Series

## Introduction

Even elementary functions can be difficult to work with. Even simple functions like can be difficult to evaluate because every integer input results in an irrational. A small subset, however, known as the polynomials are much easier to work with because they are

* Easy to integrate
* Easy to evaluate for any value of x
* Infinitely differentiable

The tactic of expressing complicated functions as infinite series motivates much of the study of infinite series

## Sums

### Properties of Sums

Listing Properties of Sums

|  |  |
| --- | --- |
|  |  |
|  |  |
| 1. Sum of first n integers |  |

#### Proofs

##### Sum of first n integers

Because

and

therefore

## Sequences

### Definition

* List of numbers in a definitive order 
* If has a limit L we can make as close to L as we like by increasing n
* is bounded above if there is some M such that 
* is bounded below if there is some M such that 
* Every bounded monotonic sequence is convergent



### Notation

The following notations are equivalent

Some sequences can be defined by a formula for the nth term.

## Series

### Definition

* Obtained by adding the terms of an infinite sequence
* Given a series we can generate a sequence of its partial sums
* If the resulting sequence is convergent we say the series is convergent
* With any series we associate two sequences
* the terms
* the partial sums
* If is convergent
* If does not exist is divergent
* thenis divergent

### Geometric Series

A geometric series is a series where the ratio of each term to its predecessor is constant.

#### Value of A Geometric Series

We prove this useful result by noting that

Subtracting the second expression from the first we get

Factoring both sides

Dividing both sides by

#### Infinite Geometric Series

If the first N elements in a geometric series are given by then the first N-1 elements will be given by and in the case where x is greater than -1 and less than one then in the limit as the number of elements tends to infinity we get

Since in the case where the absolute value of x is less than zerowill tend to zero as N tends to infinity the second term also tends to zero and we have an expression for an infinite geometric series.

## Power Series

* For each x the series is a series of constants which we can test for convergence
* The sum of the series is a function
* The domain of this function is the set of x for which the series converges

## Taylor and Maclaurin Series

We can approximate a functionfor small values around some anchor point a, as a power series.

For a giventhe Taylor series will only be valid for a given subset of the domain, known as the radius of convergence.

## Value of an infinite geometric Series having r < 1



We prove this useful result by looking at the limit



And noting that the numerator will tend to a as n tends to infinity

### Value of an infinite geometric Series having r < 1 (2)

