Interest Rates

And discount factors

## Introduction

This document covers

Introduction

Interest rates are the bedrock of finance. Whether it is the man on the street taking out a mortgage to purchase a house or an investment bank trading complex derivatives, it is interest rates that specify the cost of capital and the return on investment of financial products.

In this article we discuss how interest rates are quoted. We show how to convert rates quoted in one quoting convention to an equivalent rate in a different convention. The important concept of a discount factor is also introduced at this stage. Building on this foundation we move on to the concept of a forward interest rate and show the relationship between spot starting rates and forward starting rates. Finally, we introduce the concept of a yield or discount curve and show how different interpolation schemes can be used to extract rates for maturities that do not have actual data points on the curve.

## Rate Definitions

There are three aspects that make interests rates more complicated than simple returns or growth factors

1. Quoted on an annualized basis
2. Compounding frequency
3. Day count conventions

Most people intuitively understand the first point. When we say the rate on our mortgage is 5%, we know it means 5% per annum. The second aspect is trickier. We consider that now

### Rate Expressions and compounding

The compounding frequency defines the units in which an interest rate is measured. A rate expressed with one compounding frequency can be converted into an equivalent rate with a different compounding frequency. They are two different units of measurement. We can convert a rate in one compounding frequency to the equivalent rate in another compounding frequency. Simple interest has no compounding and its rate expression is given by

#### Simple Interest

Moving on from simple interest we add compounding a given number of times per annum

#### M times per annum

#### Continuous

If we increase the compounding frequency without limit, we get continuous compounding

#### Converting Between Rate Types

Given two rate expressions withrespectiverates we can create expressions for converting between rates by letting and solving forin terms **of** We use this technique to convert in the following situations.

##### M Times per annum to continuous

Letting

Taking the natural logarithm of each side

Dividing through by

##### Continuous to m times per annum

##### N time per annum to m times per annum

Taking natural logarithms of each side

Note that 

Divide each side by mt

Note that 

Taking exponents

Subtract one from each side

Multiply both sides by m

Questions - Rate Definitions

What are the three aspects that make interest rates more complicated than simple returns?

Quoted on an annualized basis

Compounding frequency

Day count conventions

Give an expression for simple interest

Give an expression for m times per times per annum

Give an expression for continuous compounding

Give an expression for to convert a rate compounding m times per annum to a continuous rate?

Show the derivation

Letting

Taking the natural logarithm of each side

Dividing through by

Give an expression to convert from n times per annum to m times per annum?

Show the derivation

## Discount Factors

We are interested in how we can convert from a discount factor to each rate type and back again. In general, we have an equation of the form

So to convert from a given rate expression a discount factor we solve algebraically from the expression

Questions - Rate Definitions

Give a general expression to convert a rate expression to a discount factor?

Give a general expression to convert a rate m times per annum to a discount factor?

Give a general expression to convert a continuous rate to discount factor