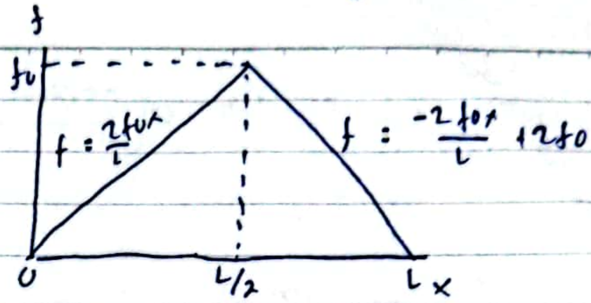
Nama : Muhammad Suryadika   Tugas 5 Solusi pers. Gelombang dengan PDE.



1. Dik. $L = 10$,
   $c = 12$,
   $f_0 = 1$

dengan kondisi batas ;

1. $y(0,t) = 0$, $(0 < t < \infty)$
2. $y(L,t) = 0$, $(0 < t < \infty)$
3. $y(x,0)$: $\begin{cases} \dfrac{2f_0x}{L}, & 0 < x < \dfrac{L}{2} \\ -\dfrac{2f_0x}{L} + 2f_0, & \dfrac{L}{2} < x < L \end{cases}$
4. $y_t(x,0) = 0$, $0 < x < L$

Hitung solusi persoalan gelombang pada gambar 1 dengan metode pemisahan variabel (gunakan kondisi batas diatas).

A : pendekatan $n = 1$
1. $y(3,3)$
2. $y(3,6)$

※ Menentukan R_n dan S_n

$$R_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

$$= \frac{2}{L}\left(\int_0^{L/2}\frac{2f_0x}{L}\sin\frac{n\pi x}{L}\,dx + \int_{L/2}^{L}\left(-\frac{2f_0x}{L}+2f_0\right)\sin\frac{n\pi x}{L}\,dx\right)$$

$$= \frac{4f_0}{L^2}\left(\int_0^{L/2}x\sin\frac{n\pi x}{L}\,dx + \int_{L/2}^{L}(-x+L)\sin\frac{n\pi x}{L}\,dx\right)$$

$$\boxed{R_n = \frac{4F_0\left(\sin(n\pi) - 2\sin(n\pi/2)\right)}{n^2\pi^2} = \frac{8F_0}{n^2\pi^2}\sin\frac{n\pi}{2}}$$

$$S_n = \frac{2}{n\pi c}\int_0^L g(x)\sin\frac{n\pi x}{L}\,dx$$

$g(x) = 0$

$S_n = 0$ //

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Solusi khusus.

$$y(x,t) = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \left(R_n \cos \frac{n\pi ct}{L} + S_n \sin \frac{n\pi ct}{L} \right)$$

nilainya 0

Maka

$$y(x,t) = \frac{8F_0}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{2} \sin \frac{n\pi x}{L} \cos \frac{n\pi ct}{L}$$

$L = 10 \quad c = 12 \quad f_0 = 1 \quad n = 1$

1. $$y(3,3) = \frac{8 \cdot 1}{\pi^2} \sum_{n=1}^{1} \sin \frac{\pi}{2} \sin \frac{\pi 3}{10} \cos \frac{\pi 12 \cdot 3}{10}$$

$$= 0{,}81 \cdot (1 \cdot 0{,}809 \cdot 0{,}309)$$

$$= 0{,}20248$$

//

2. $$y(3,6) = \frac{8 \cdot 1}{\pi^2} \sum_{n=1}^{1} \sin \frac{\pi}{2} \sin \frac{\pi 3}{10} \cos \frac{\pi 12 \cdot 6}{10}$$

$$= 0{,}81 \; (1 \cdot 0{,}809 \times (-0{,}809))$$

$$= -0{,}52012$$

//

B. Interpretasi traveling wave pada ;

1. $y(3,3)$
2. $y(3,6)$

- Solusi khusus ;

$$y(x,t) = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \left(R_n \cos \frac{n\pi c t}{L} + S_n \sin \frac{n\pi c t}{L} \right)$$

Jika

$$y(x,0) = f(x) \quad \text{dan} \quad y_t(x,0) = 0$$

$$y(x,t) = \sum_{n=1}^{\infty} R_n \sin \frac{n\pi x}{L} \cos \frac{n\pi c t}{L} \quad 0 < x < L \;, 0 < t < \infty$$

- Identitas trigono.

$$y(x,t) = \frac{1}{2} \left[\sum_{n=1}^{\infty} R_n \sin \frac{n\pi}{L}(x-ct) + \sum_{n=1}^{\infty} R_n \sin \frac{n\pi}{L}(x+ct) \right]$$

Fourier series expansion

- 2L periode
- anti simetri pada $x = 0$ dan $x = L$

$$y(x,t) = \frac{1}{2} \left[F_{ext}(x-ct) + f_{ext}(x+ct) \right]$$

$L = 10 \quad c = 12 \quad f_0 = 1$

$$y(3,3) = \frac{1}{2} \left[F_{ext}(3 - 12(3)) + f_{ext}(3 + 12(3)) \right]$$

$$= \frac{1}{2} \left[f_{ext}(-33) + f_{ext}(39) \right]$$

$$= \frac{1}{2} \left[-f(3) + f(9) \right]$$

$$= \frac{1}{2} \left[-f(1) + f(7) \right]$$

$$F(7) = \frac{-2 f_0 x}{L} + 2 f_0$$

$$= \frac{-2 \cdot 1 \cdot 7}{10} + 2 \cdot 1$$

$$= 0,6$$

$$F(1) = \frac{2 f_0 . x}{L} + 2 f_0$$

$$= \frac{2 . 1 , 1}{10} + 2 . 1$$

$$= 0,2$$

$$y(3,3) = \frac{1}{2}(-0,2 + 0,6)$$

$$= \frac{1}{2}(0,4)$$

$$= 0,2 \;//$$

$$y(3,6) = \frac{1}{2}\left[F_{ext}(3-12\cdot 6) + F_{ext}(3+12\cdot 6)\right]$$

$$= \frac{1}{2}\left[F_{ext}(-69) + F_{ext}(75)\right]$$

$$= \frac{1}{2}\left[-f(29) + f(35)\right]$$

$$= \frac{1}{2}\left[-f(9) + f(15)\right]$$

$$= \frac{1}{2}\left[-f(1) + f(5)\right]$$

$$F(1) = \frac{2 f_0 x}{L} = \frac{2\cdot 1\cdot 1}{10} = 0.2$$

$$F(7) = \frac{2 f_0 \cdot x}{L} = \frac{2\cdot 1\cdot 5}{10} = \frac{10}{10} = 1$$

$$y(x,t) = \frac{1}{2}\left[-F(1) + F(5)\right]$$

$$= \frac{1}{2}\left[\cancel{2008} - 0,2 + 1\right]$$

$$= \frac{1}{2}\left[0,8\right]$$

$$= 0,4$$

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