1. **John, Jim, Jay and Jack have formed a band consisting of four instruments. If each of the boys can play four instruments how many different arrangements are possible?**

The first thing in handing any such problem in counting is to reformulate it such that it is more easily amenable to the techniques of counting. We need to decide what is to be chosen. We first assign each band member a number

* Jim 1
* John 2
* Jay 3
* Jack 4

These will correspond to elements in a sequence

We then want to decide how many choices there are for each element in the sequence. We will be selecting from the instruments

Selecting T into the first element in the sequence is equivalent to Jim playing the Trumpet. There are hence four such selections possible for the first element in the sequence.

Each of these choices has three available choices for the second element (band member) and so on.



1. **Consider a set of antennas of which m are defective and n-m are functional and assume all of the defectives and all of the functional are considered indistinguishable. How many linear orderings are there in which no two defectives are consecutive.**

Consider the n-m functional antennas lined up in a row with a space before and after each antenna

...

In this case it is the spaces we are interested in. We can number the spaces from 1 to n-m+1 as follows

...

1

2

3

4

n-m

n-m+1

Now what we are selecting is which of these spaces we will insert the defective antennas. Since we have m defective antennas we need to select m spaces from a possible n-m+1 spaces. Order is not important so we have



1. **Prove the following identity**

First we use a combinatorial argument. If we fix the first element in the sequence to contain a particular object then there will be different sequences with that object in the first element of the sequence. Now consider all the other combinations with a different object in the first element of the sequence. We are choosing from n-1 items takes r at a time 

Second we use an analytic argument

**4 An urn contains n balls of which one is special. If k of the balls are withdrawn one at a time. What is the probability a special ball is chosen?**

Let us first start concrete, say 5 balls taken three at a time one of which is special

As we are looking at groups/sets where order is not important there arecombinations in total. Of these will have two non-special balls and one special ball. The probability of getting the special ball is 

In general we have 

One other way of looking at this if we let the event be the probability of drawing the special ball on the ith draw then since the events are clearly mutually exclusive and we have n draws the probability is 

**2 A poker hand consists of five cards. What is the probability we are dealt a straight**

We first note that there are different combinations of cards. How many five cards deals are considered a straight? First consider only the straight 1,2,3,4,5 As there are four aces each of which may have four twos each of which may have four 3’s etc we have different combinations which count as a straight using these cards. We then have ten different straights 2,3,4,5,6 etc So the probability is



**If n people are present in a room, what is the probability that no two of them celebrate their birthday on the same day of the year? How large need n be so this probability is greater than ½ ?**

**A five card poker hand is said to be a full house if it consists of 3 cards of the same denomination and 2 cards of the same denomination (That is, a full house is three of a kind plus a pair) what is the probability that one is dealt a full house?**

**A =** 

**In a game of bridge the entire deck of 52 cards is dealt out to four players. What is the probability that**

1. **One of the players receives all 13 spades**
2. **Each player receives one ace**

**A a-** 

Ab 