

14 Jan 20

## Hash Functions

Key is the major factor in cryptography.

## Public Key Crypto System

(A)

PA  
PrA

(B)

(B)

PB  
PrB

he can unlock  
it, becoz he  
knows

## # Types of cryptography

wallet associated to a user  
whenever you are about to receive it is  
going to generate key pub & priv. key  
whenever it has enough bitcoins it makes the  
block and put it above the chain.

Mining: Anyone can become

## Permissionless Blockchain

~~out~~ (P) 17 Jan 20

⑧ 17 [Jan] 20  
RSA deals with the integers of size, 2000 bits

- multiple cryptosystems

  - 1)  $p, q \rightarrow$  large primes      choosey  
2 nos      & comput.
  - 2)  $N = p * q$        $(\log p \text{ or } \log q)$       & comput.
  - 3)  $\phi(N) = (p-1) * (q-1)$

Euler's Function

function

$\phi(n) = \left\{ a \mid a \in \mathbb{Z}^+, 1 \leq a < n, \text{afn } B \right\}$

- 4) choose such e. s.t gcd(e,  $\phi(N)$ ) = 1

5) compute d s.t  $e d \equiv 1 \pmod{\phi(N)}$

$$a \equiv b \pmod{d}$$

$$d \mid (a - b)$$

$$\exists q \text{ s.t } d = q_r(a-b)$$

$$\mathbb{Z}[N^2] \subseteq \{0, 1, 2\}$$

3

21

## Key Parameters

$K^2$  (P  
get

Public

privat

encryption

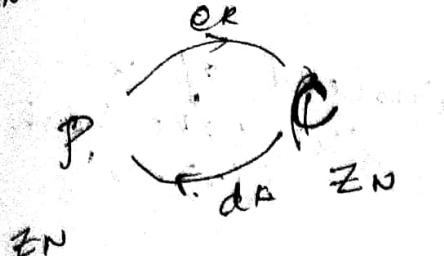
$$ER =$$

$$x \in F$$

decryption

$$dx(y)^2$$

$\mathbb{Z}_N^2 \subseteq \{0, 1, 2, \dots, N-1\}$



key parameters

$K = (p, q, e, d, \phi(N), N)$

get it off from them

public info :  $(e, N)$

private part :  $(d)$

encryption :-

$$EK = P \rightarrow C$$

$$\begin{aligned} x \in P & \quad EK(x) \equiv x^e \pmod{N} \\ & \equiv y \end{aligned}$$

decryption :-

$$dE(y) = y^d \pmod{N} \approx x$$

$$dE(y) \stackrel{?}{=} y^d \pmod{N}$$

$$\stackrel{?}{=} x^d \pmod{N}$$

$$\stackrel{?}{=} x^{d(\phi(N)+1)} \pmod{N}$$

$$\stackrel{?}{=} x^{d\phi(N)+d} \pmod{N}$$

$$\stackrel{?}{=} (x^{\phi(N)})^d \cdot x^d \pmod{N}$$

$$\stackrel{?}{=} 1 \cdot x^d \pmod{N}$$

$$\stackrel{?}{=} x^d \pmod{N}$$

Encryption will use this

# compute exponentiation  
sq. & mult algo.

# extended Euclid's algorithm

↑ to compute  
 $d$ ,  
Euler's quotient.  
 $(x^{\phi(N)}) \equiv 1$  always

$$(x^{\phi(N)}) \equiv 1$$

signature  $\rightarrow$  msg, sig

(x, s)

$$s = x^d \text{ Mod } N$$

$$\text{if } s^e = x \text{ Mod } N$$

s is valid

instead of x  
i'll take hash  
 $h(x) = x'$

Now send this

Problem?

$$s = (x')^d \text{ Mod. } N$$

$$\text{if } s^e = x' \text{ mod. } N.$$

s is valid.

(2)

$$\log_b a$$

$$\log_2 4$$

group

(S, \*)

closed

Associativity

id

inverse

1)  $(S, *, *)$  in Ab. group with id "o"

2)  $(S - \{o\}, *)$  in Ab. group

$$3) a * (b + c) = a * b + a * c$$

\* a, b, c es

field  
complex no, Rational no.

finite field

$\mathbb{Z}_p, +_p, *_p$  prime p are defined as

$$a +_p b = (a+b) \text{ Mod } p$$

$$a *_p b = (a * b) \text{ Mod } p$$

$$\mathbb{Z}_2 = \{0, 1\}^2$$

	0	1
0	0	1
1	1	0
	0	1

corresponds to XOR operation

*	1
1	1

Ex:-

$$(\mathbb{Z}_3, +_3, *_3)$$

is field with 3 elements

$$|\mathbb{Z}_p^*| = p-1 \quad \text{coz } p \text{ is prime}$$

cardinality of the group

and since  $a^{(p-1)}$  mod p

$a \in \mathbb{Z}_p^*$  a  $\in \mathbb{Z}_p$  mod p

Lagrange's theorem

$\mathbb{Z}_p^* = \langle g \rangle$  there will be relevant g in the group such that all power of g will give you  $\mathbb{Z}_p^*$

$$g^{p-1} \equiv 1$$

Discrete log problem.

$Z_7^*$  &  $C_7$  ?

$Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$$Z_7^* = 6$$

$$6 = (2)^g$$

$$\alpha^{p-1}$$

$$S^1 = 5$$

$$S^2 = \frac{25}{7} = 4$$

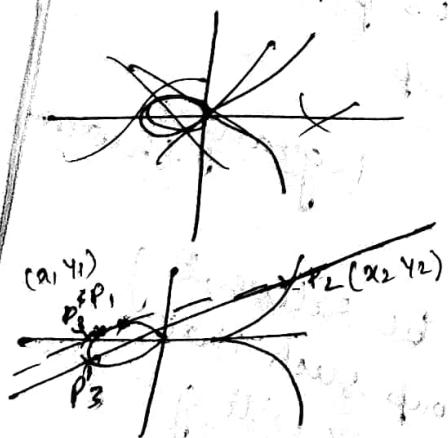
$$S^3 = \frac{125}{7} = 6.$$

$$S^4 = \frac{625}{7} = 2$$

$$S^5 = 3.$$

In 'S' all the elements are coming that are inside the set  $Z_7$ .

So, that's the generator.



How to add these 2 points

$$P_1 + P_2 = P_3$$

$$P_1 + P_2$$

wherever it will be cutting take the mirror image &

Alic

① C

② C

SS

③ C

④ C

The

21/Jan/20

## Elgamal Crypto System

y - public  
x - private

$$y = g^x \pmod{p}$$

Based on difficulty of discrete learning algorithm

Alice

- ① Generate a random prime  $p$ .
- ② choose an arbitrary integer 'a', which is a primitive root modulo  $p$ .
- ③ choose a random no.  $x$  from interval  $(1, p)$  which is co-prime to  $(p-1)$ .
- ④ calculate  $y = a^x \pmod{p}$ . Then the public key is a triple  $(a, p, y)$ , and private key  $= x$ .

Alice

Encryption

Let  $M$

- ① Select a random secret no. 'k', which is coprime to  $(p-1)$ .

- ② Calculate

$$V = a^k \bmod p$$

$$\sigma = M \cdot V^k \bmod p$$

Ciphertext  $(V, \sigma)$

Decryption

we need to know here the private key  $'x'$ , and then  $\sigma^{-x} \bmod p$

$$M = (\sigma \cdot V^{p-1-x}) \bmod p$$

$$= V^{-x} \sigma \bmod p$$

(A)

receives

$$y = a^x \bmod p$$

$$x - m$$

m

Eig

Key

\* Ch

\* Com

\* PW

\* Sec

Ch

(A)  
receiver

$a, p, y$

(B)

sender

$$y = a^x \bmod p$$

$x$  - random value  
 $1 < x < (p-2)$

preliminary  
calculation

$$K\text{-random } 1 \leq R \leq (p-2)$$

$$v^R = a^R \bmod p$$

$$\sigma = M \cdot y^R \bmod p$$

$$M = v^{-x} \bmod p$$

2nd step

$[v, \sigma]$

Elgamal signature scheme  $\rightarrow$  generator

Key Generation

\* choose randomly a secret key  $x$   
with  $1 < x < p-1$

\* compute  $y = g^x \bmod p$

» Public Key  $(p, g, y)$

\* Secret key  $(x)$

Signature Generation Scheme

To sign a msg  $M$ , the signer performs  
the following steps

\* choose a random 'k'

\* compute  $r = g^x \mod p$

\* compute  $t = g^{H(m)} \mod p$

\* compute  $s = (H(m) - x \cdot r) k^{-1} \mod p$

The pair  $(r, s)$  is the signature of  $m$ .

\* You can use any hash func' here.

### Verification

A signature  $(r, s)$  of a  $m$  is

verified as follows

\*  $0 < r < p$  and  $0 < s < (p-1)$

\*  $g^{H(m)} \equiv y^r r^s \mod p$

$$P - (x_1, y_1)$$

$$x_1 \neq x_2$$

$$Q - (x_2, y_2)$$

$$\textcircled{1} \quad P+Q = (x_1, y_1) + (x_2, y_2)$$
$$= (x_3, \underline{y_3})$$

$$x_3 = \lambda^2 - x_1 - x_2$$

$$y_3 = \lambda(x_1 - x_3) - y_1 \text{ and}$$

$$\lambda = \frac{(y_2 - y_1)}{x_2 - x_1}$$

Case-2

$$x_1 = x_2, \quad y_1 = y_2$$

$$(x, y) = (x, y) + (x - y) = 0^\circ$$

28/Jan

## Block structure

### Block Header

### Merkle Tree

### Bitcoin Mining

## Transaction Script

Empty stack → True

① push 2      ② push 3  
                ADD 5 EQUAL

③ ADD 5 EQUAL  
(pop) 2 & 3 push 5

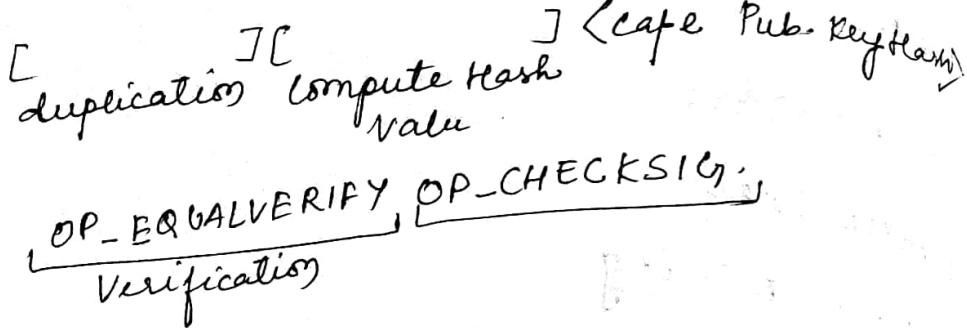
④ 5 EQUAL  
    ↑  
    push .

⑤ EQUAL  
pop (5 & 5)

True

## Locking Script form

You store public key  
in hashed form



Verify  $\rightarrow$  PubKey + Msg + signature.  
Part need only

Double

gm o

one

so, h

solution

Every  
unsp

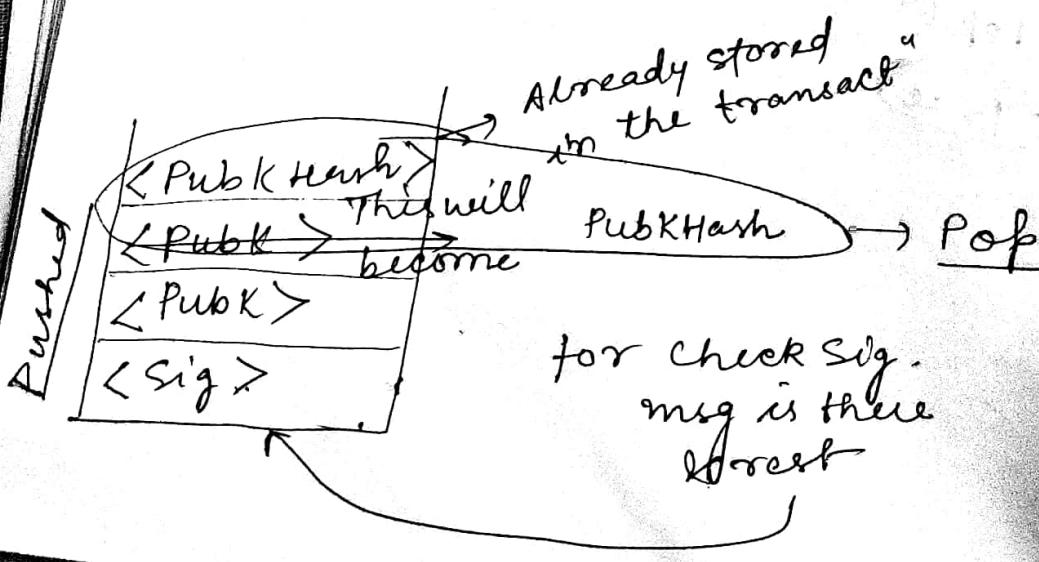
UTXO

## Unlocking Script

<cafe signature> <cafe Publickey>

## Evaluating Combined Script

<sig> <pubk> DUP HASH160 <PubKHash>  
EQUALVERIFY CHECKSIG..



Every  
new

Bon

1 sat

0.010

gf

## Double Spending

In digital transaction I can send one 100 among 2 people.

So, how can we avoid that?

Solution :-  
Every miner will maintain a list of unspent transaction o/p.

### UTXO

Once you make the transaction automatically it is deleted from it. So that's how we take care of double spending.

Every miner has a responsibility of making new coins which gives

trans '0' → reward for miner. ] created after each trans.

Reward

1 satoshi =  $10^{-8}$  BC,

$0.01 \text{ mBC} \rightarrow 0.01 \times 10^8 \text{ } 10^8 \text{ satoshis}$

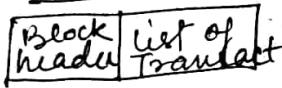
If the flow is more, the reward will come down.

## Consensus

(Kind of Agreement)

31 Jan 2020

Block

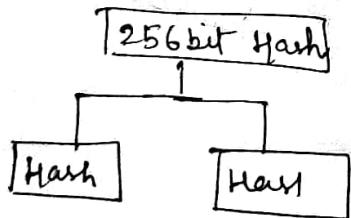


connected via  
hash value — SHA256

of Backed linked list

Hash is 2cf----9824  
digest

Merkle root: To aggregate the transaction  
→ It comprises of hash values.



Transactions can be  
of any size (may be in  
4Bs) but finally they  
are converted into  
256 bit

Address of the block: Hash of the header block.

→ Our tree header

Nonce:  
by b

Block is  
block 1  
to make

What is

H (pr)

→ Suppose we  
make  
with l  
so this

→ Difficult

Mining

Tamper  
Proof

→ Our transactions are also part of the header

Nonce: Miner will calculate the nonce by bruteforce method.

Block is invalid because appending the block you need to change the nonce also to make it valid.

What is there in <sup>prev of</sup> genesis block? → NULL

list of trans<sup>n</sup>

↑

$H(\text{prev-Hash} \parallel \text{Tx} \parallel \text{nonce})$

< difficulty  
↓  
this is also  
hash value.

→ Suppose we have to make a hash value

with leading four bit as 0.

so this is the difficulty.

→ Difficulty rises as blocks rises.

Mining process → is finding the nonce & we have to spend too much computational power

# Tampering last block is a bit easy task.  
But not in the middle

# As the blocks rises, security also rises

# Everyone can become a miner, it

was those

but nowadays we cannot becomes  
because ~~set~~ they are not only using  
GPUs but ASICs which has <sup>very</sup> high  
computational power.

SHA-3 → 512 bits

500

$2^{512}$

ip domain

output domain

output changes about 50%.

DviDas 2018

7/2/08

Bitco

1) gm

2) De

3) Tr

script

→ gt

# why

LOCK

(Locking

Vitali

201

He

Date :-  
1/2/08

Bitcoin features :-

- 1) Immutability
- 2) Decentralization
- 3) Traceability

`<script>` is involved in writing Bitcoin programming.

It can hold only 256 opcodes.  
OP-ADD  
OP-SUB  
OP-EQUAL-VERIFY

# why Bitcoin is made turing incomplete?

LOCK  $\xrightarrow{\text{send}}$  unlock  
(Locking script) (Unlocking script)

Vitalik

2014

He released a white paper in the name

Ethereum

We can't write locking & unlocking script in other application.

Bitcoin  
Nakamoto  
2007  
script-256

Ethereum  
Vitalik  
2014

Solidity

This is used to write  
smart contracts  
↳ set of instructions  
↳ special kind of  
programs.

We are not gonna  
explicitly run  
this program.  
only its triggered.

You can implement our own  
idea in the form of smart contract  
and can deploy in Ethereum. Which  
can't be done in Bitcoin.

# Smart Contract has  
↳ functions  
↳ state variable.

$x=5$   
 $x=10$  } Ethereum maintains both  
the states using time stamp  
concept.

functions which are associated with objects  
are methods.

- ⇒ We can assume our Ethereum BlockChain  
as Server &  
MetaMask → To create a link b/w our  
Ethereum & Browser.

3 Types of n/w

mainnet    Test    private

Block Height → no. of blocks.

Smart Contract &  
Pragma Solidity > 0.4.0; → which compiler  
we are going  
to use to comp  
Smart Contract

pyboard  
↳ Contract - SampleData

of

    uint storeData

    function set(uint x)

        storeData = x

    }

    function get()

        public view returns (uint)

        return storeData

    }

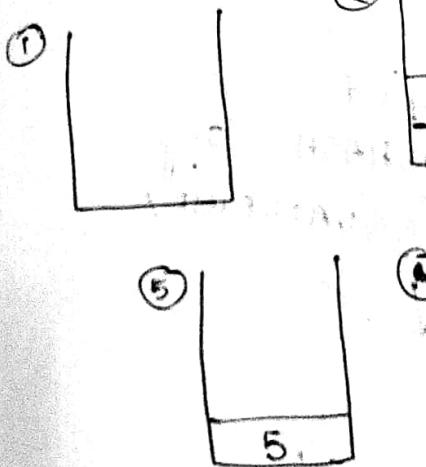
    }

    }

To compile Ethereum we need  
EVM.

14/2/20

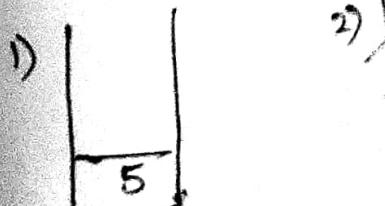
OP-2 OP-3 OP-  
T  
P  
stack pointer



Result : false

Ex-2

OP-5 OP-



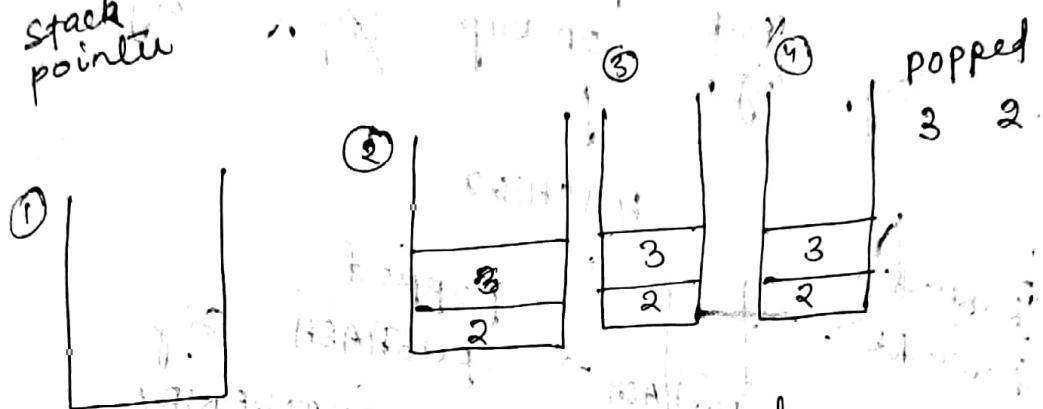
(8) OP-EQUA  
popped  
S

Result :

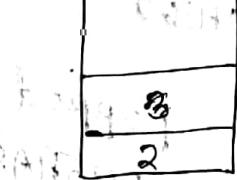
14/2/20

OP-2 OP-3 OP-ADD OP-6 OP-EQUAL  
→ ↑

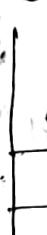
stack  
pointer



⑤



③



④



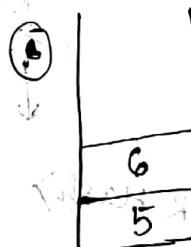
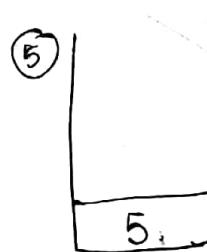
popped

3 2

popped

6 5

OP-EQUAL



Result : False

Ex-2

OP-5 OP-DUP OP-EQUALVERIFY



popped

5

OP-DUP

5



⑧ OP-EQUALVERIFY →  
popped  
5 5

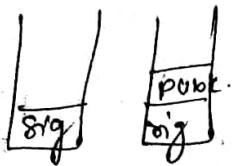
If the result is  
false then it  
stops or else  
it will go on.

Result : True

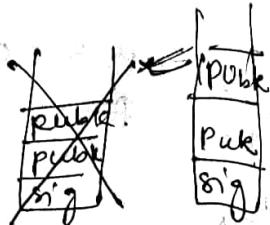
These are just values

<sig> <pubK> DUP HASH160 <pubKHash>  
EQUALVERIFY CHECKSIG

①



pop  
pubK  
OP-DUP



solidity read

gas limit : 100.185  
gas used by Tx =  
gas price = 0.1

### Assignment

student contract

- Details of student
- Attendance

TF = QUT X

GP for 1 uni

popped

<pubHash> <pubK Hash>  
EQUALVERIFY

popped  
pubHash Sig  
EQUALVERIFY

if(true)

popped

<pubK> + <sig>

CHECKSIG

if(true)

unlocked

soldiary read mode.

Gas limit : 100.185

Gas used by Tca = 100.185

Gas Price = 0.0750 00., 1 Eru.

### ⑧ Assignment

student Attendance Management  
contract

- Details of students
- Attendance values

$$F.P = Q.U.T \times G.P$$

GP for 1 unit.