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Properties of the Hodgkin-Huxley equations

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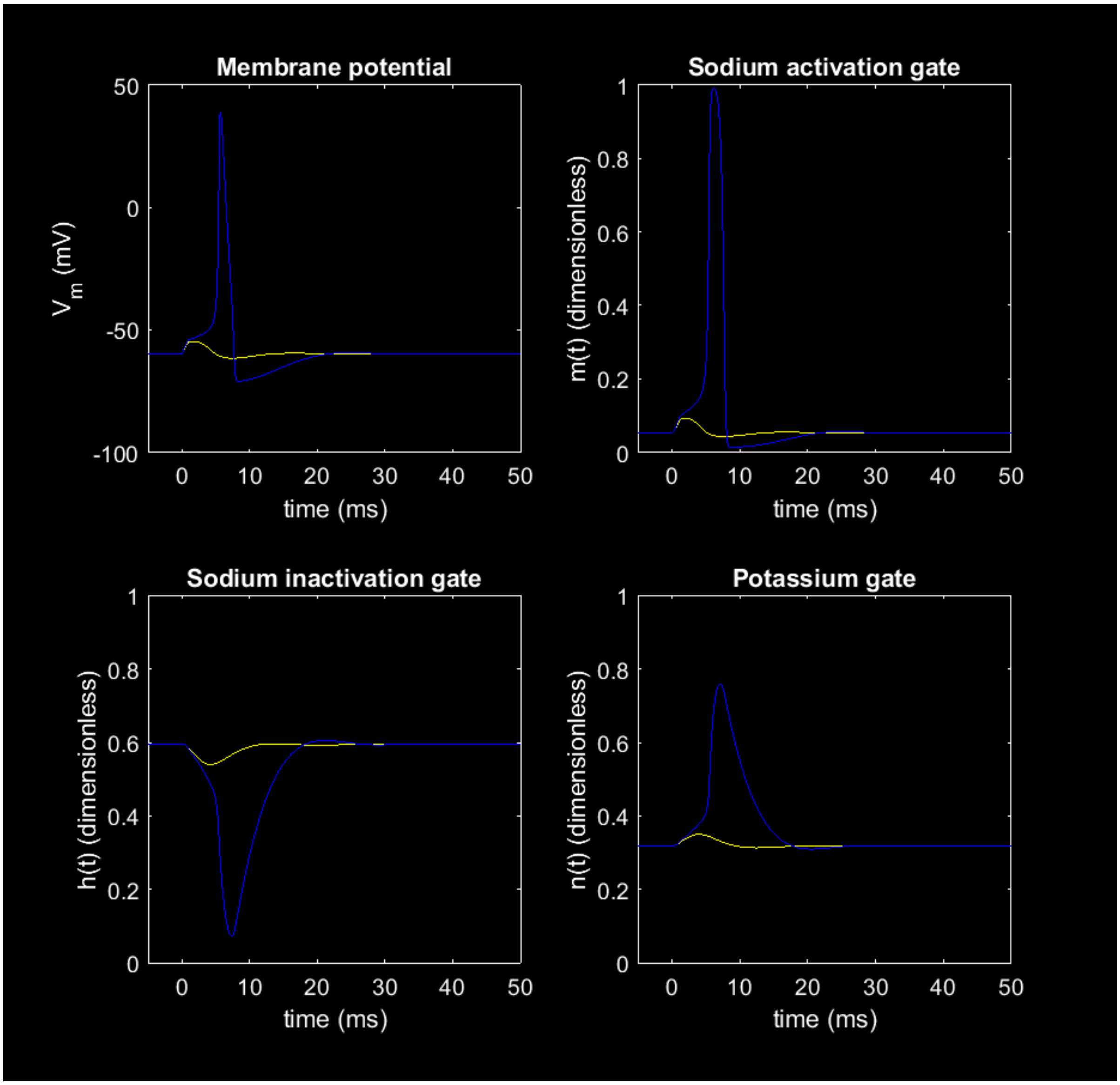
In [30]:
```
hhconst;
```

1. Threshold

In [31]:
```
amp1 = 6;
width1 = 1;
hhmplot(0,50,0);
amp1 = 7;
hhmplot(0,50,1);
```

Out[31]:



Question 01

In [32]:
```
amp1=6;
width1=1;
disp(amp1);
for n= 1:6
 amp1=(amp1+7)/2;
```

```
 disp(amp1);
end
```

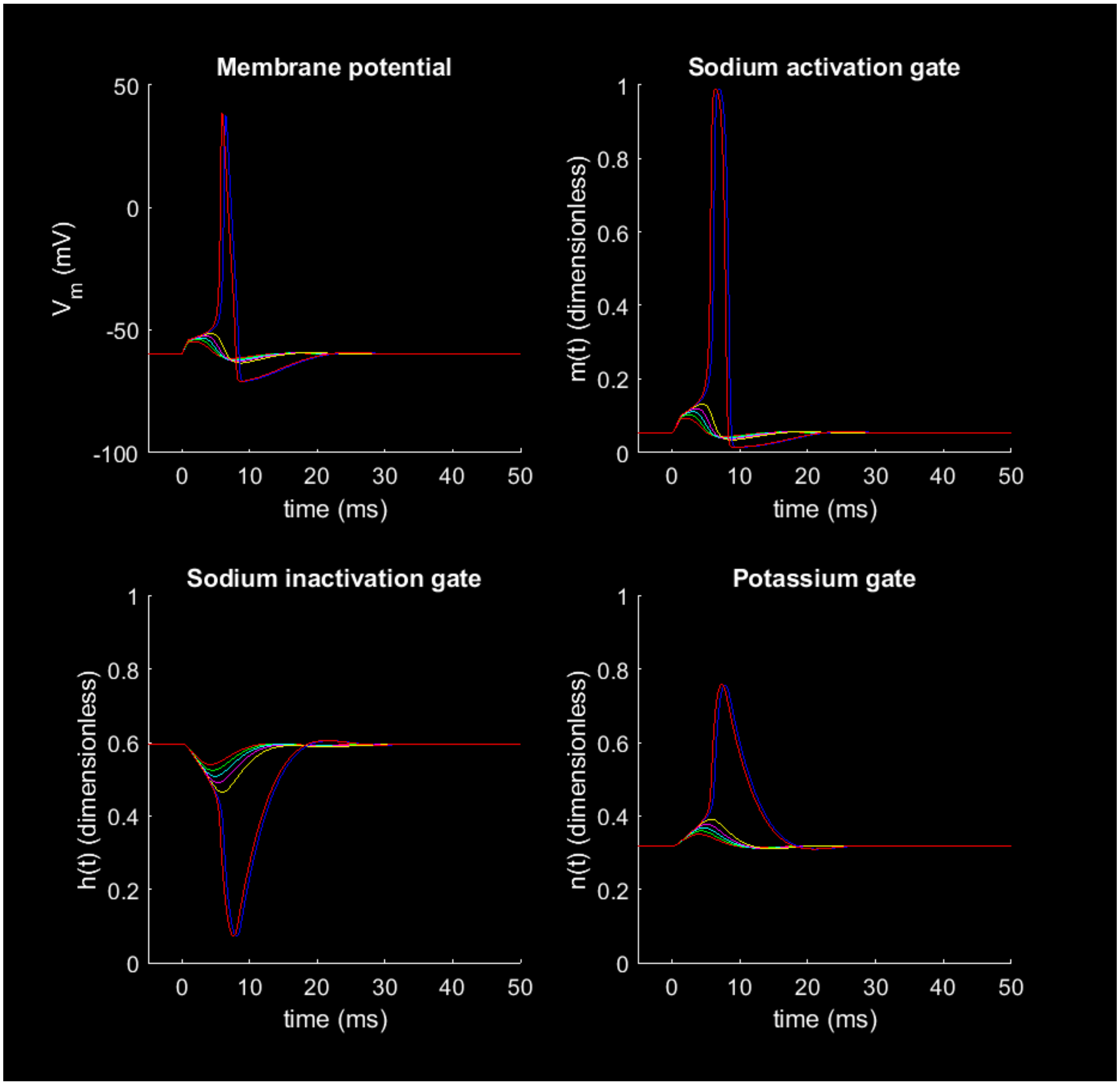
```
    6

   6.5000

   6.7500

   6.8750

   6.9375

   6.9688

   6.9844
```

Action potentials for the above values

In [33]:

```
amp1=6;
width1=1;
hhmplot(0,50,1);
for n= 1:6
 amp1=(amp1+7)/2;
 hhmplot(0,50,1);
end
```

Out[33]:

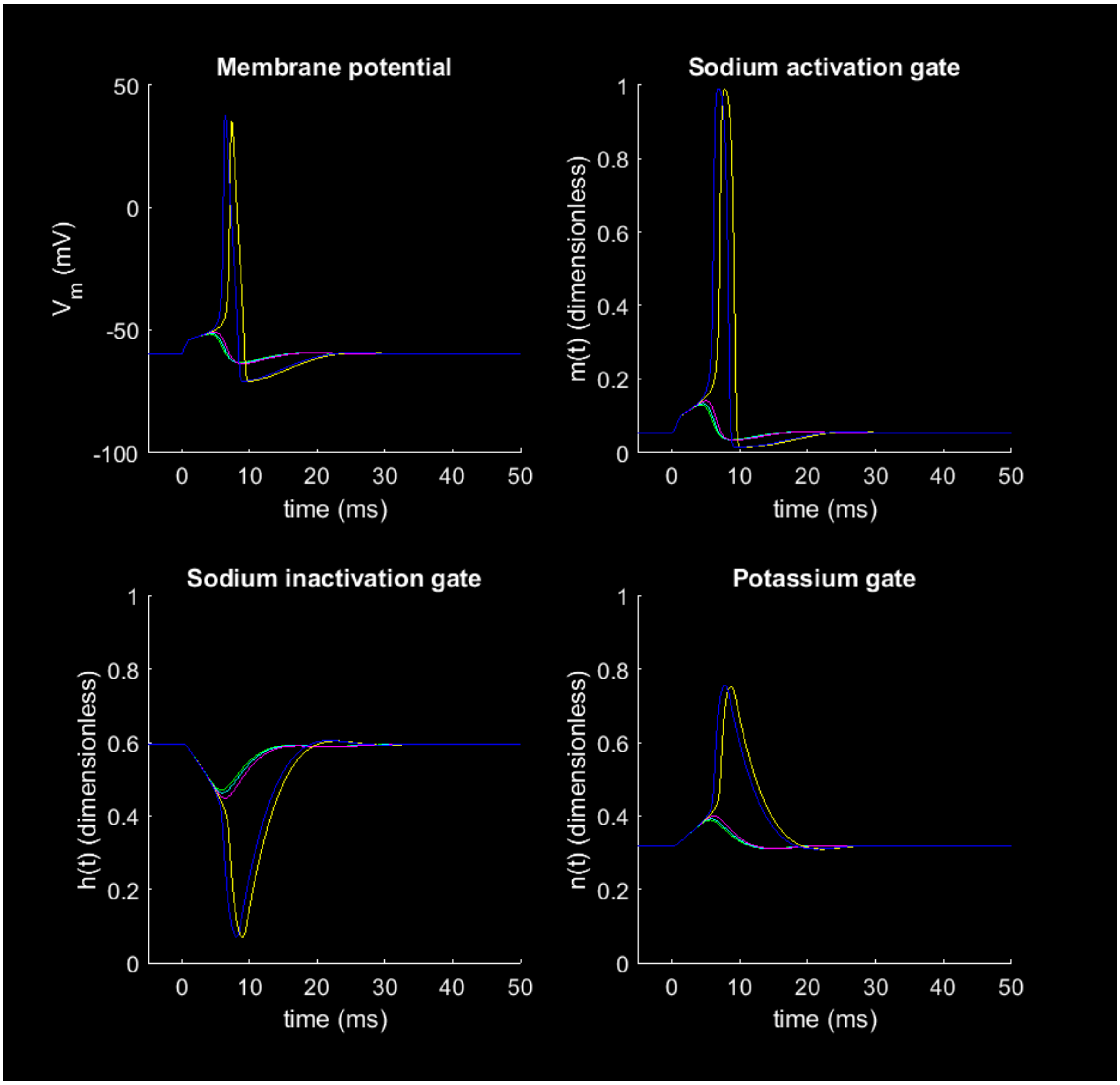


Based on the presented data in amp1, it is evident that all values up to 6.9375 do not surpass the threshold. Only the value 6.9688 surpasses the threshold. So, among the given values, 6.9688 is the amplitude that meets the threshold requirement. To precisely determine the closest threshold value to two decimal places, the following procedure is employed.

In [34]:

```
amp1=6.93;
width1=1;
for n= 1:5
 hhmplot(0,50,1);
 disp(amp1);
 amp1=(amp1+0.01);
end
```

Out[34]:




```
6.9300

6.9400

6.9500

6.9600

6.9700
```

Based on these plots, the threshold value is **6.96 uAcm^(-2)**

Question 02

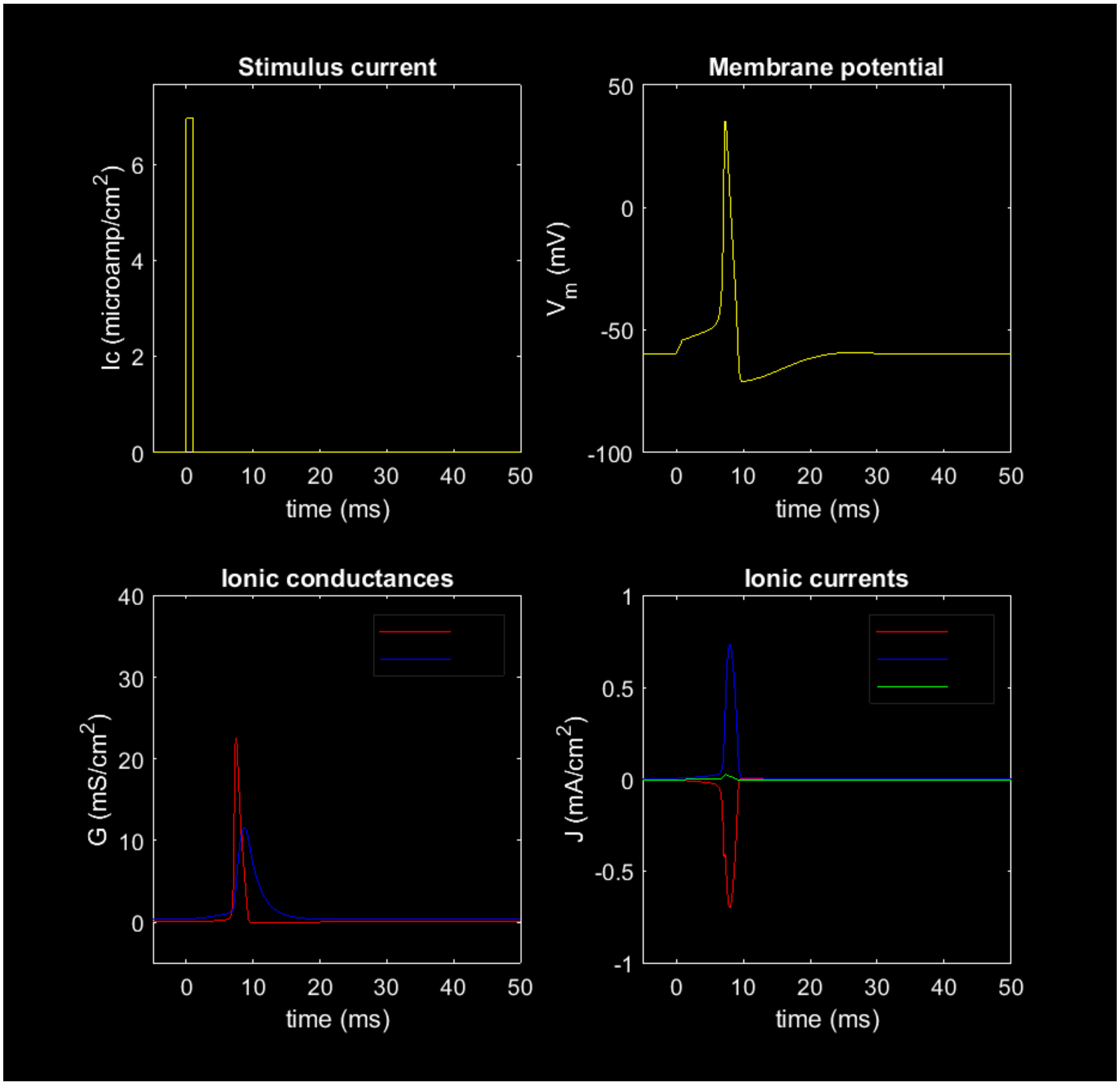
The relationship between $\int_{t_o}^{t_f} \sum_k J_k \, dt$ and $\int_{t_o}^{t_f} J_{ei} \, dt$

Finding values for these for different amplitudes

In [35]:

```
%relationship between sum of Jie and Current density
amp1 = 6.90;
for n=1:7
    [qna, qk, ql] = hhsplot(0, 50);
    display(amp1);
    Sum_of_Jie = width1 * amp1;
    Total_current_density = qna + qk + ql;
    fprintf('Sum_of_Jie: %f\n', Sum_of_Jie);
    fprintf('Total_current_density: %f\n', Total_current_density);
    fprintf('\n');
    amp1 = amp1 + 0.01;
end
```

Out[35]:



Out[35]: amp1 = 6.9000

```
Sum_of_Jie: 6.900000
Total_current_density: 6.899811
```

Out[35]: amp1 = 6.9100

```
Sum_of_Jie: 6.910000
Total_current_density: 6.910011
```

Out[35]: amp1 = 6.9200

```
Sum_of_Jie: 6.920000
Total_current_density: 6.919854
```

Out[35]: amp1 = 6.9300

```
Sum_of_Jie: 6.930000
Total_current_density: 6.929953
```

Out[35]: amp1 = 6.9400

```
Sum_of_Jie: 6.940000
Total_current_density: 6.939887
```

Out[35]: amp1 = 6.9500

```
Sum_of_Jie: 6.950000
Total_current_density: 6.950007
```

Out[35]: amp1 = 6.9600

```
Sum_of_Jie: 6.960000
Total_current_density: 6.962006
```

From the above values it is evident that $\int_{t_o}^{t_f} \sum_k J_k \, dt \text{ and } \int_{t_o}^{t_f} J_{ei} \, dt$ are equal

2. Refractoriness

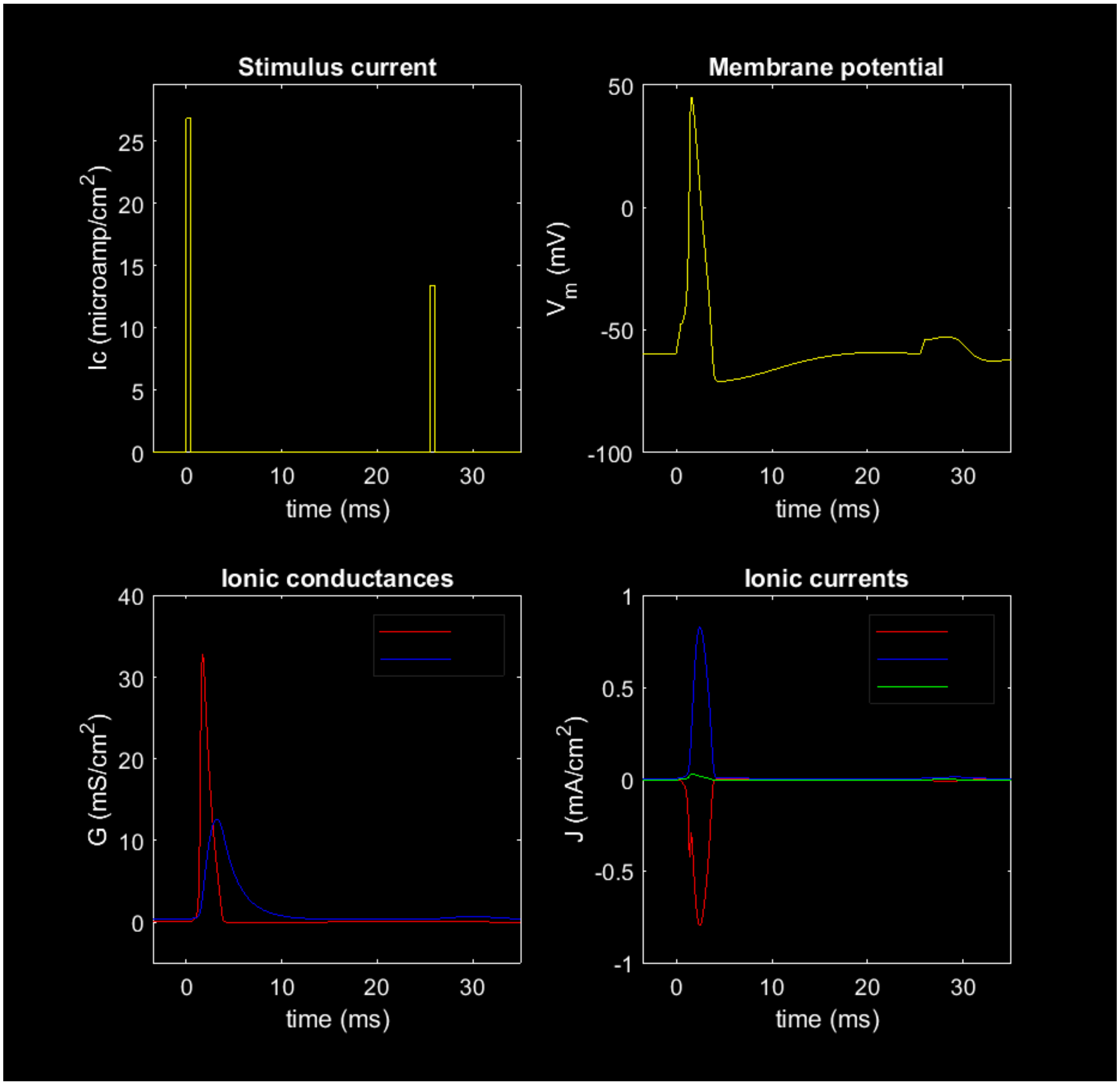
Question 3

Obtained $I2th$ for delay of 25ms

In [36]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 25;
amp2 = 13.4;
width2 = 0.5;
hhsplot(0,35);
```

Out[36]:



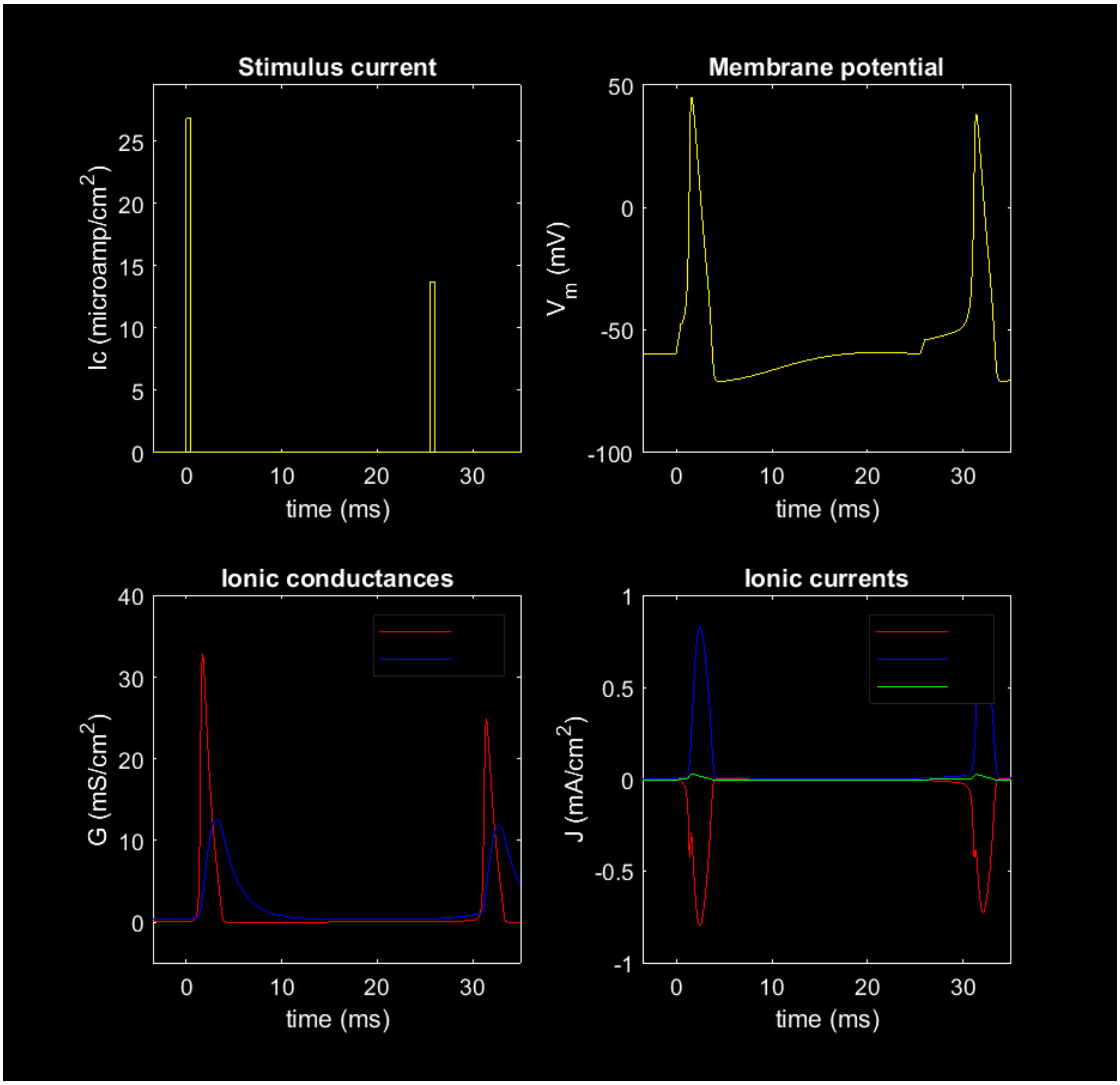
13.4 uAcm^(-2) is not enough to elicit an action potential. So by increasing the value for amp2, threshold value should be found.

In [37]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 25;
amp2 = 13.7;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 13.7
```

Out[37]:



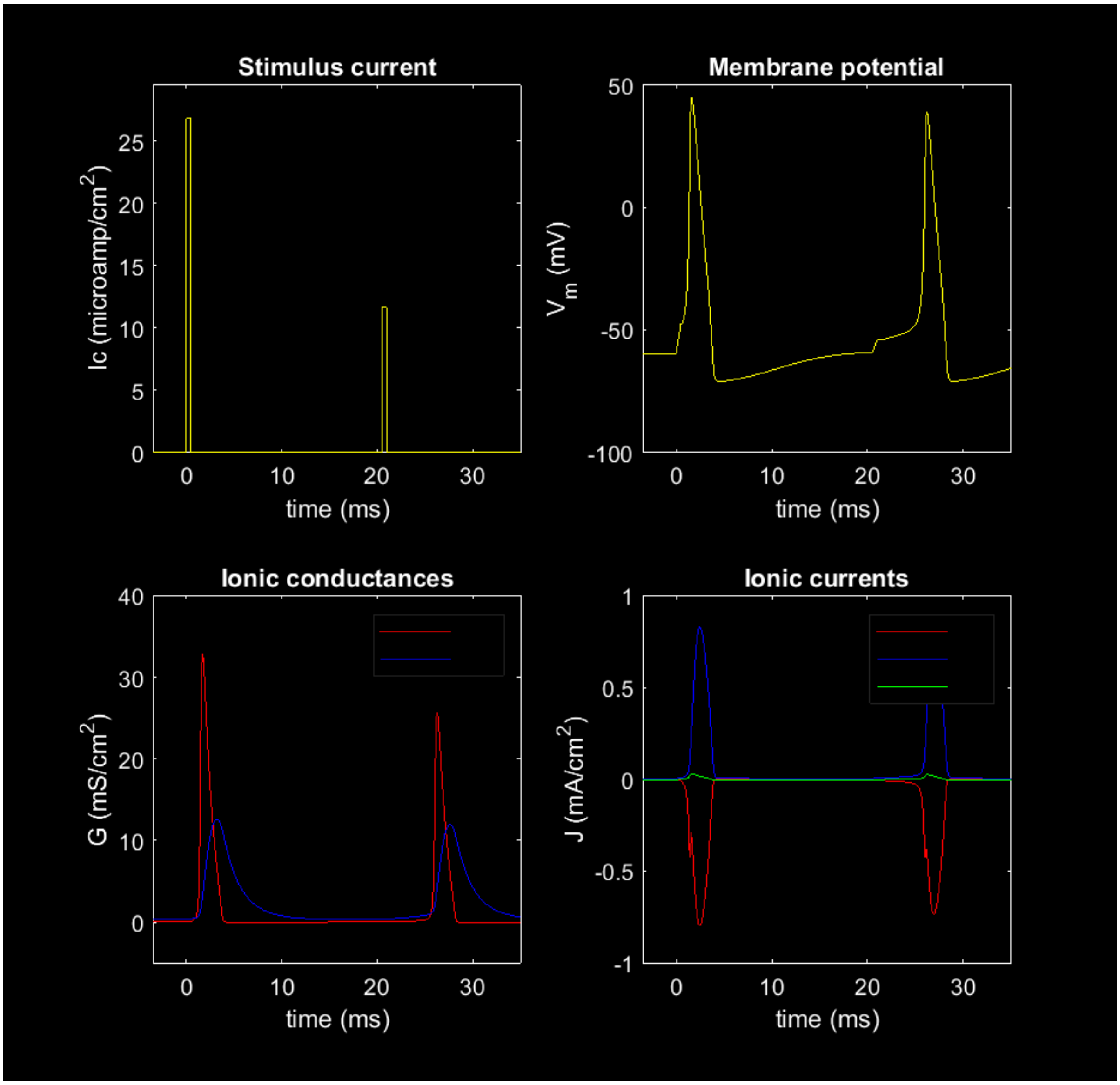
Obtained $I2th$ for delay of 20ms

In [38]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 20;
amp2 = 11.6;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 11.6
```

Out[38]:



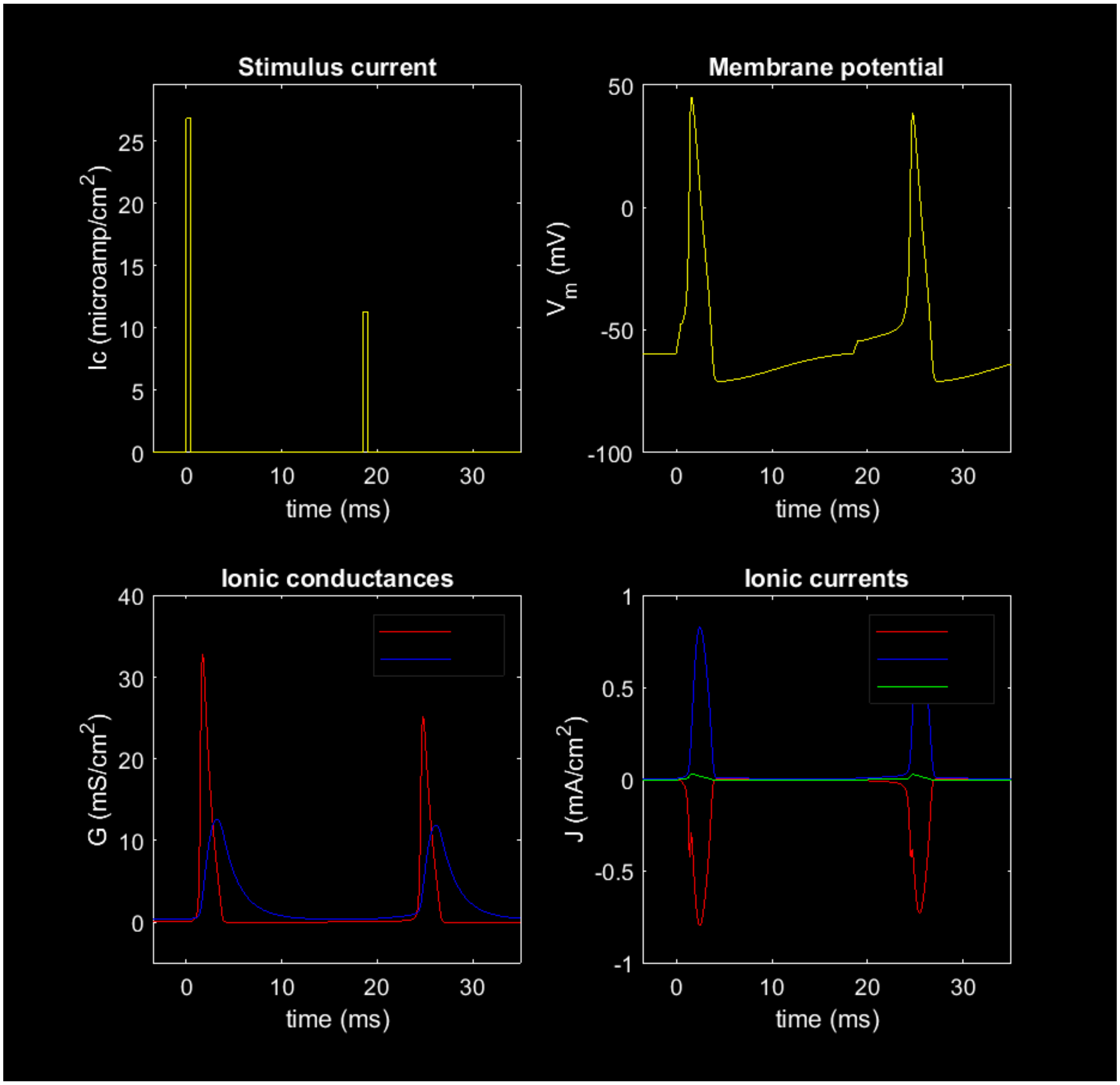
Obtained $I2th$ for delay of 18ms

In [39]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 18;
amp2 = 11.3;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 11.3
```

Out[39]:



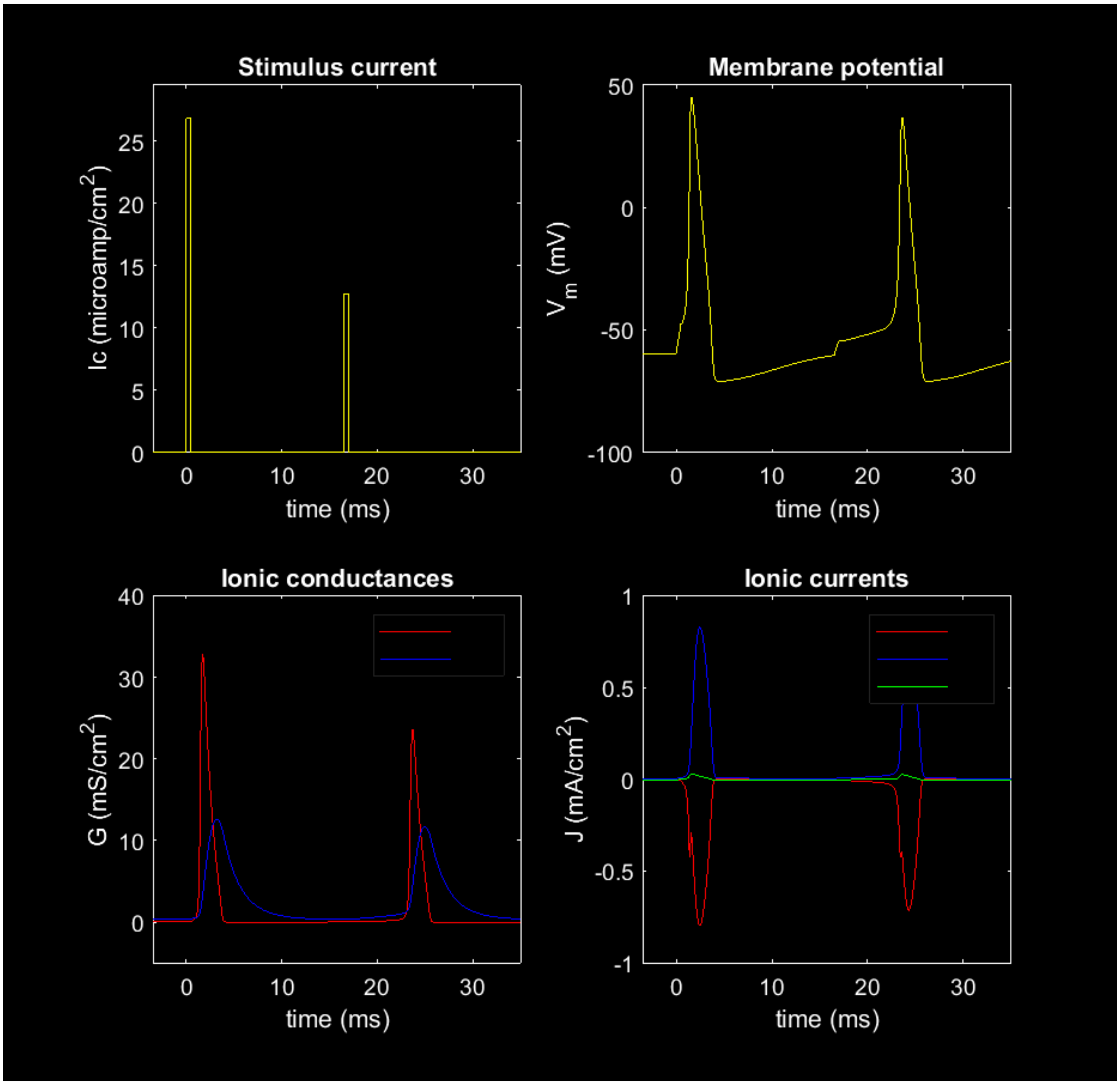
Obtained $I2th$ for delay of 16ms

In [40]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 16;
amp2 = 12.7;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 12.7
```

Out[40]:



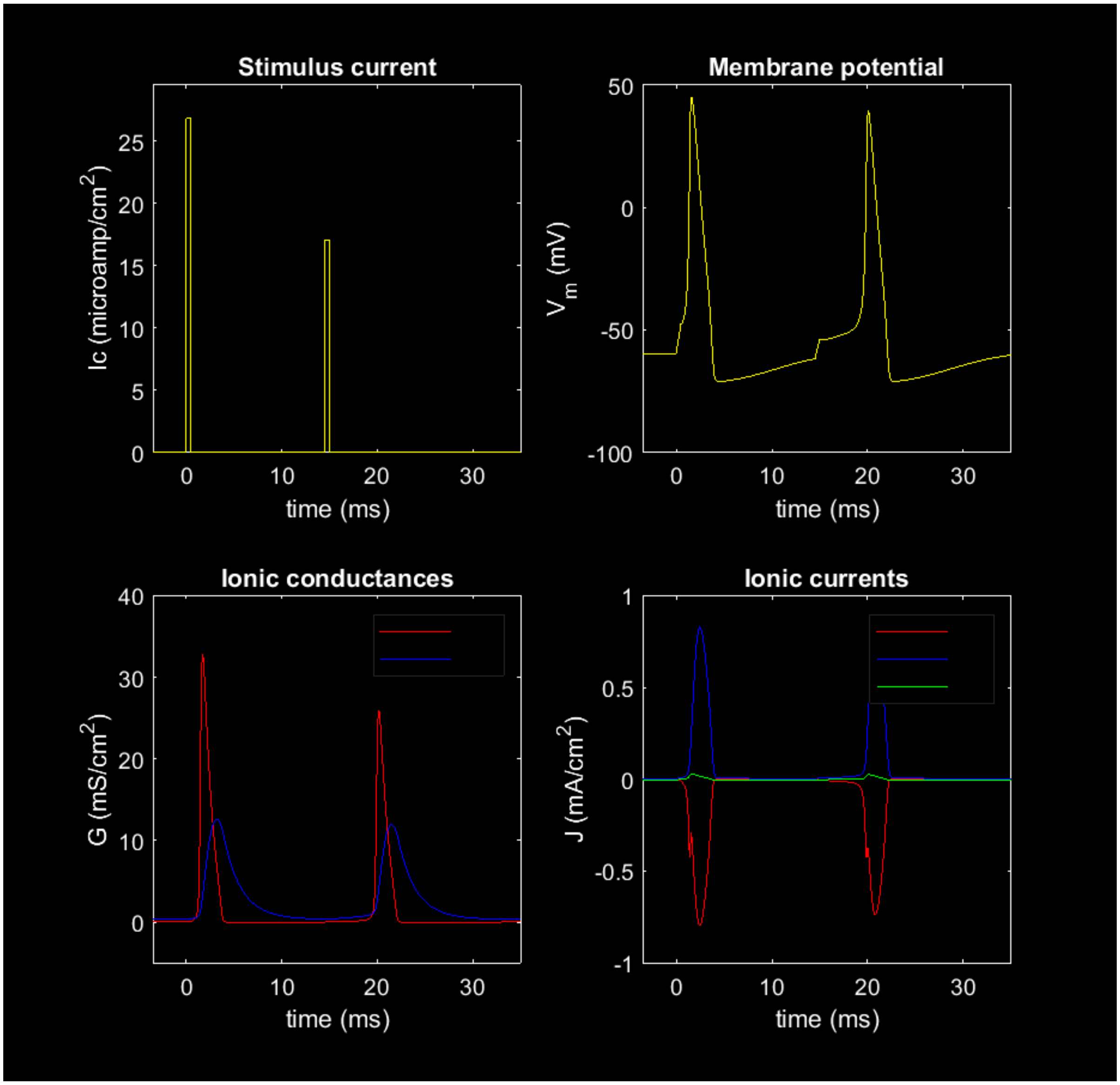
Obtained $I2th$ for delay of 14ms

In [41]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 14;
amp2 = 17;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 17
```

Out[41]:



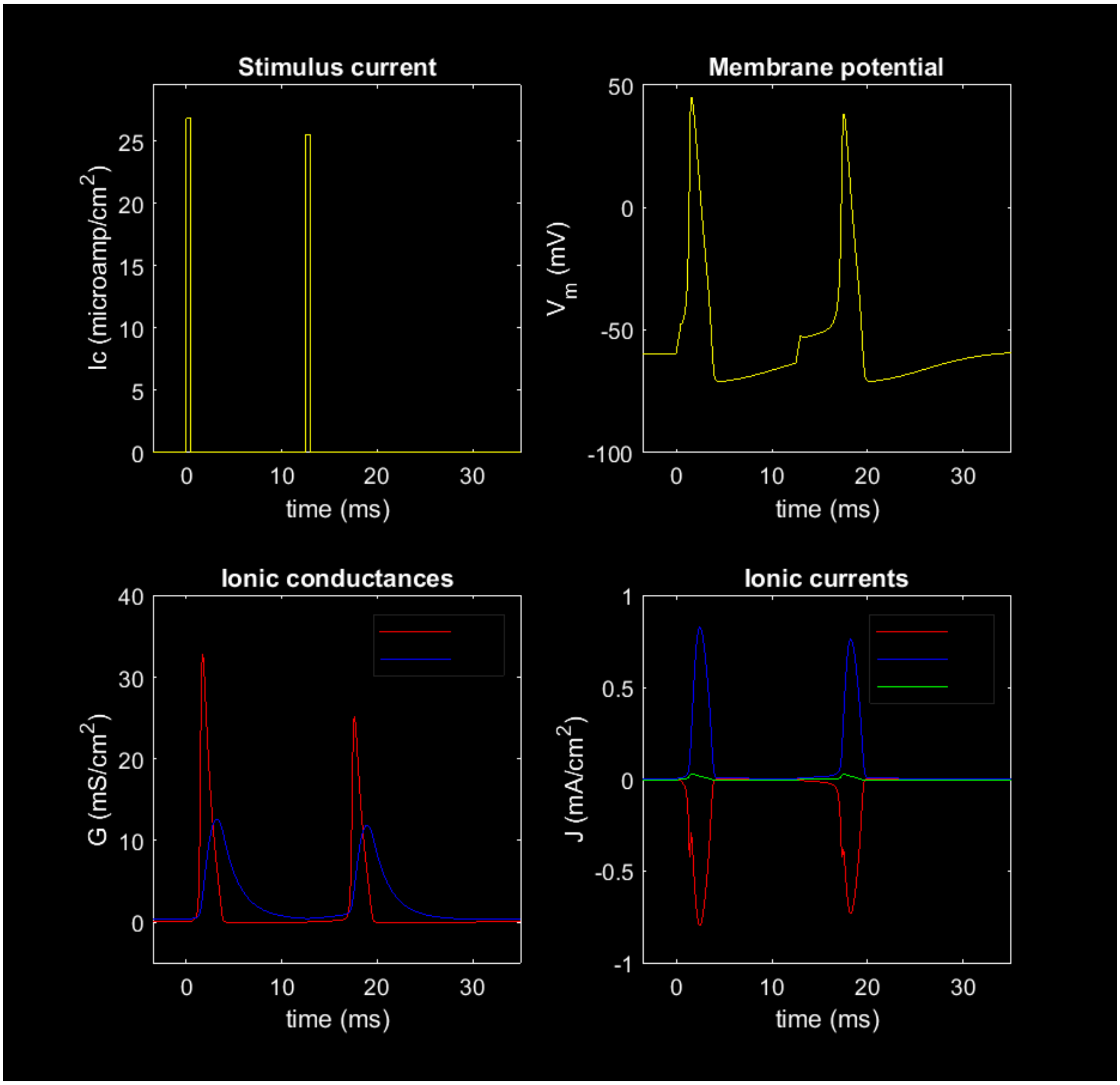
Obtained $I2th$ for delay of 12ms

In [42]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 12;
amp2 = 25.5;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 25.5
```

Out[42]:



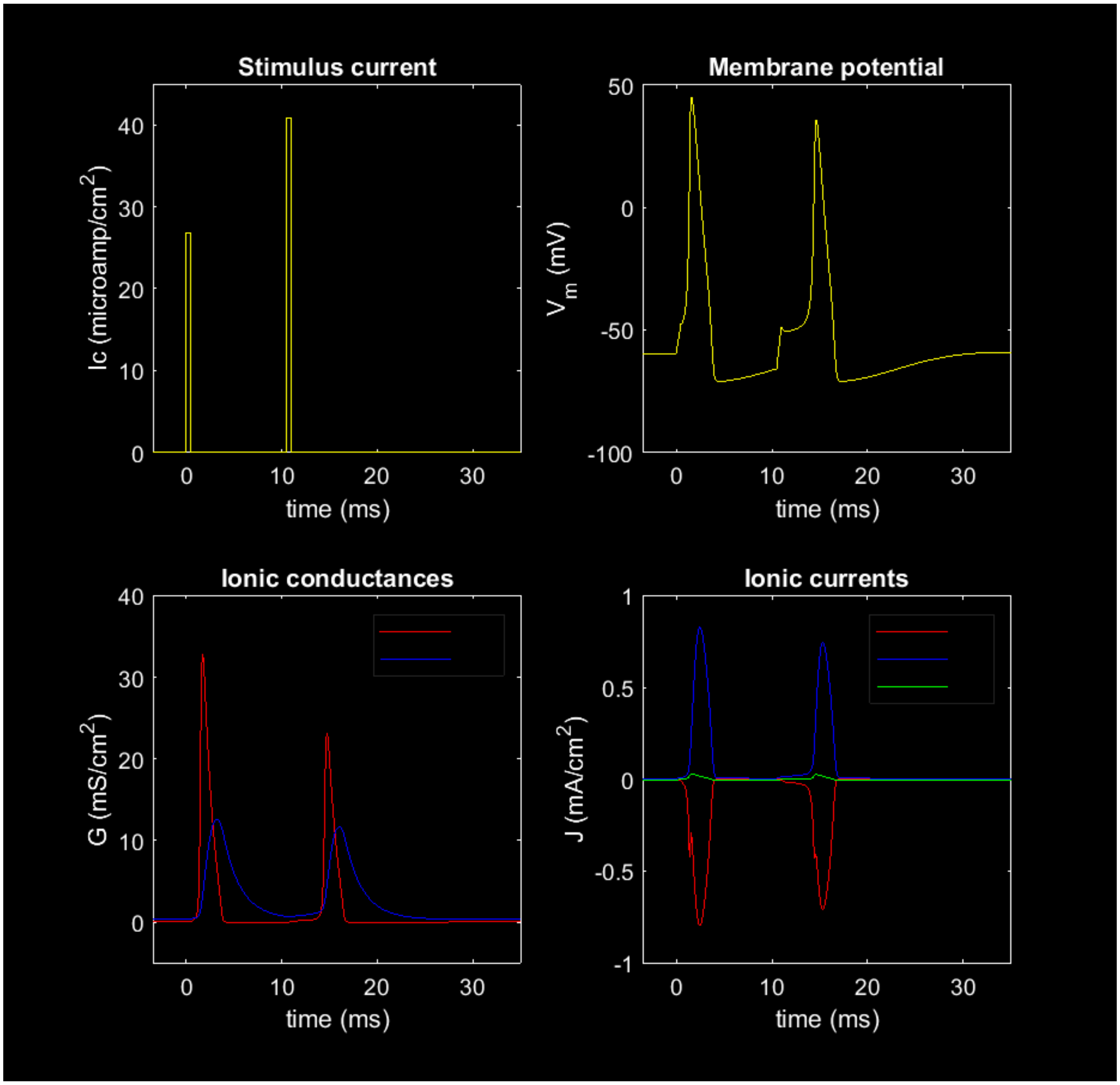
Obtained $I2th$ for delay of 10ms

In [43]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 10;
amp2 = 40.8;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 40.8
```

Out[43]:



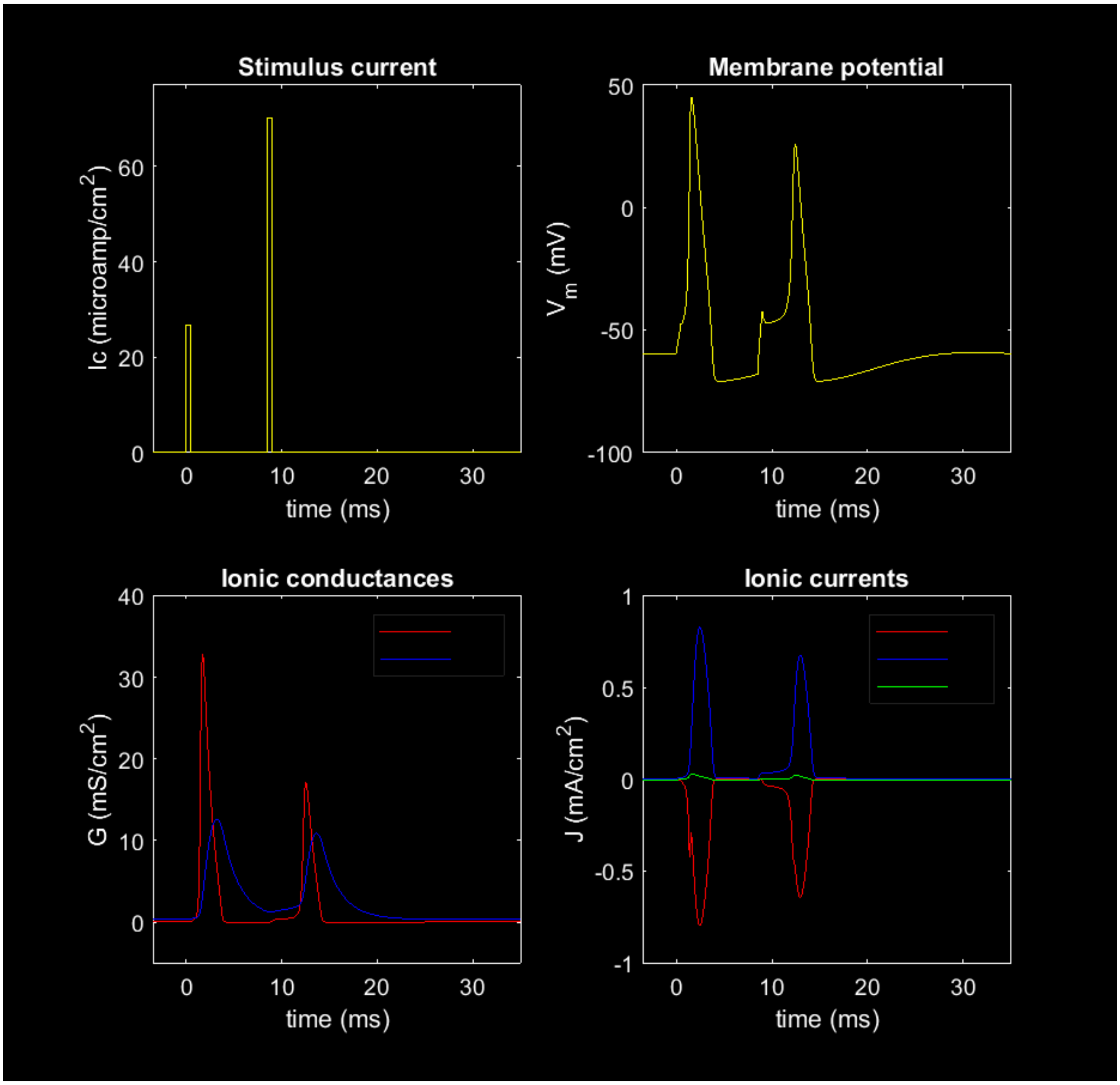
Obtained $I2th$ for delay of 8ms

In [44]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 8;
amp2 = 70.1;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 70.1
```

Out[44]:



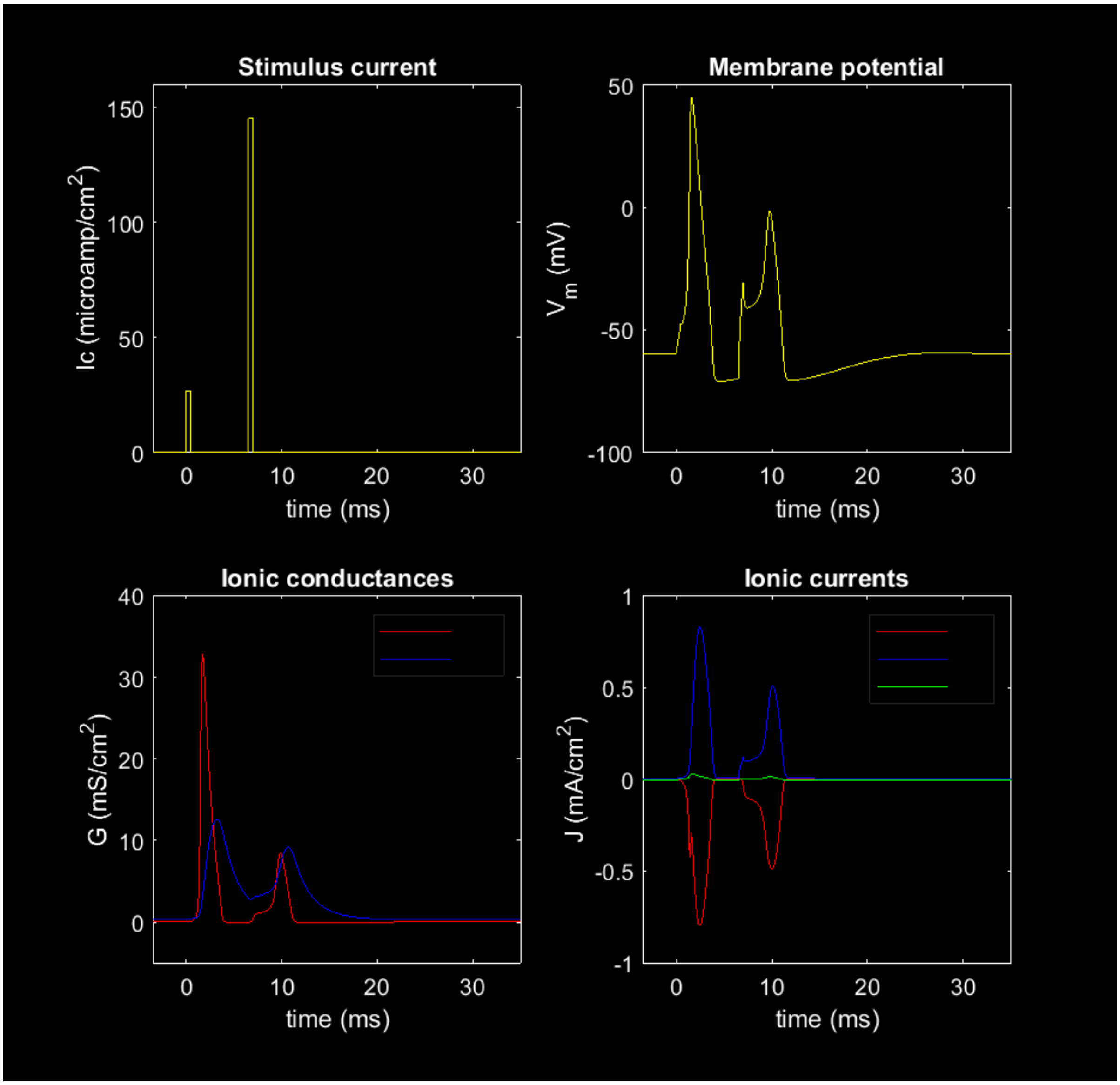
Obtained $I2th$ for delay of 6ms

In [45]:

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 6;
amp2 = 145.2;
width2 = 0.5;
disp("Threshold value: " + amp2);
hhsplot(0,35);
```

```
Threshold value: 145.2
```

Out[45]:



Threshold values and the ratio I2th/I1th with respective delay

In [46]:

```
delay_2 = [6, 8, 10, 12, 14, 16, 18, 20, 25];
I2th = [145.2, 70.1, 40.8, 25.5, 17, 12.7, 11.3, 11.6, 13.7];
I1th = 26.8 * ones(size(I2th));
Ratio = I2th ./ I1th;
% Display the table
disp(table(delay_2', I2th', Ratio', 'VariableNames', {'Delay_2', 'Second_Impulse',
```

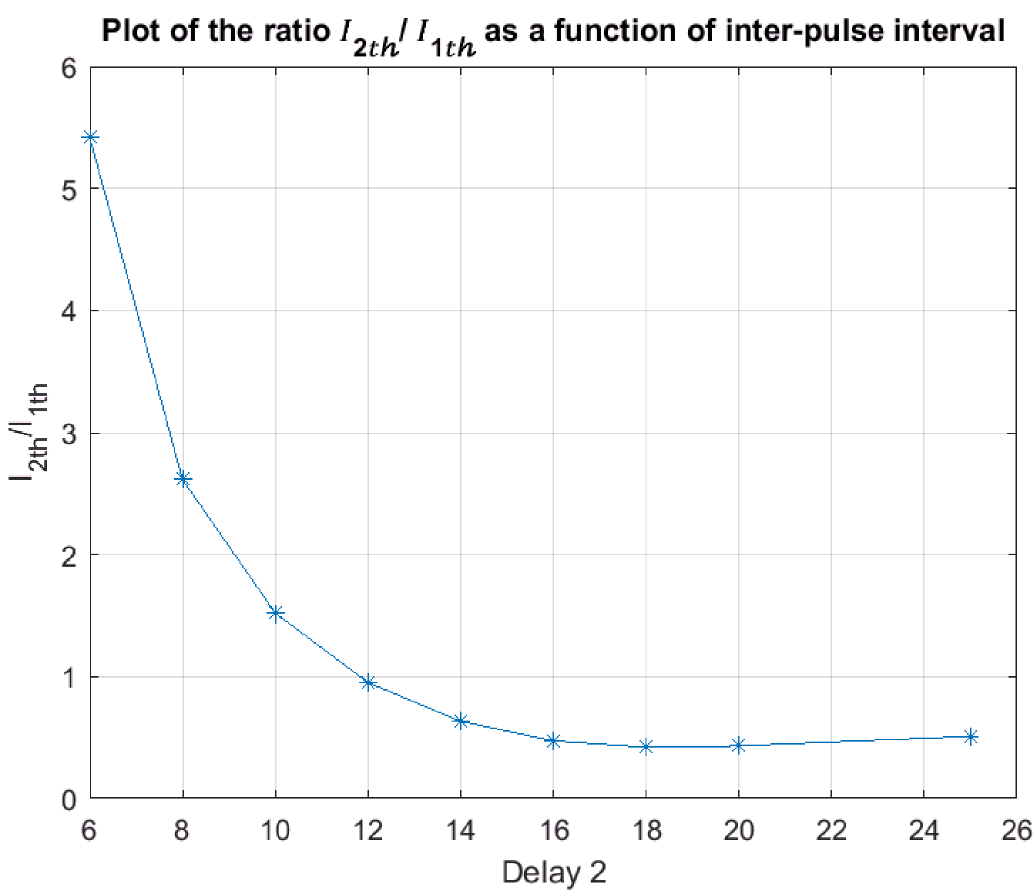
Delay_2	Second_Impulse	Ratio
6	145.2	5.4179
8	70.1	2.6157
10	40.8	1.5224
12	25.5	0.95149
14	17	0.63433
16	12.7	0.47388
18	11.3	0.42164
20	11.6	0.43284
25	13.7	0.51119

Question 04

Plot of the ratio $I2th/\ I1th$ as a function of inter-pulse interval

In [47]:
```
delay2=[6 8 10 12 14 16 18 20 25];
Ratio=[145.2 70.1 40.8 25.5 17 12.7 11.3 11.6 13.7]./26.8;
figure;
plot(delay2,Ratio,'*-');
xlabel("Delay 2");
ylabel('I_{2th}/I_{1th}');
grid on;
xlim([6, 26])
title('Plot of the ratio I_{2th}/ I_{1th} as a function of inter-pulse interval')
```

Out[47]:



- **Absolute refractory period- from 0ms to 6ms :** The absolute refractory period, denoted by the absence of a second action potential regardless of stimulus intensity, is characterized by a region where the ratio $I2th/I1th$ becomes infinite. This region signifies the period during which the neuron is unresponsive to additional stimuli due to the refractory state induced by the preceding action potential. The graph illustrates an exponential increase in current as the time delay decreases. Notably, at a time delay of 6 ms, the value is more than five times the current, indicating a relatively substantial delay. This prolonged delay signifies the absolute refractory period, estimated at 6 ms.

- **Relative refractory period- from 6ms to 10ms:** Conversely, the relative refractory period, distinguished by a progressively increasing threshold for eliciting a second action potential, manifests as a region where the ratio I2th$I$1th exceeds 1 but decreases as the inter-pulse interval increases. Beyond the 6 ms mark, it becomes evident that the value surpasses the current at 10 ms for the first instance. Consequently, the relative refractory period extends to 10 ms.

3. Repetitive activity

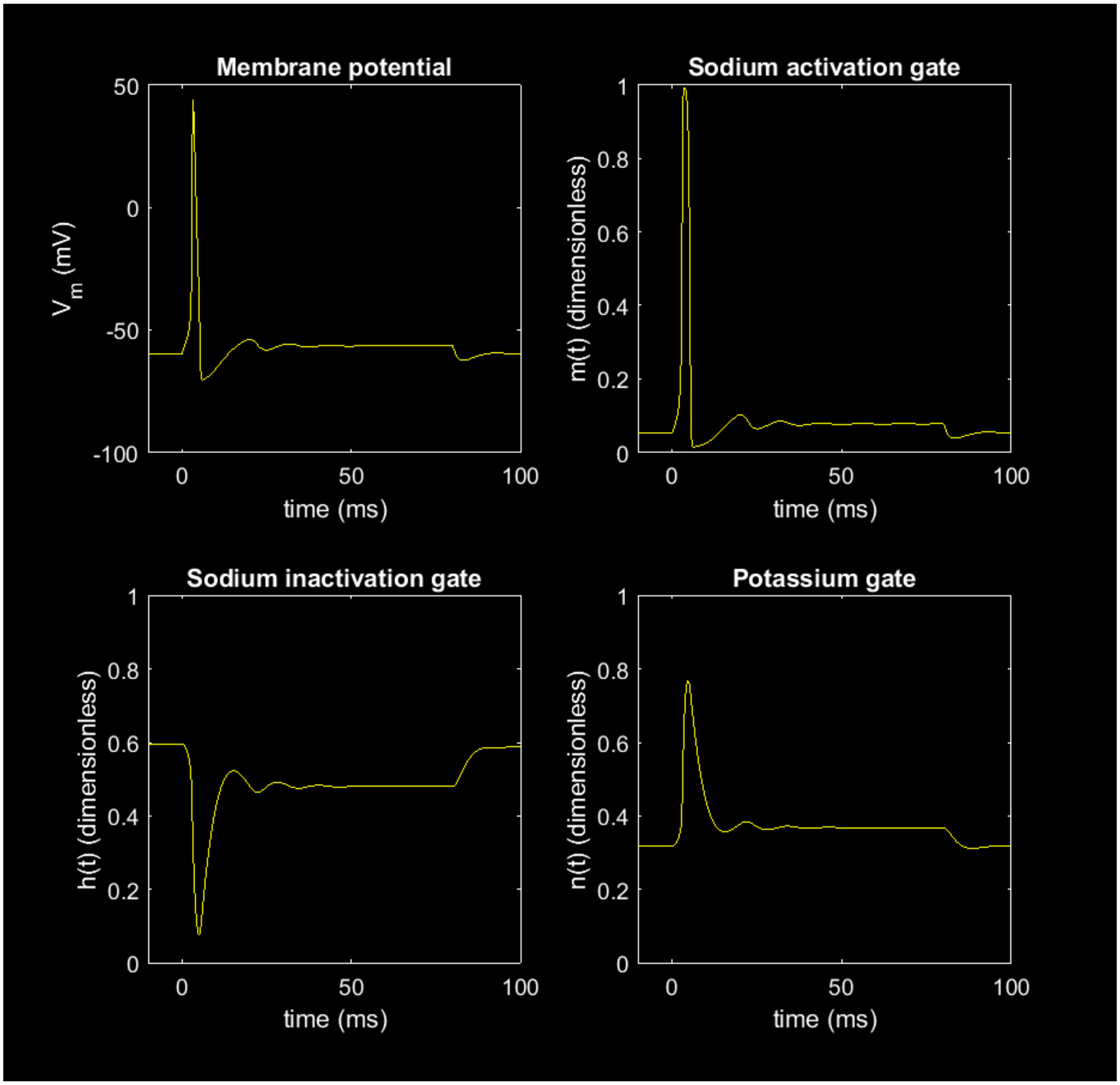
Long duration supra-threshold currents elicit multiple action potentials, a phenomenon called repetitive activity.

Question 05

Stimulating current amplitude= 5 μAcm^-2

In [48]:
```
amp1=5;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Out[48]:

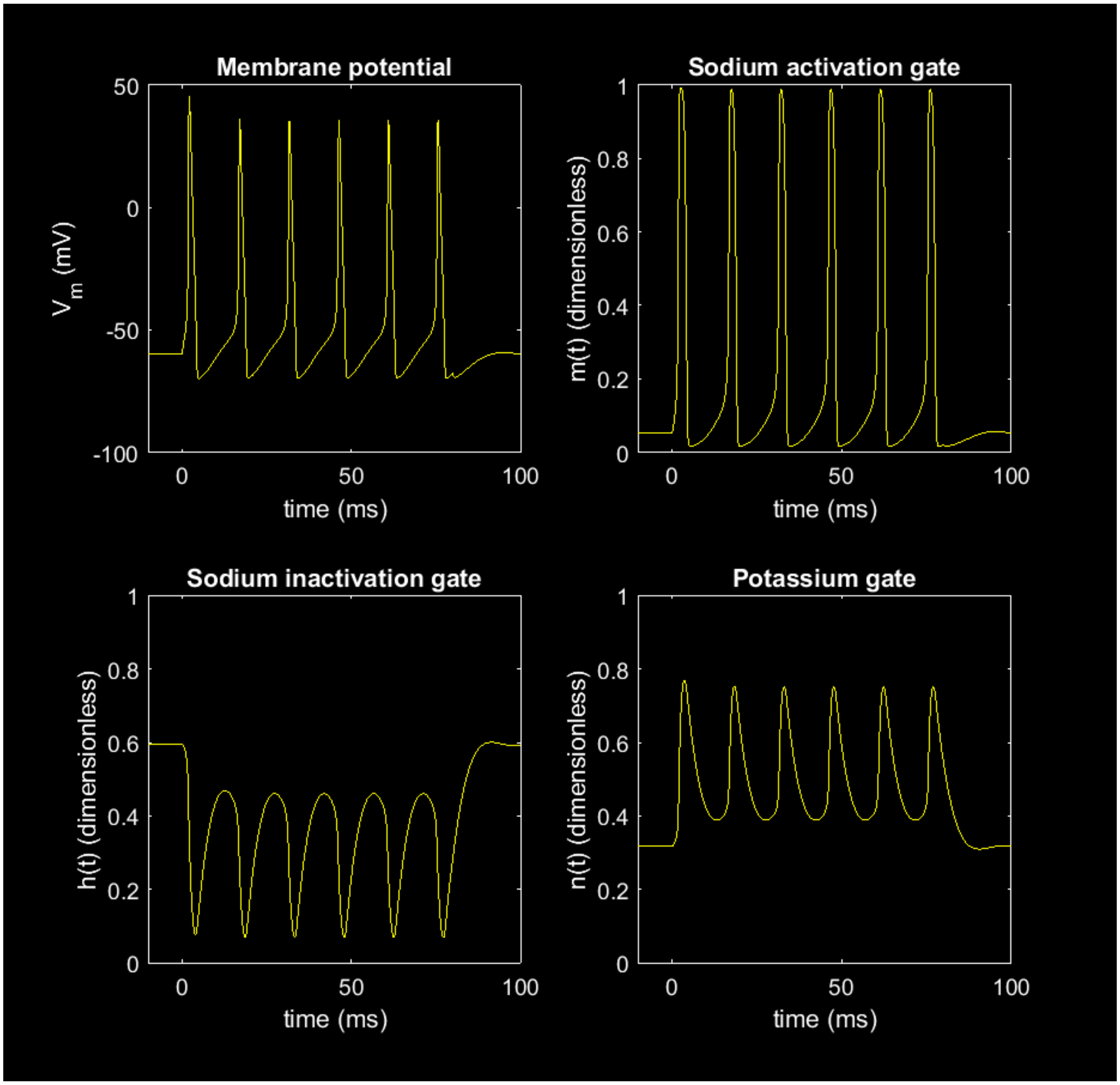


Stimulating current amplitude= 10 μAcm^-2

In [49]:

```
amp1=10;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Out[49]:

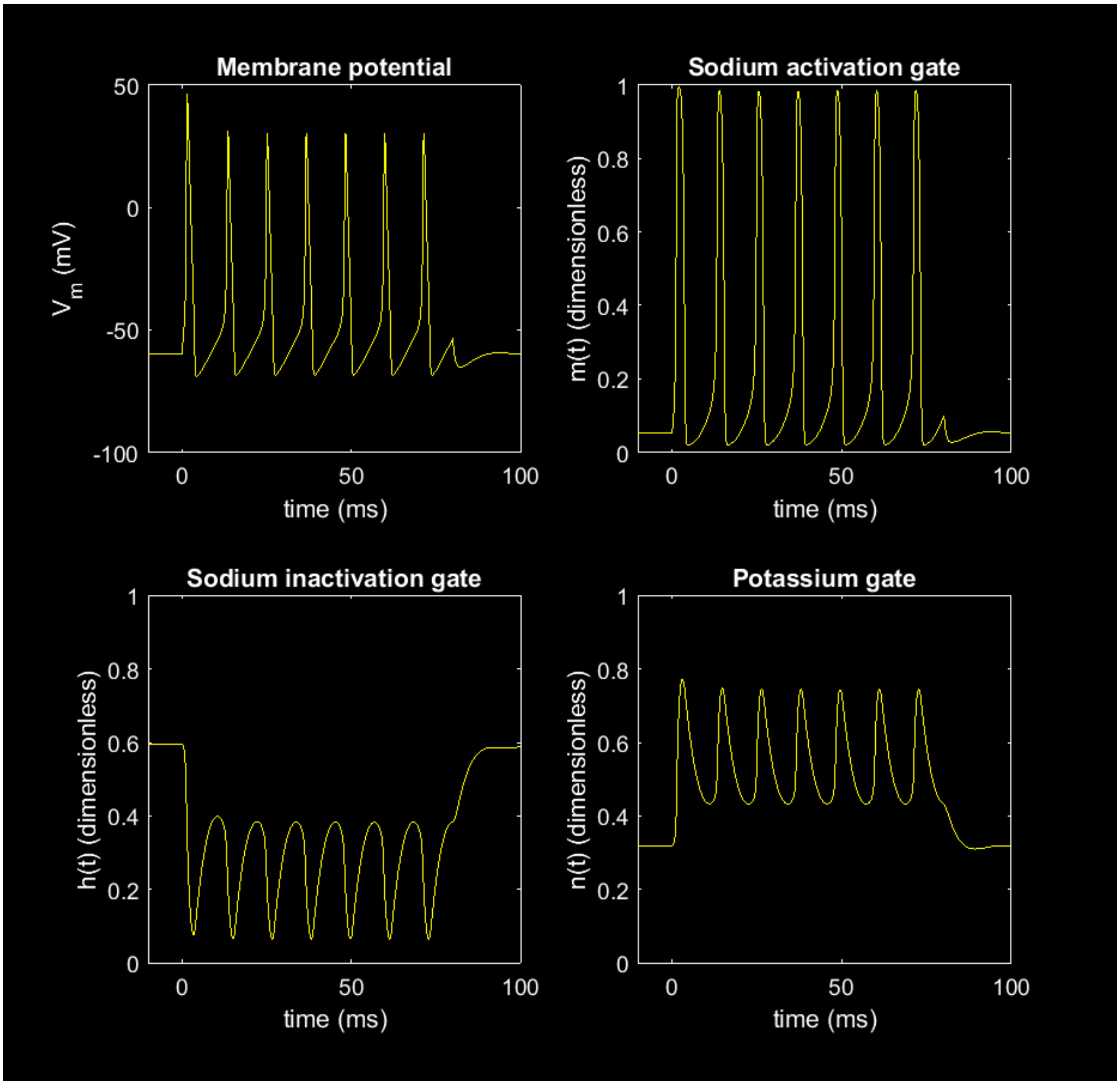


Stimulating current amplitude= 20 μAcm^-2

In [50]:

```
amp1=20;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Out[50]:

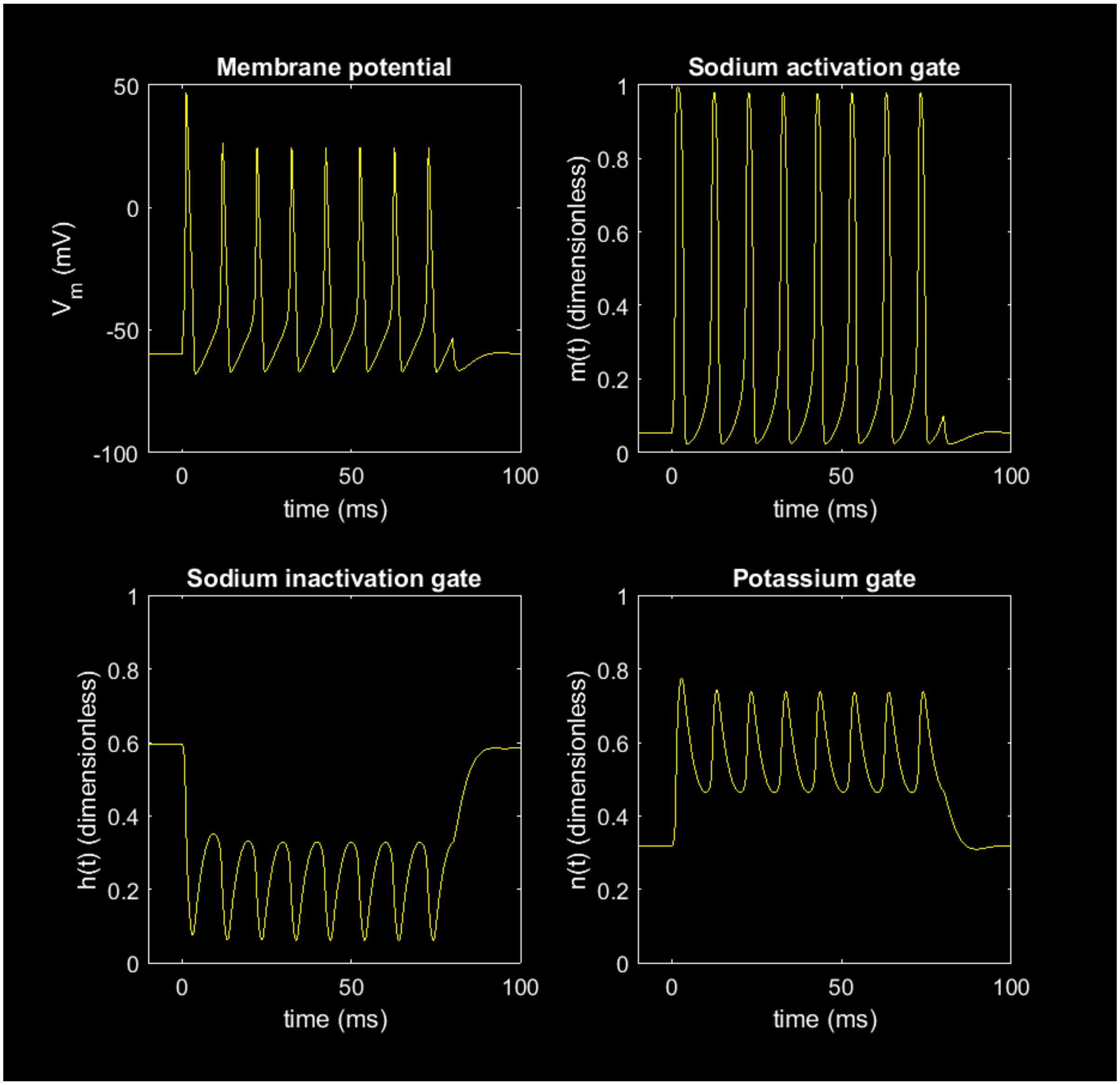


Stimulating current amplitude= 30 μAcm^-2

In [51]:

```
amp1=30;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Out[51]:

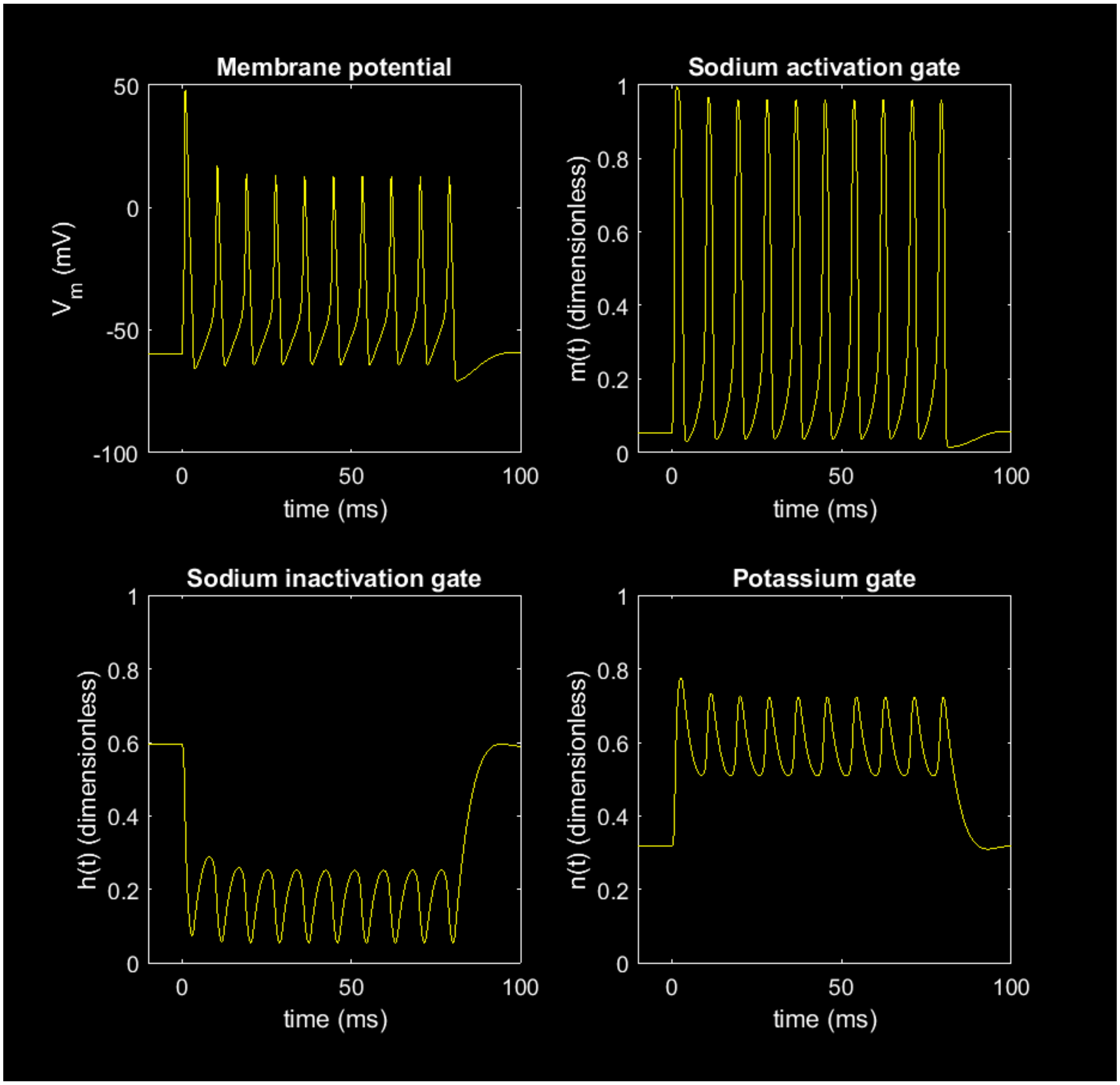


Stimulating current amplitude= 50 μAcm^-2

In [52]:

```
amp1=50;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Out[52]:

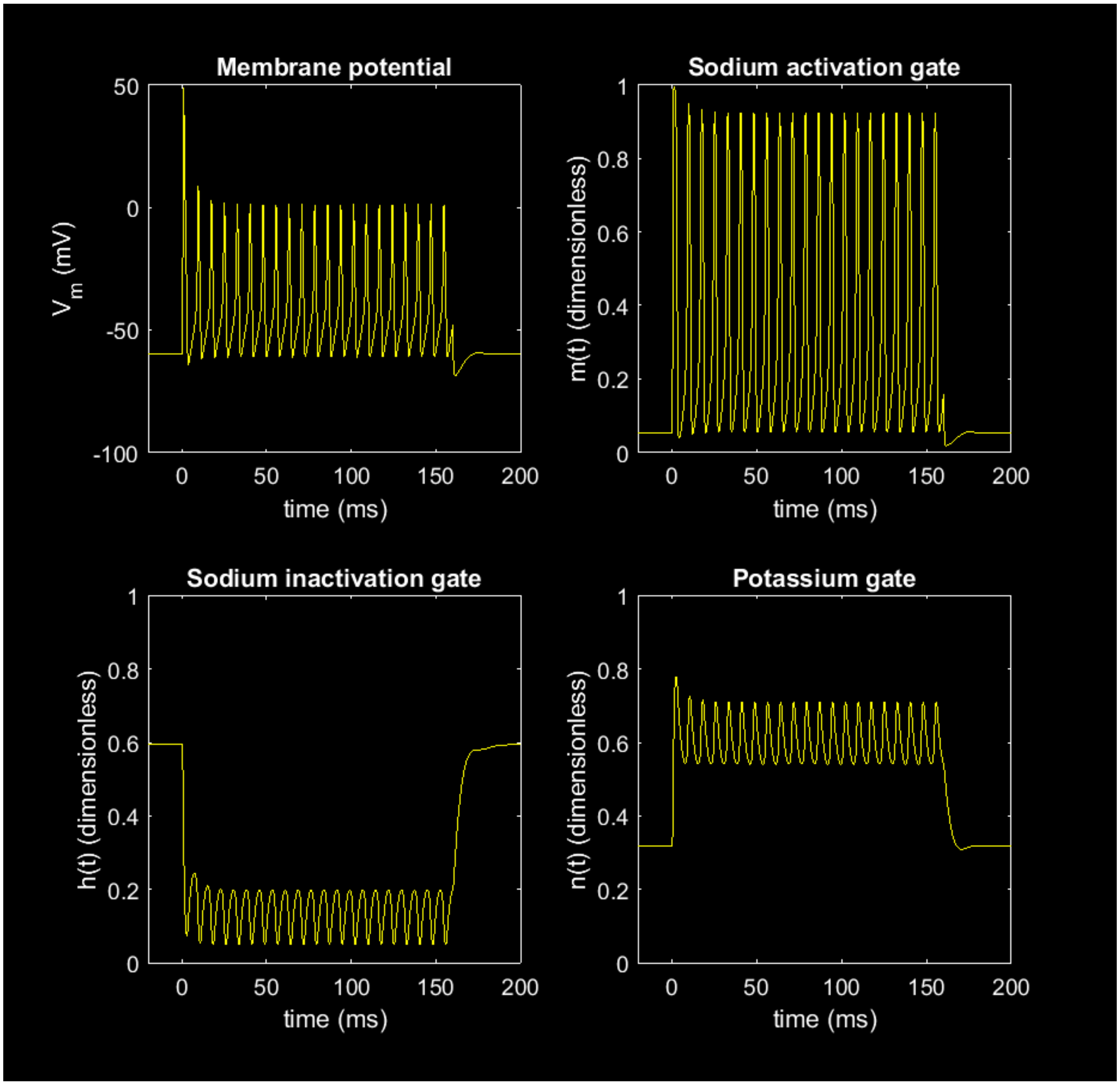


Stimulating current amplitude= 70 µAcm^-2

In [53]:

```
amp1=70;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Out[53]:



In [58]:

```
Amplitudes = [5, 10, 20, 30, 50, 70, 100]; % Stimulating current amplitudes (μA/cm^
No_of_AP = [1, 6, 7, 8, 10, 11, 12]; % Number of action potentials observed within

stim_duration_s = 80 / 1000;

AP_frequency = No_of_AP / stim_duration_s; % Action potential frequencies (Hz)

T = table(Amplitudes', No_of_AP', AP_frequency', ...
    'VariableNames', {'Amplitude', 'No_of_Action_Potentials', 'Frequency'});

disp(T);
```

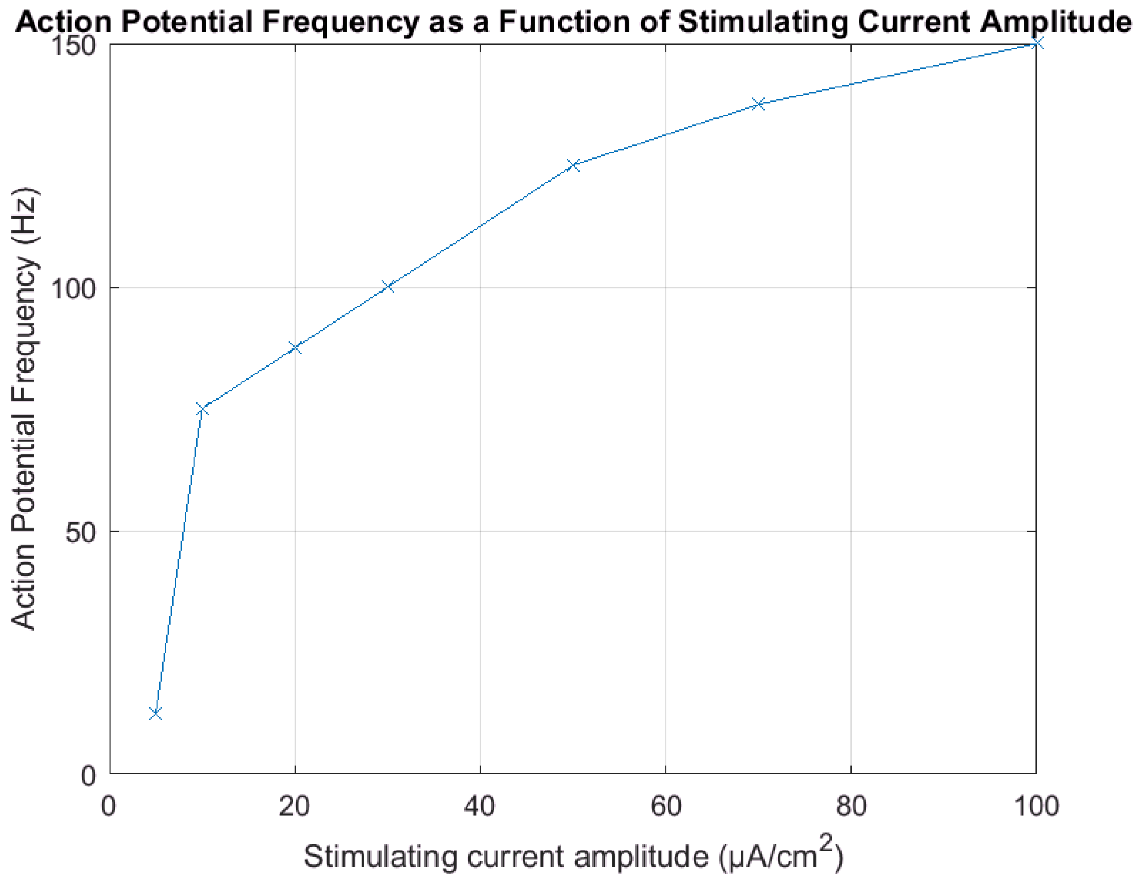
Amplitude	No_of_Action_Potentials	Frequency
5	1	12.5
10	6	75
20	7	87.5
30	8	100
50	10	125
70	11	137.5
100	12	150

Plotting action potential frequency as a function of stimulating current amplitude.

In [60]:

```
% Given data
Amplitudes = [5, 10, 20, 30, 50, 70, 100];
No_of_AP = [1, 6, 7, 8, 10, 11, 12];

stim_duration_s = 80 / 1000;


AP_frequency = No_of_AP / stim_duration_s;

figure;
plot(Amplitudes, AP_frequency, 'x-');
xlabel('Stimulating current amplitude (μA/cm^2)');
ylabel('Action Potential Frequency (Hz)');
title('Action Potential Frequency as a Function of Stimulating Current Amplitude');
grid on;
```

Out[60]:



Following are the observations from the grapgs above,

- **Amplitude of Action Potentials:** As stimulus intensity or amplitude increases, the amplitude of action potentials decreases.
- **Action Potential Frequency:** Conversely, when stimulus intensity or amplitude increases, the frequency of action potentials increases.

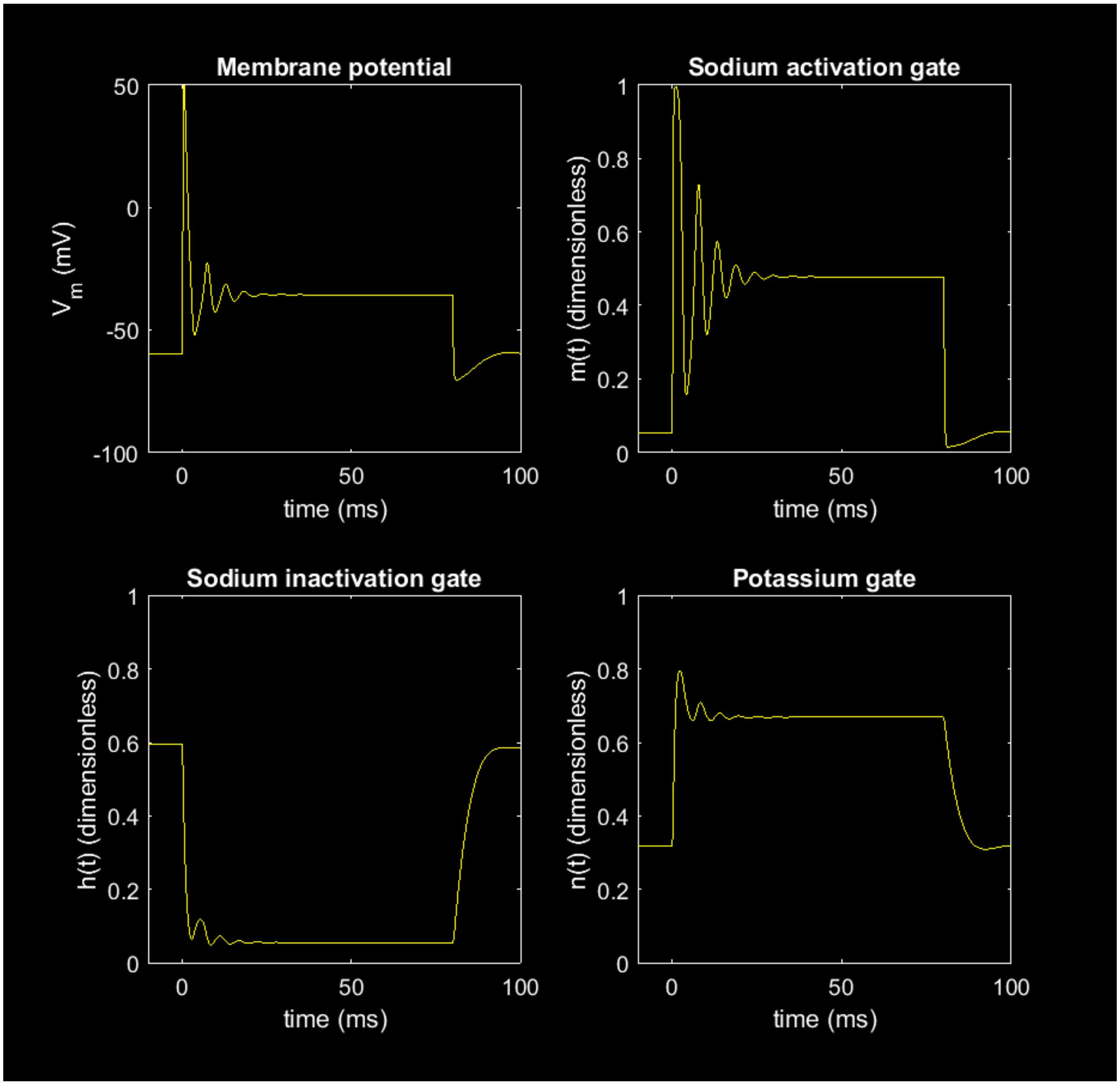
Qusetion 06

Stimulating current amplitude = 200 μAcm^-2. This result is known as a depolarisation block.

In [55]:

```
amp1=200;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Out[55]:



In response to increased electrical current intensity, the neuron undergoes depolarization, primarily mediated by the opening of voltage-gated sodium channels (referred to as the "m" factor). This depolarization leads to a rapid influx of sodium ions into the neuron, initiating an action potential. However, heightened depolarization also affects the "h" factor, which regulates the activation and inactivation of sodium channels. As membrane potential increases, the "h" factor becomes more pronounced, causing the inactivation of a portion of sodium channels. This results in fewer sodium channels available to contribute to the rising phase of the action potential, leading to a diminished amplitude.

Moreover, the "n" factor in the Hodgkin-Huxley equations represents the activation state for potassium channels, crucial for repolarizing the neuron's membrane. At extremely high levels of depolarization, the "n" factor decreases significantly, leading to reduced potassium channel activity. Consequently, the repolarization process is hindered by the "h" factors, as potassium ions remain inactive, thus maintaining the membrane voltage in a depolarized state.

In summary, heightened depolarization not only influences sodium channel dynamics through the "m" and "h" factors but also affects potassium channel activity via the "n" factor. These interactions contribute to the modulation of action potential characteristics, including amplitude and repolarization kinetics.

Question 07

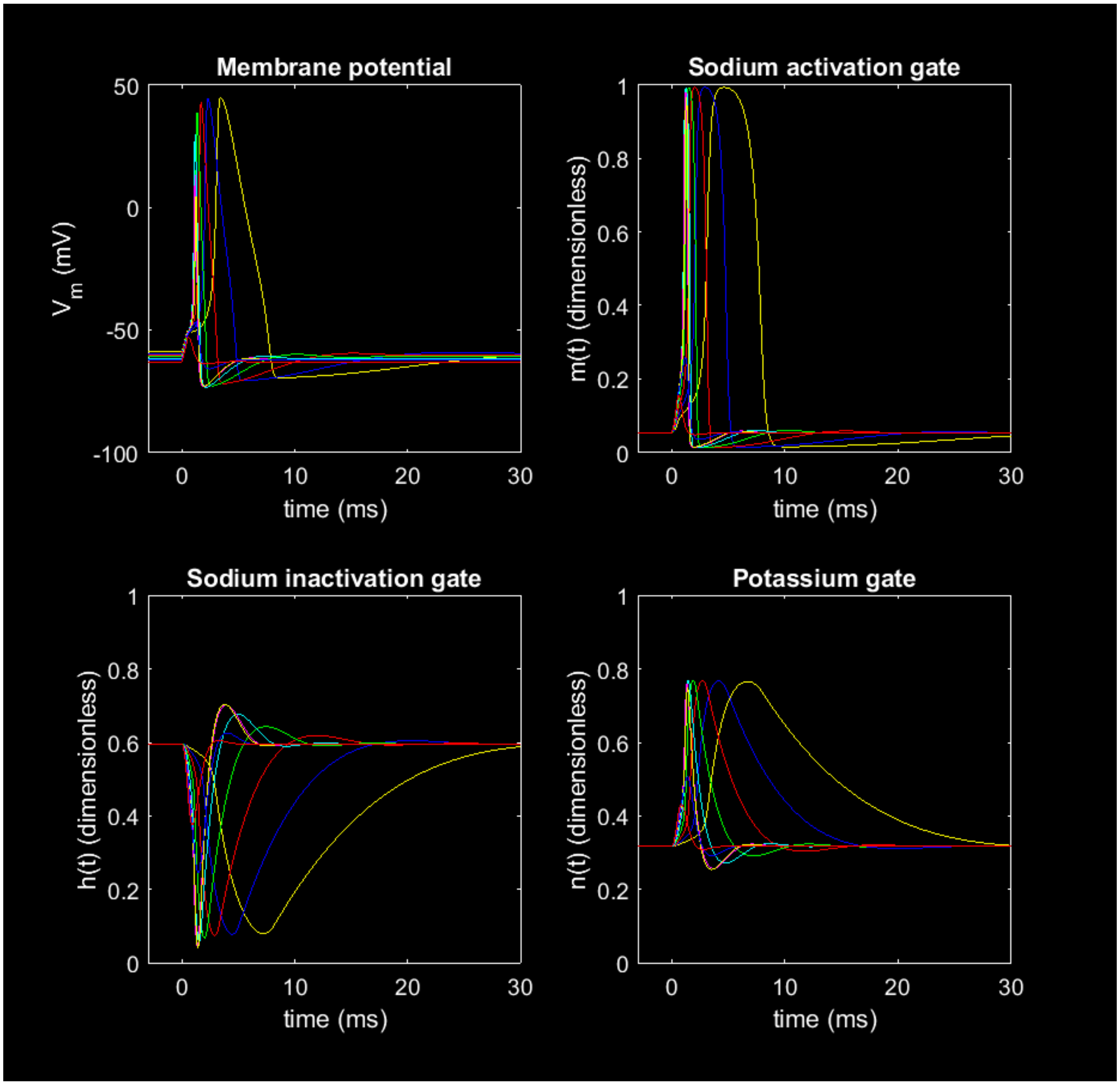
4. Temperature dependence

The effects of the following temperatures on the duration and amplitude of the resulting action potential: 0, 5, 10, 15, 20, 24, 25, 26 and 30 Celsius

In [56]:

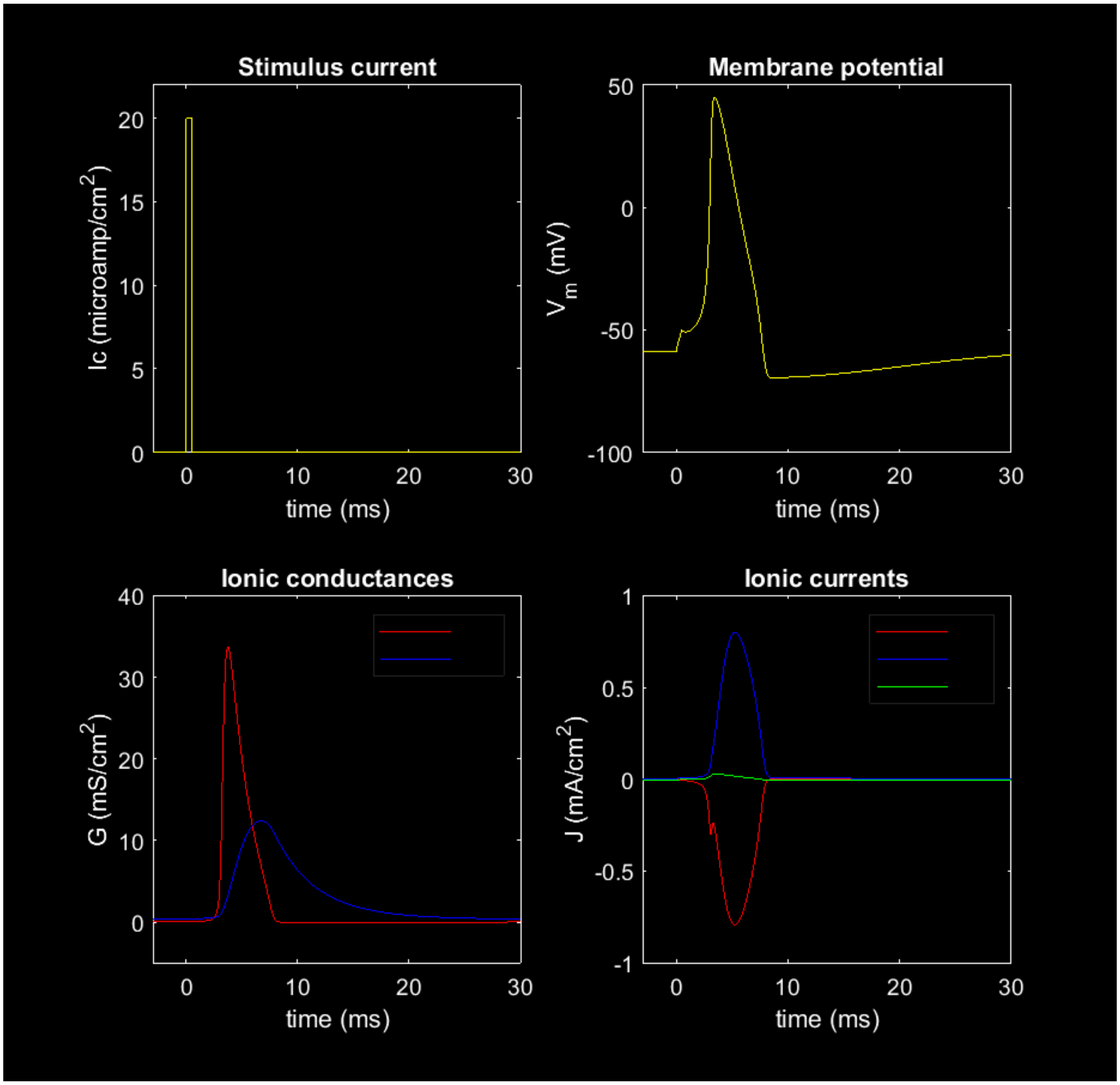
```
vc1amp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 0;
hhmplot(0,30,0);
hhsplot(0,30)

tempc = 5;
hhmplot(0,30,1);


tempc = 10;
hhmplot(0,30,2);


tempc = 15;
hhmplot(0,30,3);


tempc = 20;
hhmplot(0,30,4);


tempc = 24;
hhmplot(0,30,5);


tempc = 25;
hhmplot(0,30,6);


tempc = 26;
hhmplot(0,30,7);

tempc = 30;
hhmplot(0,30,8);
```

Out[56]:



Membrane potential
Sodium activation gate
Sodium inactivation gate
Potassium gate
V_m (mV)
m(t) (dimensionless)
h(t) (dimensionless)
n(t) (dimensionless)
time (ms)

Out[56]:



Out[56]: ans = -2.7016e+03

Observations

Amplitude of action potential decreases with the increase in temperature

Features of the action potential tha are affected by increasing temperature

Increasing temperature affects various features of the action potential. Some of the key effects of temperature on action potentials include:

- **Action Potential Duration:** The duration of the action potential can decrease with increasing temperature. At temperatures below approximately 37°C, the after-hyperpolarization (AHP) duration decreases slowly, while it decreases rapidly for higher temperatures. This change in duration can impact the overall firing pattern of neurons.
- **Firing Rate:** Temperature can influence the firing rate of neurons. Warm-induced increases in temperature can lead to a substantial increase in firing rate in warm-

sensitive neurons, while the firing rate of temperature-insensitive neurons may be less affected by temperature changes.

- **Amplitude:** Changes in temperature can affect the amplitude of action potentials. Warming can decrease the action potential amplitudes, while cooling can increase them. This alteration in amplitude can impact the strength and efficiency of signal transmission.
- **Threshold:** Temperature can influence the threshold for action potential generation. Cooling can increase the threshold for impulse propagation, making it more difficult for the impulse to supply the necessary charge for excitation of the region ahead. Conversely, warming can cause a transient hyperpolarization, affecting the threshold stimulus required for excitation.
- **Refractory Period:** The relative refractory period, which is the period after an action potential during which the threshold is elevated, is highly sensitive to temperature changes. Cooling can increase the relative refractory period, affecting the neuron's ability to generate subsequent action potentials.
- **Conductance and Channel Gating:** Temperature affects the kinetics of channel gating, including the rates at which ion channels open, close, and inactivate. The speed of ionic conductances turning on and off is influenced by temperature changes, impacting the overall dynamics of action potential generation. In summary, increasing temperature can have significant effects on the duration, firing rate, amplitude, threshold, refractory period, and ion conductances of action potentials. These temperature-induced changes play a crucial role in modulating the excitability and signaling properties of neurons.

In [ ]:

```

```