4.7.7 $\mu_2 = 10 \quad \mu_1 = 0 \quad \hat{\sigma} = 36 \quad \pi_2 = 0.8 \quad \pi_1 = 0.2$

$$\text{prob}(Y=2 \mid X=4) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{k} \pi_l f_l(x)} \; ; \quad f_k(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)$$

$$f_1(0.4) = \frac{1}{6\sqrt{2\pi}} \exp\left(-\frac{1}{72}(4-0)^2\right) = \frac{e^{-2/9}}{6\sqrt{2\pi}}$$

$$f_2(0.4) = \frac{1}{6\sqrt{2\pi}} \exp\left(-\frac{1}{72}(4-10)^2\right) = \frac{e^{-1/2}}{6\sqrt{2\pi}}$$

$$= \frac{0.8\left(\frac{e^{-1/2}}{6\sqrt{2\pi}}\right)}{0.2\left(\frac{e^{-2/9}}{6\sqrt{2\pi}}\right) + 0.8\left(\frac{e^{-1/2}}{6\sqrt{2\pi}}\right)}$$

$$= 0.7518$$