Random Processes

via coin tossing

## Summing multipleidentical Independent random variables

What if we toss the coin multiple times? If we toss the coin n times our sample space becomes. We can define a random variable that counts the number of heads. Such a random variable can take any value between 0 and n. What is the probability that in n tosses we will obtain k heads? Consider the following decision tree.



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To calculate the probability of obtaining k heads in n tosses we need to take into account the probability of a head on a single toss which we call p and the number of paths through the tree that come to that number of heads. The paths are given by the binomial co-efficients

and the probability becomes

If we let be the number of tails we have another random variable defined on the sample space. Clearly in all cases .We can define another random variable D that counts the number of heads minus the number of tails .Our distribution depends on both the random variable and the probability measure. In the case where n is equal to three, then under the measure P our distribution of becomes.

-3

-1

3

1

Under the same measure the distribution of becomes

0

1

3

2

We can intepret D as the distance from the origin if we move one unit in a positive direction whenever we obtain a head and one unit in a negative direction whenever we obtain a tail. This is the ‘random walk’ interpretation. What is the expectation of our random variables and ? If our coin is fairly weighted then and

We can create a new game by playing the original games multiple times. If we play the original game twice then our new game effectively involves tossing the coin twice and our sample space becomes . We can define a new random variable X as the sum of two identical independent random variables

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If we use the original measure P we get



And our distribution becomes

-2

0

2

By setting up our random variable in this way  is actually measuring the number of heads less the number of tails. The reason for the jump of 2 between the possible values is that if we go from 1 head to 2 heads then the number of tails decreases from one tail to zero tails and the value  increases by two.

If we then perform n identical tosses of the coin and define n identical random variables each will also have mean zero, and variance, of one. hen the distribution of our profit and loss is as follows. The mean of the distribution is zero and the variance is two.

* Summing three Identical Independent random variables

Let us go one-step further and look at the event obtained by summing three of the original events. We get the following distribution, whose mean is zero and whose variance is three.

-3

-1

3

1



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