Risk Systems

Introduction

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A risk and P&L reporting tool's calculations can be roughly grouped into three categories.

1. P&L – How much have I made or lost on this position, account, portfolio
2. Risk – what will happen if – market data inputs changes
3. P&L attribution – breaking down and attributing unrealized P&L to market data input

Notation

In general, we will use subscripts to denote time and superscripts to denote currency. If a value is not dependant on time or currency, we may use the subscripts or superscripts for other things. For example, we denote the current fair value of a derivative security in the currency of the derivative as F_0^S Similarly we denote the fair value of a derivative security yesterday in the currency of the underlying as F_{-1}^U

Building Blocks

In order to calculate position level risk, value, and P&L a system needs some numerical building blocks. The most fundamental piece of information is the unit value of the security which we denote F_t. Similarly, we use S_t to denote the market price of the underlying security at time t.

The subscript denotes the time to which the number corresponds. A subscript of zero indicates a live number and a subscript of -1 corresponds the value as of yesterday. Since such a number corresponds to the raw per unit value, we need to scale it up to the position level. There are several scaling factors which are applied to the raw value in order to scale it up to the position level. While complex products may require many multipliers, in practice most vanilla type products require two multipliers which map to the number of contracts Q_t and the contract multiplier M.

Often, we quote position level values in US dollars, so we need a set of exchange rates. We let X_t^S be the value of 1$ in the (derivative) security currency at time t and X_t^U be the value of 1$ in the currency of the derivative security's underlying.

When calculating the unrealized P&L on a position we need to know what was paid for the position. For this we need the average unit cost paid which we denote as a_t.

Value

Position Fair Value

The fair value of the position in us dollars denoted V_t^{usd} is given by

$$V_t^{usd} = F_t^S \times M \times Q_t \times \frac{1}{X_t^S}$$

Position Market Value

Similarly, if we let P_t^{usd} be the unit market price in us dollars at time t, the position market values as

$$P_t^S \times M \times Q_t \times \frac{1}{X_t^S}$$

P&L

The total P&L numbers consist of several parts the most important of which are

- Realized P&L – caused by trading activity
- Unrealized P&L – caused by market data changing.

UNREALIZED P&L

The inception to date unrealized P&L of the position is defined as the difference between the current value of the position and what we paid for it. If we use the fair value as the unit worth, we can define the inception to date fair unrealized P&L as follows.

$$U_t^f \times Q_t \times \left[\left(F_t \times M \times \frac{1}{X_t^S}\right) - \left(a_t \times M \times \frac{1}{X_t^S}\right)\right]$$

If we use the market price as the unit worth we can define the inception to date market unrealized P&L as follows

$$U_t^p \times Q_t \times \left[\left(P_t \times M \times \frac{1}{X_t^S}\right) - \left(a_t \times M \times \frac{1}{X_t^S}\right)\right]$$

The day to day fair unrealized P&L is then defined as the inception to date fair unrealized P&L less the inception to yesterday fair unrealized P&L

$$U_0^f - U_{-1}^f$$

The day to day market unrealized P&L is then defined as the inception to date market unrealized P&L less the inception to yesterday market unrealized P&L

$$U_0^p - U_{-1}^p$$

Realized P&L

We calculate the realized P&L by tracking trades. If the trade is in the opposite direction to the position, we adjust the running realized P&L by the product of the quantity traded and the difference between the average cost and the trade price.

Ignoring the case where the direction on the position changes sign the basic adjustment to the realized P&L of any trading will be

$$Realized = Realized + [(Trade\ Price - Average\ Cost) \times Quatity\ Bought/Sold]$$

Unrealized P&L

Unrealized P&L can be calculated from the average cost and the current market value or price (depending on instrument). Trades have no real impact on the unrealized P&L other than as adjustments to the average cost.

Unrealized P&L as an inception to date number is then simply the difference between the market value or price and the average cost, multiplied by the quantity. We can calculate a day to day unrealized number by calculating the inception to date number and the inception to yesterday number and taking the difference.

Calculating the Average Cost

At each trade we adjust the average accost according to the following algorithm.

1. If the trade is in the same direction as the position
    a. Realized P&L is not changed
    b. Unrealized is changed due to the average cost changing
2. If the trade is in the opposite direction to the position quantity
    a. If trade size is less than position size average cost stays the same
    b. If trade causes sign change average cost becomes trade price

c. Realized P&L is adjusted but unrealized remains the same

Trades in the same direction cause adjustments to the unrealized P&L via changes to the average cost. Trades in the opposite direction cause changes to the realized P&L but don't affect the average cost or unrealized P&L (unless the sign of the position changes)

Attribution

The total derivative is calculated from the Taylor series as follows.

$$\Delta f \cong \frac{\partial f}{\partial S}\Delta S + \frac{\partial f}{\partial \sigma}\Delta\sigma$$

We can then add second order terms as needed.

Delta P&L

The delta P&L uses a first order Taylor series approximation of the change in value due to the change in spot. As such it is given by

$$\frac{\delta_{t-1} \times (S_t - S_{t-1})}{X_t} \times P_{t-1} \times M \times Q_t$$

Gamma P&L

The gamma P&L is given by

$$\frac{1}{2}\gamma_{t-1}(S_t - S_{t-1})^2 \times P_{t-1} \times M \times Q_t$$

Rather than use yesterdays gamma we can also use a combination of todays and yesterdays delta via a finite difference approximation

$$\frac{1}{2}\frac{\delta_t - \delta_{t-1}}{(S_t - S_{t-1})}(S_t - S_{t-1})^2 \times P_{t-1} \times M \times Q_t$$

Vega P&L

$$\frac{\delta_{t-1} \times 100 \times (\sigma_t - \sigma_{t-1})}{X_t} P_{t-1} \times M$$

Risk

Risk numbers use partial derivatives to provide approximations of how the value of a position will change as the market data changes. Risk numbers, known as ‘the Greeks’, look to the future and answer the questions such as “what will happen if the spot price moves by 1%”, “what will happen if rates decrease by 1%”

Raw Greeks

Delta

The raw delta is defined as the sensitivity of the derivative security to a one-unit change in the underlying spot level. The underlying price is quoted in the underlying currency and the raw delta is quoted in the currency of the security and is hence given by

$$\boldsymbol{\delta_t = \frac{X_t^S}{X_t^u}\left(\frac{\partial F^u}{\partial S^u}\right)_t}$$

For non-cross-currency options $\frac{X_t^S}{X_t^u}$ is equal to one.

Gamma

The raw gamma is defined as the sensitivity of the securities delta to a one-unit change in the spot level. This is the model’s raw gamma.

$$\gamma_t = \frac{X_t^S}{X_t^u}\left(\frac{\partial^2 F^u}{(\partial S^u)^2}\right)$$

Vega

The raw Vega is the change in value for a one percentage point move in the volatility, expressed in the derivative currency

$$\left(\frac{\partial F^u}{\partial \sigma_{\%}}\right)_t$$

Theta

The raw theta is the change in value of a unit of the derivative security for a one day decrease in time to expiry

$$\vartheta_t = \left(\frac{\partial \mathrm{F}^{\mathrm{u}}}{\partial \mathrm{T}_{day}}\right)_t$$

Cash Greeks

Cash Delta

The cash delta is expressed in the currency of the derivative security

$$\delta_{\mathrm{t}} \times S_t^s$$

It can be interpreted as

1. The currency value of the stock position to be held against the derivative position
2. The change in value of the derivative for a 100% move in the spot

Cash Gamma

The cash gamma per 1% is defined as $\frac{\gamma_t \times (S_t^S)^2}{100}$ As such it represents the change in the cash delta for a 1% move in the spot due to the need to change position. Note that while

$$\frac{d^2 F^u}{d(lnS^u)^2} = (S_t^2)^2 \frac{d^2 F^u}{d(S^u)^2} + S_t^S \frac{dF^u}{dS^u}$$

We do not include the term $S_t^S \frac{dF^u}{dS^u}$in the cash gamma

Delta 1%

$$\boldsymbol{RawDelta_{\%}} = \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{S}} \times \frac{\boldsymbol{S}}{\boldsymbol{100}}$$

Position Level Greeks

Position Delta

Position Delta means the total delta of the position as a currency amount in US dollars.

$$\boldsymbol{\delta}_{\mathbf{t}} \times S_t^S \times M \times Q_t \times \frac{1}{X_t^S}$$

Position Gamma

The position gamma 1% defines the change in value of the delta for a 1% move in the spot level

$$\gamma_t \times M \times Q_t \times (S_t^S)^2 \times \frac{1}{100} \times \frac{1}{X_t^S}$$

Position Vega

The Vega 1% defines the change in value of the position for a 1% move in volatility, expressed in USD.

$$\left(\frac{\partial \mathrm{F}^{\mathrm{u}}}{\partial \sigma_{\%}}\right)_t \times Q_t \times M \times \frac{1}{X_t^S}$$

Position Theta

The position theta defines the change in value of the position for a one day decrease in time to expiry. We express the result in USD

$$\vartheta_t \times Q_t \times M \times \frac{1}{X_t^S}$$