Sets

Overview

Operators

Symbol	Meaning	Example	Description/Example
$\in$	Is in the set	$1 \in \{1,2\}$	1 is in the set $\{1,2\}$
$\notin$	Is not in the set	$3 \notin \{1,2\}$	3 is not in $\{1,2\}$
$\lvert\ \ \rvert$	Cardinality	$\lvert\emptyset\rvert = \mathbf{0}, \lvert\{\emptyset\}\rvert = \mathbf{1},$	The cardinality of the empty set is zero.
$\subseteq$	Subset	$\{1\} \subseteq \{1,2\}$, $\{1,2\} \subseteq \{1,2\}$,	
$\subset$	Proper subset	$\{1\} \subset \{1,2\}$	
$\cap$	Intersection	$\boldsymbol{A \cap B}$	
$\cup$	Union		
$\setminus$	Difference/Relative Complement	$A \setminus B$	
$\boldsymbol{A^C}$	Absolute complement	$\boldsymbol{A^C}$	
$\boldsymbol{A \Delta B}$	Symmetric difference		
$[\boldsymbol{a}, \boldsymbol{b}]$	Open Interval	$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$	
$(\boldsymbol{a}, \boldsymbol{b})$	Closed Iterval	$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$	
$\times$	Cross Product / Cartesian Product	$A \times B$	

Rules

Law	**Name**	**Description/Example**
$\mathbf{A} \cap \mathbf{A} = \mathbf{A}$	Idempotency	
$\mathbf{A} \cup \mathbf{A} = \mathbf{A}$	Idempotency	
$\mathbf{A} \cap \mathbf{B} = \mathbf{B} \cap \mathbf{A}$	Commutative	
$\mathbf{A} \cup \mathbf{B} = \mathbf{B} \cup \mathbf{A}$	Commutative	
$(\boldsymbol{A} \cup \boldsymbol{B}) \cup \boldsymbol{C} = \mathbf{A} \cup (\boldsymbol{B} \cup \boldsymbol{C})$	Associative	
$(\boldsymbol{A} \cap \boldsymbol{B}) \cap \boldsymbol{C} = \mathbf{A} \cap (\boldsymbol{B} \cap \boldsymbol{C})$	Associative	
$\mathbf{A} \cap (\boldsymbol{A} \cup \boldsymbol{B}) = \boldsymbol{A}$	Absorption	
$\mathbf{A} \cup (\boldsymbol{A} \cap \boldsymbol{B}) = \boldsymbol{A}$	Absorption	
$\mathbf{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\mathbf{A} \cap \boldsymbol{B}) \cup (\mathbf{A} \cap \boldsymbol{C})$	Distributive	
$\mathbf{A} \cup (\boldsymbol{B} \cap \boldsymbol{C}) = (\boldsymbol{A} \cup \boldsymbol{B}) \cap (\boldsymbol{A} \cup \boldsymbol{C})$	Distributive	
$\overline{(\boldsymbol{A} \cup \boldsymbol{A})} = \overline{\boldsymbol{A}} \cap \overline{\mathbf{B}}$	De Morgan	
$\overline{(\boldsymbol{A} \cap \boldsymbol{A})} = \overline{\boldsymbol{A}} \cup \overline{\mathbf{B}}$	De Morgan	
$\mathbf{A} \setminus (\mathbf{B} \cap \mathbf{C}) = (\mathbf{A} \setminus \boldsymbol{B}) \cup (\mathbf{A} \setminus \boldsymbol{C})$	De Morgan	
$\mathbf{A} \setminus (\mathbf{B} \cup \mathbf{C}) = (\mathbf{A} \setminus \boldsymbol{B}) \cap (\mathbf{A} \setminus \boldsymbol{C})$	De Morgan	

Common Sets

Set	Meaning	Example	Description/Example
$\mathbb{N}$	The natural numbers	$\{\mathbf{1},\mathbf{2},\mathbf{3},\ldots\}$	Can sometimes be assumed to include 0, especially by computer scientists
$\mathbb{Z}$	The integers	$\{\ldots-2,-1,0,1,2\ldots\}$	
$\mathbb{Z}^+$	The positive integers	$\{0,1,2\ldots\}$	
$\mathbb{Q}$	The rational numbers		
$\mathbb{R}$	The real numbers		
$\mathbb{C}$	Complex numbers		

Note that:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

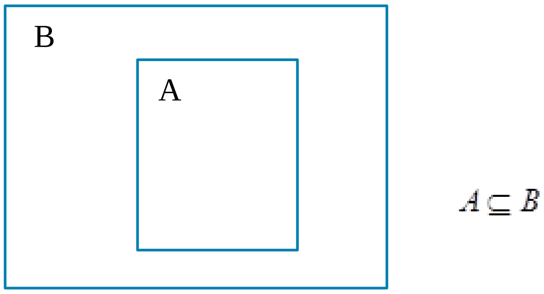
Defining Sets

A set is a collection of things. We call the things elements of the set. If a set consists of the difference faces of a die we can write.

$$S = \{x: 1 \leq x \leq 6, x \in Z\}$$

⊆ - Subset

$If\ x \in A \rightarrow x \in B\ then\ A \subseteq B$ A



⊂ - Proper Subset

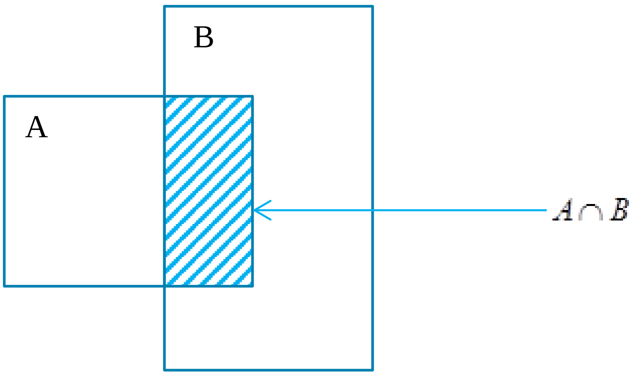
$A \subseteq B\ and\ A\ \neq B \rightarrow A \subset B$

= Equality

$A = B\ \leftrightarrow A \subseteq B\ and\ B \subseteq A$

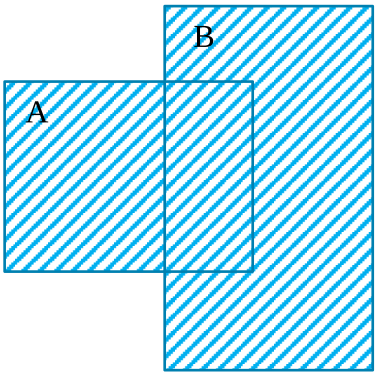
∩ - Intersection

$A \cap B \rightarrow\ x \in A\ and\ x \in B$

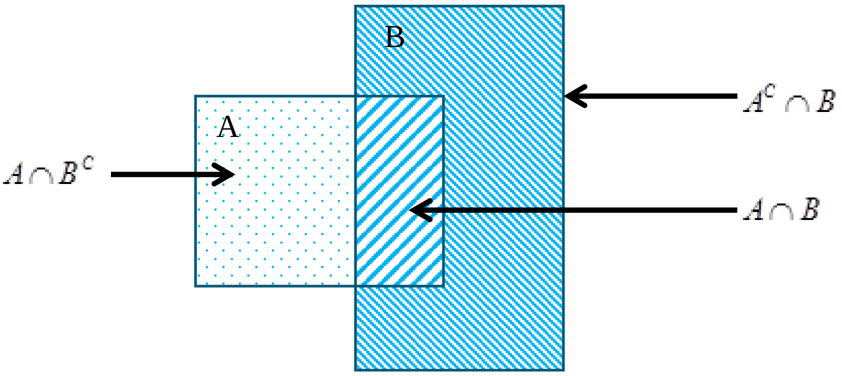


∪ - Union

$A \cup B =\ x \in A\ or\ x \in B$



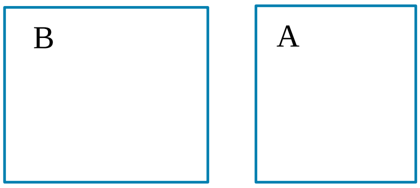
Note the following about the union of A and B



$$A \cup B = (A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$$
$$= A \backslash B \cup B \backslash A \cup (A \cap B)$$

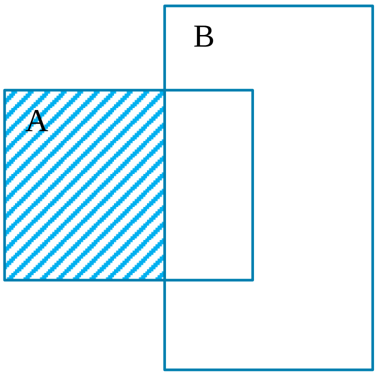
Disjoint Sets

$A \cap B = \emptyset$



$A \backslash$B - Difference (Relative Complement)

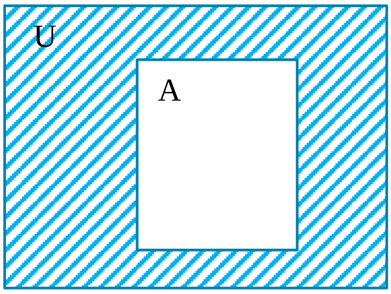
$A \backslash B = \ x \in A \ and \ x \notin B = A \cap B^C$



A^c – Absolute Complement

$A^C = \bar{A} = \ x \in U \ and \ x \notin A \ = U \backslash A$

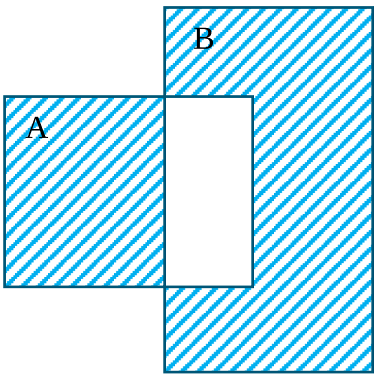
The set of elements in the universe U which are not in the set A



$A\Delta B$ Symmetric Difference

The elements in A or B but not in both

$A\Delta B = (A \backslash B) \cup (B \backslash A)$



$A\Delta B = (A \backslash B) \cup (B \backslash A)$

Cross Product / Cartesian Product

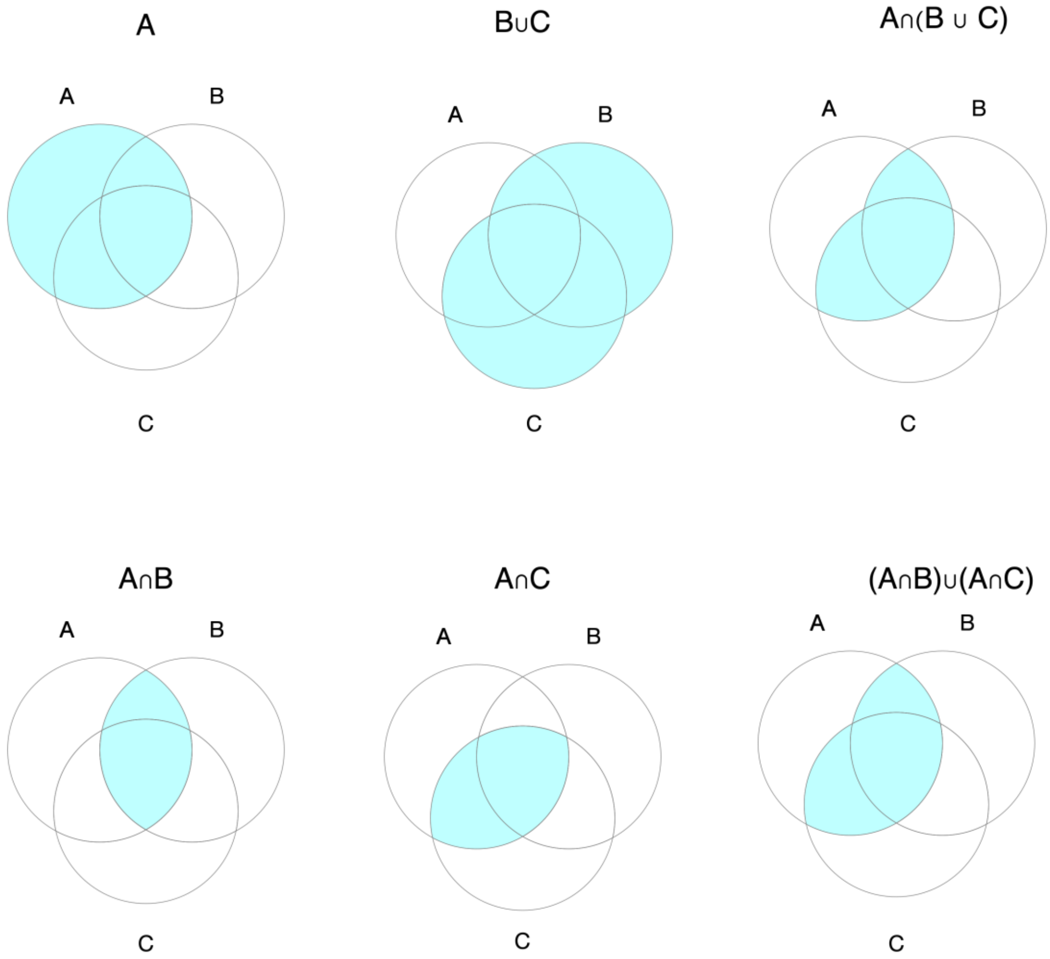
If A and B are sets we can form the product C as

$$C = \{(a, b): a \in A, b \in B\}$$

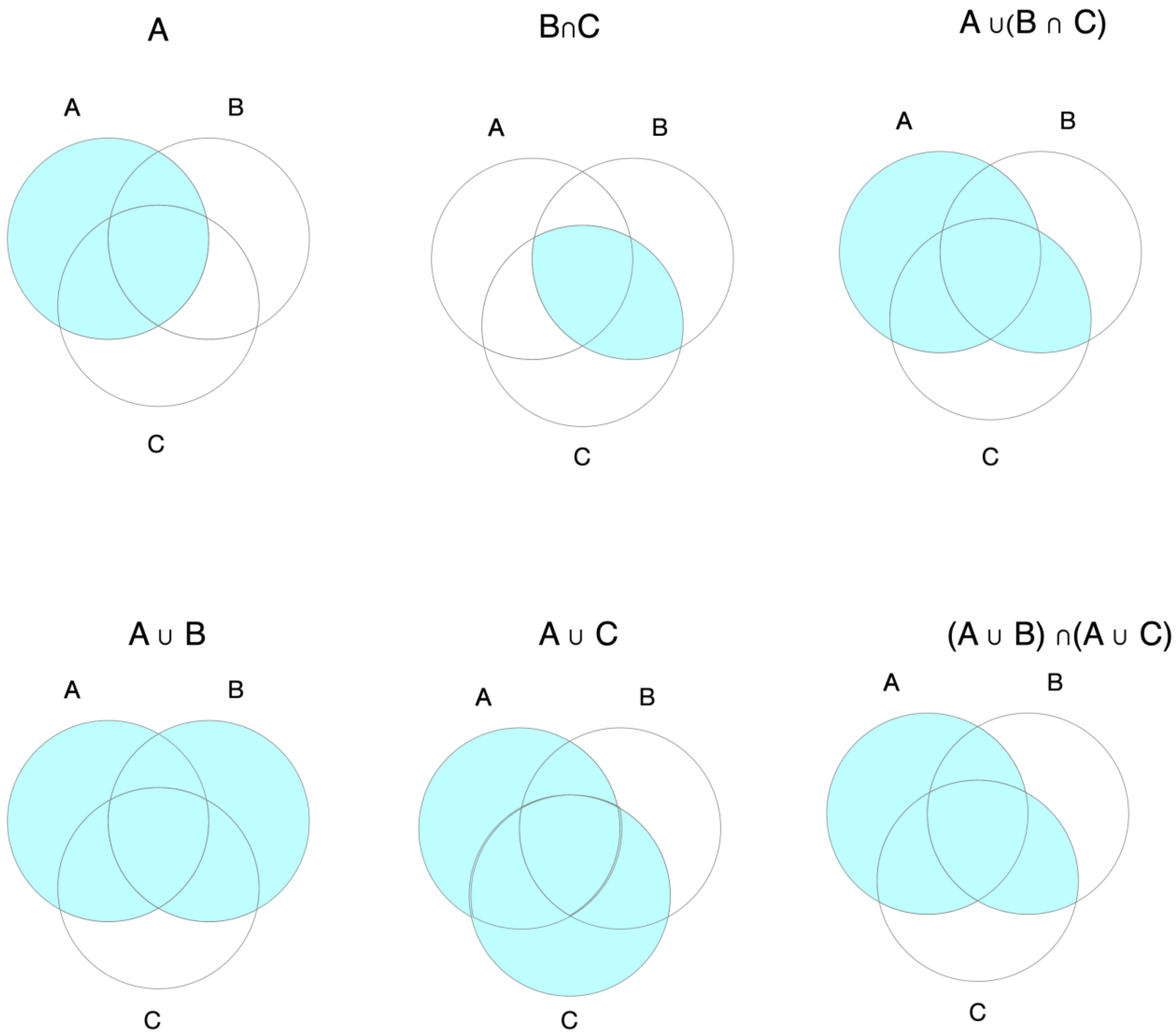
And we write

$$C = A \times B$$

DISTRIBUTIVE LAW 1 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



Distributive Law 2 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



DeMorgan Law $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$