# Module 2 : A.C. Circuits

# Contents-

- Representation of sinusoidal waveform
- Peak, RMS values
- Phasor representation real, reactive and apparent power.
- Analysis of single-phase, ac circuits consisting of R, L, C, RL, RC, RLC (series and parallel) circuits and three-phase balanced circuits.
- Voltage and current relations in star and delta.

# Introduction

- The majority of electrical power in the world is generated, distributed and consumed in the form of 50-60Hz sinusoidal alternating current and voltage.
- The major advantage that AC electricity has over DC electricity is that AC voltages can be readily transformed to higher or lower voltage levels, while it is difficult to do that with DC voltages. Since high voltages are more efficient for sending electricity over great distances, AC electricity has an advantage over DC.

- **Faraday's laws of electromagnetic induction** explains the relationship between electric circuit and magnetic field. This law is the basic working principle of the most of the electrical motors, generators, transformers, inductors etc.
- Whenever a conductor is moved in the magnetic field , an emf is induced and the magnitude of the induced emf is directly proportional to the rate of change of flux linkage.

- Consider a coil with N turns, let flux through it changes from initial value $\phi_1$ to final value $\phi_2$ Wb.
- Flux Linkages (initial)= $N^* \phi_1$
- Flux Linkages (final)= $N^* \phi_2$
- Induced emf is proportional to

$$N^* \phi_1 - N^* \phi_2 / \text{time}$$

$$N(\phi_1 - \phi_2)/t$$

Emf (e)= $d/dt\,(N\phi) = N\, d\phi/dt$ Volts

# Lenz’s Law

The direction of induced emf is given by Lenz’s law .

According to this law, the induced emf will be acting in such a way so as to oppose the very cause of production of it .

- $$e = -N\ (d\text{Ø}/dt) \text{ volts}$$

## Induced emf

### Dynamically induced e.m.f
(e.g.- a.c/d.c generator)

When conductor is moving in a stationary magnetic field such that the flux linking the conductor changes in magnitude. The term dynamic is used since conductor is in motion.

### Statically induced e.m.f
(e.g. transformer)

Here the conductor is stationary and magnetic field is changing. The word static means stationary.

# STATICALLY **INDUCED EMF**

- **Self-induced electromotive force** (emf which is induced in the coil due to the change of flux produced by it linking with its own turns.)

- **Mutually induced electromotive force**(emf which is induced in the coil due to the change of flux produced by another coil, linking with it.)

# Generation of AC quantity

- Alternating Current: An alternating current is the current which changes periodically both in magnitude and direction.
- The machines which are used to generate electrical voltages are called "**GENERATORS**".
- The generators which generate purely sinusoidal A.C. voltage are called "**ALTERNATORS**"
- Sinusoidal Voltage: A sinusoidal voltage is an oscillating voltage that can be described mathematically through the use of a sine function.
- A.C voltage may be generated by rotating a coil in a magnetic field or by rotating a magnetic field within a stationary coil. (Faraday's Law)

# Construction of single wave alternator:

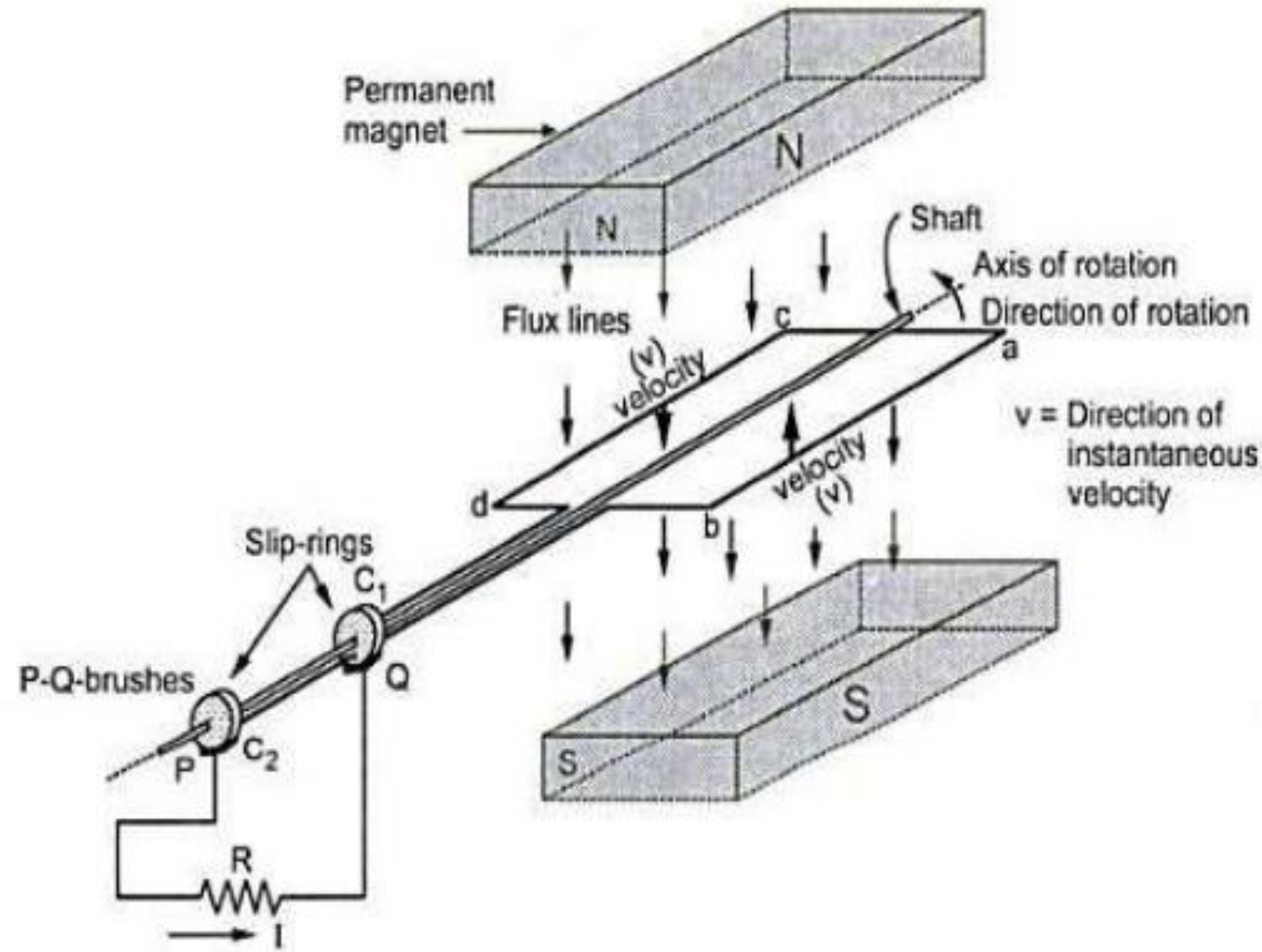
- The basic principle of an alternator is the principle of Electromagnetic Induction. It says that whenever there is a relative motion between the conductor and the magnetic field in which it is kept, an e.m.f gets induced in the conductor.

- It consists of a permanent magnet of two poles. A single turn rectangular coil is kept on the vicinity of the permanent magnet. This coil is made up of same conducting material like copper or aluminium.

- The coil is made up of two conductors namely a-b and c-d. Such two conductors are connected at one end to a coil.

- The coil is so placed that it can be rotated about its own axis in clockwise or anticlockwise direction. The remaining two ends C1 and C2 of the coil are connected to the rings mounted on the shaft called slip rings.

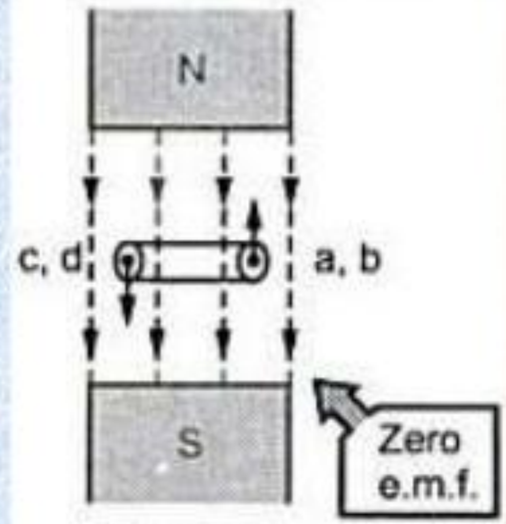
- Slip rings are also rotating members of the alternator. The two brushes P and Q are resting on the slip rings. The brushes are stationary and are just making contacts with slip rings. The slip rings and brush assembly is necessary to collect the current induced in the rotating coil and make it available to the stationary external resistance.



Permanent magnet
N
N
Shaft
Axis of rotation
Direction of rotation
Flux lines
c
a
(v) velocity
v = Direction of instantaneous velocity
velocity (v)
d
b
Slip-rings
$C_1$
P-Q-brushes
Q
S
S
P
$C_2$
R
I

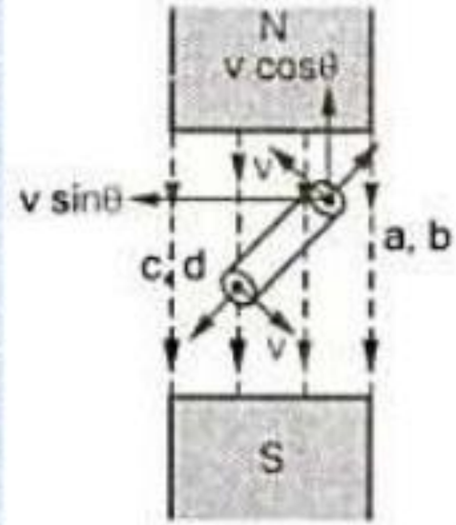
- Working: The coil is rotated in anticlockwise direction. While rotating, the conductors ab and cd cut the lines of flux of the permanent magnet.
- Due to faraday’s law of electromagnetic induction, an e.m.f gets induced in the conductors. The e.m.f. drives a current through resistance R connected across the brush P and Q.
- The magnitude of the induced e.m.f depends on the position of the coil in magnetic field. Let us see the relation between magnitude of the induced e.m.f and the position of the coil. Consider different instants and the different position of the coil.

**Instant 1:** The plane of the coil is perpendicular to the direction of the magnetic field.The instantaneous component of velocity of the conductors ab and cd is parallel to the magnetic field.So there cannot be the cutting of the flux lines by the conductors.Hence,no e.m.f will be generated in the conductors ab and cd and no current will flow through the external resistance R.



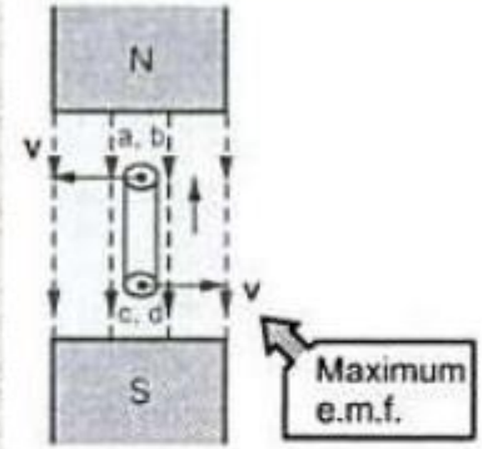
(a) θ = 0°

**Instant 2:** When the coil is rotated in
anticlockwise direction through some angle θ, then the velocity will have
two components v sinθ (perpendicular to flux lines) and v cosθ (parallel to the flux lines). Due to v sinθ component, there will be cutting of the flux and proportionally, there will be induced e.m.f un the conductors ab and cd. This e.m.f will drive a current through the externam resistance R.




(b) $0° < \theta < 90°$

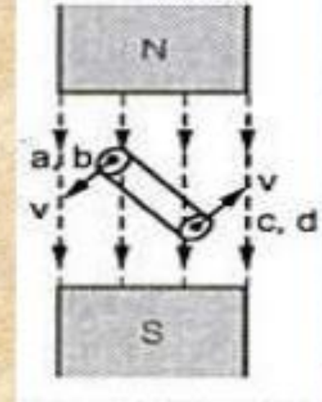
**Instant 3:** As angle 'θ' increases, the component of velocity acting perpendicular to flux lines increases, hence inducesd e.m.f also increases. At θ=90°, the plane of the coil is parallel to the plane of the magnetic field while the component of velocity cutting the lines of flux is at its maximum. So induced e.m.f in this position, is at its maximum value.



(c) θ = 90°

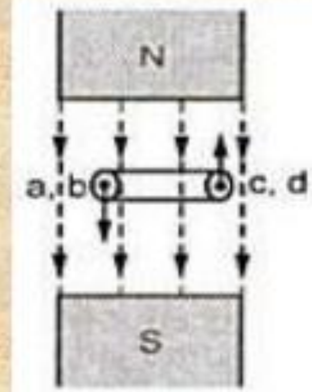
-So, as θ increases from 0° to 90°, e.m.f induces in the conductors increases gradually from o to maximum value.

**Instant 4:** As the coil continues to rotate further from $\theta=90^\circ$ to $180^\circ$, the component of velocity perpendicular to magnetic field starts decreasing. Hence, gradually decreasing the magnitude of the induced e.m.f.



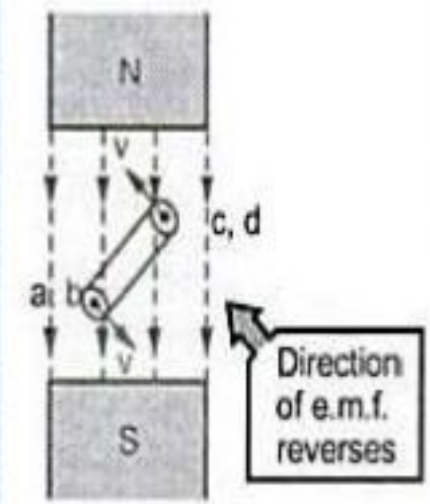
(d) $90^\circ < \theta < 180^\circ$

**Instant 5:** In this position, the velocity component is fully parallel to the lines of flux similar to instant1. There is no cutting of flux, so no induced e.m.f in both the conductors. Hence, current through external circuit is also zero.



(e) $\theta = 180^\circ$

Instant 6:As the coil rotates beyond $\theta$=180°,the conductor ab uptill now cutting flux lines in one particular direction reverses the direction of cutting the flux lines.Similiar is the behaviour of conductor cd.

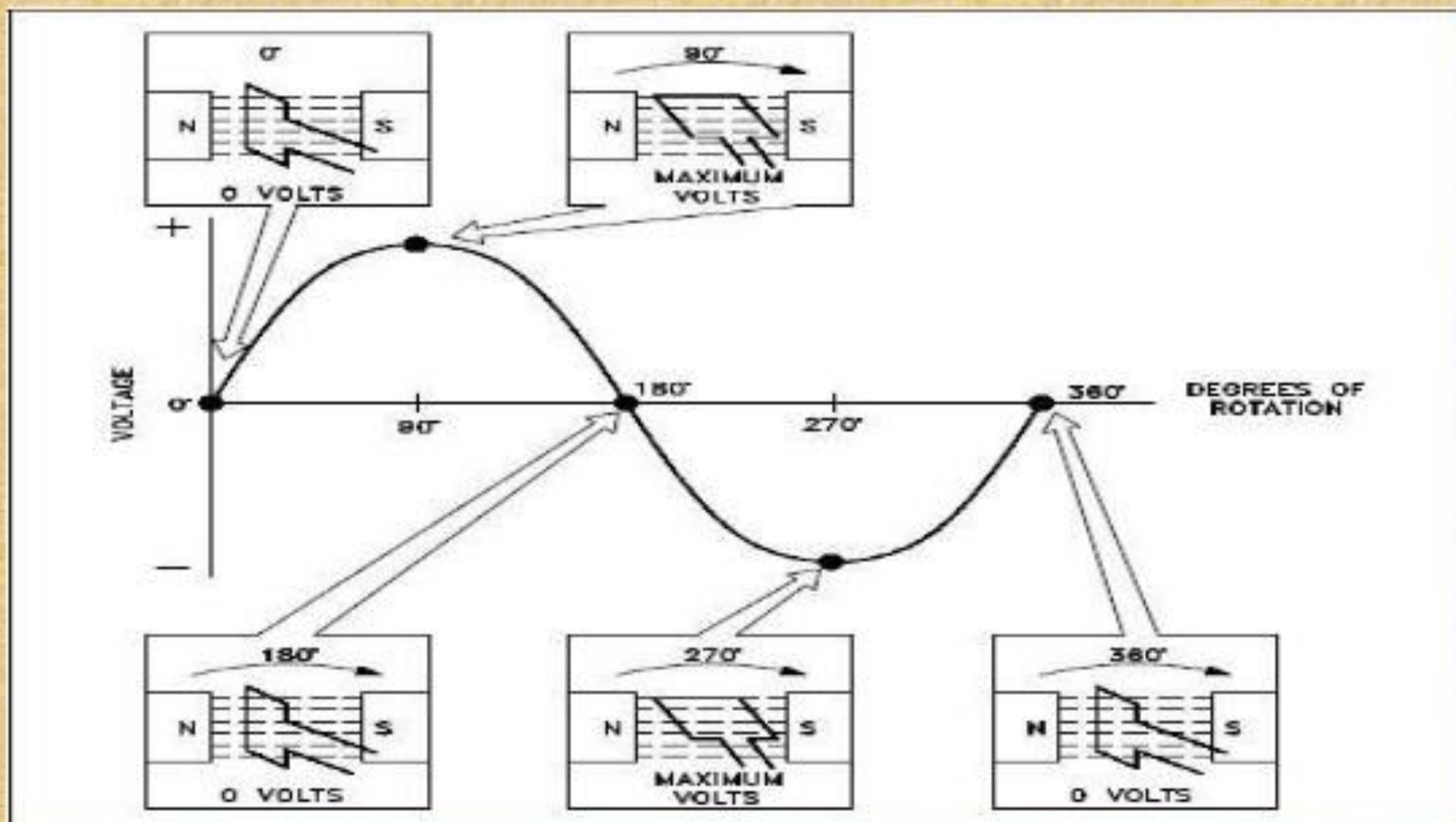


(f) $180° < \theta < 270°$

-So,direction of induced e.m.f in conductor ab is opposite to the direction of induced e.m.f in it for the rotation of $\theta$=0° to 180°.Similirly,the direction of induced e.m.f in conductor cd also reverses.The change in direction of induced e.m.f occurs because the direction of rotation of conductors ab and cd reverses with respect to the field as $\theta$ varies from 180° to 360°.

- This process continues a coil rotates further. At $\theta=270^\circ$ again, the induced e.m.f achieves itz maximum value but the direction of this e.m.f in both the conductors is opposite to the previous maximum position i.e. at $\theta=90^\circ$. From $\theta=270^\circ$ to $360^\circ$, induced e.m.f decreases without change in direction and at $\theta=360^\circ$, coil achieves the starting position with zero induced e.m.f.
- So, as $\theta$ varies from $0^\circ$ to $360^\circ$, the e.m.f in the conductor ab or cd varies in an alternating manner i.e. zero, increasing to achieve maximum in one direction, decreasing to zero, increasing to achieve maximum in other direction and again decreasing to zero. This set of variation repeats for every revolution as the conductor rotate in a circular motion within a certain speed.

The instantaneous value of the induced e.m.f in any conductor,as it is rotated from θ=0° to 360°,i.e. through one complete revolution can be represented as shown in the below figure.



-To derive the equation of an alternating quantity,consider single turn,2 pole alternator.The coil is rotated with constant angular velocity in the magnetic field.

Let,

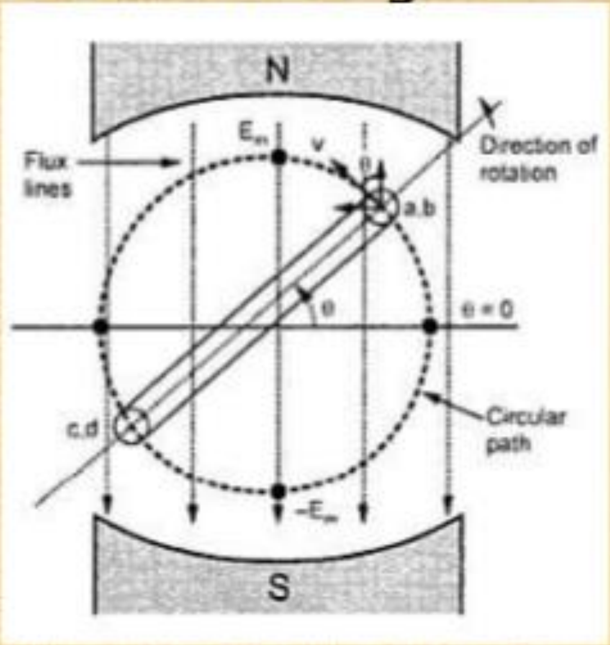
- -B=Flux density of the magnetic field
- -l=Active length of the each conductor
- -r=radius of circular path traced by conductors
- -$\omega$=Angular velocity of coil
- -v=linear velocity of the each conductor

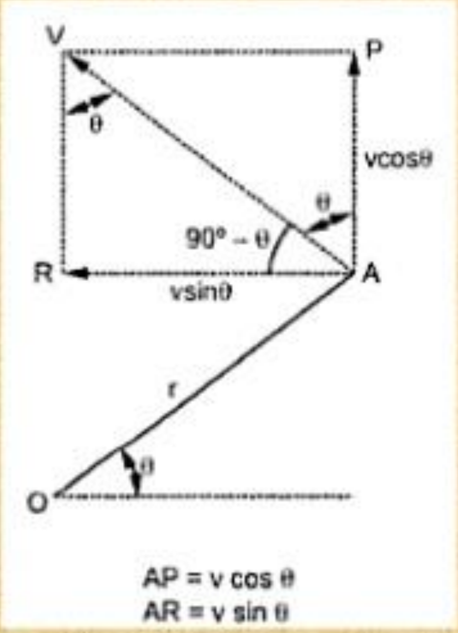
Consider an instant where coil has rotated through angle $\theta$ from the position corrresponding to $\theta=0^\circ$ i.e. from the instant where induced e.m.f is zero.It requires time t to rotate through $\theta$.

So, $\theta$ in radians can be expressed as,

$$\theta = \omega t \quad \text{(radians)}$$

The position of the coil is shown in the below figure. The instantaneous peripheral velocity of any conductor can be resolved into two components as shown in the figure.



-The components of velocity(v) are,
(1) Parallel to the magnetic flux lines($v \cos\theta$)
(2)Perpendicular to the magnetic flux lines($v \sin\theta$)

- Out of the two,due to the component parallel to the flux lines,there cannot be the generation of e.m.f as there cannot be the cutting of the flux lines.Hence,the component which is acting perpendicular to the magnetic flux lines i.e. $v \sin\theta$ is responsible for the generation of the em.f.

- According to the faraday's law of electromagnetic induction,the expression for the generated e.m.f in each conductor is,

$$E = B\, l\, v \sin\theta \quad \text{(volts)}$$

-The active length 'l' means the length of the conductor which is under the influence of the magnetic field

Now, $E_m = B\,l\,v$ (volts)

= Maximum value of the induced em.f in the conductor

- This is achieved at $\theta = 90^\circ$ and is the peak value or amplitude of the sinusoidal induced e.m.f.

- Hence,equation giving instantaneous value of the generated e.m.f can be expressed as,

$$e = E_m \sin\theta \text{ (volts)}$$

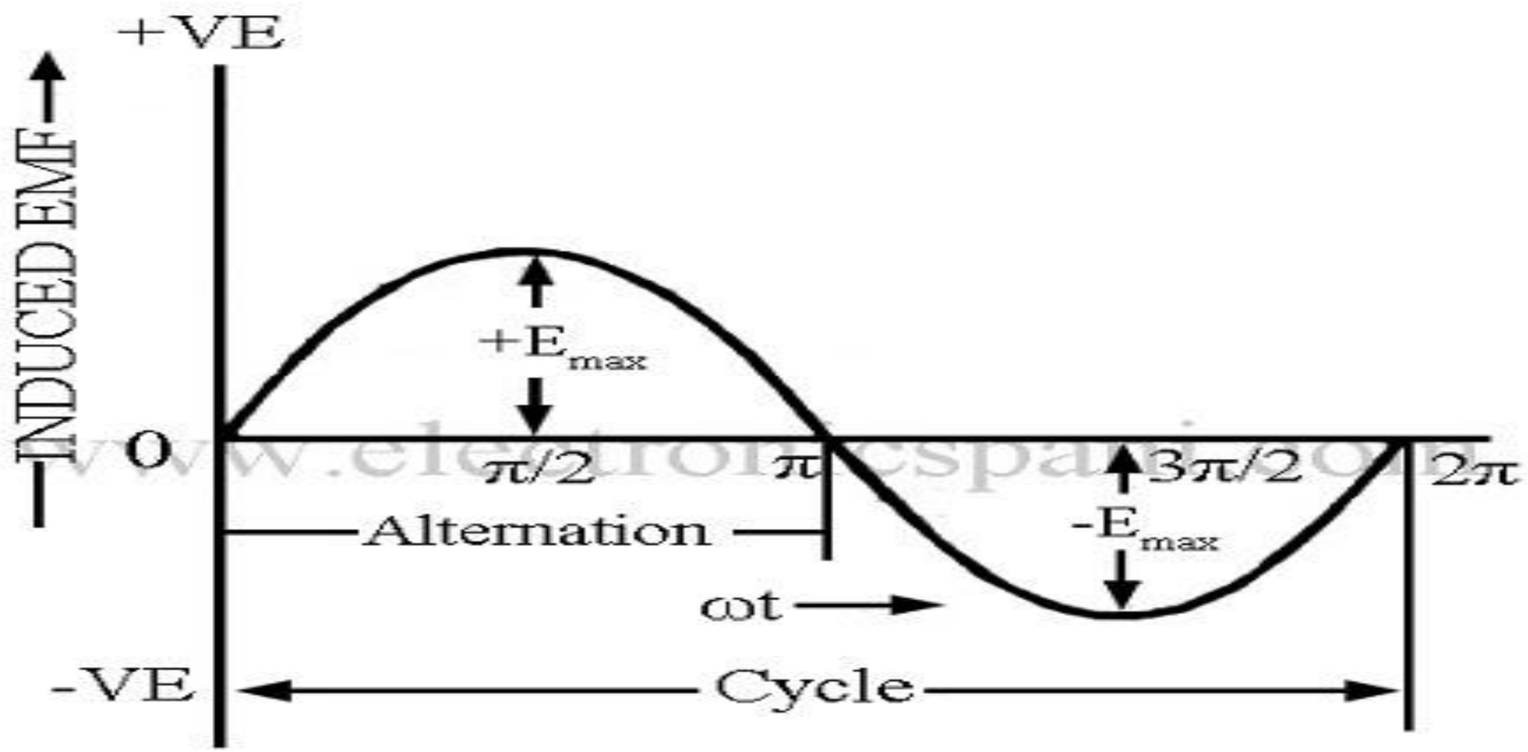
- This alternating e.m.f drives a current through the electrical load which also varies in similar manner.
- Its frequency as the same as the frequency of the generated e.m.f. Hence, it can be expressed as,

$$i=Im\ \sin\theta\ (Ampere)$$

- Where Im is the maximum or peak value of the current. This maximum value depends on the resistance of the electric circuit to which an e.m.f is applied. The instantaneous value of the sinusoidal current set by the e.m.f can be expressed as,

$$i=Im\ \sin(wt)\ (Ampere)$$

# $e = E_m \sin\theta$ (volts)



# Basic Terminologies

- Cycle:- A complete set of positive and negative values of an alternating quantity is known as cycle.
- Time period: The time taken by an alternating quantity to complete one cycle is called time T.
- Frequency:  It is the number of cycles that occur in one second. f = 1/T
- Waveform: A curve which shows the variation of voltage and current w.r.t time or rotation.

- **Peak Value**

Peak is the maximum value, either positive or negative, that a waveform attains.

- **Average Value**

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value.

$$Average\ value = \frac{Area\ under\ one\ cycle}{Base}$$

The average value of a whole sinusoidal waveform over one complete cycle is zero as the two halves cancel each other out, so the average value is taken over half a cycle.

$$i = i_m \sin(\omega t)$$

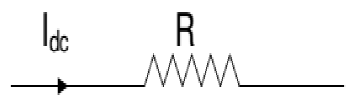
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i\, d(\omega t)$$

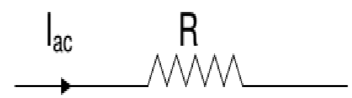
$$i_{avg} = \frac{1}{\pi}\int_0^{\pi} i_m \sin(\omega t) d(\omega t)$$

$$i_{avg} = \frac{2i_m}{\pi} = 0.637 i_m$$

## RMS or Effective Value

The effective or RMS (Root Mean Square) value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.




$$RMS = \sqrt{\frac{Area\ under\ squared\ curve}{Base}}$$

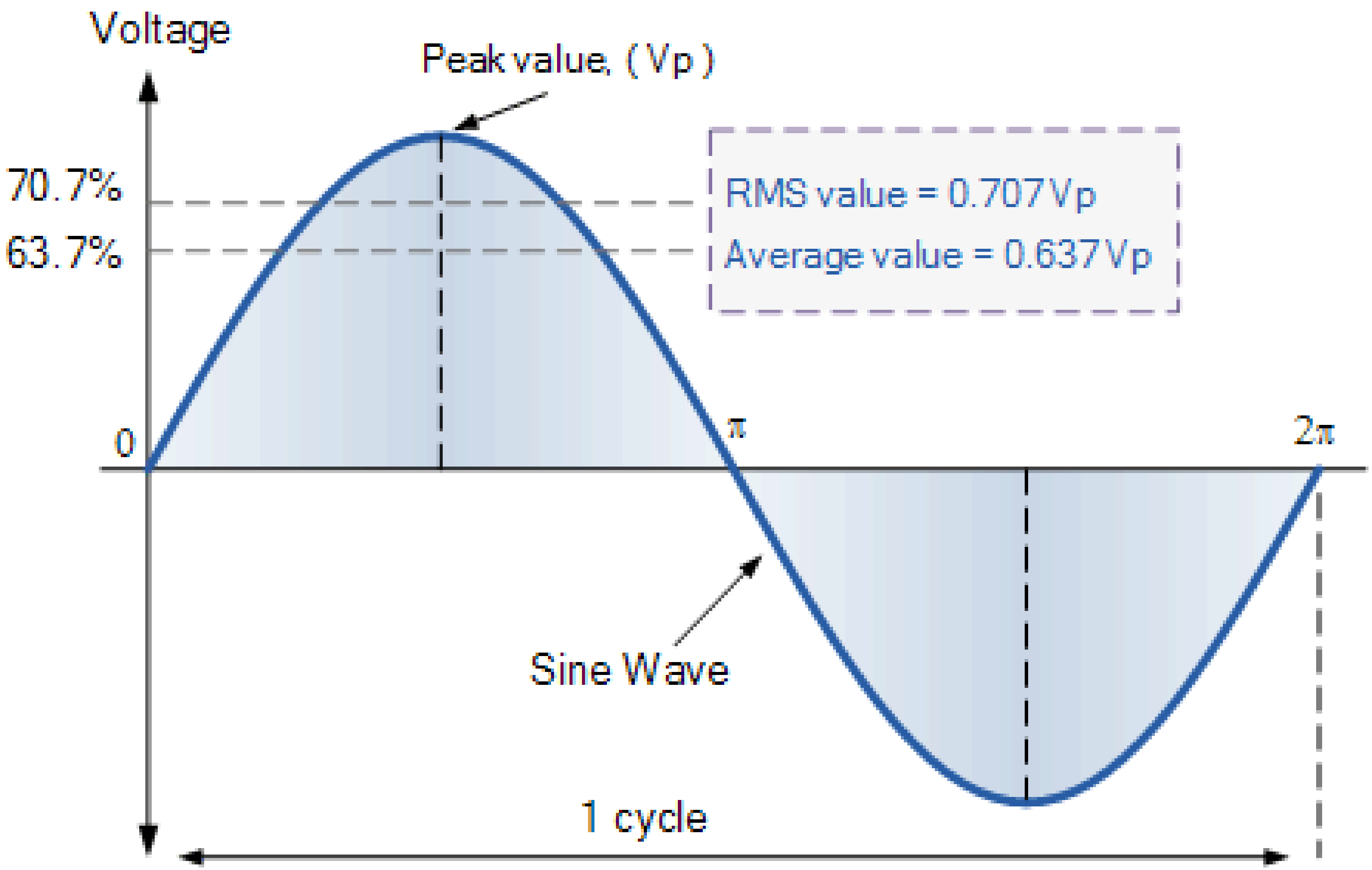
RMS value of a sinusoidal current:

$$i_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d(\omega t)}$$

$$i_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2 sin^2(\omega t)\, d(\omega t)}$$

$$i_{rms} = \frac{i_m}{\sqrt{2}} = 0.707\ i_m$$

$$i_{rms} = \sqrt{\frac{i_m^2}{\pi}\int_0^{\pi} \frac{(1 - \cos(2\omega t))}{2}\, d(\omega t)}$$


Voltage
Peak value, ( Vp )
70.7%
63.7%
RMS value = 0.707 Vp
Average value = 0.637 Vp
0
π
2π
Sine Wave
1 cycle

- **Form Factor:**

It is the ratio of RMS value to the average value of an alternating quantity.

$$FF = \frac{RMS\ \ value}{Avg\ Value}$$

- **Peak Factor or Crest Factor**:

It is the ratio of maximum value to the RMS value of an alternating quantity.

$$PF = \frac{Maximum\ \ value}{RMS\ \ Value}$$

For a sinusoidal waveform

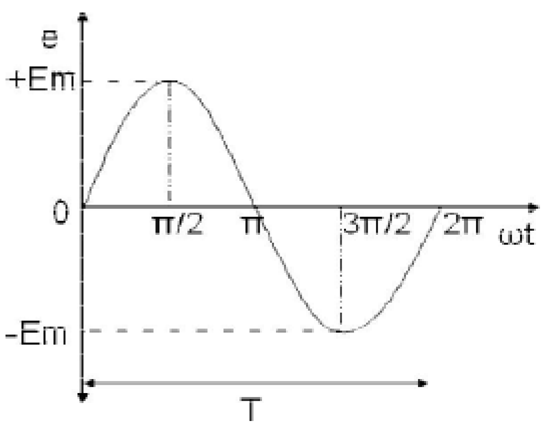
$$FF = \frac{i_{rms}}{i_{avg}} = \frac{0.637 i_m}{0.707 i_m} = 1.11$$

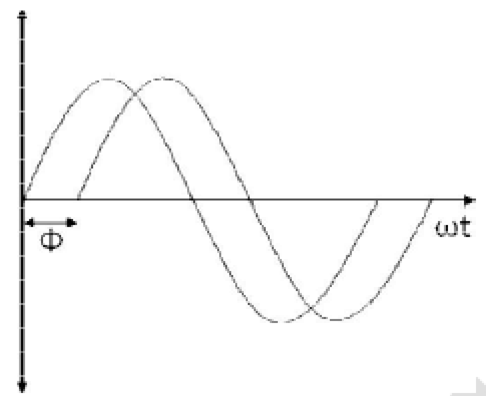
$$PF = \frac{i_{rms}}{i_{avg}} = \frac{i_m}{0.707 i_m} = 1.414$$

- Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference

Phase of +Em is π/2rad or T/4 sec

Phase of -Em is 3π/2rad or 3T/4 sec

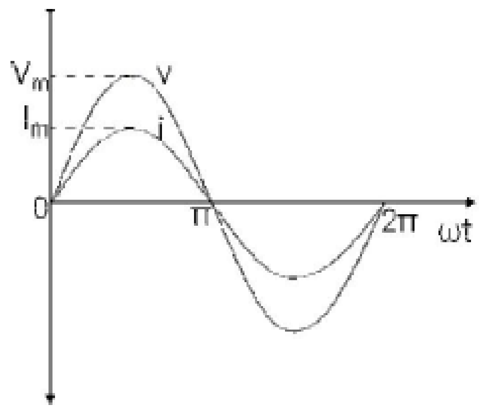



- When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

- **In Phase**

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same.

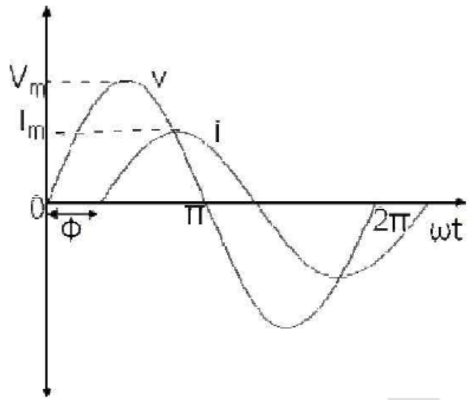


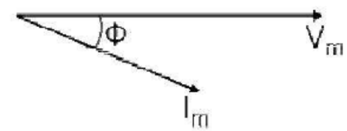


$$v = v_m \sin(\omega t)$$
$$i = i_m \sin(\omega t)$$

- **Lagging**

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage.

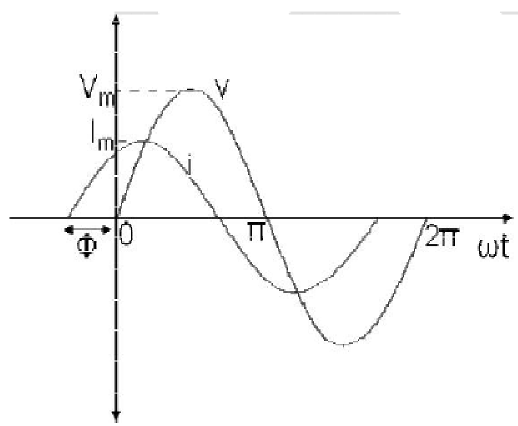


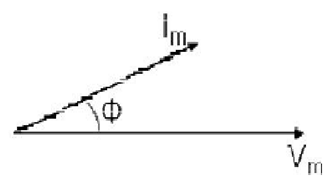




$$v = v_m \sin(\omega t) => \bar{V} = V_m\angle 0^0$$
$$i = i_m \sin(\omega t - \theta) => \bar{I} = I_m\angle -\theta$$

- **Leading**

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage..

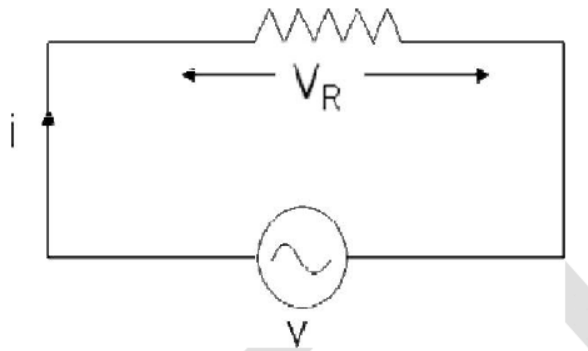


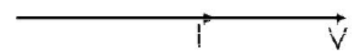



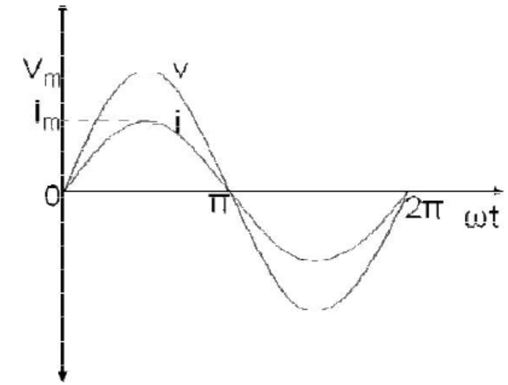

$$v = v_m \sin(\omega t) => \bar{V} = V_m\angle 0^0$$
$$i = i_m \sin(\omega t + \theta) => \bar{I} = I_m\angle \theta^0$$

# Numericals

# AC Circuit with pure resistance




Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

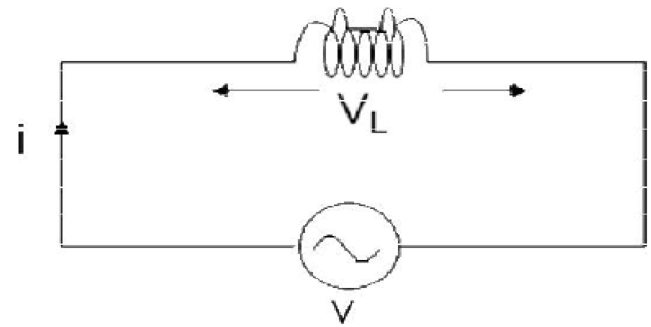
The current flowing in the circuit is i. The voltage across the resistor is given as $V_R$ which is the same as V. Using ohms law, we can write the following relations

$$i = \frac{v}{R} = \frac{v_m \sin(\omega t)}{R} \qquad \text{where} \qquad i_m = \frac{v_m}{R}$$

$$i = i_m \sin(\omega t)$$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase.

# AC Circuit with pure inductance




Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the inductor is given as $V_L$ which is the same as V. We can find the current through the inductor as follows

$$v = L\frac{di}{dt}$$

$$v_m \sin(\omega t) = L\frac{di}{dt}$$

$$di = \frac{v_m}{L} * \sin(\omega t)\, dt$$

$$i = \frac{v_m}{L} * \int \sin(\omega t)\, dt$$

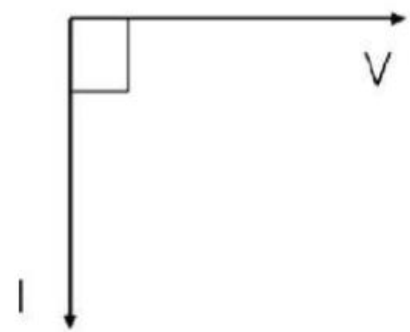
$$i = \frac{v_m}{\omega L} * -\cos(\omega t)$$

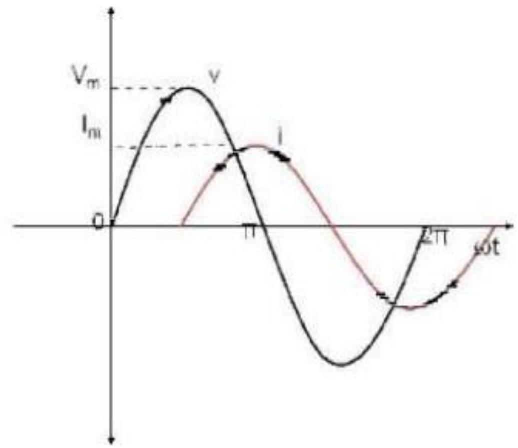
$$i = \frac{v_m}{\omega L} * \sin(\omega t - \frac{\pi}{2})$$

$$i = i_m * \sin(\omega t - \frac{\pi}{2})$$

where $i_m = \frac{v_m}{\omega L}$

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90˚. Hence the voltage and current waveforms and phasors can be drawn as below.




- Inductive reactance - The inductive reactance $X_L$ is given as

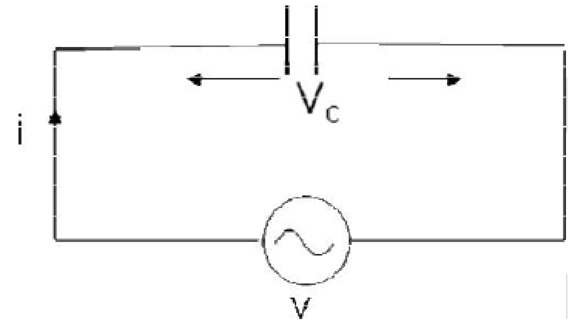
$$X_L = 2\pi f L$$

$$\bar{V} = j\omega L \bar{I}$$

$$i_m = \frac{v_m}{X_L}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω).

# AC Circuit with pure capacitance




Consider an AC circuit with a pure capacitance C as shown in the figure. The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the capacitor is given as $V_C$ which is the same as V. We can find the current through the capacitor as follows

$$q = Cv$$

$$q = Cv_m \sin(\omega t)$$

$$\frac{dq}{dt} = \omega C v_m \cos(\omega t)$$

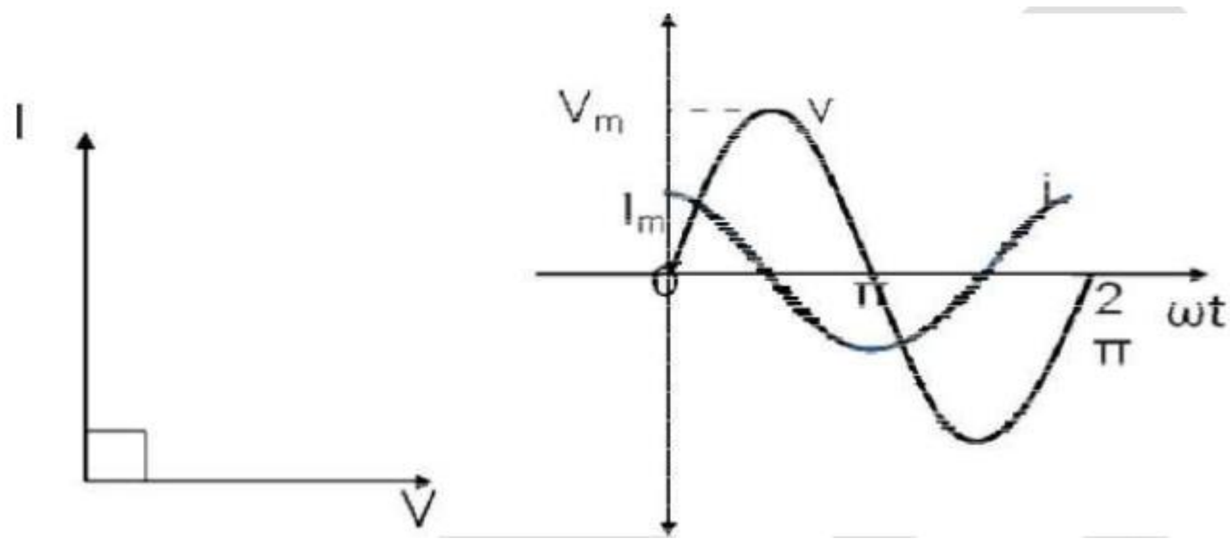
$$i = \omega C v_m \cos(\omega t)$$

$$i = \omega C v_m \sin(\omega t + \frac{\pi}{2}).$$

$$i = i_m \sin(\omega t + \frac{\pi}{2})$$

$$i_m = \omega C v_m \Rightarrow X_C = \frac{v_m}{i_m} = \frac{1}{\omega C}$$

From equation (1) and (2) we observe that in a pure capacitive circuit, the current leads the voltage by 90˚. Hence the voltage and current waveforms and phasors can be drawn as below.

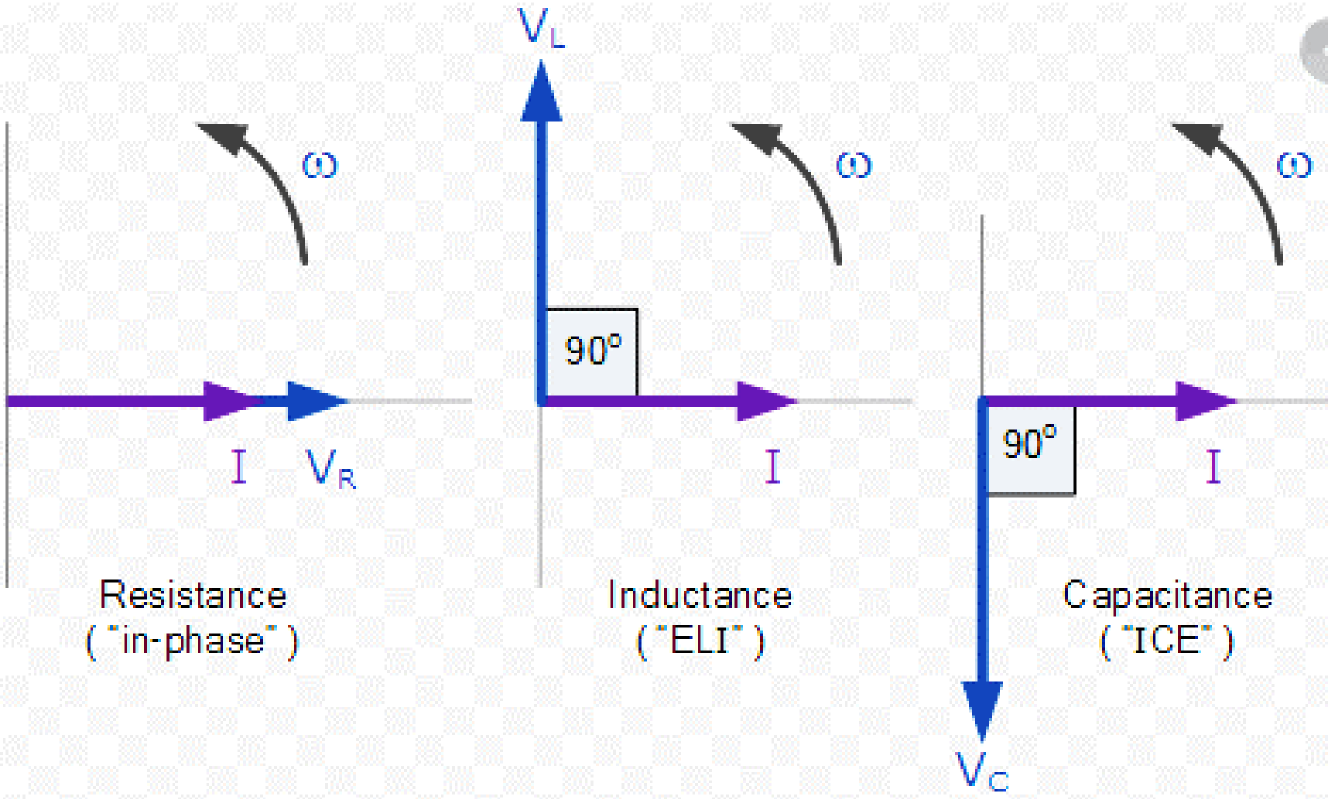



- Capacitive reactance - The capacitive reactance $X_C$ is given as

$$X_C = \frac{1}{2\pi f C}$$

$$i_m = \frac{v_m}{X_C}$$

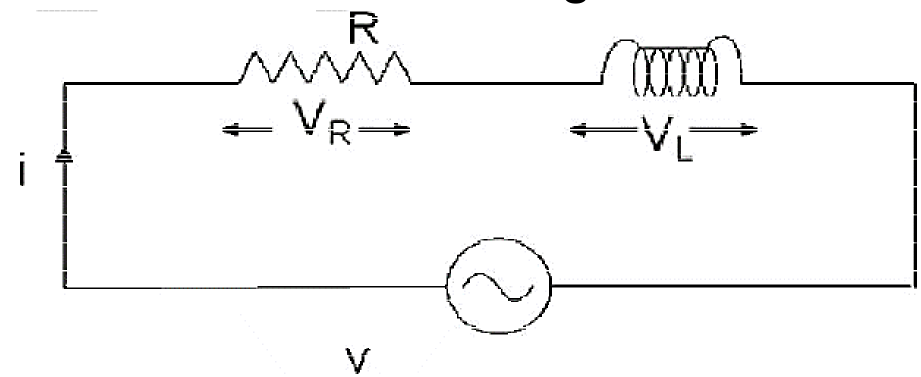
It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω).



$V_L$
ω
ω
ω
90°
90°
I
$V_R$
I
I
Resistance
( "in-phase" )
Inductance
( "ELI" )
Capacitance
( "ICE" )
$V_C$

# R-L Series Circuit:

Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure.




The alternating voltage v is given by

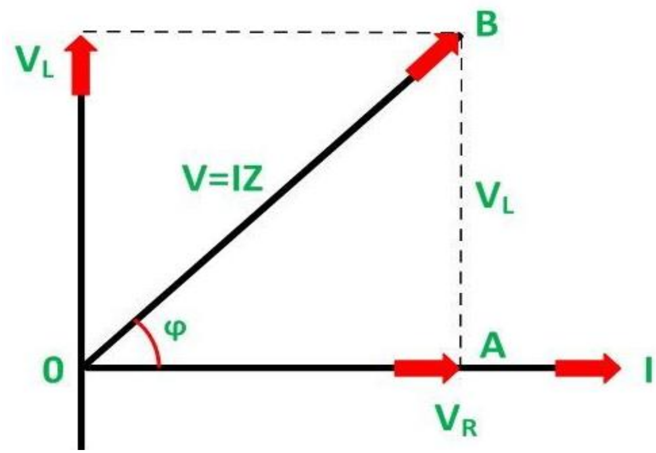
$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the resistor is $V_R$ and that across the inductor is $V_L$

$V_R$ =IR is in phase with I

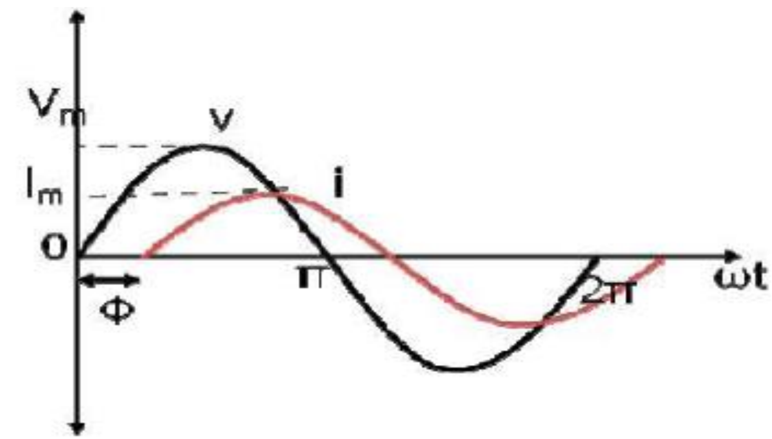
$V_L$ =I$X_L$ leads current by 90 degrees

Phasor diagram:




From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.

$$V = V_m \sin(\omega t)$$
$$I = I_m \sin(\omega t - \emptyset)$$




From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V_R = IR$$

$$V_L = IX_L$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L{}^2}$$

Where $$Z = \sqrt{R^2 + X_L{}^2}$$

$$V = IZ$$

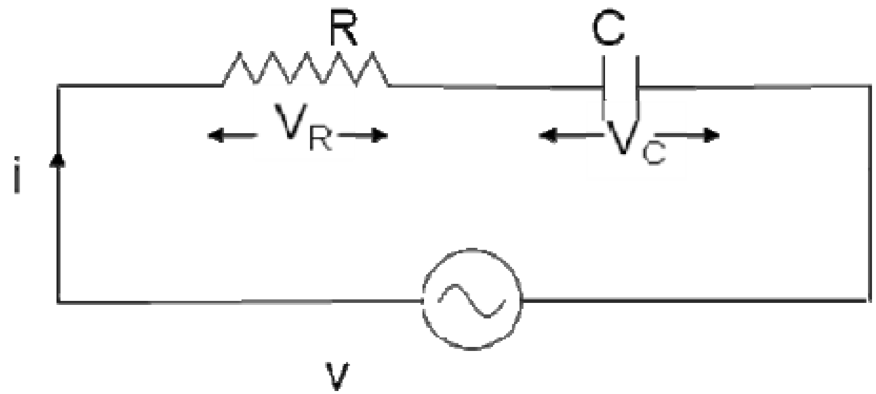
The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for Impedance is ohms(Ω).

Phase angle:

$$\emptyset = tan^{-1}\left(\frac{V_L}{V_R}\right)$$

# R-C Series Circuit:

Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure.




The alternating voltage v is given by

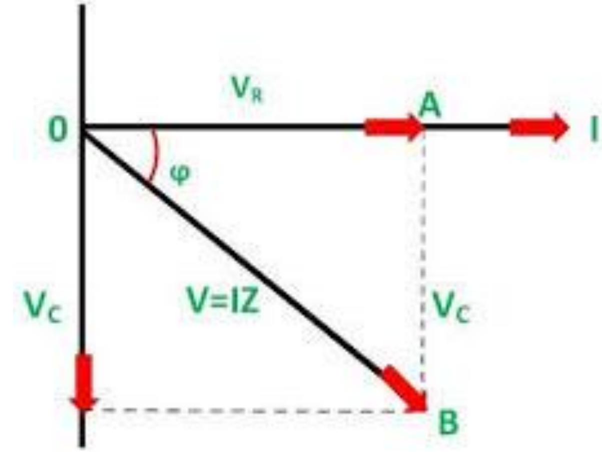
$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the resistor is $V_R$ and that across the capacitor is $V_c$

$V_R$ =IR is in phase with I

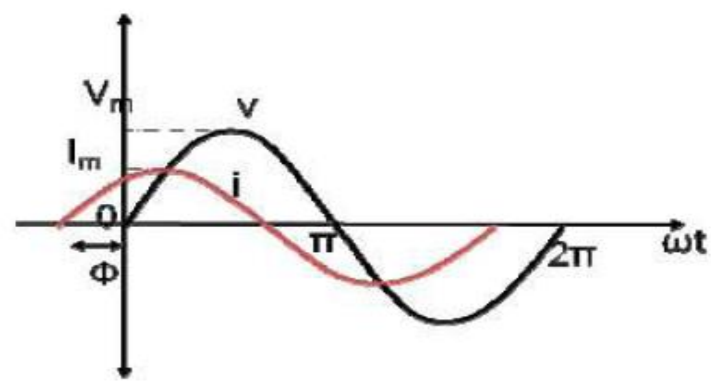
$V_c$ =I$X_c$ lags current by 90 degrees

Phasor diagram:




From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

$$V = V_m \sin(\omega t)$$
$$I = I_m \sin(\omega t + \emptyset)$$




From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows

$$V = \sqrt{V_R^2 + V_C^2}$$
$$V_R = IR$$

$$V_C = IX_C$$

$$V = \sqrt{(IR)^2 + (IX_C)^2}$$
$$V = I\sqrt{R^2 + X_C{}^2}$$
$$V = IZ$$ where $$Z = \sqrt{R^2 + X_C{}^2}$$

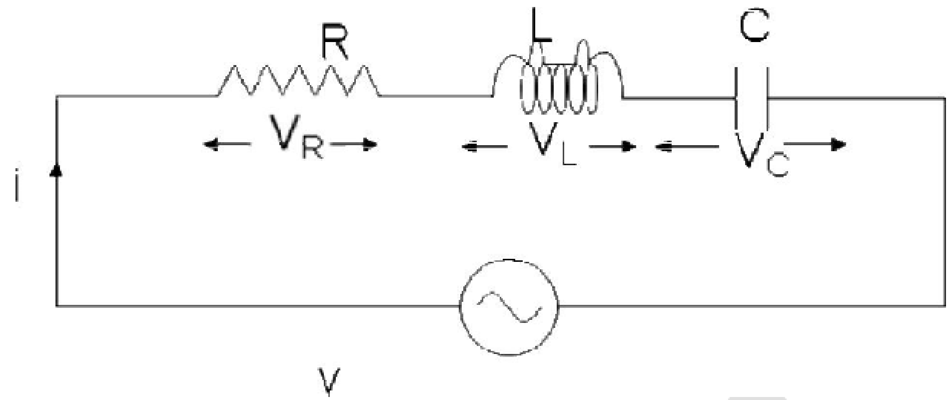
The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for Impedance is ohms(Ω).

Phase angle:

$$\varnothing = tan^{-1}(\frac{V_C}{V_R})$$

# R-L-C Series circuit:

Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure.




The alternating voltage v is given by

$$v = v_m \sin(\omega t)$$

The current flowing in the circuit is i. The voltage across the resistor is $V_R$, the voltage across the inductor is $V_L$ and that across the capacitor is $V_C$.

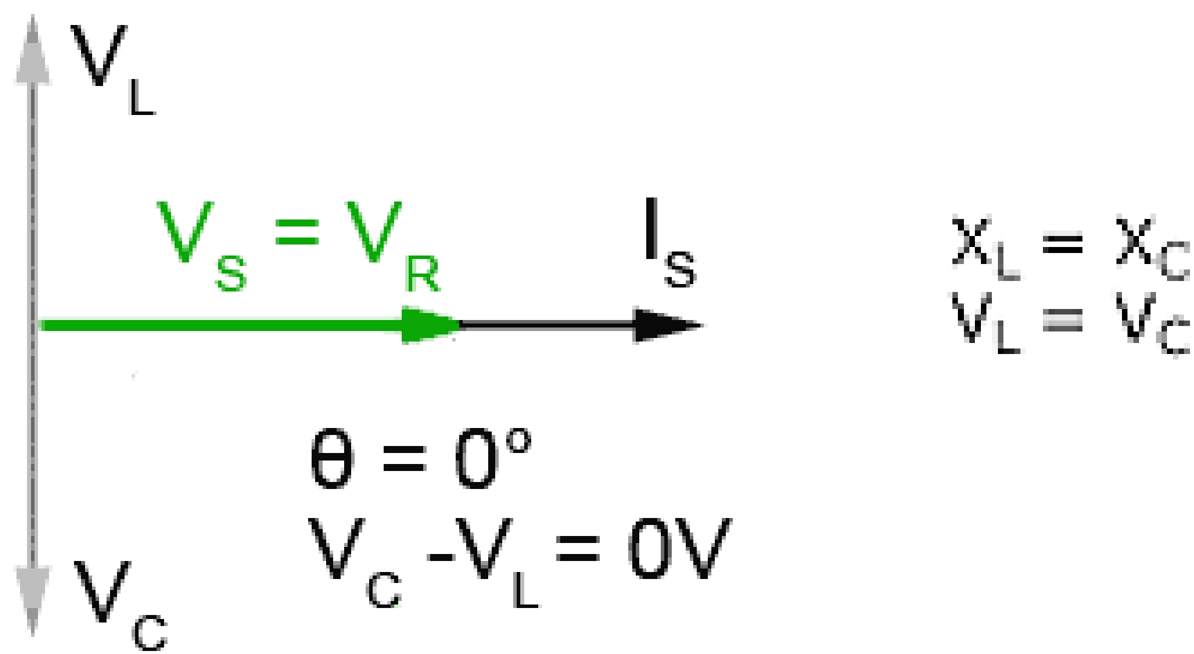
- $V_R$=IR is in phase with I
- $V_L$=I$X_L$ leads the current by 90 degrees
- $V_C$=I$X_C$ lags behind the current by 90 degrees

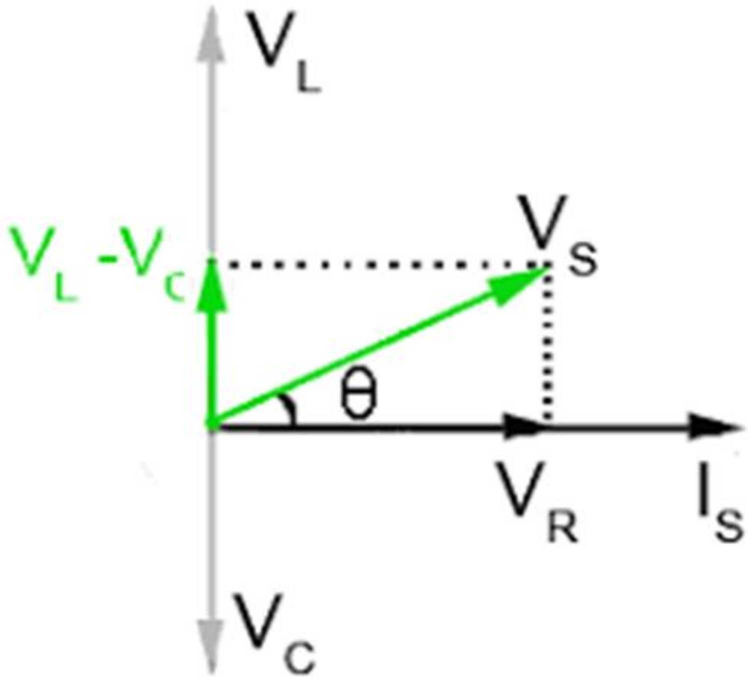
- Phasor Diagram:

There are two cases that can occur $V_L>V_C$ and $V_L<V_C$ depending on the values of $X_L$ and $X_C$. And hence there are two possible phasor diagrams.

$$V = V_m \sin(\omega t)$$
$$I = I_m \sin(\omega t \pm \emptyset)$$

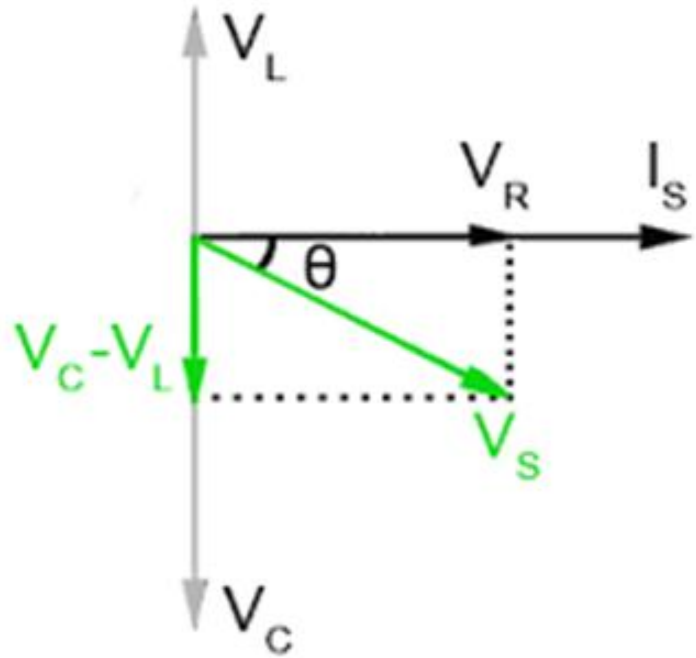
The phase angle Φ = 0 and the circuit is purely resistive. The circuit behaves like a pure resistive circuit.






$X_L > X_C$
$V_L > V_C$

From the phasor diagram we observe that when $V_L > V_C$ , the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.




$X_L < X_C$
$V_L < V_C$

When $V_L$<$V_C$ ,the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$
$$V = IZ$$

Where impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$

- Phase angle:

$$\emptyset = Tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = Tan^{-1}\left(\frac{IX_L - IX_C}{IR}\right) = Tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- Case (i): When $X_L > X_C$

The phase angle Φ is positive and the circuit is inductive. The circuit behaves like a series RL circuit.

- Case (ii): When $X_L < X_C$

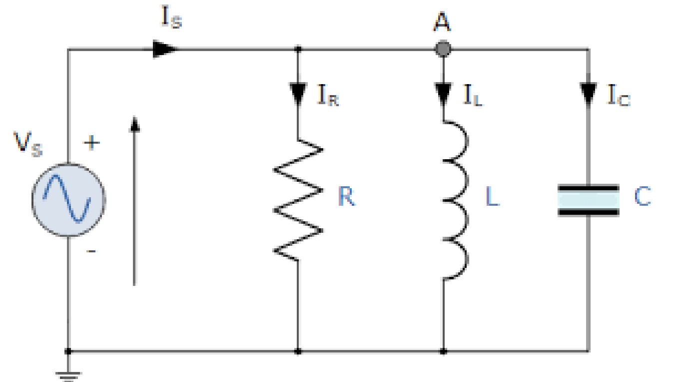
The phase angle Φ is negative and the circuit is capacitive. The circuit behaves like a series RC circuit.

- Case (iii): When $X_L = X_C$

The phase angle Φ = 0 and the circuit is purely resistive. The circuit behaves like a pure resistive circuit.

# R-L-C Parallel Circuit

Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in parallel as shown in the figure.

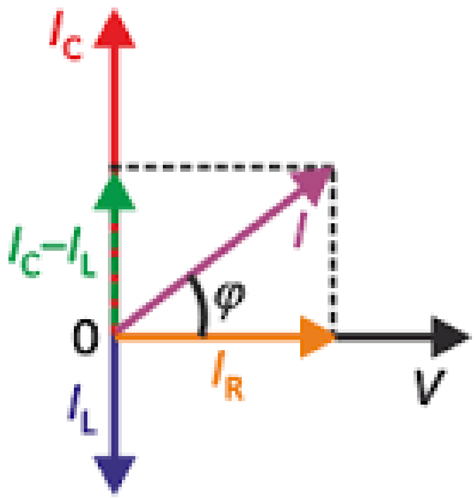


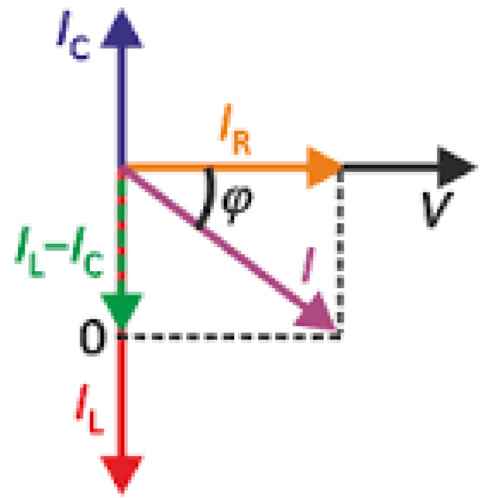

In parallel circuit, the voltage across each element remains the same and the current gets divided in each component depending upon the impedance of each component.

$$I_R = \frac{V}{R},\ I_C = \frac{V}{X_C},\ and\ I_L = \frac{V}{X_L}$$

The total current, $I_S$ drawn from the supply is equal to the vector sum of the resistive, inductive and capacitive current, not the mathematic sum of the three individual branch.

- Phasor Diagram:




$$I_S^2 = I_R^2 + \left(I_L - I_C\right)^2$$

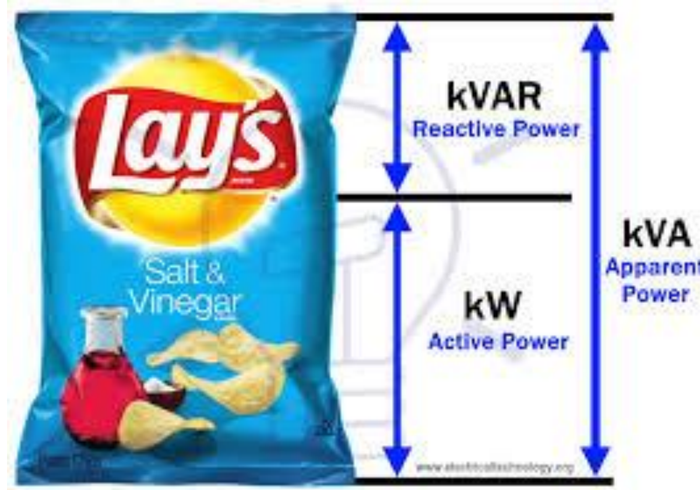
$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2}$$

- Phase angle:

$$\Phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

$$\therefore\ I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$




- P= Responsible for conversion of electrical energy into other forms like mechanical energy, heat energy etc. i.e. useful work

(True Power/ Active power/ Useful power)

- Q= Power can't be used to convert into mechanical work. It is used to produce magnetic flux in machine.

(Watt less power/ Useless power)

- Real Power:

The power due to the active component of current is called as the active power or real power. It is denoted by P.

$$P = V * I cos(\emptyset) = I^2 R cos(\emptyset)$$

Real power is the useful power. It is the power that is consumed by the resistance. The unit for real power in Watt(W).

- Reactive Power:

The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

$$Q = V * I sin(\emptyset) = I^2 X_L \sin(\emptyset)$$

Reactive power does not do any useful work. It is the circulating power in the L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

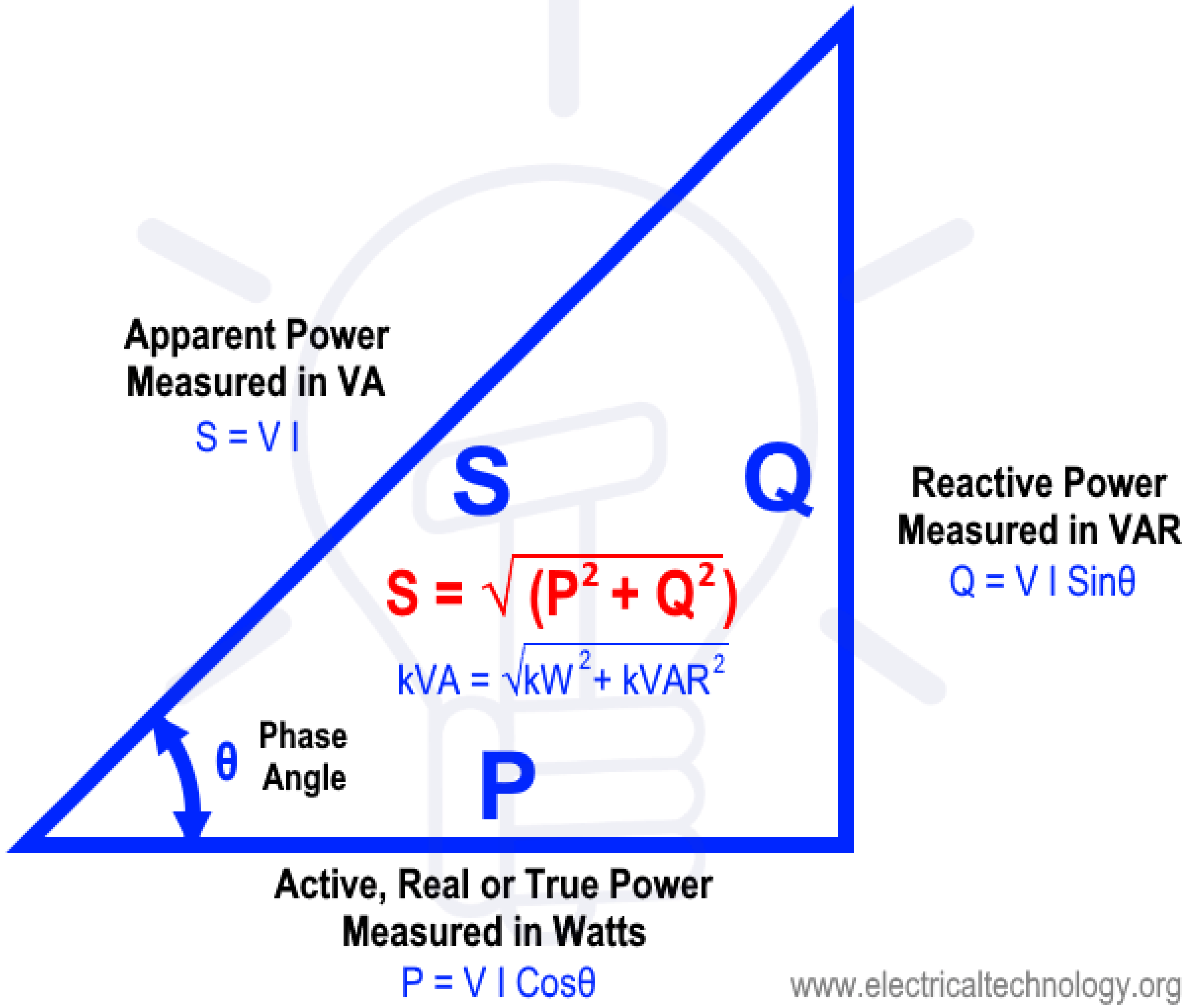
- Apparent Power:

The apparent power is the total power in the circuit. It is denoted by S. The unit for apparent power is Volt Amperes(VA).

$$S = VI = I^2 Z$$

$$S = \sqrt{P^2 + Q^2}$$

# Power Triangle



# Power Factor

- Power factor indicates that portion of apparent power which is utilized as a true power for energy conversion.

$$P.F. = \frac{\text{Actual Power}}{\text{Apparent Power}} \quad \text{or} \quad P.F. = \text{Cos}\Theta$$

- Power Factor:

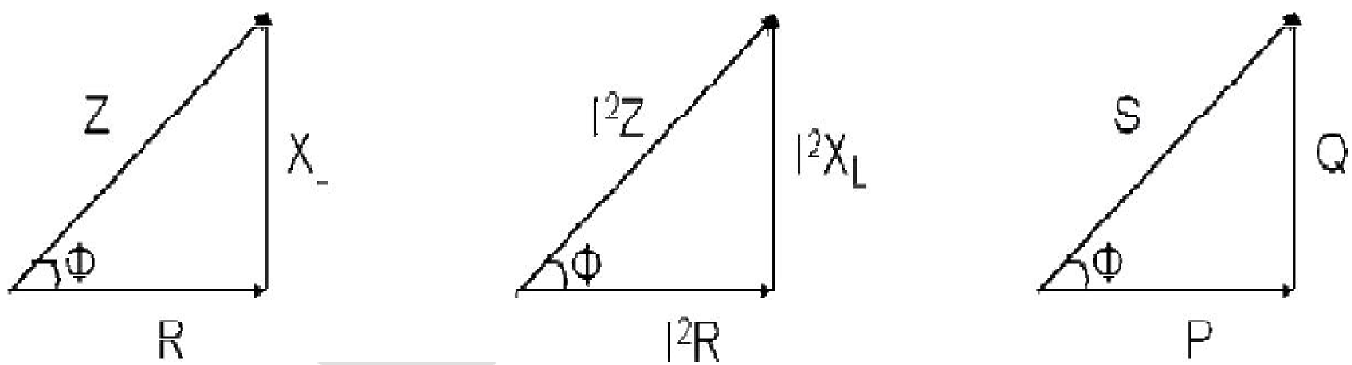
The power factor in an AC circuit is defined as the cosine of the angle between voltage and current.

$$\cos(\emptyset) = Cosine\ of\ angle\ between\ V\ and\ I$$

$$= \frac{Resistance}{Impedance} = \frac{R}{Z}$$

$$= \frac{Real\ power}{Apparent\ power}$$

- Power Triangle




Apparent Power

$$S = \sqrt{P^2 + Q^2}$$

# Mathematical representation of vectors:

- Rectangular or Cartesian form- $\vec{V} = a \pm jb$
- Polar form- $\vec{V} = V \angle \pm \theta$
- Trignometrical form- $\vec{V} = V\left(\cos\theta \pm j\sin\theta\right)$
- Exponential form- $\vec{V} = Ve^{\pm j\theta}$

# Phasor Operations

addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$

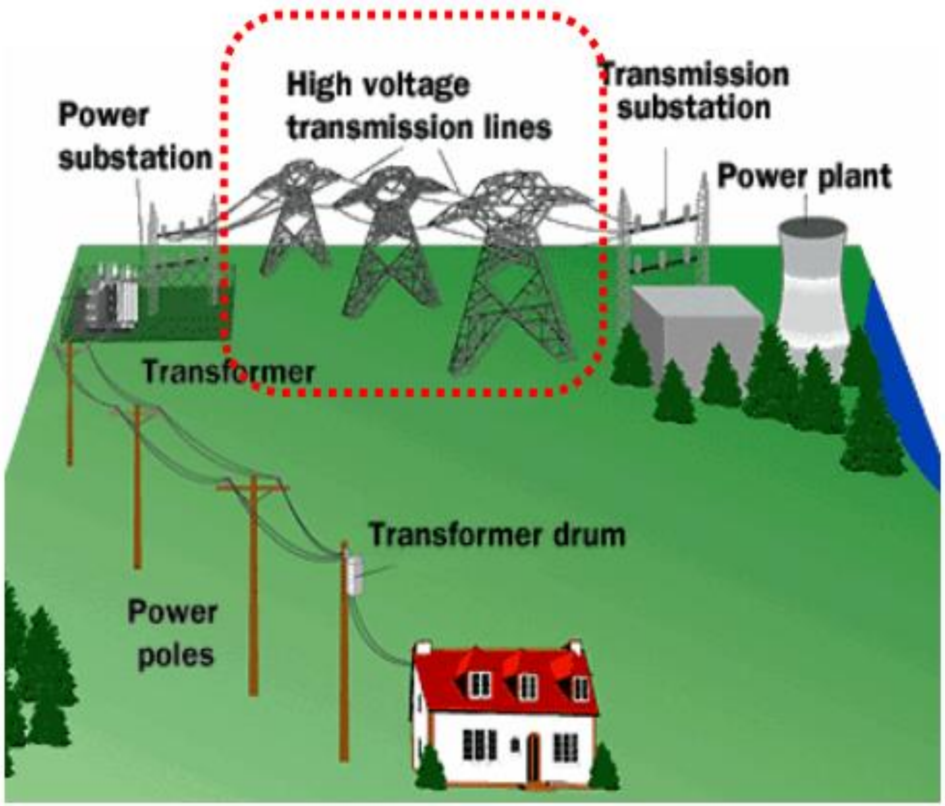
multiplication: $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$

division: $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \angle \phi_1 - \phi_2$

# Numericals

# Three phase balanced circuits

- Electrical power distribution system




# Why Three Phase is preferred Over Single Phase?

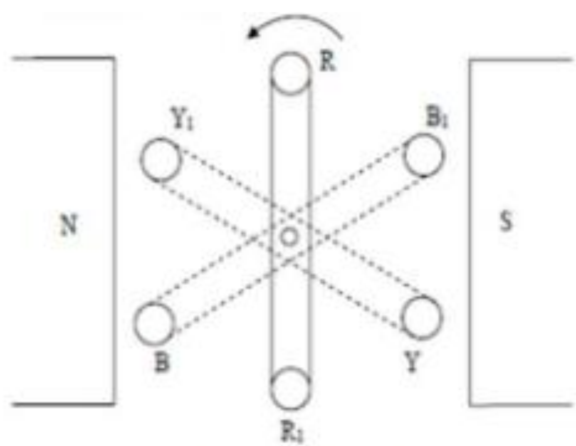
- Power delivered is constant. In single phase circuit the power delivered is pulsating.
- The output of three phase system is higher than a single phase system.
- Comparing with single phase motor, three phase induction motor has higher power factor and efficiency. Three phase motors are very robust, relatively cheap, generally smaller, have self-starting properties, provide a steadier output and require little maintenance compared with single phase motors.
- For transmitting the same amount of power at the same voltage, a three phase transmission line requires less conductor material than a single phase line. The three phase transmission system is so cheaper.

# Generation of 3 Phase E.M.Fs

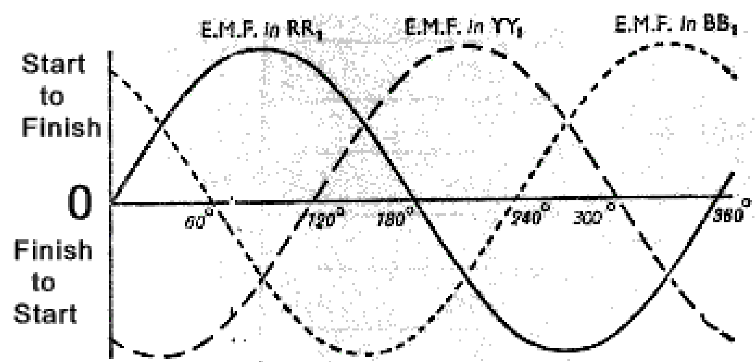
In a 3 phase balanced system, there are three equal voltages or EMFs of the same frequency having a phase difference of 120 degrees. These voltages can be produced by a three-phase AC generator having three identical coils RR1,YY1,BB1 displaced apart from each other by 120 degrees.

Let the three coils mentioned above be rotated anti-clockwise with constant angular velocity in a uniform magnetic field.

Three EMFs are induced in the three coils respectively.



Generation of three-phase e.m.f.s.

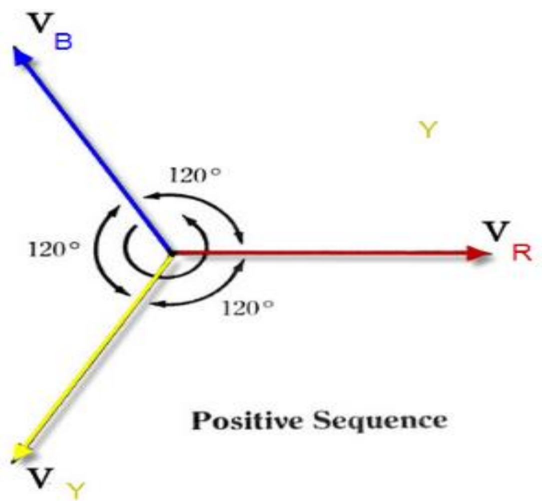


Three Phase Waveform

# Phasor Sequence

The sequence in which three phase voltages attain their positive maximum values is defined as the phase sequence.

The generated voltages of a 3 phase circuits can be expressed as




Red Phase: $V_{RN} = V_m \sin\theta$

Yellow Phase: $V_{YN} = V_m \sin(\theta - 120^o)$

Blue Phase: $V_{BN} = V_m \sin(\theta - 240^o)$

or

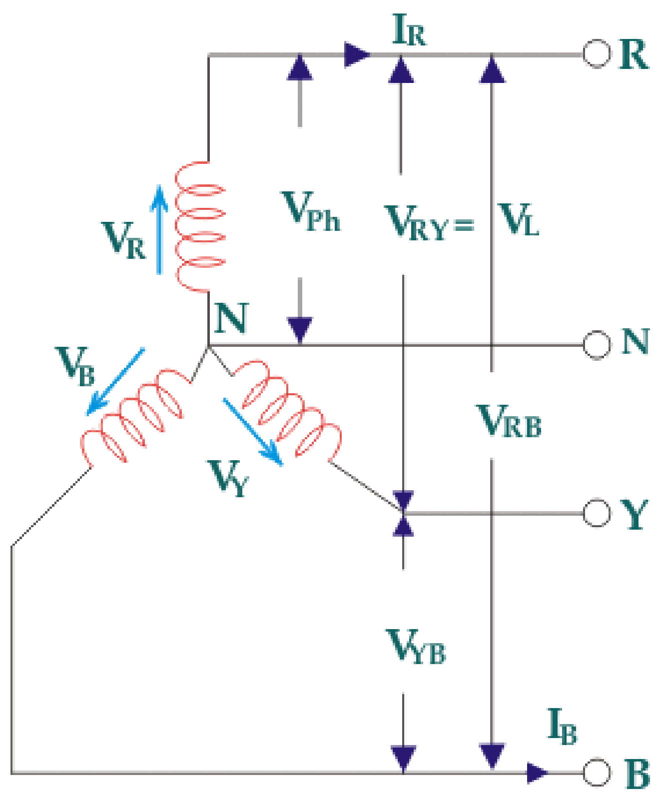
$V_{BN} = V_m \sin(\theta + 120^o)$

# Voltage and current relations in star connected system

- $V_R = V_Y = V_B = V_{ph}$

In the star connection, line current is same as phase current. The magnitude of this current is same in all three phases and say it is $I_L$.

- $I_R = I_Y = I_B = I_L = I_{ph}$.



Thus, for the star-connected system

- Line voltage = √3 × phase voltage.
- Line current = Phase current

As, the angle between voltage and current per phase is ϕ, the electric power per phase is

$$V_{ph} I_{ph} cos\phi = \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

- So the total power of three phase system is

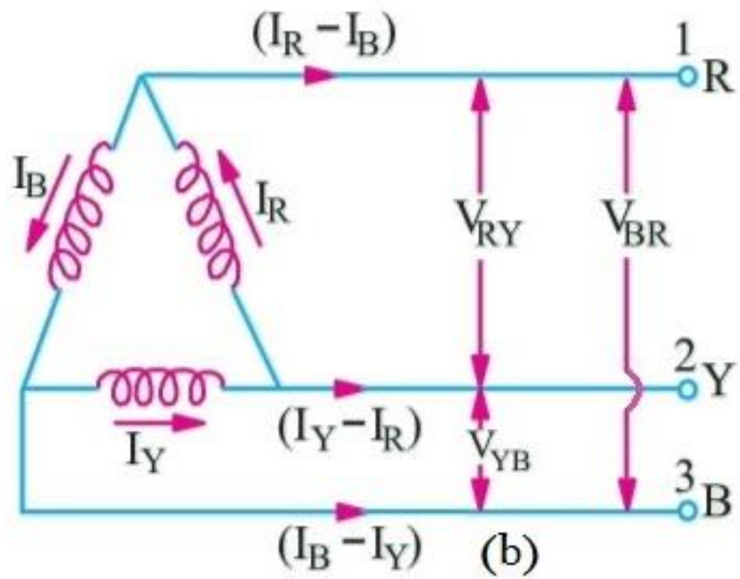
$$3 \times \frac{V_L}{\sqrt{3}} I_L \cos\phi = \sqrt{3} V_L I_L \cos\phi$$

# Voltage and current relations in delta connected system

In Delta connection, the Line Voltage is equal to the Phase Voltage.

$V_{RY} = V_{YB} = V_{BR} = V_L = V_{PH}$

$I_R = I_Y = I_B = I_{PH}$



From the diagram, it is found that

$$\text{Current in Line } 1 = I_1 = I_R - I_B$$

$$\text{Current in Line } 2 = I_2 = I_Y - I_R$$

$$\text{Current in Line } 3 = I_3 = I_B - I_Y$$

Thus, for the delta-connected system

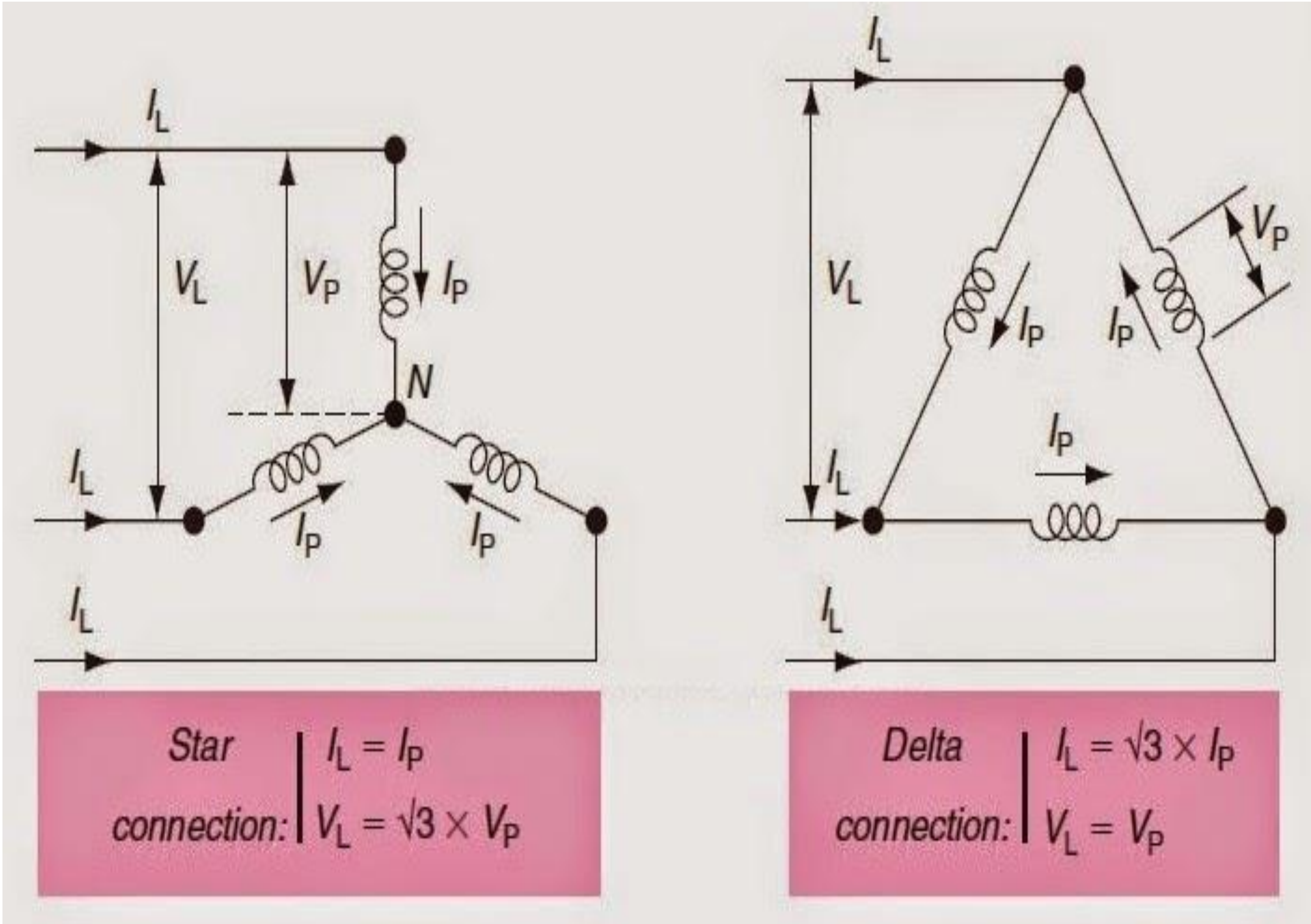
- Line voltage = phase voltage.
- Line current = √3 × Phase current

As, the angle between voltage and current per phase is ϕ, the electric power per phase is

$$V_{PH} \times I_{PH} \times \cos\Phi = V_L \times (I_L/\sqrt{3}) \times \cos\Phi$$

So the total power of three phase system is

$$3 \times V_L \times (I_L/\sqrt{3}) \times \cos\Phi = \sqrt{3} \times V_L \times I_L \times \cos\Phi$$


$I_L$
$V_L$
$V_P$
$I_P$
$N$
Star connection: $I_L = I_P$
$V_L = \sqrt{3} \times V_P$
Delta connection: $I_L = \sqrt{3} \times I_P$
$V_L = V_P$

# Thank You