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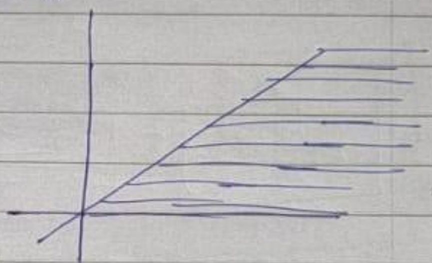
1) $$I = \int_0^\infty \int_0^x e^{-xy} \cdot y \, dy \, dx$$

given limits $y = 0$ to $y = x$
$x = 0$ to $y = \infty$

New limits $x = y$ to $x = \infty$
$y = 0$ to $y = \infty$



$$\therefore I = \int_0^\infty \int_y^\infty e^{-xy} \cdot y \, dy \, dx$$

$$I = \int_0^\infty y \left[e^{-yx} \times \frac{1}{y} \right]_y^\infty dy$$

$$I = \int_0^\infty y \left\{ \left[\frac{e^{-y^2}}{y} \right] - \left[\frac{e^{-\infty}}{y} \right] \right\} dy$$

$$\therefore I = \int_0^\infty e^{-y^2} \, dy$$

let $y^2 = t$
$\therefore y = \sqrt{t}$
$dy = \frac{1}{2\sqrt{t}} dt$

$y = 0$	$t = 0$
$y \to \infty$	$t \to \infty$

$$\therefore I = \int_0^\infty \frac{e^{-t} t^{-1/2}}{2} dt$$

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$$\therefore I = \frac{1}{2} \int e^{-t} t^{-1/2} \, dt$$

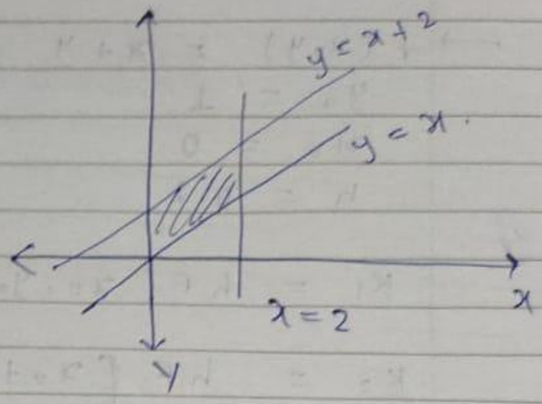
$$\therefore I = \frac{1}{2} \Gamma(1/2)$$

$$\therefore I = \sqrt{\pi} / 2.$$

② $\iint (x+y)\,dy\,dx$

$x=0 \quad y=x$

→ $x=2 \quad y=x+2$



$$\int_0^2 \int_x^{x+2} (x+y)\,dy\,dx$$

$$\therefore\ I = \int_0^2 \left[xy + \frac{y^2}{2}\right]_x^{x+2} dx$$

$$= \int_0^2 \left[x(x+2) + \frac{(x+2)^2}{2}\right] - \left[x^2 + \frac{x^2}{2}\right] dx$$

$$= \int_0^2 (4x+2)\,dx$$

$$= \left[\frac{4x^2}{2} + 2x\right]_0^2$$

$$= \left[\frac{4\times 4}{2} + 2\times 2\right]$$

$$\therefore\ \underline{\underline{I = 12}}$$

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(3) let value of $h = 0.2$

$\longrightarrow f(x,y) = x + y$

$y_0 = 1$

$x_0 = 0$

$h = 0.2$

$K_1 = hf(x_0, y_0) = 0.2(0+1) = \underline{0.2}$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= 0.2\left(0 + \frac{0.2}{2} + 1 + \frac{0.2}{2}\right)$

$= 0.2 \times 1.2$

$\boxed{K_2 = 0.24}$

$K_3 = hf\left(x_0 + h/2, y_0 + K_2/2\right)$

$= 0.2(0 + 0.1 + 1 + 0.12)$

$= 0.2(1.22)$

$\boxed{K_3 = 0.244}$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.2(0.2 + 1 + 0.244)$

$= 0.2(1.444)$

$\boxed{K_4 = 0.2888}$

$K = \frac{K_1 + 2K_2 + 2K_3 + K_4}{6} = 0.2428$

$y_1 = y_0 + K$

$y_1 = 1 + 0.2428$

$\boxed{\therefore y_1 = 1.2428}$

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Approx value of $y = 1.2428$.