Sets

Overview

Operators

Symbol	Meaning	Example	Description/Example
€	Is in the set	1 ∈ {1,2}	1 is in the set {1,2}
∉	Is not in the set	3 ∉ {1,2}	3 is not in {1,2}
	Cardinality	$ \emptyset = 0, \{\emptyset\} = 1,$	The cardinality of the empty set is zero.
⊆	Subset	$\{1\} \subseteq \{1,2\},$	
		$\{1,2\}\subseteq\{1,2\},$	
С	Proper subset	{1} ⊂ {1,2}	
Λ	Intersection	$A \cap B$	
U	Union		
\	Difference/Relative Complement	$A \setminus B$	
A^{C}	Absolute complement	$A^{\mathcal{C}}$	
$A\Delta B$	Symmetric difference		
[a, b]	Open Interval	$[a,b] = \{x \in \mathbb{R} a \le x \\ \le b\}$	
(a , b)	Closed Iterval	$(a,b) = \{x \in \mathbb{R} a < x < b\}$	
×	Cross Product / Cartesian Product	$A \times B$	

Rules

Law	Name	Description/Example
$\mathbf{A} \cap \mathbf{A} = \mathbf{A}$	Idempotency	
$\mathbf{A} \cup \mathbf{A} = \mathbf{A}$	Idempotency	
$\mathbf{A} \cap \mathbf{B} = \mathbf{B} \cap \mathbf{A}$	Commutative	
$\mathbf{A} \cup \mathbf{B} = \mathbf{B} \cup \mathbf{A}$	Commutative	
$(A \cup B) \cup C = A \cup (B \cup C)$	Associative	
$(A \cap B) \cap C = A \cap (B \cap C)$	Associative	
$\mathbf{A}\cap(\mathbf{A}\cup\mathbf{B})=\mathbf{A}$	Absorption	
$\mathbf{A}\cup(\mathbf{A}\cap\mathbf{B})=\mathbf{A}$	Absorption	
$\mathbf{A} \cap (\mathbf{B} \cup \mathbf{C}) = (\mathbf{A} \cap \mathbf{B}) \cup (\mathbf{A} \cap \mathbf{C})$	Distributive	
$\mathbf{A} \cup (\mathbf{B} \cap \mathbf{C}) = (\mathbf{A} \cup \mathbf{B}) \cap (\mathbf{A} \cup \mathbf{C})$	Distributive	
$\overline{(A \cup A)} = \overline{A} \cap \overline{B}$	De Morgan	
$\overline{(A \cap A)} = \overline{A} \cup \overline{B}$	De Morgan	
$\mathbf{A}\setminus(\mathbf{B}\cap\mathbf{C})=(\mathbf{A}\setminus\mathbf{B})\cup(\mathbf{A}\setminus\mathbf{C})$	De Morgan	
$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$	De Morgan	

Common Sets

Set	Meaning	Example	Description/Example
N	The natural numbers	{1, 2, 3,}	Can sometimes be assumed to include 0, especially by computer scientists
Z	The integers	{ – 2, –1,0,1,2}	
\mathbb{Z}^+	The positive integers	{0,1,2}	
Q	The rational numbers		
R	The real numbers		
C	Complex numbers		

Note that:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

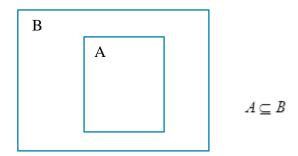
Defining Sets

A set is a collection of things. We call the things elements of the set. If a set consists of the difference faces of a die we can write.

$$S = \{x: 1 \le x \le 6, x \in Z\}$$

⊆ - SUBSET

If $x \in A \rightarrow x \in B$ then $A \subseteq B$ A



⊂ - Proper Subset

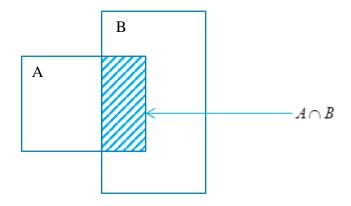
 $A \subseteq B \ and \ A \neq B \rightarrow A \subset B$

= **EQUALITY**

 $A = B \leftrightarrow A \subseteq B \text{ and } B \subseteq A$

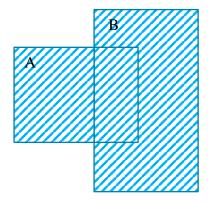
∩ - Intersection

 $A \cap B \rightarrow x \in A \ and \ x \in B$

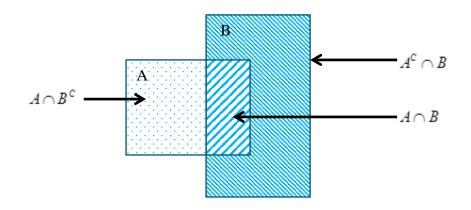


U - UNION

 $A \cup B = x \in A \text{ or } x \in B$



Note the following about the union of A and B



$$A \cup B = (A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$$
$$= A \backslash B \cup B \backslash A \cup (A \cap B)$$

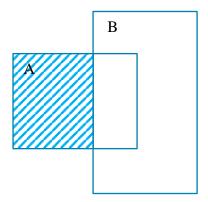
Disjoint Sets

$$A \cap B = \emptyset$$

В

 $A \setminus B$ -Difference (Relative Complement)

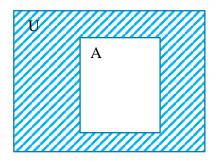
$$A \backslash B = x \in A \ and \ x \notin B = A \cap B^C$$



A^c – ABSOLUTE COMPLEMENT

$$A^C = \bar{A} = x \in U \text{ and } x \notin A = U \backslash A$$

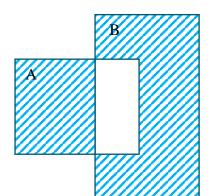
The set of elements in the universe U which are not in the set A



AΔB SYMMETRIC DIFFERENCE

The elements in A or B but not in both

$$A\Delta B=(A\backslash B)\cup(B\backslash A)$$



$$A\Delta B = (A \setminus B) \cup (B \setminus A)$$

CROSS PRODUCT / CARTESIAN PRODUCT

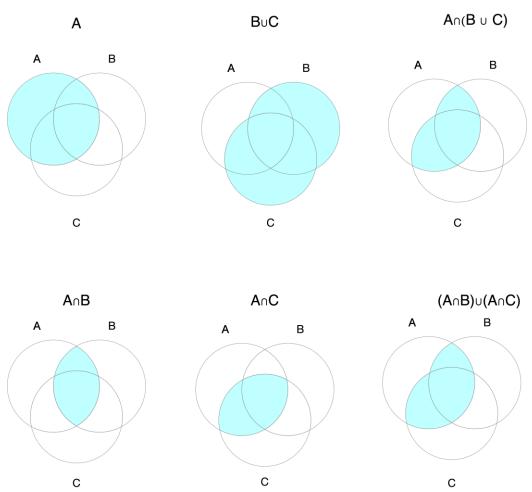
If A and B are sets we can form the product C as

$$C = \{(a,b) \colon a \in A, b \in B\}$$

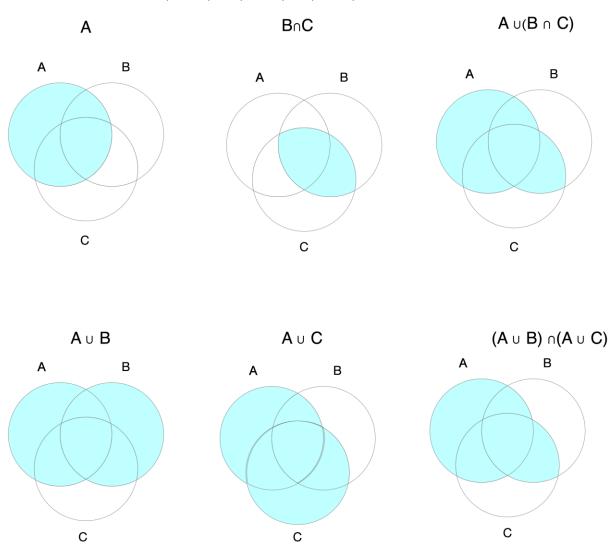
And we write

$$C = A \times B$$

DISTRIBUTIVE LAW 1 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



DISTRIBUTIVE LAW 2 A \cup (B \cap C) = (A \cup B) \cap (A \cup C)



DEMORGAN LAW $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$