Stochatics Processes

## Basic Probability

### Terminology

**Outcome**

Each thing that can occur in the expiriment is called an outcome. In the example of tossing a coin we have two outcomes ‘heads’ or ‘tails’ which we can denote by the letters H and T respectively.

**Sample Space**

The set of all possible outcomes of an expiriment is called the sample space. By convention we label it.In our simple coin tossing scenario we would have  If we toss two coins our sample space would become 

**Event**

A subset of the probability space is called an event. We define an event using the following notation.

 “This means the set of all outcomes such that is a head”.

**Probability Measure**

A probability measure P is a function that assigns to each element  in  a probability such that



Since an event A is a subset of then the probability of an event is given by

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**Probability Space**

A probability space  consists of a sample space and a probability measure. The sample space is the set of outcomes and the probability measure is a function that assigns to each element in  a value in  such that



**Random Variable**

A random variable x is a real valued function defined on  . Put another way a random variable maps each outcome from the sample space  to a real number.

### Axioms

**Conditional Probability**

Given two events E and F



**Independent Events**

If  then the events E and F are said to be independent. The occurent of E does not change the probability of F occuring.

## An example

Consider a game where we toss a fair coin and if it lands heads we receive one dollar and if it lands tails we loose one dollar. We have the following.

 (Sample Space)

And our probability measure is then given by

 (Probability Measure)

We define a random variable, on the outcomes that takes the value of plus one dollar if we obtain a head and minus one dollar if we obtain a tail.

 (Random Variable)

We now introduce one more important concept, that of a probability distribution. A random variable is a function defined on  whereas its distribution is a tabulation of the probabilities that the random variable takes its various values. A random variable is **not** a distribution.

x1

(Random Variable)

Distribution of

x1

Probability measure

Under the probability measure P defined on  either a head or tail are equally likely so our distribution becomes



-1

1

We can then calculate the mean and the variance,  of our distribution.



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## Summing two Identical Independent random variables

We can create a new game by playing the original games twice. We define  as the profit/loss from the first coin toss and  as the profit/loss from the second toss. We can then define the random variable  giving the total profit or loss at the end of the two tosses.

We obtain the following sample space



With the following measure



Giving us the random variable



Applying our measure to the random variable we get our probability distribution



-2

0

2



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## Summing three Identical Independent random variables

Let us go one-step further and look at the event obtained by summing three of the original events. We get the following distribution, whose mean is zero and whose variance is three.

-3

-1

3

1



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## Sum of n identical Independent random variables

Taking this process to its logical conclusion by summing n of our independent, identically distributed random variables we obtain the random variable which is distributed with mean zero and variance n.



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From a proof of why the sum of n independent, identically distributed random variables with mean and variance is a random variable with mean and variance see below

## Expectation of sum of I.I.D random variables



### Highlighting the principle



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We have random variable with sample space and another identically distributed random variable with sample space. The sample space of the joint distribution is given by the set of pairs





The expectation of the sum of the variables is then given by



Multiplying out



Noting that and 



Therefore we can note that



### Generalising

If  is a random variable with sample space and  is an independent random variable with sample space then the sample space of the joint distribution will be given by a set of pairs





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The expectation of the sum of the two variables in then given by



Multiplying out we get



Noting that and 



Therefore we can note that



Furthermore



And



## Variance of sum of I.I.D random variables



### Highlighting the principle



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We have random variable with sample space and another identically distributed random variable with sample space. The sample space of the joint distribution is given by the set of pairs





### Proof













