Module 1: DC Circuits

Basic Electrical Engineering- 5EL 101

Contents:

- Review of R-L-C- Electrical circuit elements
- KCL and KVL
- Star- delta conversion
- Voltage and current sources
- Thevenin, Norton and Superposition, Maximum power transfer Theorems.

Revision of Basic Concepts

Electricity generation, transmission, distribution

- Electricity Generation 11 kV, 50 Hz, 3 Phase
- Conversion of mechanical, solar, wind, nuclear, hydel, thermal energy into electrical energy
- > Thermal Power Plant
- ➤ Hydel Power Plant
- ➤ Nuclear Power Plant
- ➤ Solar Power Plant
- ➤ Wind Power Plant etc.

Thermal Power Plant

- ➤ Coal burning and boiler
- ➤ Superheated steam and turbine
- ➤ Rotational energy and generation of electricity
- > Heat-Mechanical-Electrical

Hydel Power Plant

- > Water head is used to drive the turbine
- ➤ Potential energy of water through Penstock
- ➤ Water turbine and Turbogenerator
- ➤ Construction of dam

Nuclear Power plant

- ➤ Nuclear Fission U²³⁵
- ➤ Bombardment of neutrons
- >Lot of heat energy and chain reaction (controlled)
- ➤ Heat exchanger- steam turbine- generation

Solar Power plant

- ➤ Solar power to electricity
- ➤ Photovoltaic cells
- >Stored in batteries and then used.

Wind Power plant

- ➤ Kinetic energy of air
- > Rotation of propeller like blades around rotor

Electricity Transmission

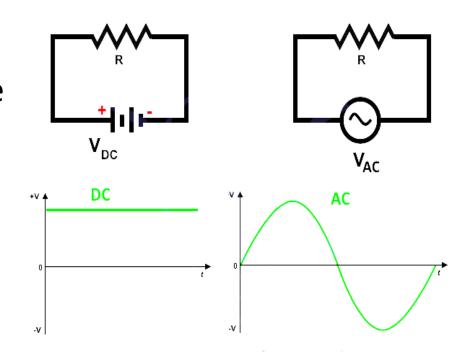
- ➤ Power in MWs to be transported to long distance
- >Transmission tower- transmission lines, disc insulators, tower structure
- ➤ 132kV,220kV,400kV,765kV transmission
- ➤ Step up or Step down transformers
- ➤ Substation- Transformers, Switches, Circuit Breakers, Relays, Meters, Control, Protection
- ➤ Big/Medium/Small industries

Electricity Distribution

- ≥33kV at substation
- >LT consumers
- ➤ Distribution transformer with 400V- star connected secondary
- $>400/\sqrt{3} = 230V$
- ➤ Phase-Neutral-Earth
- ➤ Overhead conductors and underground cables

Basic definitions:

- 1. Potentional Difference-It is the work done when an unit charge is moved from one point to another point in an electric field. i.e. Voltage
- 2. Current- Rate of flow of charge through a conducting path. Direction of current-flow of positive charge, Unit-Ampere
- a) Direct current Unidirectional current
- b) Alternating current- Flow of current in both positive and negative direction



Electrical Energy and Power

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P = VI,

P = I^2R,

P = V^2/R

1 watt (W) = 1 joule/second (J/s)
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Electrical Energy= Power * Time = V*I*t Joules

Basic Laws

- Ohm's Law
- KCL
- KVL
- Faradays law
- Lenz's law
- Thumb rule
- Fleming's rule

Ohm's Law

• Ohm's Law states that the current flowing through a conductor is directly proportional to the potential difference applied across its ends, provided the temperature and other physical conditions remain unchanged.

Or

• The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant. • Potential difference ∝ Current i.e. V ∝ I (When the value of V increases the value of I increases simultaneously)

$$V = IR$$

Where,

V is Voltage in volts (V)

R is Resistance in ohm (Ω)

I is Current in Ampere (A)

Numerical-

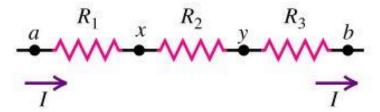
1. The electric current in a resistor wire is 4 A. When both ends are given a potential of 12 Volts. What is the electrical resistance?

Solution: R=V/I, R=12/4=3 ohms

2. In an electric circuit, current of 1.5 microamps (1.5 μ A) were to go through a resistance of 2.3 mega-ohms (2.3 M Ω). How much voltage would be "dropped" across this resistance?

Solution: V = IR, $V = 1.5 \times 10^{-6} A * 2.3 \times 10^{6} \Omega = 3.45 V$

Resistors in Series



Since these resistors are in series, we have the same current in all three resistors

$$I_1 = I_2 = I_3 = I$$

We also have that the sum of the potential differences across the three resistors must be the same as the potential difference between points *a* and *b*

$$V_{ab} = V_{ax} + V_{xy} + V_{yb}$$

Then using
$$V_{ax} = I R_1$$
; $V_{xy} = I R_2$; $V_{yb} = I R_3$

We have that
$$oldsymbol{V_{ab}} = oldsymbol{I} \left(oldsymbol{R}_1 + oldsymbol{R}_2 + oldsymbol{R}_3
ight)$$

Now the equivalent resistor, R, will also have the same potential difference across it as V_{ab} , and it will also have the same current I

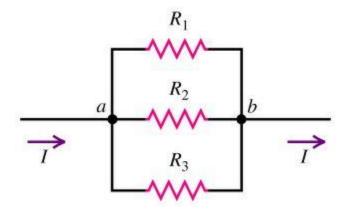
$$V_{ab} = IR$$

Equating these last two results, we then have that

$$\mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3 = \sum_{i} \mathbf{R}_{i}$$

The equivalent resistance for a sequence of resistors in series is just the sum of the individual resistances

Resistors in Parallel



Here we have that the voltage across each resistor has to be the same (work done in going from a to b is independent of the path, independent of which resistor you go through)

$$V_1 = V_2 = V_3 = V_{ab}$$

Resistors in Parallel

We now deal with currents through the resistors

At point *a* the current splits up into three distinct currents

We have that the sum of theses three currents must add to the value coming into this point

$$\boldsymbol{I} = \boldsymbol{I}_1 + \boldsymbol{I}_2 + \boldsymbol{I}_3$$

We also have that

$$I_1 = \frac{V_{ab}}{R_1}; \quad I_2 = \frac{V_{ab}}{R_2}; \quad I_3 = \frac{V_{ab}}{R_3}$$

The equivalent resistor, R, will have also have the current I going through it

Resistors in Parallel

Using
$$I=rac{V_{ab}}{R}$$

or

and combining with the previous equations, we then have

$$\frac{\boldsymbol{V_{ab}}}{\boldsymbol{R}} = \frac{\boldsymbol{V_{ab}}}{\boldsymbol{R_1}} + \frac{\boldsymbol{V_{ab}}}{\boldsymbol{R_2}} + \frac{\boldsymbol{V_{ab}}}{\boldsymbol{R_3}}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \sum_{i} \frac{1}{R_i}$$

The inverse of the effective resistance is given by the sum of the inverses of the individual resistances

Complex numbers

Convert Complex Number from Rectangular Form to Polar (Trigonometric) Form

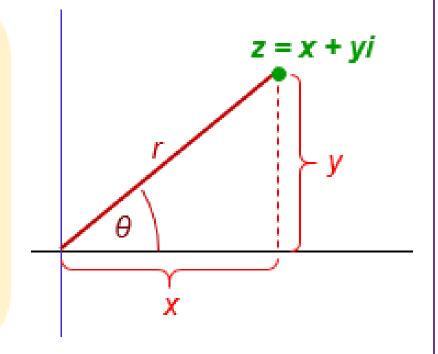
$$z = x + yi$$
 (rectangular form)

$$r = |z| = \sqrt{x^2 + y^2}$$

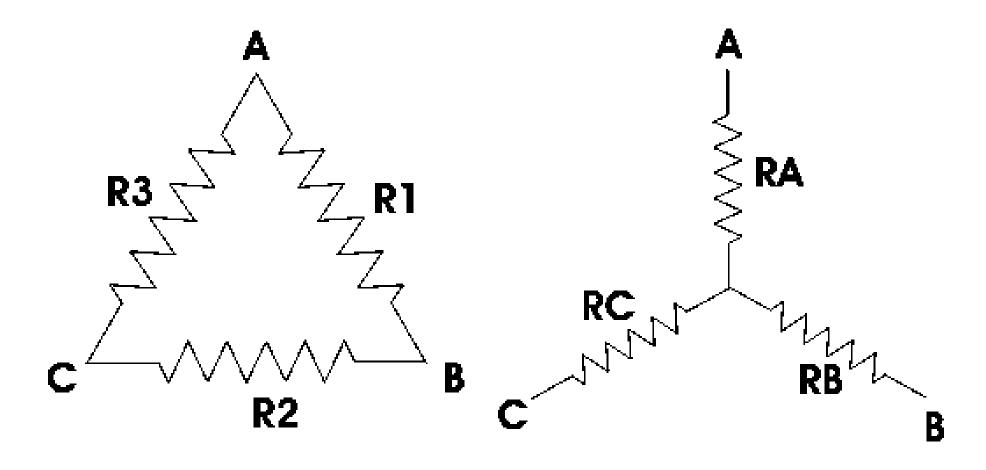
$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = r(\cos\theta + i\sin\theta)$$
 (polar form)



DELTA AND STAR CONNECTED RESISTORS

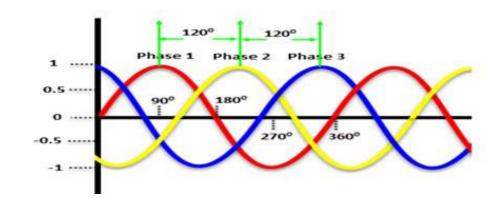


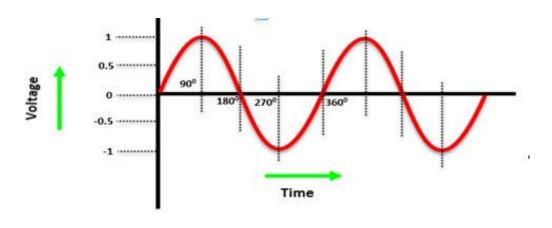
• Loads or sources can be star or delta connected.

3 phase and 1 phase AC power

• In a **single-phase power supply**, it only requires two wires, namely **Phase** and Neutral.

 On the other hand, a three-phase power supply only works through three wires, including threeconductor wires and a neutral wire





Cramer's Rule for Three Equations in Three Unknowns

The solution to the system

$$a_1x + b_1y + c_1z = d_1$$

 $a_2x + b_2y + c_2z = d_2$
 $a_3x + b_3y + c_3z = d_3$

is given by
$$x = \frac{D_x}{D}$$
, $y = \frac{D_y}{D}$, and $z = \frac{D_z}{D}$, where

provided that $D \neq 0$.

Module 1 Electric circuit elements

- 1. Resistance
- 2. Inductance
- 3. Capacitance

Resistance

- Resistance is a measure of the opposition to current flow in an electrical circuit. Resistance is measured in ohms(Ω).
- Temperature dependent

$$R = \frac{V}{I}$$
 $R \ge \frac{1}{V_R}$
 $R = \frac{\rho L}{A}$
 $R = \frac{\rho}{A}$
 $R = \frac{\rho}{A}$

(Ohm's law is valid when temp. is constant)

- Resistance increases with temperature-Pure Metals
- Resistance decreases with temperature-Carbon, Insulating material etc.
- Resistance converts electrical energy into heat.
- Power Dissipation:

$$V = I \times R$$
 Now Power, $W = I^2 \times R = \left(\frac{V}{R}\right)^2 \times R$

$$\therefore I = \frac{V}{R}$$
 Power, $W = \frac{V^2}{R}$



Inductor

- Stores energy in the form of magnetic field and delivers it as and when required.
- Whenever current passes through a conductor, lines of magnetic flux are generated around it. This magnetic flux opposes any change in current due to the induced e.m.f.
- This opposition to the flow of current in an inductor is known as inductance.

"Ohm's Law" for an inductor

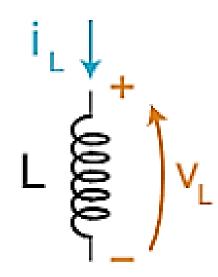
$$\mathbf{v} = \mathbf{L} \frac{\mathbf{di}}{\mathbf{dt}}$$

Where.

 \mathbf{v} = Instantaneous voltage across the inductor

L = Inductance in Henrys

$$\frac{di}{dt} = Instantaneous rate of current change (amps per second)$$



FACTORS THAT AFFECT INDUCTANCE OF A COIL.

- Number of turns in the coil
- Permeability of the core material.
- Size of core.

Inductance is given by,

$$L = \frac{\mu_o \mu_r A N^2}{l} H$$

where μ_o = permeability of free space = $4\pi \times 10^{-7}$ H/m = 1.257×10^{-6} H/m

 μ_r = relative permeability of the core material

A =area of cross-section of the core

N = number of turns of the coil

l = length of the core.



Capacitor

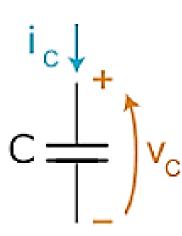
- Capacitor is a device that stores electrical energy in an electric field. Unit-Farad
- Two parallel plates separated by an insulating material.
- The non-conductive region can either be a vacuum or an electrical insulator material known as a dielectric. Examples of dielectric media are glass, air, paper, plastic, ceramic etc.

"Ohm's Law" for a capacitor

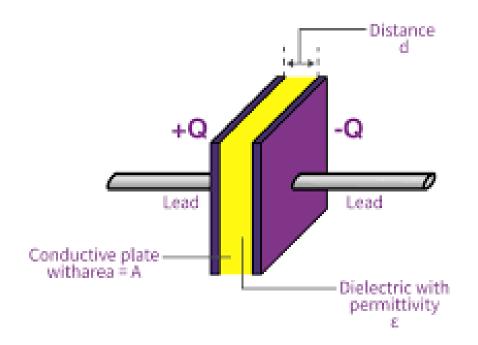
$$i = C \frac{dv}{dt}$$
 Where,
$$i = \text{Instantaneous current through the capacitor}$$

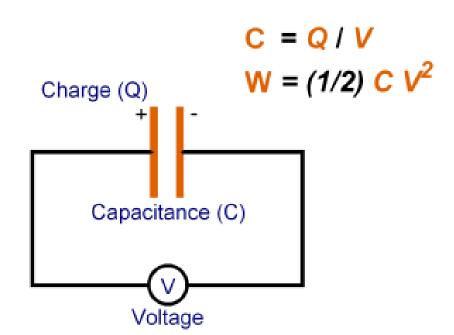
$$C = \text{Capacitance in Farads}$$

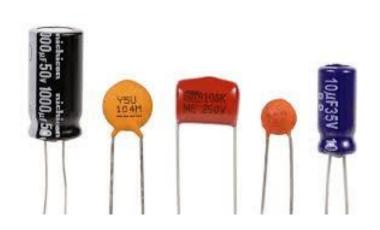
$$\frac{dv}{dt} = \text{Instantaneous rate of voltage change}$$
 (volts per second)



Mathematically,
$$C = \frac{\epsilon A}{d}$$



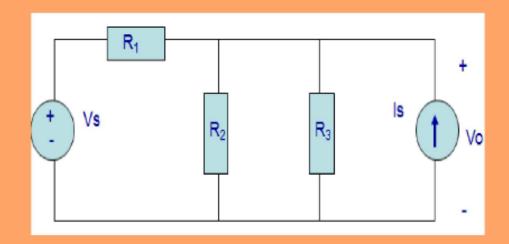




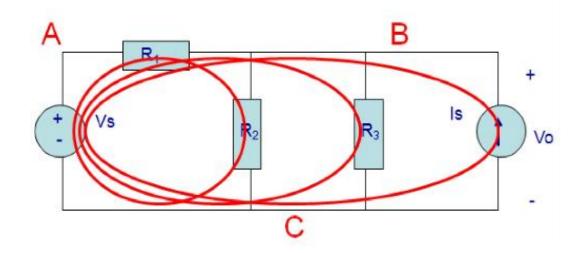
Circuit Definitions

- Node- It is a junction where two or more elemental points meet.
- Path- It is a traversal through elements from one node to another without going through the same node twice.
- Branch- It is a path between two adjoining nodes.
- Loop- It is a closed path where the transversal ends upon the starting node.
- Mesh- It is a loop that does not contain any other loop or within it.

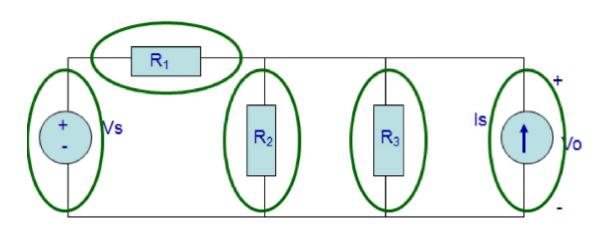
How many nodes, branches & loops?



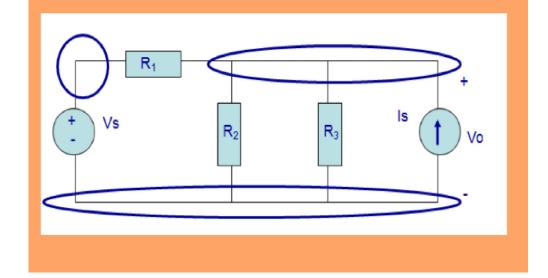
Three Loops, if starting at node A

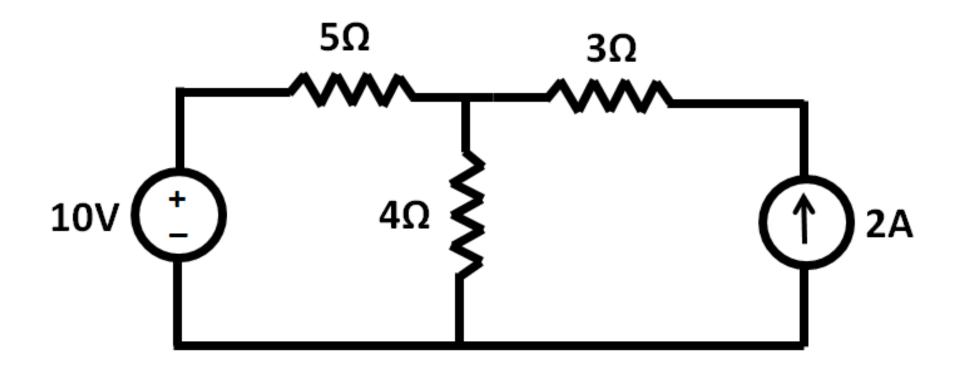


5 Branches



Three nodes

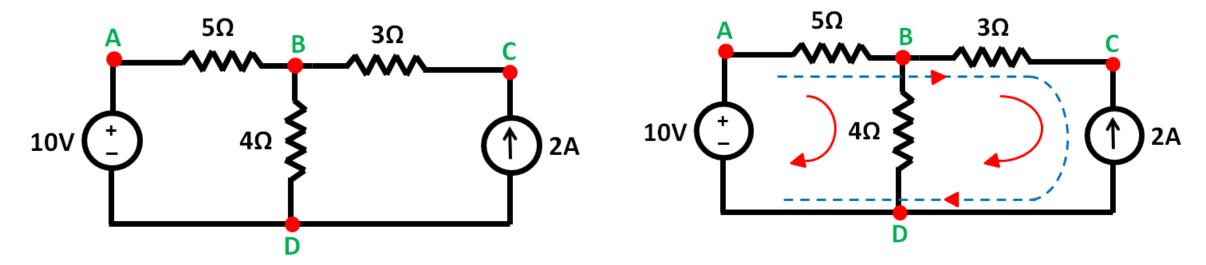




• 5 branches-A 10 V voltage source, 2A current source, 4 Ω , 5 Ω , and 3 Ω resistors.

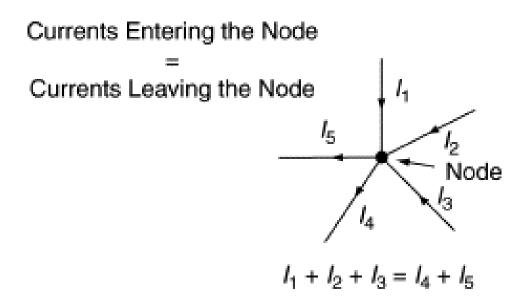
4 Nodes

- **3 Loops** The first is loop A-B-D-A, the second loop is B-C-D-B. And the third loop is A-B-C-D-A.
- 2 Meshes Loop 1(A-B-D-A) and loop 2 (B-C-D-B) does not contain any other closed path within them. And they are the example of the Mesh. While loop 3 (A-B-C-D-A) contains loop 1 and loop 2 within it. So, it can't be called as a Mesh.



Kirchhoff's Current Law (KCL)

- The algebraic sum of all currents meeting at a junction point is zero.
- (Add each branch current entering the junction and subtract each branch current leaving the junction)



- Σ currents in Σ currents out = 0 i.e. Σ I at junction = 0
- A current heading towards a junction, is considered to be positive
- A current heading away from a junction, is considered to be negative

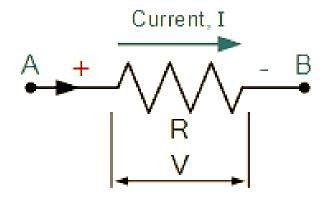
Kirchhoff's Voltage Law (KVL)

The algebraic sum of all the voltages around any closed loop in a circuit is equal to zero. $\Sigma V = 0$.

Or

In any electrical network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f.s in the path

Important Sign Convention



- Here the flow of current through the resistor is from point A to point B, that is from positive terminal to a negative terminal. Thus as we are travelling in the same direction as current flow, there will be a fall in potential across the resistive element giving rise to a
 - -IR voltage drop across it.
- If the flow of current was in the opposite direction from point B to point A, then there would be a *rise* in potential across the resistive element as we are moving from a potential to a + potential giving us a +I*R voltage drop.

$$V_S + (-IR_1) + (-IR_2) = 0$$

$$\therefore V_S = IR_1 + IR_2$$

$$V_S = I(R_1 + R_2)$$

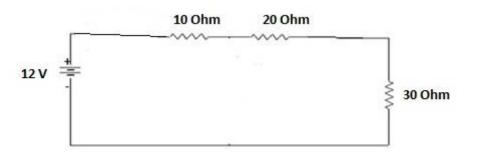
$$V_S = IR_T$$

Where: $R_T = R_1 + R_2$

Example

• Three resistor of values: 10 ohms, 20 ohms and 30 ohms, respectively are connected in series across a 12 volt battery supply. Calculate: a) the total resistance, b) the circuit current, c) the current through each resistor, d) the voltage drop across each resistor, e) verify that Kirchhoff's voltage law, KVL holds true.

Solution



a) Total Resistance (R_T)

$$R_T = R_1 + R_2 + R_3 = 10\Omega + 20\Omega + 30\Omega = 60\Omega$$

Then the total circuit resistance R_T is equal to 60Ω

b) Circuit Current (I)

$$I = \frac{V_S}{R_T} = \frac{12}{60} = 0.2A$$

Thus the total circuit current I is equal to 0.2 amperes or 200mA

c) Current Through Each Resistor

The resistors are wired together in series, they are all part of the same loop and therefore each experience the same amount of current. Thus:

$$I_{R1} = I_{R2} = I_{R3} = I_{SERIES} = 0.2$$
 amperes

d) Voltage Drop Across Each Resistor

$$V_{R1} = I \times R_1 = 0.2 \times 10 = 2 \text{ volts}$$

$$V_{p_2} = I \times R_2 = 0.2 \times 20 = 4 \text{ volts}$$

$$V_{R3} = I \times R_3 = 0.2 \times 30 = 6 \text{ volts}$$

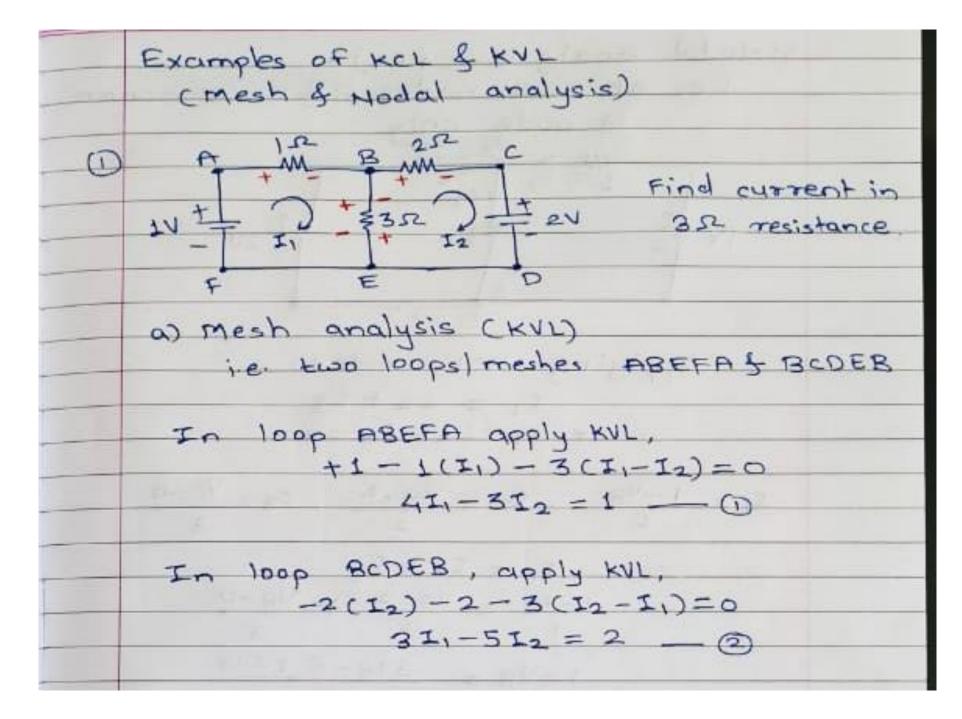
e) Verify Kirchhoff's Voltage Law

$$V_S + (-IR_1) + (-IR_2) + (-IR_3) = 0$$

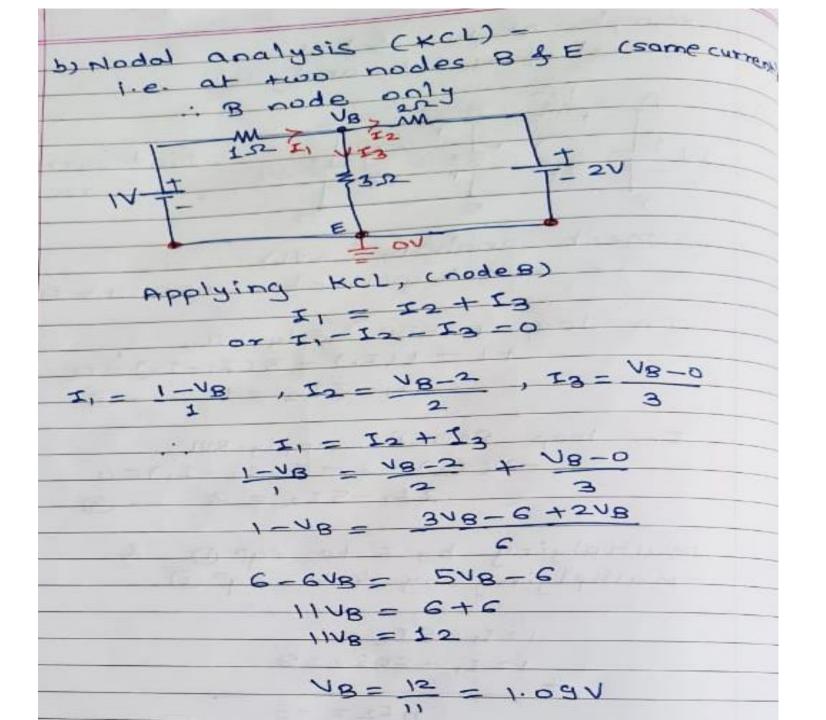
$$12 + (-0.2 \times 10) + (-0.2 \times 20) + (-0.2 \times 30) = 0$$

$$12 + (-2) + (-4) + (-6) = 0$$

$$\therefore 12 - 2 - 4 - 6 = 0$$

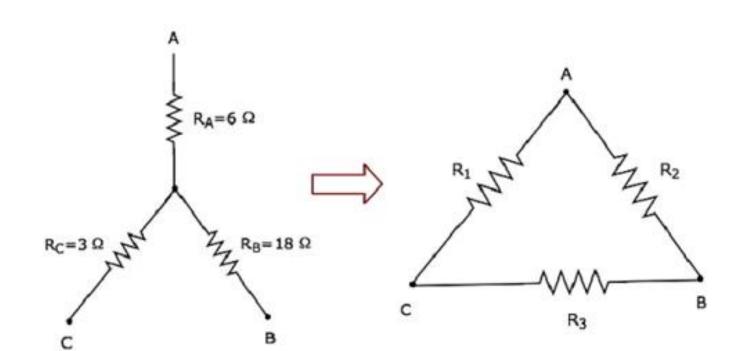


multiplying by 3 to eqn 10 & Multiplying by 4 to eq @ 124-912=3 121, -2012=8 1112 = -5 I2 = -0.45 A I = -0.09A 8 to E -> [I 32 = I1-I2 = [0.36A] [13=0.36A]



: H= 1-1.09 = -0.09A - V8-2 - 1.09-2 = 0.363 A (B to E) i.e. current in 3-12 rel

Star-Delta conversion

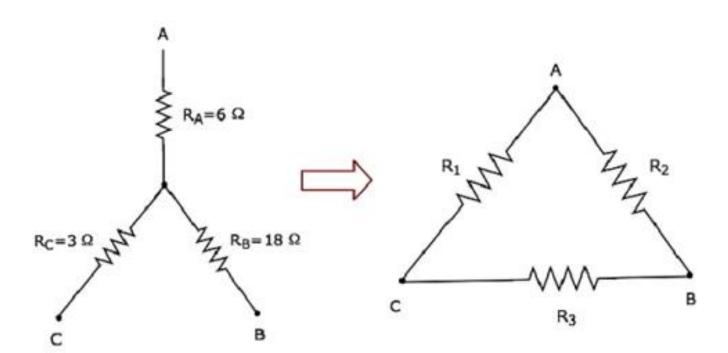


$$R_1 = R_C + R_A + \frac{R_C R_A}{R_B}$$

$$R_2 = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_3 = R_B + R_C + \frac{R_B R_C}{R_A}$$

Example



$$R_1 = R_C + R_A + \frac{R_C R_A}{R_B}$$

$$R_2 = R_A + R_B + \frac{R_A R_B}{R_C}$$

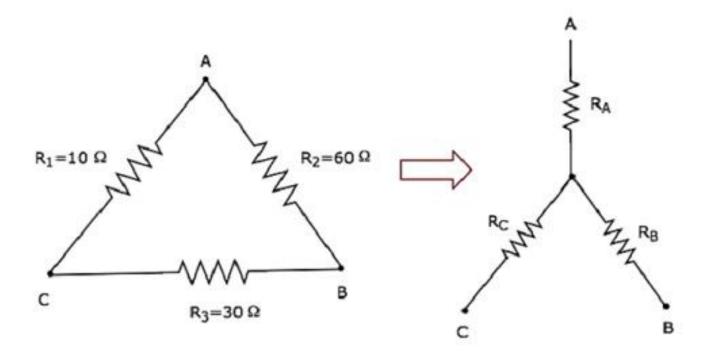
$$R_3 = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_1 = 3 + 6 + \frac{3 \times 6}{18} = 9 + 1 = 10\Omega$$

$$R_2 = 6 + 18 + rac{6 imes 18}{3} = 24 + 36 = 60 \Omega$$

$$R_3 = 18 + 3 + rac{18 imes 3}{6} = 21 + 9 = 30 \Omega$$

Delta-Star conversion

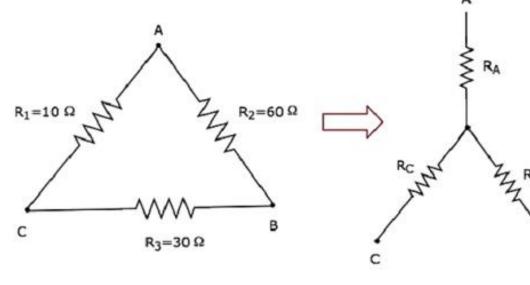


$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_B = rac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

Example



$$R_A = rac{10 imes 60}{10 + 60 + 30} = rac{600}{100} = 6\Omega$$

$$R_B = \frac{60 \times 30}{10 + 60 + 30} = \frac{1800}{100} = 18\Omega$$

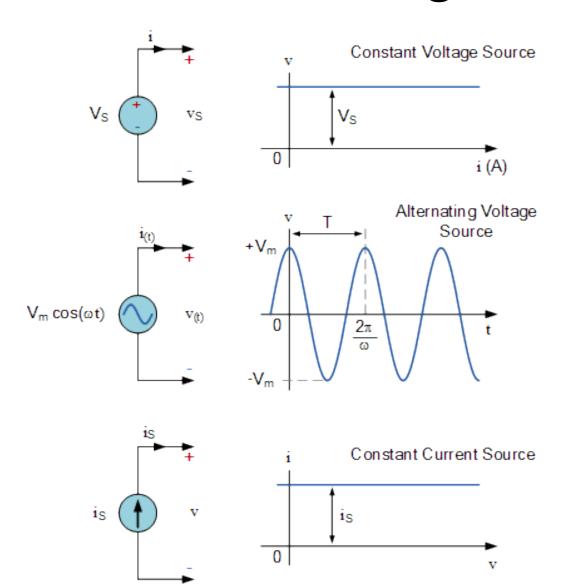
$$R_C = \frac{30 \times 10}{10 + 60 + 30} = \frac{300}{100} = 3\Omega$$

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

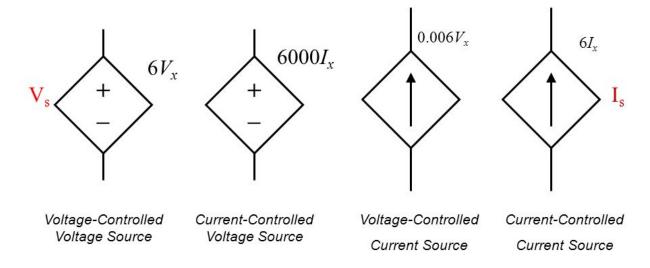
$$R_C = rac{R_3 R_1}{R_1 + R_2 + R_3}$$

Voltage and current sources



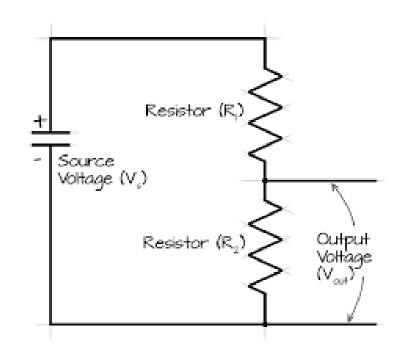
Dependent Sources

The *dependent source* magnitude is a function of another voltage or current in the circuit



Voltage Division Rule

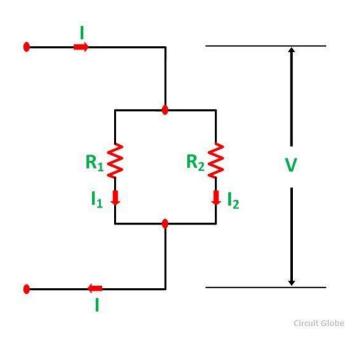
$$\begin{split} V_{\mathrm{S}} &= V_{\mathrm{R1}} + V_{\mathrm{R2}} \quad \text{(KVL)} \\ V_{\mathrm{R1}} &= I \times R_{1} \quad \text{and} \quad V_{\mathrm{R2}} = I \times R_{2} \\ \text{Then:} \quad V_{\mathrm{S}} &= I \times R_{1} + I \times R_{2} \\ & \therefore \quad V_{\mathrm{S}} = I \Big(R_{1} + R_{2} \Big) \\ \text{So:} \quad I &= \frac{V_{\mathrm{S}}}{\left(R_{1} + R_{2} \right)} \\ I_{\mathrm{R1}} &= \frac{V_{\mathrm{R1}}}{R_{1}} = \frac{V_{\mathrm{S}}}{\left(R_{1} + R_{2} \right)} \\ & \therefore \quad V_{\mathrm{R1}} = V_{\mathrm{S}} \Big(\frac{R_{1}}{R_{1} + R_{2}} \Big) \end{split}$$



$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{V_{S}}{(R_1 + R_2)}$$

$$\therefore V_{R2} = V_{S} \left(\frac{R_{2}}{R_{1} + R_{2}} \right)$$

Current Division Rule



$$I_{1} = \frac{V}{R_{1}} \quad \text{and} \quad I_{2} = \frac{V}{R_{2}}$$

$$R = \frac{R_{1}R_{2}}{R_{1} + R_{2}} \quad I = V/R$$

$$I = \frac{V(R_{1} + R_{2})}{R_{1}R_{2}}$$

$$V = I_{1}R_{1} = I_{2}R_{2}$$

$$I = \frac{I_1 R_1 (R_1 + R_2)}{R_1 R_2} = \frac{I_1}{R_2} (R_1 + R_2) \qquad I = \frac{I_2 R_2 (R_1 + R_2)}{R_1 R_2} = \frac{I_1}{R_1} (R_1 + R_2)$$

$$I_1 = I \frac{R_2}{R_1 + R_2}$$
 and $I_2 = I \frac{R_1}{R_1 + R_2}$

Superposition theorem

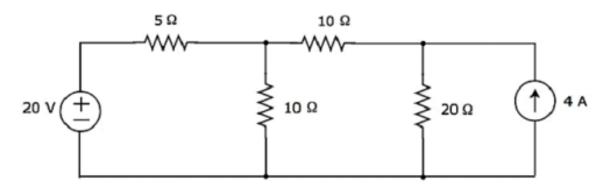
Statement

Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance.

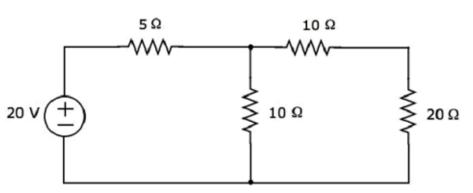
Procedure of Superposition Theorem

- **Step 1** Find the response in a particular branch by considering one independent source and eliminating the remaining independent sources present in the network.
- Step 2 Repeat Step 1 for all independent sources present in the network.
- **Step 3** Add all the responses in order to get the overall response in a particular branch when all independent sources are present in the network.

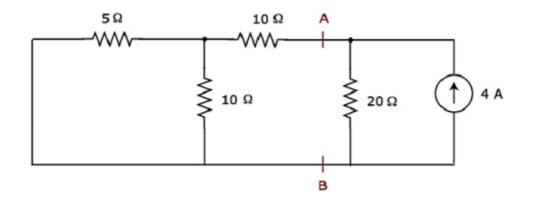
Find the current flowing through 20 Ω resistor of the following circuit using **superposition theorem**.



Step 1 – Let us find the current flowing through 20 Ω resistor by considering only 20 V voltage source. In this case, we can eliminate the 4 A current source by making open circuit of it.

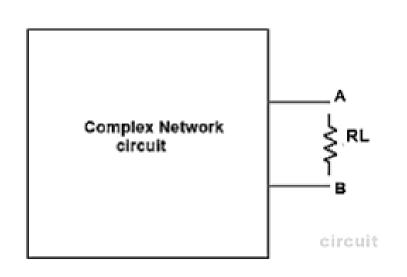


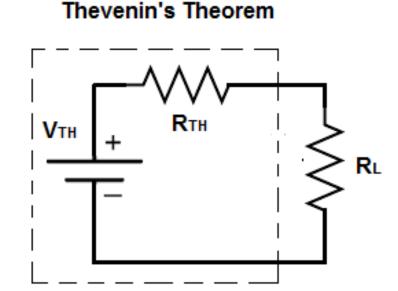
Step 2 – Let us find the current flowing through 20 Ω resistor by considering only **4 A current source**. In this case, we can eliminate the 20 V voltage source by making short-circuit of it.



Thevenin's theorem

 Statement- Any linear active network consisting of independent or dependent voltage and current source and the network elements can be replaced by an equivalent circuit having a voltage source in series with a resistance.



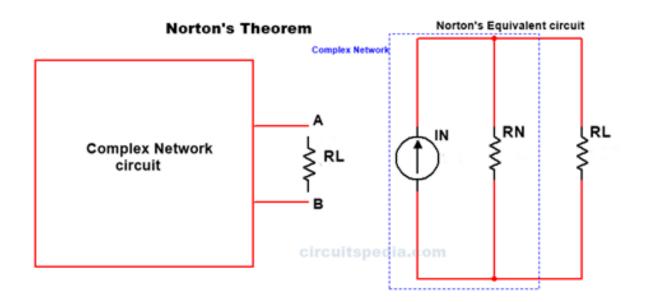


Steps

- Remove load resistance R_L
- Find open circuit voltage V_{th}
- Find R_{th} by replacing voltage source by short circuit and current source by open circuit.
- Replace the network by V_{th} in series with R_{th}.
- Find current through R_L

Norton's theorem

• Statement- A linear active network consisting of the independent or dependent voltage source and current sources and the various circuit elements can be substituted by an equivalent circuit consisting of a current source in parallel with a resistance. The current source being the short-circuited current across the load terminal and the resistance being the internal resistance of the source network.



Steps

- Remove load resistance R_L
- Find short circuit current I_N
- Find the resistance R_N by replacing voltage source by short circuit and current source by open circuit.
- Replace network by I_N in parallel with R_N .
- Reconnect the load resistance R_L of the circuit across the load terminals and find the current through it known as load current I_L

Maximum Power Transfer Theorem

• Statement- The maximum power transfer theorem states that in a linear, bilateral DC network, maximum power is transferred from the source to the load when the load resistance is equal to the Thevenin's equivalent resistance.

i.e,
$$R_L = R_{TH}$$

$$V_{Th} = R_{Th}$$

$$V_{Th} = R_{Th}$$

Steps for Solving Network Using Maximum Power Transfer Theorem

- Step 1 Remove the load resistance of the circuit. Find open circuit voltage V_{th} .
- Step 2 Find the Thevenin's resistance (R_{TH}) of the source network looking through the open-circuited load terminals.
- Step 3 As per the maximum power transfer theorem, this R_{TH} is the load resistance of the network, i.e., $R_L = R_{TH}$ that allows maximum power transfer.
- Step 4 Maximum Power Transfer is calculated by the equation shown below

$$P_{\text{max}} = \frac{V_{\text{TH}}^2}{4R_{\text{TH}}}$$