Interest Rate Products

Introduction

THIS DOCUMENT COVERS

- Introduction

Bond Pricing

Fair Value

A bonds fair value is defined as the Net Present Value of all cashflows including coupons and principle.

Bond Yield

The single discount rate that when applied to all cash flows gives the bond its market value

$$2.5e^{-0.5y} + 2.5e^{-1.0y} + 2.5e^{-1.5y} + 102.5e^{-2.0y} = 97.45$$

Requires a numerical solver.

Par Yield

The coupon rate that gives a bond a fair value of par. If a 1-year semi-annual bond gives coupons of c per annum we are solving for c. Given a set of zero rates

6 months = 0.04 1 year = 0.045 1.5 years = 0.05 2 years = 0.052

$$\frac{c}{2}e^{-0.04 \times 0.5} + \frac{c}{2}e^{-0.045 \times 1.0} + \frac{c}{2}e^{-0.05 \times 1.5} + \left(100 + \frac{c}{2}\right)e^{-0.052 \times 2.0} = 100$$

Risk and Pricing Solutions

Zero Rate

The zero-coupon interest rate is the rate of interest earned on an investment that starts today and lasts for n years. All interest and principal is earned at the end of n years. If a five year zero rate with continuous compounding is quoted as 5% per annum. This means that $100, if invested for 5 years grows to

$$e^{0.05*5} = e^{.25} = \$128.4$$

Zero treasury rates can be calculated in two ways

- Observe the yield on strips
- Bootstrap from treasury bills and bonds

Duration

Measures how long on average a bond holder must wait before receiving cash payments. For a 3-year zero coupon bond the duration would be 3 years. For coupon bearing bonds we use the following

$$D = \sum_{i=1}^{n} t^i \left[\frac{c_i e^{-yt_i}}{B} \right]$$

Where B is the market price of the Bond and y is the bond yield

Questions - Bond Pricing

How does one calculate the fair value of a bond?

As the NPV of all coupons and principle

What is the bond yield?

The single discount rate that when applied to all cash ones give the bond its market value

$$2.5e^{-0.5y} + 2.5e^{-1.0y} + 2.5e^{-1.5y} + 102.5e^{-2.0y} = 97.45$$

How does one calculate the bond yield?

By using a numeric solver

What is the par yield?

Risk and Pricing Solutions

The coupon rate that gives a bond a fair value of par

How does one calculate the par yield?

By simply solving equations of the following form

$$\frac{c}{2}e^{-0.04 \times 0.5} + \frac{c}{2}e^{-0.045 \times 1.0} + \frac{c}{2}e^{-0.05 \times 1.5} + \left(100 + \frac{c}{2}\right)e^{-0.052 \times 2.0} = 100$$

What is a zero rate?

T rate of interest earned on an investment that starts today and lasts for n years.

All interest and principal are earned at the end of n years

How does one calculate the zero rates?

Observe the yield on strips

Bootstrap from treasure bills and bonds.

FRA

Forward rates can be calculated from spot rates using arbitrage arguments

$$R_2 T_2 = R_1 T_1 + F_{T_2 - T_1}(T_2 - T_1)$$

$$F_{T_2 - T_1} = \frac{R_2 T_2 - R_1 T_1}{(T_2 - T_1)}$$

A FRA is an agreement to switch a fixed rate for a floating rate as applied to some future period. The amount is settled at time T_1 The single cashflow of an FRA from the perspective of the counterparty receiving fixed is as follows. Note the FRA is usually settled at time T_1 so the payment must be discounted from time T_2 to time T_1

$$F_{T_2 - T_1} = \frac{Principle \times (R_{fixed} - R_{floating})(T_2 - T_1) -}{1 + R_{floating}(T_2 - T_1)}$$

Risk and Pricing Solutions

Valuing a FRA involves assuming that forward rates are realised and then discounting the cashflow using realized forward LIBOR rates in places of the unknown LIBOR rates. The present value is simply this payment discounted back to T_0 using discount rates

$$Principle \times \left(R_{fixed} - R_{floating}\right)(T_2 - T_1) \times e^{R_2 T_2}$$

Questions – FRA

How does one calculate a forward rate?

From a set of spot rates

Show the derivation?

$$R_2T_2 = R_1T_1 + F_{T_2-T_1}(T_2 - T_1)$$

$$F_{T_2-T_1} = \frac{R_2T_2 - R_1T_1}{(T_2 - T_1)}$$

What is an FRA?

An agreement to switch a fixed rate for floating rate as applied to some future period

Typically the payment is discounted back to the start of the period

Show the calculation?

$$F_{T_2-T_1} = \frac{Principle \times (R_{fixed} - R_{floating})(T_2 - T_1)}{1 + R_{floating}(T_2 - T_1)}$$

How does one value an FRA?

Assume the forward rates are realized and use that for floating leg and discount the payment back to the present date

$$Principle \times (R_{fixed} - R_{floating})(T_2 - T_1) \times e^{R_2T_2}$$

Interest Rate Swaps

Fixed for floating

Can be used to

1. Transform fixed liabilities and assets to floating
2. Transform floating liabilities and assets to fixed

Can be hedges with bonds, FRAs and IR futures

DETERMINING LIBOR/SWAP ZERO RATES

At inception the fixed rate is set such that the swap has zero value

$$B_{fixed} = B_{floating}$$

A newly issued floating rate bond that pays 6-month LIBOR has a value of par when LIBOR is used for discounting. Since at inception of the swap the fixed leg has the same value as the floating leg it must also be worth par. The swap rate at inception is then given by a par yield.

A set of swap rates can then be interpreted as a set of par yield bonds.

VALUATION

Portfolio of FRA's

If we assume the LIBOR forward rates will be realized we can calculate the cashflows and discount them to calculate the bond value.

Questions - Interest Rate Swaps

What is a vanilla fixed for floating swap used for?

Transform floating rate assets and liabilities to fixed rate

Transform fixed rate assets and liabilities to floating rate

How does one calculate LIBOR/SWAP Zero rates?

At inception a fixed rate is set such that the swap has zero value

$$B_{fixed} = B_{floating}$$

The floating leg will have a value of par when it pays LIBOR and LIBOR is used for the discounting. If the floating leg has value of par then so much the fixed leg. The swap rate is then given by a par yield bond

Interest Rate Futures

Interest rate futures, when used in conjunction with the duration measure can be used to hedge exposure to interest rate movements.

Clean and Dirty price

The clean price is the quoted price which excludes accrued interest.

Dirty Price = Clean Price + Accrued Interest

How does one value a swap?

Assume the LIBOR forward rates will be realized and calcite the cashflows and discount them

How does one value a swap?