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1 Introduction

In this set of notes I collect the technical aspects concerning generalised parton distributions (GPDs). Since the computation GPDs introduces new kinds of convolution integrals, a strategy aimed at optimising the numerics needs to be devised.

2 Evolution equation

The evolution equation for GPDs reads:

$$\mu^2\frac{d}{d\mu^2}f(x,\xi)=\int_{-\infty}^{+\infty}\frac{dx'}{|2\xi|}\mathbb{P}\left(\frac{x}{\xi},\frac{x'}{\xi}\right)f(x',\xi)\,. \tag{2.1}$$

In general, the GPD f and the evolution kernel $\mathbb{P}$ should be respectively interpreted as a vector and a matrix in flavour space. However, for now, we will just be concerned with the integral in the r.h.s. of Eq. (2.1) regardless of the flavour structure.

The support of the evolution kernel $\mathbb{P}\left(\frac{x}{\xi},\frac{x'}{\xi}\right)$ is shown in Fig. 2.1. The Knowledge of the support region of

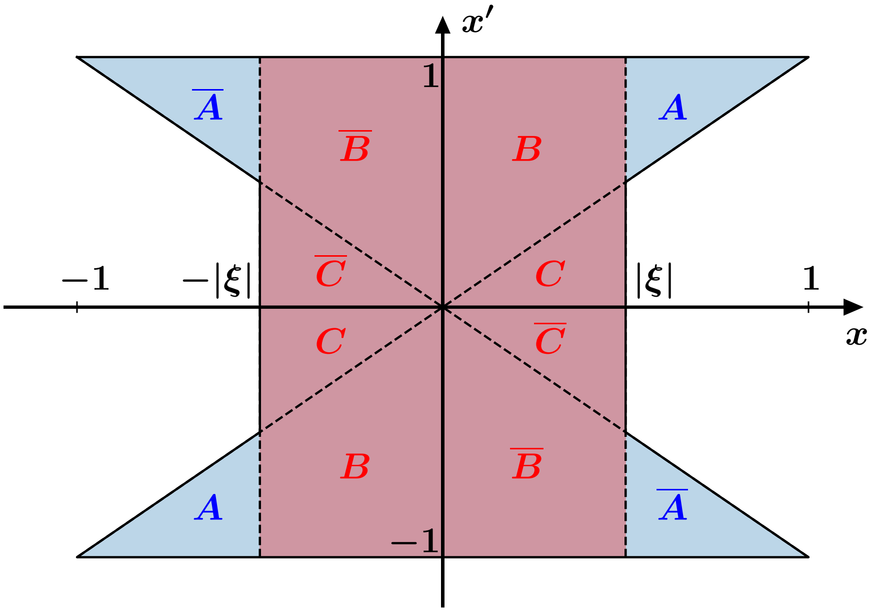

Fig. 2.1: Support domain of the evolution kernel $\mathbb{P}\left(\frac{x}{\xi},\frac{x'}{\xi}\right)$.

the evolution kernel allows us to rearrange Eq. (2.1) as follows:

$$\mu^2\frac{d}{d\mu^2}f(\pm x,\xi)=\int_{b(x)}^{1}\frac{dx'}{x'}\left[\frac{x'}{|2\xi|}\mathbb{P}\left(\pm\frac{x}{\xi},\frac{x'}{\xi}\right)f(x',\xi)+\frac{x'}{|2\xi|}\mathbb{P}\left(\mp\frac{x}{\xi},\frac{x'}{\xi}\right)f(-x',\xi)\right]\,, \tag{2.2}$$

with:

$$b(x)=|x|\theta\left(\left|\frac{x}{\xi}\right|-1\right)\,, \tag{2.3}$$

and where we have used the symmetry property of the evolution kernels: $\mathbb{P}(y, y') = \mathbb{P}(-y, -y')$. In the unpolarised case, it is useful to define:[1]

$$\begin{aligned} f^{\pm}(x,\xi) &= f(x,\xi) \mp f(-x,\xi)\,, \\ \mathbb{P}^{\pm}(y,y') &= \mathbb{P}(y,y') \mp \mathbb{P}(-y,y')\,, \end{aligned} \tag{2.4}$$

so that the evolution equation for $f^{\pm}$ reads:

$$\mu^2 \frac{d}{d\mu^2} f^{\pm}(x,\xi) = \int_{b(x)}^{1} \frac{dx'}{x'} \frac{x'}{|2\xi|} \mathbb{P}^{\pm}\left(\frac{x}{\xi}, \frac{x'}{\xi}\right) f^{\pm}(x',\xi)\,. \tag{2.5}$$

The $f^{\pm}$ distributions can be regarded as the GPD analogous of the $\pm$ forward distributions that can then be used to construct the usual singlet and non-singlet distributions in the QCD evolution basis. This uniquely determines the flavour structure of the evolution kernels $\mathbb{P}^{\pm}$.

It is relevant to observe that the presence of the θ-function in the lower integration bound b, Eq. (2.3), is such that for $|x| > |\xi|$ the evolution equation has the exact form of the DGLAP evolution equation which corresponds to integrating over the blue regions in Fig. 2.1 (DGLAP region, henceforth). Conversely, for $|x| \leq |\xi|$ the lower integration bound becomes zero and the evolution equation assumes the form of the so-called ERBL equation that describes the evolution of meson distribution amplitudes (DAs). This corresponds to integrating over the red region (ERBL region, henceforth). Crucially, in the limits $\xi \to 0$ and $\xi \to \pm 1$ Eq. (2.5) needs to recover the DGLAP and ERBL equations, respectively.

GPD anomalous dimensions are generally tricky to integrate numerically because of the intricate support. In order to simplify the integration procedure, we can decompose the anomalous dimensions using the labels given in Fig. 2.1 as a guide:

$$\begin{aligned} \mathbb{P}(y,y') &= \theta(y') \\ &\times \Big[\theta(y-1)\theta(y'-y)\mathbb{P}_A(y,y') + \theta(1-y)\theta(y'-y)\mathbb{P}_B(y,y') + \theta(1-y)\theta(y-y')\mathbb{P}_C(y,y') \\ &+ \theta(-y-1)\theta(y+y')\mathbb{P}_{\overline{A}}(y,y') + \theta(1+y)\theta(y+y')\mathbb{P}_{\overline{B}}(y,y') + \theta(1+y)\theta(-y'-y)\mathbb{P}_{\overline{C}}(y,y')\Big] \\ &+ \theta(-y') \\ &\times \Big[\theta(y-1)\theta(-y-y')\mathbb{P}_{\overline{A}}(y,y') + \theta(1-y)\theta(-y-y')\mathbb{P}_{\overline{B}}(y,y') + \theta(1-y)\theta(y'+y)\mathbb{P}_{\overline{C}}(y,y') \\ &+ \theta(-y-1)\theta(-y'+y)\mathbb{P}_A(y,y') + \theta(1+y)\theta(-y'+y)\mathbb{P}_B(y,y') + \theta(1+y)\theta(-y+y')\mathbb{P}_C(y,y')\Big]\,, \end{aligned} \tag{2.6}$$

where the functions $\mathbb{P}_I$ and $\mathbb{P}_{\overline{I}}$, with $I = A, B, C$, are defined on the respective regions in Fig. 2.1.[2] Next, we take the combinations given in Eq. (2.8) relevant to implement the evolution equation in Eq. (2.5). By doing this, one obtains:

$$\mathbb{P}^{\pm}(y,y') = \theta(y-1)\mathbb{P}^{\pm}_{A}(y,y') + \theta(1-y)\left[\theta(y'-y)\mathbb{P}^{\pm}_{B}(y,y') + \theta(y-y')\mathbb{P}^{\pm}_{C}(y,y')\right]\,, \tag{2.7}$$

where we have defined:

$$\mathbb{P}^{\pm}_{I}(y,y') = \mathbb{P}_I(y,y') \mp \mathbb{P}_{\overline{I}}(-y,y')\,, \tag{2.8}$$

and omitted all the irrelevant/redundant terms and factors for the computation of the integral in the r.h.s. of Eq. (2.5). From Eq. (2.7), it should be clear that the anomalous dimension $\mathbb{P}^{\pm}_{A}$ is responsible for the evolution in the DGLAP region while $\mathbb{P}^{\pm}_{B}$ and $\mathbb{P}^{\pm}_{C}$ are responsible for the evolution in the ERBL region. The latter observation suggests that $\mathbb{P}^{\pm}_{B}$ and $\mathbb{P}^{\pm}_{C}$ are related. The relation can easily be established by observing that the general structure of the ERBL anomalous dimensions is:

$$V^{\rm ERBL}(y,y') = \theta(y'-y)F(y,y') + \theta(y-y')F(-y,-y')\,, \tag{2.9}$$

[1] Notice the seemingly unusual fact that f^{+} is defined as difference and f^{-} as sum of GPDs computed at opposite values of x. This can be understood from the fact that, in the forward limit, $f(-x) = -\overline{f}(x)$, *i.e.* the PDF of a quark computed at $-x$ equals the PDF of the corresponding antiquark computed at x with opposite sign. The opposite sign is absent in the longitudinally polarised case.

[2] Note that $\mathbb{P}_I(y,y')$ and $\mathbb{P}_{\overline{I}}(y,y')$ are not required to be symmetric upon the transformation $(y \to -y, y' \to -y')$.

which immediately implies that:

$$\mathbb{P}_C^{\pm}(y,y') = \mathbb{P}_B^{\pm}(-y,-y')\,. \tag{2.10}$$

Finally, one finds that a convenient decomposition for the anomalous dimension in Eq. (2.5) is:

$$\mathbb{P}^{\pm}(y,y') = \theta(y-1)\mathbb{P}_A^{\pm}(y,y') + \theta(1-y)\left[\theta(y'-y)\mathbb{P}_B^{\pm}(y,y') + \theta(y-y')\mathbb{P}_B^{\pm}(-y,-y')\right]\,. \tag{2.11}$$

Eq. (2.5) can be further manipulated to make it resemble the structure of the DGLAP equation as much as possible. To this purpose, we define the parameter:

$$\kappa(x) = \frac{\xi}{x}\,, \tag{2.12}$$

so that:

$$\frac{x'}{|2\xi|}\mathbb{P}_I^{\pm}\left(\pm\frac{x}{\xi},\pm\frac{x'}{\xi}\right) = \mathrm{sign}(\xi)\frac{1}{2\kappa}\frac{x'}{x}\mathbb{P}_I^{\pm}\left(\pm\frac{1}{\kappa},\pm\frac{1}{\kappa}\frac{x'}{x}\right) \equiv \mathrm{sign}(\xi)\mathcal{P}_I^{\pm}\left(\pm\kappa,\frac{x}{x'}\right)\,, \tag{2.13}$$

where the last equality effectively defines the *DGLAP-like* splitting function:

$$\mathcal{P}_I^{\pm}(\pm\kappa,y) = \frac{1}{2\kappa y}\mathbb{P}_I^{\pm}\left(\pm\frac{1}{\kappa},\pm\frac{1}{\kappa y}\right)\,. \tag{2.14}$$

In the following we will assume $\xi > 0$ as, so far, this is the only experimentally accessible region. This allows us to get rid of $\mathrm{sign}(\xi)$ in Eq. (2.13). In addition, without loss of generality, we can also restrict ourselves to positive values of x because negative values can be easily accessed by symmetry using Eq. (2.8), *i.e.* $f^{\pm}(-x,\xi) = \mp f^{\pm}(x,\xi)$. Using the definition in Eq. (2.14) in the integral in the r.h.s. of Eq. (2.5) and finally performing a change of variable gives:

$$\mu^2\frac{d}{d\mu^2}f^{\pm}(x,\xi) = \int_{b(x)}^{1}\frac{dx'}{x'}\mathcal{P}^{\pm}\left(\kappa,\frac{x}{x'}\right)f^{\pm}\left(x',\xi\right) = \int_{x}^{x/b(x)}\frac{dy}{y}\mathcal{P}^{\pm}\left(\kappa,y\right)f^{\pm}\left(\frac{x}{y},\xi\right)\,, \tag{2.15}$$

with:

$$b(x) = x\,\theta(1-\kappa)\,, \tag{2.16}$$

and:

$$\mathcal{P}^{\pm}\left(\kappa,y\right) = \theta(1-\kappa)\mathcal{P}_A^{\pm}(\kappa,y) + \theta(\kappa-1)\left[\theta(1-y)\mathcal{P}_B^{\pm}(\kappa,y) + \theta(y-1)\mathcal{P}_B^{\pm}(-\kappa,y)\right]\,. \tag{2.17}$$

Plugging Eq. (2.17) into Eq. (2.15), one obtains:

$$\begin{aligned}\mu^2\frac{d}{d\mu^2}f^{\pm}(x,\xi) &= \theta(1-\kappa)\int_x^1\frac{dy}{y}\mathcal{P}_A^{\pm}\left(\kappa,y\right)f^{\pm}\left(\frac{x}{y},\xi\right)\\ &+ \theta(\kappa-1)\int_x^{\infty}\frac{dy}{y}\left[\theta(1-y)\mathcal{P}_B^{\pm}\left(\kappa,y\right) + \theta(y-1)\mathcal{P}_B^{\pm}\left(-\kappa,y\right)\right]f^{\pm}\left(\frac{x}{y},\xi\right)\,.\end{aligned} \tag{2.18}$$

Eq. (2.18) has almost the form of a “standard” DGLAP equation except for the upper bound of the integral in the second line that extends up to infinity. However, this kind of integrals can be handled within APFEL with minor modifications of the integration strategy and up to a numerical approximation to be assessed.

2.1 On continuity of GPDs

It is well known that GPDs are required to be continuous at $x=\xi$ for factorisation to be valid [3]. It is thus interesting to consider the consequence of this constraint. To this end, let us consider the limits of Eq. (2.18) for $x\to\xi^{\pm}$, which corresponds to $\kappa\to 1^{\pm}$:

$$\lim_{x\to\xi^+}\mu^2\frac{d}{d\mu^2}f^{\pm}(x,\xi) = \int_x^1\frac{dy}{y}\mathcal{P}_B^{\pm}\left(1,y\right)f^{\pm}\left(\frac{x}{y},\xi\right) + \int_1^{\infty}\frac{dy}{y}\mathcal{P}_B^{\pm}\left(-1,y\right)f^{\pm}\left(\frac{x}{y},\xi\right)\,, \tag{2.19}$$

and:

$$\lim_{x\to\xi^-}\mu^2\frac{d}{d\mu^2}f^{\pm}(x,\xi) = \mu^2\frac{d}{d\mu^2}f^{\pm}(\xi,\xi) = \int_x^1\frac{dy}{y}\mathcal{P}_A^{\pm}\left(1,y\right)f^{\pm}\left(\frac{x}{y},\xi\right)\,. \tag{2.20}$$

Taking the difference between Eqs. (2.19) and (2.20), using the continuity of f at $x = \xi$, and considering that:[3]

$$\mathcal{P}^{\pm}_{A}(1,y) = \mathcal{P}^{\pm}_{B}(1,y)\ , \tag{2.21}$$

one finds:

$$\int_1^\infty \frac{dy}{y}\mathcal{P}^{\pm}_{B}(-1,y)\, f^{\pm}\left(\frac{x}{y},\xi\right) = 0\,, \tag{2.22}$$

which has to be valid at any scale and for any $f^{\pm}$. This immediately implies that:

$$\mathcal{P}^{\pm}_{B}(-1,y) = 0\,, \tag{2.23}$$

for all values of y and order-by-order in perturbation theory. We will explicitly verify this constraint when we will discuss the explicit expressions.

2.2 End-point contributions

Some of the expressions for the anomalous dimensions discussed below contain +-prescribed terms. It is thus important to treat these terms properly accounting for additional local terms stemming from the "incompleteness" of the convolution integrals. However, own to the support region in Fig. 2.1, such incomplete convolutions take place in the DGLAP region ($\kappa < 1$) only where the standard procedure already implemented in APFEL can be used. Conversely, in the ERBL region ($\kappa > 1$) the convolution integrals are always complete therefore no additional local terms have to be included.

3 Anomalous dimensions

A crucial ingredient for an efficient implementation of GPD evolution is the availability of the DGLAP-like splitting functions $\mathcal{P}^{\pm}_{I}$, Eq. (2.14), in a closed form amenable to be easily integrated as in Eq. (2.18).

As a first step, we need to make the flavour structure of the evolution kernels explicit. Working in the QCD evolution basis, we have:

$$\mathcal{P}^{+}_{I} = \begin{pmatrix} P_{I,qq} & P_{I,qg} \\ P_{I,gq} & P_{I,gg} \end{pmatrix}\ , \tag{3.1}$$

and:

$$\mathcal{P}^{-}_{I} = P^{\rm NS}_{I}\,, \tag{3.2}$$

where $P^{\rm NS}_{I}$ is the appropriate evolution kernel for the following particular non-singlet distribution. Each single function P admits the perturbative expansion in powers of α_s:

$$P(y,\kappa,\mu) = \sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu)}{4\pi}\right)^{n+1} P^{(n)}(y,\kappa)\,. \tag{3.3}$$

At one loop it turns out that all the non-singlet splitting functions are equal amongst themselves and to $P^{(0)}_{I,qq}$, *i.e.* $P^{{\rm NS},(0)}_{I} = P^{(0)}_{I,qq}$.[4]

Explicit expressions for the one-loop anomalous dimensions can be found, for example, in Ref. [2]. However, these expressions require some algebraic manipulation to make them suitable for our purpose (se the Mathematica notebook in doc/src/codes/GPDKernels.nb). Let us first consider the non-singlet unpolarised anomalous dimension at one loop. Using Eqs. (2.8) and (2.14), one finds:

$$P^{{\rm NS},(0)}_{I}(\kappa,y) = \begin{cases} I = A: \quad 2C_F\left[\dfrac{1+(1-2\kappa^2)y^2}{(1-y)(1-\kappa^2y^2)}\right]_+\,, \\[2ex] I = B: \quad 2C_F\left[\dfrac{1}{1-y}+\dfrac{1-\kappa}{2\kappa}\dfrac{1}{1+\kappa y}\right]_+\,. \end{cases} \tag{3.4}$$

[3] We will prove this equality case by case.

[4] When going beyond one loop, three different non-singlet structures emerge. In the QCD evolution basis, they are associated to the total-valence and ±-like distributions.

We now verify Eq. (2.21), finding:

$$P_A^{\mathrm{NS},(0)}(1,y) = P_B^{\mathrm{NS},(0)}(1,y) = 2C_F \left[\frac{1}{1-y}\right]_+ . \tag{3.5}$$

Also Eq. (2.23) is easily verified:

$$P_B^{\mathrm{NS},(0)}(-1,y) = 0 . \tag{3.6}$$

It is finally interesting to take the DGLAP limit $\kappa \to 0$ of $P_A^{\mathrm{NS},(0)}$ that gives:

$$P_A^{\mathrm{NS},(0)}(0,y) = 2C_F \left[\frac{1+y^2}{1-y}\right]_+ , \tag{3.7}$$

that coincides with the usual DGLAP splitting function, as it should. Since $P_{I,qq}^{(0)} = P_I^{\mathrm{NS},(0)}$, the same findings apply to $P_{I,qq}^{(0)}$.

Let us now turn to the remaining splitting functions $P_{I,qg}^{(0)}$, $P_{I,gq}^{(0)}$, and $P_{I,gg}^{(0)}$. Their explicit expressions read[5]:

$$P_{I,qg}^{(0)}(\kappa,y) = \begin{cases} I = A: \quad 4n_f T_R \dfrac{1-2y+(2-\kappa^2)y^2}{(1-\kappa^2y^2)^2} , \\[2ex] I = B: \quad 2n_f T_R \left[\dfrac{1+\kappa}{\kappa^2(1+\kappa y)}\right]\left[\dfrac{1-\kappa}{\kappa y} + \dfrac{1}{1+\kappa y}\right] , \end{cases} \tag{3.8}$$

$$P_{I,gq}^{(0)}(\kappa,y) = \begin{cases} I = A: \quad 2C_F \left[\dfrac{2-2y+(1-\kappa^2)y^2}{y(1-\kappa^2y^2)}\right] , \\[2ex] I = B: \quad C_F \left[\dfrac{2(1+\kappa)-(1-\kappa^2)y}{\kappa y(1+\kappa y)}\right] , \end{cases} \tag{3.9}$$

$$P_{I,gg}^{(0)}(\kappa,y) = \begin{cases} I = A: \quad 4C_A \left[\dfrac{-2+y(1-y+\kappa^2(1+y))}{(1-\kappa^2y^2)^2} + \dfrac{1}{y} + \dfrac{1}{(1-y)_+}\right] + \delta(1-x)\dfrac{11C_A - 4n_f T_R}{3} , \\ I = B: \quad C_A \left[\dfrac{-1+3\kappa^2-\kappa(2+(1-\kappa)^2\kappa)y}{\kappa^3 y(1+\kappa y)^2} + \dfrac{2}{y} + \dfrac{2}{(1-y)_+}\right] + \delta(1-x)\dfrac{11C_A - 4n_f T_R}{3} . \end{cases} \tag{3.10}$$

The continuity condition at $\kappa = 1$ is obeyed, producing:

$$P_{A,qg}^{(0)}(1,y) = P_{B,qg}^{(0)}(1,y) = \frac{4n_f T_R}{(1+y)^2} , \tag{3.11}$$

$$P_{A,gq}^{(0)}(1,y) = P_{B,gq}^{(0)}(1,y)\frac{4C_F}{y(1+y)} , \tag{3.12}$$

$$P_{A,gg}^{(0)}(1,y) = P_{B,gg}^{(0)}(1,y) = 4C_A \left[\frac{1+y^2}{y(1+y)^2\,(1-y)_+}\right] + \delta(1-y)\frac{11C_A - 4n_f T_R}{3} , \tag{3.13}$$

as well as the condition in Eq. (2.23) (NOT TRUE FOR P_{gg}. TO BE CHECKED!). Finally, their forward limit ($\kappa \to 0$) is:

$$\begin{aligned} P_{A,qg}^{(0)}(0,y) &= 4n_f T_R \left[y^2 + (1-y)^2\right] \\ P_{A,gq}^{(0)}(0,y) &= 2C_F \left[\frac{1+(1-y)^2}{y}\right] , \\ P_{A,gg}^{(0)}(0,y) &= 4C_A \left[-2 + y(1-y) + \frac{1}{y} + \frac{1}{(1-y)_+}\right] + \delta(1-x)\frac{11C_A - 4n_f T_R}{3} , \end{aligned} \tag{3.14}$$

which correctly reproduces the one-loop DGLAP splitting functions.

[5] The expression for $P_{I,gq}^{(0)}$ derived from Ref. [2] seems to be wrong because it does not seem to respect the condition in Eq. (2.21). A seemingly correct expression can be found in Ref. [4] by performing the replacement $\xi \to 2\kappa y$ in Eq. (24).

A On Vinnikov's code

The purpose of this Appendix is to draw the attention on a possible incongruence of the GPD evolution code developed by Vinnikov and presented in Ref. [5]. For definiteness, we concentrate on the non-singlet $H_{\rm NS}$ GPD in the DGLAP region $x > \xi$, whose evolution equation is given in Eq. (29). For completeness, I report that equation here:

$$\begin{aligned}\frac{dH_{\rm NS}(x,\xi,Q^2)}{d\ln Q^2} &= \frac{2\alpha_s(Q^2)}{3\pi}\left[\int_x^1 dy\frac{x^2+y^2-2\xi^2}{(y-x)(y^2-\xi^2)}\left(H_{\rm NS}(y,\xi,Q^2)-H_{\rm NS}(x,\xi,Q^2)\right)\right.\\ &+ H_{\rm NS}(x,\xi,Q^2)\left(\frac{3}{2}+2\ln(1-x)+\frac{x-\xi}{2\xi}\ln((x-\xi)(1+\xi))\right.\\ &- \left.\left.\frac{x+\xi}{2\xi}\ln((x+\xi)(1-\xi))\right)\right].\end{aligned} \tag{A.1}$$

The limit for $\xi \to 0$ of the equation above should reproduce the usual DGLAP evolution equation:

$$\frac{dH_{\rm NS}(x,0,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{4\pi}\int_x^1\frac{dz}{z}\left[\hat{P}_{\rm NS}(z)\right]_+ H_{\rm NS}\left(\frac{x}{z},0,Q^2\right), \tag{A.2}$$

where:

$$\hat{P}_{\rm NS}(z) = 2C_F\frac{1+z^2}{1-z} = 2C_F\left[\frac{2}{1-z}-(1+z)\right], \tag{A.3}$$

with $C_F = 4/3$. Written explicitly:

$$\begin{aligned}\frac{dH_{\rm NS}(x,0,Q^2)}{d\ln Q^2} &= \frac{\alpha_s(Q^2)}{4\pi}2C_F\left[\int_x^1 dz\,\frac{2}{1-z}\left(\frac{1}{z}H_{\rm NS}\left(\frac{x}{z},0,Q^2\right)-H_{\rm NS}(x,0,Q^2)\right)\right.\\ &- \int_x^1\frac{dz}{z}(1+z)H_{\rm NS}\left(\frac{x}{z},0,Q^2\right)\\ &+ \left.H_{\rm NS}(x,0,Q^2)\left(\frac{3}{2}+2\ln(1-x)\right)\right].\end{aligned} \tag{A.4}$$

Now I explicitly take the limit for $\xi \to 0$ of Eq. (A.1). The result is:

$$\begin{aligned}\frac{dH_{\rm NS}(x,0,Q^2)}{d\ln Q^2} &= \frac{2\alpha_s(Q^2)}{3\pi}\left[\int_x^1 dy\frac{x^2+y^2}{y^2(y-x)}\left(H_{\rm NS}(y,0,Q^2)-H_{\rm NS}(x,0,Q^2)\right)\right.\\ &+ \left.H_{\rm NS}(x,0,Q^2)\left(\frac{3}{2}+2\ln(1-x)\right)\right].\end{aligned} \tag{A.5}$$

Some further simple algebraic manipulation finally gives:

$$\begin{aligned}\frac{dH_{\rm NS}(x,0,Q^2)}{d\ln Q^2} &= \frac{\alpha_s(Q^2)}{4\pi}2C_F\left[\int_x^1 dz\,\frac{2}{1-z}\left(\frac{1}{z}H_{\rm NS}\left(\frac{x}{z},0,Q^2\right)-H_{\rm NS}(x,0,Q^2)\right)\right.\\ &- \int_x^1\frac{dz}{z}(1+z)H_{\rm NS}\left(\frac{x}{z},0,Q^2\right)\\ &+ \left.H_{\rm NS}(x,0,Q^2)\left(\frac{3}{2}+2\ln(1-x)+\ln(x)+(1-x)\right)\right],\end{aligned} \tag{A.6}$$

that is close to the correct results, Eq. (A.4), except for the two additional terms in the third line. Since the $\xi \to 0$ limit does not seem to produce the correct result, this suggests that the evolution code presented in Ref. [5] may not be entirely correct.[6]

[6] It is, in fact, possible to correct Eq. (A.1) in such a way that its $\xi \to 0$ limit gives the correct DGLAP evolution equation.

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