

Jet TMD

1 Definition of the jet TMD

Following the notes written by Lorenzo and Yannis, the impact-parameter-space inclusive jet-production cross section in DIS in TMD factorisation takes the standard form:

$$d\sigma \sim H_{ij}(Q; \mu) D_{i \rightarrow \text{jet}}(b, Q, t_{\mathcal{R}}; \mu, \zeta_1) F_{j \leftarrow P}(x, b, Q; \mu, \zeta_2), \quad (1.1)$$

where H_{ij} is the DIS hard function, $F_{j \leftarrow P}$ is the usual TMD PDF of the quark flavour j inside the proton, and $D_{i \rightarrow \text{jet}}$ is the TMD of the jet generated by the quark flavour i . x and Q are the usual DIS variable corresponding to the Bjorken variable and the negative virtuality of the vector boson while b is the Fourier conjugate variable of the partonic transverse momentum k_T . The scale μ is the resummation scale and must be $\mu = C_f Q$, with C_f order one, while ζ_1 and ζ_2 are the rapidity scale that in this case must obey the momentum-space equality $\zeta_1 \zeta_2 = Q^2 t_{\mathcal{R}} k_T^2$, with k_T being the partonic transverse momentum. In impact parameter space this equality naturally turns into $\zeta_1 \zeta_2 = Q^2 t_{\mathcal{R}}^2 b_0^2 / b^2$ with $b_0 = 2e^{-\gamma_E}$. Without loss of generality one can choose $\zeta_2 = Q^2$ such that $\zeta_1 = t_{\mathcal{R}}^2 b_0^2 / b^2$. Finally, the variable $t_{\mathcal{R}} = \tan(\mathcal{R}/2)$ depends on the jet opening \mathcal{R} and for TMD factorisation to be valid one needs $\mathcal{R} \sim 1$.

Using the definition devised by Yannis and Lorenzo, both jet TMD and TMD PDFs evolve multiplicatively through the standard Sudakov form factor R as:

$$G(\mu, \zeta) = R[(\mu, \zeta) \leftarrow (\mu_0, \zeta_0)] G(\mu_0, \zeta_0) \quad G = D_{i \rightarrow \text{jet}}, F_{j \leftarrow P}, \quad (1.2)$$

where we simplified the notation by dropping the unnecessary variables. In the following we take:

$$\mu_0 = \sqrt{\zeta_0} = C_i \mu_b, \quad \text{with} \quad \mu_b = \frac{b_0}{b}, \quad (1.3)$$

where C_i is a constant of order one. As is well know, $F_{j \leftarrow P}(\mu_0, \zeta_0)$ can be matched onto collinear PDFs for small values of b while a non-perturbative component needs to be accounted for larger values of b . This is the standard procedure and will not be discussed any further here. Let us now turn to the initial-scale jet TMD that can be further factorised as:

$$D_{i \rightarrow \text{jet}}(\mu_0, \zeta_0) = D_{i \rightarrow \text{jet}}(\mu_0, \zeta_0; \mu_0) = U_J[\mu_0 \leftarrow \mu_J] D_{i \rightarrow \text{jet}}(\mu_0, \zeta_0; \mu_J), \quad (1.4)$$

with $\mu_J = C_J Q t_{\mathcal{R}}$, $C_J \sim 1$. The evolution factor U_J takes the following explicit form:

$$U_J[\mu_0 \leftarrow \mu_J] = \exp \left[- \int_{\mu_0}^{\mu_J} \frac{d\mu'}{\mu'} \gamma_J(\mu') \right]. \quad (1.5)$$

The anomalous dimension valid up to NLL reads:

$$\gamma_J(\mu') = \left(\frac{\alpha_s(\mu')}{4\pi} \right) \gamma_F^{(0)} - \left[\left(\frac{\alpha_s(\mu')}{4\pi} \right) \gamma_K^{(0)} + \left(\frac{\alpha_s(\mu')}{4\pi} \right)^2 \gamma_K^{(1)} \right] \ln \frac{\mu_J}{\mu'}, \quad (1.6)$$

where $\gamma_F^{(i)}$ and $\gamma_K^{(i)}$ are the coefficients of the perturbative expansion of the non-cusp and cusp anomalous dimensions, respectively. Finally, the initial-scale jet TMD reads:

$$D_{i \rightarrow \text{jet}}(\mu_0, \zeta_0; \mu_J) = 1 + \frac{\alpha_s(\mu_J)}{4\pi} \left[\frac{1}{2} \gamma_K^{(0)} \ln^2 C_J + \gamma_F^{(0)} \ln C_J + d_J^{q, \text{alg}} \right], \quad (1.7)$$

where the coefficient $d_J^{q, \text{alg}}$ depends on the jet algorithm. For cone and k_T algorithms respectively reads:

$$d_J^{q, \text{cone}} = 7 + 6 \ln 2 - \frac{5\pi^2}{6}, \quad \text{and} \quad d_J^{q, k_T} = 13 - \frac{3\pi^2}{2}. \quad (1.8)$$