

$$4.7.7 \quad \mu_2 = 10 \quad \mu_1 = 0 \quad \hat{\sigma} = 36 \quad \pi_2 = 0.8 \quad \pi_1 = 0.2$$

$$\begin{aligned} \text{prob}(Y=2 | X=4) &= \frac{\pi_k f_k(x)}{\sum_{k=1}^K \pi_k f_k(x)} ; \quad f_k(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right) \\ &= 0.8 \left(\frac{e^{-1/2}}{6\sqrt{2\pi}} \right) \\ &\quad \frac{0.2 \left(\frac{e^{-2/9}}{6\sqrt{2\pi}} \right) + 0.8 \left(\frac{e^{-1/2}}{6\sqrt{2\pi}} \right)}{0.2 \left(\frac{e^{-2/9}}{6\sqrt{2\pi}} \right) + 0.8 \left(\frac{e^{-1/2}}{6\sqrt{2\pi}} \right)} \\ &= 0.7518 \end{aligned}$$

$$\begin{aligned} f_1(0.4) &= \frac{1}{6\sqrt{2\pi}} \exp\left(-\frac{1}{72}(4-0)^2\right) \\ &= \frac{e^{-1/9}}{6\sqrt{2\pi}} \end{aligned}$$

$$\begin{aligned} f_2(0.4) &= \frac{1}{6\sqrt{2\pi}} \exp\left(-\frac{1}{72}(4-10)^2\right) \\ &= \frac{e^{-1/2}}{6\sqrt{2\pi}} \end{aligned}$$