Sets

Introduction

Common Sets

TABLE 1 COMMON SETS

Set	Meaning	Example	Description/Example
$\mathbb{N}$	The natural numbers	$\{\mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots\}$	Can sometimes be assumed to include 0, especially by computer scientists
$\mathbb{Z}$	The integers	$\{\ldots -2, -1,0,1,2 \ldots\}$	
$\mathbb{Z}^{+}$	The positive integers	$\{0,1,2 \ldots\}$	
$\mathbb{Q}$	The rational numbers		
$\mathbb{R}$	The real numbers		
$\mathbb{C}$	Complex numbers		

Note that:

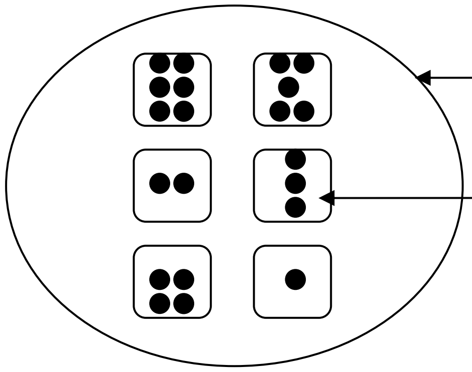
$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

Defining Sets

TABLE 2 DEFINING SETS

A set is a collection of things. We call the things elements of the set. If a set consists of the difference faces of a die we can write.

$$S = \{x: 1 \leq x \leq 6, x \in Z\}$$



Operations

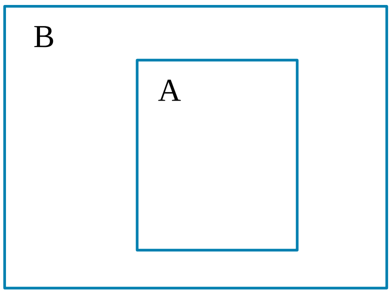
TABLE 3 SET DEFITINITONS

Symbol	Meaning	Example	Description/Example
$\in$	Membership	$1 \in \{1,2\}$	1 is in the set $\{1,2\}$
$\notin$	Not a member	$3 \notin \{1,2\}$	3 is not in $\{1,2\}$
$\lvert\ \ \rvert$	Cardinality	$\lvert\emptyset\rvert = \mathbf{0}$, $\lvert\{\emptyset\}\rvert = \mathbf{1}$,	The cardinality of the empty set is zero. The cardanlity of a set containing the empty set is one.
$\subseteq$	Subset	$\{1\} \subseteq \{1,2\}$, $\{1,2\} \subseteq \{1,2\}$,	
$\subset$	Proper subset	$\{1\} \subset \{1,2\}$	
$\cap$	Intersection	$\boldsymbol{A} \cap \boldsymbol{B}$	
$\cup$	Union		
$\setminus$	Difference/Relative Complement	$A \setminus B$	
A^C	Absolute complement	$\boldsymbol{A^C}$	

$[\boldsymbol{a},\boldsymbol{b}]$	Open Interval	$[a,b]=\{x\in\mathbb{R}\vert a\leq x \leq b\}$	
$(\boldsymbol{a},\boldsymbol{b})$	Closed Iterval	$(a,b)=\{x\in\mathbb{R}\vert a< x < b\}$	
$\times$	Cross Product / Cartesian Product	$A\times B$	

⊆ - Subset

$If\ x\in A\to x\in B\ then\ A\subseteq B$ A



$A\subseteq B$

⊂ - Proper Subset

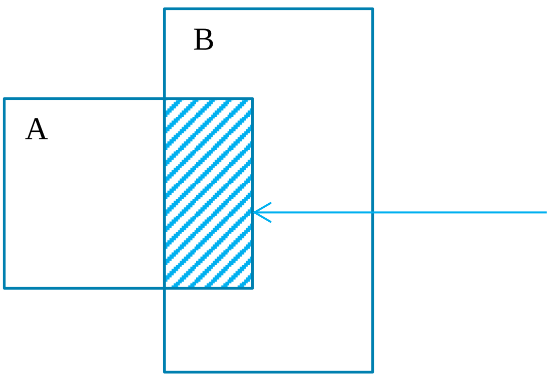
$A\subseteq B\ and\ A\ \neq B\to A\subset B$

= Equality

$A=B\ \leftrightarrow A\subseteq B\ and\ B\subseteq A$

∩ - Intersection

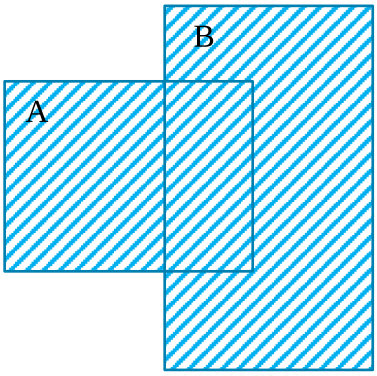
$A\cap B\to\ x\in A\ and\ x\in B$



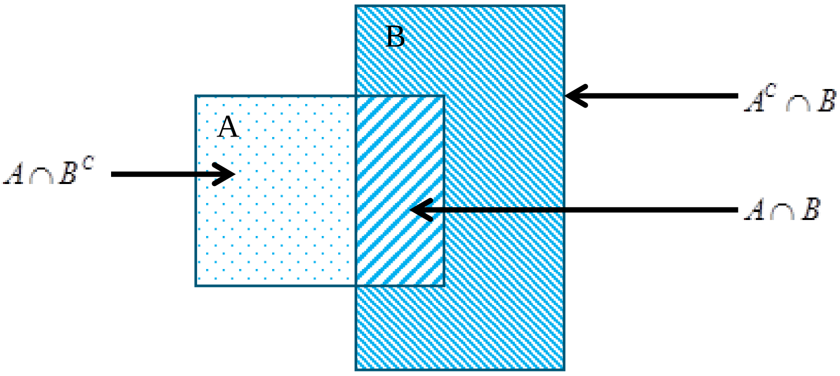
$A\cap B$

∪ - UNION

$A \cup B = \ x \in A \ or \ x \in B$



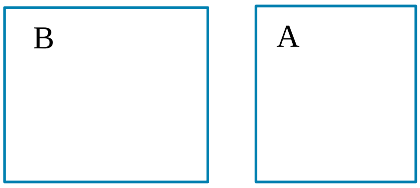
Note the following about the union of A and B



$$A \cup B = (A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$$
$$= A \backslash B \cup B \backslash A \cup (A \cap B)$$

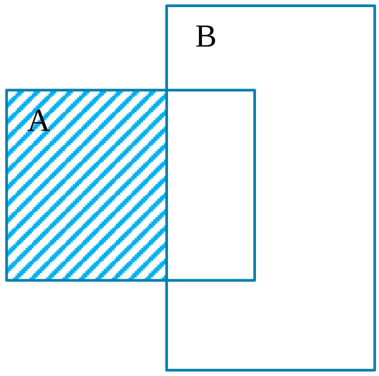
Disjoint Sets

$A \cap B = \emptyset$



$A\backslash B$ -Difference (Relative Complement)

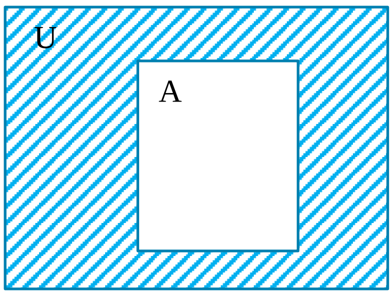
$A\backslash B = \; x \in A \; and \; x \notin B = A \cap B^C$



A^c – Absolute Complement

$A^C = \bar{A} = \; x \in U \; and \; x \notin A \; = U\backslash A$

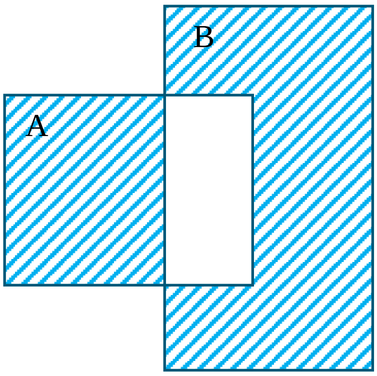
The set of elements in the universe U which are not in the set A



$A\Delta B$ Symmetric Difference

The elements in A or B but not in both

$A\Delta B = (A\backslash B) \cup (B\backslash A)$



$$A \Delta B = (A \setminus B) \cup (B \setminus A)$$

Cross Product / Cartesian Product

If A and B are sets we can form the product C as

$$C = \{(a, b): a \in A, b \in B\}$$

And we write

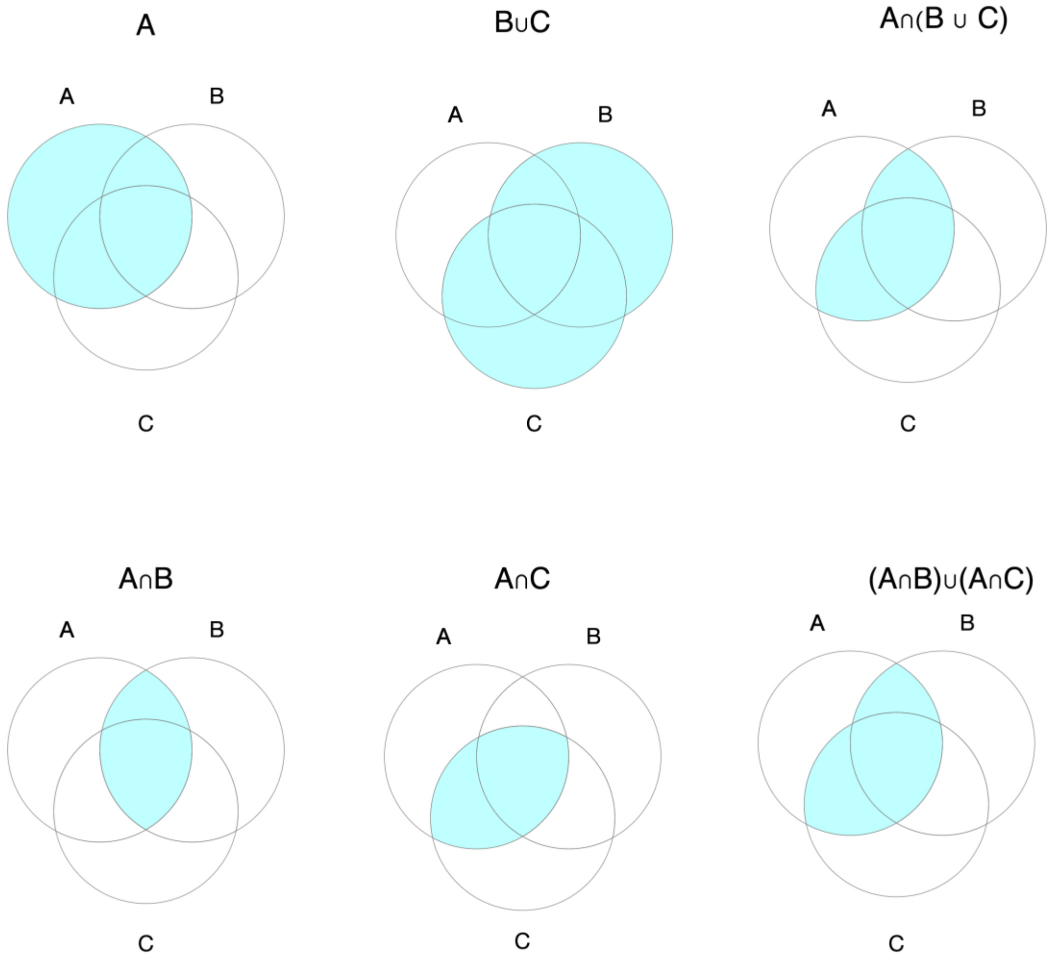
$$C = A \times B$$

Rules

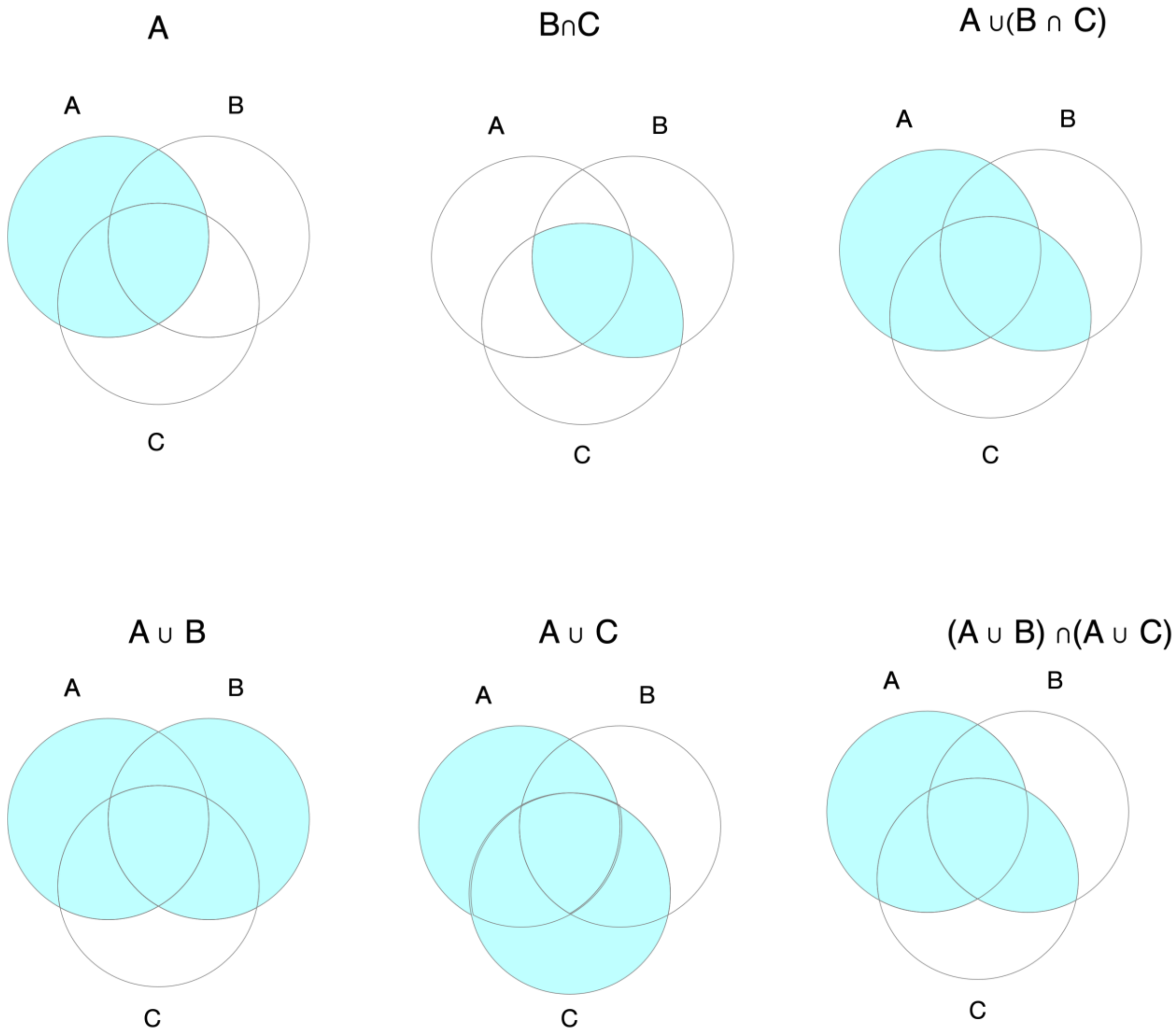
Table 4 Rules

Law	Name	Description/Example
$\mathrm{A} \cap \mathrm{A} = \mathrm{A}$	Idempotency	
$\mathrm{A} \cup \mathrm{A} = \mathrm{A}$	Idempotency	
$\mathrm{A} \cap \mathrm{B} = \mathrm{B} \cap \mathrm{A}$	Commutative	
$\mathrm{A} \cup \mathrm{B} = \mathrm{B} \cup \mathrm{A}$	Commutative	
$(A \cup B) \cup C = \mathrm{A} \cup (B \cup C)$	Associative	
$(A \cap B) \cap C = \mathrm{A} \cap (B \cap C)$	Associative	
$\mathrm{A} \cap (A \cup B) = A$	Absorption	
$\mathrm{A} \cup (A \cap B) = A$	Absorption	
$\mathrm{A} \cap (B \cup C) = (\mathrm{A} \cap B) \cup (\mathrm{A} \cap C)$	Distributive	
$\mathrm{A} \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive	
$\overline{(A \cup A)} = \bar{A} \cap \overline{\mathrm{B}}$	De Morgan	
$\overline{(A \cap A)} = \bar{A} \cup \overline{\mathrm{B}}$	De Morgan	
$\mathrm{A} \setminus (\mathrm{B} \cap \mathrm{C}) = (\mathrm{A} \setminus B) \cup (\mathrm{A} \setminus C)$	De Morgan	
$\mathrm{A} \setminus (\mathrm{B} \cup \mathrm{C}) = (\mathrm{A} \setminus B) \cap (\mathrm{A} \setminus C)$	De Morgan	

Distributive Law 1 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



DISTRIBUTIVE LAW 2 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



DEMORGAN LAW $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$