

$$P_r(Y=k | X=x) = \frac{\pi_k f_k(x)}{\sum_{i=1}^K \pi_i f_i(x)}$$

Bayes' Theorem

4.7.3

plugging in:

$$f_k(x) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)$$

results in:

$$p_k(x) = \frac{\pi_k \frac{1^*}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)}{\sum_{i=1}^K \pi_i \frac{1^*}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{1}{2\sigma_i^2}(x-\mu_i)^2\right)}$$

* when $\sigma_1 = \sigma_2 = \sigma_k$ (LDA),
these terms cancelled out.
Now they do not.

rearranging...

$$\ln(p_k(x)) = \frac{\ln(\pi_k) \cdot \ln\left(\frac{1}{\sqrt{2\pi}\sigma_k}\right) \cdot \left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)}{\ln\left(\sum_{i=1}^K \pi_i\right) \cdot \ln\left(\sum_{i=1}^K \frac{1}{\sqrt{2\pi}\sigma_i}\right) \cdot \left(\sum_{i=1}^K -\frac{1}{2\sigma_i^2}(x-\mu_i)^2\right)}$$

→ which is quadratic.