ENGINEERING A COMPILER

SECOND EDITION

MORGAN KAUFMANN

Keith D. Cooper & Linda Torczon

In Praise of *Engineering a Compiler* Second Edition

Compilers are a rich area of study, drawing together the whole world of computer science in one, elegant construction. Cooper and Torczon have succeeded in creating a welcoming guide to these software systems, enhancing this new edition with clear lessons and the details you simply must get right, all the while keeping the big picture firmly in view. Engineering a Compiler *is an invaluable companion for anyone new to the subject.*

Michael D. Smith
Dean of the Faculty of Arts and Sciences
John H. Finley, Jr. Professor of Engineering and Applied Sciences, Harvard University

The Second Edition of Engineering a Compiler *is an excellent introduction to the construction of modern optimizing compilers. The authors draw from a wealth of experience in compiler construction in order to help students grasp the big picture while at the same time guiding them through many important but subtle details that must be addressed to construct an effective optimizing compiler. In particular, this book contains the best introduction to Static Single Assignment Form that I've seen.*

Jeffery von Ronne
Assistant Professor
Department of Computer Science
The University of Texas at San Antonio

Engineering a Compiler *increases its value as a textbook with a more regular and consistent structure, and with a host of instructional aids: review questions, extra examples, sidebars, and marginal notes. It also includes a wealth of technical updates, including more on nontraditional languages, real-world compilers, and nontraditional uses of compiler technology. The optimization material—already a signature strength—has become even more accessible and clear.*

Michael L. Scott
Professor
Computer Science Department
University of Rochester
Author of *Programming Language Pragmatics*

Keith Cooper and Linda Torczon present an effective treatment of the history as well as a practitioner's perspective of how compilers are developed. Theory as well as practical real world examples of existing compilers (i.e. LISP, FORTRAN, etc.) comprise a multitude of effective discussions and illustrations. Full circle discussion of introductory along with advanced "allocation" and "optimization" concepts encompass an effective "life-cycle" of compiler engineering. This text should be on every bookshelf of computer science students as well as professionals involved with compiler engineering and development.

David Orleans
Nova Southeastern University

Engineering a Compiler

Second Edition

About the Authors

Keith D. Cooper is the Doerr Professor of Computational Engineering at Rice University. He has worked on a broad collection of problems in optimization of compiled code, including interprocedural data-flow analysis and its applications, value numbering, algebraic reassociation, register allocation, and instruction scheduling. His recent work has focused on a fundamental reexamination of the structure and behavior of traditional compilers. He has taught a variety of courses at the undergraduate level, from introductory programming through code optimization at the graduate level. He is a Fellow of the ACM.

Linda Torczon, Senior Research Scientist, Department of Computer Science at Rice University, is a principal investigator on the Platform-Aware Compilation Environment project (PACE), a DARPA-sponsored project that is developing an optimizing compiler environment which automatically adjusts its optimizations and strategies to new platforms. From 1990 to 2000, Dr. Torczon served as executive director of the Center for Research on Parallel Computation (CRPC), a National Science Foundation Science and Technology Center. She also served as the executive director of HiPerSoft, of the Los Alamos Computer Science Institute, and of the Virtual Grid Application Development Software Project (VGrADS).

Engineering a Compiler

Second Edition

Keith D. Cooper

Linda Torczon

Rice University
Houston, Texas

ELSEVIER

AMSTERDAM • BOSTON • HEIDELBERG • LONDON
NEW YORK • OXFORD • PARIS • SAN DIEGO
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

Morgan Kaufmann Publishers is an imprint of Elsevier

ELSEVIER

Acquiring Editor: Todd Green
Development Editor: Nate McFadden
Project Manager: Andre Cuello
Designer: Alisa Andreola

Cover Image: "The Landing of the Ark," a vaulted ceiling-design whose iconography was narrated, designed, and drawn by John Outram of John Outram Associates, Architects and City Planners, London, England. To read more visit *www.johnoutram.com/rice.html*.

Morgan Kaufmann is an imprint of Elsevier.
30 Corporate Drive, Suite 400, Burlington, MA 01803, USA

Library of Congress Cataloging-in-Publication Data
Application submitted

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

ISBN: 978-0-12-088478-0

Printed in the United States of America
11 12 13 14 10 9 8 7 6 5 4 3 2 1

We dedicate this volume to

- *our parents, who instilled in us the thirst for knowledge and supported us as we developed the skills to follow our quest for knowledge;*
- *our children, who have shown us again how wonderful the process of learning and growing can be; and*
- *our spouses, without whom this book would never have been written.*

About the Cover

The cover of this book features a portion of the drawing, “The Landing of the Ark,” which decorates the ceiling of Duncan Hall at Rice University. Both Duncan Hall and its ceiling were designed by British architect John Outram. Duncan Hall is an outward expression of architectural, decorative, and philosophical themes developed over Outram’s career as an architect. The decorated ceiling of the ceremonial hall plays a central role in the building’s decorative scheme. Outram inscribed the ceiling with a set of significant ideas—a creation myth. By expressing those ideas in an allegorical drawing of vast size and intense color, Outram created a signpost that tells visitors who wander into the hall that, indeed, this building is not like other buildings.

By using the same signpost on the cover of *Engineering a Compiler*, the authors intend to signal that this work contains significant ideas that are at the core of their discipline. Like Outram’s building, this volume is the culmination of intellectual themes developed over the authors’ professional careers. Like Outram’s decorative scheme, this book is a device for communicating ideas. Like Outram’s ceiling, it presents significant ideas in new ways.

By connecting the design and construction of compilers with the design and construction of buildings, we intend to convey the many similarities in these two distinct activities. Our many long discussions with Outram introduced us to the Vitruvian ideals for architecture: commodity, firmness, and delight. These ideals apply to many kinds of construction. Their analogs for compiler construction are consistent themes of this text: function, structure, and elegance. Function matters; a compiler that generates incorrect code is useless. Structure matters; engineering detail determines a compiler’s efficiency and robustness. Elegance matters; a well-designed compiler, in which the algorithms and data structures flow smoothly from one pass to another, can be a thing of beauty.

We are delighted to have John Outram’s work grace the cover of this book.

Duncan Hall’s ceiling is an interesting technological artifact. Outram drew the original design on one sheet of paper. It was photographed and scanned at 1200 dpi yielding roughly 750 MB of data. The image was enlarged to form 234 distinct 2 × 8 foot panels, creating a 52 × 72 foot image. The panels were printed onto oversize sheets of perforated vinyl using a 12 dpi acrylic-ink printer. These sheets were precision mounted onto 2 × 8 foot acoustic tiles and hung on the vault’s aluminum frame.

Contents

Preface to the Second Edition

The practice of compiler construction changes continually, in part because the designs of processors and systems change. For example, when we began to write *Engineering a Compiler* (EAC) in 1998, some of our colleagues questioned the wisdom of including a chapter on instruction scheduling because out-of-order execution threatened to make scheduling largely irrelevant. Today, as the second edition goes to press, the rise of multicore processors and the push for more cores has made in-order execution pipelines attractive again because their smaller footprints allow the designer to place more cores on a chip. Instruction scheduling will remain important for the near-term future.

At the same time, the compiler construction community continues to develop new insights and algorithms, and to rediscover older techniques that were effective but largely forgotten. Recent research has created excitement surrounding the use of chordal graphs in register allocation (see Section 13.5.2). That work promises to simplify some aspects of graph-coloring allocators. Brzozowski's algorithm is a DFA minimization technique that dates to the early 1960s but has not been taught in compiler courses for decades (see Section 2.6.2). It provides an easy path from an implementation of the subset construction to one that minimizes DFAS. A modern course in compiler construction might include both of these ideas.

How, then, are we to structure a curriculum in compiler construction so that it prepares students to enter this ever changing field? We believe that the course should provide each student with the set of base skills that they will need to build new compiler components and to modify existing ones. Students need to understand both sweeping concepts, such as the collaboration between the compiler, linker, loader, and operating system embodied in a linkage convention, and minute detail, such as how the compiler writer might reduce the aggregate code space used by the register-save code at each procedure call.

CHANGES IN THE SECOND EDITION

The second edition of *Engineering a Compiler* (EAC2e) presents both perspectives: big-picture views of the problems in compiler construction and detailed discussions of algorithmic alternatives. In preparing EAC2e, we focused on the usability of the book, both as a textbook and as a reference for professionals. Specifically, we

- Improved the flow of ideas to help the student who reads the book sequentially. Chapter introductions explain the purpose of the chapter, lay out the major concepts, and provide a high-level overview of the chapter's subject matter. Examples have been reworked to provide continuity across chapters. In addition, each chapter begins with a summary and a set of keywords to aid the user who treats EAC2e as a reference book.
- Added section reviews and review questions at the end of each major section. The review questions provide a quick check as to whether or not the reader has understood the major points of the section.

- Moved definitions of key terms into the margin adjacent to the paragraph where they are first defined and discussed.
- Revised the material on optimization extensively so that it provides broader coverage of the possibilities for an optimizing compiler.

Compiler development today focuses on optimization and on code generation. A newly hired compiler writer is far more likely to port a code generator to a new processor or modify an optimization pass than to write a scanner or parser. The successful compiler writer must be familiar with current best-practice techniques in optimization, such as the construction of static single-assignment form, and in code generation, such as software pipelining. They must also have the background and insight to understand new techniques as they appear during the coming years. Finally, they must understand the techniques of scanning, parsing, and semantic elaboration well enough to build or modify a front end.

Our goal for EAC2e has been to create a text and a course that exposes students to the critical issues in modern compilers and provides them with the background to tackle those problems. We have retained, from the first edition, the basic balance of material. Front ends are commodity components; they can be purchased from a reliable vendor or adapted from one of the many open-source systems. At the same time, optimizers and code generators are custom-crafted for particular processors and, sometimes, for individual models, because performance relies so heavily on specific low-level details of the generated code. These facts affect the way that we build compilers today; they should also affect the way that we teach compiler construction.

ORGANIZATION

EAC2e divides the material into four roughly equal pieces:

- The first major section, Chapters 2 through 4, covers both the design of a compiler front end and the design and construction of tools to build front ends.
- The second major section, Chapters 5 through 7, explores the mapping of source-code into the compiler's intermediate form—that is, these chapters examine the kind of code that the front end generates for the optimizer and back end.
- The third major section, Chapters 8 through 10, introduces the subject of code optimization. Chapter 8 provides an overview of optimization. Chapters 9 and 10 contain deeper treatments of analysis and transformation; these two chapters are often omitted from an undergraduate course.
- The final section, Chapters 11 through 13, focuses on algorithms used in the compiler's back end.

THE ART AND SCIENCE OF COMPILATION

The lore of compiler construction includes both amazing success stories about the application of theory to practice and humbling stories about the limits of what we can do. On the success side, modern scanners are built by applying the theory of regular languages to automatic construction of recognizers. LR parsers use the same techniques to perform the handle-recognition that drives a shift-reduce parser. Data-flow analysis applies lattice theory to the analysis of programs in clever and useful ways. The approximation algorithms used in code generation produce good solutions to many instances of truly hard problems.

On the other side, compiler construction exposes complex problems that defy good solutions. The back end of a compiler for a modern processor approximates the solution to two or more interacting NP-complete problems (instruction scheduling, register allocation, and, perhaps, instruction and data placement). These NP-complete problems, however, look easy next to problems such as algebraic reassociation of expressions (see, for example, Figure 7.1). This problem admits a huge number of solutions; to make matters worse, the desired solution depends on context in both the compiler and the application code. As the compiler approximates the solutions to such problems, it faces constraints on compile time and available memory. A good compiler artfully blends theory, practical knowledge, engineering, and experience.

Open up a modern optimizing compiler and you will find a wide variety of techniques. Compilers use greedy heuristic searches that explore large solution spaces and deterministic finite automata that recognize words in the input. They employ fixed-point algorithms to reason about program behavior and simple theorem provers and algebraic simplifiers to predict the values of expressions. Compilers take advantage of fast pattern-matching algorithms to map abstract computations to machine-level operations. They use linear diophantine equations and Pressburger arithmetic to analyze array subscripts. Finally, compilers use a large set of classic algorithms and data structures such as hash tables, graph algorithms, and sparse set implementations.

In EAC2e, we have tried to convey both the art and the science of compiler construction. The book includes a sufficiently broad selection of material to show the reader that real tradeoffs exist and that the impact of design decisions can be both subtle and far-reaching. At the same time, EAC2e omits some techniques that have long been part of an undergraduate compiler construction course, but have been rendered less important by changes in the marketplace, in the technology of languages and compilers, or in the availability of tools.

■ APPROACH

Compiler construction is an exercise in engineering design. The compiler writer must choose a path through a design space that is filled with diverse alternatives, each with distinct costs, advantages, and complexity. Each decision has an impact on the resulting compiler. The quality of the end product depends on informed decisions at each step along the way.

Thus, there is no single right answer for many of the design decisions in a compiler. Even within "well understood" and "solved" problems, nuances in design and implementation have an impact on both the behavior of the compiler and the quality of the code that it produces. Many considerations play into each decision. As an example, the choice of an intermediate representation for the compiler has a profound impact on the rest of the compiler, from time and space requirements through the ease with which different algorithms can be applied. The decision, however, is often given short shrift. Chapter 5 examines the space of intermediate representations and some of the issues that should be considered in selecting one. We raise the issue again at several points in the book—both directly in the text and indirectly in the exercises.

EAC2e explores the design space and conveys both the depth of the problems and the breadth of the possible solutions. It shows some ways that those problems have been solved, along with the constraints that made those solutions attractive. Compiler writers need to understand both the problems and their solutions, as well as the impact of those decisions on other facets of the compiler's design. Only then can they make informed and intelligent choices.

■ PHILOSOPHY

This text exposes our philosophy for building compilers, developed during more than twenty-five years each of research, teaching, and practice. For example, intermediate representations should expose those details that matter in the final code; this belief leads to a bias toward low-level representations. Values should reside in registers until the allocator discovers that it cannot keep them there; this practice produces examples that use virtual registers and store values to memory only when it cannot be avoided. Every compiler should include optimization; it simplifies the rest of the compiler. Our experiences over the years have informed the selection of material and its presentation.

■ A WORD ABOUT PROGRAMMING EXERCISES

A class in compiler construction offers the opportunity to explore software design issues in the context of a concrete application—one whose basic functions are well understood by any student with the background for a compiler construction course. In most versions of this course, the programming exercises play a large role.

We have taught this class in versions where the students build a simple compiler from start to finish—beginning with a generated scanner and parser and ending with a code generator for some simplified RISC instruction set. We have taught this class in versions where the students write programs that address well-contained individual problems, such as register allocation or instruction scheduling. The choice of programming exercises depends heavily on the role that the course plays in the surrounding curriculum.

In some schools, the compiler course serves as a capstone course for seniors, tying together concepts from many other courses in a large, practical, design and implementation project. Students in such a class might write a complete compiler for a simple language or modify an open-source compiler to add support for a new language feature or a new architectural feature. This class might present the material in a linear order that closely follows the text's organization.

In other schools, that capstone experience occurs in other courses or in other ways. In such a class, the teacher might focus the programming exercises more narrowly on algorithms and their implementation, using labs such as a local register allocator or a tree-height rebalancing pass. This course might skip around in the text and adjust the order of presentation to meet the needs of the labs. For example, at Rice, we have often used a simple local register allocator as the first lab; any student with assembly-language programming experience understands the basics of the problem. That strategy, however, exposes the students to material from Chapter 13 before they see Chapter 2.

In either scenario, the course should draw material from other classes. Obvious connections exist to computer organization, assembly-language programming, operating systems, computer architecture, algorithms, and formal languages. Although the connections from compiler construction to other courses may be less obvious, they are no less important. Character copying, as discussed in Chapter 7, plays a critical role in the performance of applications that include network protocols, file servers, and web servers. The techniques developed in Chapter 2 for scanning have applications that range from text editing through URL-filtering. The bottom-up local register allocator in Chapter 13 is a cousin of the optimal offline page replacement algorithm, MIN.

■ ADDITIONAL MATERIALS

Additional resources are available that can help you adapt the material presented in EAC2e to your course. These include a complete set of lectures from the authors' version of the course at Rice University and a set of solutions to the exercises. Your Elsevier representative can provide you with access.

Acknowledgments

Many people were involved in the preparation of the first edition of EAC. Their contributions have carried forward into this second edition. Many people pointed out problems in the first edition, including Amit Saha, Andrew Waters, Anna Youssefi, Ayal Zachs, Daniel Salce, David Peixotto, Fengmei Zhao, Greg Malecha, Hwansoo Han, Jason Eckhardt, Jeffrey Sandoval, John Elliot, Kamal Sharma, Kim Hazelwood, Max Hailperin, Peter Froehlich, Ryan Stinnett, Sachin Rehki, Sağnak Taşırlar, Timothy Harvey, and Xipeng Shen. We also want to thank the reviewers of the second edition, who were Jeffery von Ronne, Carl Offner, David Orleans, K. Stuart Smith, John Mallozzi, Elizabeth White, and Paul C. Anagnostopoulos. The production team at Elsevier, in particular, Alisa Andreola, Andre Cuello, and Megan Guiney, played a critical role in converting the a rough manuscript into its final form. All of these people improved this volume in significant ways with their insights and their help.

Finally, many people have provided us with intellectual and emotional support over the last five years. First and foremost, our families and our colleagues at Rice have encouraged us at every step of the way. Christine and Carolyn, in particular, tolerated myriad long discussions on topics in compiler construction. Nate McFadden guided this edition from its inception through its publication with patience and good humor. Penny Anderson provided administrative and organizational support that was critical to finishing the second edition. To all these people go our heartfelt thanks.

This page intentionally left blank

Chapter 1

Overview of Compilation

CHAPTER OVERVIEW

Compilers are computer programs that translate a program written in one language into a program written in another language. At the same time, a compiler is a large software system, with many internal components and algorithms and complex interactions between them. Thus, the study of compiler construction is an introduction to techniques for the translation and improvement of programs, and a practical exercise in software engineering. This chapter provides a conceptual overview of all the major components of a modern compiler.

Keywords: Compiler, Interpreter, Automatic Translation

1.1 INTRODUCTION

The role of the computer in daily life grows each year. With the rise of the Internet, computers and the software that runs on them provide communications, news, entertainment, and security. Embedded computers have changed the ways that we build automobiles, airplanes, telephones, televisions, and radios. Computation has created entirely new categories of activity, from video games to social networks. Supercomputers predict daily weather and the course of violent storms. Embedded computers synchronize traffic lights and deliver e-mail to your pocket.

All of these computer applications rely on software computer programs that build virtual tools on top of the low-level abstractions provided by the underlying hardware. Almost all of that software is translated by a tool called a *compiler*. A compiler is simply a computer program that translates other computer programs to prepare them for execution. This book presents the fundamental techniques of automatic translation that are used

Compiler
a computer program that translates other computer programs

Engineering a Compiler. DOI: 10.1016/B978-0-12-088478-0.00001-3

to build compilers. It describes many of the challenges that arise in compiler construction and the algorithms that compiler writers use to address them.

Conceptual Roadmap

A compiler is a tool that translates software written in one language into another language. To translate text from one language to another, the tool must understand both the form, or syntax, and content, or meaning, of the input language. It needs to understand the rules that govern syntax and meaning in the output language. Finally, it needs a scheme for mapping content from the source language to the target language.

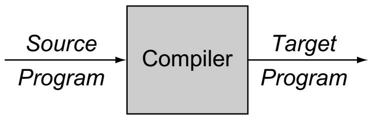
The structure of a typical compiler derives from these simple observations. The compiler has a front end to deal with the source language. It has a back end to deal with the target language. Connecting the front end and the back end, it has a formal structure for representing the program in an intermediate form whose meaning is largely independent of either language. To improve the translation, a compiler often includes an optimizer that analyzes and rewrites that intermediate form.

Overview

Computer programs are simply sequences of abstract operations written in a *programming language*—a formal language designed for expressing computation. Programming languages have rigid properties and meanings—as opposed to natural languages, such as Chinese or Portuguese. Programming languages are designed for expressiveness, conciseness, and clarity. Natural languages allow ambiguity. Programming languages are designed to avoid ambiguity; an ambiguous program has no meaning. Programming languages are designed to specify computations—to record the sequence of actions that perform some task or produce some results.

Programming languages are, in general, designed to allow humans to express computations as sequences of operations. Computer processors, hereafter referred to as processors, microprocessors, or machines, are designed to execute sequences of operations. The operations that a processor implements are, for the most part, at a much lower level of abstraction than those specified in a programming language. For example, a programming language typically includes a concise way to print some number to a file. That single programming language statement must be translated into literally hundreds of machine operations before it can execute.

The tool that performs such translations is called a compiler. The compiler takes as input a program written in some language and produces as its output an equivalent program. In the classic notion of a compiler, the output program is expressed in the operations available on some specific processor, often called the target machine. Viewed as a black box, a compiler might look like this:



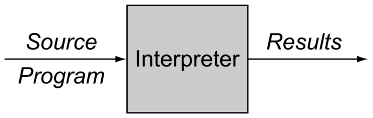
Typical "source" languages might be C, C++, FORTRAN, Java, or ML. The "target" language is usually the instruction set of some processor.

Instruction set
The set of operations supported by a processor; the overall design of an instruction set is often called an *instruction set architecture* or ISA.

Some compilers produce a target program written in a human-oriented programming language rather than the assembly language of some computer. The programs that these compilers produce require further translation before they can execute directly on a computer. Many research compilers produce C programs as their output. Because compilers for C are available on most computers, this makes the target program executable on all those systems, at the cost of an extra compilation for the final target. Compilers that target programming languages rather than the instruction set of a computer are often called *source-to-source translators*.

Many other systems qualify as compilers. For example, a typesetting program that produces PostScript can be considered a compiler. It takes as input a specification for how the document should look on the printed page and it produces as output a PostScript file. PostScript is simply a language for describing images. Because the typesetting program takes an executable specification and produces another executable specification, it is a compiler.

The code that turns PostScript into pixels is typically an *interpreter*, not a compiler. An interpreter takes as input an executable specification and produces as output the result of executing the specification.



Some languages, such as Perl, Scheme, and APL, are more often implemented with interpreters than with compilers.

Some languages adopt translation schemes that include both compilation and interpretation. Java is compiled from source code into a form called *bytecode*, a compact representation intended to decrease download times for Java applications. Java applications execute by running the bytecode on the corresponding Java Virtual Machine (JVM), an interpreter for bytecode. To complicate the picture further, many implementations of the JVM include a

Virtual machine
A virtual machine is a simulator for some processor. It is an *interpreter* for that machine's instruction set.

compiler that executes at runtime, sometimes called a *just-in-time compiler*, or JIT, that translates heavily used bytecode sequences into native code for the underlying computer.

Interpreters and compilers have much in common. They perform many of the same tasks. Both analyze the input program and determine whether or not it is a valid program. Both build an internal model of the structure and meaning of the program. Both determine where to store values during execution. However, interpreting the code to produce a result is quite different from emitting a translated program that can be executed to produce the result. This book focuses on the problems that arise in building compilers. However, an implementor of interpreters may find much of the material relevant.

Why Study Compiler Construction?

A compiler is a large, complex program. Compilers often include hundreds of thousands, if not millions, of lines of code, organized into multiple subsystems and components. The various parts of a compiler interact in complex ways. Design decisions made for one part of the compiler have important ramifications for other parts. Thus, the design and implementation of a compiler is a substantial exercise in software engineering.

A good compiler contains a microcosm of computer science. It makes practical use of greedy algorithms (register allocation), heuristic search techniques (list scheduling), graph algorithms (dead-code elimination), dynamic programming (instruction selection), finite automata and push-down automata (scanning and parsing), and fixed-point algorithms (data-flow analysis). It deals with problems such as dynamic allocation, synchronization, naming, locality, memory hierarchy management, and pipeline scheduling. Few software systems bring together as many complex and diverse components. Working inside a compiler provides practical experience in software engineering that is hard to obtain with smaller, less intricate systems.

Compilers play a fundamental role in the central activity of computer science: preparing problems for solution by computer. Most software is compiled, and the correctness of that process and the efficiency of the resulting code have a direct impact on our ability to build large systems. Most students are not satisfied with reading about these ideas; many of the ideas must be implemented to be appreciated. Thus, the study of compiler construction is an important component of a computer science education.

Compilers demonstrate the successful application of theory to practical problems. The tools that automate the production of scanners and parsers apply results from formal language theory. These same tools are used for text searching, website filtering, word processing, and command-language interpreters. Type checking and static analysis apply results from lattice theory, number theory, and other branches of mathematics to understand and improve programs. Code generators use algorithms for tree-pattern matching, parsing, dynamic programming, and text matching to automate the selection of instructions.

Still, some problems that arise in compiler construction are open problems—that is, the current best solutions have room for improvement. Attempts to design high-level, universal, intermediate representations have foundered on complexity. The dominant method for scheduling instructions is a greedy algorithm with several layers of tie-breaking heuristics. While it is obvious that compilers should use commutativity and associativity to improve the code, most compilers that try to do so simply rearrange the expression into some canonical order.

Building a successful compiler requires expertise in algorithms, engineering, and planning. Good compilers approximate the solutions to hard problems. They emphasize efficiency, in their own implementations and in the code they generate. They have internal data structures and knowledge representations that expose the right level of detail—enough to allow strong optimization, but not enough to force the compiler to wallow in detail. Compiler construction brings together ideas and techniques from across the breadth of computer science and applies them in a constrained setting to solve some truly hard problems.

The Fundamental Principles of Compilation

Compilers are large, complex, carefully engineered objects. While many issues in compiler design are amenable to multiple solutions and interpretations, there are two fundamental principles that a compiler writer must keep in mind at all times. The first principle is inviolable:

The compiler must preserve the meaning of the program being compiled.

Correctness is a fundamental issue in programming. The compiler must preserve correctness by faithfully implementing the "meaning" of its input program. This principle lies at the heart of the social contract between the compiler writer and compiler user. If the compiler can take liberties with meaning, then why not simply generate a `nop` or a `return`? If an incorrect translation is acceptable, why expend the effort to get it right?

The second principle that a compiler must observe is practical:

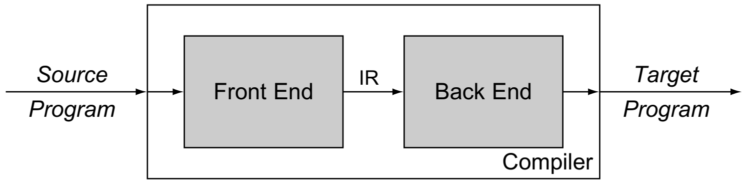
The compiler must improve the input program in some discernible way.

A traditional compiler improves the input program by making it directly executable on some target machine. Other "compilers" improve their input in different ways. For example, `tpic` is a program that takes the specification for a drawing written in the graphics language `pic` and converts it into LaTeX; the "improvement" lies in LaTeX's greater availability and generality. A source-to-source translator for C must produce code that is, in some measure, better than the input program; if it is not, why would anyone invoke it?

1.2 COMPILER STRUCTURE

A compiler is a large, complex software system. The community has been building compilers since 1955, and over the years, we have learned many lessons about how to structure a compiler. Earlier, we depicted a compiler as a simple box that translates a source program into a target program. Reality, of course, is more complex than that simple picture.

As the single-box model suggests, a compiler must both understand the source program that it takes as input and map its functionality to the target machine. The distinct nature of these two tasks suggests a division of labor and leads to a design that decomposes compilation into two major pieces: a *front end* and a *back end*.



The front end focuses on understanding the source-language program. The back end focuses on mapping programs to the target machine. This separation of concerns has several important implications for the design and implementation of compilers.

IR
A compiler uses some set of data structures to represent the code that it processes. That form is called an *intermediate representation*, or IR.

The front end must encode its knowledge of the source program in some structure for later use by the back end. This *intermediate representation* (IR) becomes the compiler's definitive representation for the code it is translating. At each point in compilation, the compiler will have a definitive representation. It may, in fact, use several different IRs as compilation progresses, but at each point, one representation will be the definitive IR. We think of the definitive IR as the version of the program passed between independent phases of the compiler, like the IR passed from the front end to the back end in the preceding drawing.

In a two-phase compiler, the front end must ensure that the source program is well formed, and it must map that code into the IR. The back end must map

MAY YOU STUDY IN INTERESTING TIMES

This is an exciting era in the design and implementation of compilers. In the 1980s, almost all compilers were large, monolithic systems. They took as input one of a handful of languages and produced assembly code for some particular computer. The assembly code was pasted together with the code produced by other compilations—including system libraries and application libraries—to form an executable. The executable was stored on a disk, and at the appropriate time, the final code was moved from the disk to main memory and executed.

Today, compiler technology is being applied in many different settings. As computers find applications in diverse places, compilers must cope with new and different constraints. Speed is no longer the sole criterion for judging the compiled code. Today, code might be judged on how small it is, on how much energy it consumes, on how well it compresses, or on how many page faults it generates when it runs.

At the same time, compilation techniques have escaped from the monolithic systems of the 1980s. They are appearing in many new places. Java compilers take partially compiled programs (in Java "bytecode" format) and translate them into native code for the target machine. In this environment, success requires that the sum of compile time plus runtime must be less than the cost of interpretation. Techniques to analyze whole programs are moving from compile time to link time, where the linker can analyze the assembly code for the entire application and use that knowledge to improve the program. Finally, compilers are being invoked at runtime to generate customized code that capitalizes on facts that cannot be known any earlier. If the compilation time can be kept small and the benefits are large, this strategy can produce noticeable improvements.

the IR program into the instruction set and the finite resources of the target machine. Because the back end only processes IR created by the front end, it can assume that the IR contains no syntactic or semantic errors.

The compiler can make multiple passes over the IR form of the code before emitting the target program. This should lead to better code, as the compiler can, in effect, study the code in one phase and record relevant details. Then, in later phases, it can use these recorded facts to improve the quality of translation. This strategy requires that knowledge derived in the first pass be recorded in the IR, where later passes can find and use it.

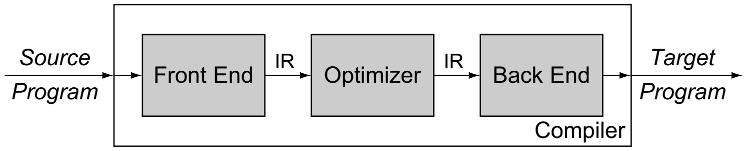
Finally, the two-phase structure may simplify the process of *retargeting* the compiler. We can easily envision constructing multiple back ends for a single front end to produce compilers that accept the same language but target different machines. Similarly, we can envision front ends for different

Retargeting
The task of changing the compiler to generate code for a new processor is often called *retargeting* the compiler.

languages producing the same IR and using a common back end. Both scenarios assume that one IR can serve for several combinations of source and target; in practice, both language-specific and machine-specific details usually find their way into the IR.

Optimizer
The middle section of a compiler, called an *optimizer*, analyzes and transforms the IR to improve it.

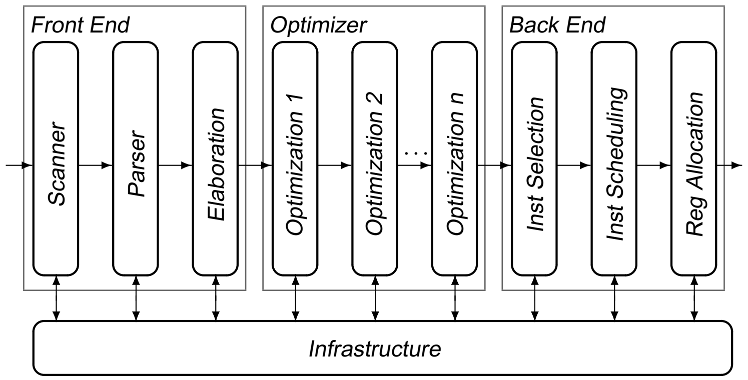
Introducing an IR makes it possible to add more phases to compilation. The compiler writer can insert a third phase between the front end and the back end. This middle section, or *optimizer*, takes an IR program as its input and produces a semantically equivalent IR program as its output. By using the IR as an interface, the compiler writer can insert this third phase with minimal disruption to the front end and back end. This leads to the following compiler structure, termed a *three-phase compiler*.



The optimizer is an IR-to-IR transformer that tries to improve the IR program in some way. (Notice that these transformers are, themselves, compilers according to our definition in Section 1.1.) The optimizer can make one or more passes over the IR, analyze the IR, and rewrite the IR. The optimizer may rewrite the IR in a way that is likely to produce a faster target program from the back end or a smaller target program from the back end. It may have other objectives, such as a program that produces fewer page faults or uses less energy.

Conceptually, the three-phase structure represents the classic optimizing compiler. In practice, each phase is divided internally into a series of passes. The front end consists of two or three passes that handle the details of recognizing valid source-language programs and producing the initial IR form of the program. The middle section contains passes that perform different optimizations. The number and purpose of these passes vary from compiler to compiler. The back end consists of a series of passes, each of which takes the IR program one step closer to the target machine's instruction set. The three phases and their individual passes share a common infrastructure. This structure is shown in Figure 1.1.

In practice, the conceptual division of a compiler into three phases, a front end, a middle section or optimizer, and a back end, is useful. The problems addressed by these phases are different. The front end is concerned with understanding the source program and recording the results of its analysis into IR form. The optimizer section focuses on improving the IR form.



■ **FIGURE 1.1** Structure of a Typical Compiler.

The back end must map the transformed IR program onto the bounded resources of the target machine in a way that leads to efficient use of those resources.

Of these three phases, the optimizer has the murkiest description. The term *optimization* implies that the compiler discovers an optimal solution to some problem. The issues and problems that arise in optimization are so complex and so interrelated that they cannot, in practice, be solved optimally. Furthermore, the actual behavior of the compiled code depends on interactions among all of the techniques applied in the optimizer and the back end. Thus, even if a single technique can be proved optimal, its interactions with other techniques may produce less than optimal results. As a result, a good optimizing compiler can improve the quality of the code, relative to an unoptimized version. However, an optimizing compiler will almost always fail to produce optimal code.

The middle section can be a single monolithic pass that applies one or more optimizations to improve the code, or it can be structured as a series of smaller passes with each pass reading and writing IR. The monolithic structure may be more efficient. The multipass structure may lend itself to a less complex implementation and a simpler approach to debugging the compiler. It also creates the flexibility to employ different sets of optimization in different situations. The choice between these two approaches depends on the constraints under which the compiler is built and operates.

1.3 OVERVIEW OF TRANSLATION

To translate code written in a programming language into code suitable for execution on some target machine, a compiler runs through many steps.

NOTATION

Compiler books are, in essence, about notation. After all, a compiler translates a program written in one notation into an equivalent program written in another notation. A number of notational issues will arise in your reading of this book. In some cases, these issues will directly affect your understanding of the material.

Expressing Algorithms We have tried to keep the algorithms concise. Algorithms are written at a relatively high level, assuming that the reader can supply implementation details. They are written in a *slanted, sans-serif font*. Indentation is both deliberate and significant; it matters most in an *if-then-else* construct. Indented code after a *then* or an *else* forms a block. In the following code fragment

```
if Action [s,word] = "shift $s_i$" then
   push word
   push $s_i$
   word ← NextWord()
else if ···
```

all the statements between the *then* and the *else* are part of the *then* clause of the *if-then-else* construct. When a clause in an *if-then-else* construct contains just one statement, we write the keyword *then* or *else* on the same line as the statement.

Writing Code In some examples, we show actual program text written in some language chosen to demonstrate a particular point. Actual program text is written in a `monospace` font.

Arithmetic Operators Finally, we have forsaken the traditional use of * for × and of / for ÷, except in actual program text. The meaning should be clear to the reader.

To make this abstract process more concrete, consider the steps needed to generate executable code for the following expression:

```
a ← a × 2 × b × c × d
```

where `a`, `b`, `c`, and `d` are variables, ← indicates an assignment, and × is the operator for multiplication. In the following subsections, we will trace the path that a compiler takes to turn this simple expression into executable code.

1.3.1 The Front End

Before the compiler can translate an expression into executable target-machine code, it must understand both its form, or *syntax*, and its meaning, or *semantics*. The front end determines if the input code is well formed, in terms of both syntax and semantics. If it finds that the code is valid, it creates a representation of the code in the compiler's intermediate representation; if not, it reports back to the user with diagnostic error messages to identify the problems with the code.

Checking Syntax

To check the syntax of the input program, the compiler must compare the program's structure against a definition for the language. This requires an appropriate formal definition, an efficient mechanism for testing whether or not the input meets that definition, and a plan for how to proceed on an illegal input.

Mathematically, the source language is a set, usually infinite, of strings defined by some finite set of rules, called a *grammar*. Two separate passes in the front end, called the scanner and the parser, determine whether or not the input code is, in fact, a member of the set of valid programs defined by the grammar.

Programming language grammars usually refer to words based on their parts of speech, sometimes called syntactic categories. Basing the grammar rules on parts of speech lets a single rule describe many sentences. For example, in English, many sentences have the form

$$\textit{Sentence} \rightarrow \textit{Subject}\ \texttt{verb}\ \textit{Object}\ \texttt{endmark}$$

where `verb` and `endmark` are parts of speech, and *Sentence*, *Subject*, and *Object* are syntactic variables. *Sentence* represents any string with the form described by this rule. The symbol "→" reads "derives" and means that an instance of the right-hand side can be abstracted to the syntactic variable on the left-hand side.

Consider a sentence like "Compilers are engineered objects." The first step in understanding the syntax of this sentence is to identify distinct words in the input program and to classify each word with a part of speech. In a compiler, this task falls to a pass called the *scanner*. The scanner takes a stream of characters and converts it to a stream of classified words—that is, pairs of the form (*p*,*s*), where *p* is the word's *part of speech* and *s* is its spelling. A scanner would convert the example sentence into the following stream of classified words:

Scanner
the compiler pass that converts a string of characters into a stream of words

(`noun`,"Compilers"), (`verb`,"are"), (`adjective`,"engineered"),
(`noun`,"objects"), (`endmark`,".")

In practice, the actual spelling of the words might be stored in a hash table and represented in the pairs with an integer index to simplify equality tests. Chapter 2 explores the theory and practice of scanner construction.

In the next step, the compiler tries to match the stream of categorized words against the rules that specify syntax for the input language. For example, a working knowledge of English might include the following grammatical rules:

1	*Sentence*	→	*Subject* `verb` *Object* `endmark`
2	*Subject*	→	`noun`
3	*Subject*	→	*Modifier* `noun`
4	*Object*	→	`noun`
5	*Object*	→	*Modifier* `noun`
6	*Modifier*	→	`adjective`
	...		

By inspection, we can discover the following *derivation* for our example sentence:

Rule	Prototype Sentence
—	*Sentence*
1	*Subject* `verb` *Object* `endmark`
2	`noun verb` *Object* `endmark`
5	`noun verb` *Modifier* `noun endmark`
6	`noun verb adjective noun endmark`

The derivation starts with the syntactic variable *Sentence*. At each step, it rewrites one term in the prototype sentence, replacing the term with a right-hand side that can be derived from that rule. The first step uses Rule 1 to replace *Sentence*. The second uses Rule 2 to replace *Subject*. The third replaces *Object* using Rule 5, while the final step rewrites *Modifier* with `adjective` according to Rule 6. At this point, the prototype sentence generated by the derivation matches the stream of categorized words produced by the scanner.

Parser
the compiler pass that determines if the input stream is a sentence in the source language

The derivation proves that the sentence "Compilers are engineered objects." belongs to the language described by Rules 1 through 6. The sentence is grammatically correct. The process of automatically finding derivations is called *parsing*. Chapter 3 presents the techniques that compilers use to parse the input program.

A grammatically correct sentence can be meaningless. For example, the sentence "Rocks are green vegetables" has the same parts of speech in the same order as "Compilers are engineered objects," but has no rational meaning. To understand the difference between these two sentences requires contextual knowledge about software systems, rocks, and vegetables.

Type checking
the compiler pass that checks for type-consistent uses of names in the input program

The semantic models that compilers use to reason about programming languages are simpler than the models needed to understand natural language. A compiler builds mathematical models that detect specific kinds of inconsistency in a program. Compilers check for consistency of type; for example, the expression

$$a \leftarrow a \times 2 \times b \times c \times d$$

might be syntactically well-formed, but if b and d are character strings, the sentence might still be invalid. Compilers also check for consistency of number in specific situations; for example, an array reference should have the same number of dimensions as the array's declared rank and a procedure call should specify the same number of arguments as the procedure's definition. Chapter 4 explores some of the issues that arise in compiler-based type checking and semantic elaboration.

Intermediate Representations

The final issue handled in the front end of a compiler is the generation of an IR form of the code. Compilers use a variety of different kinds of IR, depending on the source language, the target language, and the specific transformations that the compiler applies. Some IRs represent the program as a graph. Others resemble a sequential assembly code program. The code in the margin shows how our example expression might look in a low-level, sequential IR. Chapter 5 presents an overview of the variety of kinds of IRs that compilers use.

$$\begin{array}{lcl} t_0 & \leftarrow & a \times 2 \\ t_1 & \leftarrow & t_0 \times b \\ t_2 & \leftarrow & t_1 \times c \\ t_3 & \leftarrow & t_2 \times d \\ a & \leftarrow & t_3 \end{array}$$

For every source-language construct, the compiler needs a strategy for how it will implement that construct in the IR form of the code. Specific choices affect the compiler's ability to transform and improve the code. Thus, we spend two chapters on the issues that arise in generation of IR for source-code constructs. Procedure linkages are, at once, a source of inefficiency in the final code and the fundamental glue that pieces together different source files into an application. Thus, we devote Chapter 6 to the issues that surround procedure calls. Chapter 7 presents implementation strategies for most other programming language constructs.

TERMINOLOGY

A careful reader will notice that we use the word *code* in many places where either *program* or *procedure* might naturally fit. Compilers can be invoked to translate fragments of code that range from a single reference through an entire system of programs. Rather than specify some scope of compilation, we will continue to use the ambiguous, but more general, term, *code*.

1.3.2 The Optimizer

When the front end emits IR for the input program, it handles the statements one at a time, in the order that they are encountered. Thus, the initial IR program contains general implementation strategies that will work in any surrounding context that the compiler might generate. At runtime, the code will execute in a more constrained and predictable context. The optimizer analyzes the IR form of the code to discover facts about that context and uses that contextual knowledge to rewrite the code so that it computes the same answer in a more efficient way.

Efficiency can have many meanings. The classic notion of optimization is to reduce the application's running time. In other contexts, the optimizer might try to reduce the size of the compiled code, or other properties such as the energy that the processor consumes evaluating the code. All of these strategies target efficiency.

Returning to our example, consider it in the context shown in Figure 1.2a. The statement occurs inside a loop. Of the values that it uses, only a and d change inside the loop. The values of 2, b, and c are invariant in the loop. If the optimizer discovers this fact, it can rewrite the code as shown in Figure 1.2b. In this version, the number of multiplications has been reduced from $4 \cdot n$ to $2 \cdot n + 2$. For $n > 1$, the rewritten loop should execute faster. This kind of optimization is discussed in Chapters 8, 9, and 10.

Analysis

Data-flow analysis
a form of compile-time reasoning about the runtime flow of values

Most optimizations consist of an analysis and a transformation. The analysis determines where the compiler can safely and profitably apply the technique. Compilers use several kinds of analysis to support transformations. *Data-flow analysis* reasons, at compile time, about the flow of values at runtime. Data-flow analyzers typically solve a system of simultaneous set equations that are derived from the structure of the code being translated. *Dependence analysis* uses number-theoretic tests to reason about the values that can be

```
b ← ···
c ← ···
a ← 1
for i = 1 to n
  read d
  a ← a × 2 × b × c × d
  end
```

(a) Original Code in Context

```
b ← ···
c ← ···
a ← 1
t ← 2 × b × c
for i = 1 to n
  read d
  a ← a × d × t
  end
```

(b) Improved Code

■ **FIGURE 1.2** Context Makes a Difference.

assumed by subscript expressions. It is used to disambiguate references to array elements. Chapter 9 presents a detailed look at data-flow analysis and its application, along with the construction of static-single-assignment form, an IR that encodes information about the flow of both values and control directly in the IR.

Transformation

To improve the code, the compiler must go beyond analyzing it. The compiler must use the results of analysis to rewrite the code into a more efficient form. Myriad transformations have been invented to improve the time or space requirements of executable code. Some, such as discovering loop-invariant computations and moving them to less frequently executed locations, improve the running time of the program. Others make the code more compact. Transformations vary in their effect, the scope over which they operate, and the analysis required to support them. The literature on transformations is rich; the subject is large enough and deep enough to merit one or more separate books. Chapter 10 covers the subject of scalar transformations—that is, transformations intended to improve the performance of code on a single processor. It presents a taxonomy for organizing the subject and populates that taxonomy with examples.

1.3.3 **The Back End**

The compiler's back end traverses the IR form of the code and emits code for the target machine. It selects target-machine operations to implement each IR operation. It chooses an order in which the operations will execute efficiently. It decides which values will reside in registers and which values will reside in memory and inserts code to enforce those decisions.

ABOUT ILOC

Throughout the book, low-level examples are written in a notation that we call ILOC—an acronym derived from "intermediate language for an optimizing compiler." Over the years, this notation has undergone many changes. The version used in this book is described in detail in Appendix A.

Think of ILOC as the assembly language for a simple RISC machine. It has a standard set of operations. Most operations take arguments that are registers. The memory operations, `loads` and `stores`, transfer values between memory and the registers. To simplify the exposition in the text, most examples assume that all data consists of integers.

Each operation has a set of operands and a target. The operation is written in five parts: an operation name, a list of operands, a separator, a list of targets, and an optional comment. Thus, to add registers 1 and 2, leaving the result in register 3, the programmer would write

$$\texttt{add } r_1, r_2 \Rightarrow r_3 \quad \texttt{// example instruction}$$

The separator, ⇒, precedes the target list. It is a visual reminder that information flows from left to right. In particular, it disambiguates cases where a person reading the assembly-level text can easily confuse operands and targets. (See `loadAI` and `storeAI` in the following table.)

The example in Figure 1.3 only uses four ILOC operations:

ILOC Operation		Meaning
loadAI	$r_1, c_2 \Rightarrow r_3$	Memory$(r_1+c_2) \rightarrow r_3$
loadI	$c_1 \Rightarrow r_2$	$c_1 \rightarrow r_2$
mult	$r_1, r_2 \Rightarrow r_3$	$r_1 \times r_2 \rightarrow r_3$
storeAI	$r_1 \Rightarrow r_2, c_3$	$r_1 \rightarrow$ Memory(r_2+c_3)

Appendix A contains a more detailed description of ILOC. The examples consistently use r_{arp} as a register that contains the start of data storage for the current procedure, also known as the *activation record pointer*.

Instruction Selection

$$\begin{aligned} t_0 &\leftarrow a \times 2 \\ t_1 &\leftarrow t_0 \times b \\ t_2 &\leftarrow t_1 \times c \\ t_3 &\leftarrow t_2 \times d \\ a &\leftarrow t_3 \end{aligned}$$

The first stage of code generation rewrites the IR operations into target machine operations, a process called *instruction selection*. Instruction selection maps each IR operation, in its context, into one or more target machine operations. Consider rewriting our example expression, $a \leftarrow a \times 2 \times b \times c \times d$, into code for the ILOC virtual machine to illustrate the process. (We will use ILOC throughout the book.) The IR form of the expression is repeated in the margin. The compiler might choose the operations shown in Figure 1.3. This code assumes that a, b, c, and d

```
loadAI  r_arp, @a ⇒ r_a        // load 'a'
loadI   2         ⇒ r_2        // constant 2 into r_2
loadAI  r_arp, @b ⇒ r_b        // load 'b'
loadAI  r_arp, @c ⇒ r_c        // load 'c'
loadAI  r_arp, @d ⇒ r_d        // load 'd'
mult    r_a, r_2  ⇒ r_a        // r_a ← a × 2
mult    r_a, r_b  ⇒ r_a        // r_a ← (a × 2) × b
mult    r_a, r_c  ⇒ r_a        // r_a ← (a × 2 × b) × c
mult    r_a, r_d  ⇒ r_a        // r_a ← (a × 2 × b × c) × d
storeAI r_a       ⇒ r_arp,@a   // write r_a back to 'a'
```

■ **FIGURE 1.3** ILOC Code for a ← a × 2 × b × c × d.

are located at offsets @a, @b, @c, and @d from an address contained in the register r_{arp}.

The compiler has chosen a straightforward sequence of operations. It loads all of the relevant values into registers, performs the multiplications in order, and stores the result to the memory location for a. It assumes an unlimited supply of registers and names them with symbolic names such as r_a to hold a and r_{arp} to hold the address where the data storage for our named values begins. Implicitly, the instruction selector relies on the register allocator to map these symbolic register names, or *virtual registers*, to the actual registers of the target machine.

Virtual register
a symbolic register name that the compiler uses to indicate that a value can be stored in a register

The instruction selector can take advantage of special operations on the target machine. For example, if an immediate-multiply operation (multI) is available, it might replace the operation mult $r_a, r_2 \Rightarrow r_a$ with multI $r_a, 2 \Rightarrow r_a$, eliminating the need for the operation loadI $2 \Rightarrow r_2$ and reducing the demand for registers. If addition is faster than multiplication, it might replace mult $r_a, r_2 \Rightarrow r_a$ with add $r_a, r_a \Rightarrow r_a$, avoiding both the loadI and its use of r_2, as well as replacing the mult with a faster add. Chapter 11 presents two different techniques for instruction selection that use pattern matching to choose efficient implementations for IR operations.

Register Allocation

During instruction selection, the compiler deliberately ignored the fact that the target machine has a limited set of registers. Instead, it used virtual registers and assumed that "enough" registers existed. In practice, the earlier stages of compilation may create more demand for registers than the hardware can support. The register allocator must map those virtual registers onto actual target-machine registers. Thus, the register allocator decides, at each point in the code, which values should reside in the target-machine registers. It then rewrites the code to reflect its decisions. For example, a register allocator might minimize register use by rewriting the code from Figure 1.3 as follows:

```
loadAI  rarp, @a ⇒ r1          // load ‘a’
add     r1, r1   ⇒ r1          // r1 ← a × 2
loadAI  rarp, @b ⇒ r2          // load ‘b’
mult    r1, r2   ⇒ r1          // r1 ← (a × 2) × b
loadAI  rarp, @c ⇒ r2          // load ‘c’
mult    r1, r2   ⇒ r1          // r1 ← (a × 2 × b) × c
loadAI  rarp, @d ⇒ r2          // load ‘d’
mult    r1, r2   ⇒ r1          // r1 ← (a × 2 × b × c) × d
storeAI r1       ⇒ rarp, @a    // write ra back to ‘a’
```

This sequence uses three registers instead of six.

Minimizing register use may be counterproductive. If, for example, any of the named values, a, b, c, or d, are already in registers, the code should reference those registers directly. If all are in registers, the sequence could be implemented so that it required no additional registers. Alternatively, if some nearby expression also computed a × 2, it might be better to preserve that value in a register than to recompute it later. This optimization would increase demand for registers but eliminate a later instruction. Chapter 13 explores the problems that arise in register allocation and the techniques that compiler writers use to solve them.

Instruction Scheduling

To produce code that executes quickly, the code generator may need to reorder operations to reflect the target machine's specific performance constraints. The execution time of the different operations can vary. Memory access operations can take tens or hundreds of cycles, while some arithmetic operations, particularly division, take several cycles. The impact of these longer latency operations on the performance of compiled code can be dramatic.

Assume, for the moment, that a loadAI or storeAI operation requires three cycles, a mult requires two cycles, and all other operations require one cycle. The following table shows how the previous code fragment performs under these assumptions. The **Start** column shows the cycle in which each operation begins execution and the **End** column shows the cycle in which it completes.

Start	End				
1	3	loadAI	r_{arp}, @a	$\Rightarrow r_1$	// load 'a'
4	4	add	r_1, r_1	$\Rightarrow r_1$	// $r_1 \leftarrow$ a $\times$ 2
5	7	loadAI	r_{arp}, @b	$\Rightarrow r_2$	// load 'b'
8	9	mult	r_1, r_2	$\Rightarrow r_1$	// $r_1 \leftarrow$ (a $\times$ 2) $\times$ b
10	12	loadAI	r_{arp}, @c	$\Rightarrow r_2$	// load 'c'
13	14	mult	r_1, r_2	$\Rightarrow r_1$	// $r_1 \leftarrow$ (a $\times$ 2 $\times$ b) $\times$ c
15	17	loadAI	r_{arp}, @d	$\Rightarrow r_2$	// load 'd'
18	19	mult	r_1, r_2	$\Rightarrow r_1$	// $r_1 \leftarrow$ (a $\times$ 2 $\times$ b $\times$ c) $\times$ d
20	22	storeAI	r_1	$\Rightarrow r_{arp}$, @a	// write r_a back to 'a'

This nine-operation sequence takes 22 cycles to execute. Minimizing register use did not lead to rapid execution.

Many processors have a property by which they can initiate new operations while a long-latency operation executes. As long as the results of a long-latency operation are not referenced until the operation completes, execution proceeds normally. If, however, some intervening operation tries to read the result of the long-latency operation prematurely, the processor delays the operation that needs the value until the long-latency operation completes. An operation cannot begin to execute until its operands are ready, and its results are not ready until the operation terminates.

The instruction scheduler reorders the operations in the code. It attempts to minimize the number of cycles wasted waiting for operands. Of course, the scheduler must ensure that the new sequence produces the same result as the original. In many cases, the scheduler can drastically improve the performance of "naive" code. For our example, a good scheduler might produce the following sequence:

Start	End				
1	3	loadAI	r_{arp}, @a	$\Rightarrow r_1$	// load 'a'
2	4	loadAI	r_{arp}, @b	$\Rightarrow r_2$	// load 'b'
3	5	loadAI	r_{arp}, @c	$\Rightarrow r_3$	// load 'c'
4	4	add	r_1, r_1	$\Rightarrow r_1$	// $r_1 \leftarrow$ a $\times$ 2
5	6	mult	r_1, r_2	$\Rightarrow r_1$	// $r_1 \leftarrow$ (a $\times$ 2) $\times$ b
6	8	loadAI	r_{arp}, @d	$\Rightarrow r_2$	// load 'd'
7	8	mult	r_1, r_3	$\Rightarrow r_1$	// $r_1 \leftarrow$ (a $\times$ 2 $\times$ b) $\times$ c
9	10	mult	r_1, r_2	$\Rightarrow r_1$	// $r_1 \leftarrow$ (a $\times$ 2 $\times$ b $\times$ c) $\times$ d
11	13	storeAI	r_1	$\Rightarrow r_{arp}$, @a	// write r_a back to 'a'

COMPILER CONSTRUCTION IS ENGINEERING

A typical compiler has a series of passes that, together, translate code from some source language into some target language. Along the way, the compiler uses dozens of algorithms and data structures. The compiler writer must select, for each step in the process, an appropriate solution.

A successful compiler executes an unimaginable number of times. Consider the total number of times that GCC compiler has run. Over GCC's lifetime, even small inefficiencies add up to a significant amount of time. The savings from good design and implementation accumulate over time. Thus, the compiler writer must pay attention to compile time costs, such as the asymptotic complexity of algorithms, the actual running time of the implementation, and the space used by data structures. The compiler writer should have in mind a budget for how much time the compiler will spend on its various tasks.

For example, scanning and parsing are two problems for which efficient algorithms abound. Scanners recognize and classify words in time proportional to the number of characters in the input program. For a typical programming language, a parser can build derivations in time proportional to the length of the derivation. (The restricted structure of programming languages makes efficient parsing possible.) Because efficient and effective techniques exist for scanning and parsing, the compiler writer should expect to spend just a small fraction of compile time on these tasks.

By contrast, optimization and code generation contain several problems that require more time. Many of the algorithms that we will examine for program analysis and optimization will have complexities greater than $\mathbf{O}(n)$. Thus, algorithm choice in the optimizer and code generator has a larger impact on compile time than it does in the compiler's front end. The compiler writer may need to trade precision of analysis and effectiveness of optimization against increases in compile time. He or she should make such decisions consciously and carefully.

This version of the code requires just 13 cycles to execute. The code uses one more register than the minimal number. It starts an operation in every cycle except 8, 10, and 12. Other equivalent schedules are possible, as are equal-length schedules that use more registers. Chapter 12 presents several scheduling techniques that are in widespread use.

Interactions Among Code-Generation Components

Most of the truly hard problems that occur in compilation arise during code generation. To make matters more complex, these problems interact. For example, instruction scheduling moves `load` operations away from the arithmetic operations that depend on them. This can increase the period over which the values are needed and, correspondingly, increase the number of registers needed during that period. Similarly, the assignment of particular values to specific registers can constrain instruction scheduling by creating a "false" dependence between two operations. (The second operation cannot be scheduled until the first completes, even though the values in the common register are independent. Renaming the values can eliminate this false dependence, at the cost of using more registers.)

1.4 SUMMARY AND PERSPECTIVE

Compiler construction is a complex task. A good compiler combines ideas from formal language theory, from the study of algorithms, from artificial intelligence, from systems design, from computer architecture, and from the theory of programming languages and applies them to the problem of translating a program. A compiler brings together greedy algorithms, heuristic techniques, graph algorithms, dynamic programming, DFAS and NFAS, fixed-point algorithms, synchronization and locality, allocation and naming, and pipeline management. Many of the problems that confront the compiler are too hard for it to solve optimally; thus, compilers use approximations, heuristics, and rules of thumb. This produces complex interactions that can lead to surprising results—both good and bad.

To place this activity in an orderly framework, most compilers are organized into three major phases: a front end, an optimizer, and a back end. Each phase has a different set of problems to tackle, and the approaches used to solve those problems differ, too. The front end focuses on translating source code into some IR. Front ends rely on results from formal language theory and type theory, with a healthy dose of algorithms and data structures. The middle section, or optimizer, translates one IR program into another, with the goal of producing an IR program that executes efficiently. Optimizers analyze programs to derive knowledge about their runtime behavior and then use that knowledge to transform the code and improve its behavior. The back end maps an IR program to the instruction set of a specific processor. A back end approximates the answers to hard problems in allocation and scheduling, and the quality of its approximation has a direct impact on the speed and size of the compiled code.

This book explores each of these phases. Chapters 2 through 4 deal with the algorithms used in a compiler's front end. Chapters 5 through 7 describe background material for the discussion of optimization and code generation. Chapter 8 provides an introduction to code optimization. Chapters 9 and 10 provide more detailed treatment of analysis and optimization for the interested reader. Finally, Chapters 11 through 13 cover the techniques used by back ends for instruction selection, scheduling, and register allocation.

CHAPTER NOTES

The first compilers appeared in the 1950s. These early systems showed surprising sophistication. The original FORTRAN compiler was a multipass system that included a distinct scanner, parser, and register allocator, along with some optimizations [26, 27]. The Alpha system, built by Ershov and his colleagues, performed local optimization [139] and used graph coloring to reduce the amount of memory needed for data items [140, 141].

Knuth provides some interesting recollections of compiler construction in the early 1960s [227]. Randell and Russell describe early implementation efforts for Algol 60 [293]. Allen describes the history of compiler development inside IBM with an emphasis on the interplay of theory and practice [14].

Many influential compilers were built in the 1960s and 1970s. These include the classic optimizing compiler FORTRAN H [252, 307], the Bliss-11 and Bliss-32 compilers [72, 356], and the portable BCPL compiler [300]. These compilers produced high-quality code for a variety of CISC machines. Compilers for students, on the other hand, focused on rapid compilation, good diagnostic messages, and error correction [97, 146].

The advent of RISC architecture in the 1980s led to another generation of compilers; these focused on strong optimization and code generation [24, 81, 89, 204]. These compilers featured full-blown optimizers structured as shown in Figure 1.1. Modern RISC compilers still follow this model.

During the 1990s, compiler-construction research focused on reacting to the rapid changes taking place in microprocessor architecture. The decade began with Intel's *i*860 processor challenging compiler writers to manage pipelines and memory latencies directly. At its end, compilers confronted challenges that ranged from multiple functional units to long memory latencies to parallel code generation. The structure and organization of 1980s RISC compilers proved flexible enough for these new challenges, so researchers built new passes to insert into the optimizers and code generators of their compilers.

While Java systems use a mix of compilation and interpretation [63, 279], Java is not the first language to employ such a mix. Lisp systems have long included both native-code compilers and virtual-machine implementation schemes [266, 324]. The Smalltalk-80 system used a bytecode distribution and a virtual machine [233]; several implementations added just-in-time compilers [126].

EXERCISES

1. Consider a simple Web browser that takes as input a textual string in HTML format and displays the specified graphics on the screen. Is the display process one of compilation or interpretation?
2. In designing a compiler, you will face many tradeoffs. What are the five qualities that you, as a user, consider most important in a compiler that you purchase? Does that list change when you are the compiler writer? What does your list tell you about a compiler that you would implement?
3. Compilers are used in many different circumstances. What differences might you expect in compilers designed for the following applications?
   - **a.** A just-in-time compiler used to translate user interface code downloaded over a network
   - **b.** A compiler that targets the embedded processor used in a cellular telephone
   - **c.** A compiler used in an introductory programming course at a high school
   - **d.** A compiler used to build wind-tunnel simulations that run on a massively parallel processor (where all processors are identical)
   - **e.** A compiler that targets numerically intensive programs to a large number of diverse machines

This page intentionally left blank

Chapter 2

Scanners

CHAPTER OVERVIEW

The scanner's task is to transform a stream of characters into a stream of words in the input language. Each word must be classified into a syntactic category, or "part of speech." The scanner is the only pass in the compiler to touch every character in the input program. Compiler writers place a premium on speed in scanning, in part because the scanner's input is larger, in some measure, than that of any other pass, and, in part, because highly efficient techniques are easy to understand and to implement.

This chapter introduces regular expressions, a notation used to describe the valid words in a programming language. It develops the formal mechanisms to generate scanners from regular expressions, either manually or automatically.

Keywords: Scanner, Finite Automaton, Regular Expression, Fixed Point

2.1 INTRODUCTION

Scanning is the first stage of a three-part process that the compiler uses to understand the input program. The scanner, or lexical analyzer, reads a stream of characters and produces a stream of words. It aggregates characters to form words and applies a set of rules to determine whether or not each word is legal in the source language. If the word is valid, the scanner assigns it a syntactic category, or part of speech.

The scanner is the only pass in the compiler that manipulates every character of the input program. Because scanners perform a relatively simple task, grouping characters together to form words and punctuation in the source language, they lend themselves to fast implementations. Automatic tools for scanner generation are common. These tools process a mathematical

Engineering a Compiler. DOI: 10.1016/B978-0-12-088478-0.00002-5

description of the language's lexical syntax and produce a fast recognizer. Alternatively, many compilers use hand-crafted scanners; because the task is simple, such scanners can be fast and robust.

Conceptual Roadmap

Recognizer
a program that identifies specific words in a stream of characters

This chapter describes the mathematical tools and programming techniques that are commonly used to construct scanners—both generated scanners and hand-crafted scanners. The chapter begins, in Section 2.2, by introducing a model for *recognizers*, programs that identify words in a stream of characters. Section 2.3 describes *regular expressions*, a formal notation for specifying syntax. In Section 2.4, we show a set of constructions to convert a regular expression into a recognizer. Finally, in Section 2.5 we present three different ways to implement a scanner: a table-driven scanner, a direct-coded scanner, and a hand-coded approach.

Both generated and hand-crafted scanners rely on the same underlying techniques. While most textbooks and courses advocate the use of generated scanners, most commercial compilers and open-source compilers use hand-crafted scanners. A hand-crafted scanner can be faster than a generated scanner because the implementation can optimize away a portion of the overhead that cannot be avoided in a generated scanner. Because scanners are simple and they change infrequently, many compiler writers deem that the performance gain from a hand-crafted scanner outweighs the convenience of automated scanner generation. We will explore both alternatives.

Overview

Syntactic category
a classification of words according to their grammatical usage

Microsyntax
the lexical structure of a language

A compiler's scanner reads an input stream that consists of characters and produces an output stream that contains words, each labelled with its *syntactic category*—equivalent to a word's part of speech in English. To accomplish this aggregation and classification, the scanner applies a set of rules that describe the lexical structure of the input programming language, sometimes called its *microsyntax*. The microsyntax of a programming language specifies how to group characters into words and, conversely, how to separate words that run together. (In the context of scanning, we consider punctuation marks and other symbols as words.)

Western languages, such as English, have simple microsyntax. Adjacent alphabetic letters are grouped together, left to right, to form a word. A blank space terminates a word, as do most nonalphabetic symbols. (The word-building algorithm can treat a hyphen in the midst of a word as if it were an alphabetic character.) Once a group of characters has been aggregated together to form a potential word, the word-building algorithm can determine its validity with a dictionary lookup.

Most programming languages have equally simple microsyntax. Characters are aggregated into words. In most languages, blanks and punctuation marks terminate a word. For example, Algol and its descendants define an *identifier* as a single alphabetic character followed by zero or more alphanumeric characters. The identifier ends with the first nonalphanumeric character. Thus, `fee` and `f1e` are valid identifiers, but `12fum` is not. Notice that the set of valid words is specified by rules rather than by enumeration in a dictionary.

In a typical programming language, some words, called *keywords* or *reserved words*, match the rule for an identifier but have special meanings. Both `while` and `static` are keywords in both C and Java. Keywords (and punctuation marks) form their own syntactic categories. Even though `static` matches the rule for an identifier, the scanner in a C or Java compiler would undoubtedly classify it into a category that has only one element, the keyword `static`. To recognize keywords, the scanner can either use dictionary lookup or encode the keywords directly into its microsyntax rules.

Keyword
a word that is reserved for a particular syntactic purpose and, thus, cannot be used as an identifier

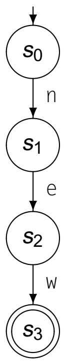
The simple lexical structure of programming languages lends itself to efficient scanners. The compiler writer starts from a specification of the language's microsyntax. She either encodes the microsyntax into a notation accepted by a scanner generator, which then constructs an executable scanner, or she uses that specification to build a hand-crafted scanner. Both generated and hand-crafted scanners can be implemented to require just **O**(1) time per character, so they run in time proportional to the number of characters in the input stream.

2.2 RECOGNIZING WORDS

The simplest explanation of an algorithm to recognize words is often a character-by-character formulation. The structure of the code can provide some insight into the underlying problem. Consider the problem of recognizing the keyword `new`. Assuming the presence of a routine *`NextChar`* that returns the next character, the code might look like the fragment shown in Figure 2.1. The code tests for `n` followed by `e` followed by `w`. At each step, failure to match the appropriate character causes the code to reject the string and "try something else." If the sole purpose of the program was to recognize the word `new`, then it should print an error message or return failure. Because scanners rarely recognize only one word, we will leave this "error path" deliberately vague at this point.

The code fragment performs one test per character. We can represent the code fragment using the simple transition diagram shown to the right of the code. The transition diagram represents a recognizer. Each circle represents an abstract state in the computation. Each state is labelled for convenience.

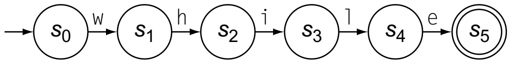
```
c ← NextChar();
if (c = ‘n’)
   then begin;
      c ← NextChar();
      if (c = ‘e’)
          then begin;
            c ← NextChar();
            if (c = ‘w’)
                then report success;
                else try something else;
         end;
         else try something else;
   end;
   else try something else;
```



■ **FIGURE 2.1** Code Fragment to Recognize "new".

The initial state, or start state, is s_0. We will always label the start state as s_0. State s_3 is an accepting state; the recognizer reaches s_3 only when the input is new. Accepting states are drawn with double circles, as shown in the margin. The arrows represent transitions from state to state based on the input character. If the recognizer starts in s_0 and reads the characters n, e, and w, the transitions take us to s_3. What happens on any other input, such as n, o, and t? The n takes the recognizer to s_1. The o does not match the edge leaving s_1, so the input word is not new. In the code, cases that do not match new *try something else*. In the recognizer, we can think of this action as a transition to an error state. When we draw the transition diagram of a recognizer, we usually omit transitions to the error state. Each state has a transition to the error state on each unspecified input.

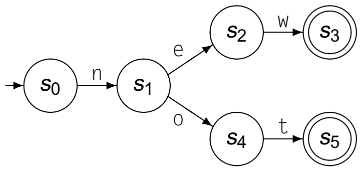
Using this same approach to build a recognizer for while would produce the following transition diagram:



If it starts in s_0 and reaches s_5, it has identified the word while. The corresponding code fragment would involve five nested *if-then-else* constructs.

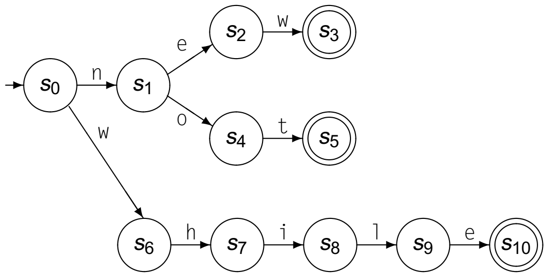
To recognize multiple words, we can create multiple edges that leave a given state. (In the code, we would begin to elaborate the *do something else* paths.)

One recognizer for both `new` and `not` might be



The recognizer uses a common test for `n` that takes it from s_0 to s_1, denoted $s_0 \xrightarrow{\texttt{n}} s_1$. If the next character is `e`, it takes the transition $s_1 \xrightarrow{\texttt{e}} s_2$. If, instead, the next character is `o`, it makes the move $s_1 \xrightarrow{\texttt{o}} s_4$. Finally, a `w` in s_2, causes the transition $s_2 \xrightarrow{\texttt{w}} s_3$, while a `t` in s_4 produces $s_4 \xrightarrow{\texttt{t}} s_5$. State s_3 indicates that the input was `new` while s_5 indicates that it was `not`. The recognizer takes one transition per input character.

We can combine the recognizer for `new` or `not` with the one for `while` by merging their initial states and relabeling all the states.



State s_0 has transitions for `n` and `w`. The recognizer has three accepting states, s_3, s_5, and s_{10}. If any state encounters an input character that does not match one of its transitions, the recognizer moves to an error state.

2.2.1 A Formalism for Recognizers

Transition diagrams serve as abstractions of the code that would be required to implement them. They can also be viewed as formal mathematical objects, called *finite automata*, that specify recognizers. Formally, a finite automaton (FA) is a five-tuple $(S, \Sigma, \delta, s_0, S_A)$, where

Finite automaton
a formalism for recognizers that has a finite set of states, an alphabet, a transition function, a start state, and one or more accepting states

- S is the finite set of states in the recognizer, along with an error state s_e.
- Σ is the finite alphabet used by the recognizer. Typically, Σ is the union of the edge labels in the transition diagram.

- $\delta(s,c)$ is the recognizer's transition function. It maps each state $s \in S$ and each character $c \in \Sigma$ into some next state. In state s_i with input character c, the FA takes the transition $s_i \xrightarrow{c} \delta(s_i,c)$.
- $s_0 \in S$ is the designated start state.
- S_A is the set of accepting states, $S_A \subseteq S$. Each state in S_A appears as a double circle in the transition diagram.

As an example, we can cast the FA for *new* or *not* or *while* in the formalism as follows:

$$S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_e\}$$

$$\Sigma = \{\texttt{e}, \texttt{h}, \texttt{i}, \texttt{l}, \texttt{n}, \texttt{o}, \texttt{t}, \texttt{w}\}$$

$$\delta = \left\{\begin{array}{lllll} s_0 \xrightarrow{\texttt{n}} s_1, & s_0 \xrightarrow{\texttt{w}} s_6, & s_1 \xrightarrow{\texttt{e}} s_2, & s_1 \xrightarrow{\texttt{o}} s_4, & s_2 \xrightarrow{\texttt{w}} s_3, \\ s_4 \xrightarrow{\texttt{t}} s_5, & s_6 \xrightarrow{\texttt{h}} s_7, & s_7 \xrightarrow{\texttt{i}} s_8, & s_8 \xrightarrow{\texttt{l}} s_9, & s_9 \xrightarrow{\texttt{e}} s_{10} \end{array}\right\}$$

$$s_0 = s_0$$

$$S_A = \{s_3, s_5, s_{10}\}$$

For all other combinations of state s_i and input character c, we define $\delta(s_i,c) = s_e$, where s_e is the designated error state. This quintuple is equivalent to the transition diagram; given one, we can easily re-create the other. The transition diagram is a picture of the corresponding FA.

An FA accepts a string x if and only if, starting in s_0, the sequence of characters in the string takes the FA through a series of transitions that leaves it in an accepting state when the entire string has been consumed. This corresponds to our intuition for the transition diagram. For the string `new`, our example recognizer runs through the transitions $s_0 \xrightarrow{\texttt{n}} s_1$, $s_1 \xrightarrow{\texttt{e}} s_2$, and $s_2 \xrightarrow{\texttt{w}} s_3$. Since $s_3 \in S_A$, and no input remains, the FA accepts `new`. For the input string `nut`, the behavior is different. On `n`, the FA takes $s_0 \xrightarrow{\texttt{n}} s_1$. On `u`, it takes $s_1 \xrightarrow{\texttt{u}} s_e$. Once the FA enters s_e, it stays in s_e until it exhausts the input stream.

More formally, if the string x is composed of characters $x_1\, x_2\, x_3 \ldots x_n$, then the FA $(S, \Sigma, \delta, s_0, S_A)$ accepts x if and only if

$$\delta(\delta(\ldots\delta(\delta(\delta(s_0,x_1),x_2),x_3)\ldots,x_{n-1}),x_n) \in S_A.$$

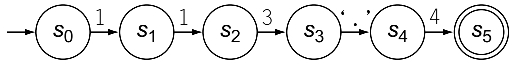
Intuitively, this definition corresponds to a repeated application of δ to a pair composed of some state $s \in S$ and an input symbol x_i. The base case, $\delta(s_0,x_1)$, represents the FA's initial transition, out of the start state, s_0, on the character x_1. The state produced by $\delta(s_0, x_1)$ is then used as input, along with x_2, to δ to produce the next state, and so on, until all the input has been consumed. The result of the final application of δ is, again, a state. If that state is an accepting state, then the FA accepts $x_1\, x_2\, x_3 \ldots x_n$.

Two other cases are possible. The FA might encounter an error while processing the string—that is, some character x_j might take it into the error state s_e. This condition indicates a lexical error; the string $x_1\, x_2\, x_3 \ldots x_j$ is not a valid prefix for any word in the language accepted by the FA. The FA can also discover an error by exhausting its input and terminating in a nonaccepting state other than s_e. In this case, the input string is a proper prefix of some word accepted by the FA. Again, this indicates an error. Either kind of error should be reported to the end user.

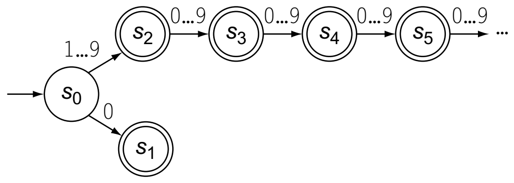
In any case, notice that the FA takes one transition for each input character. Assuming that we can implement the FA efficiently, we should expect the recognizer to run in time proportional to the length of the input string.

2.2.2 Recognizing More Complex Words

The character-by-character model shown in the original recognizer for `not` extends easily to handle arbitrary collections of fully specified words. How could we recognize a number with such a recognizer? A specific number, such as 113.4, is easy.



To be useful, however, we need a transition diagram (and the corresponding code fragment) that can recognize any number. For simplicity's sake, let's limit the discussion to unsigned integers. In general, an integer is either zero, or it is a series of one or more digits where the first digit is from one to nine, and the subsequent digits are from zero to nine. (This definition rules out leading zeros.) How would we draw a transition diagram for this definition?



The transition $s_0 \xrightarrow{0} s_1$ handles the case for zero. The other path, from s_0 to s_2, to s_3, and so on, handles the case for an integer greater than zero. This path, however, presents several problems. First, it does not end, violating the stipulation that S is finite. Second, all of the states on the path beginning with s_2 are equivalent, that is, they have the same labels on their output transitions and they are all accepting states.

```
char ← NextChar( );
state ← $s_0$ ;

while (char ≠ eof and state ≠ $s_e$) do
   state ← δ(state,char);
   char ← NextChar( );
end;

if (state ∈ $S_A$)
    then report acceptance;
    else report failure;
```

$S = \{s_0, s_1, s_2, s_e\}$

$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$\delta = \left\{ \begin{array}{ll} s_0 \overset{0}{\rightarrow} s_1, & s_0 \overset{1-9}{\rightarrow} s_2 \\ s_2 \overset{0-9}{\rightarrow} s_2, & s_1 \overset{0-9}{\rightarrow} s_e \end{array} \right\}$$

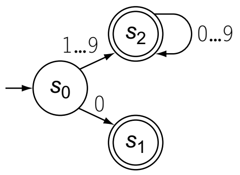
$S_A = \{s_1, s_2\}$

■ **FIGURE 2.2** A Recognizer for Unsigned Integers.

Lexeme
the actual text for a word recognized by an FA

This FA recognizes a class of strings with a common property: they are all unsigned integers. It raises the distinction between the class of strings and the text of any particular string. The class "unsigned integer" is a syntactic category, or part of speech. The text of a specific unsigned integer, such as `113`, is its *lexeme*.

We can simplify the FA significantly if we allow the transition diagram to have cycles. We can replace the entire chain of states beginning at s_2 with a single transition from s_2 back to itself:



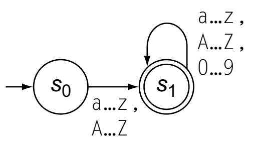
This cyclic transition diagram makes sense as an FA. From an implementation perspective, however, it is more complex than the acyclic transition diagrams shown earlier. We cannot translate this directly into a set of nested `if-then-else` constructs. The introduction of a cycle in the transition graph creates the need for cyclic control flow. We can implement this with a `while` loop, as shown in Figure 2.2. We can specify δ efficiently using a table:

δ	0	1	2	3	4	5	6	7	8	9	**Other**
s_0	s_1	s_2	s_2	s_2	s_2	s_2	s_2	s_2	s_2	s_2	s_e
s_1	s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e
s_2	s_2	s_2	s_2	s_2	s_2	s_2	s_2	s_2	s_2	s_2	s_e
s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e	s_e

Changing the table allows the same basic code skeleton to implement other recognizers. Notice that this table has ample opportunity for compression.

The columns for the digits `1` through `9` are identical, so they could be represented once. This leaves a table with three columns: `0`, `1`...`9`, and *other*. Close examination of the code skeleton shows that it reports failure as soon as it enters s_e, so it never references that row of the table. The implementation can elide the entire row, leaving a table with just three rows and three columns.

We can develop similar FAs for signed integers, real numbers, and complex numbers. A simplified version of the rule that governs identifier names in Algol-like languages, such as C or Java, might be: *an identifier consists of an alphabetic character followed by zero or more alphanumeric characters*. This definition allows an infinite set of identifiers, but can be specified with the simple two-state FA shown to the left. Many programming languages extend the notion of "alphabetic character" to include designated special characters, such as the underscore.



FAs can be viewed as specifications for a recognizer. However, they are not particularly concise specifications. To simplify scanner implementation, we need a concise notation for specifying the lexical structure of words, and a way of turning those specifications into an FA and into code that implements the FA. The remaining sections of this chapter develop precisely those ideas.

SECTION REVIEW

A character-by-character approach to scanning leads to algorithmic clarity. We can represent character-by-character scanners with a transition diagram; that diagram, in turn, corresponds to a finite automaton. Small sets of words are easily encoded in acyclic transition diagrams. Infinite sets, such as the set of integers or the set of identifiers in an Algol-like language, require cyclic transition diagrams.

Review Questions

Construct an FA to accept each of the following languages:

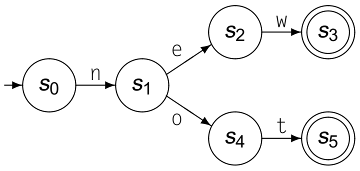
1. A six-character identifier consisting of an alphabetic character followed by zero to five alphanumeric characters
2. A string of one or more pairs, where each pair consists of an open parenthesis followed by a close parenthesis
3. A Pascal comment, which consists of an open brace, {, followed by zero or more characters drawn from an alphabet, Σ, followed by a close brace, }

2.3 REGULAR EXPRESSIONS

The set of words accepted by a finite automaton, $\mathcal{F}$, forms a language, denoted $L(\mathcal{F})$. The transition diagram of the FA specifies, in precise detail, that language. It is not, however, a specification that humans find intuitive. For any FA, we can also describe its language using a notation called a *regular expression* (RE). The language described by an RE is called a *regular language*.

Regular expressions are equivalent to the FAs described in the previous section. (We will prove this with a construction in Section 2.4.) Simple recognizers have simple RE specifications.

- The language consisting of the single word `new` can be described by an RE written as *new*. Writing two characters next to each other implies that they are expected to appear in that order.
- The language consisting of the two words `new` or `while` can be written as *new* or *while*. To avoid possible misinterpretation of *or*, we write this using the symbol | to mean *or*. Thus, we write the RE as *new* | *while*.
- The language consisting of `new` or `not` can be written as *new* | *not*. Other REs are possible, such as *n(ew* | *ot)*. Both REs specify the same pair of words. The RE *n(ew* | *ot)* suggests the structure of the FA that we drew earlier for these two words.



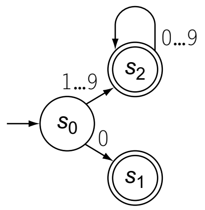
To make this discussion concrete, consider some examples that occur in most programming languages. Punctuation marks, such as colons, semicolons, commas, and various brackets, can be represented by their character representations. Their REs have the same "spelling" as the punctuation marks themselves. Thus, the following REs might occur in the lexical specification for a programming language:

: ; ? => ( ) { } [ ]

Similarly, keywords have simple REs.

if while this integer instanceof

To model more complex constructs, such as integers or identifiers, we need a notation that can capture the essence of the cyclic edge in an FA.



The FA for an unsigned integer, shown at the left, has three states: an initial state s_0, an accepting state s_1 for the unique integer zero, and another accepting state s_2 for all other integers. The key to this FA's power is the transition from s_2 back to itself that occurs on each additional digit. State s_2 folds the specification back on itself, creating a rule to derive a new unsigned integer from an existing one: add another digit to the right end of the existing number. Another way of stating this rule is: *an unsigned integer is either a zero, or a nonzero digit followed by zero or more digits.* To capture the essence of this FA, we need a notation for this notion of "zero or more occurrences" of an RE. For the RE x, we write this as x^*, with the meaning "zero or more occurrences of x." We call the * operator *Kleene closure*, or *closure* for short. Using the closure operator, we can write an RE for this FA:

$$0 \mid (1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)\ (0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)^*.$$

2.3.1 Formalizing the Notation

To work with regular expressions in a rigorous way, we must define them more formally. An RE describes a set of strings over the characters contained in some alphabet, Σ, augmented with a character ϵ that represents the empty string. We call the set of strings a *language*. For a given RE, r, we denote the language that it specifies as $L(r)$. An RE is built up from three basic operations:

1. *Alternation* The alternation, or union, of two sets of strings, R and S, denoted $R \mid S$, is $\{x \mid x \in R \text{ or } x \in S\}$.
2. *Concatenation* The concatenation oftwo sets R and S, denoted RS, contains all strings formed by prepending an element of R onto one from S, or $\{xy \mid x \in R \text{ and } y \in S\}$.
3. *Closure* The Kleene closure of a set R, denoted R^*, is $\bigcup_{i=0}^{\infty} R^i$. This is just the union of the concatenations of R with itself, zero or more times.

For convenience, we sometimes use a notation for *finite closure*. The notation R^i denotes from one to i occurrences of R. A finite closure can be always be replaced with an enumeration of the possibilities; for example, R^3 is just $(R \mid RR \mid RRR)$. The *positive closure*, denoted R^+, is just RR^* and consists of one or more occurrences of R. Since all these closures can be rewritten with the three basic operations, we ignore them in the discussion that follows.

Finite closure
For any integer i, the RE R^i designates one to i occurrences of R.

Positive closure
The RE R^+ denotes one or more occurrences of R, often written as $\bigcup_{i=1}^{\infty} R^i$.

Using the three basic operations, alternation, concatenation, and Kleene closure, we can define the set of REs over an alphabet Σ as follows:

1. If $a \in \Sigma$, then a is also an RE denoting the set containing only a.
2. If r and s are REs, denoting sets $L(r)$ and $L(s)$, respectively, then

REGULAR EXPRESSIONS IN VIRTUAL LIFE

Regular expressions are used in many applications to specify patterns in character strings. Some of the early work on translating REs into code was done to provide a flexible way of specifying strings in the "find" command of a text editor. From that early genesis, the notation has crept into many different applications.

Unix and other operating systems use the asterisk as a wildcard to match substrings against file names. Here, $*$ is a shorthand for the RE Σ^*, specifying zero or more characters drawn from the entire alphabet of legal characters. (Since few keyboards have a Σ key, the shorthand has stayed with us.) Many systems use **?** as a wildcard that matches a single character.

The `grep` family of tools, and their kin in non-Unix systems, implement regular expression pattern matching. (In fact, `grep` is an acronym for global regular-expression pattern match and print.)

Regular expressions have found widespread use because they are easily written and easily understood. They are one of the techniques of choice when a program must recognize a fixed vocabulary. They work well for languages that fit within their limited rules. They are easily translated into an executable form, and the resulting recognizer is fast.

$r \mid s$ is an RE denoting the union, or alternation, of $L(r)$ and $L(s)$,
rs is an RE denoting the concatenation of $L(r)$ and $L(s)$, respectively, and
r^* is an RE denoting the Kleene closure of $L(r)$.

3. ϵ is an RE denoting the set containing only the empty string.

To eliminate any ambiguity, parentheses have highest precedence, followed by closure, concatenation, and alternation, in that order.

As a convenient shorthand, we will specify ranges of characters with the first and the last element connected by an ellipsis, "…". To make this abbreviation stand out, we surround it with a pair of square brackets. Thus, $[0 \ldots 9]$ represents the set of decimal digits. It can always be rewritten as $(0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)$.

2.3.2 Examples

The goal of this chapter is to show how we can use formal techniques to automate the construction of high-quality scanners and how we can encode the microsyntax of programming languages into that formalism. Before proceeding further, some examples from real programming languages are in order.

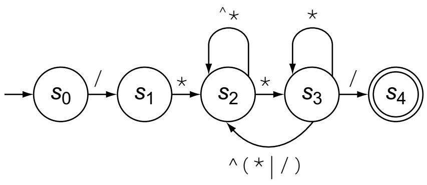
1. The simplified rule given earlier for identifiers in Algol-like languages, an alphabetic character followed by zero or more alphanumeric characters, is just $([A\ldots Z] \mid [a\ldots z])\ ([A\ldots Z] \mid [a\ldots z] \mid [0\ldots 9])^*$. Most languages also allow a few special characters, such as the underscore (_), the percent sign (%), or the ampersand (&), in identifiers.
   If the language limits the maximum length of an identifier, we can use the appropriate finite closure. Thus, identifiers limited to six characters might be specified as $([A\ldots Z] \mid [a\ldots z])\ ([A\ldots Z] \mid [a\ldots z] \mid [0\ldots 9])^5$. If we had to write out the full expansion of the finite closure, the RE would be much longer.
2. An unsigned integer can be described as either zero or a nonzero digit followed by zero or more digits. The RE $0 \mid [1\ldots 9]\, [0\ldots 9]^*$ is more concise. In practice, many implementations admit a larger class of strings as integers, accepting the language $[0\ldots 9]^+$.
3. Unsigned real numbers are more complex than integers. One possible RE might be $(0 \mid [1\ldots 9]\, [0\ldots 9]^*)\ \ (\epsilon \mid .\, [0\ldots 9]^*)$ The first part is just the RE for an integer. The rest generates either the empty string or a decimal point followed by zero or more digits.
   Programming languages often extend real numbers to scientific notation, as in $(0 \mid [1\ldots 9]\, [0\ldots 9]^*)\ (\epsilon \mid .\, [0\ldots 9]^*)\ E\ (\epsilon \mid + \mid -)$
   $(0 \mid [1\ldots 9]\, [0\ldots 9]^*)$.
   This RE describes a real number, followed by an E, followed by an integer to specify the exponent.
4. Quoted character strings have their own complexity. In most languages, any character can appear inside a string. While we can write an RE for strings using only the basic operators, it is our first example where a complement operator simplifies the RE. Using complement, a character string in C or Java can be described as " (^")* ".
   C and C++ do not allow a string to span multiple lines in the source code—that is, if the scanner reaches the end of a line while inside a string, it terminates the string and issues an error message. If we represent newline with the escape sequence `\n`, in the C style, then the RE " (^(" | `\n`))* " will recognize a correctly formed string and will take an error transition on a string that includes a newline.
5. Comments appear in a number of forms. C++ and Java offer the programmer two ways of writing a comment. The delimiter `//` indicates a comment that runs to the end of the current input line. The RE for this style of comment is straightforward: // (^`\n`)* `\n`, where `\n` represents the newline character.
   Multiline comments in C, C++, and Java begin with the delimiter `/*` and end with `*/`. If we could disallow `*` in a comment, the RE would be

Complement operator
The notation $^\wedge c$ specifies the set $\{\Sigma - c\}$, the complement of c with respect to Σ.

Complement has higher precedence than $*$, $|$, or $^+$.

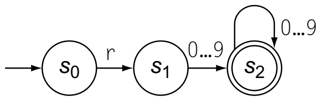
Escape sequence
Two or more characters that the scanner translates into another character. Escape sequences are used for characters that lack a glyph, such as newline or tab, and for ones that occur in the syntax, such as an open or close quote.

simple: /* (^*)* */. With *, the RE is more complex: /* (^* | *$^+$ ^/)* */. An FA to implement this RE follows.



The correspondence between the RE and this FA is not as obvious as it was in the examples earlier in the chapter. Section 2.4 presents constructions that automate the construction of an FA from an RE.
The complexity of the RE and FA for multiline comments arises from the use of multi-character delimiters. The transition from s_2 to s_3 encodes the fact that the recognizer has seen a * so that it can handle either the appearance of a / or the lack thereof in the correct manner. In contrast, Pascal uses single-character comment delimiters: { and }, so a Pascal comment is just { ^}* }.

Trying to be specific with an RE can also lead to complex expressions. Consider, for example, that the register specifier in a typical assembly language consists of the letter r followed immediately by a small integer. In ILOC, which admits an unlimited set of register names, the RE might be $r[0\ldots 9]^+$, with the following FA:

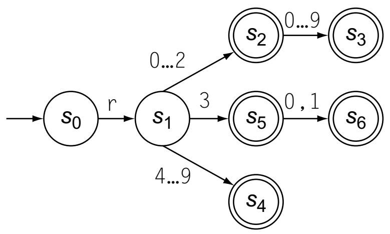


This recognizer accepts `r29`, and rejects `s29`. It also accepts `r99999`, even though no currently available computer has 100,000 registers.

On a real computer, however, the set of register names is severely limited—say, to 32, 64, 128, or 256 registers. One way for a scanner to check validity of a register name is to convert the digits into a number and test whether or not it falls into the range of valid register numbers. The alternative is to adopt a more precise RE specification, such as:

$$r\,(\,[0\ldots 2]\,([0\ldots 9]\,|\,\epsilon)\;|\;[4\ldots 9]\;|\;(3\,(0\,|\,1\,|\,\epsilon))\,)$$

This RE specifies a much smaller language, limited to register numbers 0 to 31 with an optional leading 0 on single-digit register names. It accepts `r0`, `r00`, `r01`, and `r31`, but rejects `r001`, `r32`, and `r99999`. The corresponding FA looks like:



Which FA is better? They both make a single transition on each input character. Thus, they have the same cost, even though the second FA checks a more complex specification. The more complex FA has more states and transitions, so its representation requires more space. However, their operating costs are the same.

This point is critical: the cost of operating an FA is proportional to the length of the input, not to the length or complexity of the RE that generates the FA. More complex REs may produce FAs with more states that, in turn, need more space. The cost of generating an FA from an RE may also rise with increased complexity in the RE. But, the cost of FA operation remains one transition per input character.

Can we improve our description of the register specifier? The previous RE is both complex and counterintuitive. A simpler alternative might be:

r0 | *r00* | *r1* | *r01* | *r2* | *r02* | *r3* | *r03* | *r4* | *r04* | *r5* | *r05* | *r6* | *r06* | *r7* | *r07* |
r8 | *r08* | *r9* | *r09* | *r10* | *r11* | *r12* | *r13* | *r14* | *r15* | *r16* | *r17* | *r18* | *r19* | *r20* |
r21 | *r22* | *r23* | *r24* | *r25* | *r26* | *r27* | *r28* | *r29* | *r30* | *r31*

This RE is conceptually simpler, but much longer than the previous version. The resulting FA still requires one transition per input symbol. Thus, if we can control the growth in the number of states, we might prefer this version of the RE because it is clear and obvious. However, when our processor suddenly has 256 or 384 registers, enumeration may become tedious, too.

2.3.3 **Closure Properties of REs**

Regular expressions and the languages that they generate have been the subject of extensive study. They have many interesting and useful properties. Some of these properties play a critical role in the constructions that build recognizers from REs.

Regular languages
Any language that can be specified by a regular expression is called a *regular language*.

PROGRAMMING LANGUAGES VERSUS NATURAL LANGUAGES

Lexical analysis highlights one of the subtle ways in which programming languages differ from natural languages, such as English or Chinese. In natural languages, the relationship between a word's representation—its spelling or its pictogram—and its meaning is not obvious. In English, *are* is a verb while *art* is a noun, even though they differ only in the final character. Furthermore, not all combinations of characters are legitimate words. For example, *arz* differs minimally from *are* and *art*, but does not occur as a word in normal English usage.

A scanner for English could use FA-based techniques to recognize potential words, since all English words are drawn from a restricted alphabet. After that, however, it must look up the prospective word in a dictionary to determine if it is, in fact, a word. If the word has a unique part of speech, dictionary lookup will also resolve that issue. However, many English words can be classified with several parts of speech. Examples include *buoy* and *stress*; both can be either a noun or a verb. For these words, the part of speech depends on the surrounding context. In some cases, understanding the grammatical context suffices to classify the word. In other cases, it requires an understanding of meaning, for both the word and its context.

In contrast, the words in a programming language are almost always specified lexically. Thus, any string in $[1 \ldots 9][0 \ldots 9]^*$ is a positive integer. The RE $[a \ldots z]([a \ldots z] \mid [0 \ldots 9])^*$ defines a subset of the Algol identifiers; *arz*, *are* and *art* are all identifiers, with no lookup needed to establish the fact. To be sure, some identifiers may be reserved as keywords. However, these exceptions can be specified lexically, as well. No context is required.

This property results from a deliberate decision in programming language design. The choice to make spelling imply a unique part of speech simplifies scanning, simplifies parsing, and, apparently, gives up little in the expressiveness of the language. Some languages have allowed words with dual parts of speech—for example, PL/I has no reserved keywords. The fact that more recent languages abandoned the idea suggests that the complications outweighed the extra linguistic flexibility.

Regular expressions are closed under many operations—that is, if we apply the operation to an RE or a collection of REs, the result is an RE. Obvious examples are concatenation, union, and closure. The concatenation of two REs x and y is just xy. Their union is $x \mid y$. The Kleene closure of x is just x^*. From the definition of an RE, all of these expressions are also REs.

These closure properties play a critical role in the use of REs to build scanners. Assume that we have an RE for each syntactic category in the source language, $a_0, a_1, a_2, \ldots, a_n$. Then, to construct an RE for all the valid words in the language, we can join them with alternation as $a_0 \mid a_1 \mid a_2 \mid \ldots \mid a_n$. Since REs are closed under union, the result is an RE. Anything that we can do to an RE for a single syntactic category will be equally applicable to the RE for all the valid words in the language.

Closure under union implies that any finite language is a regular language. We can construct an RE for any finite collection of words by listing them in a large alternation. Because the set of REs is closed under union, that alternation is an RE and the corresponding language is regular.

Closure under concatenation allows us to build complex REs from simpler ones by concatenating them. This property seems both obvious and unimportant. However, it lets us piece together REs in systematic ways. Closure ensures that *ab* is an RE as long as both *a* and *b* are REs. Thus, any techniques that can be applied to either *a* or *b* can be applied to *ab*; this includes constructions that automatically generate a recognizer from REs.

Regular expressions are also closed under both Kleene closure and the finite closures. This property lets us specify particular kinds of large, or even infinite, sets with finite patterns. Kleene closure lets us specify infinite sets with concise finite patterns; examples include the integers and unbounded-length identifiers. Finite closures let us specify large but finite sets with equal ease.

The next section shows a sequence of constructions that build an FA to recognize the language specified by an RE. Section 2.6 shows an algorithm that goes the other way, from an FA to an RE. Together, these constructions establish the equivalence of REs and FAs. The fact that REs are closed under alternation, concatenation, and closure is critical to these constructions.

The equivalence between REs and FAs also suggests other closure properties. For example, given a complete FA, we can construct an FA that recognizes all words *w* that are not in *L*(FA), called the complement of *L*(FA). To build this new FA for the complement, we can swap the designation of accepting and nonaccepting states in the original FA. This result suggests that REs are closed under complement. Indeed, many systems that use REs include a complement operator, such as the ^ operator in `lex`.

Complete FA
an FA that explicitly includes all error transitions

SECTION REVIEW
Regular expressions are a concise and powerful notation for specifying the microsyntax of programming languages. REs build on three basic operations over finite alphabets: alternation, concatenation, and Kleene closure. Other convenient operators, such as finite closures, positive closure, and complement, derive from the three basic operations. Regular expressions and finite automata are related; any RE can be realized in an FA and the language accepted by any FA can be described with RE. The next section formalizes that relationship.

Review Questions

1. Recall the RE for a six-character identifier, written using a finite closure.

$$([A \ldots Z] \mid [a \ldots z])\ ([A \ldots Z] \mid [a \ldots z] \mid [0 \ldots 9])^5$$

   Rewrite it in terms of the three basic RE operations: alternation, concatenation, and closure.

2. In PL/I, the programmer can insert a quotation mark into a string by writing two quotation marks in a row. Thus, the string

```
 The quotation mark, ", should be typeset in italics
```

   would be written in a PL/I program as

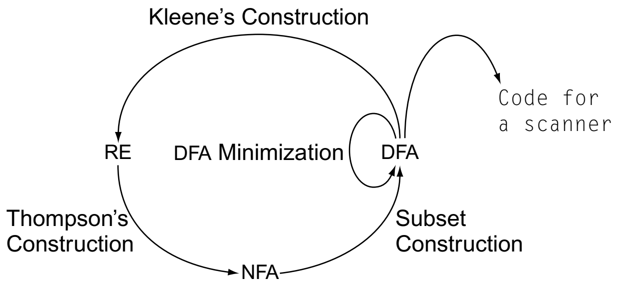
```
"The quotation mark, "", should be typeset in italics."
```

   Design an RE and an FA to recognize PL/I strings. Assume that strings begin and end with quotation marks and contain only symbols drawn from an alphabet, designated as Σ. Quotation marks are the only special case.

2.4 FROM REGULAR EXPRESSION TO SCANNER

The goal of our work with finite automata is to automate the derivation of executable scanners from a collection of REs. This section develops the constructions that transform an RE into an FA that is suitable for direct implementation and an algorithm that derives an RE for the language accepted by an FA. Figure 2.3 shows the relationship between all of these constructions.

To present these constructions, we must distinguish between *deterministic* FAs, or DFAs, and *nondeterministic* FAs, or NFAs, in Section 2.4.1. Next,

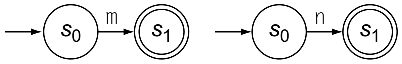


■ **FIGURE 2.3** The Cycle of Constructions.

we present the construction of a deterministic FA from an RE in three steps. Thompson's construction, in Section 2.4.2, derives an NFA from an RE. The subset construction, in Section 2.4.3, builds a DFA that simulates an NFA. Hopcroft's algorithm, in Section 2.4.4, minimizes a DFA. To establish the equivalence of REs and DFAs, we also need to show that any DFA is equivalent to an RE; Kleene's construction derives an RE from a DFA. Because it does not figure directly into scanner construction, we defer that algorithm until Section 2.6.1.

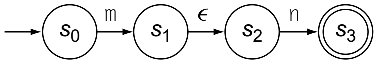
2.4.1 **Nondeterministic Finite Automata**

Recall from the definition of an RE that we designated the empty string, ϵ, as an RE. None of the FAs that we built by hand included ϵ, but some of the REs did. What role does ϵ play in an FA? We can use transitions on ϵ to combine FAs and form FAs for more complex REs. For example, assume that we have FAs for the REs *m* and *n*, called FA_m and FA_n, respectively.

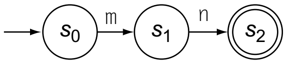


We can build an FA for *mn* by adding a transition on ϵ from the accepting state of FA_m to the initial state of FA_n, renumbering the states, and using FA_n's accepting state as the accepting state for the new FA.

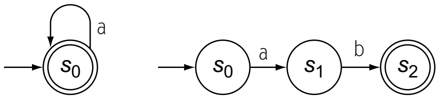
ϵ-transition
a transition on the empty string, ϵ, that does not advance the input



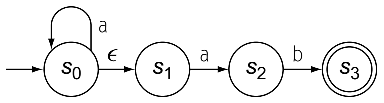
With an ϵ-transition, the definition of acceptance must change slightly to allow one or more ϵ-transitions between any two characters in the input string. For example, in s_1, the FA takes the transition $s_1 \xrightarrow{\epsilon} s_2$ without consuming any input character. This is a minor change, but it seems intuitive. Inspection shows that we can combine s_1 and s_2 to eliminate the ϵ-transition.



Merging two FAs with an ϵ-transition can complicate our model of how FAs work. Consider the FAs for the languages a^* and *ab*.



We can combine them with an ϵ-transition to form an FA for a^*ab.



The ϵ transition, in effect, gives the FA two distinct transitions out of s_0 on the letter a. It can take the transition $s_0 \xrightarrow{a} s_0$, or the two transitions $s_0 \xrightarrow{\epsilon} s_1$ and $s_1 \xrightarrow{a} s_2$. Which transition is correct? Consider the strings aab and ab. The DFA should accept both strings. For aab, it should move $s_0 \xrightarrow{a} s_0$, $s_0 \xrightarrow{\epsilon} s_1$, $s_1 \xrightarrow{a} s_2$, and $s_2 \xrightarrow{b} s_3$. For ab, it should move $s_0 \xrightarrow{\epsilon} s_1$, $s_1 \xrightarrow{a} s_2$, and $s_2 \xrightarrow{b} s_3$.

Nondeterministic FA
an FA that allows transitions on the empty string, ϵ, and states that have multiple transitions on the same character

Deterministic FA
A DFA is an FA where the transition function is single-valued. DFAs do not allow ϵ-transitions.

As these two strings show, the correct transition out of s_0 on a depends on the characters that follow the a. At each step, an FA examines the current character. Its state encodes the left context, that is, the characters that it has already processed. Because the FA must make a transition before examining the next character, a state such as s_0 violates our notion of the behavior of a sequential algorithm. An FA that includes states such as s_0 that have multiple transitions on a single character is called a *nondeterministic finite automaton* (NFA). By contrast, an FA with unique character transitions in each state is called a *deterministic finite automaton* (DFA).

To make sense of an NFA, we need a set of rules that describe its behavior. Historically, two distinct models have been given for the behavior of an NFA.

1. Each time the NFA must make a nondeterministic choice, it follows the transition that leads to an accepting state for the input string, if such a transition exists. This model, using an omniscient NFA, is appealing because it maintains (on the surface) the well-defined accepting mechanism of the DFA. In essence, the NFA guesses the correct transition at each point.
2. Each time the NFA must make a nondeterministic choice, the NFA clones itself to pursue each possible transition. Thus, for a given input character, the NFA is in a specific set of states, taken across all of its clones. In this model, the NFA pursues all paths concurrently.
   At any point, we call the specific set of states in which the NFA is active its *configuration*. When the NFA reaches a configuration in which it has exhausted the input and one or more of the clones has reached an accepting state, the NFA accepts the string.

Configuration of an NFA
the set of concurrently active states of an NFA

In either model, the NFA $(S, \Sigma, \delta, s_0, S_A)$ accepts an input string $x_1\, x_2\, x_3 \ldots x_k$ if and only if there exists at least one path through the transition diagram that starts in s_0 and ends in some $s_k \in S_A$ such that the edge labels along the path match the input string. (Edges labelled with ϵ are omitted.) In other words, the i^{th} edge label must be x_i. This definition is consistent with either model of the NFA's behavior.

Equivalence of NFAs and DFAs

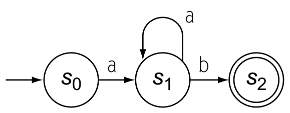
NFAs and DFAs are equivalent in their expressive power. Any DFA is a special case of an NFA. Thus, an NFA is at least as powerful as a DFA. Any NFA can be simulated by a DFA—a fact established by the subset construction in Section 2.4.3. The intuition behind this idea is simple; the construction is a little more complex.

Consider the state of an NFA when it has reached some point in the input string. Under the second model of NFA behavior, the NFA has some finite set of operating clones. The number of these configurations can be bounded; for each state, the configuration either includes one or more clones in that state or it does not. Thus, an NFA with n states produces at most $|\Sigma|^n$ configurations.

To simulate the behavior of the NFA, we need a DFA with a state for each configuration of the NFA. As a result, the DFA may have exponentially more states than the NFA. While S_{DFA}, the set of states in the DFA, might be large, it is finite. Furthermore, the DFA still makes one transition per input symbol. Thus, the DFA that simulates the NFA still runs in time proportional to the length of the input string. The simulation of an NFA on a DFA has a potential space problem, but not a time problem.

Powerset of *N*
the set of all subsets of *N*, denoted 2^N

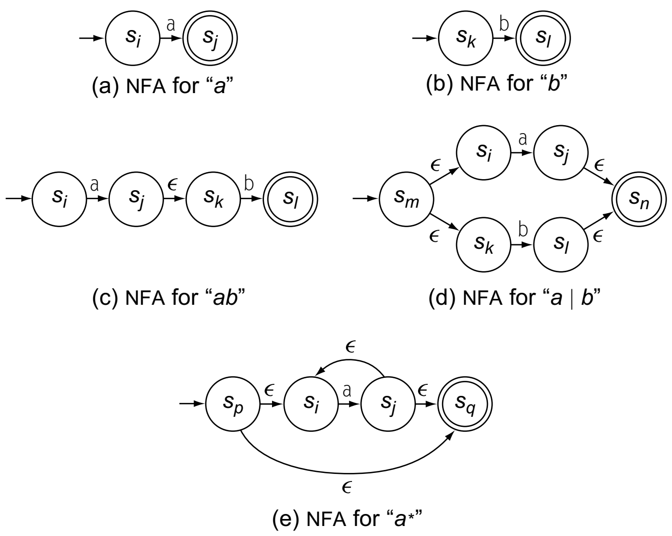
Since NFAS and DFAS are equivalent, we can construct a DFA for a^*ab:



It relies on the observation that a^*ab specifies the same set of words as aa^*b.

2.4.2 Regular Expression to NFA: Thompson's Construction

The first step in moving from an RE to an implemented scanner must derive an NFA from the RE. *Thompson's construction* accomplishes this goal in a straightforward way. It has a template for building the NFA that corresponds to a single-letter RE, and a transformation on NFAS that models the effect of each basic RE operator: concatenation, alternation, and closure. Figure 2.4

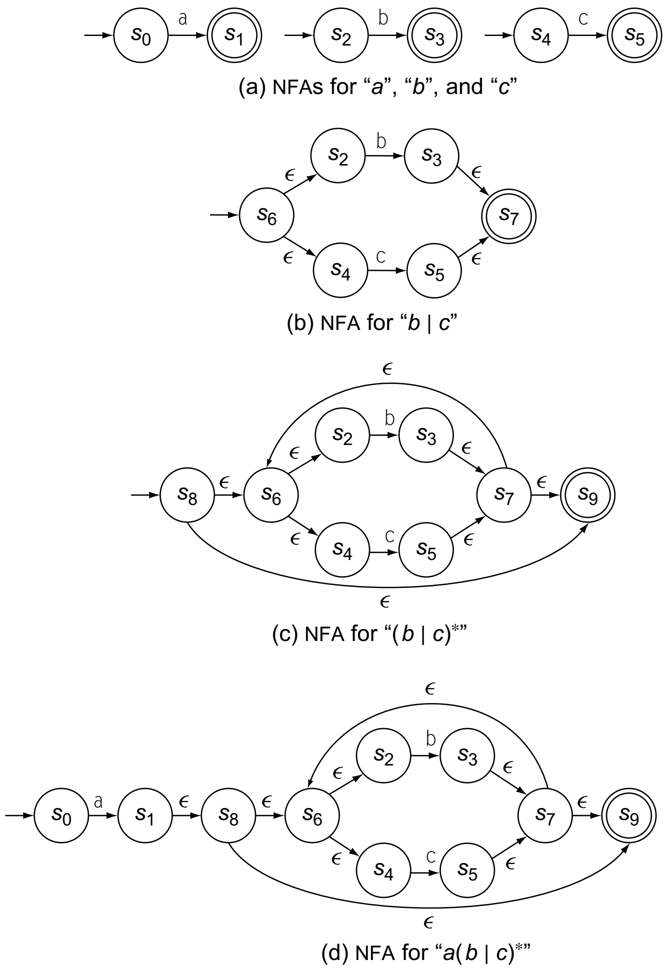


■ **FIGURE 2.4** Trivial NFAs for Regular Expression Operators.

shows the trivial NFAs for the REs *a* and *b*, as well as the transformations to form NFAs for the REs *ab*, *a*|*b*, and a^* from the NFAs for *a* and *b*. The transformations apply to arbitrary NFAs.

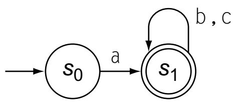
The construction begins by building trivial NFAs for each character in the input RE. Next, it applies the transformations for alternation, concatenation, and closure to the collection of trivial NFAs in the order dictated by precedence and parentheses. For the RE $a(b|c)^*$, the construction would first build NFAs for *a*, *b*, and *c*. Because parentheses have highest precedence, it next builds the NFA for the expression enclosed in parentheses, *b*|*c*. Closure has higher precedence than concatenation, so it next builds the closure, $(b|c)^*$. Finally, it concatenates the NFA for *a* to the NFA for $(b|c)^*$.

The NFAs derived from Thompson's construction have several specific properties that simplify an implementation. Each NFA has one start state and one accepting state. No transition, other than the initial transition, enters the start state. No transition leaves the accepting state. An ϵ-transition always connects two states that were, earlier in the process, the start state and the accepting state of NFAs for some component REs. Finally, each state has at most two entering and two exiting ϵ-moves, and at most one entering and one exiting move on a symbol in the alphabet. Together, these properties simplify the representation and manipulation of the NFAs. For example, the construction only needs to deal with a single accepting state, rather than iterating over a set of accepting states in the NFA.


(a) NFAs for "*a*", "*b*", and "*c*"
(b) NFA for "*b* | *c*"
(c) NFA for "(*b* | *c*)*"
(d) NFA for "*a*(*b* | *c*)*"

■ **FIGURE 2.5** Applying Thompson's Construction to $a(b|c)^*$.

Figure 2.5 shows the NFA that Thompson's construction builds for $a(b|c)^*$. It has many more states than the DFA that a human would likely produce, shown at left. The NFA also contains many ϵ-moves that are obviously unneeded. Later stages in the construction will eliminate them.

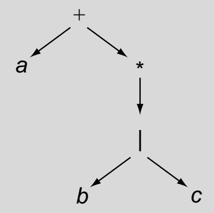


2.4.3 NFA to DFA: The Subset Construction

Thompson's construction produces an NFA to recognize the language specified by an RE. Because DFA execution is much easier to simulate than NFA execution, the next step in the cycle of constructions converts the NFA built

REPRESENTING THE PRECEDENCE OF OPERATORS

Thompson's construction must apply its three transformations in an order that is consistent with the precedence of the operators in the regular expression. To represent that order, an implementation of Thompson's construction can build a tree that represents the regular expression and its internal precedence. The RE $a(b|c)^*$ produces the following *tree*:



where + represents concatenation, | represents alternation, and $\star$ represents closure. The parentheses are folded into the structure of the tree and, thus, have no explicit representation.

The construction applies the individual transformations in a postorder walk over the tree. Since transformations correspond to operations, the postorder walk builds the following sequence of NFAs: a, b, c, $b|c$, $(b|c)^*$, and, finally, $a(b|c)^*$. Chapters 3 and 4 show how to build expression trees.

by Thompson's construction into a DFA that recognizes the same language. The resulting DFAS have a simple execution model and several efficient implementations. The algorithm that constructs a DFA from an NFA is called the *subset construction*.

The subset construction takes as input an NFA, $(N, \Sigma, \delta_N, n_0, N_A)$. It produces a DFA, $(D, \Sigma, \delta_D, d_0, D_A)$. The NFA and the DFA use the same alphabet, Σ. The DFA's start state, d_0, and its accepting states, D_A, will emerge from the construction. The complex part of the construction is the derivation of the set of DFA states D from the NFA states N, and the derivation of the DFA transition function δ_D.

Valid configuration
configuration of an NFA that can be reached by some input string

The algorithm, shown in Figure 2.6, constructs a set Q whose elements, q_i are each a subset of N, that is, each $q_i \in 2^N$. When the algorithm halts, each $q_i \in Q$ corresponds to a state, $d_i \in D$, in the DFA. The construction builds the elements of Q by following the transitions that the NFA can make on a given input. Thus, each q_i represents a valid configuration of the NFA.

The algorithm begins with an initial set, q_0, that contains n_0 and any states in the NFA that can be reached from n_0 along paths that contain only

```
$q_0$ ← $\epsilon$-closure({$n_0$});
Q ← $q_0$;
WorkList ← {$q_0$};

while (WorkList ≠ ∅) do
   remove $q$ from WorkList;

   for each character $c \in \Sigma$ do
      t ← $\epsilon$-closure(Delta($q, c$));
      T[$q, c$] ← t;

      if t ∉ Q then
         add t to Q and to WorkList;
   end;

end;
```

■ **FIGURE 2.6** The Subset Construction.

ϵ-transitions. Those states are equivalent since they can be reached without consuming input.

To construct q_0 from n_0, the algorithm computes ϵ-`closure(`n_0`)`. It takes, as input, a set S of NFA states. It returns a set of NFA states constructed from S as follows: ϵ-`closure` examines each state $s_i \in S$ and adds to S any state reachable by following one or more ϵ-transitions from s_i. If S is the set of states reachable from n_0 by following paths labelled with `abc`, then ϵ-`closure(S)` is the set of states reachable from n_0 by following paths labelled `abc`ϵ^*. Initially, `Q` has only one member, q_0 and the `WorkList` contains q_0.

The algorithm proceeds by removing a set q from the worklist. Each q represents a valid configuration of the original NFA. The algorithm constructs, for each character c in the alphabet Σ, the configuration that the NFA would reach if it read c while in configuration q. This computation uses a function `Delta(`q,c`)` that applies the NFA's transition function to each element of q. It returns $\cup_{s \in q_i} \delta_N(s,c)$.

The while loop repeatedly removes a configuration q from the worklist and uses `Delta` to compute its potential transitions. It augments this computed configuration with any states reachable by following ϵ-transitions, and adds any new configurations generated in this way to both `Q` and the worklist. When it discovers a new configuration `t` reachable from q on character c, the algorithm records that transition in the table `T`. The inner loop, which iterates over the alphabet for each configuration, performs an exhaustive search.

Notice that `Q` grows monotonically. The while loop adds sets to `Q` but never removes them. Since the number of configurations of the NFA is bounded and each configuration only appears once on the worklist, the while loop must halt. When it halts, *Q* contains all of the valid configurations of the NFA and *T* holds all of the transitions between them.

Q can become large—as large as $|2^N|$ distinct states. The amount of nondeterminism found in the NFA determines how much state expansion occurs. Recall, however, that the result is a DFA that makes exactly one transition per input character, independent of the number of states in the DFA. Thus, any expansion introduced by the subset construction does not affect the running time of the DFA.

From Q to D

When the subset construction halts, it has constructed a model of the desired DFA, one that simulates the original NFA. Building the DFA from *Q* and *T* is straightforward. Each $q_i \in Q$ needs a state $d_i \in D$ to represent it. If q_i contains an accepting state of the NFA, then d_i is an accepting state of the DFA. We can construct the transition function, δ_D, directly from *T* by observing the mapping from q_i to d_i. Finally, the state constructed from q_0 becomes d_0, the initial state of the DFA.

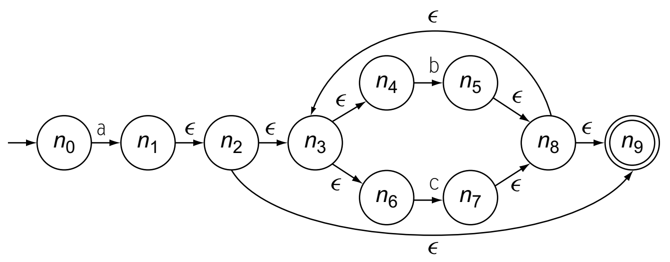
Example

Consider the NFA built for $a(b|c)^*$ in Section 2.4.2 and shown in Figure 2.7a, with its states renumbered. The table in Figure 2.7b sketches the steps that the subset construction follows. The first column shows the name of the set in *Q* being processed in a given iteration of the while loop. The second column shows the name of the corresponding state in the new DFA. The third column shows the set of NFA states contained in the current set from *Q*. The final three columns show results of computing the ϵ-*closure* of *Delta* on the state for each character in Σ.

The algorithm takes the following steps:

1. The initialization sets q_0 to ϵ-*closure*($\{n_0\}$), which is just n_0. The first iteration computes ϵ-*closure*(*Delta*(q_0,a)), which contains six NFA states, and ϵ-*closure*(*Delta*(q_0,b)) and ϵ-*closure*(*Delta*(q_0,c)), which are empty.
2. The second iteration of the while loop examines q_1. It produces two configurations and names them q_2 and q_3.
3. The third iteration of the while loop examines q_2. It constructs two configurations, which are identical to q_2 and q_3.
4. The fourth iteration of the while loop examines q_3. Like the third iteration, it reconstructs q_2 and q_3.

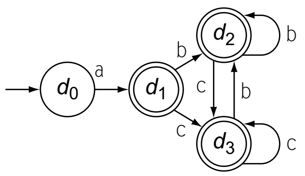
Figure 2.7c shows the resulting DFA; the states correspond to the DFA states from the table and the transitions are given by the *Delta* operations that



(a) NFA for "$a(b \mid c)^*$" (With States Renumbered)

Set Name	DFA States	NFA States	ε-closure(Delta(**q**,*))		
			a	b	c
q_0	d_0	n_0	$\begin{Bmatrix} n_1, n_2, n_3, \\ n_4, n_6, n_9 \end{Bmatrix}$	*– none –*	*– none –*
q_1	d_1	$\begin{Bmatrix} n_1, n_2, n_3, \\ n_4, n_6, n_9 \end{Bmatrix}$	*– none –*	$\begin{Bmatrix} n_5, n_8, n_9, \\ n_3, n_4, n_6 \end{Bmatrix}$	$\begin{Bmatrix} n_7, n_8, n_9, \\ n_3, n_4, n_6 \end{Bmatrix}$
q_2	d_2	$\begin{Bmatrix} n_5, n_8, n_9, \\ n_3, n_4, n_6 \end{Bmatrix}$	*– none –*	q_2	q_3
q_3	d_3	$\begin{Bmatrix} n_7, n_8, n_9, \\ n_3, n_4, n_6 \end{Bmatrix}$	*– none –*	q_2	q_3

(b) Iterations of the Subset Construction



(a) Resulting DFA

■ **FIGURE 2.7** Applying the Subset Construction to the NFA from Figure 2.5.

generate those states. Since the sets q_1, q_2 and q_3 all contain n_9 (the accepting state of the NFA), all three become accepting states in the DFA.

Fixed-Point Computations

The subset construction is an example of a *fixed-point computation*, a particular style of computation that arises regularly in computer science. These

Monotone function
a function f on domain D is *monotone* if, $\forall x,y \in D, x \le y \Rightarrow f(x) \le f(y)$

computations are characterized by the iterated application of a monotone function to some collection of sets drawn from a domain whose structure is known. These computations terminate when they reach a state where further iteration produces the same answer—a "fixed point" in the space of successive iterates. Fixed-point computations play an important and recurring role in compiler construction.

Termination arguments for fixed-point algorithms usually depend on known properties of the domain. For the subset construction, the domain D is 2^{2^N}, since $Q = \{q_0, q_1, q_2, \ldots, q_k\}$ where each $q_i \in 2^N$. Since N is finite, 2^N and 2^{2^N} are also finite. The while loop adds elements to Q; it cannot remove an element from Q. We can view the while loop as a monotone increasing function f, which means that for a set x, $f(x) \ge x$. (The comparison operator $\ge$ is $\supseteq$.) Since Q can have at most $|2^N|$ distinct elements, the while loop can iterate at most $|2^N|$ times. It may, of course, reach a fixed point and halt more quickly than that.

Computing ϵ-closure Offline

An implementation of the subset construction could compute ϵ-`closure()` by following paths in the transition graph of the NFA as needed. Figure 2.8 shows another approach: an offline algorithm that computes ϵ-`closure(`$\{n\}$`)` for each state n in the transition graph. The algorithm is another example of a fixed-point computation.

For the purposes of this algorithm, consider the transition diagram of the NFA as a graph, with nodes and edges. The algorithm begins by creating a set E for each node in the graph. For a node n, $E(n)$ will hold the current

```
for each state $n \in N$ do
    $E(n) \leftarrow \{n\}$;
end;
WorkList $\leftarrow N$;
while (WorkList $\neq \emptyset$) do
  remove $n$ from WorkList;
  t $\leftarrow \{n\} \cup \bigcup_{n \xrightarrow{\epsilon} p \in \delta_N} E(p)$;
  if t $\neq E(n)$
      then begin;
         $E(n) \leftarrow$ t;
         WorkList $\leftarrow$ WorkList $\cup \{m \mid m \xrightarrow{\epsilon} n \in \delta_N\}$;
      end;
end;
```

■ **FIGURE 2.8** An Offline Algorithm for ϵ-closure.

approximation to ϵ-`closure(n)`. Initially, the algorithm sets $E(n)$ to $\{n\}$, for each node n, and places each node on the worklist.

Each iteration of the while loop removes a node n from the worklist, finds all of the ϵ-transitions that leave n, and adds their targets to $E(n)$. If that computation changes $E(n)$, it places n's predecessors along ϵ-transitions on the worklist. (If n is in the ϵ-closure of its predecessor, adding nodes to $E(n)$ must also add them to the predecessor's set.) This process halts when the worklist becomes empty.

Using a bit-vector set for the worklist can ensure that the algorithm does not have duplicate copies of a node's name on the worklist.

See Appendix B.2.

The termination argument for this algorithm is more complex than that for the algorithm in Figure 2.6. The algorithm halts when the worklist is empty. Initially, the worklist contains every node in the graph. Each iteration removes a node from the worklist; it may also add one or more nodes to the worklist.

The algorithm only adds a node to the worklist if the E set of its successor changes. The $E(n)$ sets increase monotonically. For a node x, its successor y along an ϵ-transition can place x on the worklist at most $|E(y)| \leq |N|$ times, in the worst case. If x has multiple successors y_i along ϵ-transitions, each of them can place x on the worklist $|E(y_i)| \leq |N|$ times. Taken over the entire graph, the worst case behavior would place nodes on the worklist $k \cdot |N|$ times, where k is the number of ϵ-transitions in the graph. Thus, the worklist eventually becomes empty and the computation halts.

2.4.4 DFA to Minimal DFA: Hopcroft's Algorithm

As a final refinement to the RE→DFA conversion, we can add an algorithm to minimize the number of states in the DFA. The DFA that emerges from the subset construction can have a large set of states. While this does not increase the time needed to scan a string, it does increase the size of the recognizer in memory. On modern computers, the speed of memory accesses often governs the speed of computation. A smaller recognizer may fit better into the processor's cache memory.

To minimize the number of states in a DFA, $(D, \Sigma, \delta, d_0, D_A)$, we need a technique to detect when two states are equivalent—that is, when they produce the same behavior on any input string. The algorithm in Figure 2.9 finds equivalence classes of DFA states based on their behavior. From those equivalence classes, we can construct a minimal DFA.

The algorithm constructs a set partition, $P = \{p_1, p_2, p_3, \ldots p_m\}$, of the DFA states. The particular partition, P, that it constructs groups together DFA states by their behavior. Two DFA states, $d_i, d_j \in p_s$, have the same behavior in response to all input characters. That is, if $d_i \xrightarrow{c} d_x$, $d_j \xrightarrow{c} d_y$, and $d_i, d_j \in p_s$,

Set partition
A *set partition* of S is a collection of nonempty, disjoint subsets of S whose union is exactly S.

```
T ← {$D_A$,  {$D - D_A$} };
P ← ∅

while (P ≠ T) do
    P ← T;
    T ← ∅;

    for each set $p$ ∈ P do
        T ← T ∪ Split($p$);
    end;
end;
```

```
Split(S) {
    for each $c \in \Sigma$ do
        if $c$ splits $S$ into $s_1$ and $s_2$
            then return {$s_1, s_2$};
    end;

    return $S$;
}
```

■ **FIGURE 2.9** DFA Minimization Algorithm.

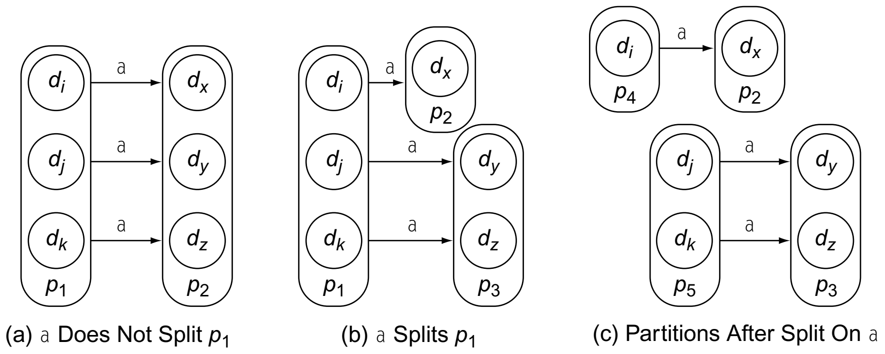
then d_x and d_y must be in the same set p_t. This property holds for every set $p_s \in P$, for every pair of states $d_i, d_j \in p_s$, and for every input character, c. Thus, the states in p_s have the same behavior with respect to input characters and the remaining sets in P.

To minimize a DFA, each set $p_s \in P$ should be as large as possible, within the constraint of behavioral equivalence. To construct such a partition, the algorithm begins with an initial rough partition that obeys all the properties *except* behavioral equivalence. It then iteratively refines that partition to enforce behavioral equivalence. The initial partition contains two sets, $p_0 = D_A$ and $p_1 = \{D - D_A\}$. This separation ensures that no set in the final partition contains both accepting and nonaccepting states, since the algorithm never combines two partitions.

The algorithm refines the initial partition by repeatedly examining each $p_s \in P$ to look for states in p_s that have different behavior for some input string. Clearly, it cannot trace the behavior of the DFA on every string. It can, however, simulate the behavior of a given state in response to a single input character. It uses a simple condition for refining the partition: a symbol $c \in \Sigma$ must produce the same behavior for every state $d_i \in p_s$. If it does not, the algorithm splits p_s around c.

This splitting action is the key to understanding the algorithm. For d_i and d_j to remain together in p_s, they must take equivalent transitions on each character $c \in \Sigma$. That is, $\forall\, c \in \Sigma$, $d_i \xrightarrow{c} d_x$ and $d_j \xrightarrow{c} d_y$, where $d_x, d_y \in p_t$. Any state $d_k \in p_s$ where $d_k \xrightarrow{c} d_z$, $d_z \notin p_t$, cannot remain in the same partition as d_i and d_j. Similarly, if d_i and d_j have transitions on c and d_k does not, it cannot remain in the same partition as d_i and d_j.

Figure 2.10 makes this concrete. The states in $p_1 = \{d_i, d_j, d_k\}$ are equivalent if and only if their transitions, $\forall\, c \in \Sigma$, take them to states that are, themselves, in an equivalence class. As shown, each state has a transition on a: $d_i \xrightarrow{a} d_x$, $d_j \xrightarrow{a} d_y$, and $d_k \xrightarrow{a} d_z$. If d_x, d_y, and d_z are all in the same set in



■ **FIGURE 2.10** Splitting a Partition around a.

the current partition, as shown on the left, then d_i, d_j, and d_k should remain together and a does not split p_1.

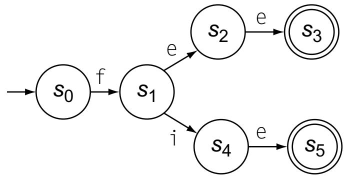
On the other hand, if d_x, d_y, and d_z are in two or more different sets, then a splits p_1. As shown in the center drawing of Figure 2.10, $d_x \in p_2$ while d_y and $d_z \in p_3$, so the algorithm must split p_1 and construct two new sets $p_4 = \{d_i\}$ and $p_5 = \{d_j, d_k\}$ to reflect the potential for different outcomes with strings that begin with the symbol a. The result is shown on the right side of Figure 2.10. The same split would result if state d_i had no transition on a.

To refine a partition P, the algorithm examines each $p \in P$ and each $c \in \Sigma$. If c splits p, the algorithm constructs two new sets from p and adds them to T. (It could split p into more than two sets, all having internally consistent behavior on c. However, creating one consistent state and lumping the rest of p into another state will suffice. If the latter state is inconsistent in its behavior on c, the algorithm will split it in a later iteration.) The algorithm repeats this process until it finds a partition where it can split no sets.

To construct the new DFA from the final partition p, we can create a single state to represent each set $p \in P$ and add the appropriate transitions between these new representative states. For the state representing p_l, we add a transition to the state representing p_m on c if some $d_j \in p_l$ has a transition on c to some $d_k \in p_m$. From the construction, we know that if d_j has such a transition, so does every other state in p_l; if this were not the case, the algorithm would have split p_l around c. The resulting DFA is minimal; the proof is beyond our scope.

Examples

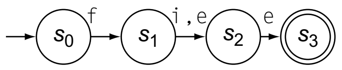
Consider a DFA that recognizes the language *fee* | *fie*, shown in Figure 2.11a. By inspection, we can see that states s_3 and s_5 serve the same purpose. Both



(a) DFA for “*fee* | *fie*”

Step	Current Partition	Examines		
		Set	Char	Action
0	$\{\{s_3,s_5\},\{s_0,s_1,s_2,s_4\}\}$	—	—	—
1	$\{\{s_3,s_5\},\{s_0,s_1,s_2,s_4\}\}$	$\{s_3,s_5\}$	*all*	*none*
2	$\{\{s_3,s_5\},\{s_0,s_1,s_2,s_4\}\}$	$\{s_0,s_1,s_2,s_4\}$	e	*split* $\{s_2,s_4\}$
3	$\{\{s_3,s_5\},\{s_0,s_1\},\{s_2,s_4\}\}$	$\{s_0,s_1\}$	f	*split* $\{s_1\}$
4	$\{\{s_3,s_5\},\{s_0\},\{s_1\},\{s_2,s_4\}\}$	*all*	*all*	*none*

(b) Critical Steps in Minimizing the DFA



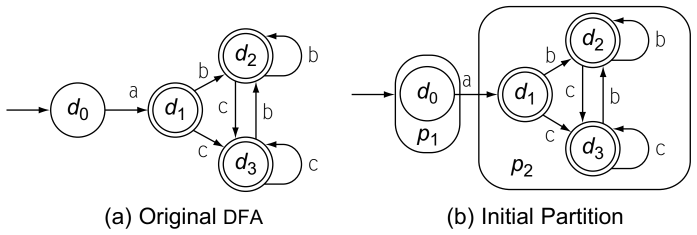
(c) The Minimal DFA (States Renumbered)

■ **FIGURE 2.11** Applying the DFA Minimization Algorithm.

are accepting states entered only by a transition on the letter e. Neither has a transition that leaves the state. We would expect the DFA minimization algorithm to discover this fact and replace them with a single state.

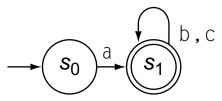
Figure 2.11b shows the significant steps that occur in minimizing this DFA. The initial partition, shown as step 0, separates accepting states from nonaccepting states. Assuming that the while loop in the algorithm iterates over the sets of P in order, and over the characters in $\Sigma = \{\texttt{e}, \texttt{f}, \texttt{i}\}$ in order, then it first examines the set $\{s_3, s_5\}$. Since neither state has an exiting transition, the state does not split on any character. In the second step, it examines $\{s_0, s_1, s_2, s_4\}$; on the character e, it splits $\{s_2, s_4\}$ out of the set. In the third step, it examines $\{s_0, s_1\}$ and splits it around the character f. At that point, the partition is $\{\{s_3, s_5\}, \{s_0\}, \{s_1\}, \{s_2, s_4\}\}$. The algorithm makes one final pass over the sets in the partition, splits none of them, and terminates.

To construct the new DFA, we must build a state to represent each set in the final partition, add the appropriate transitions from the original DFA, and designate initial and accepting state(s). Figure 2.11c shows the result for this example.



■ **FIGURE 2.12** DFA for $a(b|c^*)$.

As a second example, consider the DFA for $a\,(b\,|\,c)^*$ produced by Thompson's construction and the subset construction, shown in Figure 2.12a. The first step of the minimization algorithm constructs an initial partition $\{\{d_0\}, \{d_1,d_2,d_3\}\}$, as shown on the right. Since p_1 has only one state, it cannot be split. When the algorithm examines p_2, it finds no transitions on a from any state in p_2. For both b and c, each state has a transition back into p_2. Thus, no symbol in Σ splits p_2, and the final partition is $\{\{d_0\},\ \{d_1,d_2,d_3\}\}$.



The resulting minimal DFA is shown in Figure 2.12b. Recall that this is the DFA that we suggested a human would derive. After minimization, the automatic techniques produce the same result.

This algorithm is another example of a fixed-point computation. P is finite; at most, it can contain $|D|$ elements. The while loop splits sets in P, but never combines them. Thus, $|P|$ grows monotonically. The loop halts when some iteration splits no sets in P. The worst-case behavior occurs when each state in the DFA has different behavior; in that case, the while loop halts when P has a distinct set for each $d_i \in D$. This occurs when the algorithm is applied to a minimal DFA.

2.4.5 **Using a DFA as a Recognizer**

Thus far, we have developed the mechanisms to construct a DFA implementation from a single RE. To be useful, a compiler's scanner must recognize all the syntactic categories that appear in the grammar for the source language. What we need, then, is a recognizer that can handle all the REs for the language's microsyntax. Given the REs for the various syntactic categories, $r_1, r_2, r_3, \ldots, r_k$, we can construct a single RE for the entire collection by forming $(r_1 \mid r_2 \mid r_3 \mid \ldots \mid r_k)$.

If we run this RE through the entire process, building an NFA, constructing a DFA to simulate the NFA, minimizing it, and turning that minimal DFA into executable code, the resulting scanner recognizes the next word that matches one of the r_i's. That is, when the compiler invokes it on some input, the scanner will examine characters one at a time and accept the string if it is in an accepting state when it exhausts the input. The scanner should return both the text of the string and its syntactic category, or part of speech. Since most real programs contain more than one word, we need to transform either the language or the recognizer.

At the language level, we can insist that each word end with some easily recognizable delimiter, like a blank or a tab. This idea is deceptively attractive. Taken literally, it requires delimiters surrounding all operators, as +, `-`, (,), and the comma.

At the recognizer level, we can change the implementation of the DFA and its notion of acceptance. To find the longest word that matches one of the REs, the DFA should run until it reaches the point where the current state, *s*, has no outgoing transition on the next character. At that point, the implementation must decide which RE it has matched. Two cases arise; the first is simple. If *s* is an accepting state, then the DFA has found a word in the language and should report the word and its syntactic category.

If *s* is not an accepting state, matters are more complex. Two cases occur. If the DFA passed through one or more accepting states on its way to *s*, the recognizer should back up to the most recent such state. This strategy matches the longest valid prefix in the input string. If it never reached an accepting state, then no prefix of the input string is a valid word and the recognizer should report an error. The scanners in Section 2.5.1 implement both these notions.

As a final complication, an accepting state in the DFA may represent several accepting states in the original NFA. For example, if the lexical specification includes REs for keywords as well as an RE for identifiers, then a keyword such as `new` might match two REs. The recognizer must decide which syntactic category to return: identifier or the singleton category for the keyword `new`.

Most scanner-generator tools allow the compiler writer to specify a priority among patterns. When the recognizer matches multiple patterns, it returns the syntactic category of the highest-priority pattern. This mechanism resolves the problem in a simple way. The `lex` scanner generator, distributed with many Unix systems, assigns priorities based on position in the list of REs. The first RE has highest priority, while the last RE has lowest priority.

As a practical matter, the compiler writer must also specify REs for parts of the input stream that do not form words in the program text. In most programming languages, blank space is ignored, but every program contains it. To handle blank space, the compiler writer typically includes an RE that matches blanks, tabs, and end-of-line characters; the action on accepting blank space is to invoke the scanner, recursively, and return its result. If comments are discarded, they are handled in a similar fashion.

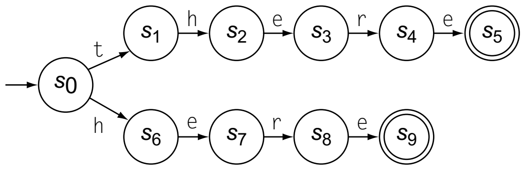
SECTION REVIEW

Given a regular expression, we can derive a minimal DFA to recognize the language specified by the RE using the following steps: (1) apply Thompson's construction to build an NFA for the RE; (2) use the subset construction to derive a DFA that simulates the behavior of the RE; and (3) use Hopcroft's algorithm to identify equivalent states in the DFA and construct a minimal DFA. This trio of constructions produces an efficient recognizer for any language that can be specified with an RE.

Both the subset construction and the DFA minimization algorithm are fixed-point computations. They are characterized by repeated application of a monotone function to some set; the properties of the domain play an important role in reasoning about the termination and complexity of these algorithms. We will see more fixed-point computations in later chapters.

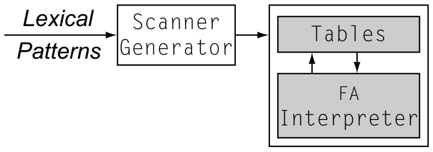
Review Questions

1. Consider the RE *who* | *what* | *where*. Use Thompson's construction to build an NFA from the RE. Use the subset construction to build a DFA from the NFA. Minimize the DFA.
2. Minimize the following DFA:



2.5 IMPLEMENTING SCANNERS

Scanner construction is a problem where the theory of formal languages has produced tools that can automate implementation. For most languages, the compiler writer can produce an acceptably fast scanner directly from a set of regular expressions. The compiler writer creates an RE for each syntactic category and gives the REs as input to a scanner generator. The generator constructs an NFA for each RE, joins them with ϵ-transitions, creates a corresponding DFA, and minimizes the DFA. At that point, the scanner generator must convert the DFA into executable code.



■ **FIGURE 2.13** Generating a Table-Driven Scanner.

This section discusses three implementation strategies for converting a DFA into executable code: a table-driven scanner, a direct-coded scanner, and a hand-coded scanner. All of these scanners operate in the same manner, by simulating the DFA. They repeatedly read the next character in the input and simulate the DFA transition caused by that character. This process stops when the DFA recognizes a word. As described in the previous section, that occurs when the current state, s, has no outbound transition on the current input character.

If s is an accepting state, the scanner recognizes the word and returns a lexeme and its syntactic category to the calling procedure. If s is a nonaccepting state, the scanner must determine whether or not it passed through an accepting state on the way to s. If the scanner did encounter an accepting state, it should roll back its internal state and its input stream to that point and report success. If it did not, it should report the failure.

These three implementation strategies, table driven, direct coded, and hand coded, differ in the details of their runtime costs. However, they all have the same asymptotic complexity—constant cost per character, plus the cost of roll back. The differences in the efficiency of well-implemented scanners change the constant costs per character but not the asymptotic complexity of scanning.

The next three subsections discuss implementation differences between table-driven, direct-coded, and hand-coded scanners. The strategies differ in how they model the DFA's transition structure and how they simulate its operation. Those differences, in turn, produce different runtime costs. The final subsection examines two different strategies for handling reserved keywords.

2.5.1 Table-Driven Scanners

The table-driven approach uses a skeleton scanner for control and a set of generated tables that encode language-specific knowledge. As shown in Figure 2.13, the compiler writer provides a set of lexical patterns, specified

```
NextWord()
  state ← $s_0$;
  lexeme ← " ";
  clear stack;
  push(bad);

  while (state≠$s_e$) do
    NextChar(char);
    lexeme ← lexeme + char;

    if state ∈ $S_A$
        then clear stack;

    push(state);

    cat ← CharCat[char];
    state ← δ[state,cat];
  end;

  while(state ∉ $S_A$ and
        state≠bad) do
    state ← pop();
    truncate lexeme;
    RollBack();
  end;

  if state ∈ $S_A$
    then return Type[state];
    else return invalid;
```

r	0,1,2,...,9	EOF	**Other**
Register	*Digit*	*Other*	*Other*

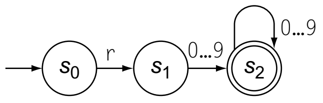
The Classifier Table, `CharCat`

	Register	***Digit***	***Other***
s_0	s_1	s_e	s_e
s_1	s_e	s_2	s_e
s_2	s_e	s_2	s_e
s_e	s_e	s_e	s_e

The Transition Table, δ

s_0	s_1	s_2	s_e
invalid	*invalid*	*register*	*invalid*

The Token Type Table, `Type`



The Underlying DFA

■ **FIGURE 2.14** A Table-Driven Scanner for Register Names.

as regular expressions. The scanner generator then produces tables that drive the skeleton scanner.

Figure 2.14 shows a table-driven scanner for the RE $r\,[0\ldots9]^+$, which was our first attempt at an RE for ILOC register names. The left side of the figure shows the skeleton scanner, while the right side shows the tables for $r\,[0\ldots9]^+$ and the underlying DFA. Notice the similarity between the code here and the recognizer shown in Figure 2.2 on page 32.

The skeleton scanner divides into four sections: initializations, a scanning loop that models the DFA's behavior, a roll back loop in case the DFA overshoots the end of the token, and a final section that interprets and reports the results. The scanning loop repeats the two basic actions of a scanner: read a character and simulate the DFA's action. It halts when the DFA enters the error state, s_e. Two tables, `CharCat` and δ, encode all knowledge about the DFA. The roll back loop uses a stack of states to revert the scanner to its most recent accepting state.

The skeleton scanner uses the variable `state` to hold the current state of the simulated DFA. It updates `state` using a two-step, table-lookup process. First, it classifies `char` into one of a small set of categories using the `CharCat` table. The scanner for $r\,[0\ldots9]^+$ has three categories: *Register, Digit*, or *Other*. Next, it uses the current state and the character category as indices into the transition table, δ.

This two-step translation, character to category, then state and category to new state, lets the scanner use a compressed transition table. The tradeoff between direct access into a larger table and indirect access into the compressed table is straightforward.A complete table would eliminate the mapping through `CharCat`, but would increase the memory footprint of the table. The uncompressed transition table grows as the product of the number of states in the DFA and the number of characters in Σ; it can grow to the point where it will not stay in cache.

For small examples, such as $r[0\ldots9]^+$, the classifier table is larger than the complete transition table. In a realistically sized example, that relationship should be reversed.

With a small, compact character set, such as ASCII, `CharCat` can be represented as a simple table lookup. The relevant portions of `CharCat` should stay in the cache. In that case, table compression adds one cache reference per input character. As the character set grows (e.g. Unicode), more complex implementations of `CharCat` may be needed. The precise tradeoff between the per-character costs of both compressed and uncompressed tables will depend on properties of both the language and the computer that runs the scanner.

To provide a character-by-character interface to the input stream, the skeleton scanner uses a macro, `NextChar`, which sets its sole parameter to contain the next character in the input stream. A corresponding macro, `RollBack`, moves the input stream back by one character. (Section 2.5.3 looks at `NextChar` and `RollBack`.)

If the scanner reads too far, `state` will not contain an accepting state at the end of the first while loop. In that case, the second while loop uses the state trace from the stack to roll the state, lexeme, and input stream back to the most recent accepting state. In most languages, the scanner's overshoot will be limited. Pathological behavior, however, can cause the scanner to examine individual characters many times, significantly increasing the overall cost of scanning. In most programming languages, the amount of roll back is small relative to the word lengths. In languages where significant amounts of roll back can occur, a more sophisticated approach to this problem is warranted.

Avoiding Excess Roll Back

Some regular expressions can produce quadratic calls to roll back in the scanner shown in Figure 2.14. The problem arises from our desire to have the scanner return the longest word that is a prefix of the input stream.

Consider the RE *ab* | (*ab*)* *c*. The corresponding DFA, shown in the margin, recognizes either *ab* or any number of occurrences of *ab* followed by a final *c*. On the input string `ababababc`, a scanner built from the DFA will read all the characters and return the entire string as a single word. If, however, the input is `abababab`, it must scan all of the characters before it can determine that the longest prefix is `ab`. On the next invocation, it will scan `ababab` to return `ab`. The third call will scan `abab` to return `ab`, and the final call will simply return `ab` without any roll back. In the worst, case, it can spend quadratic time reading the input stream.

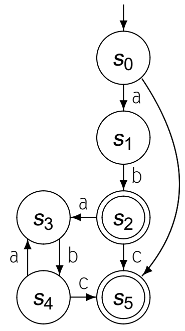

$$\begin{tikzcd}
 & s_0 \arrow[d, "a"] \arrow[dddr, bend left, "c"] & \\
 & s_1 \arrow[d, "b"] & \\
s_3 \arrow[d, shift left, "b"] & \boxed{s_2} \arrow[l, "a"'] \arrow[d, "c"] & \\
s_4 \arrow[u, shift left, "a"] \arrow[r, "c"] & \boxed{s_5} &
\end{tikzcd}$$

Figure 2.15 shows a modification to the scanner in Figure 2.14 that avoids this problem. It differs from the earlier scanner in three important ways. First, it has a global counter, `InputPos`, to record position in the input stream. Second, it has a bit-array, `Failed`, to record dead-end transitions as the scanner finds them. `Failed` has a row for each state and a column for each position in the input stream. Third, it has an initialization routine that

```
NextWord()
  state ← $s_0$;
  lexeme ← " ";
  clear stack;
  push(⟨bad, bad⟩);

  while (state≠$s_e$) do
    NextChar(char);
    InputPos ← InputPos + 1;
    lexeme ← lexeme + char;

    if Failed[state,InputPos]
        then break;

    if state ∈ $S_A$
        then clear stack;

    push(⟨state,InputPos⟩);

    cat ← CharCat[char];
    state ← δ[state,cat];
  end;

  while(state ∉ $S_A$ and state≠bad) do
    Failed[state,InputPos] ← true;
    ⟨state,InputPos⟩ ← pop();
    truncate lexeme;
    RollBack();
  end;

  if state ∈ $S_A$
    then return TokenType[state];
    else return bad;

InitializeScanner()
  InputPos=0;
  for each state s in the DFA do
      for i=0 to |input stream| do
          Failed[s,i] ← false;
      end;
  end;
```

■ **FIGURE 2.15** The Maximal Munch Scanner.

must be called before `NextWord()` is invoked. That routine sets `InputPos` to zero and sets `Failed` uniformly to false.

This scanner, called the *maximal munch scanner*, avoids the pathological behavior by marking dead-end transitions as they are popped from the stack. Thus, over time, it records specific ⟨*state*,*input position*⟩ pairs that cannot lead to an accepting state. Inside the scanning loop, the first while loop, the code tests each ⟨*state*,*input position*⟩ pair and breaks out of the scanning loop whenever a failed transition is attempted.

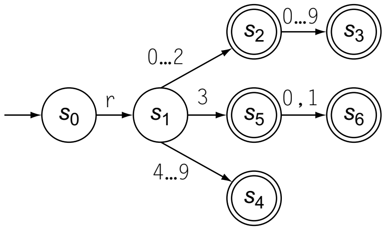
Optimizations can drastically reduce the space requirements of this scheme. (See, for example, Exercise 16 on page 82.) Most programming languages have simple enough microsyntax that this kind of quadratic roll back cannot occur. If, however, you are building a scanner for a language that can exhibit this behavior, the scanner can avoid it for a small additional overhead per character.

Generating the Transition and Classifier Tables

Given a DFA, the scanner generator can generate the tables in a straightforward fashion. The initial table has one column for every character in the input alphabet and one row for each state in the DFA. For each state, in order, the generator examines the outbound transitions and fills the row with the appropriate states. The generator can collapse identical columns into a single instance; as it does so, it can construct the character classifier. (Two characters belong in the same class if and only if they have identical columns in δ.) If the DFA has been minimized, no two rows can be identical, so row compression is not an issue.

Changing Languages

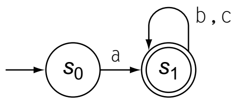
To model another DFA, the compiler writer can simply supply new tables. Earlier in the chapter, we worked with a second, more constrained specification for ILOC register names, given by the RE: $r(\,[0\ldots 2]\,([0\ldots 9]\,|\,\epsilon)\;|\;[4\ldots 9]\;|\;(3\,(0\,|\,1\,|\,\epsilon))\,)$. That RE gave rise to the following DFA:



Because it has more states and transitions than the RE for $r\,[0\ldots 9]^+$, we should expect a larger transition table.

	r	0,1	2	3	4…9	Other
s_0	s_1	s_e	s_e	s_e	s_e	s_e
s_1	s_e	s_2	s_2	s_5	s_4	s_e
s_2	s_e	s_3	s_3	s_3	s_3	s_e
s_3	s_e	s_e	s_e	s_e	s_e	s_e
s_4	s_e	s_e	s_e	s_e	s_e	s_e
s_5	s_e	s_6	s_e	s_e	s_e	s_e
s_6	s_e	s_e	s_e	s_e	s_e	s_e
s_e	s_e	s_e	s_e	s_e	s_e	s_e

As a final example, the minimal DFA for the RE $a\,(b|c)^*$ has the following table:



Minimal DFA

	a	b,c	Other
s_0	s_1	s_e	s_e
s_1	s_e	s_1	s_e

Transition Table

The character classifier has three classes: `a`, `b` or `c`, and all other characters.

2.5.2 **Direct-Coded Scanners**

To improve the performance of a table-driven scanner, we must reduce the cost of one or both of its basic actions: read a character and compute the next DFA transition. Direct-coded scanners reduce the cost of computing DFA transitions by replacing the explicit representation of the DFA's state and transition graph with an implicit one. The implicit representation simplifies the two-step, table-lookup computation. It eliminates the memory references entailed in that computation and allows other specializations. The resulting scanner has the same functionality as the table-driven scanner, but with a lower overhead per character. A direct-coded scanner is no harder to generate than the equivalent table-driven scanner.

The table-driven scanner spends most of its time inside the central while loop; thus, the heart of a direct-coded scanner is an alternate implementation of that while loop. With some detail abstracted, that loop performs the following actions:

```
while (state $\neq s_e$) do
  NextChar(char);
  cat $\leftarrow$ CharCat[char];
  state $\leftarrow$ $\delta$[state,cat];
end;
```

REPRESENTING STRINGS

The scanner classifies words in the input program into a small set of categories. From a functional perspective, each word in the input stream becomes a pair ⟨*word,type*⟩, where *word* is the actual text that forms the word and *type* represents its syntactic category.

For many categories, having both *word* and *type* is redundant. The words +, ×, and `for` have only one spelling. For identifiers, numbers, and character strings, however, the compiler will repeatedly use the *word*. Unfortunately, many compilers are written in languages that lack an appropriate representation for the *word* part of the pair. We need a representation that is compact and offers a fast equality test for two words.

A common practice to address this problem has the scanner create a single hash table (see Appendix B.4) to hold all the distinct strings used in the input program. The compiler then uses either the string's index in this "string table" or a pointer to its stored image in the string table as a proxy for the string. Information derived from the string, such as the length of a character constant or the value and type of a numerical constant, can be computed once and referenced quickly through the table. Since most computers have storage-efficient representations for integers and pointers, this reduces the amount of memory used internally in the compiler. By using the hardware comparison mechanisms on the integer or pointer proxies, it also simplifies the code used to compare them.

Notice the variable *state* that explicitly represents the DFA's current state and the tables *CharCat* and δ that represent the DFA's transition diagram.

Overhead of Table Lookup

For each character, the table-driven scanner performs two table lookups, one in *CharCat* and another in δ. While both lookups take **O**(1) time, the table abstraction imposes constant-cost overheads that a direct-coded scanner can avoid. To access the i^{th} element of *CharCat*, the code must compute its address, given by

$$@CharCat_0 + i \times w$$

Detailed discussion of code for array addressing starts on page 359 in Section 7.5.

where $@CharCat_0$ is a constant related to the starting address of *CharCat* in memory and *w* is the number of bytes in each element of *CharCat*. After computing the address, the code must load the data found at that address in memory.

Because δ has two dimensions, the address calculation is more complex. For the reference `δ(state,cat)`, the code must compute

$$@\delta_0 + (state \times number\ of\ columns\ in\ \delta + cat) \times w$$

where $@\delta_0$ is a constant related to the starting address of δ in memory and w is the number of bytes per element of δ. Again, the scanner must issue a load operation to retrieve the data stored at this address.

Thus, the table-driven scanner performs two address computations and two load operations for each character that it processes. The speed improvements in a direct-coded scanner come from reducing this overhead.

Replacing the Table-Driven Scanner's While Loop

Rather than represent the current DFA state and the transition diagram explicitly, a direct-coded scanner has a specialized code fragment to implement each state. It transfers control directly from state-fragment to state-fragment to emulate the actions of the DFA. Figure 2.16 shows a direct-coded scanner

```
s_init : lexeme ← " ";
         clear stack;
         push(bad);
         goto s_0 ;

s_0 :    NextChar(char);
         lexeme ← lexeme + char;
         if state ∈ S_A
             then clear stack;
         push(state);
         if (char='r')
            then goto s_1 ;
            else goto s_out ;

s_1 :    NextChar(char);
         lexeme ← lexeme + char;
         if state ∈ S_A
             then clear stack;
         push(state);
         if ('0'≤char≤'9')
             then goto s_2 ;
             else goto s_out ;

s_2 :    NextChar(char);
         lexeme ← lexeme + char;
         if state ∈ S_A
             then clear stack;
         push(state);
         if '0'≤char≤'9'
            then goto s_2 ;
            else goto s_out

s_out :  while (state ∉ S_A and
                state ≠ bad) do
            state ← pop();
            truncate lexeme;
            RollBack();
         end;
         if state ∈ S_A
            then return Type[state];
            else return invalid;
```

■ **FIGURE 2.16** A Direct-Coded Scanner for $r[0\ldots9]^+$.

for $r\,[0 \ldots 9]^{+}$; it is equivalent to the table-driven scanner shown earlier in Figure 2.14.

Consider the code for state s_1. It reads a character, concatenates it onto the current word, and advances the character counter. If `char` is a digit, it jumps to state s_2. Otherwise, it jumps to state s_{out}. The code requires no complicated address calculations. The code refers to a tiny set of values that can be kept in registers. The other states have equally simple implementations.

The code in Figure 2.16 uses the same mechanism as the table-driven scanner to track accepting states and to roll back to them after an overrun. Because the code represents a specific DFA, we could specialize it further. In particular, since the DFA has just one accepting state, the stack is unneeded and the transitions to s_{out} from s_0 and s_1 can be replaced with `report failure`. In a DFA where some transition leads from an accepting state to a nonaccepting state, the more general mechanism is needed.

A scanner generator can directly emit code similar to that shown in Figure 2.16. Each state has a couple of standard assignments, followed by branching logic that implements the transitions out of the state. Unlike the table-driven scanner, the code changes for each set of REs. Since that code is generated directly from the REs, the difference should not matter to the compiler writer.

Code in the style of Figure 2.16 is often called *spaghetti code* in honor of its tangled control flow.

Of course, the generated code violates many of the precepts of structured programming. While small examples may be comprehensible, the code for a complex set of regular expressions may be difficult for a human to follow. Again, since the code is generated, humans should not need to read or debug it. The additional speed obtained from direct coding makes it an attractive option, particularly since it entails no extra work for the compiler writer. Any extra work is pushed into the implementation of the scanner generator.

Classifying Characters

The continuing example, $r\,[0 \ldots 9]^{+}$, divides the alphabet of input characters into just four classes. An `r` falls in class *Register*. The digits `0`, `1`, `2`, `3`, `4`, `5`, `6`, `7`, `8`, and `9` fall in class *Digit*, the special character returned when `NextChar` exhausts its input falls in class *EndOfFile*, and anything else falls in class *Other*.

Collating sequence
the "sorting order" of the characters in an alphabet, determined by the integers assigned each character

The scanner can easily and efficiently classify a given character, as shown in Figure 2.16. State s_0 uses a direct test on '`r`' to determine if `char` is in *Register*. Because all the other classes have equivalent actions in the DFA, the scanner need not perform further tests. States s_1 and s_2 classify `char` into either *Digit* or anything else. They capitalize on the fact that the digits `0` through `9` occupy adjacent positions in the ASCII collating sequence, corresponding to the integers 48 to 57.

In a scanner where character classification is more involved, the translation-table approach used in the table-driven scanner may be less expensive than directly testing characters. In particular, if a class contains multiple characters that do not occupy adjacent slots in the collating sequence, a table lookup may be more efficient than direct testing. For example, a class that contained the arithmetic operators +, -, *, \, and ^ (43, 45, 42, 48, and 94 in the ASCII sequence) would require a moderately long series of comparisons. Using a translation table, such as `CharCat` in the table-driven example, might be faster than the comparisons if the translation table stays in the processor's primary cache.

2.5.3 **Hand-Coded Scanners**

Generated scanners, whether table-driven or direct-coded, use a small, constant amount of time per character. Despite this fact, many compilers use hand-coded scanners. In an informal survey of commercial compiler groups, we found that a surprisingly large fraction used hand-coded scanners. Similarly, many of the popular open-source compilers rely on hand-coded scanners. For example, the *flex* scanner generator was ostensibly built to support the *gcc* project, but *gcc 4.0* uses hand-coded scanners in several of its front ends.

The direct-coded scanner reduced the overhead of simulating the DFA; the hand-coded scanner can reduce the overhead of the interfaces between the scanner and the rest of the system. In particular, a careful implementation can improve the mechanisms used to read and manipulate characters on input and the operations needed to produce a copy of the actual lexeme on output.

Buffering the Input Stream

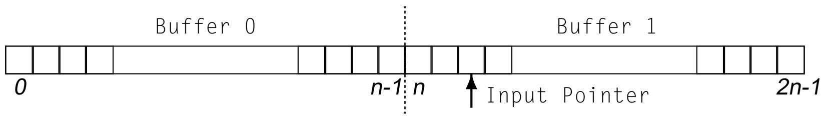
While character-by-character I/O leads to clean algorithmic formulations, the overhead of a procedure call per character is significant relative to the cost of simulating the DFA in either a table-driven or a direct-coded scanner. To reduce the I/O cost per character, the compiler writer can use buffered I/O, where each read operation returns a longer string of characters, or buffer, and the scanner then indexes through the buffer. The scanner maintains a pointer into the buffer. Responsibility for keeping the buffer filled and tracking the current location in the buffer falls to `NextChar`. These operations can be performed inline; they are often encoded in a macro to avoid cluttering the code with pointer dereferences and increments.

The cost of reading a full buffer of characters has two components, a large fixed overhead and a small per-character cost. A buffer and pointer scheme amortizes the fixed costs of the read over many single-character fetches. Making the buffer larger reduces the number of times that the scanner incurs this cost and reduces the per-character overhead.

Using a buffer and pointer also leads to a simple and efficient implementation of the `RollBack` operation that occurs at the end of both the generated scanners. To roll the input back, the scanner can simply decrement the input pointer. This scheme works as long as the scanner does not decrement the pointer beyond the start of the buffer. At that point, however, the scanner needs access to the prior contents of the buffer.

Double buffering
A scheme that uses two input buffers in a modulo fashion to provide bounded roll back is often called *double buffering*.

In practice, the compiler writer can bound the roll-back distance that a scanner will need. With bounded roll back, the scanner can simply use two adjacent buffers and increment the pointer in a modulo fashion, as shown below:



To read a character, the scanner increments the pointer, modulo *2n* and returns the character at that location. To roll back a character, the program decrements the input pointer, modulo *2n*. It must also manage the contents of the buffer, reading additional characters from the input stream as needed.

Both `NextChar` and `RollBack` have simple, efficient implementations, as shown in Figure 2.17. Each execution of `NextChar` loads a character, increments the `Input` pointer, and tests whether or not to fill the buffer. Every `n` characters, it fills the buffer. The code is small enough to be included inline, perhaps generated from a macro. This scheme amortizes the cost of filling the buffer over `n` characters. By choosing a reasonable size for `n`, such as 2048, 4096, or more, the compiler writer can keep the I/O overhead low.

`Rollback` is even less expensive. It performs a test to ensure that the buffer contents are valid and then decrements the input pointer. Again, the implementation is sufficiently simple to be expanded inline. (If we used this implementation of `NextChar` and `RollBack` in the generated scanners, `RollBack` would need to truncate the final character away from `lexeme`.)

```
Char ← Buffer[Input];
Input ← (Input+1) mod 2n;

if (Input mod n = 0)
    then begin;
        fill Buffer[Input : Input+n - 1];
        Fence ← (Input+n) mod 2n;
    end;

return Char;
```

Implementing *NextChar*

```
Input ← 0;
Fence ← 0;
fill Buffer[0 : n];
```

Initialization

```
if (Input = Fence)
    then signal roll back error;
Input ← (Input - 1) mod 2n;
```

Implementing *RollBack*

■ **FIGURE 2.17** Implementing *NextChar* and *RollBack*.

As a natural consequence of using finite buffers, *RollBack* has a limited history in the input stream. To keep it from decrementing the pointer beyond the start of that context, *NextChar* and *RollBack* cooperate. The pointer *Fence* always indicates the start of the valid context. *NextChar* sets *Fence* each time it fills a buffer. *RollBack* checks *Fence* each time it tries to decrement the *Input* pointer.

After a long series of *NextChar* operations, say, more than *n* of them, *RollBack* can always back up at least *n* characters. However, a sequence of calls to *NextChar* and *RollBack* that work forward and backward in the buffer can create a situation where the distance between *Input* and *Fence* is less than *n*. Larger values of *n* decrease the likelihood of this situation arising. Expected backup distances should be a consideration in selecting the buffer size, *n*.

Generating Lexemes

The code shown for the table-driven and direct-coded scanners accumulated the input characters into a string *lexeme*. If the appropriate output for each syntactic category is a textual copy of the lexeme, then those schemes are efficient. In some common cases, however, the parser, which consumes the scanner's output, needs the information in another form.

For example, in many circumstances, the natural representation for a register number is an integer, rather than a character string consisting of an '*r*' and a sequence of digits. If the scanner builds a character representation, then somewhere in the interface, that string must be converted to an integer. A typical way to accomplish that conversion uses a library routine, such as `atoi` in the standard C library, or a string-based I/O routine, such as sscanf. A more efficient way to solve this problem would be to accumulate the integer's value one digit at a time.

In the continuing example, the scanner could initialize a variable, *RegNum*, to zero in its initial state. Each time that it recognized a digit, it could multiply *RegNum* by 10 and add the new digit. When it reached an accepting state, *RegNum* would contain the needed value. To modify the scanner in Figure 2.16, we can delete all statements that refer to *lexeme*, add *RegNum* $\leftarrow$ *0;* to s_{init}, and replace the occurrences of goto s_2 in states s_1 and s_2 with:

```
begin;
  RegNum ← RegNum × 10 + (char - '0');
  goto s_2;
end;
```

where both *char* and *'0'* are treated as their ordinal values in the ASCII collating sequence. Accumulating the value this way likely has lower overhead than building the string and converting it in the accepting state.

For other words, the lexeme is implicit and, therefore, redundant. With singleton words, such as a punctuation mark or an operator, the syntactic category is equivalent to the lexeme. Similarly, many scanners recognize comments and white space and discard them. Again, the set of states that recognize the comment need not accumulate the lexeme. While the individual savings are small, the aggregate effect is to create a faster, more compact scanner.

This issue arises because many scanner generators let the compiler writer specify actions to be performed in an accepting state, but do not allow actions on each transition. The resulting scanners must accumulate a character copy of the lexeme for each word, whether or not that copy is needed. If compile time matters (and it should), then attention to such minor algorithmic details leads to a faster compiler.

2.5.4 **Handling Keywords**

We have consistently assumed that keywords in the input language should be recognized by including explicit REs for them in the description that generates the DFA and the recognizer. Many authors have proposed an alternative strategy: having the DFA classify them as identifiers and testing each identifier to determine whether or not it is a keyword.

This strategy made sense in the context of a hand-implemented scanner. The additional complexity added by checking explicitly for keywords causes a significant expansion in the number of DFA states. This added implementation burden matters in a hand-coded program. With a reasonable hash table (see Appendix B.4), the expected cost of each lookup should be constant. In fact, this scheme has been used as a classic application for *perfect hashing*. In perfect hashing, the implementor ensures, for a fixed set of keys, that the hash function generates a compact set of integers with no collisions. This lowers the cost of lookup on each keyword. If the table implementation takes into account the perfect hash function, a single probe serves to distinguish keywords from identifiers. If it retries on a miss, however, the behavior can be much worse for nonkeywords than for keywords.

If the compiler writer uses a scanner generator to construct the recognizer, then the added complexity of recognizing keywords in the DFA is handled by the tools. The extra states that this adds consume memory, but not compile time. Using the DFA mechanism to recognize keywords avoids a table lookup on each identifier. It also avoids the overhead of implementing a keyword table and its support functions. In most cases, folding keyword recognition into the DFA makes more sense than using a separate lookup table.

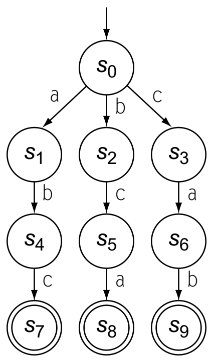
SECTION REVIEW

Automatic construction of a working scanner from a minimal DFA is straightforward. The scanner generator can adopt a table-driven approach, wherein it uses a generic skeleton scanner and language-specific tables, or it can generate a direct-coded scanner that threads together a code fragment for each DFA state. In general, the direct-coded approach produces a faster scanner because it has lower overhead per character.

Despite the fact that all DFA-based scanners have small constant costs per characters, many compiler writers choose to hand code a scanner. This approach lends itself to careful implementation of the interfaces between the scanner and the I/O system and between the scanner and the parser.

Review Questions

1. Given the DFA shown to the left, complete the following:
   a. Sketch the character classifier that you would use in a table-driven implementation of this DFA.
   b. Build the transition table, based on the transition diagram and your character classifier.
   c. Write an equivalent direct-coded scanner.



2. An alternative implementation might use a recognizer for $(a|b|c)\,(a|b|c)\,(a|b|c)$, followed by a lookup in a table that contains the three words `abc`, `bca`, and `cab`.
   - **a.** Sketch the DFA for this language.
   - **b.** Show the direct-coded scanner, including the call needed to perform keyword lookup.
   - **c.** Contrast the cost of this approach with those in question 1 above.
3. What impact would the addition of transition-by-transition actions have on the DFA-minimization process? (Assume that we have a linguistic mechanism of attaching code fragments to the edges in the transition graph.)

2.6 ADVANCED TOPICS

2.6.1 DFA to Regular Expression

The final step in the cycle of constructions, shown in Figure 2.3, is to construct an RE from a DFA. The combination of Thompson's construction and the subset construction provide a constructive proof that DFAS are at least as powerful as RES. This section presents Kleene's construction, which builds an RE to describe the set of strings accepted by an arbitrary DFA. This algorithm establishes that RES are at least as powerful as DFAS. Together, they show that RES and DFAS are equivalent.

Consider the transition diagram of a DFA as a graph with labelled edges. The problem of deriving an RE that describes the language accepted by the DFA corresponds to a path problem over the DFA's transition diagram. The set of strings in $L(\text{DFA})$ consists of the set of edge labels for every path from d_0 to d_i, $\forall\, d_i \in D_A$. For any DFA with a cyclic transition graph, the set of such paths is infinite. Fortunately, RES have the Kleene closure operator to handle this case and summarize the complete set of subpaths created by a cycle.

Figure 2.18 shows one algorithm to compute this path expression. It assumes that the DFA has states numbered from 0 to $|D| - 1$, with d_0 as the start state. It generates an expression that represents the labels along all paths between two nodes, for each pair of nodes in the transition diagram. As a final step, it combines the expressions for each path that leaves d_0 and reaches some accepting state, $d_i \in D_A$. In this way, it systematically constructs the path expressions for all paths.

The algorithm computes a set of expressions, denoted R^k_{ij}, for all the relevant values of i, j, and k. R^k_{ij} is an expression that describes all paths through the transition graph from state i to state j, without going through a state

```
for i = 0 to $|D| - 1$
    for j = 0 to $|D| - 1$
      $R^{-1}_{ij} = \{ a \mid \delta(d_i, a) = d_j \}$
      if (i = j) then
          $R^{-1}_{ij} = R^{-1}_{ij} \mid \{ \epsilon \}$

for k = 0 to $|D|-1$
    for i = 0 to $|D|-1$
      for j = 0 to $|D|-1$
          $R^k_{ij} = R^{k-1}_{ik}(R^{k-1}_{kk})^* R^{k-1}_{kj} \mid R^{k-1}_{ij}$

L = $|_{s_j \in D_A} \; R^{|D|-1}_{0j}$
```

FIGURE 2.18 Deriving a Regular Expression from a DFA.

numbered higher than k. Here, *through* means both entering and leaving, so that $R^2_{1,16}$ can be nonempty if an edge runs directly from 1 to 16.

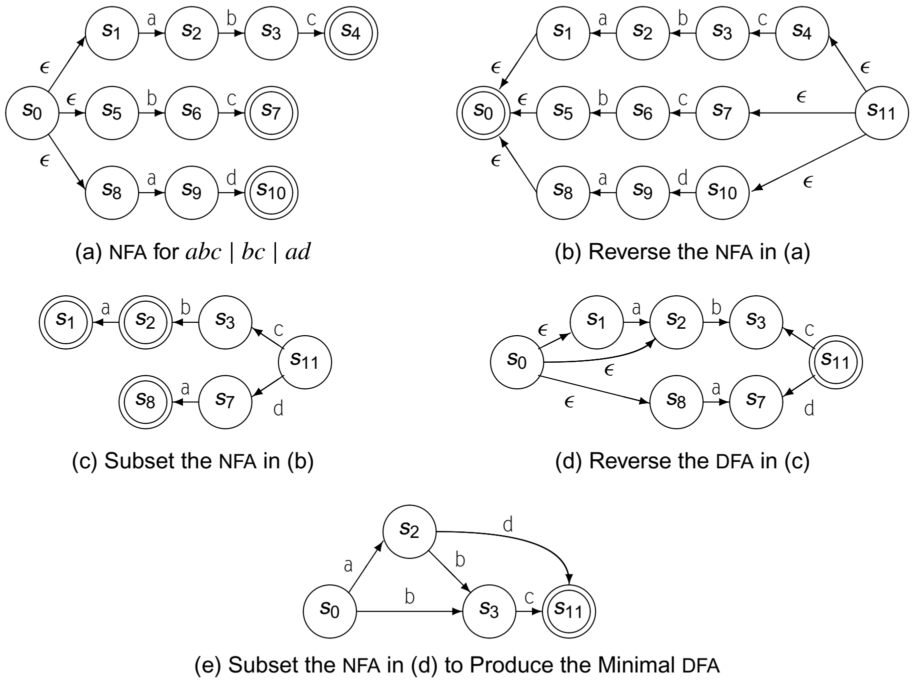
Initially, the algorithm places all of the direct paths from i to j in R^{-1}_{ij}, with $\{\epsilon\}$ added to R^{-1}_{ij} if $i = j$. Over successive iterations, it builds up longer paths to produce R^k_{ij} by adding to R^{k-1}_{ij} the paths that pass through k on their way from i to j. Given R^{k-1}_{ij}, the set of paths added by going from $k-1$ to k is exactly the set of paths that run from i to k using no state higher than $k-1$, concatenated with the paths from k to itself that pass through no state higher than $k-1$, followed by the paths from k to j that pass through no state higher than $k-1$. That is, each iteration of the loop on k adds the paths that pass through k to each set R^{k-1}_{ij} to produce R^k_{ij}.

Traditional statements of this algorithm assume that node names range from 1 to n, rather than from 0 to $n-1$. Thus, they place the direct paths in R^0_{ij}.

When the k loop terminates, the various R^k_{ij} expressions account for all paths through the graph. The final step computes the set of paths that start with d_0 and end in some accepting state, $d_j \in d_A$, as the alternation of the path expressions.

2.6.2 Another Approach to DFA Minimization: Brzozowski's Algorithm

If we apply the subset construction to an NFA that has multiple paths from the start state for some prefix, the construction will group the states involved in those duplicate prefix paths together and will create a single path for that prefix in the DFA. The subset construction always produces DFAs that have no duplicate prefix paths. Brzozowski used this observation to devise an alternative DFA minimization algorithm that directly constructs the minimal DFA from an NFA.


(a) NFA for $abc \mid bc \mid ad$
(b) Reverse the NFA in (a)
(c) Subset the NFA in (b)
(d) Reverse the DFA in (c)
(e) Subset the NFA in (d) to Produce the Minimal DFA

■ **FIGURE 2.19** Minimizing a DFA with Brzozowski's Algorithm.

For an NFA *n*, let *reverse(n)* be the NFA obtained by reversing the direction of all the transitions, making the initial state into a final state, adding a new initial state, and connecting it to all of the states that were final states in *n*. Further, let *reachable(n)* be a function that returns the set of states and transitions in *n* that are reachable from its initial state. Finally, let *subset(n)* be the DFA produced by applying the subset construction to *n*.

Now, given an NFA *n*, the minimal equivalent DFA is just

reachable(subset(reverse(reachable(subset(reverse(n)))))).

The inner application of *subset* and *reverse* eliminates duplicate suffixes in the original NFA. Next, *reachable* discards any states and transitions that are no longer interesting. Finally, the outer application of the triple, *reachable, subset*, and *reverse*, eliminates any duplicate prefixes in the NFA. (Applying *reverse* to a DFA can produce an NFA.)

The example in Figure 2.19 shows the steps of the algorithm on a simple NFA for the RE *abc* | *bc* | *ad*. The NFA in Figure 2.19a is similar to the one that Thompson's construction would produce; we have removed the ϵ-transitions that "glue" together the NFAs for individual letters. Figure 2.19b shows the result of applying *reverse* to that NFA. Figure 2.19c depicts the DFA that *subset* constructs from the *reverse* of the NFA. At this point, the algorithm applies *reachable* to remove any unreachable states; our example NFA has none. Next, the algorithm applies *reverse* to the DFA, which produces the NFA in Figure 2.19d. Applying *subset* to that NFA produces the DFA in Figure 2.19e. Since it has no unreachable states, it is the minimal DFA for *abc* | *bc* | *cd*.

This technique looks expensive, because it applies *subset* twice and we know that *subset* can construct an exponentially large set of states. Studies of the running times of various FA minimization techniques suggest, however, that this algorithm performs reasonably well, perhaps because of specific properties of the NFA produced by the first application of *reachable (subset(reverse(n)))*. From a software-engineering perspective, it may be that implementing *reverse* and *reachable* is easier than debugging the partitioning algorithm.

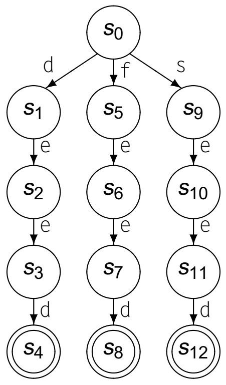
2.6.3 **Closure-Free Regular Expressions**

One subclass of regular languages that has practical application beyond scanning is the set of languages described by closure-free regular expressions. Such REs have the form $w_1 \,|\, w_2 \,|\, w_3 \,|\, \ldots \,|\, w_n$ where the individual words, w_i, are just concatenations of characters in the alphabet, Σ. These REs have the property that they produce DFAs with acyclic transition graphs.

These simple regular languages are of interest for two reasons. First, many pattern recognition problems can be described with a closure-free RE. Examples include words in a dictionary, URLs that should be filtered, and keys to a hash table. Second, the DFA for a closure-free RE can be built in a particularly efficient way.

To build the DFA for a closure-free RE, begin with a start state s_0. To add a word to the existing DFA, the algorithm follows the path for the new word until it either exhausts the pattern or finds a transition to s_e. In the former case, it designates the final state for the new word as an accepting state. In the latter, it adds a path for the new word's remaining suffix. The resulting DFA can be encoded in tabular form or in direct-coded form (see Section 2.5.2). Either way, the recognizer uses constant time per character in the input stream.

In this algorithm, the cost of adding a new word to an existing DFA is proportional to the length of the new word. The algorithm also works incrementally; an application can easily add new words to a DFA that is in use. This property makes the acyclic DFA an interesting alternative for



implementing a perfect hash function. For a small set of keys, this technique produces an efficient recognizer. As the number of states grows (in a direct-coded recognizer) or as key length grows (in a table-driven recognizer), the implementation may slow down due to cache-size constraints. At some point, the impact of cache misses will make an efficient implementation of a more traditional hash function more attractive than incremental construction of the acyclic DFA.

The DFAS produced in this way are not guaranteed to be minimal. Consider the acyclic DFA that it would produce for the RES *deed, feed*, and *seed*, shown to the left. It has three distinct paths that each recognize the suffix *eed*. Clearly, those paths can be combined to reduce the number of states and transitions in the DFA. Minimization will combine states (s_2, s_6, s_{10}), states (s_3, s_7, s_{11}), and states (s_4, s_8, s_{12}) to produce a seven state DFA.

The algorithm builds DFAS that are minimal with regard to prefixes of words in the language. Any duplication takes the form of multiple paths for the same suffix.

2.7 CHAPTER SUMMARY AND PERSPECTIVE

The widespread use of regular expressions for searching and scanning is one of the success stories of modern computer science. These ideas were developed as an early part of the theory of formal languages and automata. They are routinely applied in tools ranging from text editors to web filtering engines to compilers as a means of concisely specifying groups of strings that happen to be regular languages. Whenever a finite collection of words must be recognized, DFA-based recognizers deserve serious consideration.

The theory of regular expressions and finite automata has developed techniques that allow the recognition of regular languages in time proportional to the length of the input stream. Techniques for automatic derivation of DFAS from RES and for DFA minimization have allowed the construction of robust tools that generate DFA-based recognizers. Both generated and hand-crafted scanners are used in well-respected modern compilers. In either case, a careful implementation should run in time proportional to the length of the input stream, with a small overhead per character.

■ CHAPTER NOTES

Originally, the separation of lexical analysis, or scanning, from syntax analysis, or parsing, was justified with an efficiency argument. Since the cost of scanning grows linearly with the number of characters, and the constant costs are low, pushing lexical analysis from the parser into a separate scanner lowered the cost of compiling. The advent of efficient parsing techniques weakened this argument, but the practice of building scanners persists because it provides a clean separation of concerns between lexical structure and syntactic structure.

Because scanner construction plays a small role in building an actual compiler, we have tried to keep this chapter brief. Thus, the chapter omits many theorems on regular languages and finite automata that the ambitious reader might enjoy. The many good texts on this subject can provide a much deeper treatment of finite automata and regular expressions, and their many useful properties [194, 232, 315].

Kleene [224] established the equivalence of REs and FAs. Both the Kleene closure and the DFA to RE algorithm bear his name. McNaughton and Yamada showed one construction that relates REs to NFAs [262]. The construction shown in this chapter is patterned after Thompson's work [333], which was motivated by the implementation of a textual search command for an early text editor. Johnson describes the first application of this technology to automate scanner construction [207]. The subset construction derives from Rabin and Scott [292]. The DFA minimization algorithm in Section 2.4.4 is due to Hopcroft [193]. It has found application to many different problems, including detecting when two program variables always have the same value [22].

The idea of generating code rather than tables, to produce a direct-coded scanner, appears to originate in work by Waite [340] and Heuring [189]. They report a factor of five improvement over table-driven implementations. Ngassam et al. describe experiments that characterize the speedups possible in hand-coded scanners [274]. Several authors have examined tradeoffs in scanner implementation. Jones [208] advocates direct coding but argues for a structured approach to control flow rather than the spaghetti code shown in Section 2.5.2. Brouwer et al. compare the speed of 12 different scanner implementations; they discovered a factor of 70 difference between the fastest and slowest implementations [59].

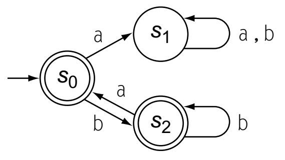
The alternative DFA minimization technique presented in Section 2.6.2 was described by Brzozowski in 1962 [60]. Several authors have compared DFA minimization techniques and their performance [328, 344]. Many authors have looked at the construction and minimization of acyclic DFAs [112, 343, 345].

■ EXERCISES

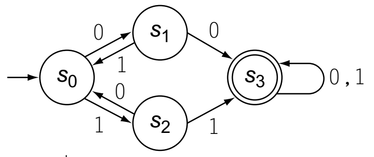
Section 2.2

1. Describe informally the languages accepted by the following FAS:

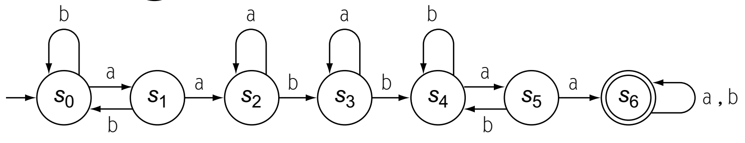
a.



b.



c.



2. Construct an FA accepting each of the following languages:

a. {$w \in \{a$, b}* | w starts with 'a' and contains '$baba$' as a substring}

b. {$w \in$ {0, 1}* | w contains '111' as a substring and does not contain '00' as a substring}

c. {$w \in \{a$, b, c}* | in w the number of 'a's modulo 2 is equal to the number of 'b's modulo 3}

3. Create FAS to recognize (a) words that represent complex numbers and (b) words that represent decimal numbers written in scientific notation.

Section 2.3

4. Different programming languages use different notations to represent integers. Construct a regular expression for each one of the following:

a. Nonnegative integers in C represented in bases 10 and 16.

b. Nonnegative integers in VHDL that may include underscores (an underscore cannot occur as the first or last character).

c. Currency, in dollars, represented as a positive decimal number rounded to the nearest one-hundredth. Such numbers begin with the character $, have commas separating each group of three digits to the left of the decimal point, and end with two digits to the right of the decimal point, for example, $8,937.43 and $7,777,777.77.

Hint
Not all the specifications describe regular languages.

5. Write a regular expression for each of the following languages:

a. Given an alphabet $\Sigma = \{0, 1\}$, L is the set of all strings of alternating pairs of 0s and pairs of 1s.

b. Given an alphabet $\Sigma = \{0, 1\}$, L is the set of all strings of 0s and 1s that contain an even number of 0s or an even number of 1s.

c. Given the lowercase English alphabet, L is the set of all strings in which the letters appear in ascending lexicographical order.

d. Given an alphabet $\Sigma = \{a, \text{b}, \text{c}, \text{d}\}$, L is the set of strings $xyzwy$, where x and w are strings of one or more characters in Σ, y is any single character in Σ, and z is the character z, taken from outside the alphabet. (Each string xyzwy contains two words xy and wy built from letters in Σ. The words end in the same letter, y. They are separated by z.)

e. Given an alphabet $\Sigma = \{+, -, \times, \div, (,), \texttt{id}\}$, L is the set of algebraic expressions using addition, subtraction, multiplication, division, and parentheses over `id`s.

6. Write a regular expression to describe each of the following programming language constructs:

a. Any sequence of tabs and blanks (sometimes called *white space*)

b. Comments in the programming language C

c. String constants (without escape characters)

d. Floating-point numbers

7. Consider the three regular expressions: Section 2.4

$$(ab \mid ac)^*$$

$$(0 \mid 1)^*\ 1100\ \ 1^*$$

$$(01 \mid 10 \mid 00)^*\ \ 11$$

a. Use Thompson's construction to construct an NFA for each RE.

b. Convert the NFAS to DFAS.

c. Minimize the DFAS.

8. One way of proving that two REs are equivalent is to construct their minimized DFAS and then compare them. If they differ only by state names, then the REs are equivalent. Use this technique to check the following pairs of REs and state whether or not they are equivalent.

a. $(0 \mid 1)^*$ and $(0^* \mid 10^*)^*$

b. $(ba)^+\ (a^*\ b^* \mid a^*)$ and $(ba)^*\ ba^+\ (b^* \mid \epsilon)$

9. In some cases, two states connected by an ϵ-move can be combined.

a. Under what set of conditions can two states connected by an ϵ-move be combined?

b. Give an algorithm for eliminating ϵ-moves.

c. How does your algorithm relate to the ϵ-closure function used to implement the subset construction?

10. Show that the set of regular languages is closed under intersection.

11. The DFA minimization algorithm given in Figure 2.9 is formulated to enumerate all the elements of *P* and all of the characters in Σ on each iteration of the while loop.

a. Recast the algorithm so that it uses a worklist to hold the sets that must still be examined.

b. Recast the *Split* function so that it partitions the set around all of the characters in Σ.

c. How does the expected case complexity of your modified algorithms compare to the expected case complexity of the original algorithm?

Section 2.5

12. Construct a DFA for each of the following C language constructs, and then build the corresponding table for a table-driven implementation for each of them:

a. Integer constants

b. Identifiers

c. Comments

13. For each of the DFAS in the previous exercise, build a direct-coded scanner.

14. This chapter describes several styles of DFA implementations. Another alternative would use mutually recursive functions to implement a scanner. Discuss the advantages and disadvantages of such an implementation.

15. To reduce the size of the transition table, the scanner generator can use a character classification scheme. Generating the classifier table, however, seems expensive. The obvious algorithm would require $\mathbf{O}(|\Sigma|^2 \cdot |states|)$ time. Derive an asymptotically faster algorithm for finding identical columns in the transition table.

16. Figure 2.15 shows a scheme that avoids quadratic roll back behavior in a scanner built by simulating a DFA. Unfortunately, that scheme requires that the scanner know in advance the length of the input stream and that it maintain a bit-matrix, *Failed*, of size $|states| \times |input|$. Devise a scheme that avoids the need to know the size of the input stream in advance. Can you use the same scheme to reduce the size of the *Failed* table in cases where the worst case input does not occur?