Identical Independent Random Variables

## Identical Independent random variables

Imagine a random event that involves the tossing of a single coin. We have two outcomes, heads or tails. Furthermore, let us define a random variable, on the outcomes that takes the value of plus one dollar if we obtain a head and minus one dollar if we obtain a tail. The distribution of our profit and loss is then as follows. The mean,  of the distribution is zero and variance,  is one.

-1

1



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## Summing two Identical Independent random variables

We can create a new game by playing the original games multiple times. If we then perform n identical tosses of the coin and define n identical random variables each will also have mean zero, and variance,  of one.

If we play the original game twice then our new game effectively involves tossing the coin twice. If we define then the distribution of our profit and loss is as follows. The mean of the distribution is zero and the variance is two.

-2

0

2



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## Summing three Identical Independent random variables

Let us go one-step further and look at the event obtained by summing three of the original events. We get the following distribution, whose mean is zero and whose variance is three.

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-1

3

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## Sum of n identical Independent random variables

Taking this process to its logical conclusion by summing n of our independent, identically distributed random variables we obtain the random variable which is distributed with mean zero and variance n.



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From a proof of why the sum of n independent, identically distributed random variables with mean and variance is a random variable with mean and variance see below

## Expectation of sum of I.I.D random variables



### Highlighting the principle



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We have random variable with sample space and another identically distributed random variable with sample space. The sample space of the joint distribution is given by the set of pairs





The expectation of the sum of the variables is then given by



Multiplying out



Noting that and 



Therefore we can note that



### Generalising

If  is a random variable with sample space and  is an independent random variable with sample space then the sample space of the joint distribution will be given by a set of pairs





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The expectation of the sum of the two variables in then given by



Multiplying out we get



Noting that and 



Therefore we can note that



Furthermore



And



## Variance of sum of I.I.D random variables



### Highlighting the principle



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We have random variable with sample space and another identically distributed random variable with sample space. The sample space of the joint distribution is given by the set of pairs





### Proof













