Foundations of Probability

## Sets Theory

### Set

A set is a collection of things. We call the things elements of the set. If a set consists of the difference faces of a die we can write.

Set S

Element

### Subsets

Every element of A is also an element of B

 B

A



### Intersection

The set containing the elements in A and B

 B

A



### Disjoint sets



 B

A

### Difference

The elements in A but not in B

 B

A



### Symmetric Difference

The elements in A or B but not in both

 B

A



### Union

Given two sets A and B we can define the set of all elements either in A or B or both is denoted AUB.

 B

A



### Product of two sets

If A and B are sets we can form the product C as

And we write

## Basic Rules

### Probability that either of two events occurs P(AB)

We can calculate the probability of either one of two events A or B occuring hence.

 B

A



Because the three sets on the right hand side are disjoint. We can get a similar result by adding

### Probability that both events

 B

A

Ω



If we consider discrete probability then where every outcome is equally likely then the probability of A∩B is simply the number of outcomes is A∩B divided by the number of outcomes in the sample space Ω



## Conditional Probability

### P(A|B)

The conditional probability P(A|B) is the probability that the event A occurs given that the event B has occurred. Of course for A to occur given that B has occurred, the two events A and B must share outcomes. We know that

But if we know that B has occurred there is a higher probability than P(A∩B) that A occurs because the extra information that B has occurred allows us to reduce the sample space.

Ω

 B

A



 Ω = B

Because.

### Multiplication Rule

Similarly if we are given we can covert it back to by multiplying it through by

We can extend this to three events

And then n events

### Partitition Rule

Any event A can be partitioned into those outsomes it shares with a second event B and those outcomes it doesn’t share with B.

 B

A



We can express this using conditional probabilities as.

### Conditional Partitition Rule

For a proof of this consider the following

Ω

 C

A



 Ω = C



But the setcan be broken up into the part that interects with a third set B and the part that doesn’t intersect with the third set B



 C

B

A



So we can insert 2 into 1

Now we need to remember that we update the numerator on the RHS of 3

Finally we note that and use this to update the numerator on the RHS

Finally we cancel the P(C)’s