Differentiation

Contents

THIS DOCUMENT COVERS

- The rules of differentiation
- Definition of the derivative

Table 1 Rules of Differentiation

1. Constant Function Rule $f'(c) = 0, if\ c\ is\ a\ constant$

2. Constant Multiple Rule
$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

3. Sum rule
$$\frac{d}{dx}f(x) + g(x) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

4. Product Rule
$$\frac{d}{dx}f(x)g(x) = f(x)\frac{d}{dx}g(x) + \frac{d}{dx}f(x)g(x)$$

5. Quotient Rule
$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{\frac{d}{dx}f(x)g(x) - \frac{d}{dx}g(x)f(x)}{[g(x)]^2}$$

6. Power Rule
$$\frac{d}{dx}[x^n] = nx^{n-1}$$

7. Chain Rule If $y = f(g(x))$ then $\frac{dy}{dx} = \frac{df}{dg}\frac{dg}{dx}$

8. Derivative of e^x
$$\frac{d}{dx}e^x = e^x$$

9. Inverse Function
$$\frac{d}{dx}f^{-1}(y) = \frac{dx}{dy} \times \frac{dy}{dx} = 1$$

10. Natural logarithm
$$\frac{dy}{dx}ln\,x = \frac{1}{x}$$

Definition of the derivative

The derivative measures the rate of change of one quantity with respect to another. Differentiation is then just the process of finding the derivative of a function. If we have a function of x then one of the many notations for specifying the derivative is as follows

$$\frac{d}{dx}f(x)$$

So if we took one or the simplest non-linear functions $f(x) = x^2$and differentiate it we see that

$$\frac{d}{dx}x^2 = 2x.$$

So in the simple case where x is equal to one

$$\frac{d}{dx}f(x) = 2$$

$$df(x) = dx \times 2$$

It is worth noting that differential calculus is concerned with finding the instantaneous rate of change of f with respect to x.

Why Bother?

Numerous problems in business, economics and finance are concerned with determining how one quantity is changing with respect to another. Differentiation also enables us to find where a function is highest and lowest both locally and across the entire domain. Also we often find where a rate of change is greatest or smallest and again differentiation provides us with this.

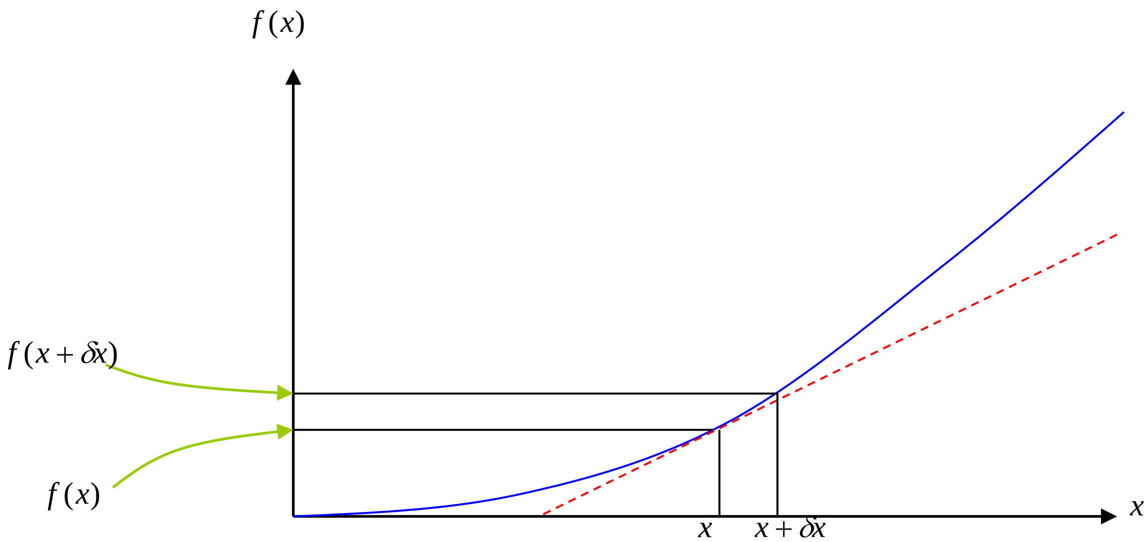
Where the derivative does not exist

There are three places where the derivative does not exist

- Discontinuity
- Cusp on a function
- Vertical inflection point

Calculation of the derivative

The approach



$$\frac{f(x+\delta x)-f(x)}{\delta x} \to \frac{df(x)}{dx}$$

Algebraic Derivation

If $y = f(x)$ then the derivative $\frac{dy}{dx} = \frac{d}{dx}\big(f(x)\big)$is defined as $lim_{x\to\alpha} \frac{f(x+\delta x)-f(x)}{\delta x}$

Simple Example

Let us consider a basic quadratic $y = f(x) = x^2$ then the derivative becomes

$$lim_{x\to\alpha} \frac{f(x+\delta x)-f(x)}{\delta x} = \frac{(x+\delta x)^2-x^2}{\delta x} = 2x$$

Proofs

Constant Function

The derivative of a constant function is zero

$$f'(c) = 0$$

$$f'(x) = lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = lim_{h \to 0} \frac{c - c}{h} = 0$$