance and impedance :



(A-5306) **Fig. P. 2.20.12**

Reactance of inductor $X_L = 2\pi f L = 2\pi \times 50 \times 30 \times 10^{-3}$ $= 9.424\ \Omega$

$$\text{Impedance (Z)} = \sqrt{R^2 + X_L^2}$$
$$= \sqrt{(40)^2 + (9.424)^2}$$
$$= 41.095\ \Omega \quad \text{...Ans.}$$

Step 2 : Calculate current :

$$I = \frac{V}{Z} = \frac{230}{41.095} = 5.596 \text{ A} \quad \text{...Ans.}$$

2.21 The Series R-C Circuit :

S-14, W-14, W-15, S-17

MSBTE Questions

Q. 1 Draw circuit diagram, waveform, phasor diagram and comment on the phase relationship between voltage and current in R-C series circuit. **(S-14, 4 Marks)**

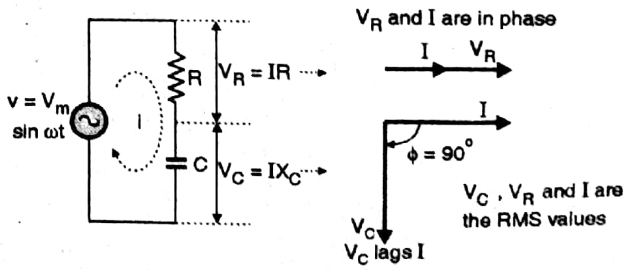
Q. 2 For R-C series circuit :
1. Draw circuit diagram 2. Its phasor diagram
3. Waveform of voltage and current
4. Impedance triangle **(W-14, 4 Marks)**

Q. 3 Draw and explain circuit diagram of R-C circuit. **(W-15, 4 Marks)**

Q. 4 Explain for series R.C. circuit :
1. Circuit diagram 2. Voltage equation
3. Current equation 4. Power **(S-17, 4 Marks)**

Circuit diagram :

- The series R-C circuit with an ac excitation is shown in Fig. 2.21.1. A pure capacitor of value C is connected in series with a resistor R and a voltage source of instantaneous voltage $v = V_m \sin \omega t$ is connected across the series combination of R and C.



(A-652) **Fig. 2.21.1 : R-C series circuit**

- Let the rms value of current flowing through R and C be equal to I amperes. The voltage drops across R and C due to this current are :

1. Voltage drop across R, $V_R = I \cdot R$ (V_R is in phase with I)
2. Voltage drop across C, $V_C = I \cdot X_C$ (V_C lags I by 90°)

2.21.1 Phasor Diagram : S-14, W-14

MSBTE Questions

Q. 1 Draw circuit diagram, waveform, phasor diagram and comment on the phase relationship between voltage and current in R-C series circuit. **(S-14, 4 Marks)**

Q. 2 For R-C series circuit :

1. Draw circuit diagram
2. Its phasor diagram
3. Waveform of voltage and current
4. Impedance triangle **(W-14, 4 Marks)**

- The applied voltage v is equal to the phasor addition of V_R and V_C.

$\therefore$ Applied voltage $\bar{V} = \bar{V}_R + \bar{V}_C$ ...(phasor addition) ...(2.21.1)

- The phasor diagram of RC series circuit is shown in Fig. 2.21.2(a) and the voltage triangle is as shown in Fig. 2.21.2(b).

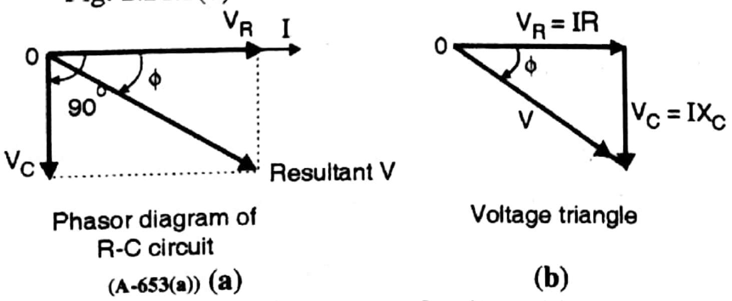


Fig. 2.21.2 : Phasor diagram and voltage triangle for the R-C circuit

- From the voltage triangle of Fig. 2.21.2(b), the resultant voltage V is given by,

$$V = \text{Vector sum of } V_R \text{ and } V_C$$

$$\therefore \quad V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} \quad ...(2.21.2)$$

$$\therefore \quad V = I\sqrt{(R^2) + (X_C^2)} \quad ...(2.21.3)$$

Let $\sqrt{R^2 + X_C^2} = |Z| =$ Impedance of the circuit

$$\therefore \quad V = I \cdot |Z| \quad ...(2.21.4)$$

2.21.2 Impedance of RC Series Circuit : W-14

MSBTE Questions

Q. 1 For R-C series circuit :

1. Draw circuit diagram
2. Its phasor diagram
3. Waveform of voltage and current
4. Impedance triangle **(W-14, 4 Marks)**

- The impedance of R-C series circuit, in the rectangular form is given by,

$$Z = R - j X_C$$

- It is expressed in the polar form as follows :

$$Z = |Z| \angle \phi$$

Where, $|Z| = \sqrt{R^2 + X_C^2}$ and $\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$

- The phase angle is negative for capacitive load.

2.21.3 Impedance Triangle (Series R-C Circuit) : W-09

MSBTE Questions

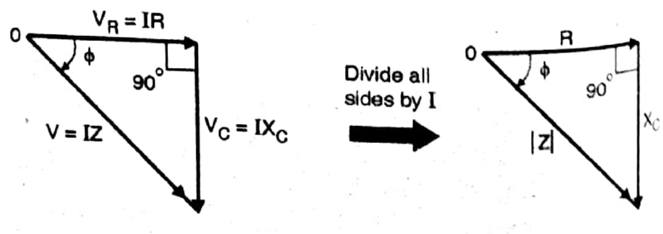
Q. 1 Draw impedance triangle for R-C series circuit. State the formula for power factor from impedance triangle. **(W-09, 2 Marks)**

Consider the voltage triangle of Fig. 2.21.3(a). We can obtain the values of sin ϕ, cos ϕ and tan ϕ from the voltage triangle as follows :

$$\sin\phi = \frac{V_C}{V} = \frac{X_C}{|Z|}, \cos\phi = \frac{V_R}{V}$$

$$= \frac{R}{|Z|} \text{ and } \tan\phi = \frac{X_C}{R}$$

- Now divide each side of the voltage triangle by the circuit current I, to obtain the impedance triangle as shown in Fig. 2.21.3(b). The sides of the impedance triangle are R, X_C and Z.



(a) Voltage triangle **(b) Impedance triangle**

(A-654) **Fig. 2.21.3**

2.21.4 Phase Relationship between Voltage and Current (Series R-C Circuit) :

S-14, W-14, S-17

MSBTE Questions

Q. 1 Draw circuit diagram, waveform, phasor diagram and comment on the phase relationship between voltage and current in R-C series circuit. **(S-14, 4 Marks)**

Q. 2 For R-C series circuit :

1. Draw circuit diagram
2. Its phasor diagram
3. Waveform of voltage and current
4. Impedance triangle **(W-14, 4 Marks)**

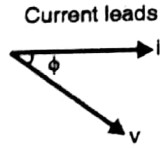
Q. 3 Explain for series R.C. circuit :

1. Circuit diagram 2. Voltage equation
3. Current equation 4. Power **(S-17, 4 Marks)**

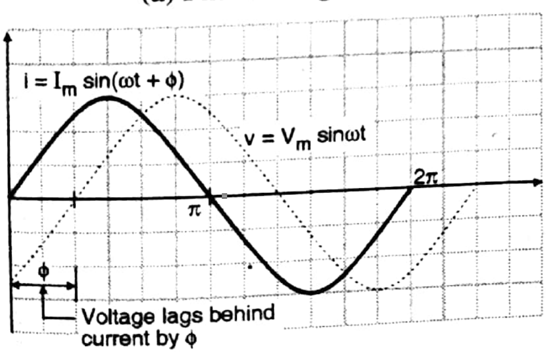
- From the phasor diagram of Fig. 2.21.4(a) it is clear that the supply voltage lags behind current i by a phase angle ϕ or current leads the voltage by ϕ.
- Hence the expressions for the instantaneous voltage and current are as follows,

$$v = V_m \sin \omega t, \quad i = I_m \sin (\omega t + \phi)$$

- Hence the voltage and currents are graphically shown in Fig. 2.21.4(b).



(a) Phasor diagram



(b) Voltage and current waveforms

(A-655)**Fig. 2.21.4 : R-C series circuit**

Voltage and Current Equations :

- Let the applied voltage be

$$v(t) = V_m \sin \omega t = V_m \angle 0° \text{ Volts}$$

- The impedance of an R-C series circuit is,

$$Z = R - j X_C = |Z| \angle -\phi$$

- Then the instantaneous current is expressed as,

$$i(t) = I_m \sin (\omega t + \phi)$$

- It shows that the current leads the applied voltage by an angle ϕ.

2.21.5 Average Power in Series R-C Circuit :

S-17

MSBTE Questions

Q. 1 Explain for series R.C. circuit :

1. Circuit diagram 2. Voltage equation
3. Current equation 4. Power **(S-17, 4 Marks)**

- The average power supplied to an RC series circuit is given by,

$$P_{av} = V_{rms} I_{rms} \times \cos \phi \quad ...(2.21.5)$$

- If we represent the rms values of voltage and current by V and I respectively, then

$$P_{av} = VI \cos \phi \text{ watts} \quad ...(2.21.6)$$

- This expression is same as the expression for the average power supplied to the L-R series circuit.
- The average power supplied to an R-C series circuit is,

$$P_{av} = (\text{Average power consumed by R}) + (\text{Average power consumed by C})$$

- But the average power consumed by a pure capacitance is zero.

$$\therefore P_{av} = \text{Average power consumed by R}$$

2.21.6 Power Triangle for R-C Series Circuit :

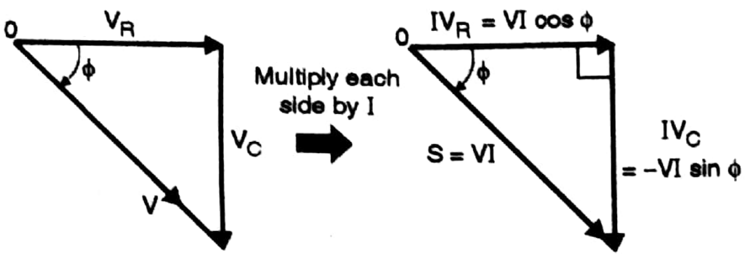
S-09, S-14

MSBTE Questions

Q. 1 Draw the voltage triangle and power triangle with all the quantities marked for R-C series circuit. **(S-09, 2 Marks)**

Q. 2 Draw a circuit diagram of R.C. series circuit. Draw impedance triangle and power triangle for same circuit. **(S-14, 4 Marks)**

- The power triangle for an R-C series circuit can be obtained from the voltage triangle by multiplying its each side by I as shown in Fig. 2.21.5.



(a) Voltage triangle **(b) Power triangle**

(A-656) **Fig. 2.21.5**

- Thus the sides of the power triangle are V_R I or VI cos ϕ, V_C I or − VI sin ϕ (since ϕ is negative) and VI. Looking at the power triangle of Fig. 2.21.5(b) we can write that,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I} \qquad ...(2.21.7)$$

$$\therefore \overline{VI} = \overline{VI \cos \phi} - \overline{VI \sin \phi} \qquad ...(2.21.8)$$

2.21.7 Various Powers and Power Factor :

W-09

MSBTE Questions

Q. 1 Draw impedance triangle for R-C series circuit. State the formula for power factor from impedance triangle. **(W-09, 2 Marks)**

Apparent Power (S) :

As defined in the previous section, the apparent power is given by,

$$S = VI \text{ Volt-Ampere} \qquad ...(2.21.9)$$

True or Real Power (P) :

The true or real or active power for the R-C series circuit is given by,

$$P = VI \cos \phi \text{ Watts} \qquad ....(2.21.10)$$

Reactive Power (Q) :

The reactive power (Q) is given by,

$$Q = VI \sin \phi \ldots \text{VAR} \qquad ...(2.21.11)$$

Power factor :

- We have already defined the power factor as,

$$\text{P.F.} = \frac{\text{True power}}{\text{Apparent power}}$$

$$= \frac{VI \cos \phi}{VI} = \cos \phi \qquad ...(2.21.12)$$

- Which is same as that for the inductive load but now it is a **leading power factor** for the RC series circuit because current leads the voltage.

Important Conclusions :

1. The expressions of average or real power for an R-C or R-L series ac circuit is same and it is given by,

   P = VI cos ϕ, where V and I are RMS values.

2. The expression for the power factor for R-C and R-L series ac circuit is, PF = cos ϕ.
3. Power factor for the RL series circuit is lagging because the circuit current lags behind the supply voltage.
4. Power factor for the RC series circuit is leading because the circuit current leads the applied voltage.
5. Impedance for R-L circuit : $Z = (R + j X_L) = |Z| \angle$ -
6. Impedance for RC circuit : $Z = (R - j X_C) = |Z| \angle$ -

2.21.8 Q-Factor :

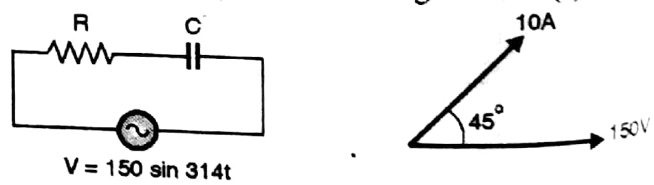
The Q factor of RC series circuit is defined as follows :

$$Q = \frac{X_C}{R} = \frac{1}{2\pi fCR}$$

Ex. 2.21.1 : Waveforms of the voltage and current of a circuit are given by the equations
V = 150 sin 314t, I = 10 sin (314t + π / 4)
Draw the circuit and vector diagram. Calculate frequency of supply and phase angle between voltage and current.

Soln. :

- From the expression of current, it is clear that current leads voltage by an angle π / 4 rad or 45°. So this is an RC series circuit as shown in Fig. P. 2.21.1(a).



(a) (b)

(A-659)**Fig. P. 2.21.1**

- The vector diagram is as shown in Fig. P. 2.21.1(b).
- From the expression for voltage,

$$2\pi ft = 314 \, t$$

$$\therefore\ f = \frac{314}{2\pi} = 50 \text{ Hz}$$ ...Ans.

From the expression of current, phase angle is,

$$\phi = \frac{\pi}{4} \text{ rad or } 45°$$ ...Ans.

Ex. 2.21.2 : The voltage and current equations in an AC circuit are given by
$v = 120 \sin \omega t$ $i = 2.5 \sin (\omega t + \pi/2)$
Find the rms value of current and voltage. Also state the type of circuit. **W-08, 4 Marks**

Soln. :

1. Rms value of voltage :

$$v = 120 \sin \omega t$$

$$\therefore\ V_m = 120 \text{ V}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{120}{\sqrt{2}} = 84.85 \text{ Volts}$$ ...Ans.

2. Rms value of current :

$$i = 2.5 \sin (\omega t + \pi/2)$$

$$\therefore\ I_m = 2.5 \text{ A}$$

$$I_{rms} = \frac{2.5}{\sqrt{2}} = 1.7677 \text{ A}$$ ...Ans.

3. Type of circuit :

The current leads voltage by 90°. So this is a purely capacitive circuit.

Ex. 2.21.3 : A resistance of 5 Ω and capacitive reactance of 10 Ω are connected in series if the current through circuit is 3 Amp.
Find :
1. Impedance
2. Total voltage
3. Phase angle **W-12, 4 Marks**

Soln. :

Given : $R = 5\ \Omega,\ X_C = 10\ \Omega,\ I = 3$ A

To find : 1. Impedance 2. Total voltage 3. Phase angle

1. Impedance Z :

$$|Z| = \sqrt{R^2 + X_C^2} = \sqrt{5^2 + 10^2} = 11.18\ \Omega$$ ...Ans.

2. Total voltage V :

$$I = \frac{V}{Z}$$

$$\therefore\ V = I \times Z = 11.18 \times 3 = 33.5 \text{ Volts}$$ ...Ans.

3. Phase angle φ :

$$\phi = \tan^{-1} [-X_C/R] = \tan^{-1} [-10/5] = -70.48°$$

$$\phi = -70.48°$$ ...Ans.

Ex. 2.21.4 : A capacitor having a capacitance of 10 microfarad is connected in series with a non-inductive resistance of 120 ohm across 100 V, 50 Hz supply. Calculate :
1. Current
2. Impedance
3. Phase difference between current and supply voltage
4. Power **S-15, 4 Marks**

Soln. :

Given : $R = 120\ \Omega$, $C = 10\ \mu F$, $V = 100$ Volts, $f = 50$ Hz.

1. Impedance :

$$|Z| = (R^2 + X_C^2)^{1/2}$$

$$\text{But } X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318.3\ \Omega$$

$$\therefore\ |Z| = [(120)^2 + (318.3)^2]^{1/2}$$

$$= 340.17\ \Omega$$ ...Ans.

2. Current :

$$\text{Circuit current } I = \frac{V}{|Z|} = \frac{100}{340.17} = 0.2939 \text{ A}$$ ...Ans.

3. Phase shift φ :

$$\phi = \tan^{-1} \frac{X_C}{R} = \tan^{-1} \frac{318.3}{120} = 69.34°$$ ...Ans.

4. Power :

$$P = V \times I \times \cos \phi = 100 \times 0.2939 \times \cos (69.34)$$

$$= 10.37 \text{ W}$$ ...Ans.

Ex. 2.21.5 : A resistance of 10 Ω and capacitance of 50 μF are connected in series across 200 V, 50 Hz AC supply. Calculate :
1. Capacitive reactance 2. Impedance
3. Current 4. Phase angle **S-16, 4 Marks**

Soln. :

Given : $R = 10\ \Omega$, $C = 50\ \mu F$, $V = 200$ Volts, $f = 50$ Hz

To find : 1. Capacitive reactance 2. Impedance 3. Current 4. Phase angle

Step 1 : Calculate capacitive reactance X_C :

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times \pi \times 50 \times 50 \times 10^{-6}}$$

$$= 63.66\ \Omega \qquad \text{...Ans.}$$

Step 2 : Calculate impedance Z :

$$|Z| = \sqrt{R^2 + X_C^2} = \sqrt{(10)^2 + (63.66)^2}$$

$$= 64.44\ \Omega \qquad \text{...Ans.}$$

Step 3 : Calculate current I :

$$I = \frac{V}{Z} = \frac{200}{64.44} = 3.10 \text{ Amp.} \qquad \text{...Ans.}$$

Step 4 : Phase angle ϕ :

$$\phi = \tan^{-1}\left[\frac{-X_C}{R}\right] = \tan^{-1}\left[\frac{-63.66}{10}\right] = \tan^{-1}[-6.366]$$

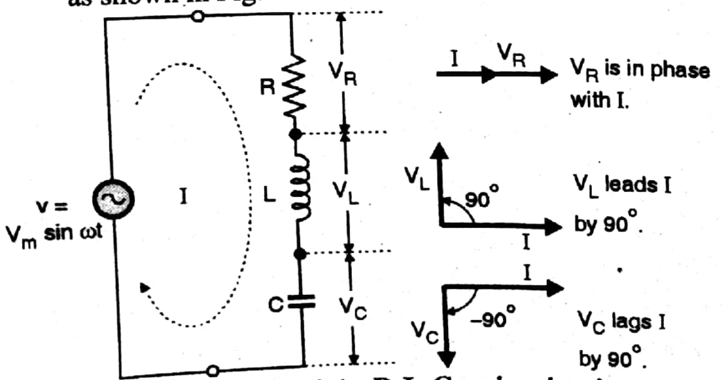
$$\phi = -81.07° \qquad \text{...Ans.}$$

2.22 An R-L-C Series Circuit :

Circuit Diagram :

- Now let us discuss the operation of an ac circuit in which R, L and C are connected in series as shown in Fig. 2.22.1.
- The ac voltage source having the instantaneous voltage $v = V_m \sin \omega t$ is connected across the series circuit. The rms current flowing through all the components is I.
- Due to current I flowing through these components, the voltage drops across them are as follows :
  1. Voltage drop across R, V_R = IR (V_R in phase with I)
  2. Voltage drop across L, $V_L = I X_L$ (V_L leads I by 90°)
  3. Voltage drop across C, $V_C = I X_C$ (V_C lags I by 90°)

  where I, V_R, V_L and V_C all are rms values.
- The phasor diagrams for the individual components are as shown in Fig. 2.22.1.



(A-661) **Fig. 2.22.1 : R-L-C series circuit**

2.22.1 Importance of the Values of Reactances :

- The relative values of the inductive reactance X_L and capacitive reactance X_C play a very important role in deciding the behaviour of the R-L-C series circuit.
- We are going to discuss the circuit behaviour and draw the corresponding phasor diagrams for the following three possible conditions :

  Condition - 1 : $X_L > X_C$

  Condition - 2 : $X_L < X_C$

  Condition - 3 : $X_L = X_C$

2.22.2 Phasor Diagrams :

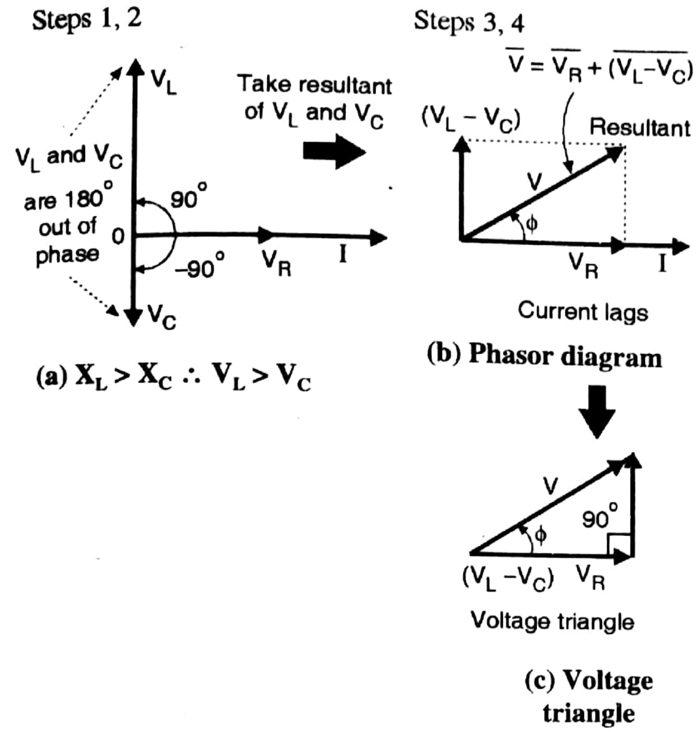
We will now draw the phasor diagram for all the three conditions mentioned earlier.

2.22.3 Phasor Diagram for : $X_L > X_C$:

- As X_L is greater than X_C, the voltage drop across X_L is greater than that across X_C. Hence $V_L > V_C$.
- The resultant voltage (supply voltage) V is obtained by the vector addition of V_R, V_L and V_C.

$$\therefore \bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{...(Phasor sum)} \quad \text{...(2.22.1)}$$

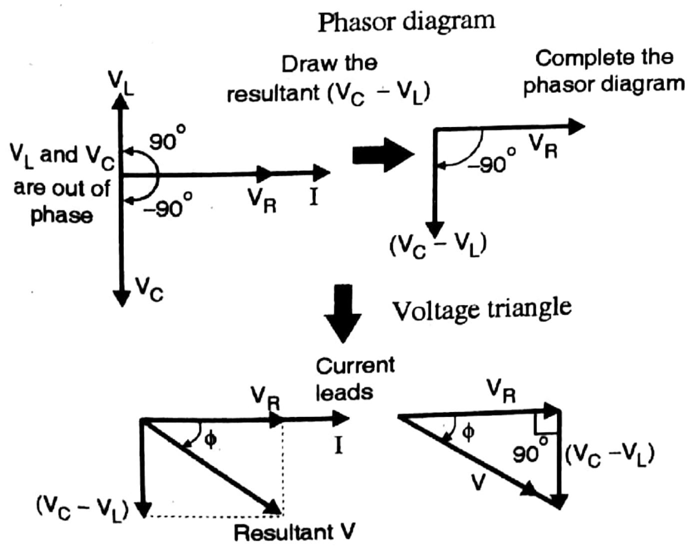
- As V_L and V_C are directly in phase opposition (180° out of phase), and as $V_L > V_C$, their resultant is $(V_L - V_C)$ i.e. the simple arithmetic subtraction of the two voltages.
- This situation is illustrated in Fig. 2.22.2(a) and the voltage triangle is as shown in Fig. 2.22.2(c).
- As shown in Fig. 2.22.2(b) the resultant of V_L and V_C i.e. $(V_L - V_C)$ will be in the direction of V_L because $V_L > V_C$. Therefore the R-L-C circuit is **inductive** if $X_L > X_C$.
- As $V_L > V_C$ the circuit is inductive and the supply current I lags behind the supply voltage V by an angle ϕ.



(A-662) **Fig. 2.22.2 : R-L-C series circuit phasor diagram and voltage triangle ($X_L > X_C$)**

2.22.4 Phasor Diagram for : $X_L < X_C$:

- When X_L is less than X_C, then V_L will be less than V_C. But V_L and V_C continue to be out of phase with respect to each other. Hence the resultant of V_L and V_C i.e. $(V_C - V_L)$ will be in the direction of V_C now, as shown in Fig. 2.22.3.



(A-663) **Fig. 2.22.3 : Phasor diagram and voltage triangle for R-L-C series circuit with $X_L < X_C$**

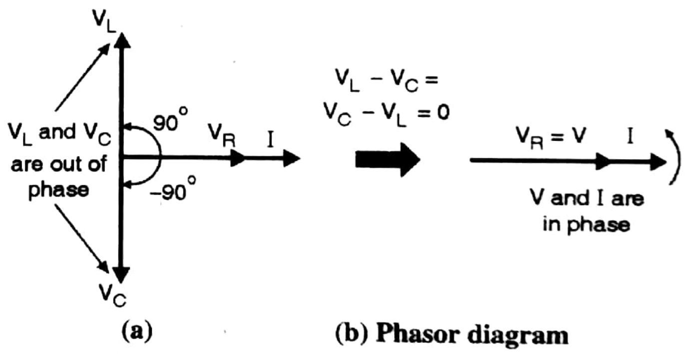
- As the resultant $(V_C - V_L)$ is in the direction of V_C, the circuit is equivalent to an R-C series circuit in which the current will lead the resultant voltage by φ radians as shown in Fig. 2.22.3.
- Draw the phasor diagram by following the steps given in Fig. 2.22.3, and using it draw the voltage triangle as shown in Fig. 2.22.3.
- Thus for the R-L-C circuit with $X_L < X_C$, the current leads the resultant voltage by φ radians. Therefore if the instantaneous applied voltage is $v = V_m \sin \omega t$ then the instantaneous current is represented by,

$$i = I_m \sin(\omega t + \phi) \qquad ...(2.22.2)$$

- RLC series circuit is equivalent to series R-C circuit, because the circuit current leads the source voltage by an angle φ.

2.22.5 Phasor Diagram for : $X_L = X_C$:

- If $X_L = X_C$ then $V_L = V_C$, and they are 180° out of phase with each other. Therefore they will cancel each other completely and their resultant will have a zero value.
- Hence the resultant (supply) voltage is $V = V_R$ and it will be in phase with I as shown in the phasor diagram of Fig. 2.22.4(b). As V and I are in phase the R-L-C circuit is equivalent to a resistive circuit.



(A-664) **Fig. 2.22.4 : Phasor diagram for R-L-C circuit with $X_L = X_C$**

- From the phasor diagram, the resultant V is given by,

$$V = V_R = IR \qquad ...(2.22.3)$$

or

$$V = I \times |Z| \qquad ....(2.22.4)$$

Because $|Z| = R$

- Thus the impedance is purely resistive for $X_L = X_C$. The instantaneous voltage $v = V_m \sin \omega t$ and $i = I_m \sin \omega t$, because they are in phase.

- The RLC series circuit is purely resistive for $X_L = X_C$ and the circuit current will be in phase with the source voltage.
- The power factor of RLC circuit is unity (1) for $X_L = X_C$.

2.22.6 Impedance of Series LCR Circuit :

The general expression for the impedance Z of the series L.C.R. circuit is as follows :

$$Z = R + jX \qquad ...(2.22.5)$$

where X = total reactance of the circuit = $(X_L - X_C)$

$$\therefore Z = R + j(X_L - X_L) \qquad ...(2.22.6)$$

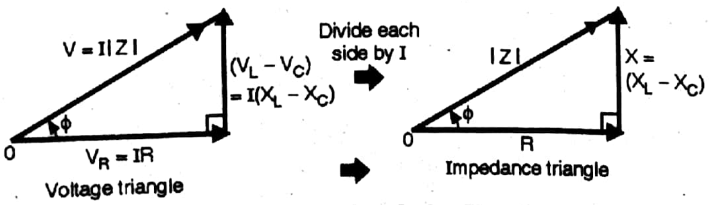
$$\therefore |Z| = \sqrt{R^2 + X^2} \text{ and } \phi = \tan^{-1}[X / R]$$

- If $X_L > X_C$ then X is positive and the L.C.R. circuit is inductive.
- If $X_L < X_C$ then X is positive and L.C.R. circuit is capacitive.
- If $X_L = X_C$ then X is zero and L.C.R. circuit is resistive.

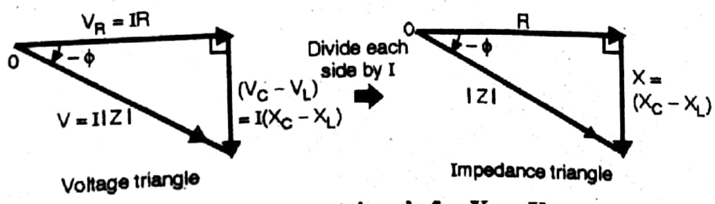
2.22.7 Impedance Triangle for an LCR Circuit :

- We know that the impedance triangle can be obtained if we divide each side of the voltage triangle by I.
- The impedance triangles for the conditions $X_L > X_C$ and $X_L < X_C$ are different and are as shown in Fig. 2.22.5(a) and (b) respectively.
- For the impedance triangles in both the cases shown in Fig. 2.22.5(a) and (b).

$$\left.\begin{aligned} R &= |Z| \cos\phi \\ \text{and} \quad X &= |Z| \sin\phi \end{aligned}\right\} \qquad ...(2.22.7)$$



(a) Impedance triangle for $X_L > X_C$



(b) Impedance triangle for $X_L < X_C$

(A-665) **Fig. 2.22.5**

2.22.8 Power Supplied to the R.L.C. Circuit :

- The average power supplied to the R.L.C. series circuit is given by :

$$P_{av} = [\text{Power consumed by R}] + [\text{Power consumed by L}] + [\text{Power consumed by C}]$$

- But the pure inductor and capacitor does not consume any power.

$$\therefore P_{av} = \text{Power consumed by } R = I^2 R = I \cdot V_R \qquad ...(2.22$$

From the voltage triangle,

$$V_R = V\cos\phi,$$

$$\therefore P_{av} = VI\cos\phi \text{ Watts.} \qquad ...(2.2$$

- In this expression, V cos ϕ represents the in phase component (X-component of V) of V, assuming current phasor to be the reference phasor.

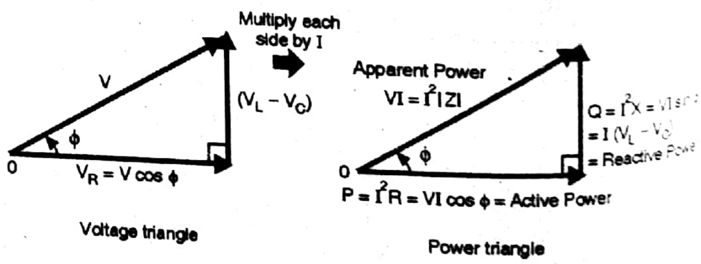
$$\therefore P_{av} = \text{In phase component of V} \times \text{Current I}$$
$$= V\cos\phi \times I = VI\cos\phi$$

2.22.9 Power Triangle :

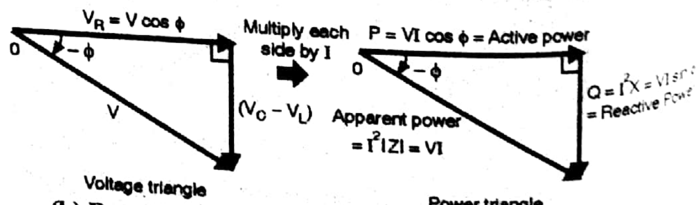
MSBTE Questions

Q. 1 Draw vector diagram, impedance triangle and power triangle for series R-L-C circuit when connected to single phase a.c. supply for the condition $X_L < X_C$. **(S-16, 2 Marks)**

- The power triangle is obtained from the impedance triangle by multiplying each side by I. The power triangles for $X_L > X_C$ and $X_L < X_C$ are as shown in Fig. 2.22.6(a) and (b).



(a) Power triangle for LCR series circuit with $X_L > X_C$



(b) Power triangle for LCR series circuit with $X_L < X_C$

(A-666) **Fig. 2.22.6 : Power triangles for LCR series circuit**

2.22.10 Various Powers and Power Factors of RLC Series Circuit :

Active power :

$$P = VI \cos \phi$$

- This is the expression for the active power. An RLC series circuit can be equivalent to RL series ($X_L > X_C$) or RC series ($X_C > X_L$) or resistive ($X_L = X_C$).
- Hence the power angle ϕ can be negative, positive or zero.

Reactive power :

The expression for reactive power is,

$$Q = VI \sin\phi$$

Total power :

Total power (apparent power) is equal to the vector sum of the active and reactive powers. That means,

$$S = \sqrt{P^2 + Q^2} = [\,(VI)^2 \cos^2 \phi + (VI)^2 \sin^2 \phi\,]^{1/2}$$

$$= VI \text{ Volt-Amp.}$$

Power factor :

- The power factor $\cos \phi$ will have different values and nature depending on the values of R, X_L and X_C.
- Table 2.22.1 summarizes the values of P.F. for different values of R, X_L and X_C.

Table 2.22.1

Sr. No.	Case	Nature of RLC circuit	P.F.
1.	$X_L > X_C$	R-L series	Lagging and less than 1.
2.	$X_C > X_L$	R-C series	Leading and less than 1.
3.	$X_L = X_C$	Purely resistive	1.

2.22.11 Figure of Merit or Q-factor :

- A resistor cannot store energy but L and C are capable of storing energy.
- The quality of the RLC series circuit is measured in terms of the efficiency of L and C to store energy.
- And the efficiency of L and C to store energy is measured in terms of a factor called Quality factor or Q factor.
- The Q-factor thus represents the efficiency of the resonant circuit to store energy. Hence it is also called as figure of merit of the RLC series circuit.

Definition :

- The Q factor is defined as the ratio of energy stored per cycle to the energy lost (or dissipated) per cycle.
- Mathematically Q can be expressed as,

$$Q = 2\pi \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}}$$

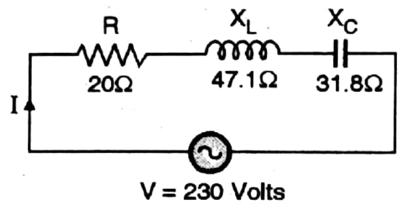
- The Q factor of an RLC series circuit is given by,

$$Q = \frac{X}{R} \qquad \text{where} \quad X = (X_L - X_C)$$

Ex. 2.22.1 : A coil having a resistance of 20 Ω and inductive reactance of 47.1 Ω is connected in series with a capacitor of reactance 31.8 Ω across an ac supply of 230 V. Determine :

1. Current drawn from the supply.
2. Active and reactive components of current.
3. Power factor.
4. Voltage across the coil.

Soln. :



(A-1782) **Fig. P. 2.22.1**

1. The current drawn from the supply :

$$I = \frac{V}{Z}$$

$$\therefore \quad Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

$$= \sqrt{(20)^2 + (47.1 - 31.8)^2}$$

$$= 25.18\ \Omega$$

$$\therefore \quad I = \frac{V}{Z} = \frac{230}{25.18} = 9.134 \text{ amp} \quad \text{...Ans.}$$

2. Power factor :

$$\cos \phi = \frac{R}{Z} = \frac{20}{25.18}$$

$$= 0.794 \text{ (lagging)} \quad \text{...Ans.}$$

$$\text{and} \quad \phi = 37.43$$

3. Active and reactive components of current :

$$\text{Active component} = I \cos \phi = 9.134 \times 0.794$$

$$= 7.252 \text{ Amp.}$$

Reactive component $= I \sin\phi = 9.134 \times 0.607$

$= 5.552$ Amp.

4. Voltage across the coil :

$$V_L = I X_L = 9.134 \times 47.1$$

$$= 430.2 \text{ V} \quad \text{...Ans.}$$

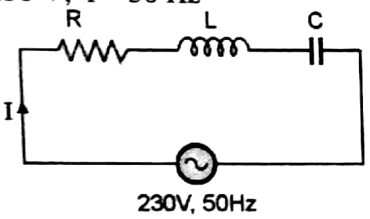
Ex. 2.22.2 : An ac circuit consists of 100 Ω resistor, 0.8 H inductor and 50 μF capacitor all in series. A voltage of 230 V, 50 Hz is applied to series combination. Determine :

1. Power factor
2. Power consumed.

Soln. :

Given : $R = 100\,\Omega,\ L = 0.8\text{ H},\ C = 50\,\mu\text{F}$

$V = 230\text{ V},\ f = 50\text{ Hz}$



(A-3399) **Fig. P. 2.22.2**

1. Power factor :

$$\cos\phi = \frac{R}{Z}$$

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 0.8 = 251.2\,\Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times 3.14 \times 50 \times 50 \times 10^{-6}} = 63.69\,\Omega$$

$$\therefore \quad Z = \sqrt{(100)^2 + (251.2 - 63.69)^2} = 212.50\,\Omega$$

$$\therefore \quad \cos\phi = \frac{R}{Z} = \frac{100}{212.50} = 0.470 \text{ (lagging)} \quad \text{...Ans.}$$

2. Power consumed :

$$P = V I \cos\phi$$

$$I = \frac{V}{Z} = \frac{230}{212.50} = 1.0823 \text{ Amp}$$

$$P = 230 \times 1.0823 \times 0.47$$

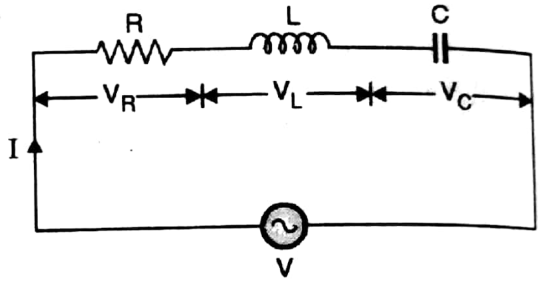
$$= 116.99 \text{ W} \quad \text{...Ans.}$$

Ex. 2.22.3 : A voltage source v(t) = 4000 sin 1000 t is applied to R-L-C series circuit where R = 100 Ω, L = 2 H and C = 100 μF. Find voltage across resistor, inductor and capacitor. Also determine expression for current through the circuit and power consumed by the circuit.

Soln. :

Given : $v(t) = 4000 \sin 1000\,t,\ R = 100\,\Omega,\ L = 2\text{ H},$

$C = 100\,\mu\text{F}$



(A-3400) **Fig. P. 2.22.3**

1. Voltage across resistor, inductor and capacitor :

$$X_L = \omega L = 1000 \times 2 = 2\text{ k}\Omega$$

$$\omega = 2\pi f = 1000$$

$$\therefore \quad f = \frac{1000}{2\pi} = 159.15\text{ Hz}$$

$$\therefore \quad X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} = \frac{1}{1000 \times 100 \times 10^{-6}}$$

$$= 10\,\Omega$$

$$I = \frac{V}{Z}$$

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(100)^2 + (2000 - 10)^2}$$

$$\therefore \quad Z = 1992.51\,\Omega$$

$$= 100 + j\,1990 = 1992.51 \angle -87.12^\circ\,\Omega$$

$$I = \frac{V}{Z} = \frac{4000/\sqrt{2} \angle 0}{1992.51 \angle 87.12}$$

$$= 1.42 \angle -87.12^\circ \text{ Amp.}$$

$$\therefore \quad V_R = IR = 141.95 \angle -87.12 \text{ Volts} \quad \text{...Ans.}$$

$$V_L = I X_L = 2840 \angle -87.12 \text{ Volts} \quad \text{...Ans.}$$

$$\therefore \quad V_C = I X_C = 14.2 \angle -87.12 \text{ Volts} \quad \text{...Ans.}$$

2. Expression for current through circuit :

$$\therefore \quad I_m = I \times \sqrt{2} = 2.01 \angle -87.12^\circ \text{ Amp.}$$

$$\therefore \quad i(t) = I_m \sin(\omega t + \phi)$$

$$= 2.01 \sin(1000t - 87.12) \text{ A}$$

3. Power consumed by the circuit :

$$\therefore \quad P = V I \cos\phi$$

$$\text{But } \cos\phi = \frac{R}{Z} = \frac{100}{1992.51} = 0.0501$$

$$\therefore \quad P = \frac{4000}{\sqrt{2}} \times 1.4210 \times 0.0501$$

$$= 201.8 \text{ Watts} \quad \text{...Ans.}$$

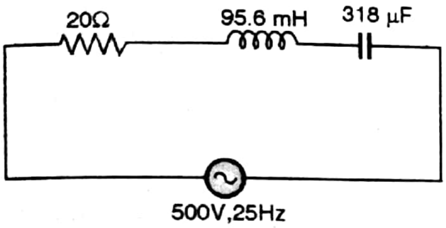
Ex. 2.22.4 : A circuit consists of a resistance of 20 Ω in series with an inductance of 95.6 mH and a capacitor of 318 μF. It is connected to a 500 V, 25 Hz supply. Find the current in the circuit and the power factor.

Soln. :

Given : $R = 20\ \Omega$, $L = 95.6$ mH, $C = 318\ \mu F$,

$V = 500$ V, $f = 25$ Hz

To find : 1. Current 2. Power factor



(A-1783) **Fig. P. 2.22.4**

Step 1 : Circuit current I :

Inductive reactance $X_L = 2\pi f L = 2 \times \pi \times 25 \times 95.6 \times 10^{-3}$

$= 15\ \Omega$

Capacitive reactance $X_C = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2\pi \times 25 \times 318 \times 10^{-6}} = 20\ \Omega$

Impedance (Z) $= \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(20)^2 + (15 - 20)^2} = 20.61\ \Omega$

Current $I = \dfrac{V}{Z} = \dfrac{500}{20.61} = 24.26$ A ...**Ans.**

Step 2 : Power factor :

Power factor $(\cos\phi) = \dfrac{R}{Z} = \dfrac{20}{20.61}$

$= 0.97$ (leading) ...**Ans.**

Ex. 2.22.5 : A voltage of e (t) = 100 sin 314 t is applied to a series circuit consisting of 10 Ω resistor, 0.0318 H inductor and capacitor of 63.6 μF.

Find :

1. Expression for i(t)
2. Phase angle between voltage and current
3. Power factor
4. Active power consumed.

Soln. :

Given : $e(t) = 100 \sin 314\, t$, $R = 10\ \Omega$,

$L = 0.0318$ H, $C = 63.6\ \mu F$

1. Current i(t) :

$\omega = 2\pi f = 314, \therefore f = 50$ Hz

Inductive reactance $X_L = 2\pi f L = 2\pi \times 50 \times 0.0318$

$= 9.99\ \Omega$

Capacitive reactance $X_C = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2\pi \times 50 \times 63.6 \times 10^{-6}} = 50\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{10^2 + (9.99 - 50)^2}$

$= 41.24\ \Omega$

Maximum current $I_m = \dfrac{V_m}{Z} = \dfrac{100}{41.24} = 2.42$ A ...**Ans.**

2. Phase angle :

$\phi = \cos^{-1} R/Z = \cos^{-1}\left(\dfrac{10}{41.24}\right) = 75.96$

3. Power factor :

$\cos\phi = \cos 75.96 = 0.24$ (leading) ...**Ans.**

$\cos\phi$ is leading because $X_C > X_L$.

4. Active power consumed :

$P = V I \cos\phi = \dfrac{100}{\sqrt{2}} \times \dfrac{2.42}{\sqrt{2}} \times 0.24$

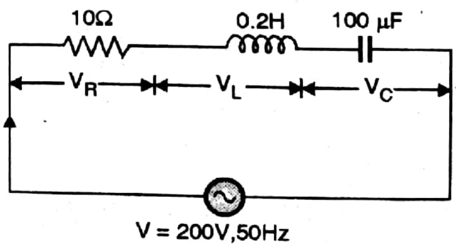
$= 29.04$ Watts ...**Ans.**

Ex. 2.22.6 : A resistance of 10 Ω, an inductance of 0.2 H and capacitance of 100 μF are connected in series across a 200 V, 50 Hz supply mains. Determine :

1. Impedance
2. Current
3. Voltage across R, L and C
4. P.F and angle of lag
5. Power consumed in watts and VA

Soln. :

Given :



(A-1785) **Fig. P. 2.22.6**

To find :

1. Impedance Z :

Inductive reactance $X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

Capacitive reactance $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$

$$= 31.83\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{10^2 + (62.83 - 31.83)^2}$$

$$= 32.57\ \Omega \qquad \text{...Ans.}$$

2. Current :

$$I = \frac{V}{Z} = \frac{200}{32.57} = 6.14 \text{ Amp.} \qquad \text{...Ans.}$$

3. Voltage across R, L and C :

Voltage across R = V_R = I.R. = 61.4 V

Voltage across L = $I \cdot X_L = 6.14 \times 62.8 = 385.77$ V

Voltage across C = $I \cdot X_C = 6.14 \times 31.83 = 195.43$ V

4. Power factor and angle of lag :

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{10}{32.57}$$

$$= 0.307 \text{ (lagging)} \qquad \text{...Ans.}$$

$$\text{Angle of lag } \phi = \cos^{-1}(0.307) = 72.1^\circ$$

5. Power consumed in watts and VA :

$$P = VI\cos\phi \text{ Watts}$$

$$= 200 \times 6.14 \times 0.307$$

$$= 376.99 \text{ Watts} \qquad \text{...Ans.}$$

$$S = VI \quad \text{...Volt Ampere}$$

$$= 200 \times 6.14 = 1230 \text{ VA} \qquad \text{...Ans.}$$

Ex. 2.22.7 : A series circuit connects of a non-inductive resistor of 10 ohm, an inductance of 0.159 H and a capacitance of 100 μF. The circuit is energized from 230 V, 50 Hz mains. Calculate current and voltage across resistor and capacitor.

Soln. :

Given : $R = 10\ \Omega$, $L = 0.159$ H, $C = 100\ \mu F$, $V = 230$ Volts, $f = 50$ Hz.

1. Inductive reactance,

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.159$$

$$= 49.95\ \Omega$$

2. Capacitive reactance,

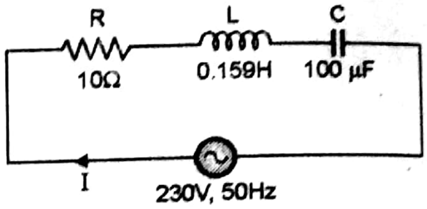
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$$

3. Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(10)^2 + (49.95 - 31.83)^2}$$

$$Z = 20.69\ \Omega$$



(A-1786) **Fig. P. 2.22.7 : Given circuit**

4. Current, $I = \frac{V}{Z} = \frac{230}{20.69} = 11.11$ Amp. ...An

5. Voltage across resistor,

$$V_R = I \times R = 11.11 \times 10 = 111.13 \text{ V} \quad \text{...An}$$

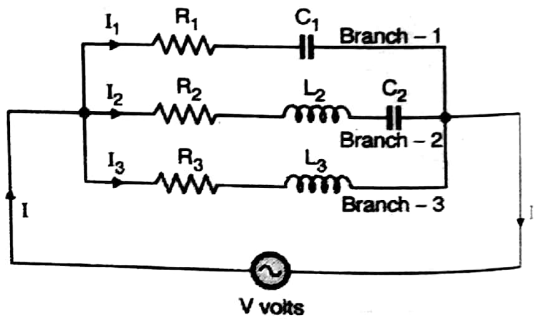
6. Voltage across capacitor,

$$V_C = I X_C = 11.11 \times 31.830$$

$$= 353.63 \text{ V} \qquad \text{...Ans}$$

2.23 A.C. Parallel Circuit :

- A parallel ac circuit has two or more series circ... connected in parallel with each other across a comm... alternating voltage source as shown in Fig. 2.23.1.



(A-691) **Fig. 2.23.1 : A parallel ac circuit**

- Each series circuit is called as a branch in the paralle circuit.
- The parallel circuit of Fig. 2.23.1 consists of three impedances connected in parallel with each other and across an ac supply voltage of V Volts.
- Let the current flowing through the three branches be I_1, I_2 and I_3 respectively. As the voltage drop across all the branches is the same i.e. V Volts, these currents are given by the following expressions :

$$I_1 = \frac{V}{Z_1},\ I_2 = \frac{V}{Z_2} \text{ and } I_3 = \frac{V}{Z_3}$$

- Applying KCL and using the phasor addition we get,

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 \quad \text{...(phasor addition)}$$

Substituting the values we get,

$$\frac{\bar{V}}{\bar{Z}} = \frac{\bar{V}}{\bar{Z}_1} + \frac{\bar{V}}{\bar{Z}_2} + \frac{\bar{V}}{\bar{Z}_3} \quad \text{...(2.23.1)}$$

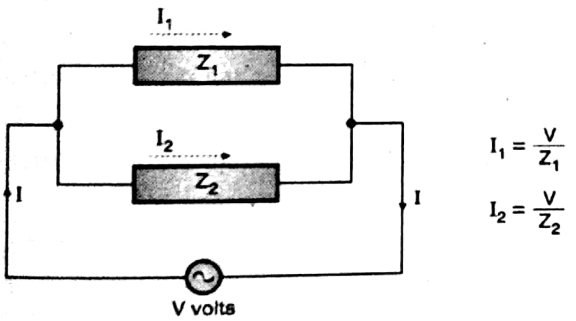
$$\therefore \frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3} \quad \text{...(2.23.2)}$$

- Equation (2.23.2) is the expression for the **equivalent impedance** of the parallel circuit. If "n" branches are connected in parallel, then the equivalent impedance is given by,

$$\frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \dots + \frac{1}{\bar{Z}_n} \quad \text{...(2.23.3)}$$

2.23.1 Two Impedances in Parallel :

- Fig. 2.23.2 shows a parallel ac circuit with two impedances connected in parallel. Let the currents through Z_1 and Z_2 be denoted by I_1 and I_2 respectively.



(A-692) **Fig. 2.23.2 : Two impedances in parallel**

Expressions for I_1 and I_2 :

- We can apply the rule of current division in order to obtain the expressions for the individual currents as follows :

$$\bar{I}_1 = \frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} \times \bar{I} \quad \text{...(2.23.4)}$$

and $$\bar{I}_2 = \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2} \times \bar{I} \quad \text{...(2.23.5)}$$

- Where I is the total current supplied by the source. Note that all the quantities in the above expressions are phasors.

2.24 Multiplication and Division of Impedances :

- To obtain the solution to an ac circuit example, we have to perform the multiplication and division operations on impedances.
- As impedance is a phasor quantity, it has angle ϕ associated with it. When we perform multiplication or division of impedances, we have to obtain the effective value of ϕ as explained below.

2.24.1 Multiplication of Impedances :

Let $Z_3 = Z_1 \times Z_2$

where, $Z_1 = (R_1 + j X_1), Z_2 = (R_2 + j X_2)$

And $Z_3 = (R_3 + j X_3)$

$$\therefore (R_3 + j X_3) = (R_1 + j X_1) \cdot (R_2 + j X_2) \quad \text{...(2.24.1)}$$

- Converting all the impedances into their polar forms we get,

$$|Z_3| \angle \phi_3 = |Z_1| \angle \phi_1 \cdot |Z_2| \angle \phi_2$$

$$\therefore |Z_3| \angle \phi_3 = |Z_1| \times |Z_2| \angle \phi_1 + \phi_2 \quad \text{...(2.24.2)}$$

- Comparing the LHS and RHS we get,

$|Z_3| = |Z_1| \times |Z_2|$ and

$$\phi_3 = \phi_1 + \phi_2 \quad \text{...(2.24.3)}$$

- Thus when two impedances are multiplied, the resultant magnitude is the product of their individual magnitudes and the resultant phase angle is equal to the addition of individual angles.
- The result can be extended for the multiplication of "n" number of impedances as follows :

$$Z_1 \times Z_2 \times \dots \times Z_n = |Z_1| \times |Z_2| \times \dots \times |Z_n| \angle \phi_1 + \phi_2 + \dots + \phi_n \quad \text{...(2.24.4)}$$

2.24.2 Division of Impedances :

Let $Z_3 = \frac{Z_1}{Z_2}$

- Writing them in the polar form we get,

$$|Z_3| \angle \phi_3 = \frac{|Z_1| \angle \phi_1}{|Z_2| \angle \phi_2}$$

Table 2.24.1 : Summary of various operations on impedances

Sr. No.	Operation	Form	Equivalent magnitude	Equivalent phase angle
1.	Addition $(Z_1 + Z_2)$	Rectangular $(R_1 + j X_1) + (R_2 + j X_2)$	$\lvert Z_1 \rvert + \lvert Z_2 \rvert$	$\tan^{-1}\left(\frac{X}{R}\right)$
2.	Subtraction $(Z_1 - Z_2)$	Rectangular $(R_1 + j X_1) - (R_2 + j X_2)$	$\lvert Z_1 \rvert + \lvert Z_2 \rvert$	$\tan^{-1}\left(\frac{X}{R}\right)$
3.	Multiplication $(Z_1 \times Z_2)$	Polar $\lvert Z_1 \rvert \angle \phi_1 \times \lvert Z_2 \rvert \angle \phi_2$	$\lvert Z_1 \rvert \times \lvert Z_2 \rvert$	$\phi_1 + \phi_2$
4.	Division (Z_1 / Z_2)	Polar $\frac{\lvert Z_1 \rvert \angle \phi_1}{\lvert Z_2 \rvert \angle \phi_2}$	$\frac{\lvert Z_1 \rvert}{\lvert Z_2 \rvert}$	$\phi_1 - \phi_2$

$$\therefore \lvert Z_3 \rvert \angle \phi_3 = \frac{\lvert Z_1 \rvert}{\lvert Z_2 \rvert} \angle \phi_1 - \phi_2 \qquad ...(2.24.5)$$

- Comparing the LHS and RHS we get,

$$\lvert Z_3 \rvert = \frac{\lvert Z_1 \rvert}{\lvert Z_2 \rvert} \text{ and } \phi_3 = \phi_1 - \phi_2 \qquad ...(2.24.6)$$

- Thus when two impedances are involved in the division operation, the resultant magnitude is obtained by taking the ratio of the individual magnitudes and the resultant phase angle is equal to the difference between individual phase angles.
- For the addition or subtraction of impedances, we have to use the rectangular form whereas for the multiplication or division of the impedances we must use the polar form.
- The process of addition, subtraction, multiplication and division of impedance has been summarized in Table 2.24.1.

2.25 Parallel AC Circuits :

- Depending on the way in which R, L and C are connected, we can get a variety of AC parallel circuits.
- However in the subsequent sections we are going to discuss the following parallel AC circuits :
  1. Resistance in parallel with pure inductor (R-L parallel).
  2. Resistance in parallel with pure capacitor (R-C parallel).
  3. Series combination of R and L in parallel with a capacitor in parallel.

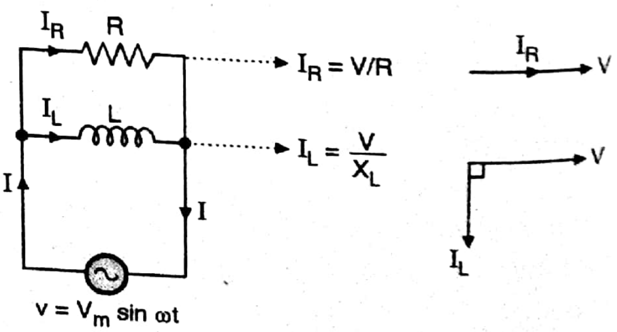
2.26 Resistance in Parallel with Pure Inductance :

- The parallel R-L circuit is as shown in Fig. 2.26.1. The ac voltage source of instantaneous voltage $v = V_m \sin \omega t$ has been connected across the parallel combination of L and R.
- Let R ohms be the value of resistance and L henries be the value of pure inductance.
- Let I_R be the current through the resistance and I_L be the current through the inductor. The total current is I amperes.
- The individual currents are as follows :

$$I_R = \frac{V}{R} \text{ (}I_R\text{ is in phase with V)}$$

$$I_L = \frac{V}{X_L} \text{ (}I_L\text{ lags behind V by }90^\circ\text{)}$$

where V = rms value of supply voltage.



(A-3412) Fig. 2.26.1 : R-L parallel circuit

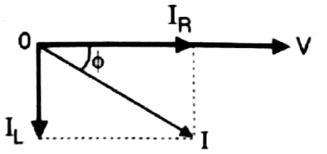
Phasor diagram :

- The supply current I is equal to the phasor addition of I_R and I_L.

$$\therefore \quad \bar{I} = \bar{I}_R + \bar{I}_L \text{....(Phasor addition)} \quad ...(2.26.1)$$

$$= \frac{V}{R} + \frac{V}{X_L} \quad ...(2.26.2)$$

- This addition is shown as phasor diagram in Fig. 2.26.2. Note that the supply voltage phasor V is treated as the reference phasor.



(A-3413) **Fig. 2.26.2 : Phasor diagram of L-R parallel circuit**

Impedance :

Impedance of R-L parallel circuit is obtained as follows :

$$Z_1 = R + j0 = R \angle 0° \,\Omega$$

and $$Z_2 = 0 + jX_L = X_L \angle 90° \,\Omega$$

$$\therefore \quad Z = Z_1 \parallel Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(R \angle 0°)(X_L \angle 90°)}{(R + j0 + 0 + jX_L)}$$

$$= \frac{R X_L (\angle 90°)}{R + jX_L}$$

$$\therefore \quad Z = \frac{R X_L (\angle 90°)}{\sqrt{R^2 + X_L^2} \angle \tan^{-1}(X_L/R)}$$

Let $\sqrt{R^2 + X_L^2} = |Z_s|$ and $\angle \tan^{-1}(X_L/R) = \phi_s$

$$\therefore \quad Z = \frac{RX_L}{|Z_s|} \angle (90° - \phi_s) \quad ...(2.26.3)$$

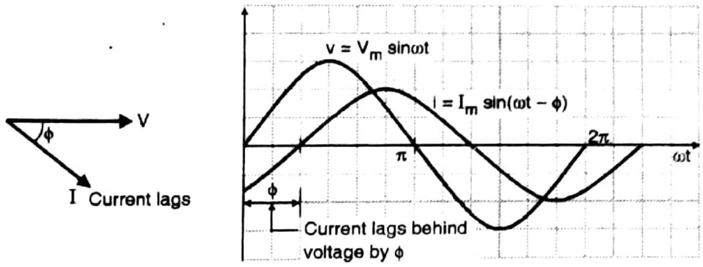
Voltage and current waveforms :

- From the phasor diagram it is evident that the supply current I lags behind the supply voltage V by a phase angle ϕ.

$$\therefore \quad i = I_m \sin(\omega t - \phi)$$

and $$v = V_m \sin \omega t$$

- The voltage and current waveforms are as shown in Fig. 2.26.3.



(a) Phasor diagram (b) Voltage and current waveforms

(A-3414) **Fig. 2.26.3 : RL parallel circuit**

Power factor :

- Power factor of R-L parallel circuit is given by,

$$PF = \cos \phi$$

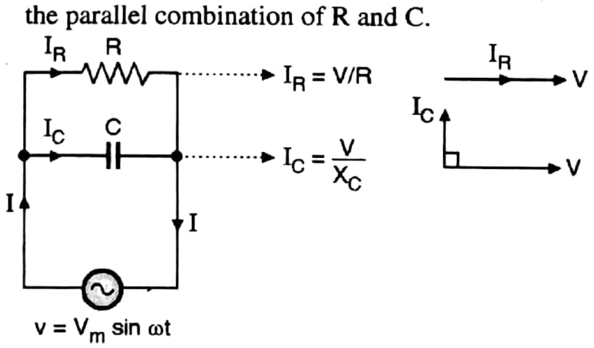
where ϕ = Phase angle between supply voltage (V) and supply current (I).

- PF of the RL parallel circuit is lagging PF because current (I) lags behind the supply voltage (V) by an angle ϕ.

2.27 Resistance in Parallel with Pure Capacitance :

Circuit Diagram :

- The parallel R-C circuit with an ac excitation is shown in Fig. 2.27.1. A pure capacitor of value C is connected in parallel with a resistor R and a voltage source of instantaneous voltage $v = V_m \sin \omega t$ is connected across the parallel combination of R and C.



(A-3415) **Fig. 2.27.1 : R–C parallel circuit**

- Let R be the value of resistance and C Farads be the value of pure capacitance.
- Let I_R be the current through the resistance and I_C be the current through the capacitor. The total current is I amperes.
- The individual currents are as follows :

$$I_R = \frac{V}{R} \qquad (I_R \text{ is in phase with V})$$

$$I_C = \frac{V}{X_C} \qquad (I_C \text{ leads V by } 90°)$$

where V = rms value of supply voltage.

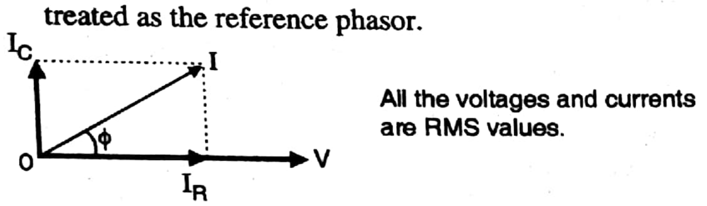
Phasor diagram :

- The supply current I is equal to the phasor addition of I_R and I_C.

$$\therefore \quad \bar{I} = \bar{I}_R + \bar{I}_C \quad \text{....(Phasor addition)...(2.27.1)}$$

$$= \frac{V}{R} + \frac{V}{X_C} \quad \text{...(2.27.2)}$$

- This addition is shown in the form of a phasor diagram in Fig. 2.27.2. Note that the supply voltage phasor V is treated as the reference phasor.



(A-3416) **Fig. 2.27.2 : Phasor diagram of R-C parallel circuit**

Impedance :

Impedance of R-C parallel circuit is obtained as follows :

$$Z_1 = R + j0 = R \angle 0° \ \Omega$$

$$\text{and} \quad Z_2 = 0 - j X_C = X_C \angle -90° \ \Omega$$

$$\therefore \quad Z = Z_1 \parallel Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(R \angle 0°)(X_C \angle -90°)}{(R + j0 + 0 - j X_C)}$$

$$= \frac{R X_C (\angle -90°)}{R - jX_C}$$

$$\therefore \quad Z = \frac{X_C (\angle -90°)}{\sqrt{R^2 + X_C^2} \angle \tan^{-1} (-X_C/R)}$$

$$\text{Let} \quad \sqrt{R^2 + X_C^2} = |Z_s| \text{ and } \angle - \tan^{-1} (X_C/R) = -\phi_s$$

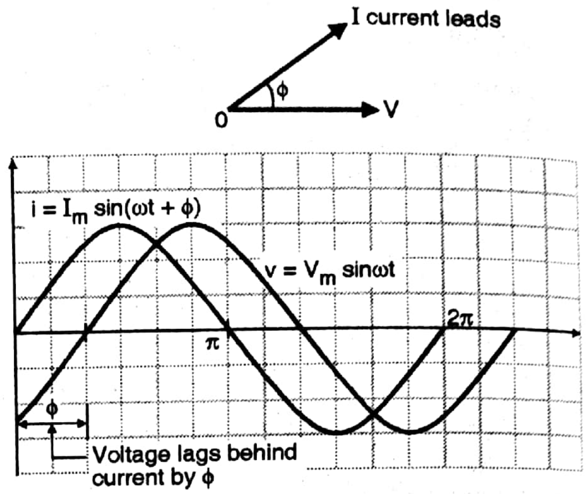
$$\therefore \quad Z = \frac{R X_L}{|Z_s|} \angle (90° + \phi_s) \quad \text{...(2.27.3)}$$

Voltage and current waveforms :

- From the phasor diagram it is evident that the supply current I leads the supply voltage V by a phase angle ϕ.

$$\therefore \quad i = I_m \sin (\omega t + \phi) \quad \text{and} \quad v = V_m \sin \omega t$$

- The voltage and current waveforms are as shown in Fig. 2.27.3.



(A-3417) **Fig. 2.27.3 : RC parallel circuit**

Power factor :

- Power factor of RC parallel circuit is given by,

$$PF = \cos \phi$$

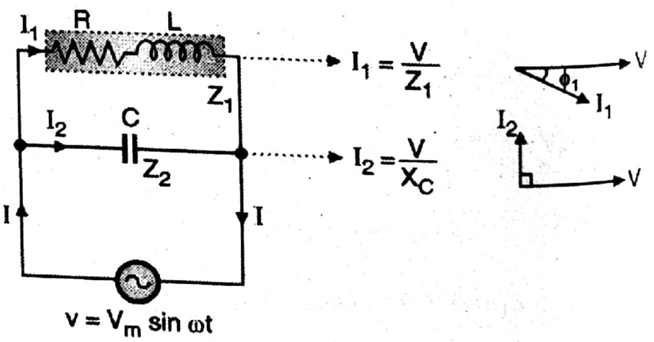
where ϕ = Angle between supply voltage (V) and supply current (I).

- The power factor of RC parallel circuit is leading because current (I) leads voltage (V) by an angle ϕ.

2.28 Series Combination of R and L in Parallel with Capacitor :

Circuit Diagram :

- Fig. 2.28.1 shows the ac parallel circuit in which a series combination of R and L is connected in parallel with a capacitor and across the ac supply voltage.



(A-3418) **Fig. 2.28.1 : RL series circuit in parallel with a capacitor**

- Let Z_1 be the impedance of the RL series combination.

$$\therefore \quad Z_1 = R + jX_L = |Z_1| \angle \phi_1$$

$$\text{where} \quad |Z_1| = \sqrt{R^2 + X_L^2} \text{ and } \phi_1 = \tan^{-1} \left(\frac{X_L}{R}\right)$$

- Let Z_2 be the impedance of the branch containing the capacitor.

$$\therefore \quad Z_2 = 0 - jX_C = |Z_2| \angle \phi_2$$

where $|Z_2| = X_C$ and $\angle \phi_2 = -90°$

- Let I_1 be the current through L-R series circuit. So it will lag behind the supply voltage by an angle ϕ_1.
- Let I_2 be the current through the capacitor C. Hence it will lead the supply voltage by 90°.
- The individual currents are given by,

$$I_1 = \frac{V}{Z_1} \quad ....I_1 \text{ lags V by an angle } \phi_1.$$

$$I_2 = \frac{V}{Z_2} \quad ....I_2 \text{ leads V by an angle } 90°.$$

where V = rms value of supply voltage.

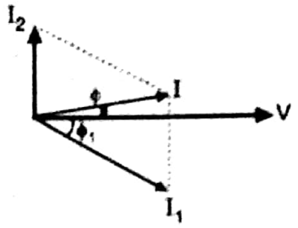
Phasor diagram :

- The supply current I is equal to the phasor addition of I_1 and I_2.

$$\therefore \quad \bar{I} = \bar{I}_1 + \bar{I}_2 \quad ....\text{(Phasor addition)}$$

$$= \frac{V}{Z_1} + \frac{V}{X_C} \quad ...(2.28.1)$$

- This addition is shown in the form of a phasor diagram in Fig. 2.28.2. Note that the supply voltage phasor V has been treated as the reference phasor.



(A-3419) **Fig. 2.28.2 : Phasor diagram of RL series in parallel with C circuit**

- Note that the value of I (supply current) and ϕ depends on the values of I_1, I_2 as well as the value of ϕ_1.
- The total current I can lead the supply voltage V, or lag behind it or be in phase with it depending on I_1, I_2 and ϕ_1 values.

Impedance :

$$Z_1 = R + jX_L = |Z_1| \angle \phi_1 \cdot \Omega, Z_2$$

$$= 0 - jX_C = X_C \angle -90° \ \Omega$$

$$\therefore \text{Total impedance } Z = Z_1 \parallel Z_2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{|Z_1| \times X_C \angle (\phi_1 - 90)°}{R + j(X_L - X_C)}$$

Power factor :

- Power factor of this circuit is given by,

$$PF = \cos \phi$$

where ϕ = Phase angle between V and I

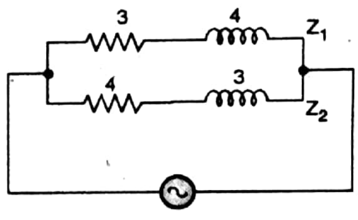
- PF of this circuit can be lagging, unity or leading depending on whether ϕ is negative, zero or positive respectively.

Ex. 2.28.1 : Impedances $Z_1 = (3 + j4)$ ohm and $Z_2 = (4 + j3)$ ohm are connected in parallel across 200 Volts, 50 Hz supply. Calculate supply current and power factor.

Soln. :

Given :

$Z_1 = (3 + j4)\ \Omega$, $Z_2 = (4 + j3)\ \Omega$, V = 200 Volts, f = 50 Hz.



(A-1787) **Fig. P. 2.28.1 : Given circuit**

1. **Supply current :**

Impedance $Z_1 = (3 + j4)\ \Omega = 5 \angle 53.13°\ \Omega$

Impedance $Z_2 = (4 + j3)\ \Omega = 5 \angle 36.86°\ \Omega$

Total impedance

$$Z_T = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$$

$$= \frac{(5 \angle 53.13) \times (5 \angle 36.86)}{(3 + j4) + (4 + j3)}$$

$$= \frac{25 \angle 89.99}{7 + j7} = \frac{25 \angle 89.99}{9.89 \angle 45}$$

$$\therefore \quad Z_T = 2.52 \angle 44.99\ \Omega$$

Supply current, $I = \frac{V}{Z_T} = \frac{200}{2.52 \angle 44.99}$

$= 79.36 \angle -44.99$ A ...Ans.

2. **Power factor of the circuit :**

$\cos \phi = \cos 44.99 = 0.707$ (lagging) ...Ans.

Ex. 2.28.2 : Find the value of $Z = \frac{(3+j4)\cdot 5\angle 30}{(6+j8)}$ in rectangular form.

Soln. :

Given : $Z = \frac{(3+j4)\cdot 5\angle 30}{(6+j8)}$

Convert first (3 + j4) and (6 + j8) in polar form.

$$Z = \frac{(5\angle 53.13)\times(5\angle 30)}{10\angle 53.13}$$

$$= \frac{25\angle 83.13}{10\angle 53.13} = 2.5\angle 30^\circ\ \Omega$$

$$Z = (2.16 + j1.25)\ \Omega \quad \text{...in rectangular form}$$

Ex. 2.28.3 : An impedance of (8 + j6) Ω is connected in parallel with an impedance of (5 – j6) Ω and this combination is supplied with alternating voltage of 200 ∠ 30°. Calculate total current from the source and power factor of the circuit.

Soln. :

Given : $V = 200\angle 30^\circ \quad Z_1 = 8 + j6 = 10\angle 36.86^\circ$

$Z_2 = 5 - j6 = 7.81\angle -50.19^\circ$

1. Total current from source :

$$\text{Total impedance} = \frac{Z_1\cdot Z_2}{Z_1 + Z_2}$$

$$= \frac{10\angle 36.86^\circ \times 7.81\angle -50.19^\circ}{8 + j6 + 5 - j6}$$

$$= \frac{78.1\angle -13.33^\circ}{13\angle 0^\circ}$$

$$= (6\angle -13.33^\circ)\ \Omega$$

$$\therefore \text{Total current} = \frac{V}{Z} = \frac{200\angle 30^\circ}{6\angle -13.33^\circ}$$

$$= 33.33\angle 43.33^\circ\ A \quad \text{...Ans.}$$

2. Power factor of the circuit :

$$\cos\phi = \cos(43.33)$$

$$= 0.727\ (\text{leading}) \quad \text{...Ans.}$$

Ex. 2.28.4 : If A = 4 + j7, B = 8 + j9; and C = 5 – j6 then calculate :

1. $\frac{A+B}{C}$ 2. $\frac{A\times B}{C}$

3. $\frac{A+B}{B+C}$ 4. $\frac{B-C}{A}$

Soln. :

Given : $A = 4 + j7, \quad B = 8 + j9 \quad \text{and} \quad C = 5 - j6$

1. $$\frac{A+B}{C} = \frac{4+j7+8+j9}{5-j6} = \frac{12+j16}{5-j6}$$

$$= \frac{20\angle 53.13}{7.81\angle -50.19}$$

$$= 2.56\angle 103.32^\circ \quad \text{...Ans.}$$

2. $$\frac{A\times B}{C} = \frac{(4+j7)\times(8+j9)}{5-j6}$$

$$= \frac{32+92j+j^2 63}{5-j6}$$

$$= \frac{-31+j92}{5-j6} \quad (\because j^2 = -1)$$

$$= \frac{97.08\angle 108.62}{7.81\angle -50.19}$$

$$= 12.43\angle 158.81^\circ \quad \text{...Ans.}$$

3. $$\frac{A+B}{B+C} = \frac{4+j7+8+j9}{8+j9+5-j6} = \frac{12+j16}{13+j3}$$

$$= \frac{20\angle 53.13}{13.34\angle 12.99}$$

$$= 1.49\angle 40.14 \quad \text{...Ans.}$$

4. $$\frac{B-C}{A} = \frac{8+j9-(5-j6)}{4+j7}$$

$$= \frac{8+j9-5+j6}{4+j7} = \frac{3+j15}{4+j7}$$

$$= \frac{15.29\angle 78.69^\circ}{8.06\angle 60.25^\circ}$$

$$= 1.89\angle 18.44^\circ \quad \text{...Ans.}$$

Ex. 2.28.5 : An inductor of 0.5 H inductance and 90 Ω resistance is connected in parallel with a 20 μF capacitor. Find :

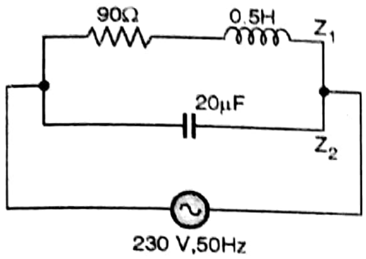
1. The total current
2. Power factor of parallel circuit
3. Total power taken from the source :

Draw the vector diagram, A voltage of 230 V at 50 Hz is maintained across this circuit.

Soln. :

Given : $L = 0.5\ H, \quad R = 90\ \Omega, \quad C = 20\ \mu F,$

$f = 50\ Hz, \quad V = 230\ \text{Volts}.$



(A-1800) **Fig. P. 2.28.5(a)**

1. **Total current I :**

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157.1\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 20 \times 10^{-6}}$$

$$= 159.15\ \Omega$$

$\therefore$ Impedance $Z_1 = 90 + j\ 157.1 = 181 \angle 60.19^\circ\ \Omega$

Impedance $Z_2 = -j\ 159.15 = 159.15 \angle -90^\circ\ \Omega$

$\therefore$ Total impedance

$$Z_T = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$$

$$= \frac{(181 \angle 60.19) \times (159.15 \angle -90)}{(90 + j\ 157.1) + (0 - j\ 159.15)}$$

$$= \frac{28806.15 \angle -29.81}{900 - j\ 2.05}$$

$$= \frac{28806.15 \angle -29.81}{90 \angle -1.31} = 320 \angle -28.5\ \Omega$$

$\therefore$ Total current drawn $= \frac{V}{Z} = \frac{230 \angle 0}{320 \angle -28.5}$

$$I = 0.72 \angle 28.5 \text{ Amp.}$$ **...Ans.**

2. **Power factor of the circuit :**

$$\cos \phi = \cos (28.5)$$

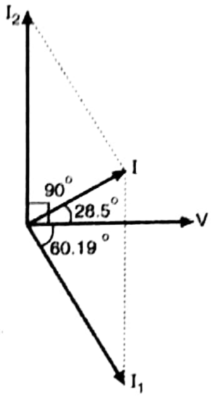
$$= 0.88 \text{ (leading)}$$ **...Ans.**

3. **Total power taken from the source :**

$$P = VI \cos \phi = 230 \times 0.72 \times 0.88$$

$$= 145.73 \text{ Watts}$$

4. **Vector diagram :**



(A-1801) **Fig. P. 2.28.5(b)**

Ex. 2.28.6 : Two coils of $R_1 = 5\ \Omega$, $L_1 = 0.03$ H and $R_2 = 3\ \Omega$, $L_2 = 0.04$ H are connected in parallel across 200 V; 50 Hz supply.

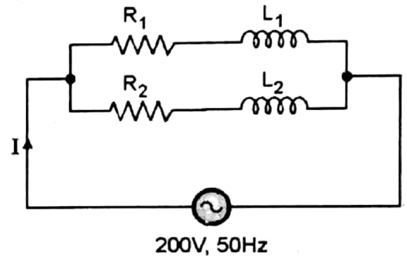
Calculate :

1. Conductance; susceptance of each coil.
2. Total current drawn by circuit.

Soln. :

Two coils : Coil 1 : $R_1 = 5\ \Omega$, $L_1 = 0.03$ H and

Coil 2 : $R_2 = 3\ \Omega$, and $L_2 = 0.04$ H.



(A-3402) **Fig. P. 2.28.6**

For coil 1 :

$$X_1 = 2\pi f L_1 = 2\ \pi f \times 0.03 = 9.424$$

$$Z_1^2 = R_1^2 + X_1^2 = (5)^2 + (9.424)^2$$

$$= 25 + 88.27 = 113.81$$

$\therefore$ Conductance $G_1 = \frac{R_1}{Z_1^2} = \frac{5}{113.81} = 0.044$ **...Ans.**

$\therefore$ Susceptance $B_1 = \frac{X_1}{Z_1^2} = \frac{9.424}{113.81} = 0.083$ **...Ans.**

$\therefore$ Admittance $Y_1 = \sqrt{G_1^2 + B_1^2}$

$$= \sqrt{(0.044)^2 + (0.083)^2}$$

$$Y_1 = \sqrt{0.00194 + 0.0069}$$

$$= \sqrt{0.00884} = 0.094$$

For coil 2 :

$$X_2 = 2\pi f L_2 = 2\ \pi f \times 0.04 = 12.57$$

$$Z_2^2 = R_2^2 + X_2^2 = 3^2 + (12.57)^2$$

$$Z_2^2 = 9 + 157.91 = 167.004$$

$\therefore$ Conductance $G_2 = \frac{R_2}{Z_2^2} = \frac{3}{167.004} = 0.018$ **...Ans.**

$\therefore$ Susceptance $B_2 = \frac{X_2}{Z_2^2} = \frac{12.57}{167.004} = 0.075$ **...Ans.**

$\therefore$ Admittance $Y_2 = \sqrt{G_2^2 + B_2^2}$

$$= \sqrt{(0.018)^2 + (0.075)^2}$$

$$Y_2 = \sqrt{0.00032 + 0.0056}$$

$$= \sqrt{0.00592} = 0.077$$

Total current drawn by circuit,

$$I = V(Y_1 + Y_2)$$

$$= 200\,(0.094 + 0.077)$$

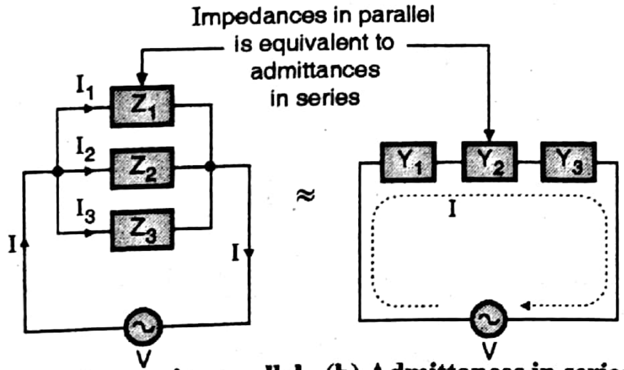
$$I = 200 \times 0.171$$

$= 34.200$ Amp **...Ans.**

2.29 The Concept of Admittance :

Definition :

- The **admittance** is defined as the reciprocal of the impedance (1/Z).
- It is denoted by Y and has units siemens (S).
- One siemens is defined as the admittance of a network having an impedance of 1 Ω.



(a) Impedances in parallel (b) Admittances in series

(A-693) **Fig. 2.29.1**

Expression for I :

- Referring to Fig. 2.29.1(a) we can write that,

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 \quad ...(2.29.1)$$

- Substituting the values of $\bar{I}_1$, $\bar{I}_2$ and $\bar{I}_3$ we get,

$$\bar{I} = \frac{\bar{V}}{\bar{Z}_1} + \frac{\bar{V}}{\bar{Z}_2} + \frac{\bar{V}}{\bar{Z}_3} \quad ...(2.29.2)$$

- But using the definition of admittance we can write that,

$$\bar{I} = \bar{V}\,\bar{Y}_1 + \bar{V}\,\bar{Y}_2 + \bar{V}\,\bar{Y}_3 \quad ...(2.29.3)$$

$$\therefore \bar{I} = \bar{V}(\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3) \quad ...(2.29.4)$$

- Hence the effective admittance $\bar{Y}$ is given by,

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 \quad ...(2.29.5)$$

- Thus the effective admittance of a parallel ac circuit is equal to the sum of the admittances of the individual branches. Hence the impedances connected in parallel are equivalent to admittances connected in series as shown.

2.29.1 Conductance and Susceptance :

- Consider the expression for the impedance in the rectangular form,

$$Z = R \pm jX$$

- Therefore the admittance Y is given by,

$$Y = \frac{1}{Z} = \frac{1}{R \pm jX}$$

- Rationalize the expression for Y as follows :

$$Y = \frac{1}{(R \pm jX)} \times \frac{R \mp jX}{(R \mp jX)} = \frac{R \mp jX}{R^2 + X^2}$$

$$= \frac{R}{(R^2 + X^2)} \mp \frac{jX}{(R^2 + X^2)}$$

But $Z^2 = (R^2 + X^2)$

$$\therefore Y = \frac{R}{Z^2} \mp \frac{jX}{Z^2} \quad ...(2.29.6)$$

Let $\frac{R}{Z^2} = G =$ Conductance and

$\frac{X}{Z^2} = B =$ Susceptance.

$$\therefore Y = G \mp jB \quad ...(2.29.7)$$

Definition of conductance (G) :

The conductance (G) is defined as the ratio of the resistance R and the squared impedance (Z^2). And it is measured in siemens.

$$G = \frac{R}{Z^2} \quad ...(2.29.7(a))$$

Definition of susceptance (B) :

- Susceptance (B) is defined as the ratio of the reactance $\mp X$ and the squared impedance (Z^2).
- It is also measured in siemens. The susceptance can be inductive or capacitive depending on the polarity of X.

$$\therefore \; B = \frac{X}{Z^2} \qquad ...(2.29.7(b))$$

- If the polarity of the susceptance is positive it is said to be capacitive (B_C) and if the polarity is negative then the susceptance is said to be inductive (B_L). Thus the sign convention for susceptance is exactly opposite to that of the reactance.

Polarity	+	–
Reactance, X	Inductive	Capacitive
Susceptance, B	Capacitive	Inductive

Phase Angle :

From Equation (2.29.7),

$$Y = G \mp jB \qquad ...(\text{Rectangular form})$$

Y can be expressed in the polar form as,

$$Y = |Y| \angle \phi \text{ siemens or mho.} \qquad ...(2.29.8)$$

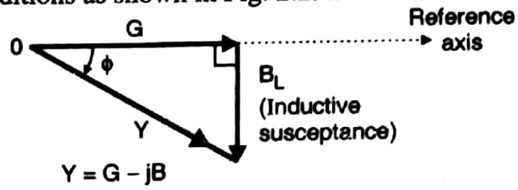
$$\text{where } |Y| = \sqrt{G^2 + B^2}$$

$$\text{and } \phi = \tan^{-1}(B/G) \qquad ...(2.29.9)$$

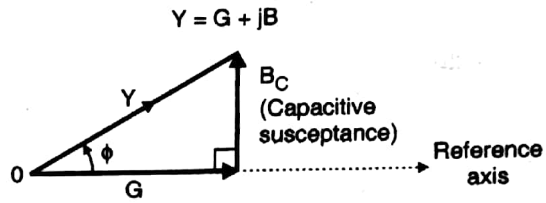
Note : Thus the impedances connected in parallel is equivalent to the admittances connected in series and the impedances connected in series is equivalent to connecting the admittances in parallel.

2.29.2 Admittance Triangle :

- We have drawn the impedance triangle earlier. Similarly we can draw the admittance triangle.
- The sides of such an admittance triangle represent the admittance, conductance and susceptance.
- The conductance (G) component will always be aligned with the reference axis direction because it corresponds to the resistive part of the impedance. But the susceptance (B) can be either inductive (negative sign) or capacitive (positive sign).
- The admittance triangles are drawn for both these conditions as shown in Fig. 2.29.2.



(a) For i.e. – B inductive susceptance



(b) For + B i.e. capacitive susceptance

(A-694) **Fig. 2.29.2 : Admittance triangle**

Ex. 2.29.1 : Explain the terms admittance, conductance and susceptance as applied to a.c. circuits. Find the admittance, in complex form, of the circuit having a resistance of 3-ohm in series with an inductive reactance of 4-ohm.

Soln. :

1. For admittance conductance and susceptance refer section 2.29.
2. The impedance of a 3 Ω resistance in series with a 4 Ω inductive reactance is given by,

$$Z = (3 + j4) = \sqrt{3^2 + 4^2} \angle \tan^{-1}(4/3)$$

$$= 5 \angle 53.13° \; \Omega$$

$$\therefore \text{ Admittance } \quad Y = \frac{1}{Z} = \frac{1}{5 \angle 53.13°}$$

$$= 0.2 \angle -53.13° \text{ siemens}$$

Converting into complex number form we get,

$$Y = (0.12 - j\,0.16) \text{ siemens} \qquad \textbf{...Ans.}$$

Ex. 2.29.2 : Calculate conductance and susceptance if Z = (6 + j8) ohm.

Soln. :

Given : $Z = (6 + j8) = \sqrt{6^2 + 8^2} = 10, R = 6, X = 8$

$$\text{Conductance, } \quad G = \frac{R}{Z^2} = \frac{6}{10^2} = 0.06 \qquad \textbf{...Ans.}$$

Susceptance, $B = \frac{X}{Z^2} = \frac{8}{10^2} = 0.08$ ...Ans.

Ex. 2.29.3 : What is the value of admittance, conductance and susceptance if Z = 4 + j6 ?

Soln. :

Given : $Z = 4 + j6$, i.e. $R = 4$,

$X = 6$, $Z = \sqrt{4^2 + 6^2} = 7.21$

1. **Admittance :**

$$Y = \frac{1}{Z} = \frac{1}{7.21} = 0.1386 \quad \text{...Ans.}$$

2. **Conductance :**

$$G = \frac{R}{Z^2} = \frac{4}{(7.21)^2} = 0.0769 \quad \text{...Ans.}$$

3. **Susceptance :**

$$B = \frac{X}{Z^2} = \frac{6}{(7.21)^2} = 0.1154 \quad \text{...Ans.}$$

Review Questions

Q. 1 Draw the phasor diagram for a purely inductive circuit.

Q. 2 Write the expression for instantaneous current in a purely capacitive circuit.

Q. 3 Define inductive and capacitive reactance.

Q. 4 Define impedance and state its units.

Q. 5 Express impedance in polar of rectangular forms.

Q. 6 Define waveform and state different types of waveforms.

Q. 7 Define cycle.

Q. 8 What is frequency and time period and state the relation between them.

Q. 9 Define angular speed and state its units.

Q. 10 Define the rms and average values of a waveform.

Q. 11 Define form factor and state its value for a sinusoid.

Q. 12 Define crest factor.

Q. 13 What is a phasor ?

Q. 14 A coil having resistance of 15 Ω and inductance of 31.8 mH is connected across a 200 V, 50 cross-sectional supply Calculate :

(a) Current taken (b) Power factor

(c) Power consumed

Q. 15 Obtain the average value of a sinusoidal signal.

Q. 16 What is the difference between a d.c. waveform and alternating waveform ?

Q. 17 Why is the sinusoidal waveform preferred over the other alternating waveforms ?

Q. 18 Explain the generation of sinusoidal alternating signal

Q. 19 Define the following terms related with an AC quantity :

1. Waveform 2. Instantaneous value

3. Cycle 4. Time period

Q. 20 Define the following terms :

1. Frequency 2. Amplitude

3. Angular velocity (ω).

Q. 21 What is meant by the peak to peak value of an ac quantity ?

Q. 22 Define and explain the effective or R.M.S. value.

Q. 23 Explain the practical significance of RMS value.

Q. 24 Explain the practical significance of average value.

Q. 25 Explain the concept of phase and phase difference.

Q. 26 Explain the following terms related to phase :

1. Leading phase difference

2. Lagging phase difference.

Q. 27 What are the advantages of using phasors rather than using waveforms ?

Q. 28 What is the relation between the rotation of a phasor and sinusoidal waveform ?

Q. 29 What is the practical use of phasors ?

Q. 30 Represent two inphase and out of phase signals with the help of waveforms and phasor diagrams.

Q. 31 With the help of waveforms and phasor diagrams, comment on the phase relationship between voltage and current in pure resistive, pure inductive and pure capacitive circuit.

Q. 32 Show that the current in a purely capacitive circuit leads the applied voltage by 90° and the current in a purely inductive circuit lags the applied voltage by 90°.

2.30 MSBTE Questions and Answers :

Summer 2014 [Total Marks - 24]

Q. 1 A voltage equation is expressed as V = 70.7 sin 314 t. Determine :

1. Maximum value of voltage
2. rms value of voltage
3. Frequency and time period of waveform.

(Ex. 2.9.8) **(4 Marks)**

Q. 2 Define inductive reactance and capacitive reactance with respect to AC circuit.

(Sections 2.13.3 and 2.13.4) **(4 Marks)**

Q. 3 A coil consist of 20 ohm resistance and 0.2 H inductance is connected across 230 V, 50 Hz supply. Calculate :

1. Impedance of coil 2. Power factor
3. Current 4. Active power.

(Ex. 2.20.5) **(4 Marks)**

Q. 4 Draw circuit diagram, waveform, phasor diagram and comment on the phase relationship between voltage and current in R-C series circuit.

(Sections 2.21, 2.21.1 and 2.21.4) **(4 Marks)**

Q. 5 Draw a circuit diagram of R.C. series circuit. Draw impedance triangle and power triangle for same circuit. **(Section 2.21.6)** **(4 Marks)**

Q. 6 Draw impedance triangle and power triangle for R.C. series circuit. **(Section 2.21.6)** **(4 Marks)**

Winter 2014 [Total Marks - 20]

Q. 7 Define frequency and time period.

(Sections 2.3.4 and 2.3.5) **(2 Marks)**

Q. 8 Define R.M.S. value in terms of a.c. circuit.

(Section 2.5) **(2 Marks)**

Q. 9 Draw and explain the labelled circuit and phasor diagram for purely inductive circuit. What is the power factor of the circuit ?

(Sections 2.17 and 2.17.2) **(4 Marks)**

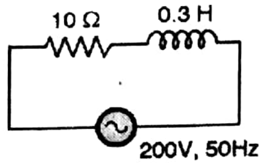
Q. 10 An alternating current is given by i = 141.4 sin 314 t. Calculate the maximum value, frequency, time period and instantaneous value when t is 3 mS.

(Ex. 2.9.9) **(4 Marks)**

Q. 11 Find :

1. Impedance, 2. Phase angle,
3. Current,
4. Total power for the circuit shown in Fig. 1.

(Ex. 2.20.6) **(4 Marks)**



(A-4202) **Fig. 1**

Q. 12 For R-C series circuit :

1. Draw circuit diagram
2. Its phasor diagram
3. Waveform of voltage and current
4. Impedance triangle

(Section 2.21.1, 2.21.2 and 2.21.4) **(4 Marks)**

Summer 2015 [Total Marks - 16]

Q. 13 Define the following terms as referred to an alternating quantity :

1. Cycle 2. Frequency
3. Maximum value 4. Average value

(Sections 2.3.3, 2.3.5 and 2.6) **(4 Marks)**

Q. 14 The current flowing through the circuit is $i = 14.14 \sin\left(314\,t - \frac{\pi}{6}\right)$.

Calculate :

1. Frequency 3. Phase difference
2. I_{rms} 4. Amplitude

(Ex. 2.9.3) **(4 Marks)**

Q. 15 Define : rms value of an alternating quantity.

(Section 2.5) **(4 Marks)**

Q. 16 A capacitor having a capacitance of 10 microfarad is connected in series with a non-inductive resistance of 120 ohm across 100 V, 50 Hz supply. Calculate :

1. Current
2. Impedance
3. Phase difference between current and supply voltage
4. Power **(Ex. 2.21.4)** **(4 Marks)**

Winter 2015 [Total Marks - 16]

Q. 17 Define RMS value of an AC quantity. Explain its practical significance. **(Section 2.5)** **(4 Marks)**

Q. 18 An alternating voltage is mathematically expressed as,

$$v = 141.42 \sin\left(157.08\, t + \frac{\pi}{12}\right) \text{ volt}$$

Find maximum value, RMS value, frequency and periodic time. **(Ex. 2.9.10)** **(4 Marks)**

Q. 19 A coil having resistance 10 ohm and an inductance 0.2 H is connected across 100 Volts, 50 Hz, supply. Calculate :

1. Reactance 2. Impedance

3. Current 4. Power consumed

(Ex. 2.20.7) **(4 Marks)**

Q. 20 Draw and explain circuit diagram of R-C circuit. **(Section 2.21)** **(4 Marks)**

Summer 2016 [Total Marks - 24]

Q. 21 Define AC and DC current. **(Section 2.1)** **(2 Marks)**

Q. 22 An alternating current is represented by $i = 50.5 \sin (314\, t + \pi/2)$ calculate :

1. Amplitude 2. Frequency 3. I_{rms} 4. Phase difference

(Ex. 2.9.11) **(4 Marks)**

Q. 23 Draw the circuit diagram and waveforms of voltage and current in RL series circuit.

(Sections 2.20 and 2.20.4) **(4 Marks)**

Q. 24 An RL series circuit consists of 100 Ω resistance and 0.22 H inductance connected across 220 V, 50 Hz AC supply. Calculate : 1. Impedance 2. Current 3. Voltage across resistor 4. Voltage across inductor

(Ex. 2.20.8) **(4 Marks)**

Q. 25 A resistance of 10 Ω and capacitance of 50 μF are connected in series across 200 V, 50 Hz AC supply. Calculate :

1. Capacitive reactance 2. Impedance

3. Current 4. Phase angle

(Ex. 2.21.5) **(4 Marks)**

Q. 26 Draw vector diagram, impedance triangle and power triangle for series R-L-C circuit when connected to single phase a.c. supply for the condition $X_L < X_C$

(Section 2.22.9) **(2 Marks)**

Q. 27 Draw a circuit diagram of pure inductive circuit and phasor diagram.

(Sections 2.17 and 2.17.2) **(4 Marks)**

Winter 2016 [Total Marks - 22]

Q. 28 Define the terms related to a.c. supply :

1. Cycle 2. Frequency

3. Period 4. Amplitude

(Sections 2.3.3, 2.3.4, 2.3.5 and 2.3.6) **(4 Marks)**

Q. 29 Define :

1. Maximum value

2. RMS value of ac supply.

(Sections 2.3.6 and 2.5) **(2 Marks)**

Q. 30 The current flowing in a circuit is $i = 28.28 \sin\left(314t - \frac{\pi}{6}\right)$

Calculate :

1. Amplitude 2. Rms current

3. Frequency 4. Phase difference

(Section 2.9.12) **(4 Marks)**

Q. 31 An alternating current is given by equation $i = 10\sqrt{2} \sin 314\, t$.

Calculate :

1. Average value
2. Instantaneous value of i at t = 5 milisec.

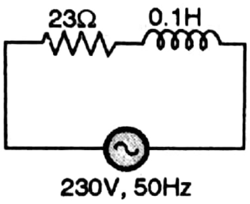
(Ex. 2.9.13) **(4 Marks)**

Q. 32 Calculate :

1. Reactance 2. Impedance
3. Current 4. Phase angle

for the circuit shown in Fig. 2.

(Ex. 2.20.9) **(4 Marks)**



(A-5181) **Fig. 2**

Q. 33 A coil having resistance 6 Ω and reactance 8 Ω is connected across 230 V, 50 Hz, a.c. supply.

Calculate :

1. Inductance 2. Impedance
3. Current 4. Active power

(Ex. 2.20.10) **(4 Marks)**

Summer 2017 [Total Marks - 14]

Q. 34 Define :

1. Frequency 2. Form factor.

(Sections 2.3.5 and 2.7) **(2 Marks)**

Q. 35 Current flowing through the circuit is $i = 141.4 \sin\left(314t - \frac{\pi}{2}\right)$ Amp.

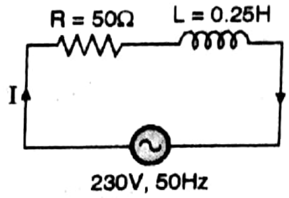
Calculate :

1. Frequency 2. RMS value
3. Phase difference 4. Amplitude

(Ex. 2.9.14) **(4 Marks)**

Q. 36 Calculate voltage across individual element for the circuit shown in Fig. 3.

(Ex. 2.20.11) **(4 Marks)**



(A-5262) **Fig. 3**

Q. 37 Explain for series R.C. circuit :

1. Circuit diagram 2. Voltage equation
3. Current equation 4. Power

(Sections 2.21, 2.21.4 and 2.21.5) **(4 Marks)**

Winter 2017 [Total Marks - 08]

Q. 38 An A.C. voltage of v(t) = 230 sin 314 t Volts is applied to a circuit. Calculate :

1. Angular frequency
2. Frequency
3. RMS value
4. Average value **(Ex. 2.9.15)** **(4 Marks)**

Q. 39 A series circuit consisting of resistance 40 Ω and inductance 30 mH is supplied by 230 V, 50 Hz, a.c. supply. Calculate impedance and current taken by the circuit. **(Ex. 2.20.12)** **(4 Marks)**

2.31 I-Scheme Solved Examples :

Ex. 2.31.1 : An inductance of 0.1 H and a resistance of 50 Ω are connected in series across a 220 V, 50 Hz AC supply.
Determine : 1. Impedance 2. Current 3. Power factor 4. Power consumed.

W-18, 6 Marks

Soln. :

Given : $L = 0.1$ H, $R = 50\ \Omega$, $V_S = 220$ V, $f = 50$ Hz

To find : 1. Z 2. I 3. $\cos\phi$ 4. P

1. Find Impedance :

$$\text{Reactance } X_L = 2\pi fL$$
$$= 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

$$\therefore \text{Impedance } |Z| = \sqrt{(R^2 + X_L^2)}$$

$$= \sqrt{(50)^2 + (31.42)^2} = 59\Omega$$

$$\phi = \tan^{-1}(X_L/R)$$

$$\phi = \tan^{-1}(31.42/50)$$

$$\therefore \phi = 32.15^\circ$$

$$\therefore Z = |Z| \angle \phi = 59 \angle 32.15^\circ \Omega \quad \text{...Ans.}$$

2. Find current :

$$I = \frac{V_S}{Z} = \frac{220\angle 0^\circ}{59\angle 32.15^\circ}$$

$$\therefore I = 3.73 \angle -32.15^\circ \text{ Amp} \quad \text{...Ans.}$$

3. Power factor :

$$\cos\phi = \cos(-32.15^\circ)$$

$$= 0.85 \text{ (lagging)} \quad \text{...Ans.}$$

4. Power consumed :

$$P = VI\cos\phi = 220 \times 3.73 \times 0.85$$

$$\therefore P = 697.51 \text{ W} \quad \text{...Ans.}$$

2.32 I-Scheme Questions and Answers :

Winter 2018 [Total Marks 10]

Q. 1 Define form factor of an alternating quantity.

(Section 2.7) **(2 Marks)**

Q. 2 Define frequency and time period of an alternating quality. **(Sections 2.3.4 and 2.3.5)** **(2 Marks)**

Q. 3 An inductance of 0.1 H and a resistance of 50 Ω are connected in series across a 220 V, 50 Hz AC supply. Determine : 1. Impedance 2. Current 3. Power factor 4. Power consumed.

(Ex. 2.31.1) **(6 Marks)**

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