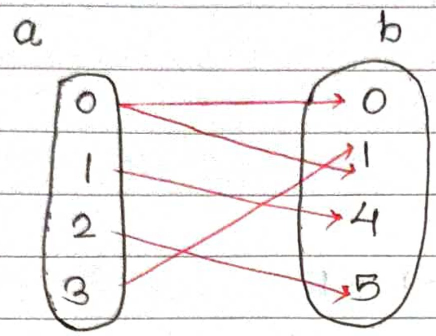
Module - 04

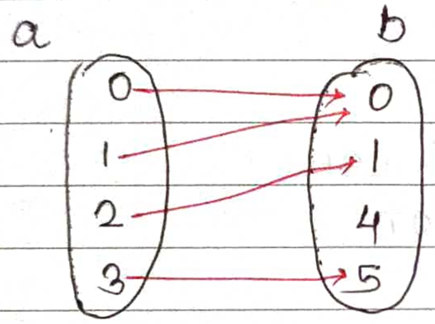
PROBABILITY

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$



not a function



Function

Probability mass function : (PMF)

discrete random variable.

$$\begin{Bmatrix} x_1, & x_2, & x_3, & \dots & x_n \\ P_1, & P_2, & P_3, & \dots, & P_n \end{Bmatrix}$$

i) $P_i \geq 0$

ii) $\sum_i P_i = 1$

probability density function : (PDF)

$f(x) \rightarrow$ contineous rv (random variable)

$f(x) \longrightarrow$ 1) $f(x) \geq 0 \quad \forall \; x \in R$

2) $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1$

Que. pmf of r.v. x is 0 except at $x = 0, 1, 2$
At these points $p(0) = 3c^3$, $p(1) = 4c - 10c^2$
$p(2) = 5c - 1$
Find c. $p(0 < x \leq 2)$

→

pmf is given
$\therefore \sum p(x) = 1$
$p(0) + p(1) + p(2) = 1$
$3c^3 + 4c - 10c^2 + 5c - 1 = 1$
$3c^3 + 4c - 10c^2 + 5c - 2 = 0$
For $c = 1$
$3 + 4 - 10 + 5 - 2 = 0$
$\therefore$ $(c=1)$ is root of eqn.
By synthetic division

$$\begin{array}{c|cccc} 1 & 3 & -10 & 9 & -2 \\ & \downarrow & 3 & -7 & 2 \\ \hline & 3 & -7 & 2 & 0 \end{array}$$

$(c-1)(3c^2 - 7c + 2) = 0$
$(c-1)[+3c^2 - 6c - c + 2] = 0$
$(c-1)[3c(c-2) - 1(c-2)] = 0$
$(c-1)(c-2)(3c-1) = 0$
$c = 1$ or $c = 2$ or $c = 1/3$

For $c = 1$ and 2 $p(x)$ is -ve
$\boxed{\therefore \; c = 1/3}$ is correct

x	0	1	2
$p(x)$	1/9	2/9	2/3

$\therefore\ p(0<x\leq 2) = p(1) + p(2)$

$= \frac{2}{9} + \frac{2}{3}$

$$\boxed{\therefore\ p(0<x\leq 2) = \frac{8}{9}}$$

Que verify whether the following function is pdf

1) $f(x) = \frac{2}{9} x\left(2 - \frac{x}{2}\right) \qquad 0 < x \leq 3$

2) $f(x) = \frac{1}{2} e^{-|x|} \qquad -\infty < x < \infty$

→ 1)

$f(x) = \frac{2}{9} x (2 - x/2)$

For all values in $(0, 3]$ function $f(x) \geq 0$
hence it satisfies condition for pdf

by using condition

$$\int_{-\infty}^{\infty} f(x)\,dx =$$

$$\cancel{\int_{-\infty}^{0} \frac{2}{9} x(2-x/2)}^{\,0} + \int_{0}^{3} \frac{2x(2-x/2)}{9} + \cancel{\int_{3}^{\infty} \frac{2(2-x/2)x}{9}}^{\,0}$$

$$= \left[\frac{2}{9} \times 2 \times \frac{x^2}{2} - \frac{1}{9} \times \frac{x^3}{3}\right]_0^3$$

$= 1$

$\therefore$ given function is pdf.

2) $\frac{1}{2} e^{-|x|} \quad -\infty < x < \infty$

$f(x) \geq 0 \quad \forall \quad -\infty < x < \infty$

$$\int_{-\infty}^{\infty} f(x)\, dx = \int_{-\infty}^{\infty} \frac{1}{2} e^{-|x|}\, dx$$

$$= \frac{1}{2}\left[\int_{-\infty}^{0} e^{-|x|}\, dx + \int_{0}^{\infty} e^{-|x|}\, dx\right]$$

we have

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$= \frac{1}{2}\left[\int_{-\infty}^{0} e^{+x}\, dx + \int_{0}^{\infty} e^{-x}\, dx\right]$$

$$= \frac{1}{2}\left[\left[e^{x}\right]_{-\infty}^{0} - \left[e^{-x}\right]_{0}^{\infty}\right]$$

$$= \frac{1}{2}\left[(1-0) - (0-1)\right]$$

$$= 1$$

$\therefore$ Given $f(x)$ is pdf.

Que. find the value of k if each of the foollowing function is pdf

a) $f(x) = kx^4 e^{-x/2}$, $0 \leq x < \infty$

b) $f(x) = \dfrac{k}{1+x^2}$, $-\infty < x < \infty$

→

a) $f(x)$ is pdf

$$\int_{-\infty}^{0} f(x)\, dx = 1$$

$$\int_{0}^{\infty} kx^4 e^{-x/2}\, dx = 1 \qquad \int_{0}^{\infty} e^{-kt} t^{n-1}\, dt = \frac{\Gamma n}{k^n}$$

$$k \frac{\Gamma 5}{(1/2)^5} = 1$$

$$k \times \frac{4 \times 3 \times 2 \times 1 \times 2^5}{1} = 1$$

$$k = \frac{1}{768}$$

b) $f(x)$ is pdf

$$\therefore \int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} \frac{k}{1+x^2}\, dx = 1 \qquad k\left[\tan^{-1} x\right]_{-\infty}^{\infty} = 1$$

$$k\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 1$$

$$\therefore k = 1/\pi$$

Page No:
Date:

Type-2

Binomial and Poisson Distribution:

1) Binomia Distribution -

If experiment results in two ways either success or failure the we are using B.D.

let p → probability of success
q → probability of failure

if expt. repeated n times then prob. of x success $p(x \text{ success}) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$, x = 0, 1, 2, 3, \ldots, n$

mean of B.D. = $\bar{x} = np$
varience of BD = $\sigma^2 = npq$

if expt each ~~has~~ n trials is replaced N times then expected frequency $= N \cdot P(x)$

Poisson Distribution -

Here no. of success only observed not number of failure.

ex. no. of accidents on road
no. of cars passing a particular point on road.

Poisson Distribution →

$p \rightarrow$ prob. of success
$n \rightarrow \infty$ and $p \rightarrow 0$

$$p(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

B.D → small n, finite
P.S → $n \rightarrow \infty$, $p \rightarrow 0$

mean of P.D $= \bar{x} = np = \lambda$
variance of P.D $= \sigma^2 = np = \lambda$

* $p(x > a) = 1 - p(x \leq a)$
$p(x \geq 0) = 1 - p(x < a)$

Page No.:
Date:

Que. If 10% bolts produced by machine are deffective. Calculate the probability that out of sample selected at random of 10 bolts, not more than 1 bolt are deffective

→

Success → deffective bolt

$p = 10\% = \frac{10}{100} \times \frac{1}{10}$, $q = \frac{9}{10}$, $n = 10$

$p(x \leq 1) = p(x=0) + p(x=1)$

$= {}^{10}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} + {}^{10}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^9$

$= 0.7361$

Que. Each of 5 sequences of MCQ has 4 choices and only one of which is correct. A student is attending to guess answer what is probability that

1) exactly 3 ans correct
2) At most 3 ans correct
3) atleast 1 ans correct

→ Here we use B.D

$n = 5$

success → no. of quenstions answered correct

$p = 1/4$ $\quad$ $p + q = 1$

$q = 3/4$

$p(x) = p$ (no. of questions answered correctly)

$$p(x) = {}^nC_x \, p^x \, q^{n-x} = {}^5C_x \, (1/4)^x \, (3/4)^{n-x}$$

1) $p(x=3) = {}^5C_3 \, (1/4)^3 (3/4)^2 = 0.0879$

2) $p(x \leq 3)$ = p(atmost 3 correct ans)

$= p(x=0) + p(x=1) + p(x=2) + p(x=3)$

or

$p(x \leq 3) = 1 - p(x>3)$

$= 1 - [\, p(x=4) + p(x=5)]$

$p(x \leq 3) = 0.9844$

3) p(atleast 1 correct) $= p(x \geq 1)$

$= 1 - p(x < 1)$

$= 1 - p(x=0)$

$= 0.7627$

Que. The probability that the person who undergoes the kidney operation will recover is 0.7 Find the prob. that of 6 patients who undergoes same operation

1) none - will recover
2) all will recover
3) half will recover
4) Atleast half will recover.

→ Here we use B.D.

success → recovery of patient

$p = 0.7$, $q = 0.3$, $n = 6$

$p(x) =$ p(no. of patients recovered)

$= {}^nC_x\, p^x\, q^{n-x}$

$p(x=0) = {}^6C_0\, p^0\, q^6$

$= 0.0007$

$p(x=6) = {}^6C_6\, (0.7)^6\, (0.3)^0$

$= 0.1176$

$p(x=3) = {}^6C_3\, (0.7)^3\, (0.3)^3$

$= 0.1852$

$p(x \geq 3) = p(x=3) + p(x=4) + p(x=5) + p(x=6)$

$p(x \geq 3) = 0.9296$

Que. Between 2 and 3 pm the average no. of phone-calls per min coming into switchboard of company is 2.5 Find the prob. during a minute there will be

i) no phone call

ii) exactly 3 calls

→

Here we have to use poisson Distri.

$np = \lambda = 2.5$

$p(x) =$ p(no. of phonecalls per min)

$$= \frac{e^{-\lambda}\lambda^x}{x!}$$

i) $p(\text{no phone call}) = \dfrac{e^{-2.5} \times (2.5)^0}{0!}$

$= 0.0821$

2) $p(x=3) = \dfrac{e^{-2.5} \times (2.5)^3}{3!} = 0.2138$

Que. If the probablity that an individual suffers a bad reaction from injuction is 0.001 Determine the prob. that out of 2000 individuals

i) exactly 3

2) more than 2 suffers a bad reaction

→

Here n is large and p is very small so we have to we P.D.

$n = 2000 \quad p = 0.001 \quad \lambda = np = 2$

$p(x) =$ p(no. of individuals suffers bad reaction)

$p(x) = \dfrac{e^{-2} 2^x}{x!}$

$p(x=3) = 0.1804$

$p(x>2) = 1 - p(x \leq 2)$

$= 1 - [p(x=0) + p(x=1) + p(x=2)]$

$p(x>2) = 0.3233$

Que. six dice are thrown 729 times. How many times do you expect atleast 3 dice to show 5 or 6.

→

Here we have to use B.D.

$n = 6$, $N = 729$

success → getting a number 5 or 6

$p = p(\text{getting } 5 \text{ or } 6)$

$= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$q = 1 - \frac{1}{3} = \frac{2}{3}$

$p(x) = p(\text{dice showing } 5 \text{ or } 6)$

$= {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$= {}^{6}C_{x}\,(1/3)^{x}\,(2/3)^{n-x}$

$p(x \geq 3) = 1 - p(x < 3)$

$= 1 - [\,p(x=0) + p(x=1) + p(x=2)\,]$

$p(x \geq 3) = 0.3196$

no. of times dice shows 5 or 6

$= N\,p(x)$

$= 729 \times 0.3196$

≈ 233

Page No.:
Date:

Que. A fair coin is tossed 8 times, find the prob. that it shows head is
a) exactly 5 times
b) larger number of time than tail
c) atleast once

→

Here we have to use B.D.

success → getting head

$p = 1/2 \qquad q = 1/2 \qquad n = 8$

$$p(x) = p(\text{getting head})$$
$$= {}^{n}C_{x}\, p^{x} q^{n-x}$$
$$= {}^{8}C_{x}\, (1/2)^{x} (1/2)^{n-x}$$
$$= {}^{8}C_{x}\, (1/2)^{8}$$

1) $p(x=5) = 0.2188$

2) $p(x \geq 5) = 0.3633$

3) $p(x \geq 1) = 1 - p(x < 1)$
$= 0.9961$

Normal Distribution :-

Probability curve →

Area under that curve between $x=\alpha$ and $x=\beta$ is $P(\alpha < x < \beta)$ and total area 1.

$$P(\alpha < x < \beta) = \int_{\alpha}^{\beta} f(x)\,dx = 1$$

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

where
$m \to$ mean
$\sigma \to$ S.D. of x

Probability of contineous random variable lies between x_1 and x_2 is area under the curve between x_1 and x_2.

$$P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} y\,dx = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$$

$$z = \frac{x-m}{\sigma}$$

$$dz = \frac{dx}{\sigma}$$

$$dx = \sigma dz$$

$x = x_1$, $z_1 = \frac{x_1 - m}{\sigma}$ $\qquad$ $x = x_2$, $z_2 = \frac{x_2 - m}{\sigma}$

$$P(z_1 \leq z \leq z_2) = \int_{z_1}^{z_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} z^2} \sigma\, dz$$

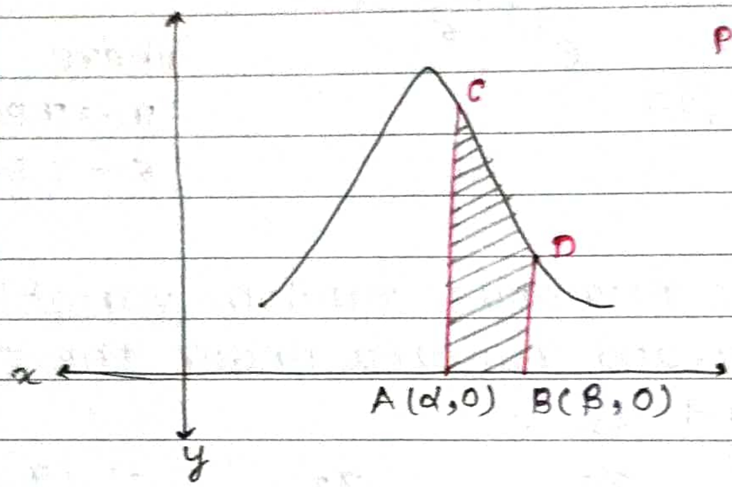
$$P(z_1 \leq z \leq z_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

For standard curve

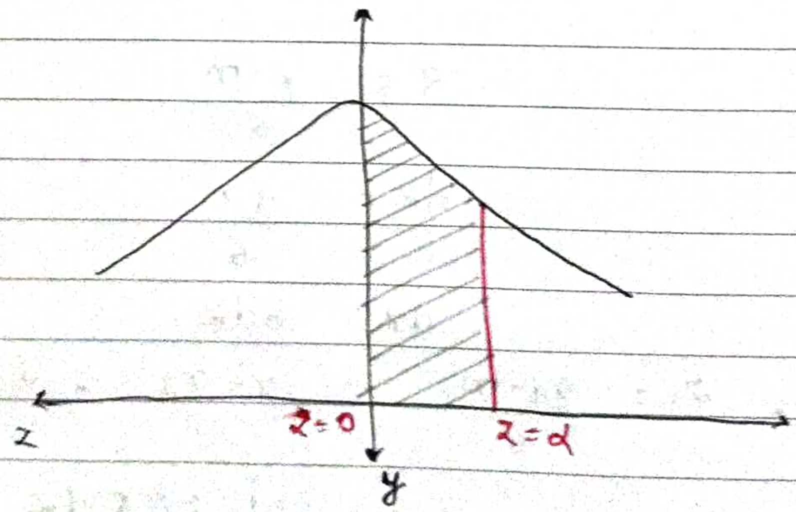
$N(m, \sigma) \longrightarrow N(0, 1)$

Normal curve is bell shaped symmetric about highest ordinate

$P(\alpha < x < \beta)$



$$P(x_1 \leq x \leq x_2) = P(z_1 \leq z \leq z_2)$$



area under curve between $z=0$ to $z=\alpha$ is $P(0 \leq z \leq \alpha)$

Area from $z=0$ to $z=2$ is 0.4772
Area from $z=0$ to $z=1$ is 0.3413
Area from $z=0$ to $z=0.525$ is 0.2
Area from $z=0$ to $z=1.28$ is 0.4
Area from $z=0$ to $z=0.25$ is 0.1
Area from $z=0$ to $z=0.52$ is 0.2
Area from $z=0$ to $z=1.645$ is 0.45
Area from $z=0$ to $z=0.5$ is 0.19
Area from $z=0$ to $z=1.4$ is 0.42

Que. students of the class give an aptitude test their marks are normally distributed with mean 60, S.D. 5 what is the %tage of students who scored more than 60.

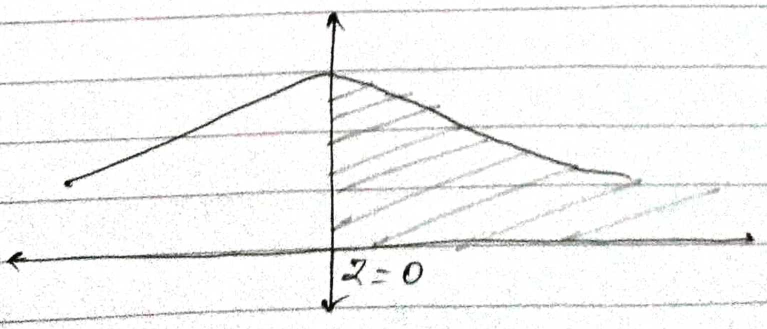
→

Given : $m = 60$, $\sigma = 5$

$x \rightarrow$ marks
we have to find $P(x > 60)$

$$P(x>60) = P\left(\frac{x-m}{\sigma} > \frac{60-m}{\sigma}\right)$$

$$= P(z > 0)$$



$P(z>0) = 1/2$
∴ 50% of students got more than 60 marks.

Que. Sacks of sugar packed by an automatic loader have an average weight 100 kg and S.D. 250 gm. Assuming a normal distribution. Find a chance of getting a sack weighing less than 99.5 kg.

→

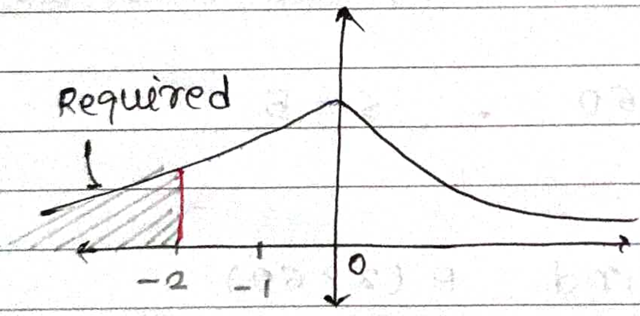
Let x be weight

$m = 100$ kg

$\sigma = 250$ gm $= 0.25$ kg

$$P(x < 99.5) = P\left(\frac{x-m}{\sigma} < \frac{99.5-m}{\sigma}\right)$$

$$= P(z < -2)$$



$$P(z < -2) = P(-\infty < z < 0) - P(-2 < z < 0)$$

$$= 0.5 - 0.4772$$

$$= 0.0228$$

Que. weights of 4000 students are found to be normally distributed with m = 50 kg and S.D. = 5 kg. Find the number of students with weights 1) less than 45 kg
2) between 45 and 60 kg

→

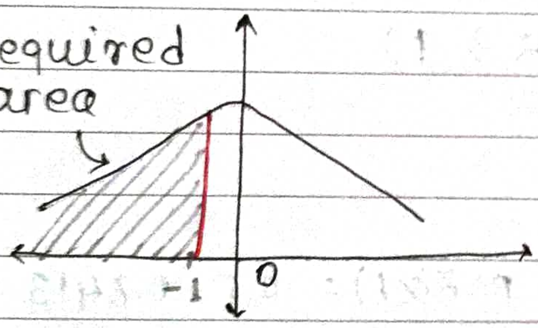
Given : $m = 50$
$\sigma = 5$

let x be weight

i) $P(x < 45) = P\left(\frac{x-m}{\sigma} < \frac{45-m}{\sigma}\right)$

$= P(z < -1)$

required area



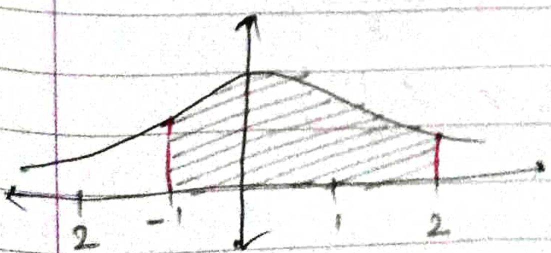
$P(z<-1) = P(-\infty < z < 0) - P(-1 < z < 0)$

$= \frac{1}{2} - 0.3413$

$P(z < -1) = 0.1587$

∴ no. of students having weight less than 45 are ≈ 635 (0.1587 × 4000)

ii) $P(45 < x < 60) = P\left(\frac{45-m}{\sigma} < \frac{x-m}{\sigma} < \frac{60-m}{\sigma}\right)$



$= P(-1 < z < 2)$

$P(-1<z<2) = 0.8185$

∴ no. of students = 4000 × 0.8185
= 3274

Que. In an examination given by 500 candidates the average and S.D of marks obtained are 40 and 10 resp. Assuming marks are distributed normally. Find approximately

i) how many will pass if 50 is fix as min.

ii) What should be the min. if 350 candidates are to pass.

→

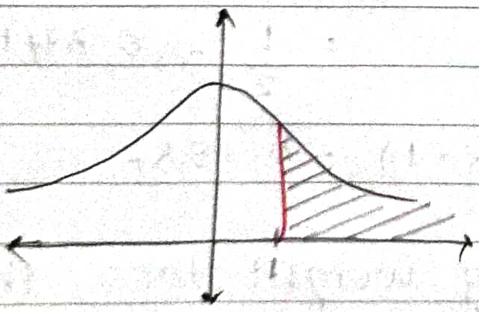
let x be marks

$m = 40$

$\sigma = 10$

i) $P(x \geq 50) = P\left(\frac{x-m}{\sigma} \geq \frac{50-m}{\sigma}\right)$

$= P(z \geq 1)$



$P(z \geq 1) = 0.5 - 0.3413$

$= 0.1587$

no. of students $= 500 \times 0.1587$

$= 79$

ii) let min marks $= a$

$\therefore P(x \geq a) = P\left(\frac{x-m}{\sigma} \geq \frac{a-m}{\sigma}\right)$

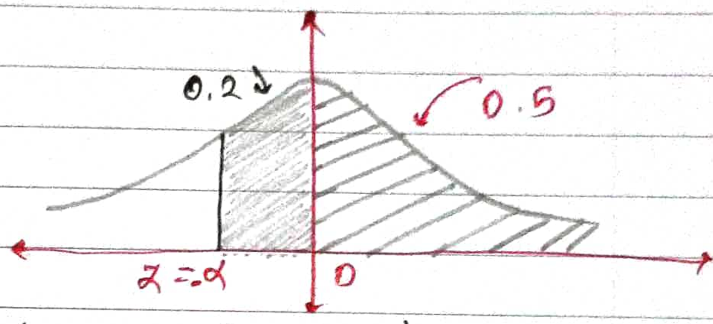
$= P\left(z \geq \frac{a-m}{\sigma}\right)$

total students = 500

passed are 350

$\therefore$ % of passed students = 70% = $\frac{70}{100} = 0.7$

$$p\left(z \geq \frac{a-m}{\sigma}\right) = 0.7$$



area from $z = -\alpha$ to $z = \infty$ is 0.7

we know area from $z = 0$ to $z = \infty$ is 0.5

$\therefore$ area between $-\alpha$ and 0 is 0.2

and also area from 0 to 0.525 is 0.2

$\therefore$ value of $-\alpha$ is -0.525 $(P(z \geq -\alpha)$

$$z = \frac{x - m}{\sigma}$$

$$x = \sigma z + m$$

$$x = 10(-0.525) + 40$$

$$x = 34.75$$

$\therefore$ minimum maks are 34.75

DATE : / /

que. 8] A machine has 14 identical components that functions independently. It will stop working if 3 or more fails. If probability that component fails is 0.1, find the probability that machine will be working.

→ We will solve this by Binomial Distribution.

$p(x = \text{machine fails}) = {}^{n}C_{x} (0.1)^{x} (0.9)^{n-x}$

$\therefore p = 0.1, \quad q = 0.9, \quad n = 14$

$p(x<3) = p(x=0) + p(x=1) + p(x=2)$

$p(x=0) = 14C_0 (0.1)^0 (0.9)^{14}$

$= 0.2288$

$p(x=1) = 14C_1 (0.1)^1 (0.9)^{13}$

$= 14 \times 0.1 \times 0.2542$

$= 0.3559$

$p(x=2) = 14C_2 (0.1)^2 (0.9)^{12}$

$= \frac{\cancel{14}^{7} \times 13}{\cancel{2} \times 1} (0.1)^2 (0.9)^{12}$

$= 0.2570$

$p(x<3) = 0.8417$

que.9] During a war, one ship out of 9 sunk on an average in making a certain journey. What is the probability that exactly 3 out of 6 ships will arrive safely.

→ $q = \frac{1}{9}$, $p = \frac{8}{9}$, $n = 6$

$$P(x=3) = {}^6C_3 \left(\frac{8}{9}\right)^3 \left(\frac{1}{9}\right)^3$$

$$= \frac{\cancel{6} \times 5 \times 4}{\cancel{3 \times 2}} \cdot \frac{8^3}{9^6}$$

$$= 20 \times 0.0010$$

$$= 0.0193$$

que.10] In a sampling, the mean no. of defective bolts manufactured by machine in a sample of 20 is 2. Determine the expected no. of sample out of such 500 samples to contain at least 2 defective bolts.

→ $p = \frac{1}{10}$, $q = \frac{9}{10}$, $n = 20$, $N = 500$

PAGE NO.:
DATE: / /

At least 2 bolts defective.

$\therefore P(x \geq 2) = 1 - P(x<2)$

$P(x=0) = 20C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20}$

$= 0.1216$

$P(x=1) = 20C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{19}$

$= \frac{20}{10} \left(\frac{9}{10}\right)^{19}$

$= 0.2702$

$P(x<2) = 1 - (0.1216 + 0.2702)$
$= 1 - 0.3918$
$= 0.6082$

$\therefore 500 \times 0.6082 = 304.1000$
$\simeq 300$

Que. 11] In a certain factory, producing cycle tyres, there is a small chance one in 500, for any tyre to be defective, the tyres are supplied in lots of 20. Calculate the approximate no. of ~~tyres~~ lots containing no defective, one defective, 2 defective tyres, in a of 20,000 tyres.

We will use Poison Distribution.

$$P = \frac{1}{500} = 0.002, \quad n=20, \quad N=20,000$$

$$\lambda = np = 20 \times 0.002$$
$$= 0.04$$

$P(x)$ = Prob. of no. of defective tyres.

$$= \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

i) No defective :⇒

$$P(x=0) = \frac{e^{-0.04} \cdot (0.04)^0}{0!}$$

$$= 0.9608$$

ii) one defective :

$$P(x=1) = \frac{e^{-0.04} \cdot (0.04)^1}{1}$$

$$= 0.0384$$

iii) Two defective :

$$P(x=2) = \frac{e^{-0.04} \cdot (0.04)^2}{2!}$$

$$= 0.0008$$

$$\text{No. of lots} = \frac{20000}{20} = 1000$$

∴ No. of lots containing no defective tyres =

1000×0.9608
$= 960.8$
$\simeq 961$

No. of lots containing one defective tyre
$= 1000 \times 0.0384$
$= 38.4$
$\simeq 38$

No. of lots containing two defective ~~lots~~ tyres
$= 1000 \times 0.0008$
$= 0.8$
$\simeq 1$

Que. 12] A company has two cars which it hires out day by day. The no. of demands for a car at each day is distributed as Poisson variate with mean 1.5. Calculate the no. of days in year for which i) neither car is in demand.
ii) Demand is refused.

→ $np = 1.5$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} = \frac{e^{-1.5} \cdot (1.5)^x}{x!}$$

i) $$P(x=0) = \frac{e^{-1.5} \cdot (1.5)^0}{0!}$$

$$= 0.2231$$

No. of days for year

365×0.2231 OR 366×0.2231

81 OR 82 days.

ii) $P(x>2) = 1 - P(x \leq 2)$

$P(x=0) = 0.2231$

$P(x=1) = \frac{e^{-1.5} \cdot (1.5)^1}{1!}$

$= 0.3347$

$P(x=2) = \frac{e^{-1.5} \cdot (1.5)^2}{2!}$

$= 0.2510$

$P(x>2) = 1 - P(x \leq 2)$

$= 1 - 0.8088$

$= 0.1912$

$365 \times 0.1912 = 69.78 \simeq 70$ days

$366 \times 0.1912 = 69.97 \simeq 70$ days.

Q. 13] 7 coins are tossed & the no. of heads obtained are noted. The exp. is repeated 128 times. Following distribution is obtained.

No. of heads	Frequency	Expected = $N\, {}^nC_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{n-x}$
0	7	1
1	6	7
2	19	21
3	35	35
4	30	35
5	23	21
6	7	7
7	1	1

Ans. of (i)

1) Fit BD if

i) if the coin is unbiased
ii) The nature of coin is not known.

→ i) $n = 7,\ p = \frac{1}{2},\ q = \frac{1}{2}$

$P(x) = {}^nC_x (p)^x (q)^{n-x}$

$P(x) = {}^7C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{n-x}$

$P(x=0) = {}^7C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^7$

$= 0.0078$

$\therefore 128 \times 0.0078 = \underline{1}$

$P(x=1) = {}^7C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^6$

$= \frac{7}{2^7}$

$= 0.0547$

$\therefore 128 \times 0.0547 = \underline{7}$

PAGE NO.:
DATE: / /

$P(x=2) = 7C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^5$

$= \frac{7\times6}{2\times1} \left(\frac{1}{2}\right)^7$

$= \frac{21}{2^7}$

$P(x=2) = 0.1641$

$\therefore N \times P(x=2) = 128 \times 0.1641$

$= 21.0048$

$\simeq 21$

$P(x=3) = 7C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^4 = \frac{7\times\cancel{6}\times5}{\cancel{3}\times\cancel{2}} \left(\frac{1}{2}\right)^7$

$= 0.2734$

$N \times P(x=3) = 128 \times 0.2734 = 35$

$P(x=4) = 7C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \frac{7\times\cancel{6}\times5}{\cancel{3}\times\cancel{2}} \left(\frac{1}{2}\right)^7$

$= 0.2734$

$N \times P(x=4) = 128 \times 0.2734 = 35$

$P(x=5) = 7C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^2 = \frac{7\times\cancel{6}^{3}}{\cancel{2}\times1} \left(\frac{1}{2}\right)^7 = 0.1641$

$N \times P(x=5) = 128 \times 0.1641 = 21$

$P(x=6) = 7C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^1 = \frac{7}{2^7} = 0.0547$

$128 \times 0.0547 = 7$

$P(x=7) = 7C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^0 = 0.0078$ | $128 \times 0.0078 = 1$

ii)

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i}$$

$=$

ii)

No. of heads x_i	Frequency f_i	$x_i f_i$
0	7	0
1	6	6
2	19	38
3	35	105
4	30	120
5	23	115
6	7	42
7	1	7
	$\sum f_i = 128$	$\sum x_i f_i = 433$

$$\therefore \bar{x} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{433}{128}$$

$$\therefore \bar{x} = 3.3828$$

Mean $= np = 3.38$

$\therefore 7p = 3.38$

$\therefore p = 0.48$

$$\therefore p = \frac{12}{25}$$

Now, Nature of coin is unknown.
Wehare $p = 0.48$, $q = 0.52$, $n = 7$

$\therefore P(x=0) = 7C_0 \left(\frac{12}{25}\right)^0 \left(\frac{13}{25}\right)^7$

$= 0.0103$

$N \times 0.0103 = 1.3159$

$N \times P(0) \simeq \underline{1}$

$P(x=1) = 7C_1 \left(\frac{12}{25}\right)^1 \left(\frac{13}{25}\right)^6$

$= 7 \times \frac{12}{25} \times \left(\frac{13}{25}\right)^6$

$= 0.0664$

$N \times P(1) = 128 \times 0.0664$

$= 8.5029 \simeq \underline{\underline{9}}$

$P(x=2) = 7C_2 \left(\frac{12}{25}\right)^2 \left(\frac{13}{25}\right)^5$

$= \frac{7 \times \cancel{6}^{3}}{\cancel{2}} \left(\frac{12}{25}\right)^2 \left(\frac{13}{25}\right)^5$

$= 21 \times 0.0088$

$= 0.1840 \simeq \cancel{2}$

$\therefore N \times P(2) = 128 \times 0.1840$

$= 23.54 \simeq \underline{\underline{24}}$

$P(x=3) = 7C_3 \left(\frac{12}{25}\right)^3 \left(\frac{13}{25}\right)^4$

$$= \frac{7 \times \cancel{6} \times 5}{\cancel{3 \times 2}} \cdot \frac{12^3 \times 13^4}{25^7}$$

$$= 0.283$$

$$N \times P(3) = 128 \times 0.283$$

$$= 36.2256 \simeq \underline{\underline{36}}$$

$$P(x=4) = 7C_4 \left(\frac{12}{25}\right)^4 \left(\frac{13}{25}\right)^3$$

$$= 0.283$$

$$N \times [P(4)] = 128 \times 0.283$$

$$= 36.2256 \simeq \underline{36}$$

$$P(x=5) = 7C_5 \left(\frac{12}{25}\right)^5 \left(\frac{13}{25}\right)^2$$

$$= \frac{21 \times 12^5 \times 13^2}{25^7}$$

$$= 0.1447$$

$$N \times P(5) = 128 \times 0.1447 = 18.52 \simeq \underline{\underline{19}}$$

$$P(x=6) = 7C_6 \left(\frac{12}{25}\right)^6 \left(\frac{13}{25}\right)^1$$

$$= 0.0445$$

$$N \times P(6) = 128 \times 0.0445 = 5.6985 \simeq \underline{\underline{6}}$$

$$P(x=7) = 7C_7 \left(\frac{12}{25}\right)^7 \left(\frac{13}{25}\right)^0$$

$$= 0.0059$$

$$N \times P(7) = 0.7514 \simeq \underline{\underline{1}}$$

DATE : / /

Heads	Expected $= N \times {}^nC_x (0.48)^x (0.52)^{n-x}$
0	1
1	9
2	24
3	36
4	36
5	19
6	6
7	1