Department of Electronic & Telecommunication Engineering

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Sri Lanka

EN4152 Assignment 2
Optimum and Adaptive Filters

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Contents

Introduction

This assignment focuses on implementing and analyzing Wiener filters and adaptive filters (LMS and RLS) for biosignal denoising. Using both ideal ECG data and synthetic signals, the tasks involve designing filters in the time and frequency domains, exploring the effect of stationary and non-stationary noise, and comparing adaptive filtering algorithms. The results are presented with supporting plots, optimum parameter values, and detailed observations.

1 Wiener Filtering

For a given filter order M, the optimum weight vector is given by:

$$\mathbf{w}_0 = (\Phi_Y + \Phi_N)^{-1}\,\Theta_{Yy} \tag{1}$$

where:

- Φ_Y is the autocorrelation matrix of the clean signal $y_i(n)$,
- Φ_N is the autocorrelation matrix of the noise $\eta(n)$,
- Θ_{Yy} is the cross-correlation vector between $y_i(n)$ and the noisy signal $x(n)$.

Data construction

Figure 1 shows the ideal ECG, noise components, and noisy input.

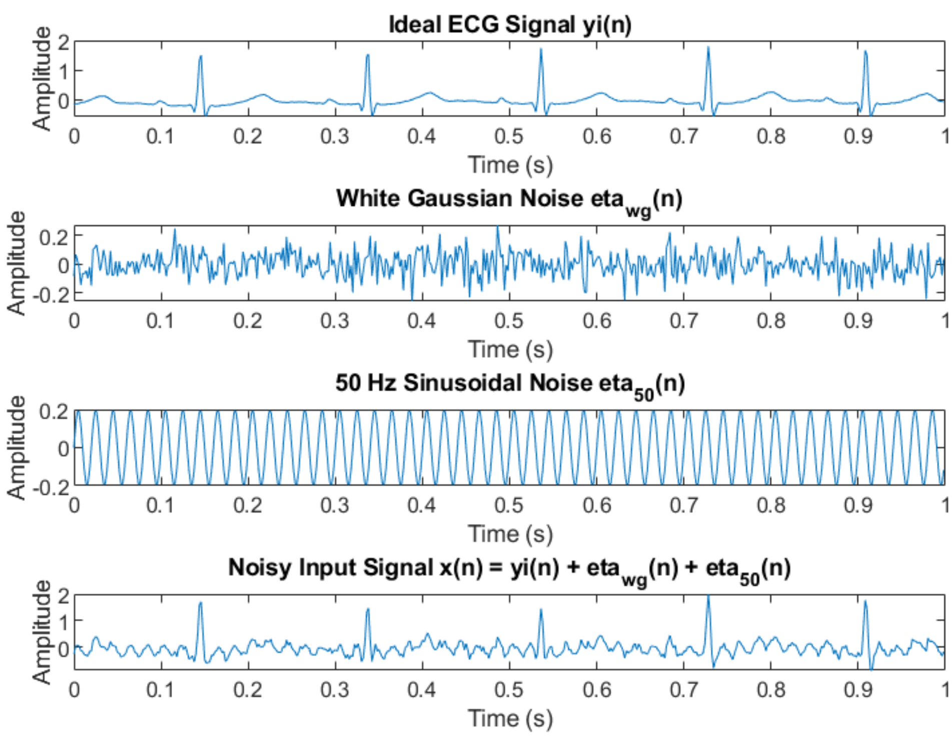

Figure 1: Ideal ECG, noise components, and noisy signal.

1.1 Discrete Time-Domain Implementation of the Wiener Filter

Part 1: Extracted ECG Beat

Figure 2 presents the extracted ECG beat segment used as the desired signal. Figure 3 shows the extracted isoelectric segment used as the noise reference.

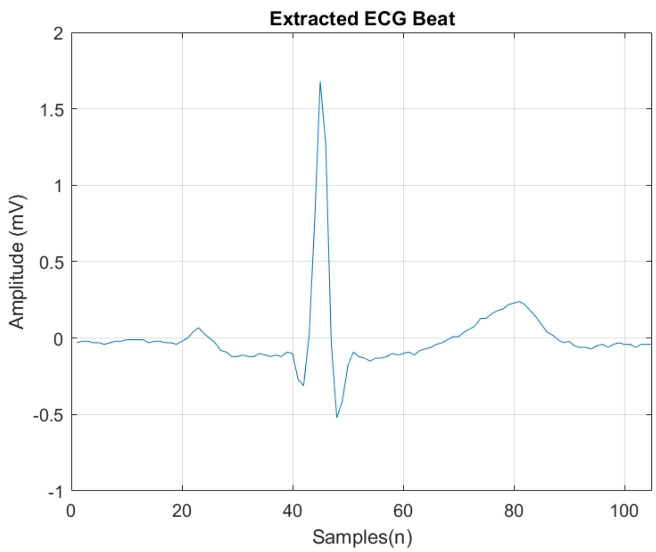

Figure 2: Extracted ECG beat (desired signal).

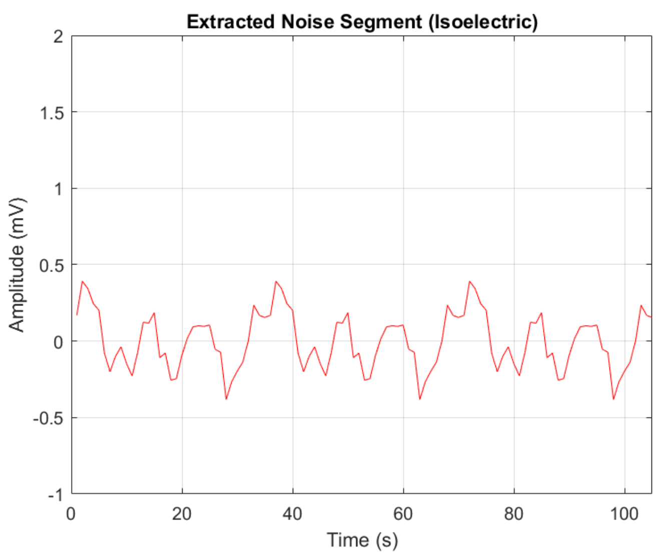

Figure 3: Extracted noise segment (isoelectric).

(a) The optimum weight vector $w_0 = (\Phi_Y + \Phi_N)^{-1}\Theta_{Yy}$ was calculated for filter order= 20.

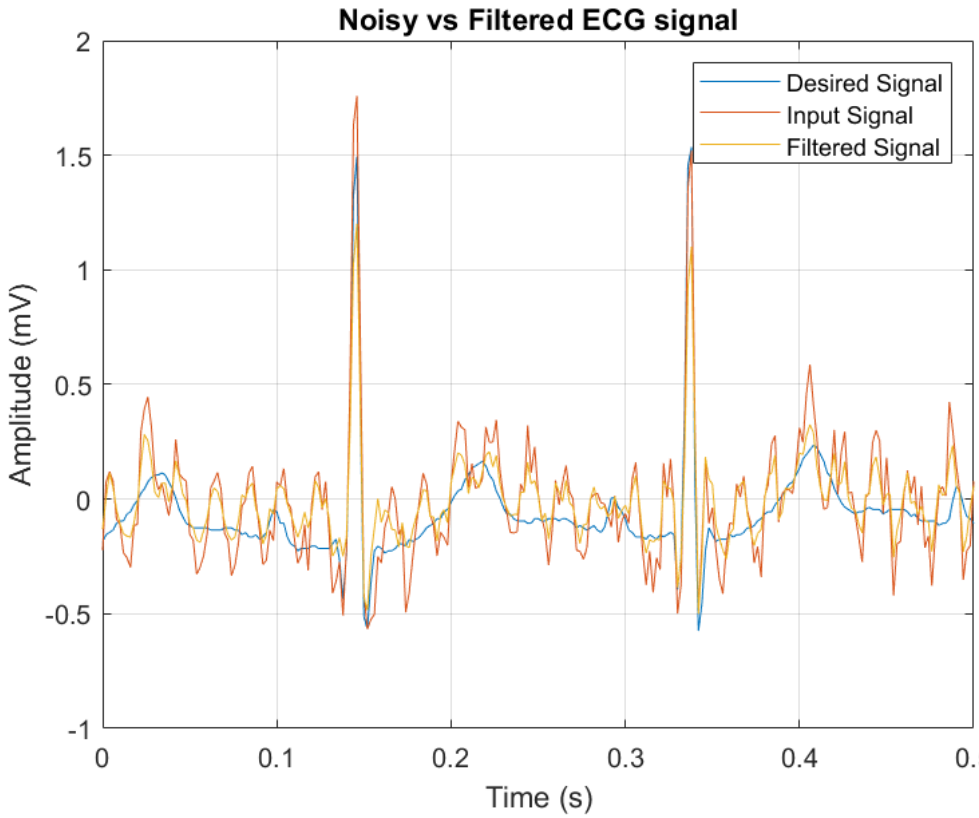

Figure 4: Noisy input vs filtered ECG

(b) The filter order was varied, and the Mean Squared Error (MSE) computed. The optimum order M^* is selected based on minimizing the mean square error (MSE):

$$\text{MSE}(M) = E\big[(y_i(n) - \hat{y}(n))^2\big] \tag{2}$$

The optimum filter order was found to be **21**, giving a minimum MSE of 0.014673 (Figure 5).

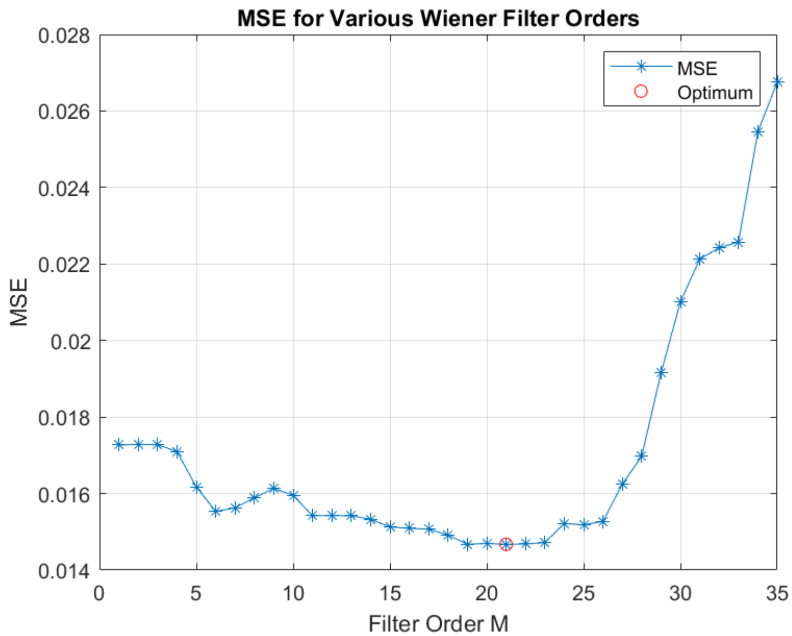

Figure 5: MSE vs filter order (optimum at 21).

The magnitude response (Figure 6) shows band-limiting characteristics consistent with ECG signal content.

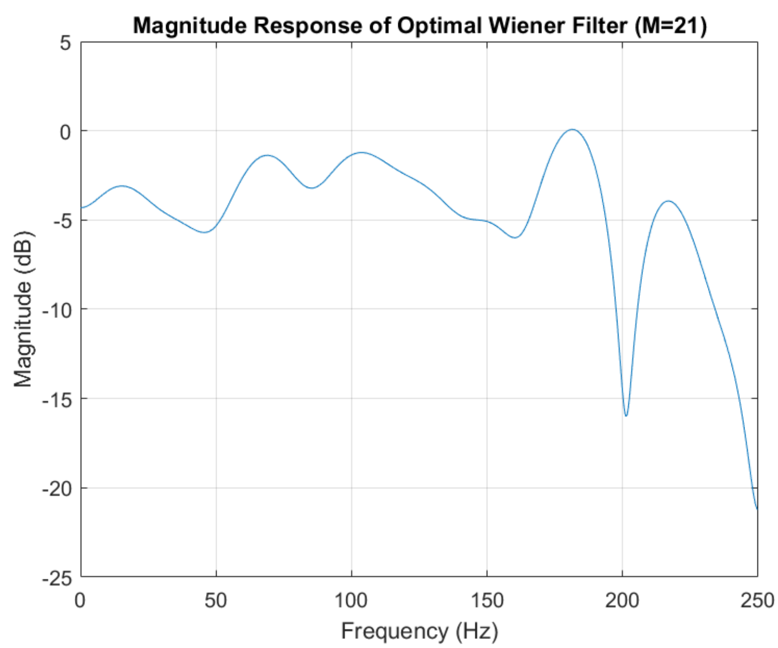

Figure 6: Magnitude response of optimal Wiener filter ($M = 21$).

(c) The filtered signal $\hat{y}(n)$ obtained with this optimal filter is shown in Figure 7, which demonstrates a small noise reduction compared to the input.

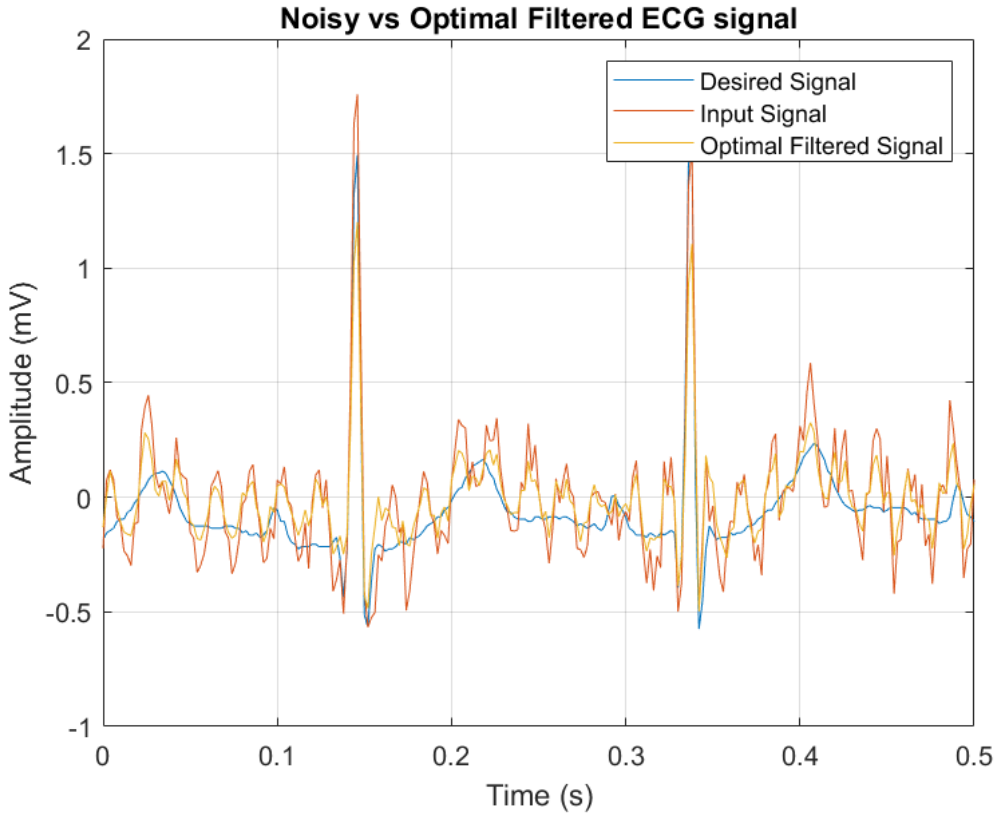

Figure 7: Noisy input vs optimal filtered ECG.

(d) The spectra of $y_i(n)$, $\eta(n)$, $x(n)$, and $\hat{y}(n)$ are plotted in Figure 8.

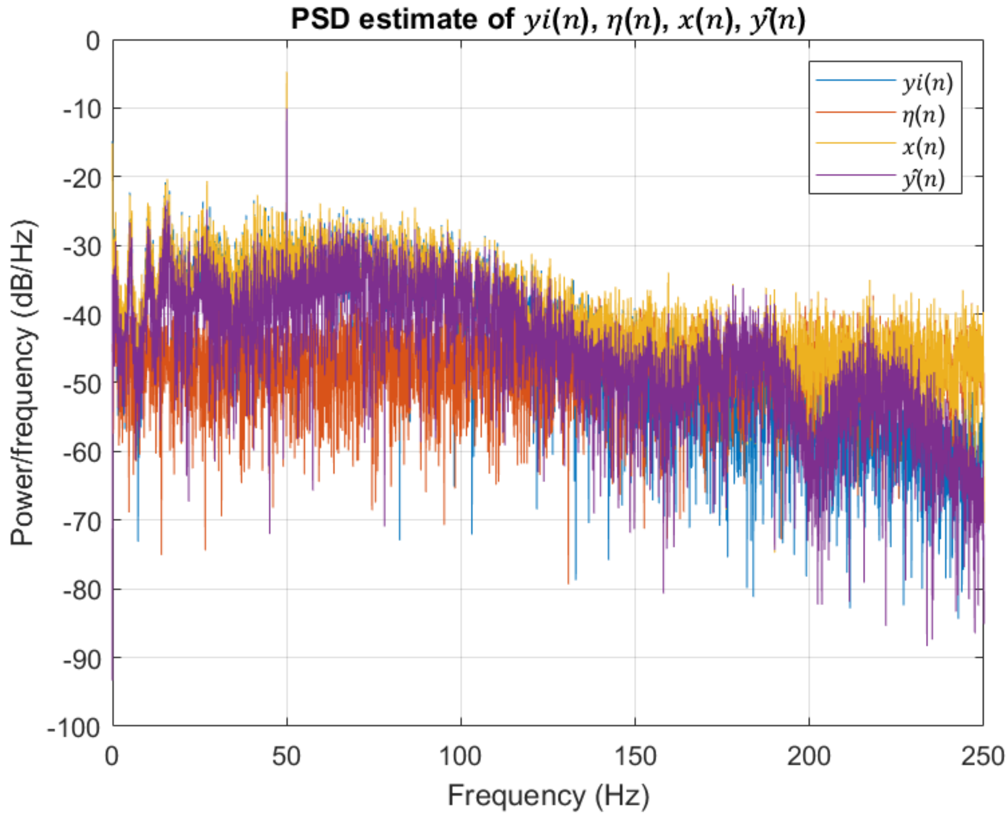

Figure 8: PSD of $y_i(n)$, $\eta(n)$, $x(n)$, and $\hat{y}(n)$.

(e) *Interpretation:* From the magnitude response plots and the power spectral density (PSD) plots, we can see noticeable attenuation around 50 Hz and 200 Hz frequencies. The peaks of the PSD plots are between 10 Hz and 100 Hz, which show the bandwidth of the ECG signal. The PSD of the filtered signal is approximately equal to the PSD of the template ECG signal. The only clear difference is observed at the 250 Hz frequency range.

Part 2: Linear ECG Model

A linear ECG model was constructed to mimic ECG morphology (Figure 9).

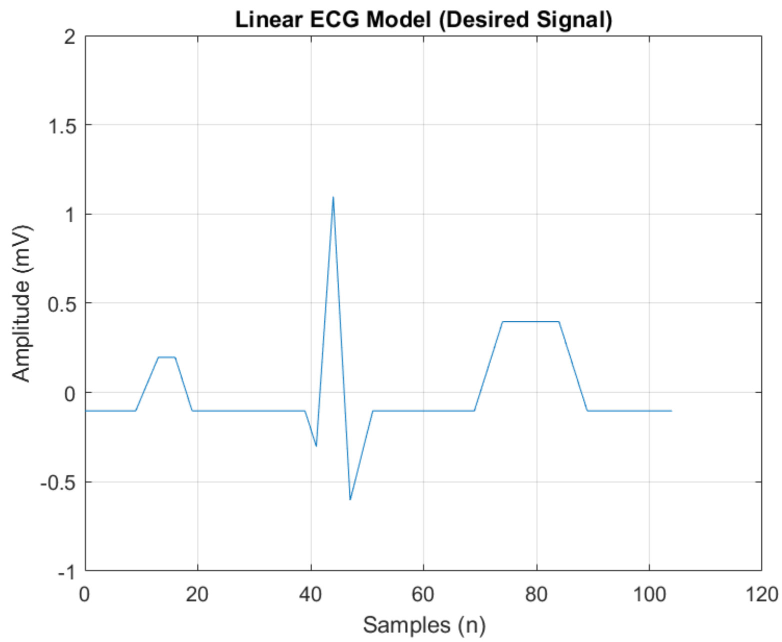

Figure 9: Linear ECG model (desired signal).

(a) Similarly, the weight vector is computed as:

$$\mathbf{w}_0 = \mathbf{R}_{xx}^{-1}\mathbf{r}_{xy} \tag{3}$$

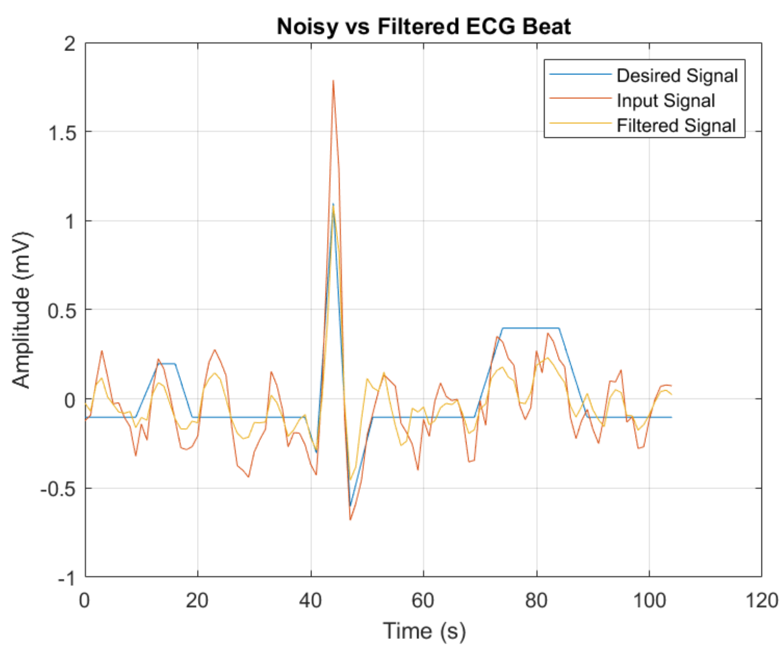

Figure 10: Filtered signal vs Noisy signal

(b) By sweeping the filter order, the optimum order was found to be **19**, with minimum MSE of 0.023813 (Figure 11).

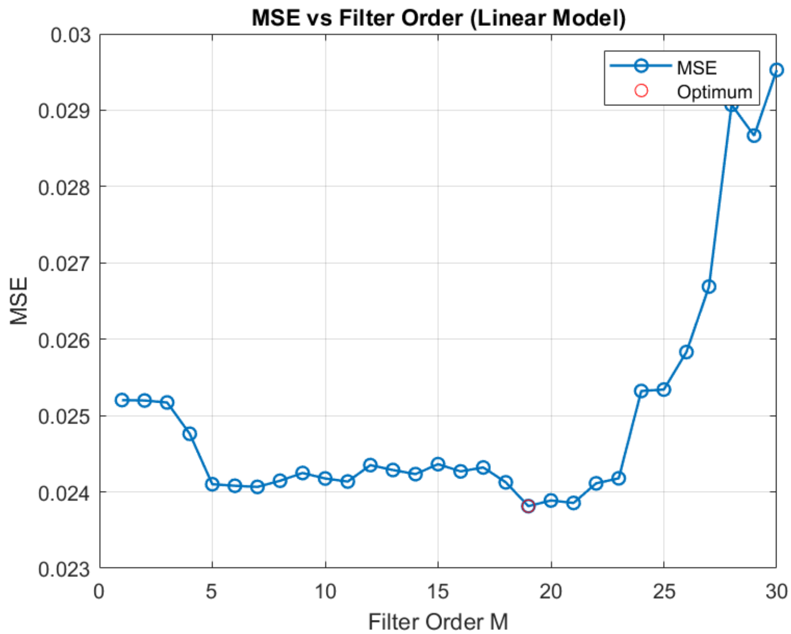

Figure 11: MSE vs filter order (linear model).

The filter's magnitude response is shown in Figure 13.

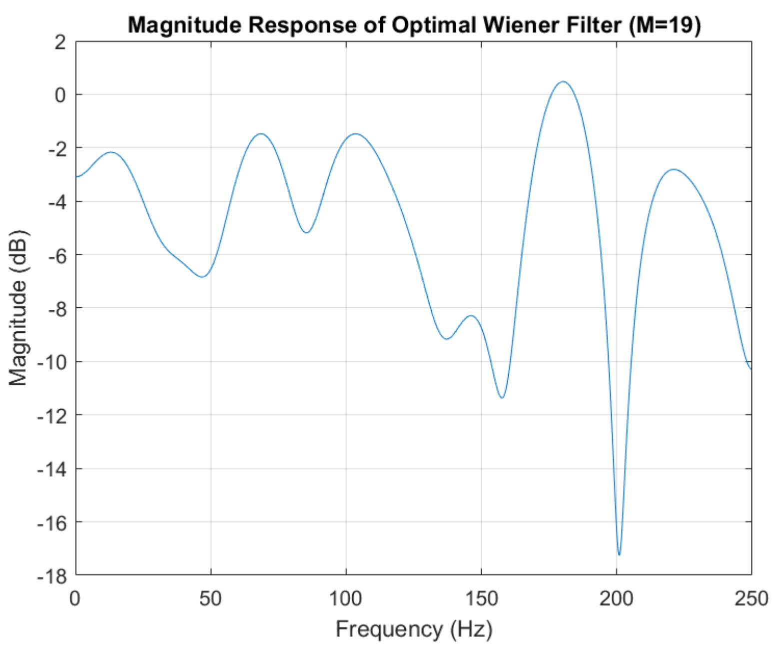

Figure 12: Magnitude response of optimal filter ($M = 19$).

(c) The optimal filtered signal is shown in Figure 12.

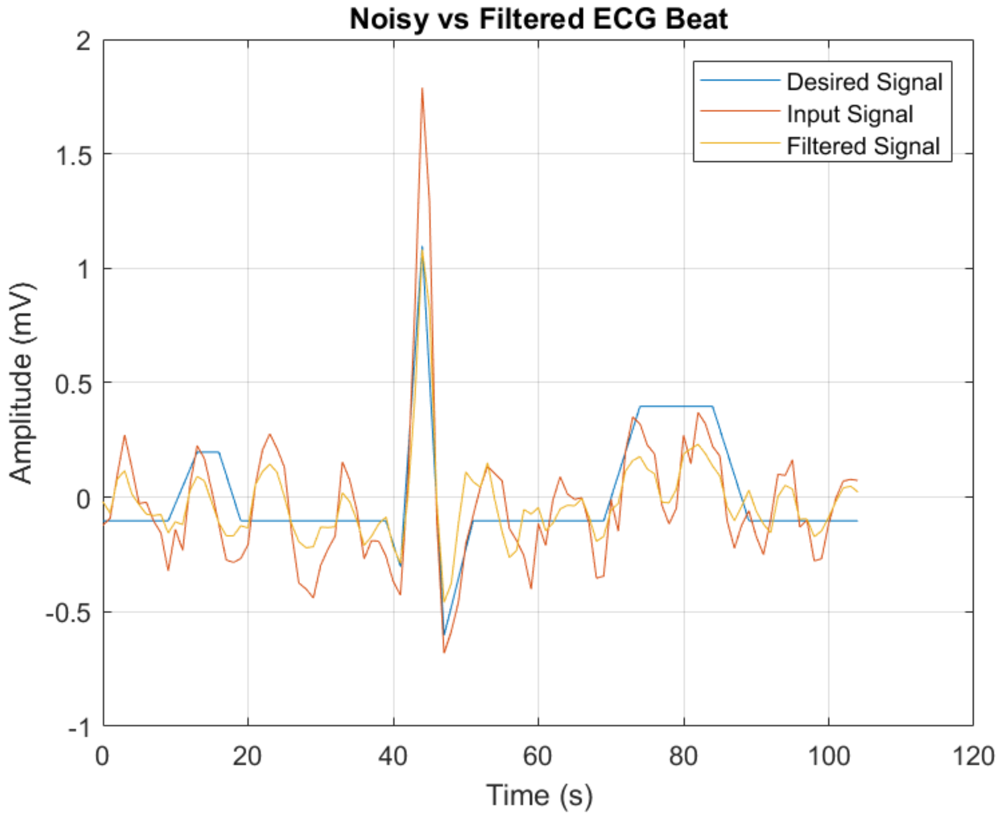

Figure 13: Filtered ECG beat using optimal filter ($M = 19$).

(d) The spectra of signals are compared in Figure 14.

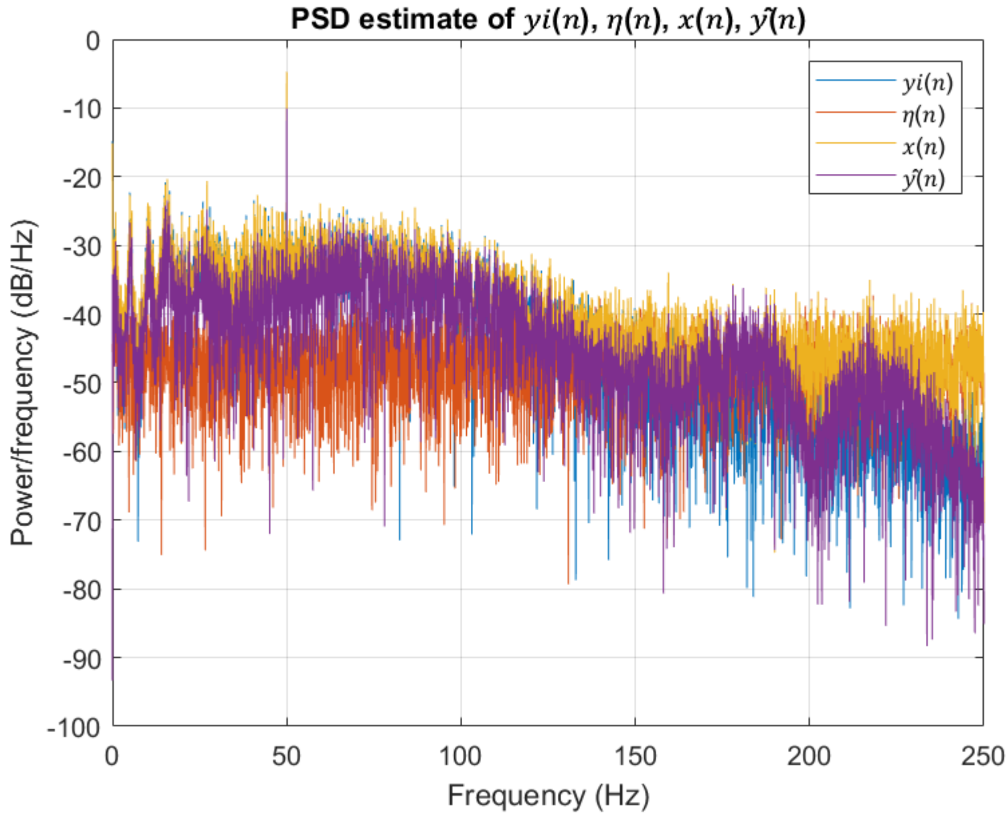

Figure 14: PSD of $y_i(n)$, $\eta(n)$, $x(n)$, and $\hat{y}(n)$ for the linear model case.

(e) *Interpretation:*The magnitude response and power spectral density (PSD) plots show the same type of observations. Even though we use a modeled ECG here, the plots appear to be similar.

1.2 Frequency-Domain Implementation

The frequency-domain Wiener filter was implemented using PSD estimates. The frequency-domain Wiener filter is defined as:

$$W(f) = \frac{S_{YY}(f)}{S_{YY}(f) + S_{NN}(f)} \tag{4}$$

where $S_{YY}(f)$ is the power spectral density (PSD) of the clean signal $y_i(n)$ and $S_{NN}(f)$ is the PSD of the noise $\eta(n)$. The PSD is estimated from the squared magnitude of the Fourier transform:

$$S_{ZZ}(f) = |\mathcal{F}\{z(n)\}|^2$$

(a) Part 1: The MSE achieved was **0.017335**, with results shown in Figure 15.

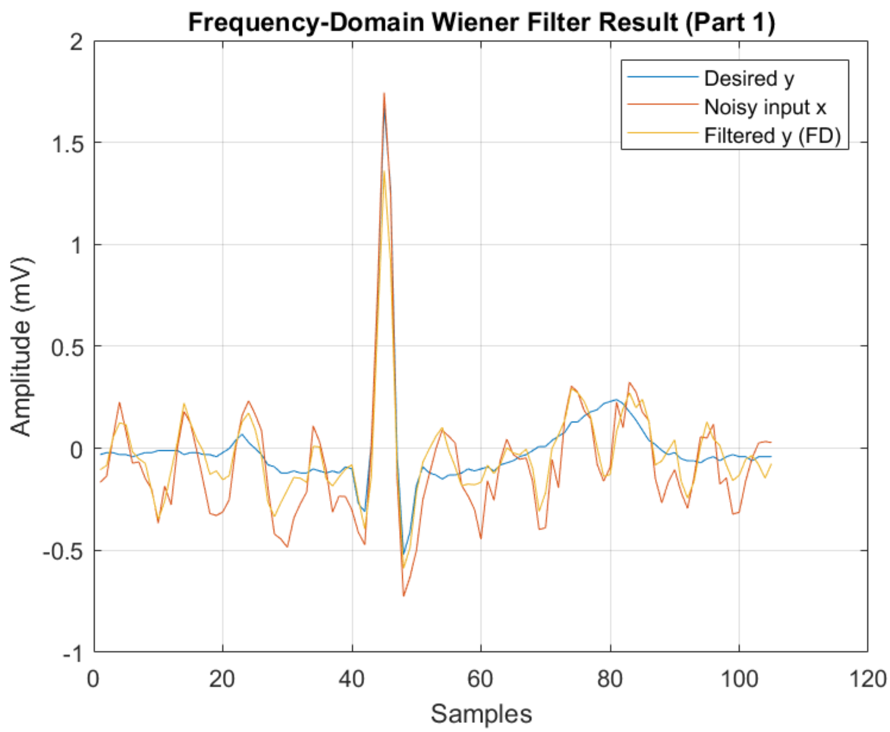

Figure 15: Frequency-domain Wiener filter (Part 1).

Part 2: The MSE was **0.025924**, shown in Figure 16.

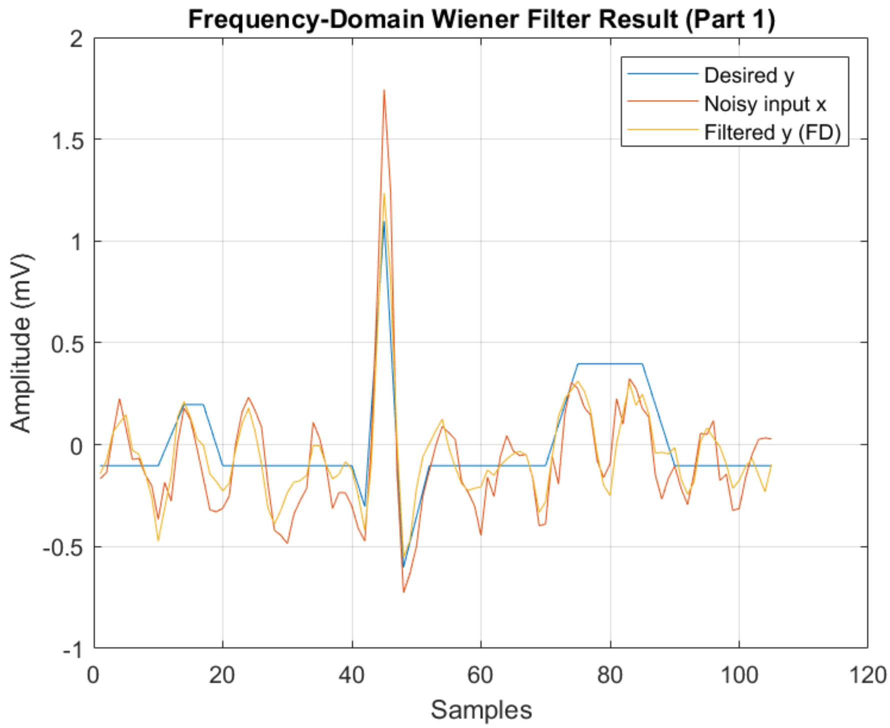

Figure 16: Frequency-domain Wiener filter (Part 2).

Observation: The MSE values of both the filtered signals are approximately equal. Linear model shows a slightly higher MSE. Both the filtered signals are almost identical. There is no noticeable difference seen in them. Using a modeled ECG segment instead of a segment of the template ECG did not make any improvement in the filtering.

1.3 Effect of Non-stationary Noise

When noise switched from 50 Hz to 100 Hz halfway through the signal, the same filter derived earlier was applied.

(a)MSE results: Overall = 0.024959, first half (50 Hz) = 0.020201, second half (100 Hz) = 0.029716.

(b)*Interpretation:* The Wiener filter, optimized for stationary noise, performs well in the first half but degrades when noise characteristics change, highlighting the need for adaptive filtering.

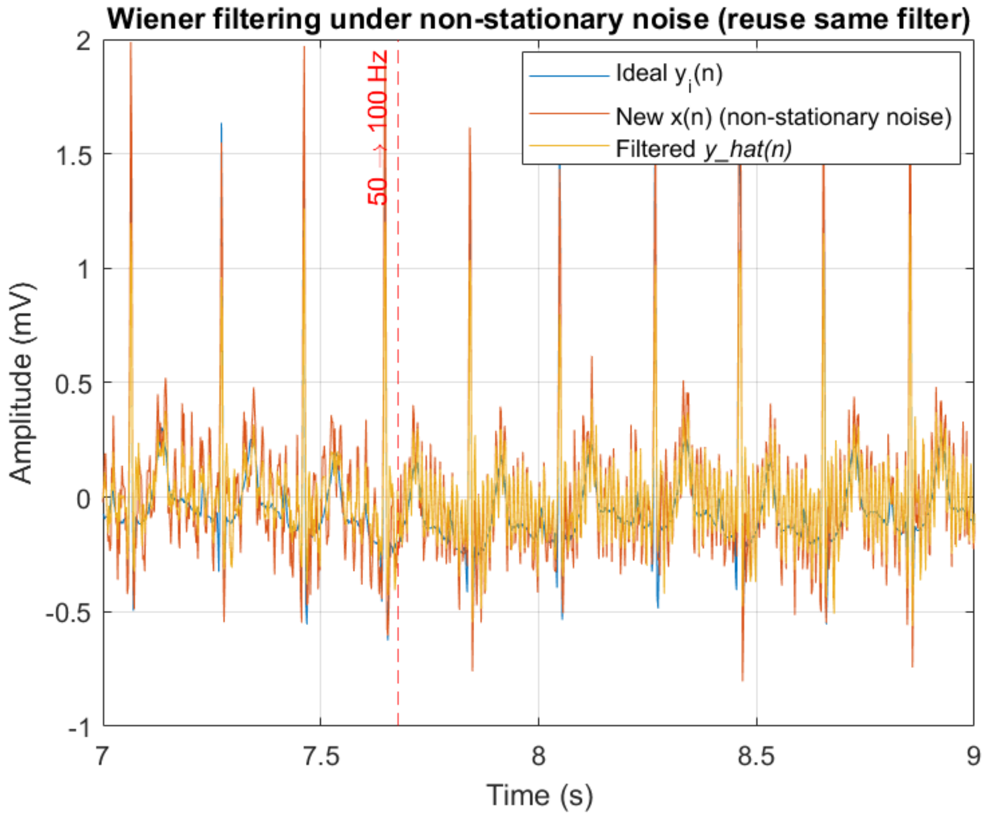

Figure 17: Filtering under non-stationary noise (50 Hz then 100 Hz).

2 Adaptive Filtering

Data Construction

In this section, adaptive filtering techniques are applied to remove non-stationary noise from the desired signal $y_i(n)$. The input signal is constructed as:

$$x(n) = y_i(n) + \eta(n) \tag{6}$$

where $y_i(n)$ is the sawtooth waveform of width 0.5, and $\eta(n)$ is non-stationary noise defined as:

$$\eta(n) = \begin{cases} \eta_{wg}(n) + \eta_{50}(n), & 0 \leq n < T/2 \\ \eta_{wg}(n) + \eta_{100}(n), & T/2 \leq n < T \end{cases}$$

The reference signal is defined as:

$$r(n) = a\Big(\eta_{wg}(n) + \sin(2\pi 50n + \phi_1) + \sin(2\pi 100n + \phi_2)\Big) \tag{7}$$

where a=0.6, ϕ_1=$\pi/4$, and ϕ_2=$\pi/6$ are constants.

2.1 LMS Method

(a) The LMS method was implemented as required.

(b) Plots of $y_i(n)$, $x(n)$, $e(n)$, and $|y_i(n) - e(n)|$ are shown in Figure 18.

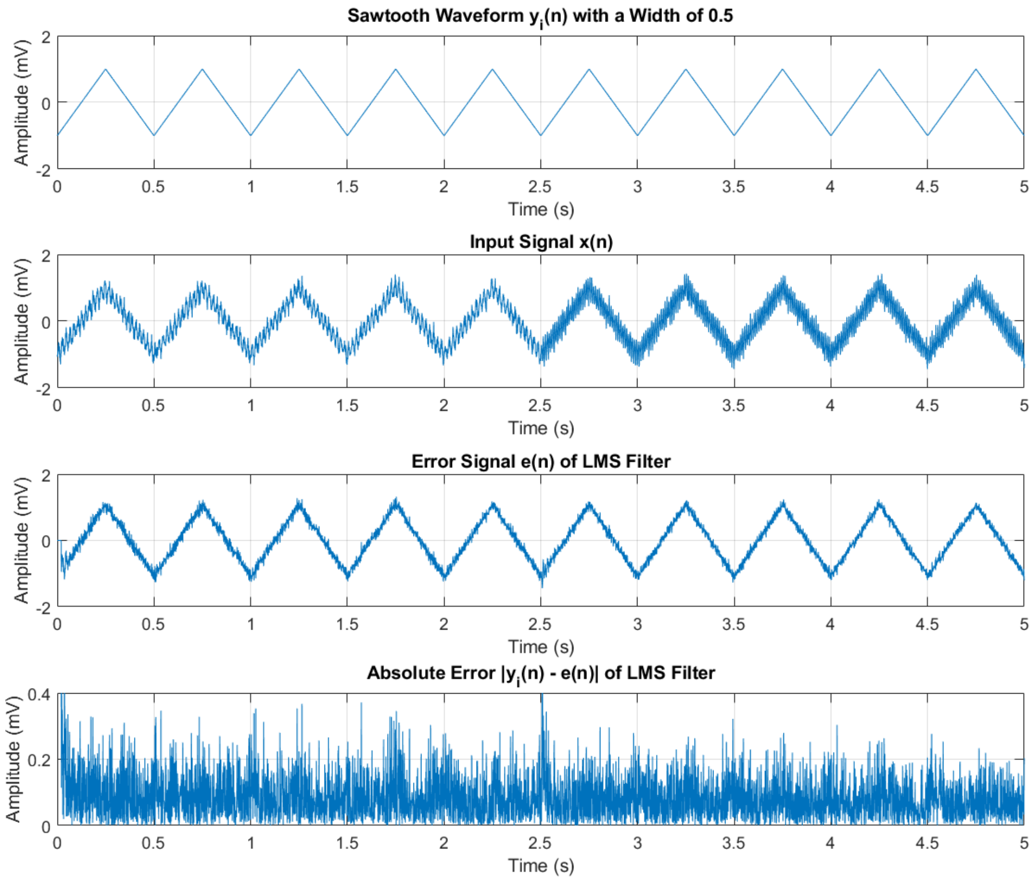

Figure 18: LMS results: signals and error plots.

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(c) By varying μ and filter order, the optimum was found at $M = 12$, $\mu = 0.009$, with MSE $= 0.012239$ (Figures 19, 20).

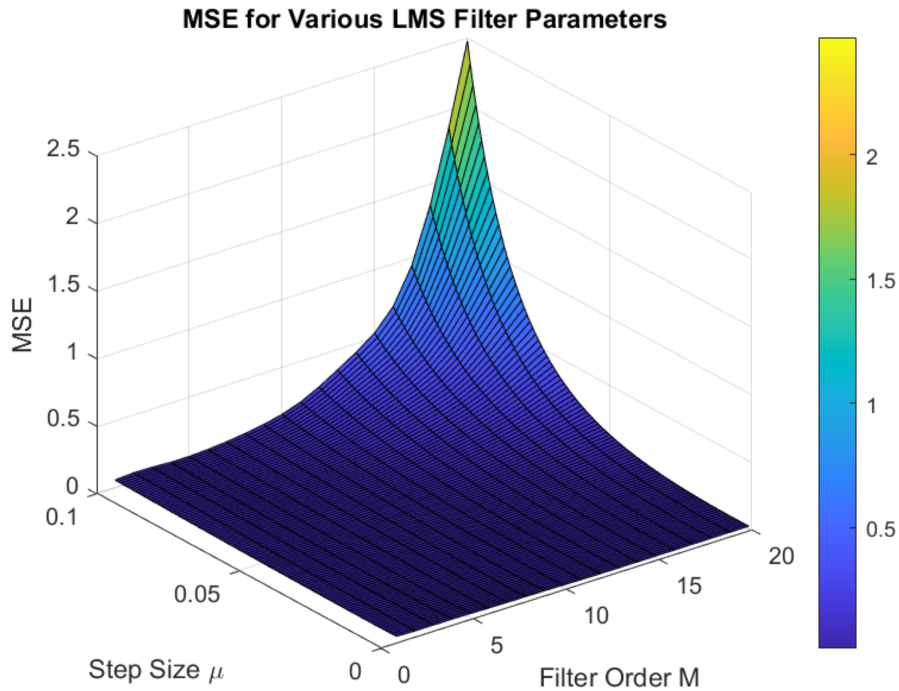

Figure 19: MSE surface for varying μ and filter order (LMS).

Observation: A small step size ensures stable convergence, while moderate filter order balances adaptation speed and accuracy.

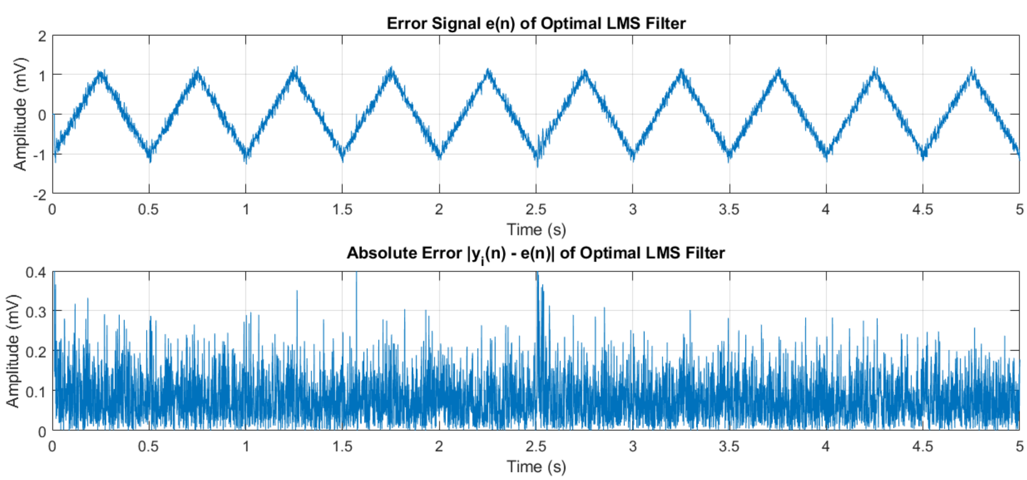

Figure 20: Error and absolute error with optimal LMS filter.

2.2 RLS Method

(a) The RLS algorithm was implemented.
(b) Figure 21 shows the error signal and absolute error for the RLS filter. Compared to LMS, the RLS method achieves faster convergence and lower steady-state error.

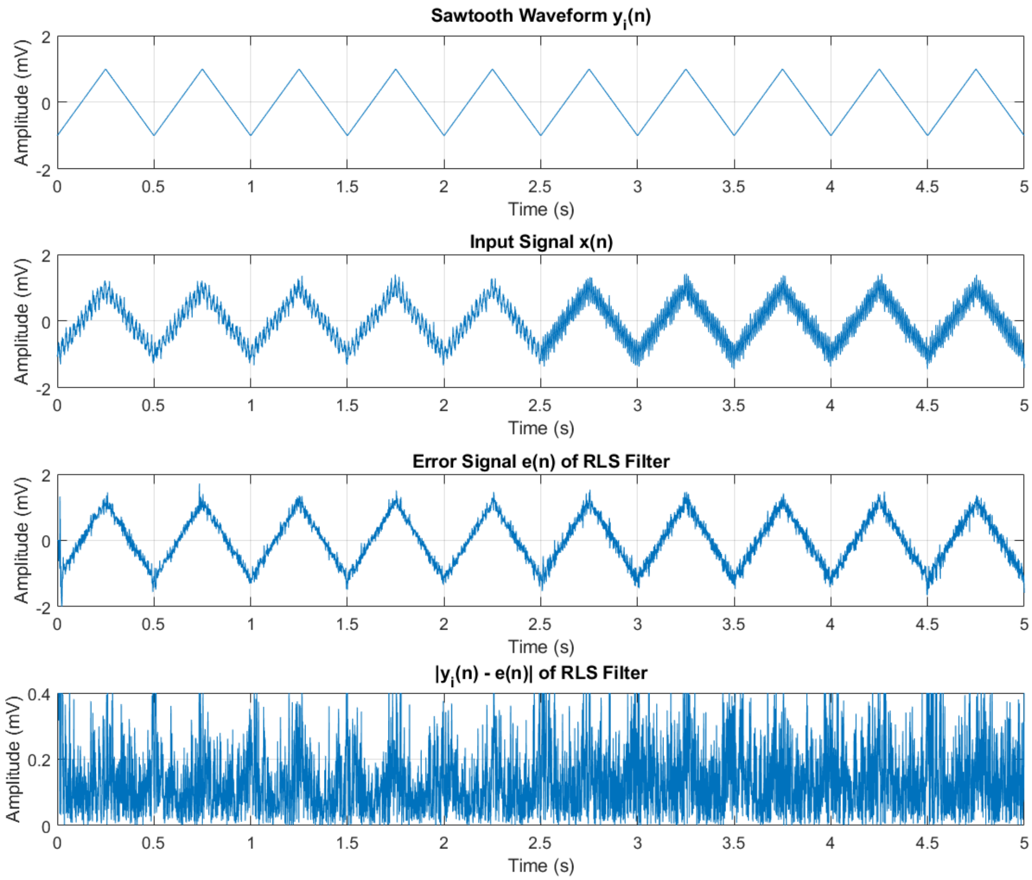

Figure 21: RLS results: signals and error plots.

(c) The forgetting factor λ and filter order $M-1$ significantly affect convergence. Figure **??** shows the variation of MSE across parameters. By sweeping parameters, optimum was found at $M = 11$, $\lambda = 0.995$, with MSE $= 0.012631$ (Figures 22, 23).

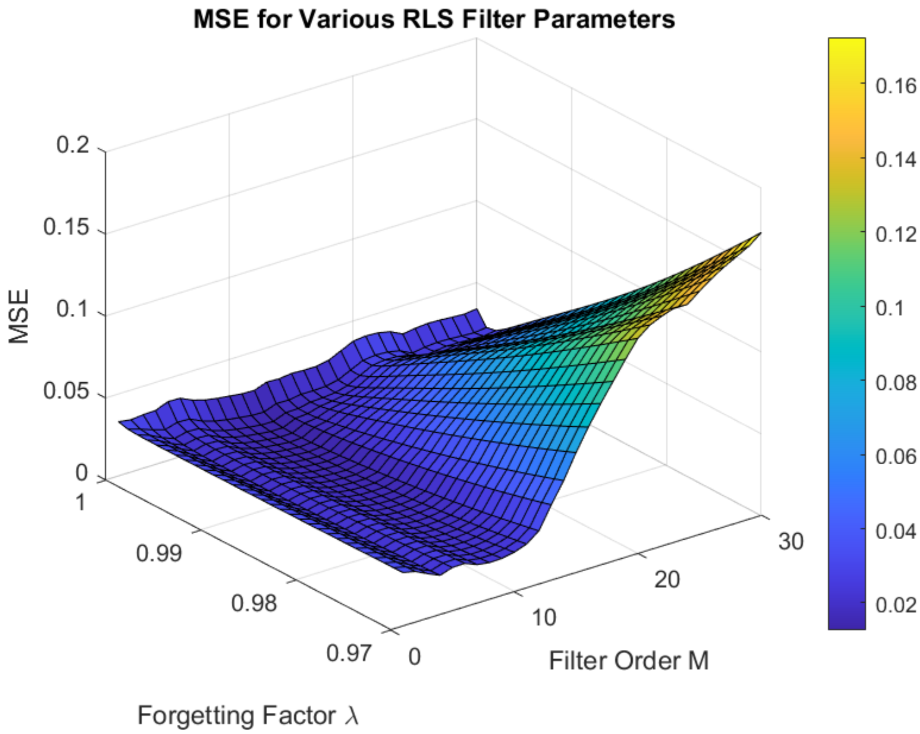

Figure 22: MSE surface for varying λ and filter order (RLS).

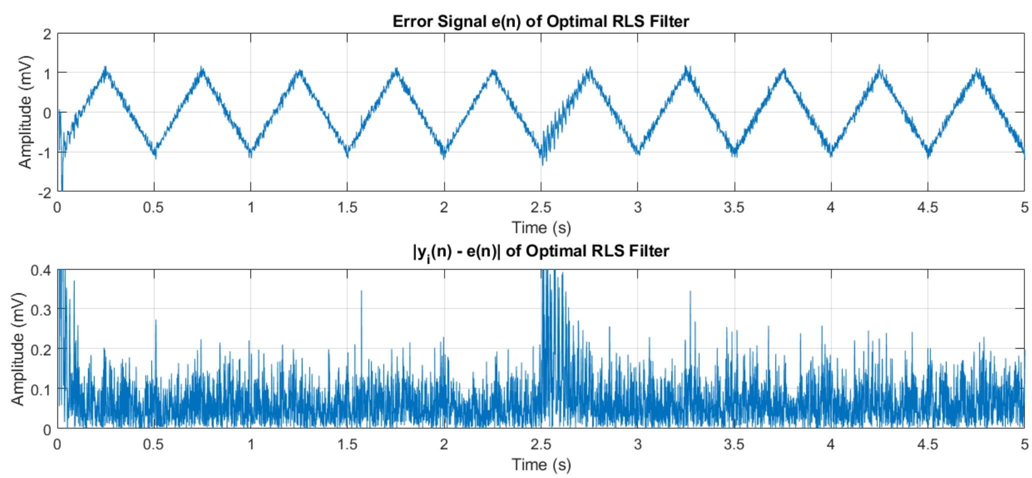

Figure 23: Error and absolute error with optimal RLS filter.

(d) LMS and RLS were applied to noisy ECG signals. Figures 24 and 25 show that both methods improve the ECG morphology, with RLS converging faster.

the LMS and RLS filters were tested using the `idealECG.mat` file as the desired signal $y_i(n)$. The noise $\eta(n)$ was kept the same as before. Figures show the filtering results.

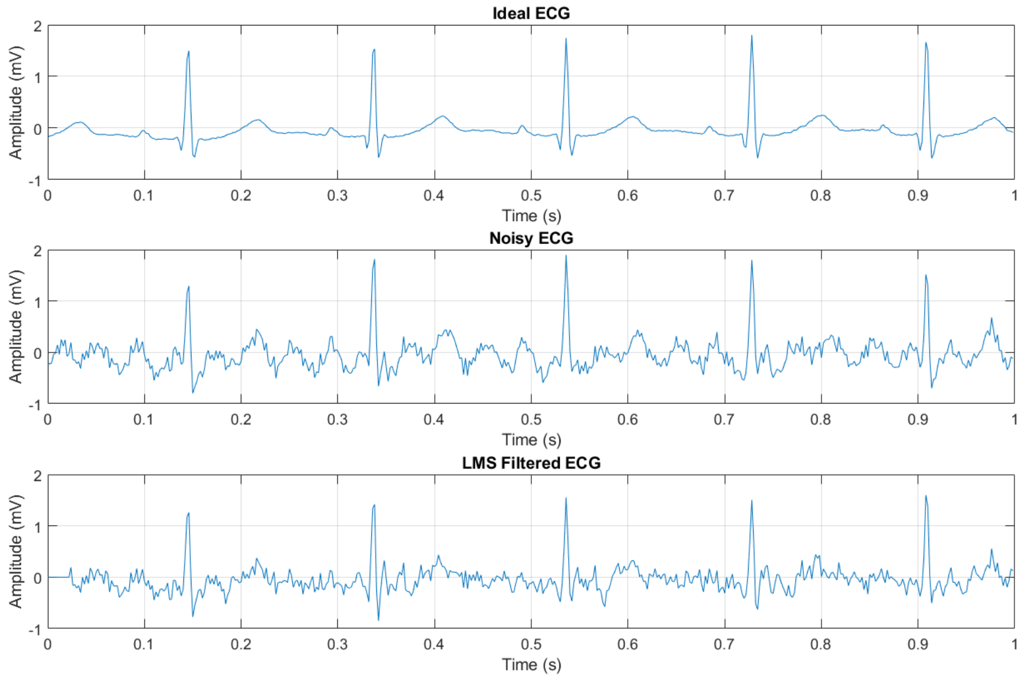

Figure 24: LMS applied to noisy ECG.

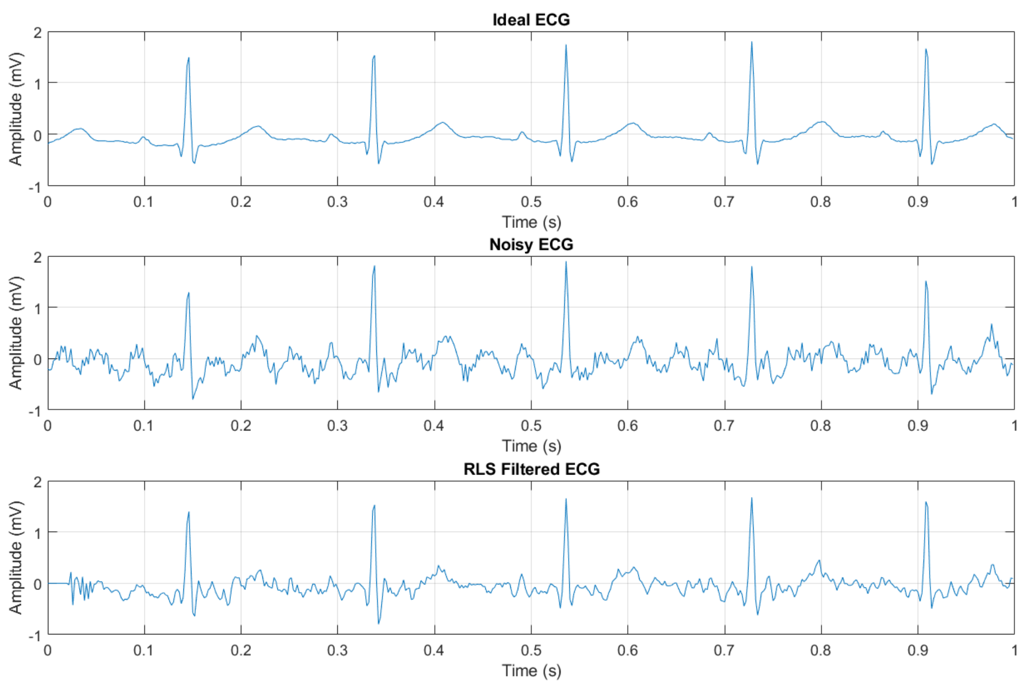

Figure 25: RLS applied to noisy ECG

The performance of both signals is approximately the same. The LMS filter provides reasonable noise suppression but converges slowly and is sensitive to the choice of μ. The RLS filter slightly outperforms LMS by achieving faster convergence and lower error, making it more suitable for tracking non-stationary noise in ECG signals.