Scilab Manual for Probability Theory and Random Processes by Prof Shital Thakkar Others Dharmsinh Desai University¹

Solutions provided by Prof Shital Thakkar Others Dharmsinh Desai University

January 17, 2020

¹Funded by a grant from the National Mission on Education through ICT, http://spoken-tutorial.org/NMEICT-Intro. This Scilab Manual and Scilab codes written in it can be downloaded from the "Migrated Labs" section at the website http://scilab.in



Contents

Li	st of Scilab Solutions	3
1	Write a program to find probability of tossing a coin and rolling a die through large no. of experimentation.	5
2	To generate Uniform, Gaussian and Exponential distributed data for given mean and variance.	9
3	Write a program to generate M trials of a random experiment having specific number of outcomes with specified probabilities.	15
4	To find estimated and true mean of Uniform, Gaussian and Exponential distributed data.	18
5	To find density and distribution function of a function of random variable $Y=2X+1.$ where X is gaussian R.V.	22
6	Estimate the mean and variance of $Y=2X+1$, where X is a gaussian random variable.	26
7	Plot Joint density and distribution function of sum of two Gaussian random variable $(Z = X + Y)$.	28
8	Estimate the mean and variance of a R.V. $Z = X+Y$. Where X and Y are also random variables.	33
9	Simulation of Central Limit Theorem.	35

List of Experiments

Calculates probability of sum of tossing two dice.	5
Finds probability of getting Head when a coin is	
tossed	6
Generation of Uniform Data	9
Gaussian Data Generation	11
Exponential Data Generation	12
Random experiment with outputs in specific range	15
Comaparison of True and estimated statics of Uni-	
form Data	18
Comaparison of True and estimated statics of Gaus-	
sian Data	19
Comaparison of True and estimated statics of Ex-	
ponential Data	20
Density and Distribution plot generation for one	
function of Random Variable	22
Statistics of Function of one random variable	26
Joint Density and Distribution of Function of two	
random varible	28
Estimation of mean and variance of sum of two ran-	
dom variable	33
Summation of two random variable leads to Gaus-	
sian density function	35
	Generation of Uniform Data Gaussian Data Generation Exponential Data Generation Random experiment with outputs in specific range Comaparison of True and estimated statics of Uniform Data Comaparison of True and estimated statics of Gaussian Data Comaparison of True and estimated statics of Exponential Data Density and Distribution plot generation for one function of Random Variable Statistics of Function of one random variable Joint Density and Distribution of Function of two random variable Estimation of mean and variance of sum of two random variable Summation of two random variable leads to Gaussummation of two random variable leads

List of Figures

1.1	Finds probability of getting Head when a coin is tossed	7
2.1 2.2 2.3	Generation of Uniform Data	10 11 13
5.1	Density and Distribution plot generation for one function of Random Variable	23
7.1 7.2	Joint Density and Distribution of Function of two random varible	29
9.1	Summation of two random variable leads to Gaussian density function	30 36

Write a program to find probability of tossing a coin and rolling a die through large no. of experimentation.

Scilab code Solution 1.1 Calculates probability of sum of tossing two dice

```
1 // Operating System : Windows XP or later,
                 : 5.3.3
2 // Scilab
4 //Program of Tossing two dice and observing
     Probability of sum of their front face.
\frac{5}{f} for e.g. Probability of sum of two dice = 2 is
     1/36 as there are 36 possibilities and sum = 2
     can // occur only one combination that is both
     face = 1
6 clc;
7 clear;
8 clf;
              // Number of times tossing of die
9 N = 10000;
     performed
10 count = 0; // Counter for counting number of times
```

```
sum of die
11 \text{ for } i = 1:N
       y1 = ceil(rand(1)*6); // output of die 1
12
       y2 = ceil(rand(1)*6); // output of die 2
13
       if ((y1+y2) == 3) // check for sum of front
14
          face of both die is = 3(\text{change sum and})
           count = count + 1; //increment the count
15
              value when sum = 3 occurs
16
       end
       prob1(i) = count/i;  // no. of times sum of
17
          die = 3/total no. trials
18 \, end
19 plot(prob1)
20 xlabel('Number of Trials');
21 ylabel('Probability');
22 title ('Probability of getting sum of dots on faces
      of a dice to be 3');
23
24
25 //Assignment : Program can be checked for other
      values of sum at line number 10.
```

Scilab code Solution 1.2 Finds probability of getting Head when a coin is tossed

```
1 // Operating System : Windows XP or later,
2 // Scilab : 5.3.3
3
4
5 //This program find probability of getting Head when
        a coin is tossed.
6 //Probability = 1/2 = 0.5 as there are two possible
        outcomes in coin tossing experiment.
```

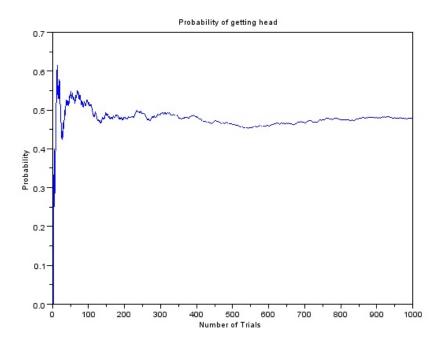


Figure 1.1: Finds probability of getting Head when a coin is tossed

```
7 clc;
8 clear all;
9 clf;
10 \text{ a1} = 1000;
11 \quad count1 = 0;
12 \text{ for } i = 1:a1
13 \times = round(rand(1));
       //round: the elements to nearest integer
14
15
       //rand:returns a pseudorandom, scalar value
          drawn from a uniform
16
       //distribution on the unit interval.
       if(x==1) // HEAD- '1', condition that detects '
17
          HEAD' comes or not
           count1 = count1 + 1; //increment the count
18
              value when head occurs
19
       p(i) = count1/i; // probability of head occuring at
20
           ith trail
21 end
22 plot(1:a1,p)
23 //plot the prob. at ith trail(plots discrete
      sequence)
24 xlabel('Number of Trials');
25 ylabel('Probability');
26 title ('Probability of getting head');
27 // Assignment:
28 //1.
          perform above experiment with n = 100,1000.
          Extend the above experiment to find
      probability of 3 heads in 4 coin tosses.
30 //
         Match the result theoretically.
```

To generate Uniform, Gaussian and Exponential distributed data for given mean and variance.

Scilab code Solution 2.1 Generation of Uniform Data

```
//To generate Uniform, Gaussian and Exponential
    distributed data.
// Operating System : Windows XP or later,
// Scilab : 5.3.3
//NOTE:EXECUTE ONE BY ONE SEGEMENT
// Uniform Data Generation
clc;
clc;
clear all;
//b = input('higher limit of uniform r.v. b = ')//
    Enter higher limit of uniform r.v.
//a = input('lower limit of uniform r.v. a =')//
    Enter lower limit of uniform r.v.
```

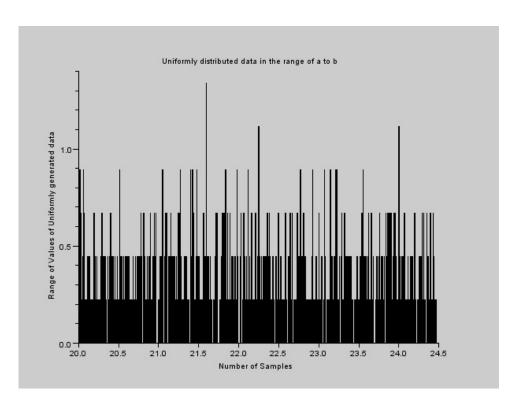


Figure 2.1: Generation of Uniform Data

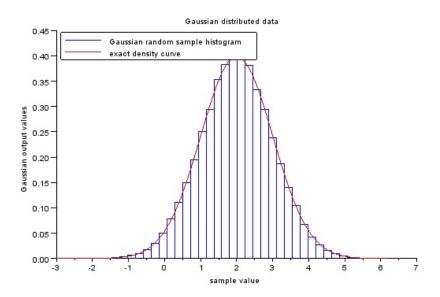


Figure 2.2: Gaussian Data Generation

Scilab code Solution 2.2 Gaussian Data Generation

```
1 //To generate Uniform, Gaussian and Exponential
      distributed data.
2 // Operating System : Windows XP or later,
3 // Scilab
                       : 5.3.3
5 / [2] Expoenential data generation & Mean and
     Variance Calcultaion of Exponential distributed
     data.
6 clc;
7 clear all;
8 clf();
9 //(i) Exponential data generation
10 lambda = 2; //or lambda = input ('enter lemda value
     for exponential r.v.')//lemda of exponential data
11 X = grand(10000, 1, "exp", 1/lambda);
12 X \max = \max(X);
13 histplot(40, X, style=2)
14 x = linspace(0, max(Xmax), 100);
15 plot2d(x,lambda*exp(-lambda*x),strf="000",style=5)
16 legend(["exponential random sample histogram" "exact
       density curve"]);
17 xlabel('sample value');
18 ylabel ('Exponential output values');
19 title ('Exponential distributed data');
```

Scilab code Solution 2.3 Exponential Data Generation

```
1 //To generate Uniform, Gaussian and Exponential
    distributed data.
2 // Operating System : Windows XP or later,
3 // Scilab : 5.3.3
```

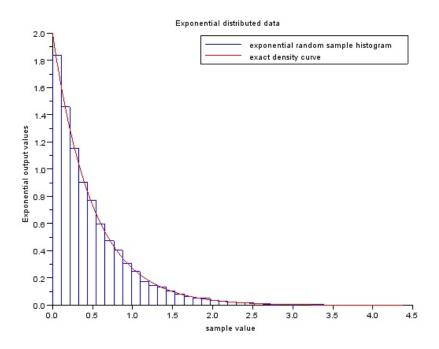


Figure 2.3: Exponential Data Generation

```
5 //[3] Gaussian data generation
6 //Gaussian data generation
7 clc;
8 clear all;
9 clf();
10 / m = input('enter mean value for Gaussian r.v.')
11 // vari = input ('enter mean value for Gaussian r.v.')
12 m = 2; //mean value of gaussian data
13 sd = 1; //standard deviation
14 \text{ vari} = sd^2;
15 X = grand(100000, 1, "nor", m, sd);
16 X \max = \max(X);
17 clf()
18 histplot(40, X, style=2)
19 x = linspace(-10, max(Xmax), 100);
20 plot2d(x,(1/(sqrt(2*\%pi*vari)))*exp(-0.5*(x-m).^2/
      vari), strf = "000", style = 5)
21 xlabel('sample value');
22 ylabel ('Gaussian output values');
23 title ('Gaussian distributed data');
24 legend(["Gaussian random sample histogram" "exact
      density curve"],2);
```

Write a program to generate M trials of a random experiment having specific number of outcomes with specified probabilities.

Scilab code Solution 3.1 Random experiment with outputs in specific range

```
// Operating System : Windows XP or later,
// Scilab : 5.3.3

// Write a program to generate M trials of a random experiment having specific
// number of outcomes with specified probabilities.
// here No. of trials = 1000, no. outcomes(rv) = 3 with specied probability entered by user
clc;
clear all;
rand('seed')//check
M = 1000; //Number of trials of random experiment
outcomes = 3; //Possible number of outcomes of
```

```
random experiment
12 for i = 1:outcomes -1
       r(i) = input ('enter upper range of probability
13
          of r.v. (values in the 0 < r < 1): ')//enter
          values in the 0 < r < 1
14
       if r(i) > 1 then
           error ('Enter values in the range 0<r<1')
15
16
       end
17 end
18 x = zeros(M,1);
19 for i = 1:M
20
       u = rand(1,1);//random outcome
21
       if u \le r(1) then
22
           x(i,1)=1;//assign v value = 1 if u <= r(1)
       elseif u > r(1) \& u \le r(2)
23
           x(i,1)=2;//second rv value
24
25
       else
26
           x(i,1)=3; //third rv value
27
       end
28 end
29 count1=0; count2=0; count3=0;
30 \text{ for } i=1:1000
31
       if x(i,1) == 1 then
32
           count1 = count1 + 1;
       elseif x(i,1) == 2
33
34
           count2 = count2 + 1;
35
       else
36
            count3 = count3 + 1;
37
       end
38 end
39 estP1 = count1/M; disp(estP1)//estimated probability
      of generated random variable
40 estP2 = count2/M; disp(estP2)//estimated probability
      of generated random variable
   estP3 = count3/M; disp(estP3) // estimated probability
      of generated random variable
42
43 // Assignment:
```

```
44 //1. Extend this program for 4 number of random variable
```

^{45 // 2.} Extend this program for more number of trials. i.e. $M=\,5000\,,10000$ etc.

To find estimated and true mean of Uniform, Gaussian and Exponential distributed data.

Scilab code Solution 4.1 Comaparison of True and estimated statics of Uniform Data

```
using function
14 Uni_true_mean=mean(x)
15 mprintf('Uniform True Mean = %f', Uni_true_mean)
16 Uni_true_var = variance(x)
17 mprintf('\n Uniform True Mean = \%f', Uni_true_var)
18 px = 1/(b-a)//pdf calcultaion of uniform r.v.
19 m_uniform=integrate('x*px', 'x',a,b)//mean
      calcultaion of uniform r.v.
20 fsq_uniform=integrate('(x^2)*px','x',a,b)//mean
     square value of uniform r.v.
21 var_uniform = fsq_uniform - (m_uniform).^2//variance
       of uniform r.v.
22
23 mprintf('\n Uniform Calculated Mean = \%f', m_uniform)
24 mprintf('\n Uniform Calculated Variance = \%f',
     var_uniform)
```

Scilab code Solution 4.2 Comaparison of True and estimated statics of Gaussian Data

```
1 // Operating System : Windows XP or later,
2 // Scilab : 5.3.3
3
4 //here we generate Gaussian distributed data compare
    its statistics with calculated using equation.
5
6 //[3] Mean & Variance calculation of Gaussian Data
7
8 clc;
9 clear all;
10 m = 2;//mean value of gaussian data
11 sd = 1;//standard deviation
12 vari = sd^2;
13 X = grand(10000,1,"nor", m,sd);//gaussian data
    generation using function
```

Scilab code Solution 4.3 Comaparison of True and estimated statics of Exponential Data

To find density and distribution function of a function of random variable Y = 2X + 1. where X is gaussian R.V.

Scilab code Solution 5.1 Density and Distribution plot generation for one function of Random Variable

```
1 // Operating System : Windows XP or later,
2 // Scilab : 5.3.3
3
4 //To find density(pdf) and distribution(cdf)
    function of a function of random variable
5 //Y = 2X + 1(having form Y = aX + b). where X is
    gaussian R.V.
6
7 clc;
8 clear all;
9 clf();
10 mean_x = 1; //mean value of gaussian data
```

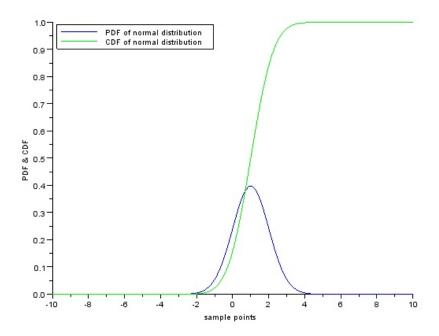


Figure 5.1: Density and Distribution plot generation for one function of Random Variable ${\bf R}$

```
11 \text{ sd_x} = 1; // \text{standard deviation}
12 vari_x = sd_x.^2;
13 \, \lg d = [];
14 //PDF and CDF of Gaussian Random Variable X
15 x = linspace(-10, 10, 100);
16 \text{ plot2d}(x,((1/(sqrt(2*\%pi*vari_x)))*exp(-0.5*(x-
      mean_x).^2/vari_x)),2);//plots pdf of X
17 set(gca(), "auto_clear", "off")
18 plot2d(x,cdfnor("PQ",x,mean_x*ones(x),sd_x*ones(x))
      ,3);//cdf of gaussian RV X
19 set(gca(), "auto_clear", "on")
20 xlabel('sample points');
21 ylabel('PDF & CDF');
22 //title ('density and distribution function for
      Gaussian function');
23 legend(['PDF of normal distribution'; 'CDF of normal
      distribution '],2);
24
25 //PDF and CDF of Y = aX + b where a = 2, b = 1
26 \ a = 2;
27 	 b = 1;
28 y = a*x+b; //Function of One Random Variable
29 mean_y=a*mean_x+b;
30 \text{ vari_y=(a*sd_x).^2};
31 figure (2, "BackgroundColor", [1,1,1]);
32 \text{ plot2d}(y,((1/(sqrt(2*\%pi*vari_y*a.^2)))*exp(-0.5*(y-
      mean_y).^2/vari_y)),2);//pdf of Y
33 set(gca(), "auto_clear", "off")
34 \text{ plot2d}(x, \text{cdfnor}("PQ", y, (a*mean_x+b)*ones(x), (a*sd_x))
      *ones(x)),3);//cdf of y
35 set(gca(), "auto_clear", "on")
36 xlabel('sample points');
37 ylabel('PDF & CDF of Y = 2X + 1');
38 legend(['PDF of Y = 2X + 1'; 'CDF of Y = 2X + 1'],2);
39
40
41 //Assignment :
42 //1. Perform the operation for function Y = 5X + 1.
```

 $43\ \ //\, 2.$ Generate pdf and cdf of nonlinear function between Y and X.

Estimate the mean and variance of Y = 2X + 1, where X is a gaussian random variable.

Scilab code Solution 6.1 Statistics of Function of one random variable

```
1 // Operating System : Windows XP or later,
2 // Scilab : 5.3.3
3
4 //True and Estimated value of mean and variance of function of one random
5 // variable having form of Y = aX +b.
6 clc;
7 clear all;
8 rand('seed',getdate('s'))
9 m = 0; // mean of random variable x
10 vari = 1; // variance of random variable x
11 m_est = 0;
12 var_est = 0;
13 for i = 1:1000
14 y(i,1) = 1 +2*rand(1,1,"normal"); // Y = 2X + 1
```

```
where x is gaussian data
       m_{est} = m_{est} + ((1/1000)*y(i)); //estimation by
15
          averaging
       var_est = var_est + ((1/1000)*(y(i)-m_est)^2);
16
17 \text{ end}
18 printf ('Estimated mean of Y(=2X + 1) is: Est_mean=\%f
      ', m_est)
19 printf('\n Estimated variance of Y) is: Est_variance
       =\%f', var_est)
  //Calculation of true mean of Y
21 y_{mean=integrate}('(2*x+1)*(1/sqrt(2*\%pi*vari)*exp(-(
      x-m)^2/(2*vari))), 'x', -100,100);
22 printf('\n True mean of Y(=2X + 1) is: True_mean=\%f'
      ,y_mean)
23 // Calculation of true variance of Y
24 //for a function like Y = aX + b the variance of Y
      is a 2 * Variance of X.
  gs_mean=integrate('x*(1/sqrt(2*\%pi*vari)*exp(-(x-m)
      ^2/(2*vari)))','x',-50,100);
  gs_fsq=integrate('((x^2)*(1/sqrt(2*\%pi*vari)*exp(-(x
      -m)^2/(2*vari))))', 'x', -50,100);
27 \text{ gs\_var} = \text{gs\_fsq} - (\text{gs\_mean}).^2;
28 \text{ var_y} = 2^2 \text{ gs_var}; // \text{here } a = 2
29 printf('\n True variance of Y(=2X + 1) is:
      True_variance=%f', var_y)
30
31 // Expectation of Y is E(Y)=E(2X+1)=2E(X)+1. That's
      why answer is 1.
32 //True variance of Y in this format is equal to a^2*
      variance of X.
33 //Assignment:
34 //1. Assume X is uniform random variable between a
      to b. find mean and variance.
```

Plot Joint density and distribution function of sum of two Gaussian random variable (Z = X + Y).

Scilab code Solution 7.1 Joint Density and Distribution of Function of two random varible

```
1 // Operating System : Windows XP or later,
2 // Scilab : 5.3.3
3
4
5 // Plot Joint density and distribution function of sum of two Gaussian random variable (Z = X + Y).
6 clc;
7 clear all;
8 clf();
```

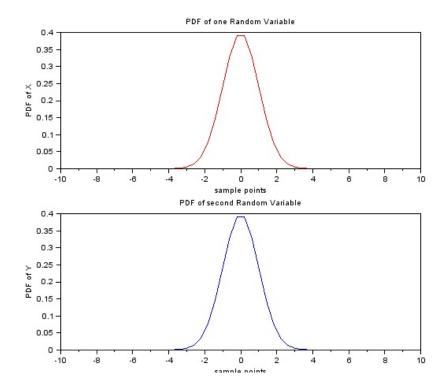


Figure 7.1: Joint Density and Distribution of Function of two random varible

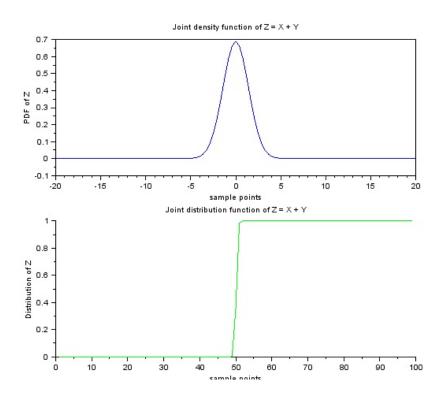


Figure 7.2: Joint Density and Distribution of Function of two random varible

```
10 //PDF of Gaussian Random Variable X
11 mean_x = 0; //mean of first gaussian RV
12 sd_x = 1; //standard deviation of first gausssian RV
13 \text{ vari}_x = \text{sd}_x.^2;
14
15 x = linspace(-10,10,50); //generating linearly spaced
       data as Random output
16 X = ((1/(sqrt(2*\%pi*vari_x)))*exp(-0.5.*(x-mean_x))
      .^2/vari_x));//finding gaussian pdf of above data
17 subplot (2,1,1);
18 plot(x, X, 'r')
19 xlabel('sample points');
20 ylabel('PDF of X');
21 title('PDF of one Random Variable')
22
23
24 //PDF of Gaussian Random Variable Y
25 \text{ mean_y} = 0;
26 \text{ sd_y} = 1;
27 \text{ vari_y} = \text{sd_y.^2};
28
29 y = linspace(-10, 10, 50);
30 \ Y = ((1/(sqrt(2*\%pi*vari_y)))*exp(-0.5.*(y-mean_y))
      .^2/vari_y));
31 subplot (2,1,2);
32 plot(y,Y,'b')
33 xlabel('sample points');
34 ylabel('PDF of Y');
35 title ('PDF of second Random Variable')
36
37 // Joint pdf of sum of random variable X & Y
38 // When two IID random variable are summen up, their
       Joint PDF is convolution between individual pdfs
       of Random variables
39 z = convol(X,Y);
40 figure (2, "BackgroundColor", [1,1,1]);
41 subplot (2,1,1); plot (linspace(-20,20,99),z)//Joint
     PDF
```

Estimate the mean and variance of a R.V. Z = X+Y. Where X and Y are also random variables.

Scilab code Solution 8.1 Estimation of mean and variance of sum of two random variable

```
1 // Operating System : Windows XP or later,
2 // Scilab : 5.3.3
3
4 //Concept : Estimation of mean and variance of sum of two random variable Z = X + Y, where X and Y are random variable.
5 // Above concept is explained with example as follows.
6 //Example: A large circular dartboard is set up with a "bullseye" at the center of the circle, which is at the coordinate(0,0). A dart is thrown at the center but lands at (X,Y) are two different Gaussian random variables. What is average distance of the dart from the bullseye? What is
```

```
variance of data?
7 // Distance from center is given as sqrt(X^2+Y^2)
9 clc;
10 clear all;
11 rand('seed',0)//setting seed of random generator to
12 \text{ m_est} = 0;
13 \text{ for } i = 1:1000
       R(i,1)=sqrt(rand(1,1,'normal')^2+rand(1,1,'
          normal')^2);//calculation of distance from
15
       m_{est} = m_{est} + (1/1000) *R(i); // estimation of mean
          from data
16 \text{ end}
17 \text{ m_est}
18 mprintf ('Mean of Sum of Two Random variable that is
      Mean of Z = \%f, m_est)
19 v_est = variance(R)//variance calculation
20 mprintf('\n Variance of Sum of Two Random variable
      that is Mean of Z = \%f', v_est)
```

Simulation of Central Limit Theorem.

Scilab code Solution 9.1 Summation of two random variable leads to Gaussian density function

```
1 // Operating System : Windows XP or later,
2 // Scilab
                    : 5.3.3
3
5 //Simulation of Central Limit Theorem.
6 //(Which says that if we keep on adding independent
     Random Variables then it density function
7 //approches to gaussian distribution)
8 //here two uniform RVs are added.
9
10 clc;
11 clear all;
12 clf();
13 \quad n = 0:0.01:1;
14 x = zeros(length(n), 1);
15 i = 1:50; //length of Uniform rv 1
```

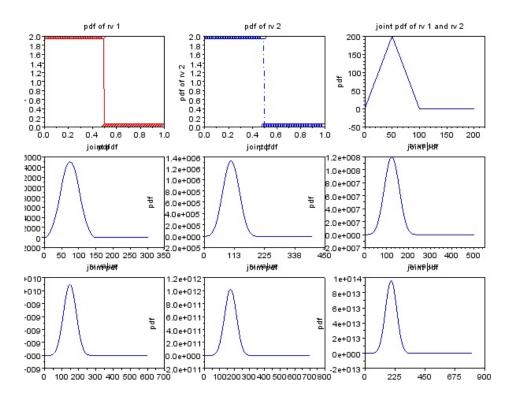


Figure 9.1: Summation of two random variable leads to Gaussian density function

```
16 x(i) = 2;
17 subplot (3,3,1);
18 plot(n,x,'r-d')
19 xlabel('pdf'); ylabel('pdf of rv 1')
20 title('pdf of rv 1 ');
21
22 y = zeros(length(n), 1);
23 \quad j = 1:50;
24 y(j) = 1*2; //length of Uniform rv 2
25 subplot(3,3,2);
26 plot(n,y,'bo-.')
27 xlabel('pdf');
28 ylabel('pdf of rv 2')
29 title('pdf of rv 2');
30
31 	 z1 = convol(x,y);
32 subplot (3,3,3)
33 //When two independent RVs are added their joint
      density is convolution of marginal density
34 plot(z1)
35 xlabel('rv value');
36 ylabel('pdf')
37 title('joint pdf of rv 1 and rv 2');
38
39 for i = 4:9 // adding rv 9 times
40
       subplot(3,3,i)
41
       z1 = convol(z1,y);
42
       plot(z1)
       xlabel('rv value');
43
       ylabel('pdf')
44
45 title('joint pdf');
46 \text{ end}
```