PROBABILITY DISTRIBUTION

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Experiment
- → Deterministic , Certain Outcome , No uncertainty
- → Non deterministic / random experiment

- **Sample Space (S) :** Set of all possible , distinct outcomes of an experiment
- **Event :** Subset of sample space .
- no. of events $= 2^n$; $n =$ no. of elements
- **Mutually exclusive event :** (Disjoint) $A \cap B = \phi$ = null set
- **Cards**
  - Red (26)
    - diamond 13
    - heart 13
  - Black (26)
    - spade 13
    - Club 13
- **Independent event :**

$$\boxed{P(A\cap B) = P(A)\cdot P(B)}$$

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

$$\boxed{P(A') = 1 - P(A)}$$

$$\boxed{0 \leq P(A) \leq 1}$$

$$\boxed{P(A\cap \bar{B}) = P(A) - P(A\cap B)}$$

$$\boxed{P(B\cap \bar{A}) = P(B) - P(A\cap B)}$$

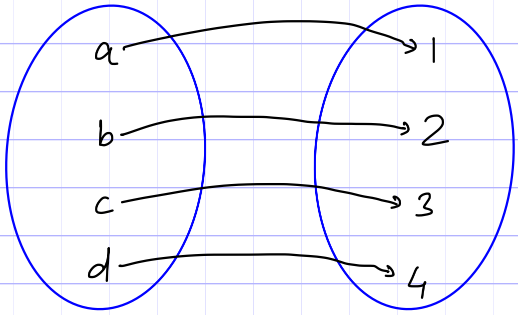
$$\boxed{\begin{aligned} P(A\cup B\cup C) &= P(A) + P(B) + P(C) \\ &\quad - P(A\cap B) - P(B\cap C) - P(A\cap C) \\ &\quad + P(A\cap B\cap C) \end{aligned}}$$

$$\boxed{P(\phi) = 0}$$

★ Random Variable :

→ Function where S is domain & R is co-domain/ Range.

→ $S \to R$, for every element in S, there is a distinct image in R.



★ Probability Distribution :

→ Random var. with its probability is called ....

★ Probability mass function (p.m.f.) :

→ X is discrete random var. defined on Sample Space S.

& Range $= \{x_1, x_2, \dots, x_n\}$

$p_1 \quad p_2 \quad \dots \quad p_n$

if 1) $P_i \geq 0 \qquad i = 0, 1, 2, \dots, n$

2) $\Sigma P_i = 1$

then, it is probability mass function.

★ Probability Density Function :

$X \to$ Continuous random variable

$f(x) \rightarrow$ 1) $f(x) \geq 0 \quad \forall\, x \in R$

2) $\int_{-\infty}^{\infty} f(x).dx = 1$

Q 1: The pmf of a random variable X is 0 except at X = 0, 1, 2. At this points, $P(0) = 3c^3$, $P(1) = 4c - 10c^2$, $P(2) = 5c - 2$

Find 1) c

2) $P(0 < X \leq 2)$

$\rightarrow$ $\Sigma P = 1$

$\therefore\ P(0) + P(1) + P(2) = 0$

$\therefore\ 3c^3 + 4c - 10c^2 + 5c - 2 = 0$

$\therefore\ 3c^3 - 10c^2 + 9c - 2 = 0$

1	3	-10	9	-2
	$-$	3	-7	2
	3	-7	2	$\boxed{0}$

$\therefore\ (c-1)(3c^2 - 7c + 2) = 0$

$\therefore\ (c-1)(c-2)(3c-1) = 0$

$c = 1,\ c = 2,\ c = 1/3$

$\boxed{\therefore\ c = 1/3}$ $\because$ for $c = 1, c = 2$, $P(0) > 1$ impossible

x	0	1	2
$p(x)$	$1/9$	$2/9$	$2/3$

ii) $p(0 < x \leq 2) = p(1) + p(2)$

$= 2/9 + 2/3$

$$\boxed{\therefore p(0 < x \leq 2) = 8/9}$$

Q 2 : Verify whether following function is P.D.F.

1) $f(x) = \frac{2}{9} x\left(2 - \frac{x}{2}\right) \qquad 0 < x \leq 3$

2) $f(x) = \frac{1}{2} e^{-|x|} \qquad -\infty < 0 < \infty$

→ 1) $f(x) = \frac{2}{9} \cdot x\left(2 - \frac{x}{2}\right)$

for $x = 0$, $f(0) = 0$

for $x = 1$, $f(1) = \frac{2}{9} \times 1\left(2 - \frac{1}{2}\right)$

$= \frac{\cancel{2}}{9} \times \frac{3}{\cancel{2}}$

$f(1) = 1/3$

for $x = 3$, $f(3) = \frac{2}{9} \times 3\left(2 - 3/2\right)$

$= \frac{1}{3}$

here, $f(x) \not\geq 0$ for $0 < x \leq 3$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\cdot dx + \int_{3}^{\infty} f(x)\cdot dx$$

$$= 0 + \int_{0}^{3} \frac{2}{9}x\left(2-\frac{x}{2}\right)dx + 0$$

$$= \frac{2}{9}\int_{0}^{3}\left(2x - \frac{x^2}{2}\right)dx$$

$$= \frac{2}{9}\left\{\left[x^2\right]_0^3 - \frac{1}{6}\left[x^3\right]_0^3\right\}$$

$$= \frac{2}{9}\left(9 - 0 - \frac{\cancel{27}\,9}{\cancel{6}\,2} + 0\right)$$

$$= \frac{2}{9}\left(9 - \frac{9}{2}\right)$$

$$= \frac{2}{9} \times \frac{9}{2}$$

$$= 1$$

$\boxed{\therefore \text{ Given } f(x) \text{ is P.D.F.}}$

2) $f(x) = \frac{1}{2}e^{-|x|}$

$f(x) \geqslant 0 \quad \forall -\infty < x < \infty$

$$\int_{-\infty}^{\infty} f(x).\,dx = \int_{-\infty}^{\infty} \frac{1}{2} e^{-|x|} dx$$

$$= \int_{-\infty}^{0} \frac{1}{2} \cdot e^{-|x|} dx + \int_{0}^{\infty} \frac{1}{2} e^{-|x|} dx$$

$x > 0,\ +x$

$x < 0,\ -x$

$$= \frac{1}{2} \int_{-\infty}^{0} e^{-(-x)} dx + \frac{1}{2} \int_{0}^{\infty} e^{-(x)} dx$$

$$= \frac{1}{2} \left\{ \left(e^{x}\right)_{-\infty}^{0} + (-1) \cdot \left(\frac{1}{e^{x}}\right)_{0}^{\infty} \right\}$$

$$= \frac{1}{2} \left\{ (1-0) - (0-1) \right\}$$

$$= \frac{1}{2} \times 2$$

$$= 1$$

$\boxed{\therefore \text{ Given } f(x) \text{ is P.D.F.}}$

Q 3: Find value of k if each of following function is p.d.f.

1) $f(x) = kx^4 \cdot e^{-x/2}$ $\quad 0 < x < \infty$

2) $f(x) = \dfrac{k}{1+x^2}$ $\quad -\infty < x < \infty$

→ 1) $\because$ given $f(x)$ is p.d.f.,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\therefore \int_0^\infty k \cdot x^4 \cdot e^{-x/2}\, dx = 1$$

$$\boxed{\int_0^\infty e^{-pt} \cdot t^{n-1}\, dt = \frac{\Gamma n}{p^n}}$$ Gamma function

$$\therefore k \int_0^\infty e^{-x/2} \cdot x^{5-1}\, dx = k \times \frac{\Gamma 5}{(1/2)^5} = 1$$

$$\boxed{\therefore k = \frac{1}{768}}$$

2) $f(x) = \dfrac{k}{1+x^2}$ $\quad -\infty < x < \infty$

$$\therefore k \int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx = 1$$

$$\therefore k \left[\tan^{-1} x\right]_{-\infty}^{\infty} = 1$$

$$\therefore k\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 1$$

$$\boxed{\therefore k = \frac{1}{\pi}}$$

★ Binomial & Poission Distribution :

• Binomial Distribution :

→ If experiment results in 2 ways, either success or failure, then we use binomial distribution.

let, p be prob. of success , q be prob. of failure

such that, $p + q = 1$

→ If experiment is repeated n times, then Prob. of x success is

$P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$ where $x = 0, 1, 2, \dots n$

→ e.g. Tossing of coin. (H/T) , Exam result (pass/fail), result of inspection of product (defective/Non defective), no. of male birth out of n births in hospital, result of game (win/loose) , no. of rainy days in a month, etc.

→ Mean of bino. distri. $= \bar{X} = n.p$

→ Variance of bino. distri. $= \sigma^2 = n.p.q$

→ If experiment with n trials is repeated N times, then expected frequency $= N \times P(x)$

- ## Poission Distribution :

→ here, no. of success is only observed, not the no. of failure.

→ e.g. No. of accidents on road, no. of cars passing particular point on road, no. of death, etc.

→ p = prob. of success. ($p \to 0$) & no. of trials are ∞ large. (finite) ($n \to \infty$)

& $np = \lambda$.

→ $$P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$ ..... for x success

$x = 0, 1, 2, \dots \infty$

→ Mean of Pois. Distri. $= \bar{X} = np = \lambda$

Variance $= \sigma^2 = np = \lambda$

→ $P(x > a) = 1 - P(x \leq a)$

→ $P(x \geq a) = 1 - P(x < a)$

Q 1: If 10 % bolts produced by machine are defective, calculate prob. that out of samples selected at random of 10 bolts, not more than 1 bolt are defective.

→ (success = defective bolt)

$p = \frac{1}{10}$, $q = \frac{9}{10}$, $n = 10$

$$\therefore P(x \leq 1) = P(0) + P(1) = {}^nC_x \cdot p^x \cdot q^{n-x} + {}^nC_x \cdot p^x \cdot q^{n-x}$$

$$= {}^{10}C_0 \cdot \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{10} + {}^{10}C_1 \cdot \left(\frac{1}{10}\right)^1 \cdot \left(\frac{9}{10}\right)^9$$

$$= \left(\frac{9}{10}\right)^{10} + \cancel{10} \times \frac{1}{\cancel{10}} \times \left(\frac{9}{10}\right)^9$$

$$= \left(\frac{9}{10}\right)^9 \left[\frac{9}{10} + 1\right]$$

$$= (0.9)^9 [1.9]$$

$$\boxed{\therefore P(x \leq 1) = 0.7361}$$

Q 2: Each of the five questions of MCQ has 4 choices. Only one of which is correct. A student is attempting to guess the answer. What is Probab. that student will get
1) Exact 3 answer correct.
2) At most 3 answer correct.
3) At least 1 correct answer.

→ here, we use binomial distribution.

$\therefore$ Success → No. of correct answers

$\therefore p = \frac{1}{4}$, $q = \frac{3}{4}$, $n = 5$

Prob. of no. of Q answered correctly ;

$$\therefore P(x) = {}^5C_x \cdot \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{5-x}$$

1) $P(x=3) = {}^5C_3 \times \left(\frac{1}{4}\right)^3 \times \left(\frac{3}{4}\right)^2 = 10 \times \frac{1}{64} \times \frac{9}{16}$

$$\boxed{P(x=3) = 0.0879}$$

2) $P(x \leq 3) = 1 - P(x > 3)$

$$= 1 - \left[P(4) + P(5)\right]$$

$$= 1 - \left[{}^5C_4 \times \left(\frac{1}{4}\right)^4 \times \left(\frac{3}{4}\right)^1 + {}^5C_5 \cdot \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0\right]$$

$$\boxed{P(x \leq 3) = 0.9844}$$

3) $P(x \geq 1) = 1 - \left[P(x<1)\right]$

$$= 1 - P(x=0)$$

$$\boxed{P(x \geq 1) = 0.7627}$$

Q3: Prob. that, the person who undergoes kidney operation will recover is 0.7. Find prob. that, of the 6 patients who undergo kidney operation,

1) None will recover

2) all will recover

3) half will recover

4) At least half will recover.

→ We use Binomial.

Success → Recovery of patient

$n = 6$, $p = 0.7$, $q = 0.3$

$$P(x) = {}^6C_x \times \left(\frac{7}{10}\right)^x \times \left(\frac{3}{10}\right)^{6-x}$$

1) $x = 0$, for none will recover.

$$\therefore P(0) = {}^6C_0 \times \left(\frac{7}{10}\right)^0 \left(\frac{3}{10}\right)^6$$

$$= 1 \times 1 \times \left(\frac{3}{10}\right)^6$$

$$\boxed{P(0) = 0.0007}$$

2) all will recover, $x = 6$

$$P(6) = {}^6C_6 \times \left(\frac{7}{10}\right)^6 \left(\frac{3}{10}\right)^0$$

$$\boxed{P(6) = 0.1176}$$

3) half recover, $x = 3$

$$\therefore P(3) = {}^6C_3 \times (0.7)^3 (0.3)^3$$

$$\boxed{P(3) = 0.1852}$$

4) $P(x \geq 3) = P(3) + P(4) + P(5) + P(6)$

$$\boxed{P(x \geq 3) = 0.9296}$$

Q4 : Betⁿ 2 & 3 pm, the avg. no. of phone calls per minute coming into a switchboard of company is 2.5. Find prob. during a minute there will be i) No phone call.
2) Exact 3 calls.

→ here, we use 'Poission distribution'.

$np = \lambda = 2.5$; Success = No. of phone call recid.

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} = \frac{e^{-2.5} \times 2.5^{x}}{x!}$$

1) $x = 0$, $$P(0) = \frac{e^{-2.5} \times 2.5^{0}}{0!}$$

$$\boxed{P(0) = 0.0821}$$

2) $x = 3$, $$P(3) = \frac{e^{-2.5} \times 2.5^{3}}{3!}$$

$$\boxed{P(3) = 0.2138}$$

Q5 : If probability that an individual suffers a bad reacⁿ from the injⁿ is 0.001. Determine the prob. that out of 2000 individuals,
1) Exactly 3
2) More than 2, suffer bad reaction.

→ We use, poission ∵ n large, p small

$n = 2000$, $p = 0.001$, $\therefore \lambda = np = 2$

$P(x)$ = prob. of no. of individ. suffer bad reacⁿ $= \dfrac{e^{-2} \times 2^{x}}{x!}$

1) $\therefore P(3) = \dfrac{e^{-2} \times 2^{3}}{3!}$

$$\boxed{P(3) = 0.1804}$$

2) $\therefore P(x > 2) = 1 - P(x \leq 2)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\dfrac{e^{-2} \times 2^{0}}{0!} + \dfrac{e^{-2} \cdot 2^{1}}{1!} + \dfrac{e^{-2} \cdot 2^{2}}{2!}\right]$

$$\boxed{P(x > 2) = 0.3233}$$

Q 6 : Six dice are thrown 729 times. How many times do you expect at least 3 dice to show 5 or 6.

→ We use, Binomial distribution.

$n = 6$, $N = 729$

success ⇒ getting 5 or 6

$\therefore p = P(\text{getting } 5 \text{ or } 6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$\therefore q = \frac{2}{3}$

$\therefore P(x) = {}^{6}C_{x} \times \left(\frac{1}{3}\right)^{x} \times \left(\frac{2}{3}\right)^{6-x}$

⇒ $\therefore P(x \geq 3) = 1 - P(x < 3)$ .... at least 3

$= 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - \left\{{}^{6}C_{0} \times \left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{6} + {}^{6}C_{1} \times \left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{5} + {}^{6}C_{2} \times \left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^{4}\right\}$

$\therefore \boxed{P(x \geq 3) = 0.3196}$

But, experiment is repeated 729 times.

$\therefore$ No. of times at least 3 dice shows 5 or 6

$= 729 \times 0.3196$

$= 232.9884$

$\boxed{\approx 233}$

Q7: A fair coin is tossed 8 times. Find prob. that, it shows ① heads exactly 5 times. ② larger no. of time than tail ③ At least once.

$\rightarrow$ $p = \frac{1}{2}, \quad q = \frac{1}{2}, \quad n = 8$ $\boxed{B.D.}$

$\therefore P(x) = {}^{8}C_{x} \times \left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{8-x}$

$= {}^{8}C_{x} \left(\frac{1}{2}\right)^{8}$

1) $\therefore P(5) = {}^{8}C_{5} \times \left(\frac{1}{2}\right)^{8}$

$\boxed{P(5) = 0.2188}$

2) $P(x \geq 5) = P(5) + P(6) + P(7) + P(8)$

$= \left(\frac{1}{2}\right)^{8}\left[{}^{8}C_{5} + {}^{8}C_{6} + {}^{8}C_{7} + {}^{8}C_{8}\right]$

$= \left(\frac{1}{2}\right)^{8}\left[56 + 28 + 8 + 1\right.$

$$\boxed{P(x \geq 5) = 0.3633}$$

3) $P(x \geq 1) = 1 - P(0)$

$$\boxed{P(x \geq 1) = 0.9961}$$

Q 8: A machine has 14 identical components that work independently. It will stop working if 3 or more fail. If prob. that component fails is 0.1, Find prob. that machine will be working.

→ Binomial, success → Machine working

$\therefore q = 0.1 \quad , p = 0.9$

$$P(x) = {}^{14}C_x \cdot (0.9)^x (0.1)^{14-x}$$

$\Rightarrow P(x < 3) = P(0) + P(1) + P(2)$

$$= {}^{14}C_0 \cdot (0.9)^0 (0.1)^{14} + {}^{14}C_1 (0.9)^1 (0.1)^{13}$$

$$+ {}^{14}C_2 \cdot (0.9)^2 (0.1)^{12}$$

$$\boxed{P(x < 3) = 0.8417}$$

Q 9: During a war, one ship out of nine sunk on an average in making a certain journey. What is probability that, exactly three out of 6 ship will arrive safely?

→ Binomial; Success → Safe arrival

$$\therefore q = \frac{1}{9} \quad , \quad p = \frac{8}{9}$$

$$\therefore P(x) = {}^6C_x \cdot \left(\frac{8}{9}\right)^x \left(\frac{1}{9}\right)^{6-x}$$

$$\Rightarrow P(x=3) = {}^6C_3 \times \left(\frac{8}{9}\right)^3 \left(\frac{1}{9}\right)^3$$

$$\boxed{P(3) = 0.0193}$$

Q 10: In a sampling, mean number of defective bolts manufactured by machine in a sample of 20 is 2. Determine expected number of samples out of such 500 samples to contain at least 2 defective bolts.

$\rightarrow$ Binomial, $N = 500$, $n = 20$, mean, $np = 2$

$$\therefore p = \frac{1}{10}, \quad q = \frac{9}{10}$$

$\therefore$ Success $\rightarrow$ Defective bolt

$$\therefore P(x) = {}^{20}C_x \left(\frac{1}{10}\right)^x \left(\frac{9}{10}\right)^{20-x}$$

$$\therefore P(x \geq 2) = 1 - P(x < 2)$$

$$= 1 - [P(0) + P(1)]$$

$$= 1 - \left[{}^{20}C_0 \times \left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^{20} + {}^{20}C_1 \times \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{19}\right]$$

$$\boxed{P(x \geq 2) = 0.6082}$$

$\therefore$ No. of samples containing at least 2 def. bolts $= N \times P(x \geq 2)$

$$= 500 \times 0.6082$$

$$= 304.1$$

$$\boxed{\approx 304}$$

Q 11: In a certain factory producing cycle tyres, there is a small chance 1 in 500 for any tyre to be defective, the tires are supplied in lots of 20. Calculate approx. number of lots containing no defective, 1 defective, 2 defective tyres in a consignment of 20000 tyres.

→ Poission $\because$ P very small

$$p = \frac{1}{500}, \quad n = 20$$

$$\therefore \lambda = np = \frac{20}{500} = \frac{1}{25} = 0.04$$

$$\therefore \lambda = 0.04$$

$$\therefore P(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \frac{e^{-0.04} \times (0.04)^{x}}{x!}$$

1) $P(0) = \dfrac{e^{-0.04} \times 1}{1}$

$$\boxed{P(0) = 0.9608}$$

2) $P(1) = \dfrac{e^{-0.04} \times 0.04}{1}$

$$\boxed{P(1) = 0.0384}$$

3) $P(2) = \dfrac{e^{-0.04} \times (0.04)^{2}}{2!}$

$$\boxed{P(2) = 0\ 0008}$$

Now, 20000 tyres $\Rightarrow$ 1000 lots ......(each lot = 20 tyres)

$\Rightarrow$ (1) No. of lots containing no defective tyres $= 1000 \times 0.9608 = 960.8 \approx 961$

(2) No. of lots containing 1 defective tyres $= 1000 \times 0.0384 = 38.4 \approx 38$

(3) No. of lots containing 2 defective tyres $= 1000 \times 0.0008 = 0.8 \approx 1$

Q12: A company has 2 cars which it hires out day by day. No. of Demands for a car at each day is distributed as a poission variate with mean 1.5. Calculate Number of days in year for which →

(1) neither car is in demand.

(2) Demand is refused.

→ Poission, $\lambda = np = 1.5$.

$$p(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$p(x) = \frac{e^{-1.5} \times (1.5)^{x}}{x!}$$

1) $\therefore p(x=0) = \dfrac{e^{-1.5} \times 1}{1}$

$$\boxed{p(x=0) = 0.2231}$$

$\therefore$ No. of days in year for which neither car is in demand $= 365 \times 0.2231$

$= 81.4425$

$\boxed{\approx 81}$

also for 366 leap year

$= 81.6656$

$\boxed{\approx 82}$

2) Demand is refused. (More than 2 cars demanded per day)

$\therefore p(x>2) = 1 - p(x \leq 2)$

$= 1 - \left[p(0) + p(1) + p(2)\right]$

$\boxed{p(x>2) = 0.1912}$

$\therefore$ No. of days on which demand is refused $= 0.1912 \times 365$

$= 69.7$

$\approx \boxed{70}$

for leap

$= 0.1912 \times 366$

$= 69.9792$

$\approx \boxed{70}$

Q13: 7 coins are tossed & number of heads obtained are noted. Experiment is repeated 128 times. Following distribution is obtained

No. of heads	0	1	2	3	4	5	6	7
frequency	7	6	19	35	30	23	7	1

1) Fit bino. distri. if coin is unbiased.

2) " " " if nature is unknown of coin

Expected freq. = ?

$\Rightarrow N \times P(x)$

$\rightarrow$ 1) $n = 7, \; p = \frac{1}{2}, \; q = \frac{1}{2}$

$p(x) = {}^7C_x \left(\frac{1}{2}\right)^{x+7-x} = {}^7C_x \left(\frac{1}{2}\right)^7$

$$N \times p(0) = {}^7C_0 \times \frac{1}{2^7} = 1$$

$$N \times p(1) = {}^7C_1 \times \frac{1}{2^7} \times 128 = 7$$

$$N \times p(2) = {}^7C_2 \times \frac{1}{2^7} \times 128 = 21$$

$$N \times p(3) = {}^7C_3 \times \frac{1}{2^7} \times 128 = 35$$

$$N . p(4) = {}^7C_4 \times \frac{1}{2^7} \times 128 = 35$$

$$N \times p(5) = {}^7C_5 \times \frac{1}{2^7} \times 128 = 21$$

$$N p(6) = {}^7C_6 \times \frac{1}{2^7} \times 128 = 7$$

$$N \times p(7) = {}^7C_7 \times \frac{1}{2^7} \times 128 = 1$$

but, $\Sigma N \times p(x)$ should be 128. ✓✓ $\boxed{\text{satisfied}}$ Correct

2) Nature of coin is unknown.

☆ $$\overline{X} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{433}{128} = 3.38$$

$\therefore$ mean $= np = 3.38$ $\quad \therefore 7 \times p = 3.38$

$\therefore p = 0.48$, $q = 0.52$

$$p(x) = {}^7C_x \times (0.48)^x \times (0.52)^{7-x}$$

Expected frequencies $\Rightarrow$

$N \times p(0) =$

$N \times p(1) =$

$N \times p(2) =$

$N \times p(3) =$

$N \times p(4) =$

$N \times p(5) =$

$N \times p(6) =$

$N \times p(7) =$

★ Normal Distribution :

(Continuous Probability distribution)

→ Probability curve : Area under curve between $x=\alpha$ & $x=\beta$ is same as $P(\alpha < x < \beta)$.
- Total area under the curve $= 1$

→ Imp. Curve (Useful) ⇒ Normal curve.

$$P(\alpha < x < \beta) = \int_{\alpha}^{\beta} f(x) \cdot dx = 1$$

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

m = mean of x
σ = Std. dev^n of x

Prob. of contin. ran. vari. which lies bet^n x_1 & x_2 is the area under curve between x_1 & x_2.

$$P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} y \cdot dx = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$$

put, $z = \dfrac{x-m}{\sigma}$

$\therefore dz = \dfrac{dx}{\sigma}$

$\therefore dx = \sigma \cdot dz$

$$\therefore P(x_1 \leq x \leq x_2) = \int_{z_1}^{z_2} \frac{1}{\sigma\sqrt{2\pi}} e^{\left(\frac{-1}{2}z^2\right)} \sigma \, dz$$

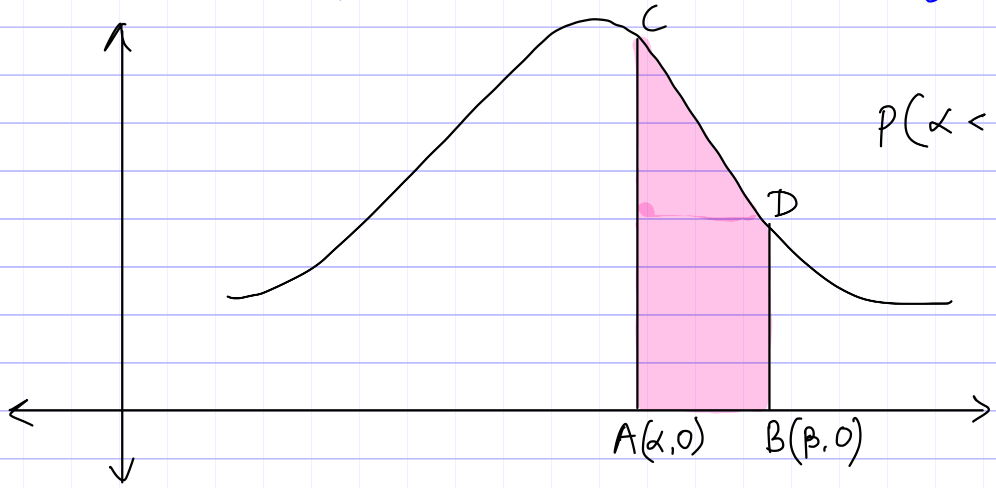
$$\therefore P(x_1 \le x \le x_2) = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$\boxed{\therefore y(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}}$$

$$N(m, \sigma) \longrightarrow N(\underset{\text{mean}}{0}, \underset{\text{s.d.}}{1})$$

Properties of Normal Curve :

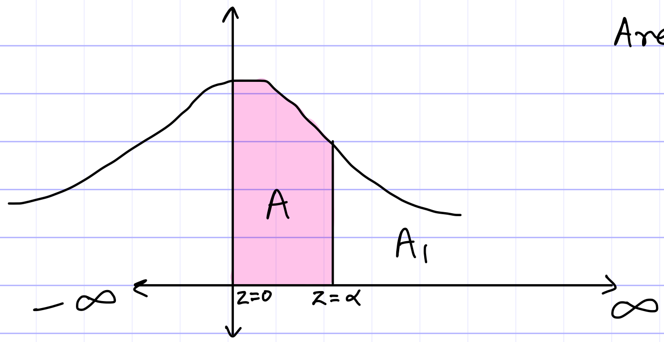
① Normal curve is bell shaped & symmetric about highest ordinate.



$P(\alpha < x < \beta)$

② $P(x_1 \le x \le x_2) = P(z_1 \le z \le z_2)$

③



$$\text{Area}(0 \to \infty) = \text{Area}(0 \to -\infty) = \frac{1}{2}$$

$$\boxed{\therefore A = \frac{1}{2} - A_1}$$

Common for all problems :

1) Area from $z=0$ to $Z=2$ is 0.4772.
2) " $z=0$ to $z=1$ is 0.3413.
3) " $z=0$ to $z=0.525$ is 0.2.
4) " $z=0$ to $z=1.28$ is 0.4.
5) " $z=0$ to $z=0.25$ is 0.1.
6) " $z=0$ to $z=0.52$ is 0.2.
7) " $z=0$ to $z=1.645$ is 0.45.
8) " $z=0$ to $z=0.5$ is 0.19.
9) " $z=0$ to $z=1.4$ is 0.42.

Q1: Students of class give an aptitude test. Their marks are normally distributed with mean 60, std. dev. 5. What is %age of student scored more than 60.

→ $x \rightarrow$ marks, (Random variable)

$$\therefore P(x>60) = P\left(\frac{x-m}{\sigma} > \frac{60-m}{5}\right)$$

$$= P\left(Z > \frac{60-60}{5}\right)$$

$$= P(Z>0)$$

$$\boxed{P(x>60) = 0.5}$$

$\therefore$ % age of student $= 50\%$

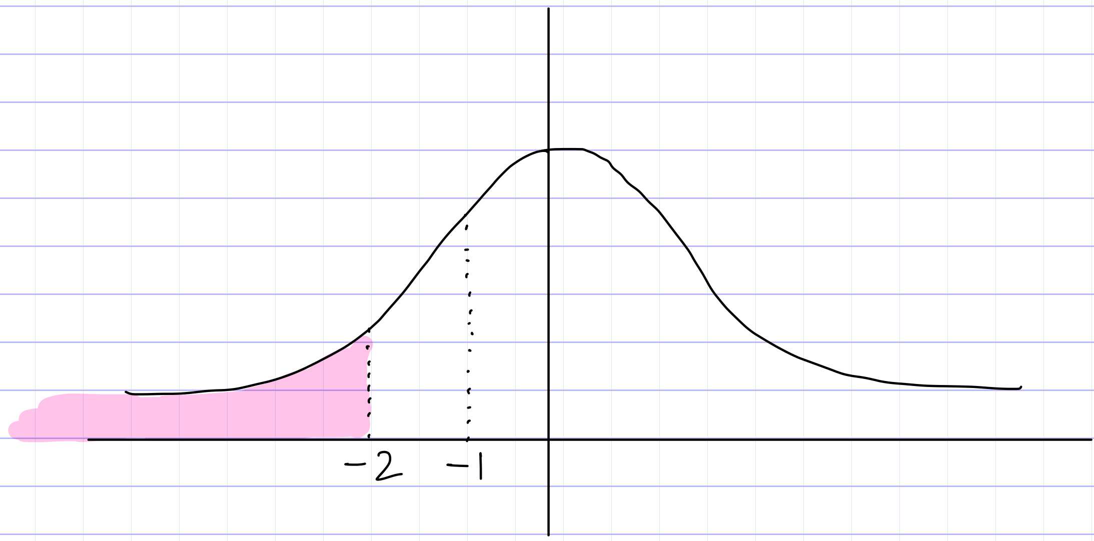
Q2: Sacks of sugar packed by an auto loader have an avg. weight 100 kg with s.d. 250 g. Assuming normal distri., find a chance of getting a sack weighing less than 99.5 kg.

→ Random var, x → weight of sack

$m = 100$ kg , $\sigma = 250$ g $= 0.25$ kg

$$P(x < 99.5) = P\left(\frac{x-m}{\sigma} < \frac{99.5 - 100}{0.25}\right)$$

$$= P(Z < -2)$$



∴ Reqd Prob $= 0.5 - P(-2 < Z < 0)$

$= 0.5 - P(0 < Z < 2)$

$= 0.5 - 0.4772$

$$\boxed{\text{Reqd Prob} = 0.0228}$$

Q3: Weights of 4000 students are found to be normally distributed with mean 50 kg, std. dev^n 5. Find no. of students with weights
1) less than 45 kg

2) between 45 & 60.

→ $X \rightarrow$ weight of student

$m = 50 \quad , \quad \sigma = 5$

1) $P(x < 45) = P\left(\frac{x-m}{\sigma} < \frac{45-50}{5}\right)$

$= P(Z < -1)$

$\therefore P(x<45) = 0.5 - P(-1 < Z < 0)$

$= 0.5 - P(0 < Z < 1)$

$= 0.5 - 0.3413$

$\therefore P(x<45) = 0.1587$

$\therefore$ No. of student with $x<45$ $= 4000 \times 0.1587$

$= 634.8$

$\boxed{\approx 635}$

2) $P(45 < x < 60) = P(-1 < Z < -2)$

$\therefore P(45 < x < 60) = P(-1 < Z < 0) + P(0 < Z < 2)$

$= P(0 < Z < 1) + P(0 < Z < 2)$

$\therefore P(45 < x < 60) = 0.8185$

$\therefore$ No. of students for $45 < x < 60$ $= 4000 \times 0.8185$

$$\boxed{= 3274}$$

Q4: In an exam given by 500 candidates, the avg. & std. dev^n of marks obtained are 40 & 100 resp. Assuming marks are distributed normally. Find approximately

1) How many will pass if 50 is fixed as a minimum.

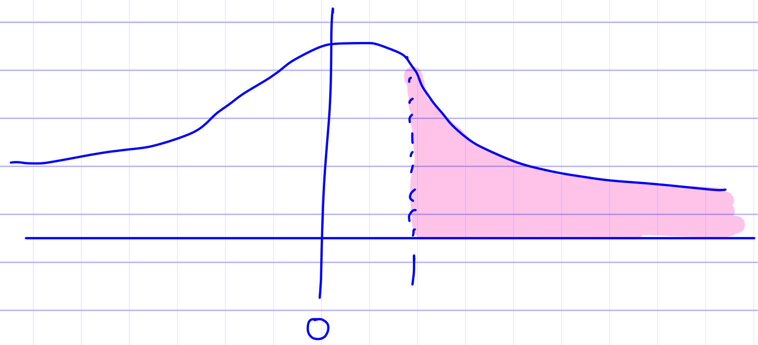
2) What should be minimum if 350 candidates are to pass.

$\rightarrow$ Let, $X \rightarrow$ marks obtained

$m = 40$, $\sigma = 10$

1) $\therefore P(x \geq 50) = P\left(\frac{x-50}{\sigma} > \frac{50-40}{10}\right)$

$\therefore P(x \geq 50) = P(Z > 1)$



$\therefore P(x \geq 50) = 0.5 - P(0 < Z < 1)$

$= 0.5 - 0.3413$

$\therefore P(x \geq 50) = 0.1587$

$\therefore$ No. of student passing for $x \geq 50$ $= 0.1587 \times 500$

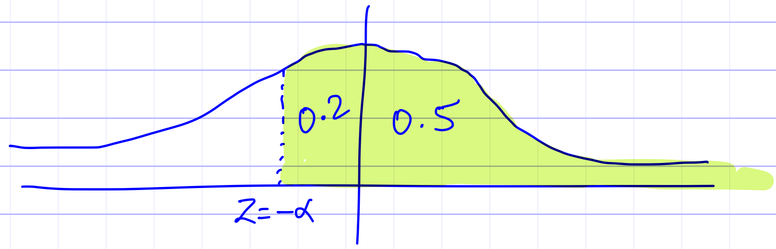
$= 79.35$

$\boxed{\approx 79}$

2) Let, x be minimum marks.

$$P(\text{passing}) = \frac{350}{500} = 0.7$$

$\therefore$



To find : z

Area from $z=-\alpha$ to $z=\infty$ $= 0.7$

Area from $z=0$ to $z=\infty$ $= 0.5$

$\therefore$ Area from $z=-\alpha$ to $z=0$ $= 0.2$

from given values, $z=0$ to $z=0.525$ is 0.2

$$\therefore z = -0.525$$

$$z = \frac{x-m}{\sigma}$$

$$\therefore (-0.525 \times 10) + 40 = x$$

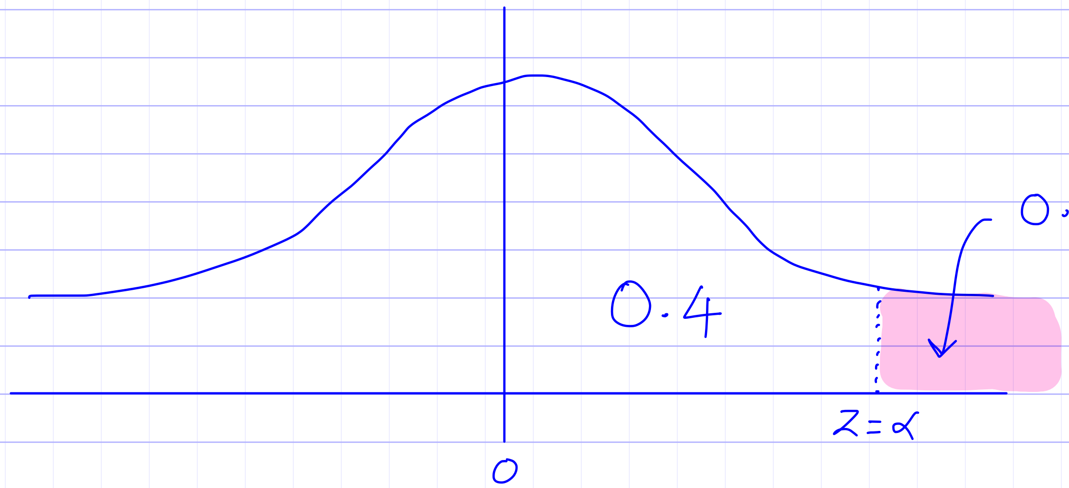
$$\boxed{\therefore x = 34.75}$$

Q5: Determine minimum marks of a student in order to get A grade if top 10 % students are awarded A grade in an exam where mean of marks is 72 & s.d. 9.

→ $X \rightarrow$ marks obtained

$m = 72$, $\sigma = 9$

$$P(\quad) = \frac{10}{100} = 0.1$$



$\therefore$ Area from $z = \alpha$ to $z = \infty$ is 0.1

& Area from $z = 0$ to $z = \alpha$ is 0.4

From table, Area from $z = 0$ to $z = 1.28$ is 0.4

$$\therefore \alpha = 1.28$$

$$\boxed{\therefore z = 1.28}$$

$$1.28 = \frac{x - 72}{9}$$

$$\therefore x = 83.52$$

$$\approx \boxed{84}$$

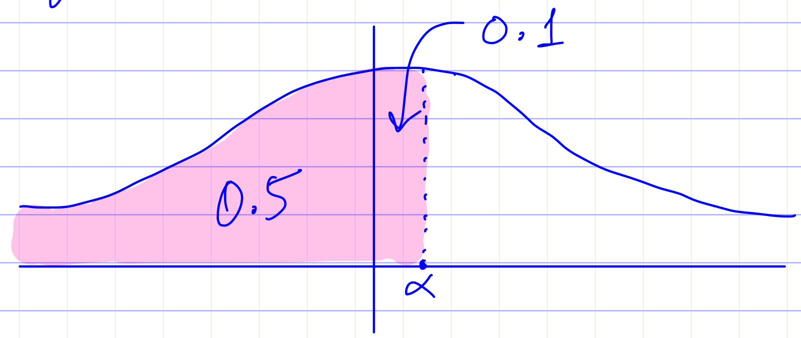
Student should get 84 marks to get A grade

Q6: When mean of marks was 50 % & s.d. 5 %, then 60 % students failed in an exam. Determine grace marks to be awarded in order to show that 70 % of students passed. Consider, marks are normally distributed.

→ $m = 50\% = 0.5$; $s.d. = 5\% = 0.05$

$X \rightarrow$ %age marks.

Before the grace marks were awarded, 60 % students failed.



Area from $z = 0$ to $z = \alpha$ is 0.1

$\therefore \alpha = 0.25$ ..... from given values/Table

$$\boxed{\therefore z = 0.25}$$

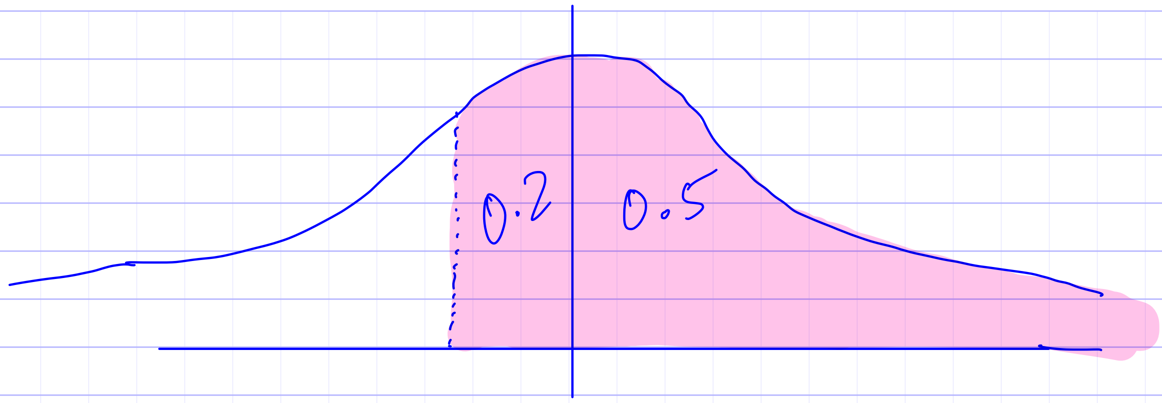
$$\therefore z = \frac{x - m}{\sigma}$$

$$\therefore 0.25 = \frac{x - 0.5}{0.05}$$

$$\boxed{\therefore x = 0.5125}$$

Minimum passing marking before grace = 51.25 %

Now, After grace marks awarded, 70 % student passed.



$Z = -\alpha$ to $Z = 0$ is 0.2

$\therefore \alpha = -0.525$ .... from table

$$\boxed{\therefore Z = -0.525}$$

$$\therefore Z = \frac{x - m}{\sigma}$$

$$\therefore (-0.525 \times 0.05) + 0.5 = x$$

$$\boxed{\therefore x = 0.4740}$$

$\therefore$ Minimum passing mark after grace $= 47.40\%$

$\therefore$ Grace marks $= 51.25 - 47.40$
$= 3.85$

$$\boxed{\approx 4\%}$$

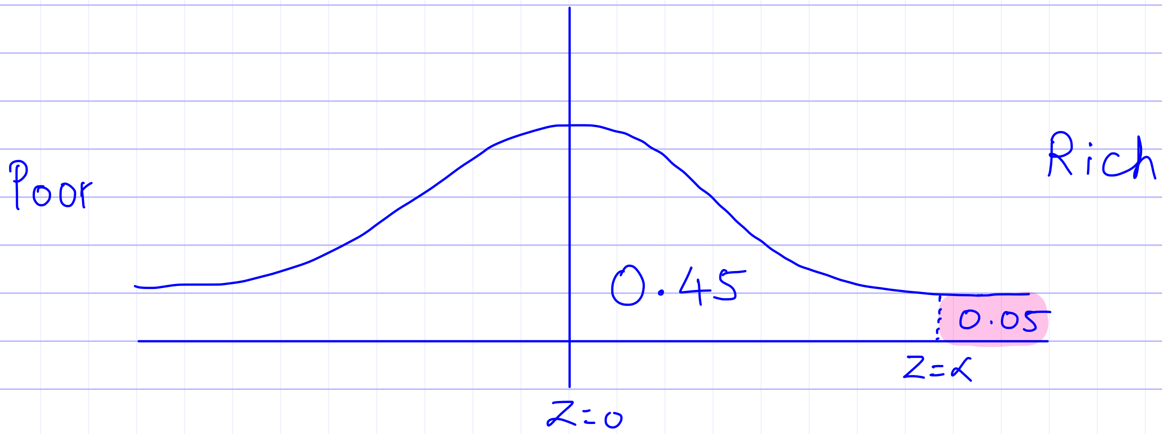
Q 7 : Income of 10000 person is normally distributed with mean Rs 520 & s.d. Rs 60. Find :
1) lowest income of richest 500.
2) Highest income of poorest 500.

→ $X \rightarrow$ income in Rs

$m = 520$, $\sigma = 60$

if we consider richest 500 persons,

Prob. that person selected at random $= \frac{500}{10000} = 0.05$



0 to α is 0.45

∴ from table, $\alpha = 1.645$

$$\boxed{\therefore Z = 1.645}$$

$$\therefore Z \times \sigma + m = x$$

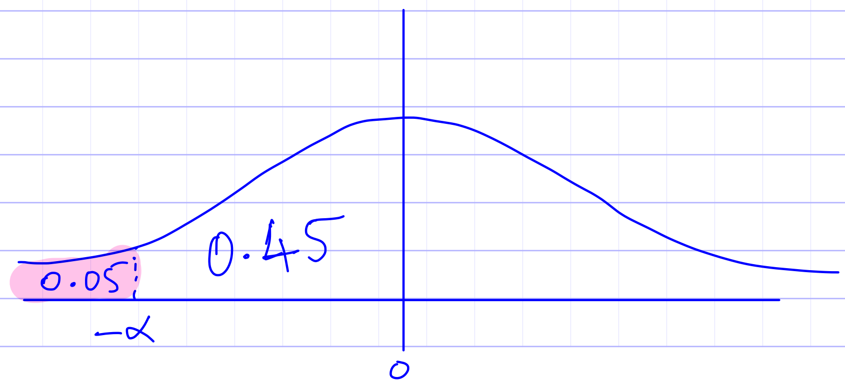
$$\therefore x = 1.645 \times 60 + 520$$

$$= 618.7$$

$$\boxed{x \approx 619}$$

$$\boxed{\text{lowest income for richest 500} = ₹\,619}$$

Now, for poor, ~~~~ $= 0.05$



Area

$\therefore -\alpha$ to 0 is 0.45

$\therefore \alpha = -1.645$ from table

$$\boxed{\therefore Z = -1.645}$$

$$\therefore x = [(-1.645) \times 60] + 520$$

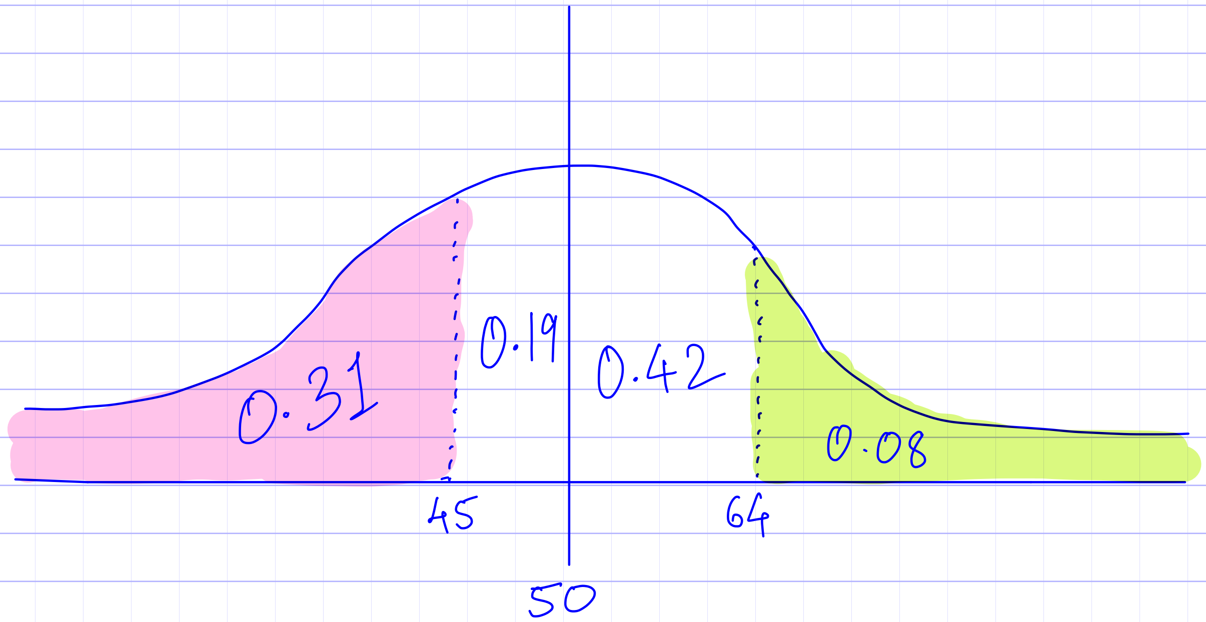
$$= 421.3$$

$$\boxed{x \approx 421}$$

$\boxed{\therefore \text{Highest income of poorest 500 is ₹ 421}}$

Q8: In normal distribution, 31% items are under 45 & 8% are over 64. Find mean & s.d.

→



$-\alpha$ to 0, Area $= 0.19$

0 to 0.5, Area $= 0.19$

$\boxed{\therefore Z = -0.5}$ corr. to $x = 45$

$\therefore\ x = -0.5 \times \sigma + m$

$$\boxed{\therefore\ m - \frac{\sigma}{2} = 45}$$

Now, α to ∞, $A = 0.08$

0 to α, $A = 0.42$

$$\boxed{\therefore\ Z = 1.4}$$ corr. to $x = 64$

$\therefore\ 64 = 1.4 \times \sigma + m$

$$\boxed{\therefore\ 1.4\sigma + m = 64}$$

Solving Simul. equ^{n}s,

$$\boxed{\therefore\ \sigma = 10}$$

$$\boxed{\therefore\ m = 50}$$