Number Systems

## Properties of Numbers

Listing 1 Properties of number

|  |  |
| --- | --- |
| 1. Associative law for addition |  |
| 1. Existence of additive identity |  |
| 1. Existence of additive inverse |  |
| 1. Commutative law of addition |  |
| 1. Associative law for multiplication |  |
| 1. Existence of multiplication identity |  |
| 1. Existence of multiplicative inverse |  |
| 1. Commutative law for multiplication |  |
| 1. Distributive law of multiplication over addition |  |
| 1. Distributive law of multiplication over subtraction |  |
| 1. Distributive law of division over addition |  |
| 1. Distributive law of division over subtraction |  |

## Types of Number

Table 1 Common Sets

|  |  |  |  |
| --- | --- | --- | --- |
| Set | Meaning | Example | Description/Example |
|  | The natural numbers |  | Can sometimes be assumed to include 0, especially by computer scientists |
|  | The integers |  |  |
|  | The positive integers |  |  |
|  | The rational numbers |  |  |
|  | The real numbers |  |  |
|  | Complex numbers |  |  |

Note that:

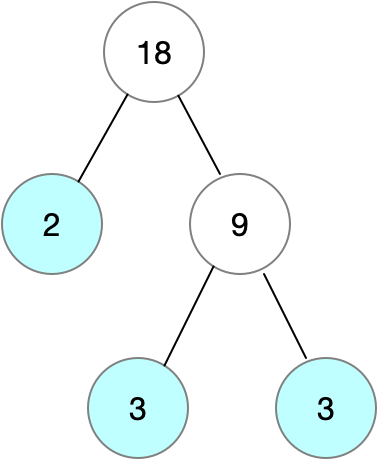
## Arithmetic

#### Fundamental Theorum Of Arithmetic

Any integer is either prime itself prime or can be expressed as a product of prime factors

Where are successive primes and are powers of that prime. For any given p, the corresponding can be zero. We can find the prime factors of any given number by continually dividing through. The following shows how to extract the prime factors of 18

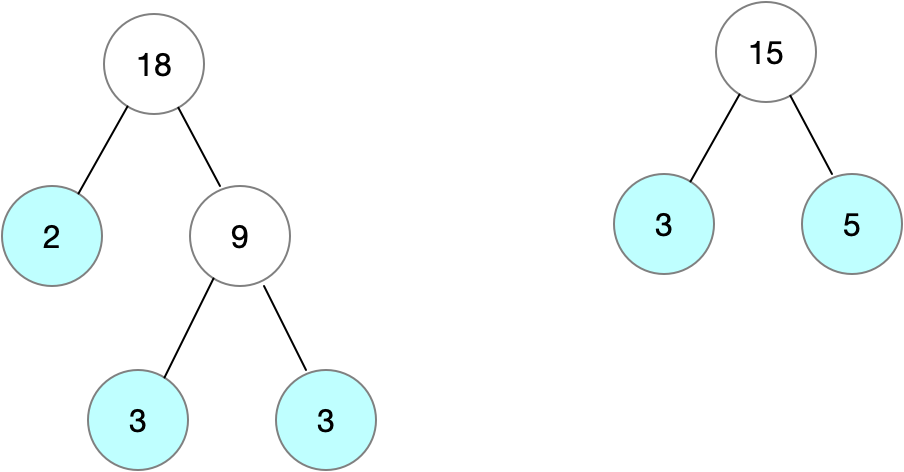
Figure 1 Prime Factorisation of 18



#### Highest Common Factor (HCF)

Given two integers x and y and their corresponding prime factorisations

We can calculate the highest common factor as

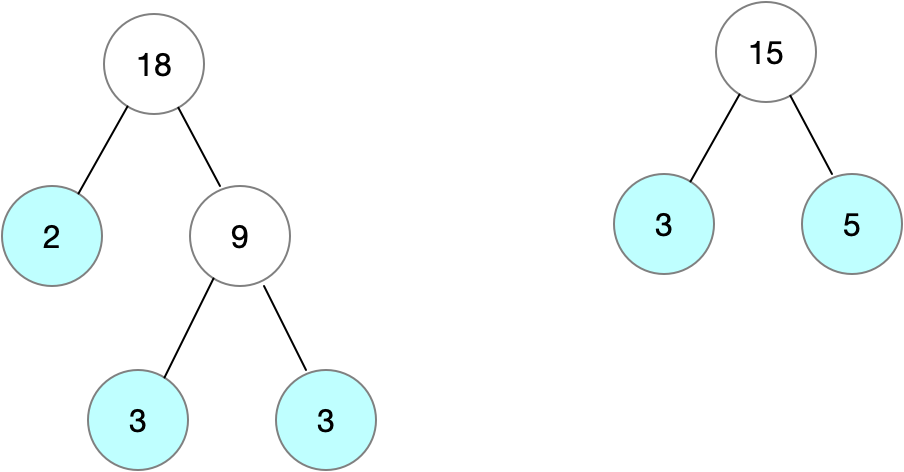


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#### Lowest Common Multiple (LCM)

Given two integers x and y and their corresponding prime factorisations

We can calculate the lowest common multiple as



#### Relating LCM and HCF

Given two integers x and y and their corresponding prime factorisations

We can show there is a relationship between lcm and hcf.

So we now know that

This is very powerful as we have efficient algorithms for calculating the hcf, whereas we do not have efficient algorithms for carrying out prime factorisation.

### Euclids Algorithm for Gcd

#### Proof

##### Show that gcd(a,b) is a divisor of a-b

By the definition of a divisor we know that

##### Show that gcd(a,b) is a common divisor of b and a-b

In the previous step we showed that gcd(a,b) is a divisor of a-b and by definition gcd(a,b) is a divisor of b. We hence know that gcd(a,b) is a common divisor of a and a-b. We know that gcd(a,b) must be less than or equal to gcd(b,a-b) by the definition of gcd(b,a-b) as the **greatest** common divisor

##### Show that gcd(b,a-b) is a divisor of a

By the definition of a divisor we know that

##### Show that gcd(b,a-b) is a common divisor of a and b

By definition gcd(b,a-b) is a divisor of b and we have shown that gcd(b,a-b) is a divisor of a. So we know that gcd(b,a-b) is a common divisor of a and b. Because gcd(a,b) is the **greatest** common divisor of a and bwe know that

Taken (4) and (9) together we have shown that

##### Show that gcd(b,a-b)=gcd(b,a%b)

**We have shown that** . We can apply the formula multiple times

The definition of the % operator is

Letting and substituting into the right hand side of (10) we have

We have now proved Euclids algorithm that

#### Implementation (C#)

