Ancillary Files for "Unpolarized Quark and Gluon TMD PDFs and FFs at N³LO"

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In this note we explain the definitions and conventions for the ancillary files associated with the arXiv submission of "Unpolarized Quark and Gluon TMD PDFs and FFs at N³LO".

TMD BEAM FUNCTION

In the MATHEMATICA file BeamFunction.m and BeamFunctionN.m, we provide matching coefficients for the transverse-momentum-dependent (TMD) beam function for all partonic channels. They correspond to Eqs. (3.4) in the paper, with

Beam["ji"] =
$$\mathcal{I}_{ji}^{(0)} + \operatorname{as} \mathcal{I}_{ji}^{(1)} + \operatorname{as}^2 \mathcal{I}_{ji}^{(2)} + \operatorname{as}^3 \mathcal{I}_{ji}^{(3)}$$
, (1)

where "as" stands for $\frac{\alpha_s(\mu)}{4\pi}$, j=q,g stands for quark or gluon distributions, and i = q, g, qp, qb, qbp stands for quark q, gluon g, a different flavor of quark q', antiquark \bar{q} , a different flavor of anti-quark \bar{q}' , respectively. For example, after loading the file, type in Beam["qqbp"] will return the result for $\mathcal{I}_{q\bar{q}'}(x,b_{\perp},\mu,\nu)$.

We use the following definitions and conventions in the file:

$$\mathrm{Lp} = L_{\perp} = \ln(|b_{\perp}^2|\mu^2/b_0^2) \,, \quad \mathrm{LQ} = L_Q = 2\ln(Q/\nu) \,, \eqno(2)$$

where $Q = xP^+$ is the invariant mass of the Drell-Yan lepton pair.

TMD PARTION DISTRIBUTION FUNCTION

The MATHEMATICA file TMDPDF.m and TMDPDFN.m give the results for the rapidity-regulator-independent TMD PDFs,

$$f_{\perp,ij}(x,b_{\perp},\mu) = \mathcal{I}_{ij}(x,b_{\perp},\mu,\nu)\sqrt{\mathcal{S}(b_{\perp},\mu,\nu)}.$$
 (3)

They have the same structure as the file BeamFunction.m, with the definition of a new symbol,

$$Lh = L_{\nu} + L_{Q} = \ln(Q^{2}/\mu^{2}), \qquad (4)$$

and

$$\label{eq:tmppdf} \begin{split} \text{TMDpdf}["ji"] &= f_{\perp,ji}^{(0)} + \mathrm{as} f_{\perp,ji}^{(1)} + \mathrm{as}^2 f_{\perp,ji}^{(2)} + \mathrm{as}^3 f_{\perp,ji}^{(3)} \,. \end{split}$$

When referring to the TMD beam functions and TMD PDFs calculated in this paper, please kindly cite Refs. [3?].

THE TIME-LIKE TMD COEFFICIENT FUNCTIONS

In the MATHEMATICA file FFMatchingKernels.m and FFMatchingKernelsN.m, we provide time-like TMD co-

efficient functions for all partonic channels. They correspond to Eqs. (3.6) in the paper, with

Frag["ij"] =
$$C_{ij}^{(0)}$$
 + as $C_{ij}^{(1)}$ + as $C_{ij}^{(2)}$ + as $C_{ij}^{(3)}$, (6)

where j=q,g stands for quark or gluon fragmentations, and i=q, g, qp, qb, qbp stands for quark q, gluon g, a different flavor of quark \bar{q}' , antiquark \bar{q} , a different flavor of anti-quark \bar{q}' , respectively. For example, after loading the file, type in Frag["qbpq"] will return the result for $C_{\bar{q}'q}(z,b_{\perp}/z,\mu,\nu)$.

We use the following definitions and conventions in the file:

$$\mathrm{Lp} = L_{\perp} = \ln(|b_{\perp}^2|\mu^2/b_0^2)\,, \quad \mathrm{LQ} = L_Q = 2\ln(Q/\nu)\,, \eqno(7)$$

where $Q = P^+/z$ is twice the energy of the mother parton.

TMD FRAGMENTATION FUNCTION

The MATHEMATICA file TMDFF.m and TMDFFN.m give the results for the rapidity-regulator-independent TMDFFs,

$$g_{\perp,ij}(x,b_{\perp}/z,\mu) = \mathcal{C}_{ij}(x,b_{\perp}/z,\mu,\nu)\sqrt{\mathcal{S}(b_{\perp},\mu,\nu)}. \quad (8)$$

They have the same structure as the file FFMatchingKernels.m, with the definition of a new symbol,

$$Lh = L_{\nu} + L_{Q} = \ln(Q^{2}/\mu^{2}), \qquad (9)$$

and

$$\label{eq:tmdff} \text{TMDFF["ji"]} = g_{\perp,ji}^{(0)} + \mathrm{as} g_{\perp,ji}^{(1)} + \mathrm{as}^2 g_{\perp,ji}^{(2)} + \mathrm{as}^3 g_{\perp,ji}^{(3)} \,. \tag{10}$$

RESUMMED TIME-LIKE COEFFICIENT FUNCTIONS

In the MATHEMATICA file resummedFFsinglets.m, we provide the Mellin N-space results for the resummed time-like coefficient functions to order $a_s^{15} = (\alpha_s/(4\pi))^{15}$, where the expressions of the respective highest three terms in the Laurent expansions about $\overline{N} = N - 1 = 0$ are given. The corresponding z-space results are obtained

by inverse Mellin transformation, and according to our definition of the Mellin moments

$$M\left[\frac{1}{z}\ln^{k}z\right] \equiv \int_{0}^{1} dz z^{N-1} \frac{1}{z}\ln^{k}z$$
$$= \frac{(-1)^{k}k!}{(N-1)^{k+1}} = \frac{(-1)^{k}k!}{\overline{N}^{k+1}}, \qquad (11)$$

the inversion rule is simply given by

$$\overline{N}^k \to -\frac{(-1)^{-k}}{(-k-1)!} \ln^{-k-1} z, \qquad \forall k < 0.$$
 (12)

TMD SOFT FUNCTION

The Mathematica file softfunction.m gives the quark TMD soft function to N^3LO with

$$Lv = L_{\nu} = \ln(\nu^2/\mu^2)$$
. (13)

Expressions for gluon TMD function is easily obtained by Casimir scaling. The soft zero-bin function is identical to TMD soft function and is universal for both space-like and time-like configurations up to $\mathcal{O}(\alpha_s^3)$.

SYMBOLS AND COLORS

For the color factors,

$$\begin{split} \text{CF} &= C_F \,, \quad \text{Nf} &= N_f \,, \quad \text{TF} &= T_F \,, \quad \text{CA} &= C_A \,, \\ \frac{\text{dabc2}}{\text{nc}} &= \frac{d^{ABC} d_{ABC}}{N_c} &= \frac{(N_c^2 - 1)(N_c^2 - 4)}{N_c^2} \,. \end{split} \tag{14}$$

plusD[n,1-z] is the conventional plus distribution

$$plusD[n,1-z] = \left(\frac{\ln^n(1-z)}{1-z}\right)_+. \tag{15}$$

Dirac delta function is represented by

$$\texttt{delta[1-z]} = \delta(1-z).$$

Harmonic polylogarithms are represented by

$$H[a1,a2,...,an,z] = HPL[{a1,a2,...,an},z],$$

and zeta[n] is Riemann zeta value ζ_n .

When referring to the unpolarized TMD distributions calculated in this paper, please kindly cite Refs. [1–3].

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