Domaći zadatak 3

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DOMAĆI 3

1.

	x_0	x_1
$t(s)$	0	1
$s(m)$	0	20
$v(\frac{m}{s})$	0	50
$a(\frac{m}{s^2})$	30	30

PODATAKA IMAMO 6
$\Rightarrow$ STEPEN HERMITOVOG POLINOMA JE 5 : $H_5(x)$

$$H_5(x) = P_1(x) + \underbrace{(x-x_0)(x-x_1)}_{\text{2. STEPENA}} \cdot \underbrace{H_3(x)}_{\text{3. STEPENA}} \quad = \text{5 STEP.}$$

$P_1(x) \downarrow$
$L_1(x)$ - NAPRAVIĆEMO LAGRANŽEOV INT. POL.

$$L_1(x) = s(x_0)\cdot\frac{(x-x_1)}{(x_0-x_1)} + s(x_1)\frac{(x-x_0)}{(x_1-x_0)} =$$

$$= 0\cdot\frac{x-1}{0-1} + 20\cdot\frac{x-0}{1-0} = \underline{\underline{20x}}$$

$$\Rightarrow H_5(x) = 20x + x(x-1)\cdot\underbrace{H_3(x)}_{?} \quad / '$$

$$H_5'(x) = 20 + [x-1+x]\cdot H_3(x) + x(x-1)\cdot H_3'(x)$$

$$H_5'(0) = 0 = 20 + (2x_0 - 1)H_3(x_0) + x_0(x_0-1)H_3'(x_0)$$

$$= 20 + (2\cdot 0 - 1)\cdot H_3(0) + 0\cdot(0-1)\cdot H_3'(0)$$

$$\Rightarrow 20 - H_3(0) = 0 \Rightarrow \boxed{H_3(0) = 20}$$

$$H_5'(x_1) = H_5'(1) = 50 = 20 + (2x_1 - 1)\cdot H_3(x_1) +$$

$$+ x_0(x_1 - 1)H_3'(x) = 20 + (2-1)H_3(1) +$$

$$+ 1\cdot(1-1)H_3'(1) \Rightarrow 50 = 20 + H_3(1)$$

$$\Rightarrow \boxed{H_3(1) = 30}$$

$H_5'(x) \quad / '$

$$\Rightarrow H_5''(x) = \left((2x-1)H_3(x)\right)' + \left(x(x-1)H_3'(x)\right)' =$$

$$= 2H_3(x) + (2x-1)H_3'(x) + x(x-1)H_3''(x)$$

$H_5''(x) = 2H_3(x) + (2x-1)H_3'(x) + x(x-1)H_3''(x)$

$H_5''(x_0) = H_5''(0) = 30 = 2\cdot H_3(0) + (2\cdot 0 - 1)\cdot H_3'(0) + 0(0-1)\cdot H_3'' = 2H_3(0) - H_3'(0)$

$\Rightarrow 30 = 2\cdot 20 - H_3'(0)$

$\Rightarrow \boxed{H_3'(0) = 10}$

$H_5''(x_1) = H_5''(1) = 30 = 2\cdot H_3(1) + (2\cdot 1 - 1)H_3'(1) + 1(1-1)H_3''(1) = 2H_3(1) + H_3'(1)$

$\Rightarrow 30 = 2\cdot 30 + H_3'(1)$

$\Rightarrow \boxed{H_3'(1) = -30}$

PONOVO REŠAVAMO PROBLEM INTERP. POLINOMA:

x	0	1
$H_3(x)$	20	30
$H_3'(x)$	10	-30

IMAMO 4 PODATKA
$\Rightarrow$ TRAŽIMO REŠENJE $\tilde{H}_3(x)$

$\tilde{H}_3(x) = \tilde{P}_1(x) + (x-0)(x-1)\tilde{H}_1(x)$

$\tilde{P}_1(x) \rightarrow \tilde{L}_1(x)$

$\tilde{L}_1(x) = H_3(0)\cdot\frac{x-1}{0-1} + H_3(1)\cdot\frac{x-0}{1-0} =$

$= -20(x-1) + 30\cdot x = -20x + 20 + 30x$

$= 10x + 20 = \underline{\underline{10(x+2)}}$

$\tilde{H}_3(x) = 10(x+2) + x(x-1)\tilde{H}_1(x) \quad /'$

$\tilde{H}_3'(x) = 10 + (x-1+x)\tilde{H}_1(x) + x(x-1)\tilde{H}_1'(x)$

$\tilde{H}_3'(x_0) = 10 + (2x-1)\cdot \tilde{H}_1(x_0) + x_0(x_0-1)\tilde{H}_1'(x) =$

$= 10 + (2\cdot 0 - 1)\tilde{H}_1(0) + 0 \cdot (0-1)\tilde{H}_1'(x) =$

$= 10 - \tilde{H}_1(0) = 10$

$\Rightarrow \boxed{\tilde{H}_1(0) = 0}$

$\tilde{H}_3'(x_1) = \tilde{H}_3'(1) = 10 + \tilde{H}_1(1) = -30$

$\Rightarrow \boxed{\tilde{H}_1(1) = -40}$

x	0	1
$\tilde{H}_1(x)$	0	-40

~~Tražimo polinom stepena 1.~~

~~$\tilde{H}_1(x) = P_1(x) + (x-x_0)(x-x_1)\cdot$~~

Pravim pravu koja prolazi kroz 2 tačke ??

$(\overset{x_0}{0}, \overset{y_0}{0}1)$, $(\overset{x_1}{1}, \overset{y_1}{-40})$:

$y - y_0 = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$

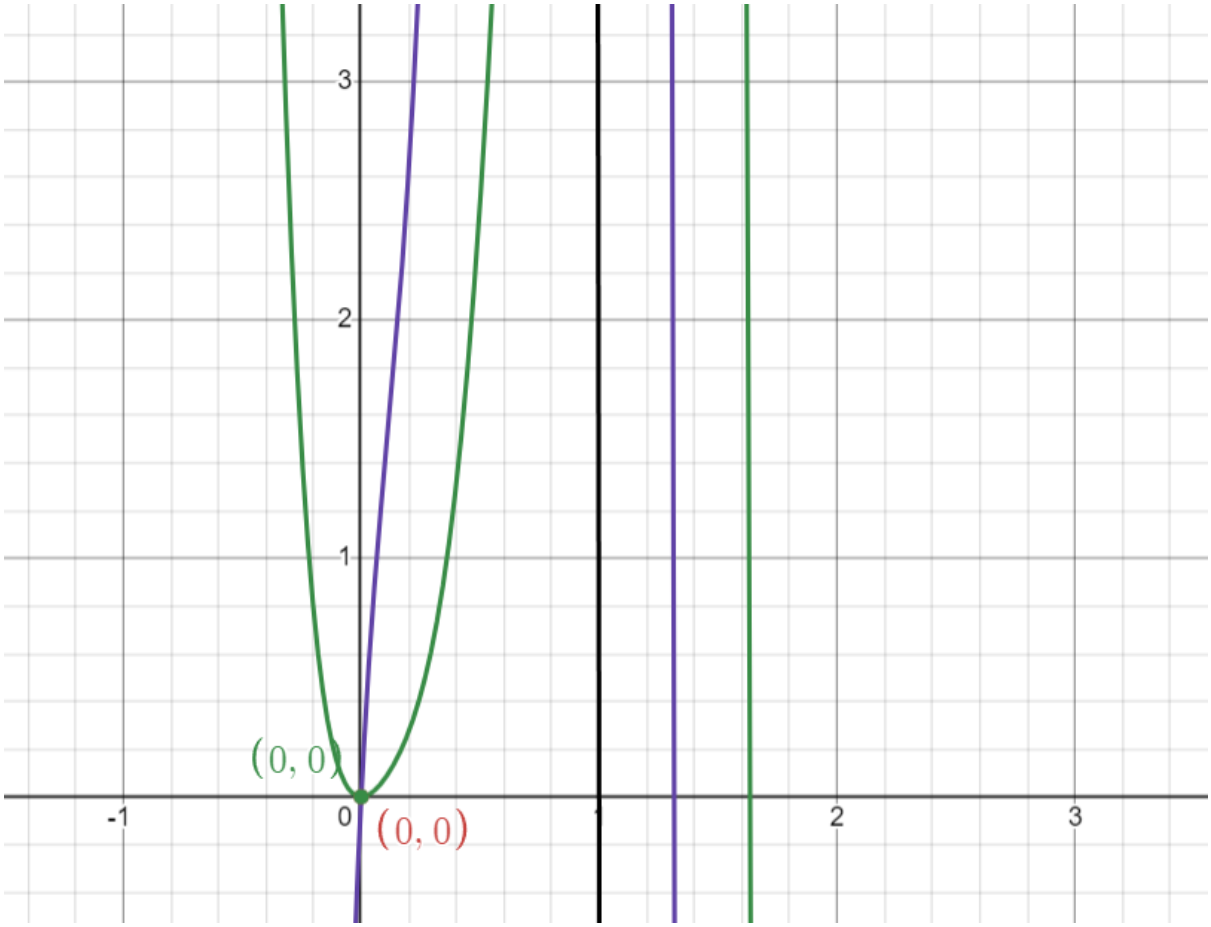
$y = \frac{-40}{1}(x-0) = -40x$

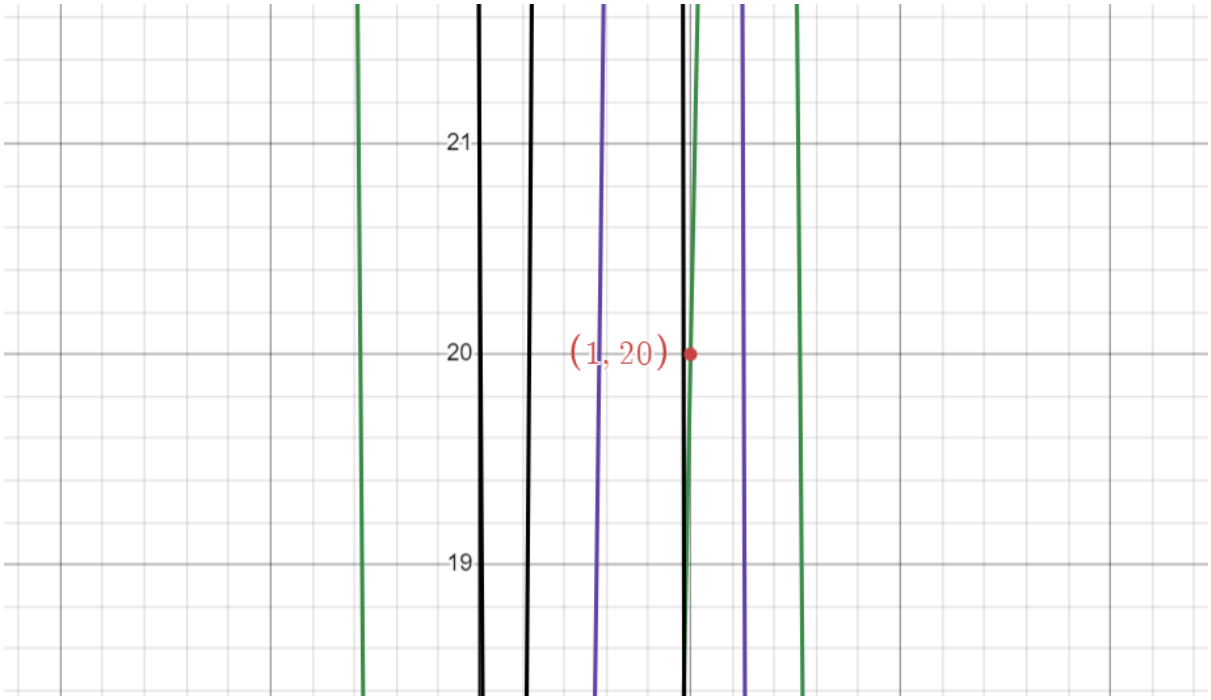
$\Rightarrow \tilde{H}_1(x) = -40x$

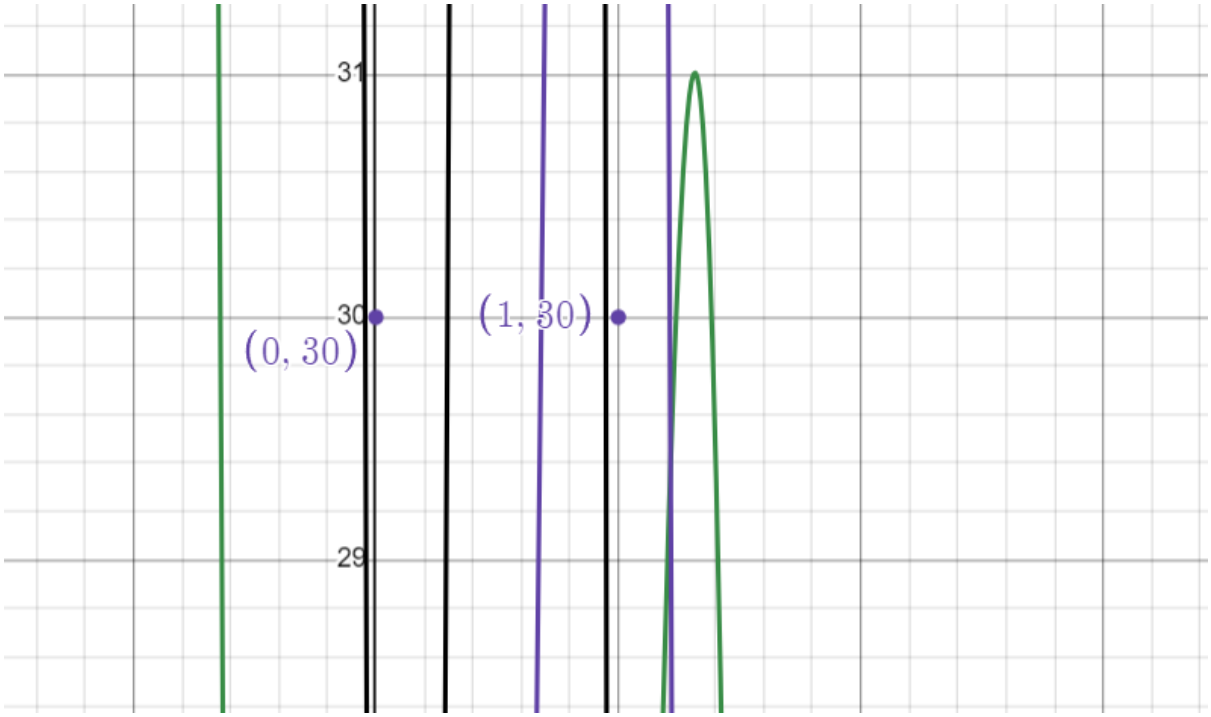
$\Rightarrow \tilde{H}_3(x) = 10(x+2) + x(x-1)\cdot(-40x) =$

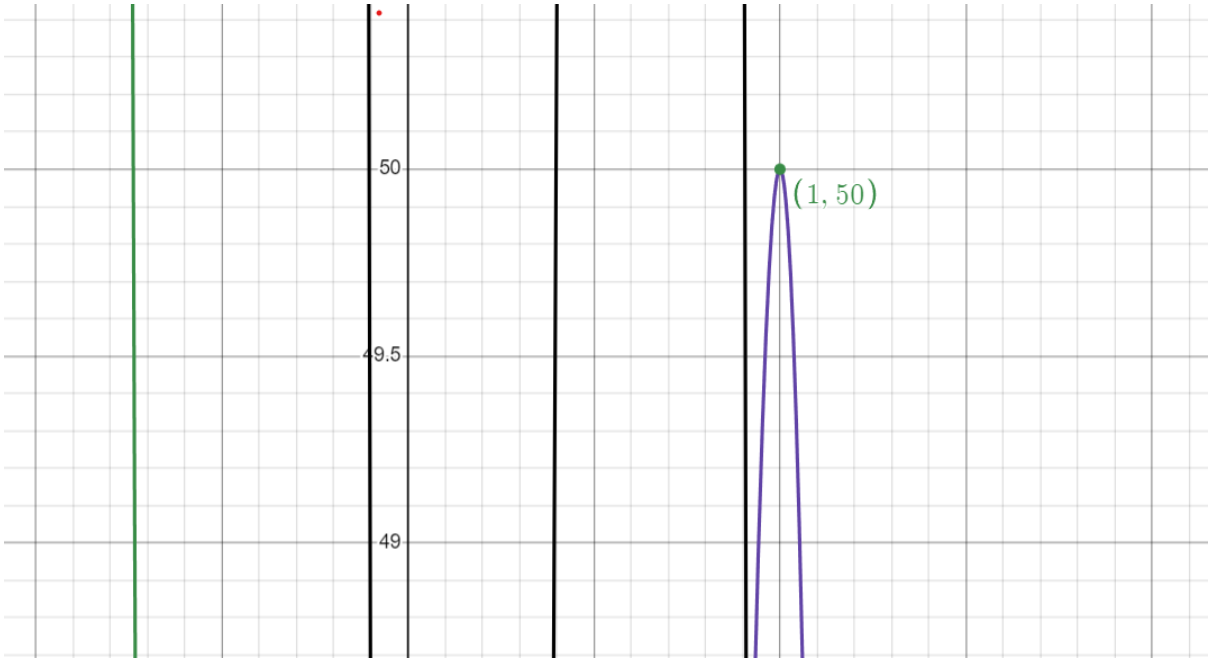
$= 10(x+2) - 40x^2(x-1) = H_3(x)$

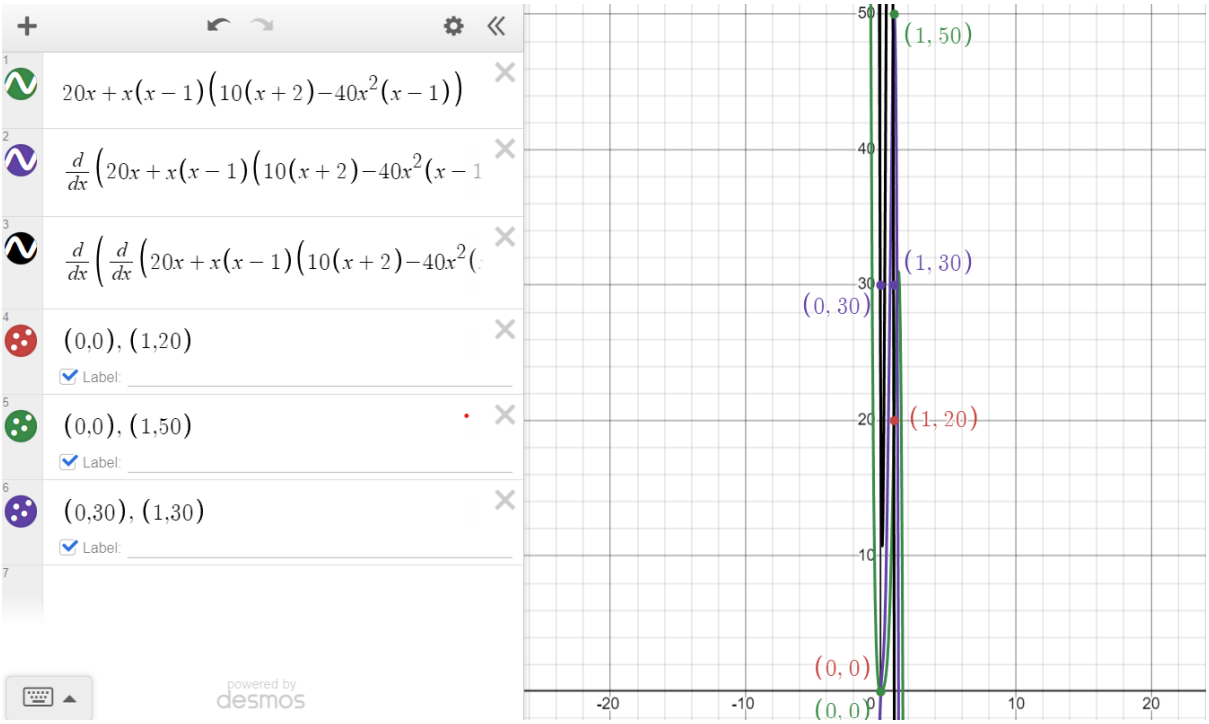
$\Rightarrow H_5(x) = 20x + x(x-1)\left[10(x+2) - 40x^2(x-1)\right]$


3
2
1
-1
0
1
2
3
(0, 0)
(0, 0)


21
20
19
(1, 20)


31
30
29
(0, 30)
(1, 30)


50
49.5
49
(1, 50)



- **Zeleno** – aproksimacija, hermitov polinom
- **Ljubičasto** – njegov prvi izvod
- **Crno** – njegov drugi izvod
- Prisutne su i **tačke** koje odgovaraju vrenostima parova vremena i puta, brzine odnosno ubrzanja

2. $f(x) = x \ln x$

NAJBOLJOM SR. KV. APR. , P_2

$[1,3]$

$p(x) = \frac{1}{x}$

VELIČINA NAJB. APR. ?

$f(x) = x \ln x \rightarrow Q(x)$; $Q \in P_2$

$(\varphi, \psi) = \int_1^3 \varphi(x) \psi(x) p(x) dx$

$\|\varphi\| = \sqrt{(\varphi, \varphi)}$

$Q(x) = a_0 Q_0(x) + a_1 Q_1(x) + a_2 Q_2(x)$

$a_k = \frac{(f, Q_k)}{(Q_k, Q_k)}$

1° FORMIRANJE ORTOGONALNE BAZE

$\{1, x, x^2\} \rightarrow \{Q_0, Q_1, Q_2\}$

$Q_0 = 1$

$Q_1 = x - \frac{(x, Q_0)}{(Q_0, Q_0)} Q_0 = \circledast$

$(x, Q_0) = \int_1^3 x \cdot Q_0 \cdot p(x) dx = \int_1^3 x \cdot 1 \cdot \frac{1}{x} dx = x|_1^3 =$

$= 3 - 1 = 2$

$(Q_0, Q_0) = \int_1^3 p(x) Q_0 Q_0 dx = \int_1^3 \frac{1}{x} \cdot 1 \cdot 1 \cdot dx = \ln|x||_1^3 =$

$= \ln|3| - \ln|1| = \ln 3 \approx 1.098$

$\circledast = x - \frac{2}{\ln 3} \cdot 1 = x - 1.82$

$$Q_2 = x^2 - \frac{(x^2, Q_0)}{(Q_0, Q_0)} Q_0(x) - \frac{(x^2, Q_1)}{(Q_1, Q_1)} Q_1(x) = (**)$$

$$(x^2, Q_0) = \int_1^3 x^2 \cdot 1 \cdot \frac{1}{x}\,dx = \int_1^3 x\,dx = \frac{x^2}{2}\Big|_1^3 = \frac{9}{2} - \frac{1}{2} = 4$$

$$(x^2, Q_1) = \int_1^3 x^1 \cdot \left(x - \frac{2}{\ln 3}\right) \cdot \frac{1}{x}\,dx = \int_1^3 \left(x^2 - \frac{2}{\ln 3} x\right) dx =$$

$$= \int_1^3 x^2\,dx - \frac{2}{\ln 3} \int_1^3 x\,dx = \frac{x^3}{3}\Big|_1^3 - \frac{2}{\ln 3} \cdot 4 =$$

$$= \left(\frac{27}{3} - \frac{1}{3}\right) - \frac{8}{\ln 3} = \frac{26}{3} - \frac{8}{\ln 3} \approx 1.38$$

$$Q_2 = x^2 - \frac{4}{\ln 3} \cdot 1 - \frac{\frac{26}{3} - \frac{8}{\ln 3}}{4 - \frac{4}{\ln 3}} \cdot \left(x - \frac{2}{\ln 3}\right) = (***)$$

$$(Q_1, Q_1) = \int_1^3 \left(x - \frac{2}{\ln 3}\right)^2 \cdot \frac{1}{x}\,dx = \int_1^3 \frac{1}{x}\left(x^2 - \frac{4}{\ln 3} x + \frac{4}{\ln^2 3}\right) dx =$$

$$= \int_1^3 x\,dx - \frac{4}{\ln 3} \int_1^3 dx + \frac{4}{\ln^2 3} \int_1^3 \frac{1}{x}\,dx =$$

$$= 4 - \frac{4}{\ln 3} \cdot x\Big|_1^3 + \frac{4}{\ln^2 3} \cdot \ln|x|\Big|_1^3 =$$

$$= 4 - \frac{8}{\ln 3} + \frac{4}{(\ln 3)^2} \cdot \ln 3 =$$

$$= 4 - \frac{8}{\ln 3} + \frac{4}{\ln 3} = 4 - \frac{4}{\ln 3} \approx 0.36$$

$$(***) = x^2 - \frac{4}{\ln 3} - \frac{\frac{26\ln 3 - 24}{3\ln 3}}{\frac{12\ln 3 - 12}{3\ln 3}} \cdot \left(x - \frac{2}{\ln 3}\right) =$$

$$= x^2 - \frac{4}{\ln 3} - \frac{26\ln 3 - 24}{12\ln 3 - 12} \cdot \left(x - \frac{2}{\ln 3}\right) =$$

$$= x^2 - \frac{4}{\ln 3} - \frac{13\ln 3 - 12}{6\ln 3 - 6} \left(x - \frac{2}{\ln 3}\right) =$$

$$= x^2 - \frac{13\ln 3 - 12}{6(\ln 3 - 1)} x - \frac{4}{\ln 3} + \frac{13\ln 3 - 12}{3(\ln 3 - 1)\ln 3}$$

$$\approx x^2 - 3.86\,x + 3.38$$

2° ODREĐIVANJE KOEF.

$$a_0 = \frac{(f,Q_0)}{(Q_0,Q_0)} = \frac{3\ln 3 - 2}{\ln 3} \approx 1.18$$

$$(f,Q_0) = \int_1^3 x\ln x \cdot 1 \cdot \frac{1}{x}\,dx - \int_1^3 \ln x\,dx =$$

$$\left| \begin{array}{ll} u = \ln x & dv = dx \\ du = \frac{1}{x}dx & v = x \end{array} \right| = x\ln x\Big|_1^3 - \int_1^3 x \cdot \frac{1}{x}dx =$$

$$= (3\ln 3 - \cancel{\ln 1}^{0}) - \int_1^3 dx = 3\ln 3 - 2 \approx 1.29$$

$$a_1 = \frac{(f,Q_1)}{(Q_1,Q_1)} = (**)$$

$$(f,Q_1) = \int_1^3 x\ln x \cdot \frac{1}{x} \cdot \left(x - \frac{2}{\ln 3}\right) dx =$$

$$= \underbrace{\int_1^3 x\ln x}_{I_1} - \frac{2}{\ln 3}\int_1^3 \ln x\,dx = (*)$$

$$I_1 = \int_1^3 x\ln x = \left| \begin{array}{ll} u = \ln x & dv = x\,dx \\ du = \frac{1}{x}dx & v = \frac{x^2}{2} \end{array} \right| =$$

$$= \frac{x^2}{2}\ln x\Big|_1^3 - \int_1^3 \frac{x^2}{2} \cdot \frac{1}{x}dx =$$

$$= \left(\frac{9}{2}\ln 3 - \frac{1}{2}\cancel{\ln 1}^{0}\right) - \frac{1}{2} \cdot \frac{x^2}{2}\Big|_1^3 =$$

$$= \frac{9}{2}\ln 3 - \frac{1}{2}\left(\frac{9}{2} - \frac{1}{2}\right) = \frac{9}{2}\ln 3 - 2 \approx 2.94$$

$$(*) = \frac{9}{2}\ln 3 - 2 - \frac{2}{\ln 3}(3\ln 3 - 2) =$$

$= \frac{9}{2}\ln 3 - 2 - 6 + \frac{4}{\ln 3} = \frac{9}{2}\ln 3 + \frac{4}{\ln 3} - 8 \approx 0.585$

(**) $a_1 = \dfrac{\frac{9}{2}\ln 3 + \frac{4}{\ln 3} - 8}{4 - \frac{4}{\ln 3}} =$

$= \dfrac{\frac{9\ln^2 3 + 8 - 16\ln 3}{2\ln 3}}{\frac{8\ln 3 - 8}{2\ln 3}} = \dfrac{9\ln^2 3 - 16\ln 3 + 8}{8(\ln 3 - 1)} \approx 1.63$

$a_2 = \dfrac{(f, Q_2)}{(Q_2, Q_2)}$

$(f, Q_2) = \int_1^3 x \ln x \cdot \frac{1}{x} \cdot (x^2 - 3.86x + 3.38)\,dx =$

$= \underbrace{\int_1^3 x^2 \ln x\,dx}_{I_2} - 3.86 \int_1^3 x \ln x + 3.38 \int_1^3 \ln x\,dx = (*)$

$I_2 = \int_1^3 x^2 \ln x\,dx = \left| \begin{array}{ll} u = \ln x & dv = x^2 dx \\ du = \frac{1}{x}dx & v = \frac{x^3}{3} \end{array} \right| =$

$= \frac{x^3}{3}\ln x \Big|_1^3 - \frac{1}{3}\int_1^3 x^2 \cdot \frac{1}{x}\,dx = \left(\frac{27}{3}\ln 3 - \frac{1}{3}\ln 1^0\right) -$

$- \frac{1}{3} \cdot \frac{x^3}{3}\Big|_1^3 = 9\ln 3 - \frac{1}{3} \cdot \frac{26}{3} =$

$= 9\ln 3 - \frac{26}{9} \approx 6.99$

(*) $= 9\ln 3 - \frac{26}{9} - 3.86 \cdot 2.94 + 3.38 \cdot 1.29 \approx 0.07$

$(Q_2, Q_2) = \int_1^3 p(x) Q_2 Q_2\,dx = \int_1^3 \frac{1}{x}(x^2 - 3.86x + 3.38)^2\,dx =$

$= \int_1^3 \frac{1}{x}(x^4 + 21.66x^2 - 7.72x^3 - 26.1x + 11.42)\,dx =$

$= \int_1^3 x^3\,dx - 7.72\int_1^3 x^2\,dx + 21.66\int_1^3 x\,dx - 26.1\int_1^3 dx + 11.42 \cdot \int_1^3 \frac{1}{x}\,dx =$

$= 20 - 66.81 + 86.64 - 52.2 + 12.55 \approx 0.08$

$\Rightarrow a_2 = \frac{0.07}{0.08} = 0.875$

$\Rightarrow Q(x) = a_0 Q_0 + a_1 Q_1 + a_2 Q_2 =$

$= 1.29 \cdot 1 + 1.63 \cdot (x - 1.82) + 0.875 (x^2 - 3.86x + 3.38) =$

$= 0.875x^2 - 1.75x + 1.28$

3° GREŠKA

$$\|\delta\|^2 = (f,f) - \frac{(f,Q_0)^2}{(Q_0,Q_0)} - \frac{(f,Q_1)^2}{(Q_1,Q_1)} - \frac{(f,Q_2)^2}{(Q_2,Q_2)}$$

$$(f,f) = \int_1^3 \frac{1}{x} \cdot x^2 \ln^2 x \, dx =$$

$$= \int_1^3 x \ln^2 x \, dx = \left| \begin{array}{ll} u = \ln^2 x & dv = x\,dx \\ du = \frac{2\ln x}{x} dx & v = \frac{x^2}{2} \end{array} \right| =$$

$$= \frac{1}{2} x^2 \ln^2 x \Big|_1^3 - \int_1^3 \frac{x^2}{2} \cdot \frac{2\ln x}{x} dx =$$

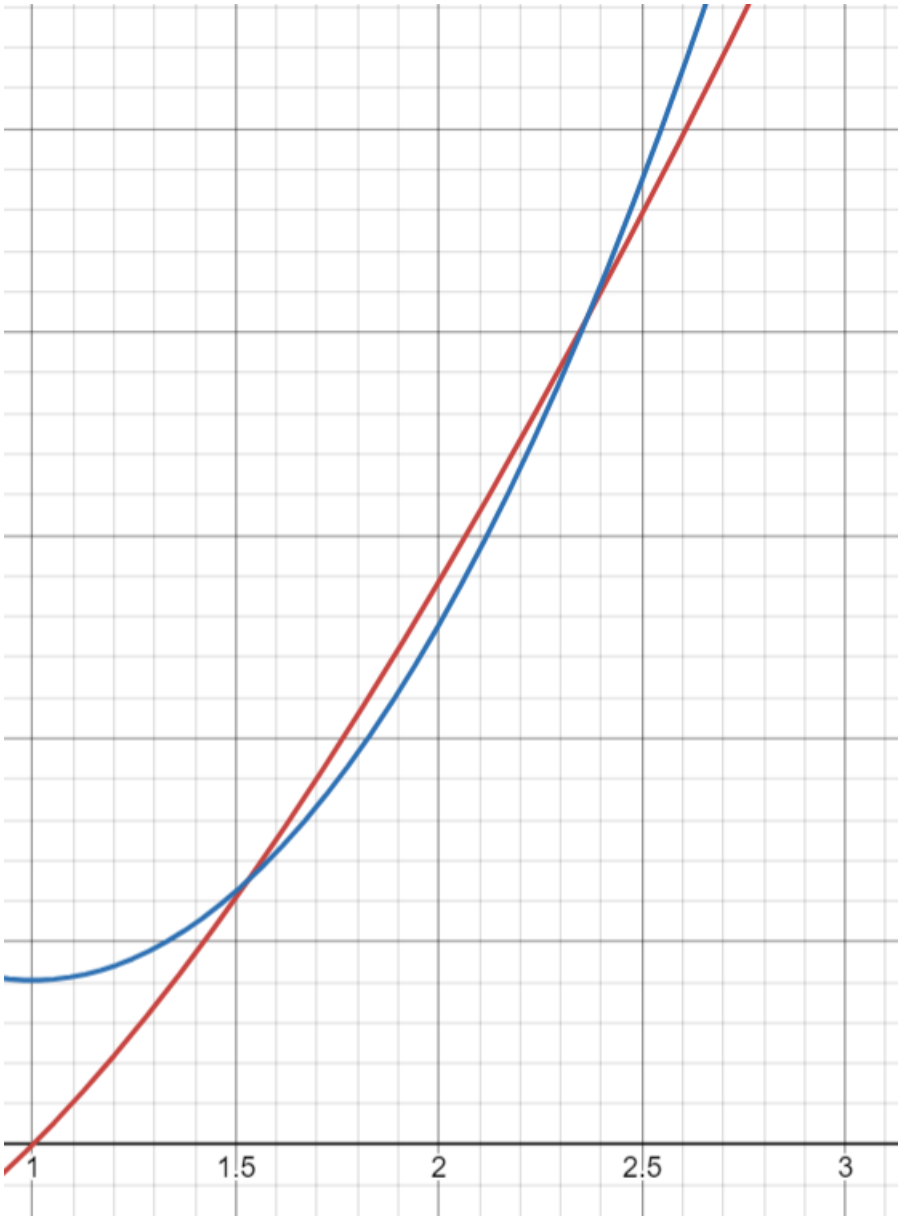
$$= \frac{1}{2}(27 \ln^2 3 - 1 \ln^2 1) - \underbrace{\int_1^3 x \ln x \, dx}_{I1} =$$

$$= \frac{27 \ln^2 3}{2} - 2.94 \approx 13.35$$

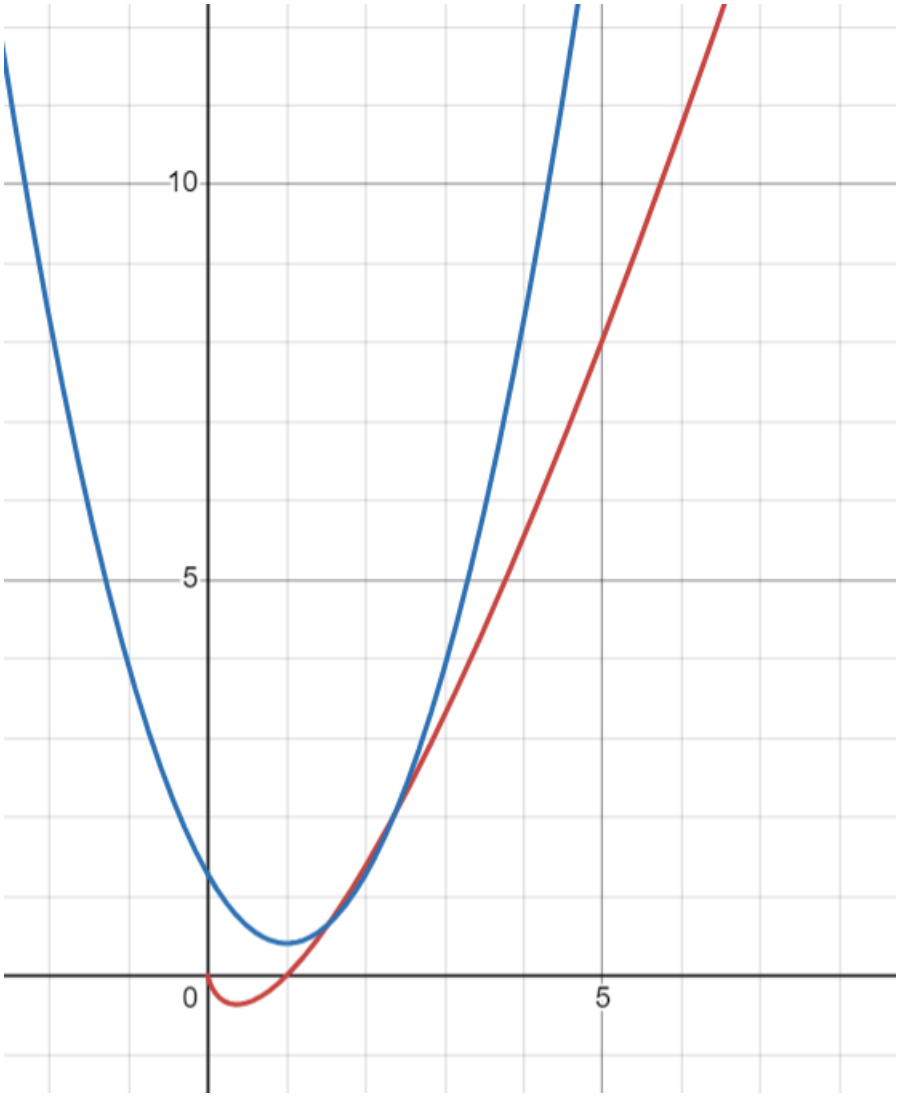
$$\Rightarrow \|\delta\|^2 = 13.35 - \frac{(1.29)^2}{1.098} - \frac{(0.585)^2}{0.36} - \frac{(0.07)^2}{0.08}$$

$$\boxed{\|\delta\|^2} = 13.35 - 1.52 - 0.95 - 0.061 = \boxed{10.82}$$

$$\Rightarrow \|\delta\| = \sqrt{10.82} \approx 3.29$$



- **Crveno** – funkcija
- **Plavo** – aproksimacija



- Van segmenta, javlja se **veliko** odstupanje.

3. $\Phi(x) = a \cdot e^{x} + c \cdot e^{-x}$

x_k	-2	-1	0	1	2
f_k	-15	-5	1	2	7

$$\Phi = a \cdot e^{x} + c \cdot e^{-x}$$

$$f = \begin{bmatrix} -15 \\ -5 \\ 1 \\ 2 \\ 7 \end{bmatrix} \qquad d = \begin{bmatrix} a \\ c \end{bmatrix}$$

$$X = \begin{bmatrix} e^{x_0} & e^{-x_0} \\ e^{x_1} & e^{-x_1} \\ e^{x_2} & e^{-x_2} \\ e^{x_3} & e^{-x_3} \\ e^{x_4} & e^{-x_4} \end{bmatrix} = \begin{bmatrix} e^{-2} & e^{2} \\ e^{-1} & e^{1} \\ e^{0} & e^{0} \\ e^{1} & e^{-1} \\ e^{2} & e^{-2} \end{bmatrix} \approx \begin{bmatrix} 0.135 & 7.389 \\ 0.368 & 2.718 \\ 1 & 1 \\ 2.718 & 0.368 \\ 7.389 & 0.135 \end{bmatrix}$$

SISTEM:

$$\begin{cases} ae^{x_0} + ce^{-x_0} = f_0 \\ \vdots \\ ae^{x_4} + ce^{-x_4} = f_4 \end{cases}$$

REZOLVARE:

$$X^{T} \cdot X \cdot d = X^{T} \cdot f$$

$$X^T \cdot X = \begin{bmatrix} 0.135 & 0.368 & 1 & 2.718 & 7.389 \\ 7.389 & 2.718 & 1 & 0.368 & 0.135 \end{bmatrix} \cdot \begin{bmatrix} 0.135 & 7.389 \\ 0.368 & 2.718 \\ 1 & 1 \\ 2.718 & 0.368 \\ 7.389 & 0.135 \end{bmatrix}$$

$$= \begin{bmatrix} 0.135^2 + 0.368^2 + 1^2 + 2.718^2 + 7.389^2 & 0.135 \cdot 7.389 + 0.368 \cdot 2.718 + 1 + \\ & + 2.718 \cdot 0.368 + 7.389 \cdot 0.135 \\ 7.389 \cdot 0.135 + 2.718 \cdot 0.368 + 1 + & 7.389^2 + 2.718^2 + 1^2 + 0.368^2 + 0.135^2 \\ + 0.368 \cdot 2.718 + 0.135 \cdot 7.389 & \end{bmatrix}$$

$$= \begin{bmatrix} 63.138 & 4.995 \\ 4.995 & 63.138 \end{bmatrix}$$

$$X^T \cdot f = \begin{bmatrix} 0.135 & 0.368 & 1 & 2.718 & 7.389 \\ 7.389 & 2.718 & 1 & 0.368 & 0.135 \end{bmatrix} \begin{bmatrix} -15 \\ -5 \\ 1 \\ 2 \\ 7 \end{bmatrix} =$$

$$= \begin{bmatrix} 0.135 \cdot (-15) + 0.368 \cdot (-5) + 1 + 2.718 \cdot 2 + 7.389 \cdot 7 \\ 7.389 \cdot (-15) + 2.718 \cdot (-5) + 1 + 0.368 \cdot 2 + 0.135 \cdot 7 \end{bmatrix} =$$

$$= \begin{bmatrix} 54.294 \\ -121.744 \end{bmatrix}$$

$$\underline{X}^T \underline{X} \cdot a = \underline{X}^T \cdot f$$

$$\begin{bmatrix} 63.138 & 4.995 \\ 4.995 & 63.138 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 54.294 \\ -121.744 \end{bmatrix}$$

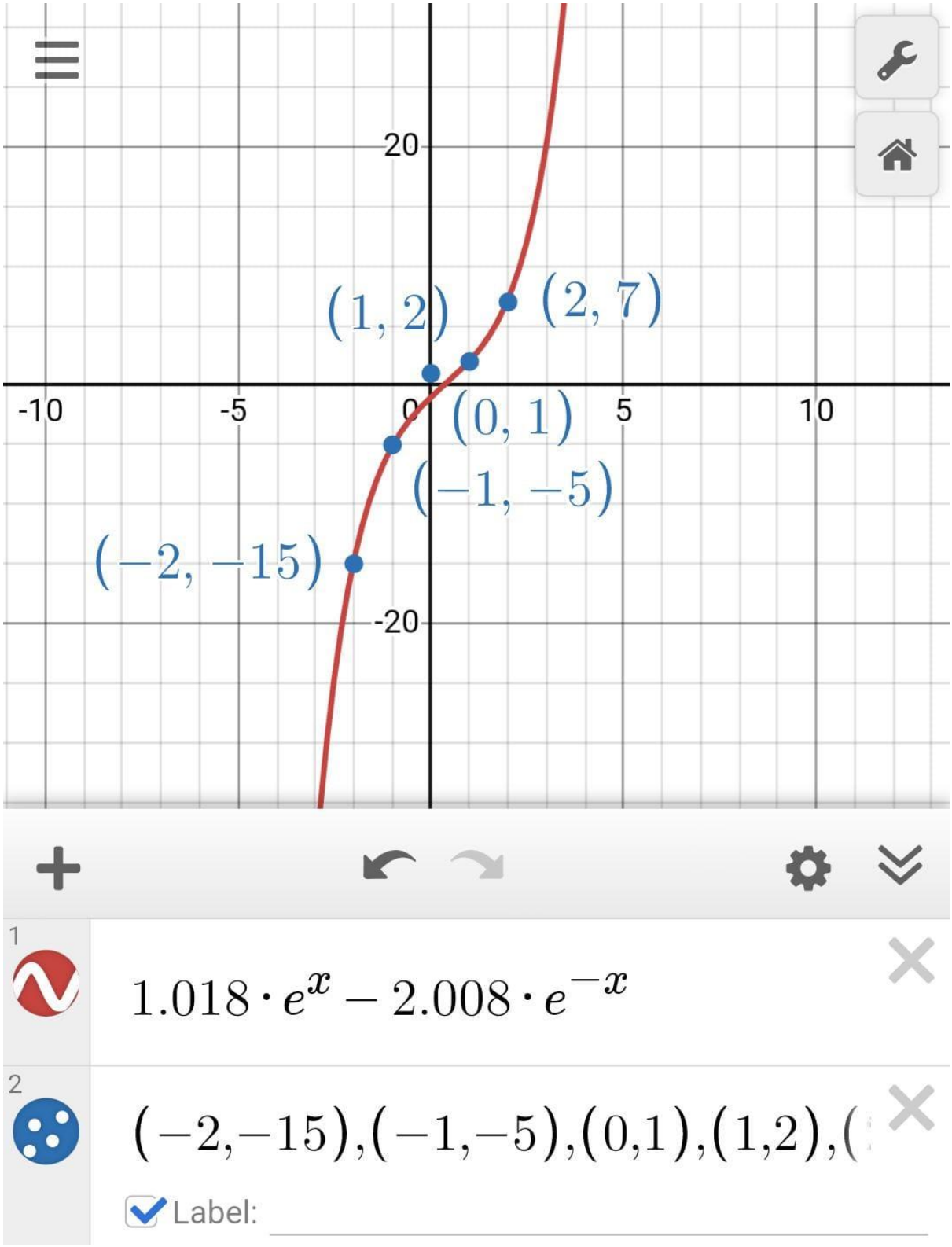
$$\Rightarrow \begin{cases} 63.138a + 4.995c = 54.294 \\ 4.995a + 63.138c = -121.744 \end{cases}$$

⋮ Kompjuter izračunao:

$\underline{a = 1.018}$

$\underline{c = -2.008}$

$$\Rightarrow \underline{\Phi(x) = 1.018e^{x} - 2.008e^{-x}}$$



- **Crveno** – aproksimacija
- **Plavo** – date vrednosti x i funkcije f(x)