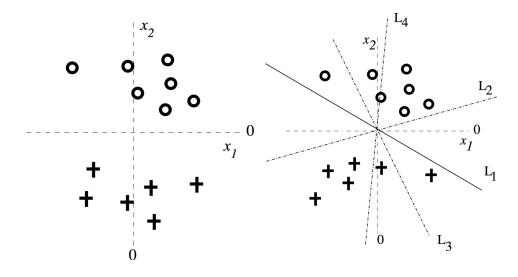
1. Assume the binary classification task depicted in Figure ??, which we attempt to solve with the simple linear logistic regression model

$$\widehat{\Pr}(Y = 1 | X = x) = \hat{p}(x) = g(\beta_1 x_1 + \beta_2 x_2) = \frac{1}{1 + \exp(\beta_1 x_1 + \beta_2 x_2)}$$

for simplicity we do not use the parameter β_0 . The training data is linearly separable, and the line L₁ is the result of logistic regression, with zero training error..



Assume that we would like to find the classifier by maximizing the following regularized objective function, in which only β_2 is regularized.

$$\prod_{i=1}^{n} [p(x_i)]^{y_i} [1 - p(x_i)]^{1-y_i} - \lambda \beta_2^2 = L(\beta_1, \beta_2) - \lambda \beta_2^2$$

- (a) Assume that λ is large. Which of the four lines L_2, L_3 or L_4 determine whether it can result from regularizing β_2 . Explain very briefly your reasons.
- (b) If we change the form of regularization to one-norm (absolute value) and also regularize β_2 we get the following penalized log-likelihood

$$\prod_{i=1}^{n} [p(x_i)]^{y_i} [1 - p(x_i)]^{1-y_i} - \lambda(|\beta_1| + |\beta_2|) = L(\beta_1, \beta_2) - \lambda(|\beta_1| + |\beta_2|)$$

As we increase the regularization parameter λ which of the following scenarios is expected to be observed? Explain why.

- i. First β_1 will become 0, then β_2 .
- ii. β_1 and β_2 will become zero simultaneously.
- iii. First β_2 will become 0, then β_1 .
- iv. None of the weights will become exactly zero, only smaller as λ increases

Solution:1

- (a) When we regularize β_2 , the resulting boundary can rely less on the value of x_2 and therefore becomes more vertical.
 - L2 here seems to be more horizontal than the unregularized solution so it cannot come as a result of penalizing β_2 .
 - When β_2 is small relative to β_1^2 (as evidenced by high slope), and even though it would assign a rather low log-probability to the observed labels, it could be forced by a large regularization parameter λ . So L3 can arise as a result.
 - For very large λ , we obtain a separator that is very close to vertical (with negative slope) and in the limit, entirely vertical (line $x_1 = 0$ or the x_2 axis). L₄ here is reflected across the x_2 axis and has a positive slope, and therefore represents a poorer solution than its counterpart on the other side. For moderate regularization we have to get the best solution that we can construct while keeping β_2 small. L₄ is not the best and thus cannot come as a result of regularizing β_2 .
- (b) The data can be classified with zero training error and therefore also with high log-probability by looking at the value of x_2 alone, i.e. making $\beta_1 = 0$. Initially we might prefer to have a non-zero value for β_1 but it will go to zero rather quickly as we increase regularization. Note that we pay a regularization penalty for a non-zero value of β_1 and if it does not help classification why should the penalty be paid? The \mathcal{L}_1 regularization ensures that β_1 will indeed go to exactly zero. As λ increases further, even β_2 will eventually become zero. We pay higher and higher cost for setting β_2 to a non-zero value. Eventually this cost overwhelms the gain from the log-probability of labels that we can achieve with a non-zero β_2 . Note that when $\beta_1 = \beta_2 = 0$, the log-probability of labels is a finite value $n\Delta \log(0.5)$.

¹Important Note: Posting the course material to online forums or sharing it with other students is strictly prohibited. Instances will be reported to USC officials as academic dishonesty for disciplinary action.

2. A statistician is working on the amount of funding that companies obtain on a crwod-sourcing website and has developed the following model. She used 26 companies to obtain the model

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5$$

$$\hat{y} = 964.8 + 700.2x_1 + 317.5x_2 - 200.2x_3 + 15.3x_4 + 17.1x_5$$

The standard errors are:

$$s_{b_1} = 12.$$

 $s_{b_2} = 22.5$
 $s_{b_3} = 101.8$
 $s_{b_4} = 45.3$
 $s_{b_5} = 2.3$

- \hat{y} : the amount of funding obtained by a company in 1000 dollars
- x_1 : the average annual salary of the founders
- x_2 : the number of employees the startup hired
- x_3 : a dummy variable that is 1 when the company's field is information technology and 0 otherwise
- x_4 : the age of the company
- x_5 is a dummy variable taking value 1 if the founders had previous failures and 0 otherwise
- (a) Interpret the estimated coefficients $b_0 = 964.8$ and $b_3 = -200.2$ (10 pts)
- (b) Test, at the 2% level, the null hypothesis that the true coefficient on the dummy variable x_5 is 0 against the alternative that it is not 0. (10 pts)
- (c) Find and interpret a 99.8% confidence interval for the parameter β_4 . (10 pts)
- (d) If for the model, SSR=18147.5 (Regression Sum of Squares) and SSE = 17136.5 (Residual Sum of Squares), test the hypothesis that all the coefficients of the model are 0 (test overal significance of the model) using $\alpha = 5\%$. (10 pts).

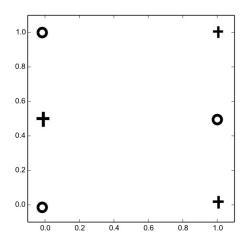
Solutions:

- (a) b_0 is the amount of the dependent variable that was not explaned by the independent variables. b_3 means that if the field of the company is information technology, on average, the funding tat it wil receive frm the website will decrease by 200.2 units (1000 dollars)
- (b) $\mathbf{H_0}: \beta_5 = 0, \ \mathbf{H_1}: \beta_5 \neq 0, \ t_{b_5} = \frac{b_5 0}{s_{b_5}} = \frac{17.1}{2.3} = 7.44$ The rejection region is $t > t_{n-K-1,\alpha/2} = t_{26-5-1,01}$ or $t < -t_{n-K-1,\alpha/2} = -t_{26-5-1,01}$.

But from the table, $t_{26-5-1,01} = 2.528$, therefore, we reject the null hypothsi that $\beta_5 = 0$.

- (c) $t_{n-K-1,\alpha/2} = t_{26-5-1,0.001} =$, so the Confidence interval for β_4 is: $[b_4 t_{n-K-1,\alpha/2} s_{b_4}, b_4 + t_{n-K-1,\alpha/2} s_{b_1}] = [15.3 (3.552)(45.3), 15.3 + (3.552)(45.3)] = [-145.61, 176.21].$
- (d) $F = \frac{SSR/K}{SSE/(n-K-1)} = \frac{18147.5/5}{17136.5/(26-5-1)} = 4.2$. The rejection region is $F > F_{K,n-K-1,\alpha} = F_{5,20,0.05} = 2.7109$, which means that we reject the null hypothesis that all coefficients are 0.

3. For the two dimensional training data shown below, determine whether or not each of the classification methods below, when trained appropriately, will have zero errors on the training set. In each case, briefly justify your answer. Moreover, provide a reasonable confusion matrix for each case.



- (a) Logistic Regression
- (b) SVM with Linear Kernel
- (c) SVM with RBF Kernel
- (d) Decision Tree
- (e) 3-Nearest-Neighbor Classifier (with Euclidean Distance).

Solution:

- (a) Logistic Regression and Linear SVM: linear decision boundaries, hence no.
- (b) SVM with RBF kernel: yes.
- (c) 3-NN: the 3 nearest neighbors of any point in our training set are 1 of the same class and 2 of the opposite class, hence 3-NN will be systematically wrong.
- (d) DT: yes, one can partition the space with lines orthogonal to the axes so that every sample ends up in a different region.

For methods with zero training error, the Confusion Matrix would be:

| | Class o | Class + |
|---------|---------|---------|
| Class o | 3 | 0 |
| Class + | 0 | 3 |

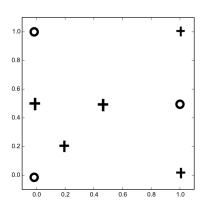
For KNN:

$$\begin{array}{ccc} & \text{Class o} & \text{Class +} \\ \text{Class o} & 0 & 3 \\ \text{Class +} & 3 & 0 \end{array}$$

For SVM, LR:

$$\begin{array}{ccc} & \text{Class o} & \text{Class} + \\ \text{Class o} & 2 & 1 \\ \text{Class} + & 1 & 2 \end{array}$$

- 4. For a numeric input, instead of a binary split in a decision tree, one can use a ternary split with two thresholds and three branches as $X_j < s_1, s_1 \le X_j < s_2, X_j \ge s_2$.
 - (a) Propose a modification of the tree learning method to adjust the two thresholds, s_1 and s_2 .
 - (b) What are the advantages and the disadvantages of such a node over a binary node?
 - (c) How would you choose between a binary and a trenary decision tree for a given data set?
 - (d) Perform two iterations of the trenary tree algoritm on the tree shown in below and draw the corresponding tree



Solution:

- (a) For the numeric attributes, instead of one split threshold, we need to try all possible pairs of split thresholds and choose the best. When there are two splits, there are three children, and in calculating the entropy/Gini index after the splits, we need to sum up over the three sets corresponding to the instances taking the three branches.
- (b) The computational complexity of finding the best pair is higher and each node stores two thresholds instead of one and has three branches instead of two. The advantage is that one ternary node splits an input into three, whereas this requires two successive binary nodes.
- (c) Which one is better depends on the data at hand; if we have a ground truth that requires bounded intervals (e.g., rectangles), a ternary node may be advantageous. One has to choose between binary and trenary nodes using *Cross Validation*.
- (d) Left to the students.

5. Consider the following dataset with one real-valued input and one binary output (+ or -). The following questions assume that we are using k- nearest-neighbor learning with Euclidean distance to predict Y for an input X. What is the leave-one-out cross-validation error of 1-NN and 3-NN on this dataset, and decide which k is better.



Solution: For each X_i , consider the majority vote of 1 nearest neighbors.

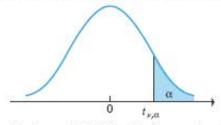
 $E_{LOOCV} = 0.4$

For each X_i , consider the majority vote of 3 nearest neighbors.

 $E_{LOOCV} = 0.8\,$

The 1-NN method is better

Upper Critical Values of Student's t Distribution with ν Degrees of Freedom



For selected probabilities, α , the table shows the values $t_{\nu,\alpha}$ such that $P(t_{\nu} > t_{\nu,\alpha}) = \alpha$, where t_{ν} is a Student's t random variable with ν degrees of freedom. For example, the probability is .10 that a Student's t random variable with 10 degrees of freedom exceeds 1.372.

| | | PROBABILE | ty of Exceeding th | E CRITICAL VALUE | | |
|-----|-------|-----------|--------------------|------------------|--------|---------|
| ν | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.313 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.20 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.782 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4,49 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.29 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.14 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.02 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.92 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.85 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.78 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.73 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.68 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.64 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.61 |
| 19 | 1,328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.57 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.55 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.52 |
| 22 | 1,321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.50 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.48 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.46 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.45 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.43 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.42 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.40 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.39 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.38 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.30 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.23 |
| 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.17 |
| 09 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.09 |
| ν | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 |

F - Distribution (α = 0.05 in the Right Tail)

| 161,45 199,50 215,71 224,58 230,16 233,99 236,77 238,88 240 238,18 238,18 2 | ۲ | df, | | , | , . | Numerator Degrees of Freedom | Degrees | of Freedo | Ĕ, | × | 0 |
|---|-----------------|-----|--------|--------|--------|------------------------------|---------|-----------|--------|--------|--------|
| 1 161,45 199.50 15.71 224.58 230.16 233.99 236.77 238.88 244 2 18,513 19,000 19.164 19.247 19.296 19.330 19.353 19.371 11 3 18,513 19,000 19.164 19.247 19.296 19.330 18.353 19.371 11 4 7,7086 9,9443 6,5914 6,382 6,2561 6,092 4.8759 4.8183 6,091 6 6094 6 6094 6 6094 6 6094 6 6094 6 6094 6 6094 6 6094 6 6 6094 6 | 5 | _[_ | - | 7 | ٦ | , | 0 | | | | |
| 18.513 19.000 19.164 19.247 19.296 19.333 19.337 19.371 10.128 9.5521 9.2766 9.1172 9.0135 8.9406 8.8867 8.8452 7.7086 9.9443 6.5094 6.382 6.2501 4.9503 4.8759 6.4110 6.6079 5.7861 5.4095 5.1922 5.0503 4.9503 4.8759 6.4110 5.9874 5.1433 4.3468 4.1203 3.8715 3.866 3.7805 4.1063 3.7876 4.1168 5.5914 4.7374 4.3468 4.1203 3.9715 3.866 3.7803 3.2577 3.2287 4.2839 4.2067 4.1468 5.7174 4.2565 3.8625 3.6331 3.4817 3.3738 3.2927 3.2296 4.7471 3.8853 3.4918 3.3584 3.2672 3.0994 3.0944 2.8466 4.4940 3.0566 2.9012 2.9142 2.7486 4.7669 4.4666 3.0017 4.7478 3.2874 3.2587 | 1 | | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
| 3 10,128 9,5521 9,2766 9,1172 9,0135 8,4966 8,8867 8,8452 4 7,7086 1,9,9443 6,5914 6,3882 6,2561 6,1631 6,0942 6,0110 6 5,8674 5,7814 5,433 4,2839 4,2859 4,2879 4,4148 7 5,874 4,7374 4,3468 4,1203 3,9715 3,860 3,7870 3,7257 9 5,1174 4,256 3,862 3,8373 3,860 3,787 3,7257 10 4,9646 4,026 3,837 3,887 3,286 3,787 3,788 3,787 3,788 3,787 3,788 3,788 3,789 3,749 3,1122< | 2 | | 18.513 | 19.000 | 19.164 | 19.247 | 19.296 | 19.330 | 19.353 | 19.371 | 19.385 |
| 4 7,7086 ⋅9,9443 6.5914 6.3882 6.2561 6.1631 6.0942 6.0410 5 6,6079 5.7861 5.4995 5.1922 5.0503 4.9503 4.8759 4.8183 6 5.9874 5.1433 4.7571 4.3877 4.3874 4.2839 4.2067 4.1468 9 5.51174 4.4590 4.0662 3.8379 3.6875 3.5866 3.7807 3.1358 10 4.9646 4.1028 3.7083 3.4780 3.2586 3.5005 3.4817 11 4.8443 3.9823 3.8417 3.2039 3.095 3.2940 3.1328 3.2172 3.1358 3.2040 3.4817 12 4.9640 4.0602 3.8373 3.2392 3.1059 2.9961 2.9137 2.3480 13 4.6672 3.8853 3.4903 3.2592 3.1059 2.9961 2.9134 2.8486 14 4.6001 3.8833 3.4903 3.2592 3.1059 | 3 | | 10.128 | 9.5521 | 9.2766 | 9.1172 | 9.0135 | 8.9406 | 8.8867 | 8.8452 | 8.8123 |
| 5 6.6079 5.7861 5.4095 5.1922 5.0503 4.9503 4.8759 4.8183 6 5.9874 4.7374 4.7371 4.5337 4.8374 4.2839 4.2839 4.2839 4.2839 4.2839 4.2839 4.2839 4.2839 4.2839 4.2839 4.2849 4.7876 4.4849 9 5.1174 4.2565 3.8625 3.6331 3.4817 3.2866 3.0057 3.4381 10 4.9446 4.1028 3.7831 3.4817 3.2927 3.2927 3.2927 11 4.8443 3.9823 3.4805 3.4906 3.0944 3.0133 3.2927 3.2939 3.0946 3.0133 3.2927 3.2946 3.0171 3.0254 2.9153 2.9467 2.9480 2.9467 2.9481 2.9482 3.0946 3.0133 3.2927 3.0946 3.0134 3.0254 2.9153 2.9406 2.9406 2.9406 2.9411 2.8486 2.9406 2.9411 2.8486 2.9411 2 | 4 | | 7.7086 | 9.9443 | 6.5914 | 6.3882 | 6.2561 | 6.1631 | 6.0942 | 6.0410 | 8866.9 |
| 6 5.9874 5.1433 4.7571 4.5337 4.3874 4.2067 4.1468 7 5.5914 4.7374 4.3468 4.1203 3.8715 3.8660 3.7870 3.7257 8 5.3177 4.4590 4.0662 3.8379 3.8675 3.5806 3.7095 3.4817 10 4.9646 4.1028 3.7083 3.4780 3.3258 3.2027 3.2296 11 4.8443 3.9823 3.8874 3.3567 3.2039 3.0946 3.1023 2.9460 12 4.6672 3.8853 3.4903 3.4780 3.2539 3.0123 2.9480 13 4.6677 3.8853 3.4903 3.1059 2.9961 2.9134 2.8486 14 4.6001 3.7389 3.3676 2.9182 2.9477 2.7413 2.6698 15 4.4430 3.6323 3.2874 3.0584 2.9182 2.7412 2.8406 16 4.4430 3.6323 3.2874 <t< th=""><th>S</th><th></th><th>6.6079</th><th>5.7861</th><th>5.4095</th><th>5.1922</th><th>5.0503</th><th>4.9503</th><th>4.8759</th><th>4.8183</th><th>4.7725</th></t<> | S | | 6.6079 | 5.7861 | 5.4095 | 5.1922 | 5.0503 | 4.9503 | 4.8759 | 4.8183 | 4.7725 |
| 7 5.5914 4.7374 4.3468 4.1203 3.9715 3.8660 3.7870 3.7257 8 5.3177 4.4590 4.0662 3.8379 3.6875 3.8660 3.7870 3.7257 10 4.9646 4.1026 3.8625 3.4817 3.3738 3.2927 3.2296 11 4.8443 3.9823 3.7083 3.4780 3.3258 3.2172 3.1355 3.0417 12 4.8443 3.9823 3.4903 3.2592 3.1059 2.9661 2.9134 2.8486 13 4.6001 3.8853 3.4903 3.2592 3.1059 2.9661 2.9153 2.9480 3.2486 14 4.6001 3.6837 3.2387 3.0592 2.9673 2.9153 2.9480 15 4.4513 3.6815 3.1988 2.9647 2.8170 2.7643 2.5480 16 4.4513 3.5219 3.1284 2.861 2.7401 2.5847 2.7402 20 <t< th=""><th>9</th><th></th><th>5.9874</th><th>5.1433</th><th>4.7571</th><th>4.5337</th><th>4.3874</th><th>4.2839</th><th>4.2067</th><th>4.1468</th><th>4.0990</th></t<> | 9 | | 5.9874 | 5.1433 | 4.7571 | 4.5337 | 4.3874 | 4.2839 | 4.2067 | 4.1468 | 4.0990 |
| 8 5.3177 4,4590 4,0662 3.8379 3.6875 3.5806 3.5005 3.4381 1.0 4,9646 4,1028 3.7083 3,4780 3.2278 3.2277 3.2296 1.1 4,8443 3,9823 3,4824 3.2328 3.2172 3.2378 3.2277 3.2296 1.1 4,8443 3,9823 3,4824 3.2358 3,2373 3,0946 3,0123 2,9480 1.2 4,4742 3.8853 3,4903 3.2552 3.1059 2.9961 2.9143 2.8486 1.1 4,6601 3.7389 3.4903 3.2552 3.1059 2.9961 2.9143 2.8486 1.1 4,6601 3.7389 3.4403 3.1122 2.9582 2.8477 2.7642 2.6987 1.1 4,4513 3.5915 3.1389 3.0069 2.8524 2.7413 2.6569 2.9961 1.2 4,4319 3.5315 3.1389 3.0069 2.8524 2.7413 2.6569 2.9961 1.2 4,4319 3.5346 3.1599 2.9677 2.7109 2.6937 2.7413 2.6572 2.5911 2.7401 2.6887 2.7413 2.5480 2.9471 2.7002 2.6943 2.5480 2.9471 2.7002 2.6943 2.5480 2.9471 2.7002 2.6943 2.5480 2.9471 2.7002 2.6943 2.5490 2.9471 2.7002 2.6943 2.5492 2.9471 2.7002 2.6943 2.5492 2.9471 2.7002 2.6943 2.5492 2.9471 2.7002 2.6943 2.5491 2.6884 2.5497 2.5492 2.3478 2.4408 3.4424 3.4424 3.4491 2.8861 2.7109 2.5590 2.5471 2.4205 2.5911 2.4409 2.9404 2.940 | 7 | _ | 5.5914 | 4.7374 | 4.3468 | 4.1203 | 3.9715 | 3.8660 | 3.7870 | 3.7257 | 3.6767 |
| 9 5.1174 4.2565 3.8625 3.6331 3.4817 3.3738 3.2927 3.2296 10 4.9646 4.1028 3.7083 3.4780 3.3258 3.2172 3.1355 3.0717 11 4.8443 3.9823 3.5874 3.3567 3.2039 3.0946 3.0123 2.9480 12 4.7472 3.8853 3.4105 3.1791 3.0946 2.9134 2.9480 13 4.6001 3.7389 3.4105 3.1791 3.0946 2.9132 2.9480 14 4.6001 3.7389 3.2874 3.0556 2.9013 2.7905 2.7402 2.6987 17 4.44940 3.6337 3.2389 3.0069 2.8524 2.7413 2.6987 2.5110 2.6408 2.641 2.6418 2.541 2.4480 2.6480 2.641 2.6413 2.4480 2.6480 2.6410 2.6438 2.4480 2.6410 2.6488 2.5411 2.6488 2.5440 2.6488 2.6410 | 90 | | 5.3177 | 4.4590 | 4.0662 | 3.8379 | 3.6875 | 3.5806 | 3.5005 | 3.4381 | 3.3881 |
| 10 4.9646 4.1028 3.7083 3.4780 3.3258 3.2172 3.1355 3.0717 11 4.8443 3.9823 3.5874 3.3567 3.2039 3.0946 3.0123 2.9480 12 4.8443 3.9823 3.4903 3.2592 3.1059 2.9961 2.0134 2.8486 13 4.6672 3.8853 3.4903 3.2524 3.1059 2.9961 2.9134 2.8486 14 4.6001 3.7389 3.0659 2.9647 2.9153 2.7066 2.6408 15 4.4490 3.6337 3.2384 3.0556 2.9013 2.7413 2.6522 2.6987 2.6413 2.6408 17 4.44139 3.546 3.1559 2.9277 2.7413 2.6527 2.5911 20 4.4439 3.5546 3.1559 2.9277 2.7413 2.5480 2.9647 2.810 2.6937 2.5410 21 4.4390 3.5219 3.7468 2.9641 2.7413 | m | _ | 5.1174 | 4.2565 | 3.8625 | 3.6331 | 3.4817 | 3.3738 | 3.2927 | 3.2296 | 3.1789 |
| 11 4.8443 3.9823 3.5874 3.3567 3.0394 3.0123 2.9480 12 4.7472 3.8853 3.4903 3.2592 3.1059 2.9961 2.9134 2.9480 13 4.6672 3.8853 3.4903 3.2592 3.1059 2.9961 2.9134 2.8486 14 4.6011 3.7389 3.3499 3.1721 2.9522 2.9153 2.8486 15 4.4540 3.6823 3.2439 3.0059 2.8477 2.7066 2.6987 16 4.4940 3.6823 3.2889 3.0069 2.8524 2.7413 2.6918 2.6987 17 4.4513 3.5546 3.1599 2.9647 2.8100 2.6987 2.6143 2.5408 18 4.4139 3.5546 3.1274 2.8061 2.7729 2.6613 2.5408 2.5408 20 4.4319 3.5546 3.1274 2.8611 2.7401 2.6833 2.3408 21 4.3248 | e op | _ | 4.9646 | 4.1028 | 3.7083 | 3.4780 | 3.3258 | 3.2172 | 3.1355 | 3.0717 | 3.0204 |
| 12 4.7472 3.8853 3.4903 3.2592 3.1059 2.961 2.9134 2.8486 13 4.6672 3.8056 3.4105 3.1791 3.0254 2.9153 2.8321 2.7669 14 4.6001 3.7389 3.1122 2.9582 2.8477 2.7642 2.6987 15 4.64940 3.7389 3.1122 2.9582 2.8477 2.7642 2.6987 17 4.4940 3.6337 3.2384 3.0699 2.8524 2.7413 2.6987 2.6408 17 4.44513 3.5516 3.1596 2.9277 2.7413 2.6432 2.4471 19 4.3807 3.5219 3.1274 2.8951 2.7401 2.6933 2.5480 2.4471 20 4.3212 3.4908 2.9641 2.6103 2.5491 2.4471 2.4058 21 4.3248 3.468 3.0725 2.8401 2.6848 2.577 2.4422 2.3748 22 4.2793 < | = | | 4.8443 | 3.9823 | 3.5874 | 3.3567 | 3.2039 | 3.0946 | 3.0123 | 2.9480 | 2.8962 |
| 13 4,6672 3,8056 3,4105 3,1791 3,0254 2,9153 2,8321 2,7669 14 4,6001 3,7389 3,3439 3,1122 2,9582 2,8477 2,7642 2,6987 15 4,5431 3,6823 3,2874 3,0556 2,9013 2,7905 2,7066 2,6408 16 4,4940 3,6337 3,2389 3,0669 2,8524 2,7413 2,6572 2,5911 17 4,4513 3,5316 3,1588 2,9647 2,8100 2,6987 2,6143 2,5480 20 4,43807 3,5219 3,1274 2,8951 2,7401 2,6987 2,6132 2,5410 21 4,43807 3,6984 2,8641 2,7401 2,6843 2,5446 2,3663 22 4,3309 3,4454 3,0491 2,8461 2,6413 2,4426 2,4406 23 4,3793 3,4221 3,0280 2,7421 2,6436 2,3463 24 4,2793 | :re | | 4.7472 | 3.8853 | 3.4903 | 3.2592 | 3.1059 | 2.9961 | 2.9134 | 2.8486 | 2.7964 |
| 14 4,6001 3.7389 3.3439 3.1122 2.9582 2.8477 2.7642 2.6987 15 4,5431 3,6823 3.2874 3,0556 2.9013 2.7905 2.7066 2,6408 16 4,4940 3,6337 3,2389 3,0069 2,8524 2.7413 2,6572 2,5911 17 4,44513 3,5346 3,1968 2,9647 2,8100 2,6987 2,6143 2,5480 19 4,44139 3,5346 3,1594 2,9647 2,8100 2,6987 2,6143 2,5480 20 4,4139 3,5219 3,1274 2,8951 2,7401 2,6837 2,6478 2,5476 2,4768 21 4,438 3,4084 2,8641 2,7109 2,5990 2,5140 2,4471 22 4,3208 3,4084 2,8641 2,7401 2,6283 2,3405 23 4,2293 3,44234 3,0491 2,8149 2,5491 2,4438 2,4405 24 | <u>"</u> t t | ,,, | 4.6672 | 3.8056 | 3.4105 | 3.1791 | 3.0254 | 2.9153 | 2.8321 | 2.7669 | 2.7144 |
| 15 4.5431 3.6823 3.2874 3.0556 2.9013 2.7905 2.7066 2.6408 16 4.4940 3.6337 3.2884 3.0669 2.8524 2.7413 2.6572 2.5911 17 4.4513 3.5316 3.1968 2.9647 2.8100 2.6987 2.6143 2.5480 18 4.4139 3.5516 3.1599 2.9277 2.7729 2.6613 2.5435 2.4768 20 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 2.5435 2.4768 21 4.3807 3.668 3.0725 2.8401 2.6848 2.5727 2.4876 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5727 2.4876 2.4205 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5727 2.4422 2.3748 24 4.2793 3.4028 2.7763 2.5082 2.4024 2.4422 2.3468 | 0 8 | | 4.6001 | 3.7389 | 3.3439 | 3.1122 | 2.9582 | 2.8477 | 2.7642 | 2.6987 | 2.6458 |
| 16 4,4940 3,6337 3,2389 3,0069 2,8524 2,7413 2,6572 2,5911 17 4,4513 3,5915 3,1968 2,9647 2,8100 2,6987 2,6143 2,5480 18 4,4513 3,5246 3,1599 2,9277 2,7129 2,6987 2,6143 2,5480 20 4,3807 3,5219 3,1274 2,8951 2,7109 2,6987 2,6143 2,5480 21 4,3807 3,5219 3,1274 2,8951 2,7401 2,6983 2,5435 2,4768 22 4,3009 3,4434 3,0994 2,8661 2,7109 2,5990 2,5140 2,4471 23 4,3049 3,0280 2,7955 2,6400 2,5327 2,4421 2,4471 24 4,2793 3,4434 3,0984 2,7643 2,5641 2,4438 2,3468 2,3468 24 4,2793 3,4434 3,088 2,7763 2,6400 2,5443 2,348 | 96 | | 4.5431 | 3.6823 | 3.2874 | 3.0556 | 2.9013 | 2.7905 | 2.7066 | 2.6408 | 2.5876 |
| 17 4.4513 3.5915 3.1968 2.9647 2.8100 2.6987 2.6143 2.5480 18 4.4139 3.5546 3.1599 2.9277 2.7729 2.6613 2.5435 2.5480 20 4.3807 3.5219 3.1274 2.8951 2.7729 2.6613 2.5767 2.5102 21 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 2.5435 2.4768 22 4.3009 3.4434 3.0491 2.8661 2.7109 2.5990 2.5140 2.4716 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5777 2.4287 2.3065 24 4.2793 3.4221 3.0280 2.7763 2.6037 2.5082 2.4226 2.3748 25 4.2417 3.3852 2.9912 2.7783 2.6077 2.5082 2.4226 2.3363 26 4.2252 3.3690 2.9726 2.7426 2.5888 2.4741 2.3883 <th>3u</th> <th></th> <th>4.4940</th> <th>3.6337</th> <th>3.2389</th> <th>3.0069</th> <th>2.8524</th> <th>2.7413</th> <th>2.6572</th> <th>2.5911</th> <th>2.5377</th> | 3u | | 4.4940 | 3.6337 | 3.2389 | 3.0069 | 2.8524 | 2.7413 | 2.6572 | 2.5911 | 2.5377 |
| 18 4.4139 3.5546 3.1599 2.9277 2.7729 2.6613 2.5767 2.5102 20 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 2.5435 2.4768 20 4.3807 3.5219 3.1274 2.8961 2.7401 2.6283 2.5435 2.4768 21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5496 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6400 2.5491 2.4876 2.4205 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4628 2.3365 24 4.2597 3.4028 2.7763 2.6007 2.5082 2.4226 2.3371 25 4.2417 3.3852 2.9912 2.7763 2.6007 2.4491 2.3463 2.3148 27 4.2252 3.3690 2.9752 2.7426 2.5868 2.4471 2.3883 2.2913 | :: G | - | 4.4513 | 3.5915 | 3.1968 | 2.9647 | 2.8100 | 2.6987 | 2.6143 | 2.5480 | 2.4943 |
| 19 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 2.5435 2.4768 20 4.3512 3.4928 3.0984 2.8661 2.7109 2.5990 2.5140 2.4471 21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5727 2.4876 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5727 2.4876 2.4471 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2793 3.4028 2.7763 2.6030 2.4904 2.4422 2.3748 25 4.2217 3.3852 2.9912 2.7783 2.6030 2.4904 2.4226 2.3551 26 4.2252 3.3690 2.9723 2.7426 2.5868 2.4741 2.3833 2.2913 27 4.1060 3.3404 2.9644 2.77426 2.5368 2.4453 2.3463 2.2463 <th><u>≃</u> . ق</th> <th>~</th> <th>4.4139</th> <th>3.5546</th> <th>3.1599</th> <th>2.9277</th> <th>2.7729</th> <th>2.6613</th> <th>2.5767</th> <th>2.5102</th> <th>2.4563</th> | <u>≃</u> . ق | ~ | 4.4139 | 3.5546 | 3.1599 | 2.9277 | 2.7729 | 2.6613 | 2.5767 | 2.5102 | 2.4563 |
| 20 4.3512 3.4928 3.0984 2.8661 2.7109 2.5990 2.5140 2.4471 21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5727 2.4876 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5491 2.4638 2.3965 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2597 3.4028 2.7763 2.6207 2.582 2.4226 2.3748 25 4.2257 3.4028 2.7763 2.6207 2.4904 2.4047 2.3371 26 4.2257 3.3852 2.9912 2.7786 2.588 2.4741 2.3883 2.3205 27 4.2100 3.3541 2.9467 2.7141 2.581 2.4453 2.363 2.2913 29 4.1830 3.3178 2.9223 2.6896 2.534 2.4324 2.3463 2.2183 | | ^ | 4.3807 | 3.5219 | 3.1274 | 2.8951 | 2.7401 | 2.6283 | 2.5435 | 2.4768 | 2.4227 |
| 21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5727 2.4876 2.4205 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5491 2.4638 2.3965 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2597 3.4028 3.0088 2.7763 2.6207 2.5082 2.4226 2.3551 25 4.2417 3.3852 2.9912 2.7763 2.6030 2.4904 2.4404 2.3478 26 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3053 27 4.1960 3.3541 2.9604 2.7778 2.5719 2.4591 2.3732 2.2913 28 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3463 2.2783 29 4.1830 3.3277 2.9340 2.7014 2.5454 2.4205 2.3490 <th></th> <th>_</th> <th>4.3512</th> <th>3.4928</th> <th>3.0984</th> <th>2.8661</th> <th>2.7109</th> <th>2.5990</th> <th>2.5140</th> <th>2.4471</th> <th>2.3928</th> | | _ | 4.3512 | 3.4928 | 3.0984 | 2.8661 | 2.7109 | 2.5990 | 2.5140 | 2.4471 | 2.3928 |
| 22 4.3009 3.4434 3.0491 2.8167 2.6613 2.5491 2.4638 2.3965 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2597 3.4028 3.0088 2.7763 2.6207 2.5082 2.4226 2.3748 25 4.2417 3.3852 2.9912 2.7587 2.6030 2.4904 2.4047 2.3371 26 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3205 27 4.2100 3.3541 2.9604 2.7278 2.5719 2.4591 2.3732 2.3053 28 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3593 2.2913 29 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3343 2.2662 20 4.0012 3.3504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 20 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 2.9340 2.3719 2.3171 2.3837 2.2899 2.1750 2.0868 2.0164 2.9400 2.3415 2.957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | | _ | 4.3248 | 3.4668 | 3.0725 | 2.8401 | 2.6848 | 2.5727 | 2.4876 | 2.4205 | 2.3660 |
| 23 4.2793 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 24 4.2597 3.4028 3.0088 2.7763 2.6207 2.5082 2.4226 2.3551 25 4.2597 3.4028 3.0088 2.7763 2.6030 2.4904 2.4047 2.3371 26 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3205 27 4.2100 3.3541 2.9604 2.7278 2.5719 2.4591 2.3732 2.3053 28 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3593 2.2913 29 4.1830 3.3277 2.9340 2.7014 2.5454 2.4453 2.3463 2.2783 40 4.0847 3.2317 2.9340 2.7014 2.5454 2.4453 2.3463 2.2490 2.1802 40 4.0847 3.2317 2.8387 2.6060 2.4495 2.3463 <th></th> <th>63</th> <th>4.3009</th> <th>3,4434</th> <th>3.0491</th> <th>2.8167</th> <th>2.6613</th> <th>2.5491</th> <th>2.4638</th> <th>2.3965</th> <th>2.3419</th> | | 63 | 4.3009 | 3,4434 | 3.0491 | 2.8167 | 2.6613 | 2.5491 | 2.4638 | 2.3965 | 2.3419 |
| 24 4.2597 3.4028 2.7763 2.6207 2.5082 2.4226 2.3551 25 4.2417 3.3852 2.9912 2.7587 2.6030 2.4904 2.4047 2.3371 26 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3205 27 4.2100 3.3541 2.9604 2.7278 2.5719 2.4591 2.3732 2.3053 28 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3593 2.2913 29 4.1830 3.3277 2.9340 2.7014 2.5454 2.4324 2.3463 2.2783 30 4.1709 3.3158 2.9223 2.6896 2.5336 2.4329 2.2490 2.1802 40 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 20 3.3201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 20 3.38415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | | ~ | 4.2793 | 3.4221 | 3.0280 | 2.7955 | 2.6400 | 2.5277 | 2.4422 | 2.3748 | 2.3201 |
| 4.2417 3.3852 2.9912 2.7587 2.6030 2.4904 2.4047 2.3371 4.2252 3.3690 2.9752 2.7426 2.5868 2.4741 2.3883 2.3205 4.2100 3.3541 2.9604 2.7278 2.5719 2.4591 2.3732 2.3053 4.1960 3.3404 2.9467 2.7141 2.5581 2.4591 2.3732 2.2913 4.1830 3.3277 2.9340 2.7014 2.5454 2.4453 2.3593 2.2913 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3463 2.2662 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | | ** | 4.2597 | 3.4028 | 3.0088 | 2.7763 | 2.6207 | 2.5082 | 2.4226 | 2.3551 | 2.3002 |
| 4,2252 3,3690 2,9752 2,7426 2,5868 2,4741 2,3883 2,3205 4,2100 3,3541 2,9604 2,7278 2,5719 2,4591 2,3732 2,3053 4,1960 3,3404 2,9467 2,7141 2,5581 2,4453 2,3593 2,2913 4,1830 3,3277 2,9340 2,7014 2,5454 2,4324 2,3463 2,2783 4,1709 3,3158 2,9223 2,6896 2,5336 2,4205 2,3443 2,2662 4,0847 3,2317 2,8387 2,6060 2,4495 2,3359 2,2490 2,1802 4,0847 3,1504 2,7581 2,5252 2,3683 2,2541 2,1665 2,0970 3,9201 3,0718 2,6802 2,4472 2,2899 2,1750 2,0868 2,0164 3,8415 2,9957 2,6049 2,3719 2,2141 2,0986 2,0096 1,9384 | 25 | 10 | 4.2417 | 3.3852 | 2.9912 | 2.7587 | 2.6030 | 2.4904 | 2.4047 | 2.3371 | 2.2821 |
| 4,2100 3,3541 2,9604 2,7278 2,5719 2,4591 2,3732 2,3053 4,1960 3,3404 2,9467 2,7141 2,5581 2,4453 2,3593 2,2913 4,1830 3,3277 2,9340 2,7014 2,5584 2,4324 2,3463 2,2783 4,1709 3,3158 2,9223 2,6896 2,5336 2,4205 2,3343 2,2662 4,0847 3,2317 2,8387 2,6060 2,4495 2,3359 2,2490 2,1802 4,0012 3,1504 2,7581 2,5252 2,3683 2,2541 2,1665 2,0970 3,9201 3,0718 2,6802 2,4472 2,2899 2,1750 2,0868 2,0164 3,8415 2,9957 2,6049 2,3719 2,2141 2,0986 2,0096 1,9384 | 2 | 100 | 4.2252 | 3.3690 | 2.9752 | 2.7426 | 2.5868 | 2.4741 | 2.3883 | 2.3205 | 2.2655 |
| 4.1960 3.3404 2.9467 2.7141 2.5581 2.4453 2.3593 2.2913 4.1830 3.3277 2.9340 2.7014 2.5454 2.4324 2.3463 2.2783 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3343 2.2662 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | 2.7 | _ | 4.2100 | 3.3541 | 2.9604 | 2.7278 | 2.5719 | 2.4591 | 2.3732 | 2.3053 | 2.2501 |
| 4.1830 3.3277 2.9340 2.7014 2.5454 2.4324 2.3463 2.2783 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3343 2.2662 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | 25 | 00 | 4.1960 | 3.3404 | 2.9467 | 2.7141 | 2.5581 | 2.4453 | 2.3593 | 2.2913 | 2.2360 |
| 4.1709 3.3158 2.9223 2.6896 2.5336 2.4205 2.3343 2.2662 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | 22 | 0 | 4.1830 | 3.3277 | 2.9340 | 2.7014 | 2.5454 | 2.4324 | 2.3463 | 2.2783 | 2.2229 |
| 4.0847 3.2317 2.8387 2.6060 2.4495 2.3359 2.2490 2.1802 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | 3 | - | 4.1709 | 3.3158 | 2.9223 | 2.6896 | 2.5336 | 2.4205 | 2.3343 | 2.2662 | 2.2107 |
| 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.1665 2.0970 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | 4 | - | 4.0847 | 3.2317 | 2.8387 | 2.6060 | 2,4495 | 2.3359 | 2.2490 | 2.1802 | 2.1240 |
| 3.9201 3.0718 2.6802 2.4472 2.2899 2.1750 2.0868 2.0164 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | 3 | 0 | 4.0012 | 3.1504 | 2.7581 | 2.5252 | 2.3683 | 2.2541 | 2.1665 | 2.0970 | 2.0401 |
| 3.8415 2.9957 2.6049 2.3719 2.2141 2.0986 2.0096 1.9384 | 121 | 0 | 3.9201 | 3.0718 | 2.6802 | 2.4472 | 2.2899 | 2.1750 | 2.0868 | 2.0164 | 1.9588 |
| | 8 | _ | 3.8415 | 2.9957 | 2.6049 | 2.3719 | 2.2141 | 2.0986 | 2.0096 | 1.9384 | 1.8799 |