

## 1 Structure of the observable

In this document we report the relevant formulas for the computation of semi-inclusive deep-inelastic scattering (SIDIS) multiplicities under the assumption that the (negative) virtuality of the  $Q^2$  of the exchanged vector boson is much smaller than the  $Z$  mass. This allows us to neglect weak contributions and write the cross section in TMD factorisation as:

$$\frac{d\sigma}{dx dQ dz dq_T} = \frac{4\pi\alpha^2 q_T}{zxQ^3} Y_+ H(Q, \mu) \sum_q e_q^2 \int_0^\infty db b J_0(bq_T) \bar{F}_q(x, b; \mu, \zeta_1) \bar{D}_q(z, b; \mu, \zeta_2), \quad (1.1)$$

with:

$$Y_+ = 1 + (1 - y)^2 = 1 + \left(1 - \frac{Q^2}{xs}\right)^2, \quad (1.2)$$

and where:

$$\bar{F}_i(x, b; \mu, \zeta) = x F_i(x, b; \mu, \zeta) = R_q(\mu_0, \zeta_0 \rightarrow \mu, \zeta; b) \sum_j \int_x^1 dy \mathcal{C}_{ij}(y; \mu_0, \zeta_0) \left[ \frac{x}{y} f_j\left(\frac{x}{y}, \mu_0\right) \right], \quad (1.3)$$

and:

$$\bar{D}_i(z, b; \mu, \zeta) = z^3 D_i(z, b; \mu, \zeta) = R_q(\mu_0, \zeta_0 \rightarrow \mu, \zeta; b) \sum_j \int_z^1 dy [y^2 \mathbb{C}_{ij}(y; \mu_0, \zeta_0)] \left[ \frac{z}{y} d_j\left(\frac{z}{y}, \mu_0\right) \right], \quad (1.4)$$

are the distributions directly computed by APFEL++.

## References