Unit - II

Chapter

3

Three Phase AC Circuits

Syllabus :

Voltage and current relationship in star and delta connections.

Chapter Contents

3.1 Introduction to Polyphase AC Circuits :

- We have already discussed about the single phase ac circuits. The domestic ac supply is single phase ac supply with 230 V / 50 Hz.
- But this ac supply is not suitable for certain applications. Some loads need a **polyphase** ac supply.
- The word poly means many. Thus a polyphase ac supply is the one which produces many phases simultaneously.

How to generate a polyphase ac supply ?

- The single phase ac voltage is generated by using a **single phase alternator.** Single phase alternator consists of only one armature winding.
- But in order to generate a polyphase voltage, we have to use many armature windings. The number of windings is equal to the number of phases.

3.1.1 Why to use a Polyphase System ?

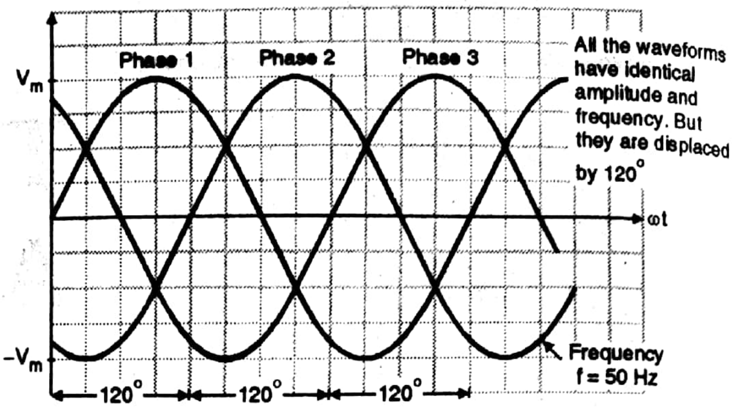
A polyphase system has certain advantages over the single phase system, hence they are preferred practically. Some of these advantages are discussed in section 3.2.

3.1.2 How Many Phases ?

- We cannot practically increase the number of phases to a very large number. The advantages related with the number of phases do not increase proportionally with number of phases, but the complications of interconnections increase.
- A three phase system has proved to be the most economical as compared to the other systems. Hence in practice the three phase system is most preferred.
- In rare cases a six phase system is also used.

3.1.3 Three Phase Supply Waveforms :

- The three armature windings used for generation of a three phase supply are placed at 120° away from each other.
- The voltages induced in these windings are of same amplitude and frequency, but they are displaced by 120° with respect to each other as shown in Fig. 3.1.1.



(A-723) **Fig. 3.1.1 : Three phase voltage waveforms**

3.2 Advantages of Three Phase Systems over Single Phase System :

S-10, S-11, W-11, W-12, S-14, S-15, S-16, W-16, S-17

MSBTE Questions

Q. 1 State the advantages of polyphase (3-phase) system over single phase system (any four). **(S-10, S-11, W-11, W-12, S-14, S-15, S-16, W-16, S-17, 4 Marks)**

The three phase system has certain advantages over the single phase system. They are as follows :

1. **More output :** For the same size, the output of the polyphase machines is always higher than that of the single phase machines. Typically the three phase machines produce about 45% higher output.
2. **Smaller size :** For producing the same output, the size of three phase machines is always smaller than that of the single phase machines.
3. **3-phase motors are self starting :** As the three phase ac supply is capable of producing a rotating magnetic field when applied to stationary windings, the three phase AC motors are self starting. The single phase motors need to use an additional starter winding.
4. **More power is transmitted :** In the transmission system it is possible to transmit more power using a three phase system, than the single phase system, by using the conductors of same cross-sectional area.
5. **Smaller cross-sectional area of conductors :** If same amount of power is to be transmitted, then the cross-sectional area of the conductors used for the three phase system is small as compared to that for the single phase system.

6. The horsepower rating of three-phase motors and the kVA (kilo-volt-amp) rating of three-phase transformers is much greater than that for single-phase motors or transformers with a similar frame size.

7. The power delivered by a single-phase system fluctuates. This power falls to zero three times during each cycle. The power delivered by a three-phase circuit also fluctuates, but it never falls to zero. This produces superior operating characteristics for three phase motors.

8. In a balanced three-phase system, the conductors need to be only about 75% the size of conductors for a single-phase two-wire system of the same KVA rating. This helps offset the cost of supplying the third conductor required by three-phase systems.

9. **Better power factor** : The power factor of three phase machines is better than that of the single phase machines.

3.2.1 Comparison of Single Phase and Three Phase Systems :

Table 3.2.1

Sr. No.	Parameter	Single phase system	Three phase system
1.	Voltage	Low (230 V)	High (415 V)
2.	Transmission efficiency	Low	High
3.	Size of machines to produce same output	Larger	Smaller
4.	Cross sectional area of conductors.	Large	Small
5.	Usage	Domestic, small power applications	Industrial, large power applications

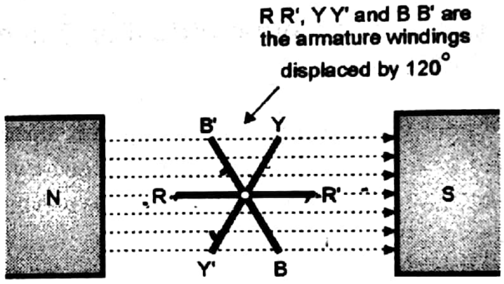
3.3 Principle of Three Phase Emf Generation and Its Waveforms :

Principle :

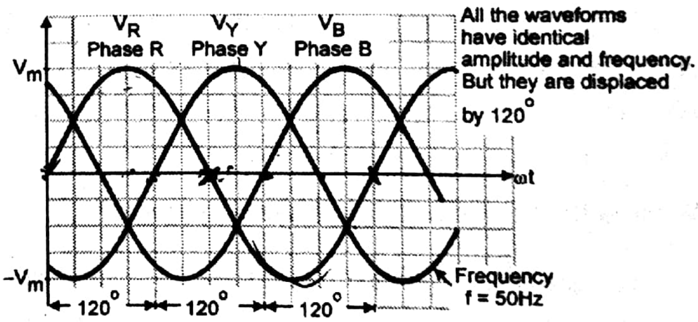
- The single phase supply is generated using a single turn alternator.
- Thus if armature consists of only one winding, then only one alternating voltage is produced.
- But if the armature winding is divided into three groups which are displaced by 120° from each other, then it is possible to generate three alternating voltages.

Construction :

- As shown in Fig. 3.3.1(a) the armature winding is divided into three groups. The three coils are $R - R'$, $Y - Y'$ and $B - B'$.
- All these coils are mounted on the same shaft and are physically placed at 120° from each other.
- When these coils rotate in the flux produced by the permanent magnet, emf is induced into these coils. These emfs as shown in Fig. 3.3.1(b) are sinusoidal, of equal amplitudes and equal frequency but they are displaced from each other by 120°.
- V_R, V_Y and V_B are the three phase voltages. If V_R is considered as the reference then V_Y lags V_R by 120° and V_B lags V_Y by 120°, in other words V_B lags V_R by 240°.



(a) Generation of a 3-phase voltage



(b) Voltages induced in the three coils

(A-724) Fig. 3.3.1

3.3.1 Mathematical Representation of the Three Phase Voltages :

- The mathematical expressions for the three induced voltages are given by :

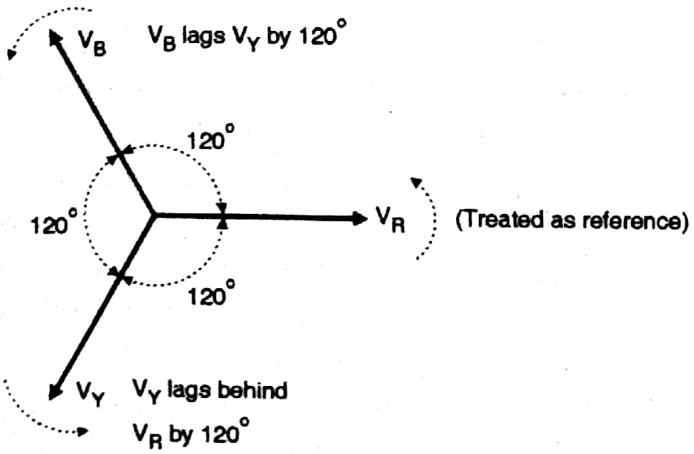
$$V_R = V_m \sin \omega t \quad \text{...(3.3.1)}$$

$$V_Y = V_m \sin (\omega t - 120°) \quad \text{...(3.3.2)}$$

$$V_B = V_m \sin(\omega t - 240°)$$

$$= V_m \sin(\omega t + 120°) \qquad ...(3.3.3)$$

- The phasor diagram for the three phase voltages is shown in Fig. 3.3.2. The phasors rotate in the anticlockwise direction. Hence V_Y lags V_R and V_B lags V_Y.



(A-725) **Fig. 3.3.2 : Phasor representation of the three phase voltages**

3.3.2 Concept of Symmetrical or Balanced System :

The three phase system in which the three phase voltages are of identical magnitudes and frequency and are displaced by 120° from each other is called as a symmetrical system.

3.3.3 Concept of Phase Sequence :

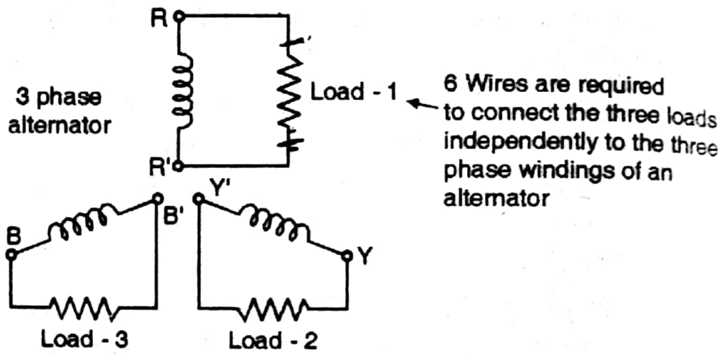
The phase sequence is defined as the sequence in which the three phases reach their maximum positive values. Normally the phase sequence is R-Y-B. The three colours used to denote 3 phases are Red, Yellow and Blue.

What is the importance of phase sequence ?

The direction of rotation of the three phase machines depends on the phase sequence. If the phase sequence is changed e.g. R-B-Y then the direction of rotation will be reversed. In order to avoid such things, the phase sequence of R-Y-B is always maintained.

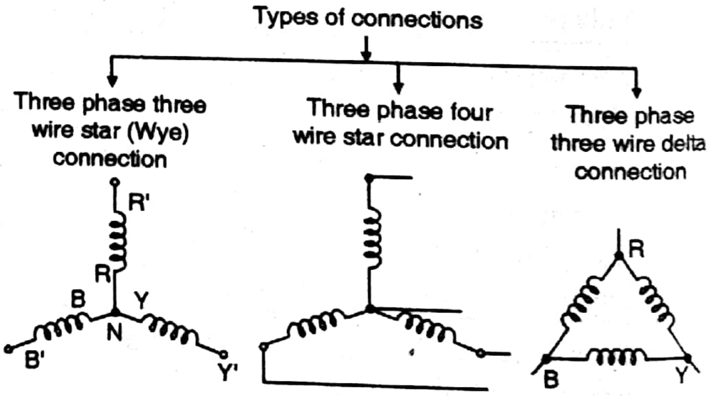
3.4 Three Phase Supply Connections :

- Similar to the dc systems which are two wire systems, only two wires are sufficient for transmitting a single phase voltage from one point to the other.
- As discussed in the previous sections, the three phase alternator has three separate windings one per each phase.
- Hence the power generated in each phase can be transmitted independently to the load using 2-wires per winding as shown in Fig. 3.4.1. But this will require 6-different wires.



(A-726) **Fig. 3.4.1 : Three phase supply system with six wires used for connection**

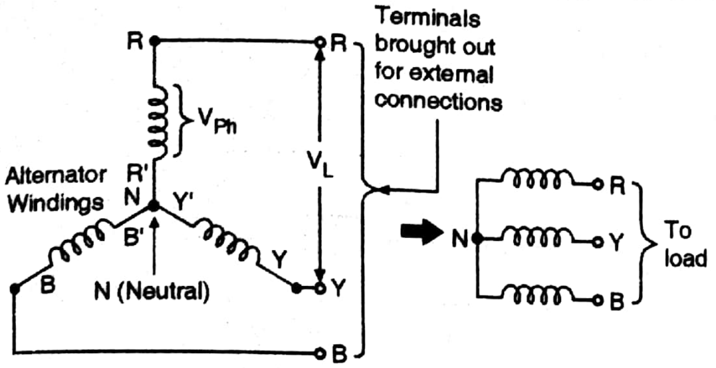
- Eventhough such a system is practically possible, it makes the system complicated and expensive.
- Hence in practice, the three windings of the alternator are interconnected in two different ways to reduce the number of wires required for the connections.



(A-727) **Fig. 3.4.2 : Different types of connections for a 3 phase ac supply**

3.4.1 Star Connection (Wye Connection) :

- The star or Wye connection of the three alternator windings is shown in Fig. 3.4.3(a). This configuration is obtained by connecting one end of the three phase windings together.
- We can connect either R Y B or R′ Y′ B′ together. This common point is called as the **Neutral Point** and it is denoted by N.



(a) Star connection or Wye connection (b) Equivalent

(A-728) **Fig. 3.4.3**

- The remaining three ends of the windings are brought out for the external connections. These ends are denoted by R-Y-B as shown in Fig. 3.4.3(b).

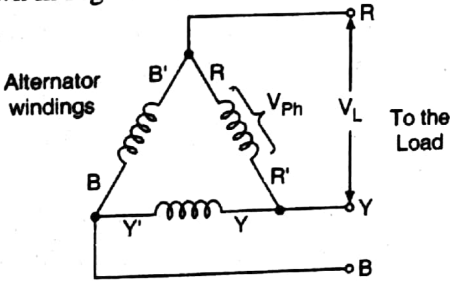
3.4.2 Delta Connection : S-13, W-15, S-17

MSBTE Questions

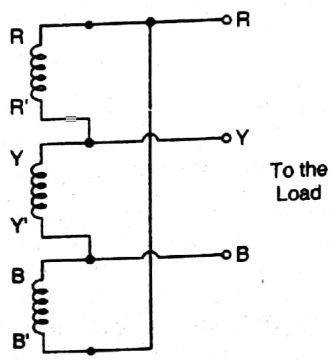
Q. 1 Draw delta connected 3-phase supply system mark-line, phase voltage, line current and phase current. Write power equation. **(S-13, W-15, 4 Marks)**

Q. 2 Draw delta connected load. State the relationship between line and phase values for the same. **(S-17, 4 Marks)**

- The delta or mesh connection of the three alternator windings is shown in Fig. 3.4.4.
- The delta or mesh configuration is obtained by connecting one end of winding to the starting end of the other winding such that it produces a closed loop as shown in Fig. 3.4.4.



(a) Delta or Mesh connection

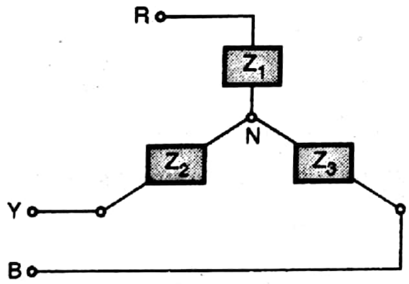


(b) Equivalent circuit

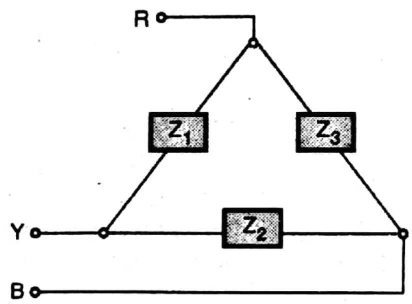
(A-729) **Fig. 3.4.4**

3.5 Types of Loads :

- For defining the phase and line currents let us define two types of load connection. The two types of load connection are :
  1. Star connection of load
  2. Delta connection of load
- The star or delta loads are known as the three phase loads.
- Fig. 3.5.1(a) shows the star connected load whereas Fig. 3.5.1(b) shows the delta connected load.



(a)



(b)

(A-730) **Fig. 3.5.1 : Types of loads**

3.5.1 Balanced or Unbalanced Load :

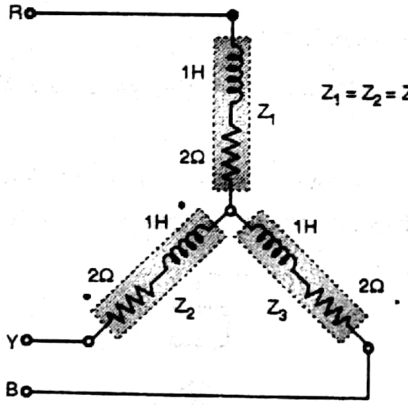
I-Scheme : W-18

- A star type or delta type load can be further classified into two categories :
  1. Balanced load 2. Unbalanced load.

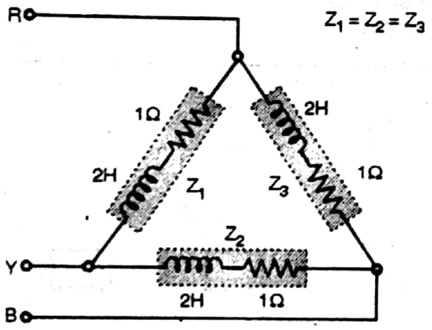
Concept of balanced load :

- A balanced load is that in which magnitudes of all impedances connected in the load are equal and the phase angles of them also are equal and of same type (inductive, resistive or capacitive).
- All the phase voltages will have equal magnitudes, all the line voltages also will have equal magnitudes, all the line voltages also will have equal magnitudes and same is true for the phase currents and line currents as well.

- Figs. 3.5.2 (a) and (b) illustrate the balanced star and balanced delta loads respectively.
- Note that the magnitude | Z | and the phase angle ϕ for all the impedances Z_1, Z_2 and Z_3 will be the same for the balanced loads.



(a) Balanced star connected load

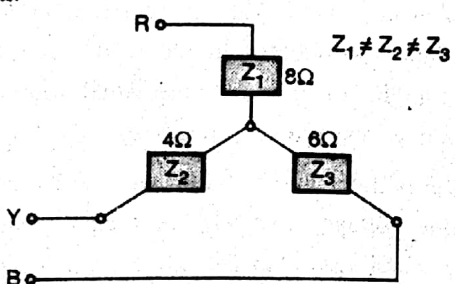


(b) Balanced delta connected load

(A-731) **Fig. 3.5.2 : Examples of balanced loads**

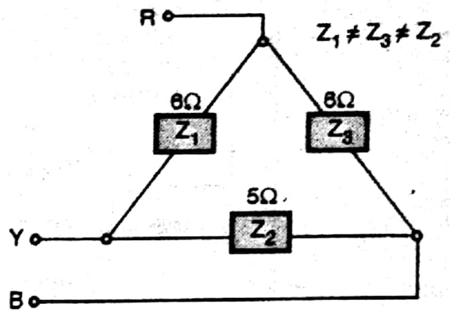
Unbalanced loads :

- If a load does not satisfy the condition of balance, then its is called as the unbalanced load.
- The magnitudes and phase angles of the three impedances Z_1, Z_2 and Z_3 differ from each other if the load is unbalanced type.
- Fig. 3.5.3 shows the unbalanced star and delta loads.
- In Fig. 3.5.3(b), $Z_1 = Z_3 = 6\ \Omega$. Still this is an unbalanced delta because all the impedances are not identical.



(a) Unbalanced star

Fig. 3.5.3



(b) Unbalanced delta

(A-732) **Fig. 3.5.3 : Unbalanced loads**

3.6 Relation between Voltages and Current for a Balanced Star Load :

3.6.1 Line Voltages and Phase Voltages :

S-15, I-Scheme : W-18

MSBTE Questions

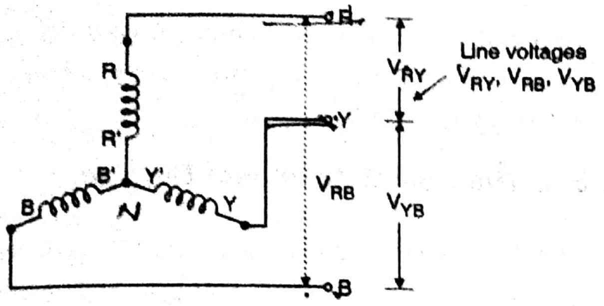
Q. 1 What is line voltage and phase voltage ? Draw star connected 3-phase supply system and mark line voltage and phase voltage. **(S-15, 4 Marks)**

Line voltage :

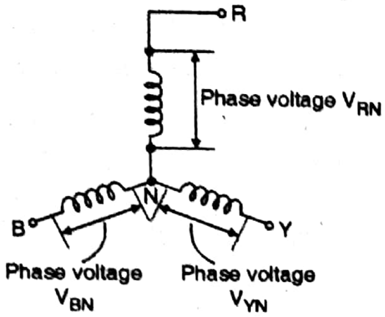
- If R, Y and B are called as the supply lines, then the potential difference between any two lines is known as the **line voltage.**
- V_{RY}, V_{RB}, V_{YB}, V_{YR}, V_{BR} and V_{BY} are six possible line voltages.
- All the line voltages are sinewaves of **50 Hz frequency** and the **phase shift** between the adjacent line voltages is **60°.**

Phase voltage :

- The voltage measured across a single winding or phase is called as **phase voltage.**
- Refer Fig. 3.6.1 to understand more about the line and phase voltages. In the star connected systems the line voltages and phase voltages are of different values.
- **The phase voltages are thus the voltages measured with respect to neutral.** They are denoted by V_{RN}, V_{YN} and V_{BN}.
- All the phase voltages are sinewaves and the phase difference between the adjacent phase voltages is 120° as discussed earlier.



(A-733) **Fig. 3.6.1(a) : Line voltages**



(A-733) **Fig. 3.6.1(b) : Phase voltages**

Note : The definitions of phase voltage and line voltage are general definitions and they are applicable to the star connection as well as the delta connection.

3.6.2 Relation between Line and Phase Voltages for Star Connected Supply :

S-12, W-14, S-15, I-Scheme : W-18

MSBTE Questions

Q. 1 Derive the relationship between line values and phase values in star connected load with reference to three phase supply. **(S-12, 4 Marks)**

Q. 2 State relation between phase and line current and phase and line voltage of the following system :

1. Star connected balanced system
2. Delta connected balanced system

(W-14, 4 Marks)

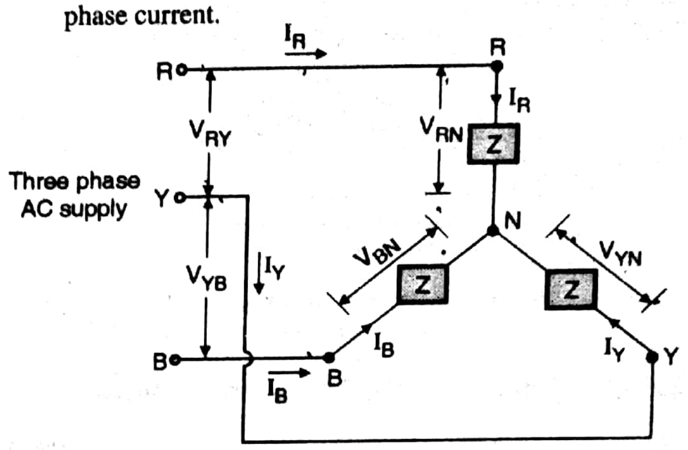
Q. 3 What is line voltage and phase voltage ? Draw star connected 3-phase supply system and mark line voltage and phase voltage. **(S-15, 4 Marks)**

- Typically the rms phase voltage is 230 or 240 V. Whereas the rms line voltage is 415 V or 440 V and the frequency of phase and line voltages is 50 Hz.
- Thus in the star connected system the line voltage is higher than the phase voltage by a factor $\sqrt{3}$.

$\therefore$ Line voltage $= \sqrt{3}$ phase voltage and Phase voltage

$$= \frac{\text{Line voltage}}{\sqrt{3}}$$

- Let us see the relation between line voltage and phase voltage and then the relation between line current and phase current.



Phase voltages : $V_{RN} = V_{YN} = V_{BN}$

Line voltages : $V_{RY} = V_{RB} = V_{YB}$

Phase currents : $I_R = I_Y = I_B$.

Line currents : $I_R = I_Y = I_B$

(A-734) **Fig. 3.6.2 : Relations between voltages and currents for the star connected load**

3.6.3 Relation between Phase and Line Current :

W-14, I-Scheme : W-18

MSBTE Questions

Q. 1 State relation between phase and line current and phase and line voltage of the following system :

1. Star connected balanced system
2. Delta connected balanced system

(W-14, 4 Marks)

Phase current :

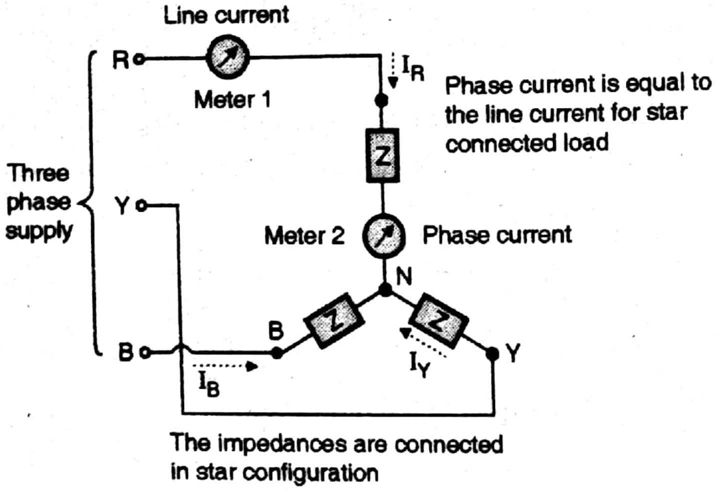
- The current passing through any branch of the star connected load is called as the phase current. I_R, I_Y and I_B are the phase currents. Meter 2 in Fig. 3.6.3 measures the phase current I_R.

$$\therefore \quad I_{ph} = I_R = I_Y = I_B$$

Line current :

- The current passing through any line R, Y, B is called as the line current. As shown in Fig. 3.6.3, meter-1 measures the line current I_R. The line current in general is denoted by I_L.
- As the current flowing through each line is equal to the current flowing through the corresponding branch, the line current is equal to the phase current.

- For star connected load $I_L = I_{ph}$.



(A-735) **Fig. 3.6.3 : Line and phase currents for a star connected load**

3.6.4 Equations for Three Phase Power :

S-13

> **MSBTE Questions**
>
> **Q. 1** Derive formula for power in 3-phase circuit for star connected load in terms of line current, line voltage and power factor. **(S-13, 4 Marks)**

- As discussed in the chapter on single phase AC circuits the power consumed in each phase is given by,

$$P_{ph} = V_{ph} I_{ph} \cos\phi \qquad ...(3.6.1)$$

Where ϕ = Angle between V_{ph} and I_{ph}

- For a balanced three phase system, the total power consumed will be given by,

$$P_T = 3 P_{ph} = 3 V_{ph} I_{ph} \cos\phi \qquad ...(3.6.2)$$

Here V_{ph} = RMS phase voltage

and I_{ph} = RMS phase current

Substituting $V_{ph} = \frac{V_L}{\sqrt{3}}$ and $I_{ph} = I_L$ we get

$$\text{Total power } P_T = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi$$

$$\therefore \quad P_T = \sqrt{3} V_L I_L \cos\phi \qquad ...(3.6.3)$$

3.6.5 Power Factor for a Star Load :

- The load power factor for a 3 phase balanced star load is equal to the power factor of each phase in the load.

$\therefore$ Overall P.F. $= \cos\phi$

Where ϕ = Angle between the phase voltage and phase current

- So overall power factor of 3-phase balanced star load is equal to the cosine of the phase angle between the phase voltage and phase current.

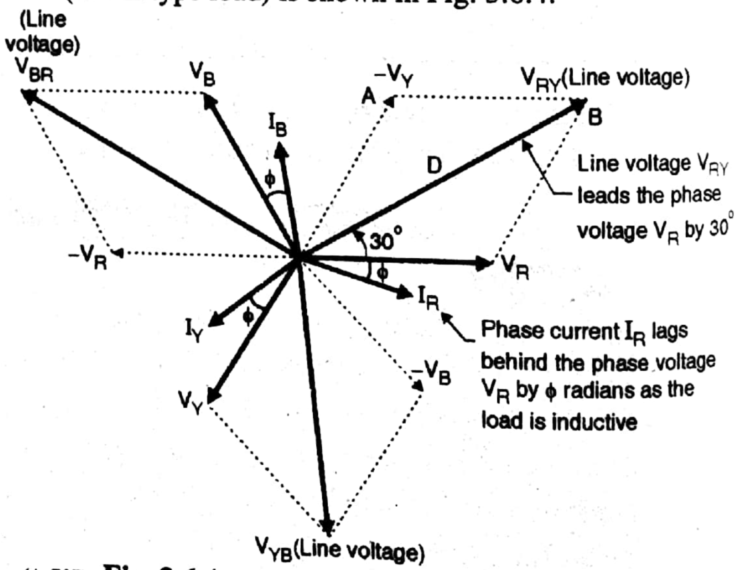
3.6.6 The Complete Phasor Diagram :

- Now let us draw the phasor diagram which includes all the phase voltages, line voltages and currents. Depending on the nature of load (resistive, inductive or capacitive) the current will be in phase, lagging or leading with respect to the corresponding voltages.
- The phase voltage and phase current are related to each other as follows :

$$|Z| = \frac{|V_{ph}|}{|I_{ph}|} \qquad ...(3.6.4)$$

$$\therefore \quad |I_{ph}| = \frac{|V_{ph}|}{|Z|} \qquad ...(3.6.5)$$

- Fig. 3.6.4 shows the complete phasor diagram for a 3 phase star connected load for lagging power factor (R + L type load) is shown in Fig. 3.6.4.



(A-737) **Fig. 3.6.4 : Complete phasor diagram for the star connected balanced load**

Conclusions from the phasor diagram :

1. Phase currents I_R, I_Y and I_B lag behind the corresponding phase voltages V_R, V_Y and V_B by ϕ radians respectively as the load is inductive.
2. The line voltages V_{RY}, V_{BR} and V_{YB} are displaced by 120° from each other.
3. The line voltages lead their respective phase voltages by 30°. So V_{RY} leads V_R, V_{BR} lead V_B and V_{YB} lead V_Y by 30°.

3.7 Voltage, Current and Power Relation in a Balanced Delta Load :

3.7.1 Line Voltages and Phase Voltages for Delta Connected Supply : W-10, S-13

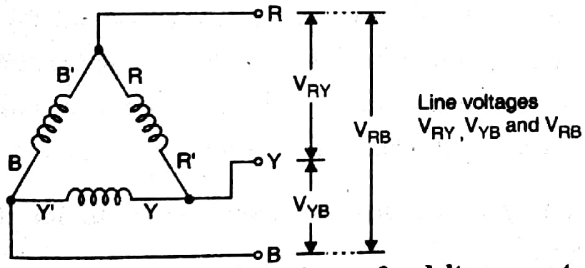
MSBTE Questions

Q. 1 Draw delta connected three phase supply system. Mark line voltage, phase voltage, line current and phase current. Write power equation. **(W-10, S-13, 4 Marks)**

- We have already defined the phase voltage and line voltage for a star connected supply.
- The same definitions are applicable to the delta connected supply as well.

Line voltages :

- Fig. 3.7.1(a) shows the **line voltages** for delta connected supply.
- The **line voltage** is defined as the potential difference between any two lines. So V_{RY}, V_{RB}, V_{YB}, V_{YR}, V_{BR} and V_{BY} are six possible line voltages.

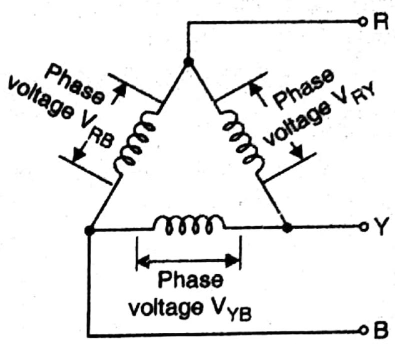


(A-742)**Fig. 3.7.1(a) : Line voltages for delta connected supply**

- All the line voltages are sinewaves of 50 Hz frequency and the phase shift between the adjacent line voltages is 60°.

Phase voltages :

- The voltage measured across a single winding is called as the phase voltage.
- Fig. 3.7.1(b) shows various phase voltages for a delta connected supply.
- Note that the phase voltages are V_{RY}, V_{YB}, V_{RB} .... etc. which are nothing but line voltage.
- Thus for a delta connected supply, the phase voltages are same as line voltages.
- Typical values of line and phase voltages for a delta connected supply are 415 V or 440 V.



(A-743)**Fig. 3.7.1(b) : Phase voltages for delta connected supply**

3.7.2 Phase and Line Voltages in Delta Connections :

S-08, W-12, S-13, W-14, W-15, S-17

MSBTE Questions

Q. 1 For delta-connected loads, state numerical relationships between : Line voltage and phase voltage **(S-08, W-14, 4 Marks)**

Q. 2 Derive the relationship between line values and phase values in delta connected load with reference to 3ϕ supply. **(W-12, 4 Marks)**

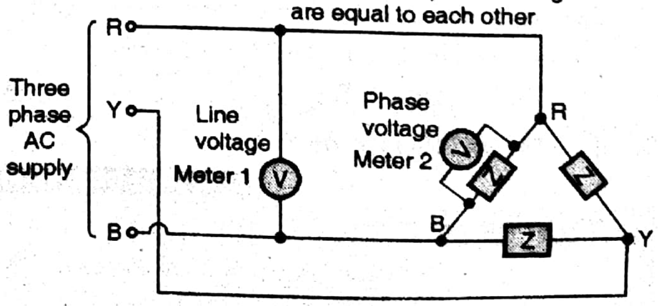
Q. 3 Draw delta connected 3-phase supply system mark line, phase voltage, line current and phase current. Write power equation. **(S-13, W-15, S-17, 4 Marks)**

- We have already defined the line voltage and phase voltages. In Fig. 3.7.2, Meter-1 measures the line voltage V_{RB} and Meter-2 measures the phase voltage.
- Note that both these voltages are identical.

Hence for the delta connection, Line voltage = Phase voltage

- The phase voltages in the delta connection are V_{RY}, V_{YB}, V_{RB} ... etc. and the line voltages also are V_{RY}, V_{RB}, V_{YB} ...etc.

$$\therefore \; V_{ph} = V_L = V_{RY}, V_{RB}, V_{YB} \qquad ...(3.7.1)$$



(A-744) **Fig. 3.7.2 : Delta connected load and phase and line voltages**

3.7.3 Line Current and Phase Current for Delta Connection :

S-08, W-12, S-13, W-14, W-15, S-17

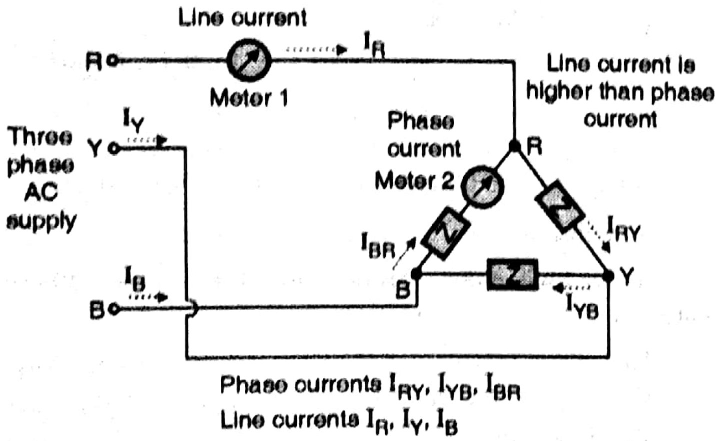
MSBTE Questions

Q. 1 For delta-connected loads, state numerical relationships between :
Line current and phase current **(S-08, 2 Marks)**

Q. 2 Derive the relationship between line values and phase values in delta connected load with reference to 3ϕ supply. **(W-12, 4 Marks)**

Q. 3 Draw delta connected 3-phase supply system mark-line, phase voltage, line current and phase current. Write power equation. **(S-13, W-14, W-15, S-17, 4 Marks)**

- We have already defined the line current and phase current. Refer Fig. 3.7.3 to get a better idea about this concept.
- The line currents are denoted by I_R, I_Y and I_B whereas the phase currents are denoted I_{RY}, I_{YB} and I_{BR}.
- From Fig. 3.7.3 it is evident that the **line current is higher than the phase current**. This is because at node R, the line current I_R gets divided into two phase currents I_{RY} and I_{RB}.



(A-745) **Fig. 3.7.3 : Phase and line currents for delta connected load**

- The relation between line current and phase current for a delta connected balanced load is as follows :

$$I_L = \sqrt{3}\, I_{ph} \qquad ...(3.7.2)$$

3.7.4 Power Relations :

S-08, S-12, S-13, W-15

MSBTE Questions

Q. 1 State the expression for power in terms of line voltage, line current and power factor. **(S-08, 2 Marks)**

Q. 2 Derive formula for power in three phase circuit for delta connected load in the form of line current, line voltage and power factor. **(S-12, 4 Marks)**

Q. 3 Draw delta connected 3-phase supply system mark-line, phase voltage, line current and phase current. Write power equation. **(S-13, W-15, 4 Marks)**

- The power consumed in each phase is given by :

$$P_{ph} = V_{ph}\, I_{ph} \cos\phi \qquad ...(3.7.3)$$

- For a balanced three phase system, the total power consumed is given by :

$$P_T = 3\, P_{ph} = 3\, V_{ph}\, I_{ph} \cos\phi \qquad ...(3.7.4)$$

$$\text{But } I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$\text{and } V_{ph} = V_L \text{, for a delta load.}$$

$$\therefore P_T = 3\, V_L \frac{I_L}{\sqrt{3}} \cos\phi$$

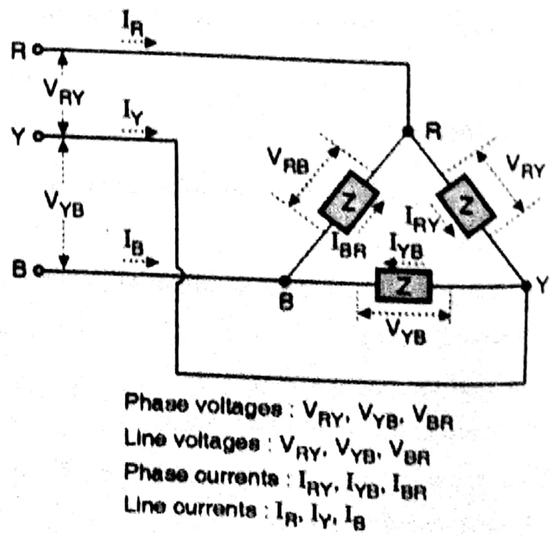
$$\therefore \text{Total power } P_T = \sqrt{3}\, V_L\, I_L \cos\phi \qquad ...(3.7.5)$$

$$\text{Where } \phi = \text{Angle between } V_{ph} \text{ and } I_{ph}$$

Thus the expression for the total power consumed for the delta connected load is same as that for the star connected load. But the values of the line currents for the two configurations will be different. This fact must be remembered while obtaining the power.

Summary of voltage / current relations for delta load :

Fig. 3.7.4 summarizes the relations between phase and line voltages and currents.



(A-1564) **Fig. 3.7.4 : Voltage and current relations for the delta connected load**

3.7.5 Power Factor :

- The load power factor for a 3 phase balanced delta load is equal to the power factor of each phase in the load.

$\therefore$ Overall P.F. = $\cos\phi$

Where ϕ = Angle between the phase voltage and phase current.

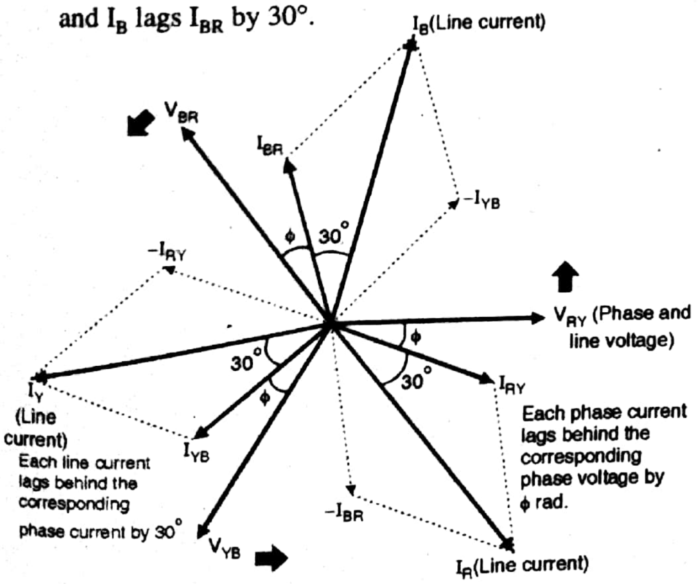
- So overall factor of 3-phase balanced delta load is equal to the cosine of the phase angle between the phase voltage and phase current.

3.7.6 The Complete Phasor Diagram for Delta Load :

- The nature of the impedances Z will decide whether the phase current lead, lag or be in phase with the corresponding phase voltages.
- The phasor diagram of Fig. 3.7.5 has been drawn for the lagging power factor load. Hence phase currents lag behind the phase voltages by an angle ϕ.

Conclusion from the phasor diagram :

1. The phase currents I_{RY}, I_{YB} and I_{BR} lag behind the corresponding phase voltages V_{RY}, V_{YB} and V_{BR} respectively by an angle ϕ.
2. Every line current lags the respective phase current by 30°. That means I_R lags I_{RY} by 30°, I_Y lags I_{YB} by 30° and I_B lags I_{BR} by 30°.



(A-747) **Fig. 3.7.5 : The complete phasor diagram for the delta connected load**

3.8 Different Types of Power and their Relations : S-10, S-11

MSBTE Questions

Q. 1 State different types of powers and their expressions for a 3-phase system. Draw power triangle. **(S-10, S-11, 4 Marks)**

We have already defined the different types of powers as,

1. Apparent power S (volt ampere VA)
2. Active power P (watts)
3. Reactive power Q (volt ampere reactive VAR)

All these are applicable to the three phase circuits also as follows :

$\therefore$ Total apparent power S = 3 × Apparent power per phase

$$\therefore S = 3 \times V_{ph} \times I_{ph}$$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \text{ ... For star load} \qquad ...(3.8.1)$$

$$\therefore S = \sqrt{3}\, V_L I_L \text{ (VA or kVA)} \qquad \text{... For star load}$$

$$\text{OR} \quad S = 3\, V_L \times \frac{I_L}{\sqrt{3}} \qquad \text{... For delta load ...(3.8.2)}$$

$$\therefore S = \sqrt{3}\, V_L I_L \text{ (VA or kVA)} \qquad \text{... For delta load}$$

$$\text{Total Active Power P} = 3 \times V_{ph} \times I_{ph} \times \cos\phi$$

$$= 3 \frac{V_L}{\sqrt{3}} \times I_L \cos\phi = 3\, V_L \times \frac{I_L}{\sqrt{3}} \cos\phi$$

$$\therefore P = \sqrt{3}\, V_L I_L \cos\phi \text{ watt}$$

...(star or delta load)

$$\text{Total Reactive Power } Q = 3\, V_{ph} I_{ph} \sin\phi$$

$$\therefore Q = \sqrt{3}\, V_L I_L \sin\phi \text{ VAR or kVAR}$$

...(star or delta load)

3.8.1 Power Triangle : S-10

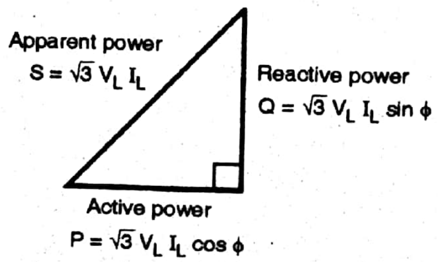
MSBTE Questions

Q. 1 State different types of powers and their expressions for a 3-phase system. Draw power triangle. **(S-10, 4 Marks)**

- The power triangle for a 3-phase system is as shown in Fig. 3.8.1, which shows that

$$\text{(Apparent Power)} = [(\text{Active Power})^2 + (\text{Reactive Power})^2]^{1/2}$$

$$S = \sqrt{P^2 + Q^2}$$



(A-748) **Fig. 3.8.1 : Power triangle for a three phase system**

3.8.2 Power Factor :

- The overall power factor of a three phase system is defined as the cosine of the angle between the phase voltage and phase current.

3.9 Comparison of Star Connection and Delta Connection :

W-07, W-09, W-10, S-11, W-12

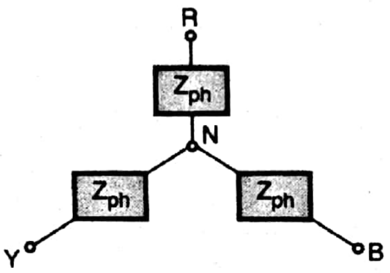
MSBTE Questions

Q. 1 Compare star connection with Delta connection. **(W-07, W-09, W-12, 4 Marks)**

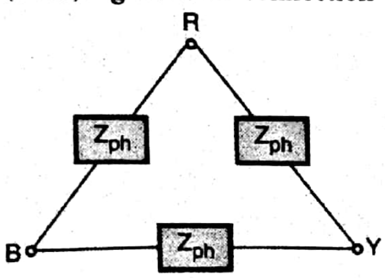
Q. 2 Compare star connected load with delta connected load. **(W-10, 4 Marks)**

Q. 3 Compare star and delta connections on the basis of connection, neutral point, relation between phase and line voltages, phase and line currents. **(S-11, 4 Marks)**

Sr. No.	Parameter	Star connection	Delta connection
1.	Connection	See Fig. A	See Fig. B
2.	Neutral point	Present	Absent
3.	Relation between phase and line voltages	$V_L = \sqrt{3}\, V_{ph}$	$V_L = V_{ph}$
4.	Relation between phase and line currents	$I_L = I_{ph}$	$I_L = \sqrt{3}\, I_{ph}$
5.	Total active power	$P = \sqrt{3}\, V_L I_L \cos\phi$	$P = \sqrt{3}\, V_L I_L \cos\phi$
6.	Total reactive power	$Q = \sqrt{3}\, V_L I_L \sin\phi$	$Q = \sqrt{3}\, V_L I_L \sin\phi$



(A-1656) **Fig. A : Star connection**



(A-1657) **Fig. B : Delta connection**

3.10 Applications of 3 Phase AC Circuits :

W-09

MSBTE Questions

Q. 1 Give applications of 3ϕ a.c. circuits. **(W-09, 4 Marks)**

1. 3 phase induction motors.
2. 3 phase synchronous motors.
3. Submersible water pumps.
4. Various machine - tool applications (lathe machine, grinder, milling machine etc.).
5. Large factories and educational institutions.

3.11 Solved Examples :

Ex. 3.11.1 : Three identical coils, each of (4.2 + j 5.6) ohms are connected in star across 415 V, 3 phase, 50 Hz supply. Determine :

1. V_{ph}. 2. I_{ph} 3. Power factor.
4. Power absorbed by each coil.
5. Power absorbed by the star connected load.

Soln. :

Given :

Impedance in each phase $Z_{ph} = (4.2 + j\,5.6) = 7\angle 53.13^\circ\ \Omega$

$V_L = 415$ V, star connected load.

1. Phase voltage $V_{ph} = \dfrac{V_L}{\sqrt{3}} = \dfrac{415\text{ V}}{\sqrt{3}} = 239.6$ V ...**Ans.**

2. Phase current $I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{239.6}{7} = 34.23$ A ...**Ans.**

Phase current is same as the line current

$\therefore\ I_L = 34.23$ A

3. Power factor $= \cos\phi = \cos(53.13^\circ)$

$= 0.6$ (lagging) ...**Ans.**

4. Power absorbed by each coil $= V_{ph} \times I_{ph} \times \cos\phi$

$= 239.6 \times 34.23 \times 0.6$

$= 4920.9$ Watts.

5. Power absorbed by each phase

$P = \sqrt{3}\, V_L I_L \cos\phi$

$= \sqrt{3} \times 415 \times 34.23 \times 0.6$

$\therefore\ P = 14762.7$ Watts ...**Ans.**

Ex. 3.11.2 : Prove that a three phase balanced load draws three times as much power when connected in delta, as it would draw when connected in star.

Soln. :

Step 1 : Calculate power in a star system :

$$\text{Power per phase} = P_{ph} = V_{ph}\, I_{ph} \cos\phi$$

$$\text{Total power} = 3\,P_{ph} = 3\,V_{ph}\, I_{ph} \cos\phi$$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times \frac{V_L}{\sqrt{3}\,Z} \cos\phi$$

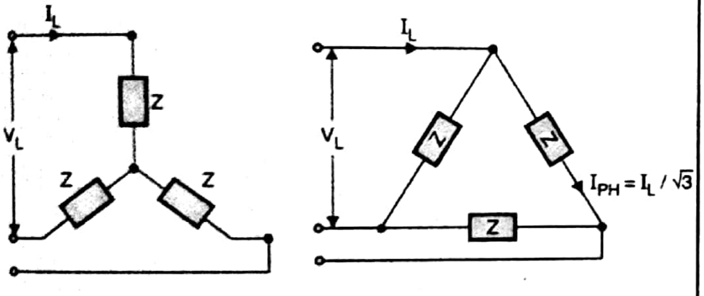
$$= \frac{V_L^2}{Z} \cos\phi \qquad \text{...(1)}$$

Step 2 : Total power in a delta circuit :

$$\text{Power per phase } P_{ph} = V_{ph}\, I_{ph} \cos\phi$$

$$\therefore \text{ Total power } P_T = 3\,P_{ph} = 3\,V_{ph}\, I_{ph} \cos\phi$$

$$\therefore P_T = 3\frac{V_L^2}{Z} \cos\phi \qquad \text{...(2)}$$



(a) Star load **(b) Delta load**

(A-2675) **Fig. P. 3.11.2**

- Compare Equations (1) and (2) assuming that the line voltage V_L and impedance per phase Z for both the types of loads are identical.
- Then

$$P_T\,(\text{Delta}) = 3\frac{V_L^2}{Z}\cos\phi = 3\,P_T\,(\text{star})$$

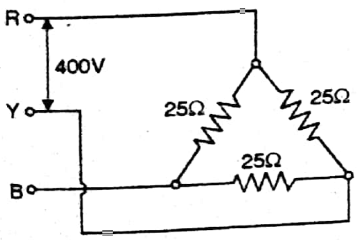
Hence it is proved that a 3 phase balanced delta load draws 3 times more power than the balance star load.

Ex. 3.11.3 : Three resistances of 25 Ω each are connected in delta across a 3-phase 400 V a.c. supply. Draw the circuit. Find phase current, line current, line voltage, phase voltage.

Soln. :

1. Circuit diagram :

The required circuit diagram is shown in Fig. P. 3.11.3.



(A-1566) **Fig. P. 3.11.3**

2. Phase and line voltages :

$$V_L = 400\text{ V} \qquad \text{...Ans.}$$

$$V_{ph} = V_L = 400\text{ V} \qquad \text{...Ans.}$$

3. Phase and line currents :

$$I_{ph} = \frac{V_{ph}}{R} = \frac{400}{25} = 16\text{ A} \qquad \text{...Ans.}$$

$$I_L = \sqrt{3}\,I_{ph} = \sqrt{3} \times 16$$

$$= 27.7\text{ A} \qquad \text{...Ans.}$$

Ex. 3.11.4 : Three impedances each of 3 ohm resistance and 4 ohm inductive reactance in series are connected in star across 3 phase, 400 Volts, 50 Hz a.c. supply. Determine :

1. Phase current 2. Line current

3. Power factor 4. Total power

Soln. :

Given : $R_{ph} = 3\ \Omega$, $X_{L\,ph} = 4\ \Omega$, star connected load, $V_L = 400$ Volts, f = 50 Hz.

To find : I_{ph}, I_L, PF and P_T

Step 1 : Find Z_{ph} and V_{ph} :

$$Z_{ph} = R_{ph} + jX_{Lph} = (3 + j4)$$

$$= \sqrt{(3)^2 + (4)^2}\ \angle \tan^{-1}(4/3)$$

$$\therefore Z_{ph} = 5 \angle 53.13^\circ\ \Omega$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\text{ Volts}$$

Step 2 : Find I_{ph} and I_L :

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{5 \angle 53.13^\circ}$$

$$= 46.19 \angle -53.13^\circ\text{ Amp.} \qquad \text{...Ans.}$$

For a star load, $I_L = I_{ph}$

$$\therefore I_L = 46.19\text{ Amp} \qquad \text{...Ans.}$$

Step 3 : Power factor :

$$PF = \cos\phi = \cos(-53.13)^\circ = 0.6\ (\text{lagging}) \qquad \text{...Ans.}$$

Step 4 : Total power :

$$P_T = \sqrt{3}\,V_L\,I_L \cos\phi = \sqrt{3} \times 400 \times 46.19 \times 0.6$$

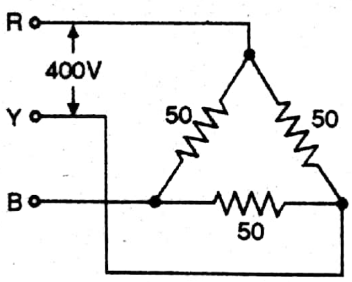
$$= 19.2\text{ kW} \qquad \text{...Ans.}$$

Ex. 3.11.5 : Three resistors of 50 Ω are connected in delta across a 400 V, 50 Hz a.c. supply. Calculate the line current, phase current, line voltage and phase voltage.

Soln. :

Given : $V_L = 400$ V, f = 50 Hz, $R_1 = R_2 = R_3 = 50\ \Omega$

To find : V_L, V_{ph}, I_L, I_{ph}



(A-3362) **Fig. P. 3.11.5 : Circuit diagram**

1. Phase and line voltages :

$$V_L = 400 \text{ V} \quad \text{...Ans.}$$
$$V_L = V_{ph} = 400 \text{ V} \quad \text{...Ans.}$$

2. Phase and line currents :

$$I_{ph} = \frac{V_{ph}}{R} = \frac{400}{50} = 8 \text{ A} \quad \text{...Ans.}$$
$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 8$$
$$= 13.856 \text{ A} \quad \text{...Ans.}$$

Ex. 3.11.6 : A balanced three-phase, star connected load is supplied from a three-phase, 400 V, 50 Hz supply. The resistance per phase is 10 Ω. Find the value of phase current, line current, power factor and total power consumed. **W-10, W-15, 4 Marks**

Soln. :

Given : $V_{ph} = 400$ V $f = 50$ Hz

$R = 10\ \Omega$, star type load

To find : I_{ph}, I_L, P.F, P_T

For star type load $I_{ph} = I_L$

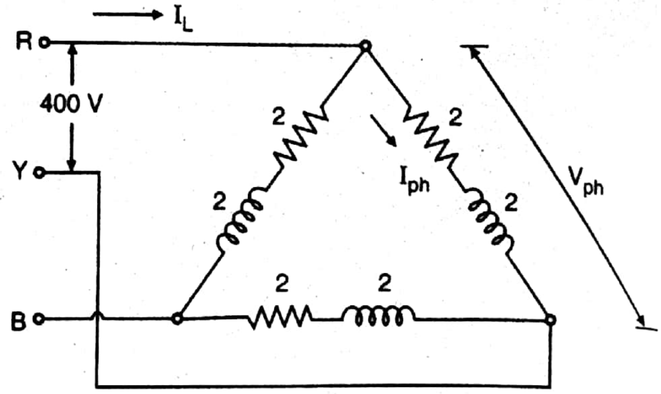
$$\therefore \quad I_{ph} = \frac{V_{ph}}{R} = \frac{400}{10} = 40 \text{ A}$$
$$\therefore \quad I_{ph} = I_L = 40 \text{ A} \quad \text{...Ans.}$$
$$Z_{ph} = 10 \angle 0°$$
$$\therefore \text{ Power factor } = \cos\phi = 1 \quad \text{...Ans.}$$
$$P_T = 3\ V_{ph}\ I_{ph} \cos\phi$$
$$= 3 \times 400 \times 40 \times 1$$
$$= 48000 \text{ Watts} = 48 \text{ kW} \quad \text{...Ans.}$$

Ex. 3.11.7 : Three impedances each of 2 Ω resistance and 2 Ω inductive reactance are connected in mesh across a 3 phase, 400 V ac supply. Determine the phase current, line current and its active and reactive power. **S-09, 4 Marks**

Soln. :

Given : $R_{ph} = 2\ \Omega, X_{ph} = 2\ \Omega$, $V_L = 400$ V, Delta type load

To find : I_{ph}, I_L, P_{active}, $P_{Reactive}$



(A-1655) **Fig. P. 3.11.7**

Step 1 : Find Z_{ph} :

$$Z_{ph} = R_{ph} + j\ X_{ph} = 2 + j2 = \sqrt{2^2 + 2^2}$$
$$= 2.83 \angle \tan^{-1}(2/2)$$
$$Z_{ph} = 2.83 \angle 45°\ \Omega$$

Step 2 : Find I_{ph} and I_L :

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}} = \frac{400}{2.83}$$
$$= 141.34 \text{ Amp} \quad \text{...Ans.}$$
$$I_L = \sqrt{3} \times I_{ph} = \sqrt{3} \times 141.34$$
$$= 244.81 \text{ Amp} \quad \text{...Ans.}$$

Step 3 : Active and reactive power :

$$\phi = 45°$$
$$P_{active} = \sqrt{3}\ V_L\ I_L \cos\phi$$
$$= \sqrt{3} \times 400 \times 244.81 \times \cos 45°$$
$$= 119.9 \text{ kW} \quad \text{...Ans.}$$
$$P_{reactive} = \sqrt{3}\ V_L\ I_L \sin\phi$$
$$= \sqrt{3} \times 400 \times 244.81 \times \sin 45°$$
$$= 119.9 \text{ kVAR} \quad \text{...Ans.}$$

Ex. 3.11.8 : A balanced star connected load is supplied from 400 V, 3ϕ, 50 Hz supply. The resistance per phase is 25 Ω. Calculate :
1. Line voltage 2. Phase voltage
3. Line current 4. Power consumed.
W-11, 4 Marks

Soln. :

Given :

$V_L = 400$ V, $f = 50$ Hz, $R_{ph} = 25\ \Omega$, Type of load : Star.

To find : 1. Line voltage 2. Phase voltage
3. Line current 4. Power consumed

Step 1 : To find line voltage :

$$V_L = 400 \text{ V} \quad \text{...Ans.}$$

Step 2 : To find phase voltage :

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V} \quad \text{...Ans.}$$

Step 3 : To find line current :

$$I_L = I_{ph} = \frac{V_{ph}}{R} = \frac{230.94}{25} = 9.23 \text{ A} \quad \text{...Ans.}$$

Step 4 : Power consumed :

$$Z_{ph} = 25 \angle 0°$$

$$\therefore \text{Power factor} = \cos\phi = 1$$

$$\therefore \quad P_T = \sqrt{3}\, V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 9.23 \times 1$$

$$P_T = 6394.7 \text{ Watts} \quad \text{...Ans.}$$

Ex. 3.11.9 : Three similar inductors, each of resistance 10 ohm and inductance 0.019 H are delta connected to a 3-phase, 415 V, 50 Hz supply. Calculate : 1. The value of line current 2. The power factor 3. Power input to the circuit.

S-13, 4 Marks

Soln. :

Given : $R = 10\ \Omega$, $L = 0.019$ H, $V_L = 415$ V, $f = 50$ Hz, delta connected load

To find : 1. The value of line current (I_L)

2. The power factor ($\cos\phi$)

3. Power input to the circuit (P_{in})

Step 1 : Calculate the value of power factor :

Reactance per phase $X = 2\pi fL = 2\pi \times 50 \times 0.019$

$$= 5.969\ \Omega$$

Impedance per phase $Z_{ph} = R + jX = (10 + j\,5.969)\ \Omega$

$$\therefore \quad Z_{ph} = [10^2 + (5.969)^2]^{1/2} \angle \tan^{-1}\left[\frac{5.969}{10}\right]$$

$$= 11.645 \angle 30.83°\ \Omega$$

$$\therefore \quad \phi = 30.83°$$

$$\therefore \quad PF = \cos\phi = 0.858 \quad \text{...Ans.}$$

Step 2 : Calculate the value of line current :

$$\text{Phase current } I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

$$\text{But } V_{ph} = V_L = 415 \text{ V} \quad \text{... For delta load}$$

$$\therefore \quad I_{ph} = \frac{415}{11.645} = 35.637 \text{ Amp}$$

$$\therefore \text{ Line current } I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 35.637$$

$$= 61.726 \text{ Amp} \quad \text{...Ans.}$$

Step 3 : Calculate power input to circuit :

$$P_{in} = 3 \times V_{ph} \times I_{ph} \times \cos\phi$$

$$= 3 \times 415 \times 35.637 \times 0.858$$

$$= 38.06 \text{ kW} \quad \text{...Ans.}$$

Ex. 3.11.10 : If $V_L = 400$ V and $I_L = 10$ A. Calculate the respective phase values for a :

1. Delta connection 2. Star connection.

(S-14, 4 Marks)

Soln. :

Given : $V_L = 400$ V, $I_L = 10$ A

To find : 1. Phase values of delta connection

2. Phase values of star connection

Step 1 : Phase values of delta connection :

For delta connection $V_L = V_{ph} = 400$ V ...Ans.

$$\text{Phase current } I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.773 \text{ Amp.}$$

Step 2 : Phase values of star connection :

For star connection, $I_{ph} = I_L = 10$ A ...Ans.

$$\text{Phase voltage } V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ Amp.} \quad \text{...Ans.}$$

Ex. 3.11.11 : A balanced three-phase delta connected load consists of three resistances each of four ohms connected to a 400 V, 3-phase, 50 Hz supply.

Find :

1. Phase voltage 2. Phase current

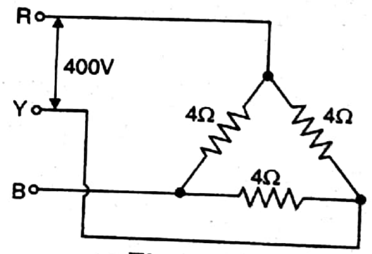
3. Line current 4. Power consumed

S-15, 4 Marks

Soln. :

1. Circuit diagram :

The required circuit diagram is shown in Fig. P. 3.11.11.



(A-4204) **Fig. P. 3.11.11**

2. Phase and line voltages :

$$V_L = 400 \text{ V} \quad \text{...Ans.}$$

$$V_{ph} = V_L = 400 \text{ V} \quad \text{...Ans.}$$

3. Phase and line currents :

$$I_{ph} = \frac{V_{ph}}{R} = \frac{400}{4} = 100 \text{ A} \quad \text{...Ans.}$$

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 100 = 173.2 \text{ A} \quad \text{...Ans.}$$

4. Power consumed :

$$P = \sqrt{3}\ V_L\ I_L \cos\phi$$

But $\cos\phi = 1$ because the load is resistive.

$$\therefore\ P = \sqrt{3} \times 400 \times 173.2$$

$$= 120\ \text{kW} \qquad \text{...Ans.}$$

Review Questions

Q. 1 Draw the phasor representation of three phase voltages.

Q. 2 Define phase sequence.

Q. 3 Explain the term : symmetrical or balanced system.

Q. 4 What is the importance of phase sequence ?

Q. 5 What is meant by a balanced load ?

Q. 6 Define power factor of a balanced 3 phase system.

Q. 7 State the advantages of polyphase systems.

Q. 8 Draw the complete phasor diagram for the balanced star load.

Q. 9 Draw the phasor diagram for the balanced delta load.

Q. 10 State different types of powers and their expressions for a 3 phase system.

Q. 11 Compare single phase and three phase system on the basis of following points :

(A) Voltage (B) Transmission efficiency

(C) Usage (D) Size of machine.

Q. 12 State the applications of 3 phase AC system.

Q. 13 Compare star and delta connections.

3.12 MSBTE Questions and Answers :

Summer 2014 [Total Marks - 08]

Q. 1 State any four advantages of three phase system over single phase system. **(Sections 3.2)** **(4 Marks)**

Q. 2 If V_L = 400 V and I_L = 10 A. Calculate the respective phase values for a :

1. Delta connection 2. Star connection.

(Ex. 3.11.10) **(4 Marks)**

Winter 2014 [Total Marks - 04]

Q. 3 State relation between phase and line current and phase and line voltage of the following system :

1. Star connected balanced system
2. Delta connected balanced system

(Sections 3.6.2, 3.6.3, 3.7.2, 3.7.3) **(4 Marks)**

Summer 2015 [Total Marks - 12]

Q. 4 State any four advantages of three phase system over single phase system.

(Section 3.2) **(4 Marks)**

Q. 5 What is line voltage and phase voltage ? Draw star connected 3-phase supply system and mark line voltage and phase voltage.

(Sections 3.6.1 and 3.6.2) **(4 Marks)**

Q. 6 A balanced three-phase delta connected load consists of three resistances each of four ohms connected to a 400 V, 3-phase, 50 Hz supply.

Find :

1. Phase voltage 2. Phase current
3. Line current
4. Power consumed **(Ex. 3.11.11)** **(4 Marks)**

Winter 2015 [Total Marks - 08]

Q. 7 Draw delta connected 3-phase supply system mark-line, phase voltage, line current and phase current. Write power equation.

(Sections 3.4.2, 3.7.1, 3.7.2, 3.7.3 and 3.7.4)

(4 Marks)

Q. 8 A balanced three-phase, star connected load is supplied from a three-phase, 400 V, 50 Hz supply. The resistance per phase is 10 Ω. Find the value of phase current, line current, power factor and total power consumed. **(Ex. 3.11.6)** **(4 Marks)**

Summer 2016 [Total Marks - 04]

Q. 9 State any four advantages of three phase system over single phase system.

(Section 3.2) **(4 Marks)**

Winter 2016 [Total Marks - 04]

Q. 10 State any four advantages of three phase system over single phase system.

(Section 3.2) **(4 Marks)**

Summer 2017 [Total Marks - 08]

Q. 11 State the advantages of polyphase (3-phase) system over single phase system (any four).

(Section 3.2) **(4 Marks)**

Q. 12 Draw delta connected load. State the relationship between line and phase values for the same.

(Sections 3.4.2, 3.7.2 and 3.7.3) **(4 Marks)**

3.13 I-Scheme Questions and Answers :

Winter 2018 [Total Marks – 04]

Q. 1 Draw a balanced 3-phase star connected load. Show various line and phase values and also state the relationship between them.

(Sections 3.5.1, 3.6.1, 3.6.2 and 3.6.3) **(4 Marks)**

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