Jet TMD

1 Definition of the jet TMD

Following the notes written by Lorenzo and Yannis, the impact-parameter-space inclusive jet-production cross section in DIS in TMD factorisation takes the standard form:

$$d\sigma \sim H_{ij}(Q;\mu) D_{i\to\text{jet}}(b, t_{\mathcal{R}};\mu,\zeta_1) F_{j\leftarrow P}(x, b;\mu,\zeta_2)\,, \tag{1.1}$$

where H_{ij} is the DIS hard function, $F_{j\leftarrow P}$ is the usual TMD PDF of the quark flavour j inside the proton, and $D_{i\to\text{jet}}$ is the TMD of the jet generated by the quark flavour i. x and Q are the usual DIS variable corresponding to the Bjorken variable and the negative virtuality of the vector boson while b is the Fourier conjugate variable of the partonic transverse momentum k_T. The scale μ is the resummation scale and must be $\mu = C_f Q$, with C_f order one, while ζ_1 and ζ_2 are the rapidity scale that in this case must obey the momentum-space equality $\zeta_1\zeta_2 = Q^2 t_{\mathcal{R}} k_T^2$, with k_T being the partonic transverse momentum. In impact parameter space this equality naturally turns into $\zeta_1\zeta_2 = Q^2 t_{\mathcal{R}}^2 b_0^2/b^2$ with $b_0 = 2e^{-\gamma_{\text{E}}}$. Without loss of generality one can choose $\zeta_2 = Q^2$ such that $\zeta_1 = t_{\mathcal{R}}^2 b_0^2/b^2$. Finally, the variable $t_{\mathcal{R}} = \tan(\mathcal{R}/2)$ depends on the jet opening $\mathcal{R}$ and for TMD factorisation to be valid one needs $\mathcal{R} \sim 1$.

Using the definition devised by Yannis and Lorenzo, both jet TMD and TMD PDFs evolve multiplicatively through the standard Sudakov form factor R as:

$$G(\mu,\zeta) = R[(\mu,\zeta) \leftarrow (\mu_0,\zeta_0)]G(\mu_0,\zeta_0) \quad G = D_{i\to\text{jet}}, F_{j\leftarrow P}\,, \tag{1.2}$$

where we simplified the notation by dropping the unnecessary variables. In the following we take:

$$\mu_0 = \sqrt{\zeta_0} = C_i\mu_b\,, \quad \text{with} \quad \mu_b = \frac{b_0}{b}\,, \tag{1.3}$$

where C_i is a constant of order one. As is well know, $F_{j\leftarrow P}(\mu_0,\zeta_0)$ can be matched onto collinear PDFs for small values of b while a non-perturbative component needs to be accounted for larger values of b. This is the standard procedure and will not be discussed any further here. Let us now turn to the initial-scale jet TMD that can be further factorised as:

$$D_{i\to\text{jet}}(\mu_0,\zeta_0) = D_{i\to\text{jet}}(\mu_0,\zeta_0;\mu_0) = U_J[\mu_0 \leftarrow \mu_J] D_{i\to\text{jet}}(\mu_0,\zeta_0;\mu_J)\,, \tag{1.4}$$

with $\mu_J = C_J\mu t_{\mathcal{R}}$, $C_J \sim 1$. The evolution factor U_J takes the following explicit form:

$$U_J[\mu_0 \leftarrow \mu_J] = \exp\left[-\int_{\mu_0}^{\mu_J}\frac{d\mu'}{\mu'}\gamma_J(\mu')\right]\,. \tag{1.5}$$

The anomalous dimension valid up to NLL reads:

$$\gamma_J(\mu') = \left(\frac{\alpha_s(\mu')}{4\pi}\right)\gamma_F^{(0)} - \left[\left(\frac{\alpha_s(\mu')}{4\pi}\right)\gamma_K^{(0)} + \left(\frac{\alpha_s(\mu')}{4\pi}\right)^2\gamma_K^{(1)}\right]\ln\frac{\mu_J}{\mu'}\,, \tag{1.6}$$

where $\gamma_F^{(i)}$ and $\gamma_K^{(i)}$ are the coefficients of the perturbative expansion of the non-cusp and cusp anomalous dimensions, respectively. Finally, the initial-scale jet TMD reads:

$$D_{i\to\text{jet}}(\mu_0,\zeta_0;\mu_J) = 1 + \frac{\alpha_s(\mu_J)}{4\pi}\left[\frac{1}{2}\gamma_K^{(0)}\ln^2 C_J + \gamma_F^{(0)}\ln C_J + d_J^{q,\text{alg}}\right]\,, \tag{1.7}$$

where the coefficient $d_J^{q,\text{alg}}$ depends on the jet algorithm. For cone a k_T algorithms respectively reads:

$$d_J^{q,\text{cone}} = C_F\left(7 + 6\ln 2 - \frac{5\pi^2}{6}\right)\,, \quad \text{and} \quad d_J^{q,k_T} = C_F\left(13 - \frac{3\pi^2}{2}\right)\,. \tag{1.8}$$