Identical Independent Random Variables

Imagine a random event that involves the tossing of a single coin. We have two outcomes, heads or tails. Let us define a random variable on the outcomes that takes the value of plus one dollar if we obtain a head and minus one dollar if we obtain a tail.



The distribution of our profit and loss is then as follows. The mean,  of the distribution is zero and variance,  is one.

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If we play the original game twice then our new game effectively involves two coin tosses.

If we define then the distribution of our profit and loss is as follows. The mean of the distribution is zero and the variance is two.



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Let us go one-step further and look at the event obtained by summing three of the original events. We get the following distribution, whose mean is zero and whose variance is three.

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Generalising, if we perform n identical tosses of a coin and define n identical random variables each will also have mean zero, and variance,  of one. By summing them we obtain the random variable which is distributed with mean zero and variance n.



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From a proof of why the sum of n independent, identically distributed random variables with mean and variance is a random variable with mean and variance see below

## Proofs

### Expectation of sum of N I.I.D random variables

The following figure shows the general approach



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We have random variable with sample space and a second identically distributed random variable with sample space. The sample space of the joint distribution is given by the set of pairs

The expectation of the sum of the variables is then given by

Multiplying out

Noting that and

Therefore we can note that

#### Generalising

If is a random variable with sample space and is an independent random variable with sample space then the sample space of the joint distribution will be given by a set of pairs

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The expectation of the sum of the two variables in then given by

Multiplying out we get

Noting that and 

Therefore we can note that

Furthermore

And

### Variance of sum of I.I.D random variables

The following diagram shows the general approach.



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We have random variable with sample space and another identically distributed random variable with sample space. The sample space of the joint distribution is given by the set of pairs

#### Proof