Logic

And Truth

Given a model (such as the natural numbers) and a set of axioms that are true statements about the model we can use inference rules to deduce new statements from the axioms. A statement is true and hence a theorem if it can be deduced from axioms and previously proved theorems using the inference rules.

## Models

Table 1 Models

|  |  |
| --- | --- |
|  | The natural numbers |
|  | The integers |
|  | The rational numbers |
|  | The real numbers |
|  | The complex numbers |
| Sets | The universe of sets |
| Lamda Calculus |  |

## Operators

Table 2 Operators

|  |  |  |  |
| --- | --- | --- | --- |
| Operation | Symbol | Alternative Symbols | Descriptions |
|  |  |  | Negation of p |
|  |  |  | And |
|  |  |  | Exclusive or |
|  |  |  | Inclusive or |
|  |  |  | Implication. is an important connective. It can only be false if p is true and q is false. |
|  |  | p |  |

### Implication and Biconditional

#### Overview

The implication means that if p is true then q is true. We say that p is the hypothesis or assumption and that q is the conclusion. The truth table is given as follows.

Table Implication truth table

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |

The key feature of implication is that if p is true then q is true (assuming the implication itself is true). If p is false then an implication says nothing about the value of q which can be either true or false. The only combination of values which can make an implication itself false are when p is true and q is false. In such case the implication itself is false.

We can describe implication in the following ways

* p implies q
* p only if q
* q if p

#### Negation

We can very simply negate an implication

The following truth table shows this

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

#### Inverse

The inverse of the implication is given by The fact that a given implication is true does not mean that its inverse is also true. There are however cases where both an implication and its inverse are true.

#### Necessary and Sufficient

If then we say that p is sufficient for p and q is necessary for p

#### Biconditional and Equivalence

If and we say that p and q are equivalent and we write

Table Biconditional truth table

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

We say p if and only if q which comes from the following observation

p only if q

p if q

p if and only if q

Equivalence can also be written as

## Theorems and proofs

### Elements

|  |  |  |
| --- | --- | --- |
| Element | Description | Example |
| Statement | A clause that is either true or false but not both |  |
| Conditional Statement | A statement whose truth or falsity is based on the value of a variable | x is even |
| Definition | Define the meaning of a mathematical word |  |
| Theorem | An important true statements |  |
| Proposition | A less important true statent |  |
| Lemma | A true statement used to prove other true statments |  |
| Corollary | A statement that is easily deduced from some other theorem/proposition |  |
| Proof | Justification or explanation of why a statement is true |  |
| Axiom | Based self evident properties of a model |  |

### Proof Types

#### Direct Proof

##### Proof 1 Prove the square of an odd integer is odd

### Proof by contradiction

#### Proof 1 Show that iF n^2 is a multiple of 3 n is a multiple of 3

Assume that n is not a multiple of 3. In such case either the remainder when dividing n by 3 is 1

Which gives a square that is not a multiple of 3

Or the remainder when dividing n by 3 is 2

Which gives a square which is not a multiple of 3

We have proved by contradition.

### Proof by induction

We prove a statement is true for all n >= by

1. Showing is true
2. Show that for all n if is true is also true

#### Proof 1 Show that sum of first n integers is n(n+1)/2