Domaći zadatak 4

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1. $\int_{-1/2}^{1/2} \underbrace{(1-2|x|)}_{p(x)} f(x)\,dx$

$$p(x) = 1-2|x| = \begin{cases} 1-2x, & x \geq 0 \\ 1+2x, & x < 0 \end{cases}$$

1° FORMIRANJE ORTOGON. POLINOMA

$Q_0 = 1$, $p(x) = 1-2|x|$

$$Q_1 = x - \frac{(x, Q_0)}{(Q_0, Q_0)} Q_0$$

$$(Q_0, Q_0) = \int_{-1/2}^{1/2} 1 \cdot 1 \cdot (1-2|x|)\,dx = \int_{-1/2}^{0} (1+2x)\,dx + \int_{0}^{1/2} (1-2x)\,dx =$$

$$= \int_{-1/2}^{0} dx + 2\int_{-1/2}^{0} x\,dx + \int_{0}^{1/2} dx - 2\int_{0}^{1/2} x\,dx = x\Big|_{-1/2}^{0} + \cancel{2} \cdot \frac{x^2}{\cancel{2}}\Big|_{-1/2}^{0} +$$

$$+ x\Big|_{0}^{1/2} - \cancel{2}\frac{x^2}{\cancel{2}}\Big|_{0}^{1/2} = \frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{4}} = \underline{\underline{\frac{1}{2}}}$$

$$(x, Q_0) = \int_{-1/2}^{1/2} x(1-2|x|)\,dx = \int_{-1/2}^{0} x(1+2x)\,dx + \int_{0}^{1/2} x(1-2x)\,dx$$

$$= \int_{-1/2}^{0} x\,dx + 2\int_{-1/2}^{0} x^2\,dx + \int_{0}^{1/2} x\,dx - 2\int_{0}^{1/2} x^2\,dx =$$

$$= \frac{x^2}{2}\Big|_{-1/2}^{0} + 2\,\frac{x^3}{3}\Big|_{-1/2}^{0} + \frac{x^2}{2}\Big|_{0}^{1/2} - 2\,\frac{x^3}{3}\Big|_{0}^{1/2} =$$

$$= -\frac{1}{8} + 2\cdot\left(0 + \frac{1}{24}\right) + \frac{1}{8} - 2\cdot\frac{1}{24} = \underline{\underline{0}}$$

$$\boxed{Q_1 = x - \frac{0}{1/2}\cdot 1 = x}$$

$$Q_2 = x^2 - \frac{(x^2, Q_0)}{(Q_0, Q_0)} Q_0 - \frac{(x^2, Q_1)}{(Q_1, Q_1)} Q_1$$

$x =$

$$(x^2, Q_0) = \int_{-1/2}^{1/2} x^2\cdot(1-2|x|)\,dx = \int_{-1/2}^{0} x^2(1+2x)\,dx +$$

$$+ \int_{0}^{1/2} x^2(1-2x)\,dx = \int_{-1/2}^{0} x^2\,dx + 2\int_{-1/2}^{0} x^3\,dx + \int_{0}^{1/2} x^2\,dx - 2\int_{0}^{1/2} x^3\,dx$$

$$= \frac{x^3}{3}\Big|_{-1/2}^{0} + 2\cdot\frac{x^4}{4}\Big|_{-1/2}^{0} + \frac{x^3}{3}\Big|_{0}^{1/2} - 2\,\frac{x^4}{4}\Big|_{0}^{1/2} =$$

$$= \frac{1}{24} + 2\left(0 - \frac{1}{64}\right) + \frac{1}{24} - 2\left(\frac{1}{64} - 0\right) =$$

$$= \frac{1}{24} - \frac{1}{32} + \frac{1}{24} - \frac{1}{32} = \frac{1}{12} - \frac{1}{16} = \underline{\underline{\frac{1}{48}}}$$

$$(x^2, Q_1) = \int_{-1/2}^{1/2} x^2\cdot x\cdot(1-2|x|)\,dx = \int_{-1/2}^{0} x^3(1+2x)\,dx + \int_{0}^{1/2} x^3(1-2x)\,dx =$$

$$= \int_{-1/2}^{0} x^3\,dx + 2\int_{-1/2}^{0} x^4\,dx + \int_{0}^{1/2} x^3\,dx - 2\int_{0}^{1/2} x^4\,dx =$$

$$= \frac{x^4}{4}\Big|_{-1/2}^{0} + 2\,\frac{x^5}{5}\Big|_{-1/2}^{0} + \frac{x^4}{4}\Big|_{0}^{1/2} - 2\,\frac{x^5}{5}\Big|_{0}^{1/2} =$$

$$= -\frac{1}{64} + 2\cdot\left(0 + \frac{1}{160}\right) + \frac{1}{64} - 2\cdot\left(\frac{1}{160} - 0\right) = \underline{\underline{0}}$$

$$(Q_1, Q_1) = \int_{-1/2}^{1/2} x\cdot x\cdot(1-2|x|)\,dx = (x^2, Q_0) = \underline{\underline{1/48}}$$

$$\boxed{Q_2(x) = x^2 - \frac{1/48}{1/12}\cdot 1 - \frac{0}{1/48}\cdot x = x^2 - \frac{1}{24}}$$

$$\Rightarrow Q_2 = \left(x - \frac{1}{\sqrt{24}}\right)\left(x + \frac{1}{\sqrt{24}}\right)$$

$$\Rightarrow x_1 = \frac{1}{\sqrt{24}}\cdot\frac{\sqrt{24}}{\sqrt{24}} = \frac{\sqrt{24}}{24} = \frac{2\sqrt{6}}{24} = \frac{\sqrt{6}}{12} \approx +0.2041$$

$$\Rightarrow x_2 = -\frac{1}{\sqrt{24}} = -\frac{\sqrt{6}}{12} \approx -0.2041$$

2° NALAŽENJE A_k

$$(Q_1, Q_1) = \|Q_1\|^2 = \frac{1}{48}$$

$$Q_1(x_1) = Q_1(+0.2041) = 0.2041$$
$$Q_1(x_2) = Q_1(-0.2041) = -0.2041$$

$$Q_2'(x) = 2x$$

$$Q_2'(x_1) = 2\cdot 0.2041 = 0.4082$$
$$Q_2'(x_2) = 2\cdot(-0.2041) = -0.4082$$

$$A_1 = \frac{(Q_1, Q_1)}{Q_1(x_1)\cdot Q_2'(x_1)} = \frac{\frac{1}{48}}{0.2041\cdot 0.4082} = 0.2501$$

$$A_2 = \frac{(Q_1, Q_1)}{Q_1(x_2)\,Q_2'(x_2)} = \frac{\frac{1}{48}}{-0.2041\cdot(-0.4082)} = 0.2501$$

$\Rightarrow$ Kvadraturna f-la:

$$\int_{-1/2}^{1/2} (1-2|x|)\, f(x)\, dx = 0.2501\cdot f(0.2041) + 0.2501\, f(-0.2041) + R_2(f)$$

$$\|Q_2\|^2 = (Q_2, Q_2) = \int_{-1/2}^{1/2} \left(x^2 - \frac{1}{24}\right)^2\cdot(1-2|x|)\, dx = \dots = 0.00121$$

$$R_2(f) = \frac{\|(Q_2)\|^2}{4!} \cdot f^{IV}(\xi) = 5.04 \cdot 10^{-5} \cdot f^{IV}(\xi)$$

3° PRIMENA

$$I_1 = \int_0^{1/2} \frac{1-2|x|}{x^2-1}\,dx = \frac{1}{2} \int_{-1/2}^{1/2} \frac{1-2|x|}{x^2-1}\,dx$$

$$\Rightarrow \quad f(x) = \frac{1}{x^2-1}$$

$$f(x_1) = f(0.2041) = \frac{1}{(0.2041)^2-1} = -1.0435$$

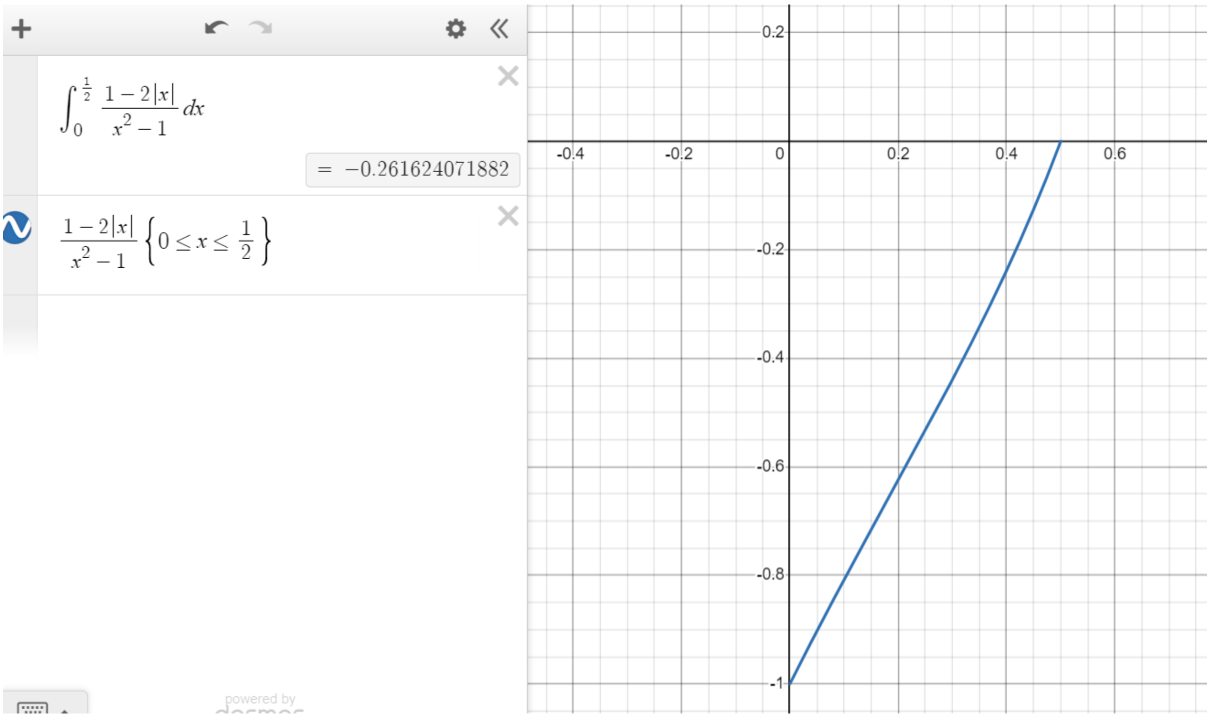
$$f(x_2) = f(-0.2041) = \frac{1}{(-0.2041)^2-1} = -1.0435$$

$$I_1 \approx (0.2501 \cdot (-1.0435) + 0.2501 \cdot (-1.0435))/2$$
$$= -0.52196/2 = -0.26098$$

aproksimacija

$$I_1 = -0.261624$$

prava vrednost


$\int_0^{\frac{1}{2}} \frac{1-2|x|}{x^2-1} dx$
$= -0.261624071882$
$\frac{1-2|x|}{x^2-1} \left\{ 0 \le x \le \frac{1}{2} \right\}$
powered by
0.2
-0.4
-0.2
0
0.2
0.4
0.6
-0.2
-0.4
-0.6
-0.8
-1

$$\int_0^3 e^{-x^2}dx \approx e^{-\frac{9}{4}(-1+1)^2} + e^{-\frac{9}{4}(1+1)^2} + \frac{2}{3}\cdot\left(-\frac{9}{2}\cdot\left(-\frac{1}{2}+1\right)\cdot e^{-\frac{9}{4}\left(-\frac{1}{2}+1\right)^2}\right) -$$

$$-\frac{2}{3}\left(-\frac{9}{2}\left(\frac{1}{2}+1\right)\cdot e^{-\frac{9}{4}\left(\frac{1}{2}+1\right)^2}\right) =$$

$$= 1 + e^{-9} + \frac{2}{3}\cdot\left(-\frac{9}{4}\cdot e^{-\frac{9}{16}}\right) - \frac{2}{3}\cdot\left(-\frac{27}{4}\cdot e^{-\frac{81}{16}}\right) =$$

$$= 1 + 0.00012 - 0.854 + 0.028 = \underline{\underline{0.17412}}$$

2. $$I = \int_0^1 \sin\left(\frac{\pi}{2}x^2\right)dx$$

a) $\varepsilon = 10^{-2}$, TRAPEZNA F-LA

1° $n = 1$:

$b = 1 \quad a = 0 \Rightarrow h_1 = \frac{(b-a)}{n} = \frac{1-0}{1} = 1$

0 1

$$\underset{\sim}{T_1} = \frac{h_1}{2}\cdot(f(a)+f(b)) = 0.5\cdot\left(\sin\left(\frac{\pi}{2}0^2\right)+\sin\left(\frac{\pi}{2}1^2\right)\right) =$$

$$= 0.5(0+1) = \underset{\sim}{0.5}$$

2° $n = 2$:

$b = 1 \quad a = 0 \Rightarrow h_2 = \frac{b-a}{n} = \frac{1-0}{2} = 0.5$

0 0.5 1

$$\underset{\sim}{T_2} = \frac{h_2}{2}\cdot(f(0)+2\cdot f(0.5)+f(1)) = 0.25\cdot\left(\sin\left(\frac{\pi}{2}\cdot 0^2\right) +\right.$$

$$\left.+2\cdot\sin(\pi/2\cdot 0.5^2)+\sin(\pi/2\cdot 1^2)\right) = 0.25\cdot(2\cdot 0.383+1) =$$

$$= \underset{\sim}{0.4415}$$

$$\underline{R_2} = \frac{T_2-T_1}{3} = \frac{0.4415-0.5}{3} = \underline{-0.0195 \geq \varepsilon}$$

veće po modulu od ε

$n = 4$:

$$a = 0,\ b = 1 \Rightarrow h_3 = \frac{1-0}{4} = 0.25$$

0 0.25 0.5 0.75 1

$$\underline{\underline{T_3}} = \frac{0.25}{2}\left(\overset{0}{\cancel{\sin}}\left(\frac{\pi}{2}\cdot 0^2\right) + 2\cdot\left[\overset{0.098}{\cancel{\sin}}\left(\frac{\pi}{2}\cdot 0.25^2\right) + \overset{0.383}{\cancel{\sin}}\left(\frac{\pi}{2}\cdot 0.5^2\right) + \overset{0.773}{\cancel{\sin}}\left(\frac{\pi}{2}\cdot 0.75^2\right)\right] + \underset{1}{\cancel{\sin}}\left(\frac{\pi}{2}\cdot 1^2\right)\right) =$$

$$= 0.125\cdot\left(1 + 2\cdot(0.098 + 0.383 + 0.773)\right) = \underline{0.4385}$$

$$\underline{\underline{R_3}} = \frac{T_3 - T_2}{3} = \frac{0.4385 - 0.4415}{3} = \underline{-0.001} \underset{=}{<} \underline{\underline{\varepsilon}}$$

$$\Rightarrow \boxed{I \approx T_3 + R_3 = 0.4385 - 0.001 = 0.4375}$$

b) $\varepsilon = 10^{-3}$, SIMPSONOVA F-LA

$n = 1$:

$$a = 0 \quad b = 1 \Rightarrow h_1 = \frac{b-a}{2n} = \frac{1-0}{2} = 0.5$$

f_0 f_1 f_2

0 0.5 1

$$\underline{\underline{S_1}} = \frac{h_1}{3}\left(f(0) + 4\cdot f(0.5) + f(1)\right) =$$

$$= \frac{0.5}{3}\left(\overset{0}{\cancel{\sin}}\left(\frac{\pi}{2}\cdot 0^2\right) + 4\cdot\overset{0.383}{\cancel{\sin}}\left(\frac{\pi}{2}\cdot 0.5^2\right) + \overset{1}{\cancel{\sin}}\left(\frac{\pi}{2}\cdot 1^2\right)\right) = \underline{\underline{0.422}}$$

$n = 2$:

$$h_2 = \frac{b-a}{2\cdot n} = \frac{1-0}{4} = 0.25$$

f_0 f_1 f_2 f_3 f_4

0 0.25 0.5 0.75 1

$$S_2 = \frac{0.25}{3} \cdot \left(f_0 + 4(f_1 + f_3) + 2f_2 + f_4 \right) =$$

$$= \frac{0.25}{3} \cdot \left(\cancelto{0}{\sin\left(\frac{\pi}{2} \cdot 0^2\right)} + 4 \cdot \left(\cancelto{0.098}{\sin\left(\frac{\pi}{2} \cdot 0.25^2\right)} + \cancelto{0.773}{\sin\left(\frac{\pi}{2} \cdot 0.75^2\right)} \right) + \right.$$

$$\left. + 2 \cdot \cancelto{0.383}{\sin\left(\frac{\pi}{2} \cdot 0.5^2\right)} + \cancelto{1}{\sin\left(\frac{\pi}{2} \cdot 1^2\right)} \right) = \underline{\underline{0.4375}}$$

$$R_2 = \frac{S_2 - S_1}{15} = \frac{0.4375 - 0.422}{15} = 0.001033 \overset{!!}{>} \varepsilon$$

$n = 4$:

$$h_3 = \frac{b-a}{2n} = \frac{1}{8} = 0.125$$

f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1

$$S_3 = \frac{0.125}{3} \cdot \left(f_0 + 4(f_1 + f_3 + f_5 + f_7) + 2(f_2 + f_4 + f_6) + f_8 \right) =$$

$$= \frac{0.125}{3} \cdot \left(\cancelto{0}{\sin\left(\frac{\pi}{2} \cdot 0^2\right)} + 4 \cdot \left(\cancelto{0.0245}{\sin\left(\frac{\pi}{2} \cdot 0.125^2\right)} + \cancelto{0.2191}{\sin\left(\frac{\pi}{2} \cdot 0.375^2\right)} + \cancelto{0.5758}{\sin\left(\frac{\pi}{2} \cdot 0.625^2\right)} + \right. \right.$$

$$\left. + \cancelto{0.933}{\sin\left(\frac{\pi}{2} \cdot 0.875^2\right)} \right) + 2 \cdot \left(\cancelto{0.098}{\sin\left(\frac{\pi}{2} \cdot 0.25^2\right)} + \cancelto{0.383}{\sin\left(\frac{\pi}{2} \cdot 0.5^2\right)} + \cancelto{0.773}{\sin\left(\frac{\pi}{2} \cdot 0.75^2\right)} \right) +$$

$$\left. + \cancelto{1}{\sin\left(\frac{\pi}{2} \cdot 1^2\right)} \right) = 0.4382$$

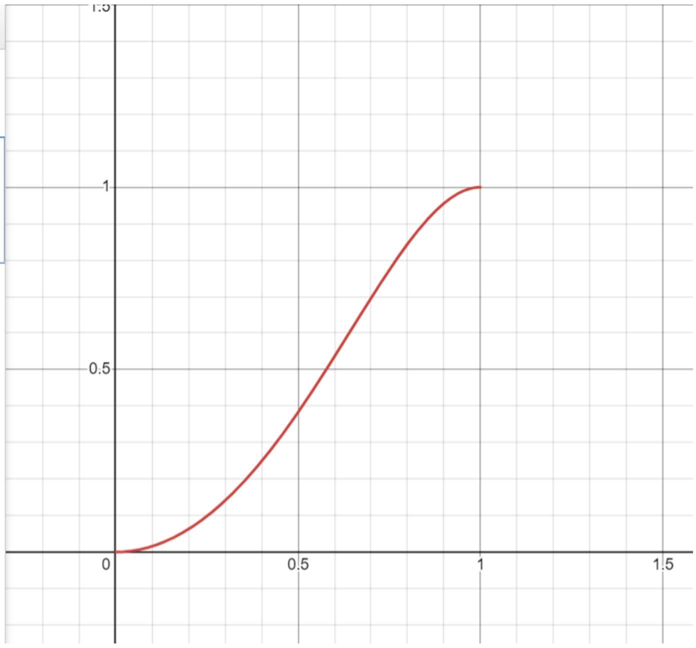
$$\underline{\underline{R_3}} = \frac{S_3 - S_2}{15} = \frac{0.4382 - 0.4375}{15} = \underline{4.7 \cdot 10^{-5}} \overset{!!}{<} \varepsilon$$

$$\Rightarrow \boxed{I \approx S_3 + R_3 = 0.4382 + 4.7 \cdot 10^{-5} = 0.438247}$$

$$\sin\left(\frac{\pi}{2}\cdot x^2\right)\{0<x<1\}$$

$$\int_0^1 \sin\left(\frac{\pi}{2}\cdot x^2\right)dx$$

$= 0.43825914739$



DOMAĆI 4

3.

a) $\int_{-1}^{1} f(x)dx = A_1 f(-1) + A_2 f(1) + A_3 f'(-1/2) + A_4 f'(1/2) + R(f)$

b) Alg. st. tačn?

c) $\int_{0}^{3} e^{-x^2} dx$

a) Kako imamo 4 nepoznate - A_1, A_2, A_3, A_4, potrebne su nam 4 linearne nezavisne j-ne. Tražićemo za početak koeficijente tako da $R(f)$ bude 0 za $f(x) = x^j$, $j = 0,1,2,3$.

$j=0$:

$$\int_{-1}^{1} x^0 dx = x\Big|_{-1}^{1} = 2 \qquad (x^0)' = 0$$

$$2 = A_1 \cdot (-1)^0 + A_2 \cdot 1^0 + A_3 \cdot 0 + A_4 \cdot 0 = A_1 + A_2$$

$j=1$:

$$\int_{-1}^{1} x^1 dx = \frac{x^2}{2}\Big|_{-1}^{1} = 0 \qquad (x^1)' = 1$$

$$0 = A_1 \cdot (-1)^1 + A_2 \cdot 1^1 + A_3 \cdot 1 + A_4 \cdot 1 = -A_1 + A_2 + A_3 + A_4$$

$j=2$:

$$\int_{-1}^{1} x^2 dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{2}{3} \qquad (x^2)' = 2x$$

$$\frac{2}{3} = A_1 \cdot (-1)^2 + A_2 \cdot 1^2 + A_3 \cdot 2 \cdot \left(-\frac{1}{2}\right) + A_4 \cdot 2 \cdot \frac{1}{2} =$$

$$= A_1 + A_2 - A_3 + A_4$$

$j = 3$:
$$\int_{-1}^{1} x^3 dx = \frac{x^4}{4}\Big|_{-1}^{1} = 0 \qquad (x^3)' = 3x^2$$

$$0 = A_1 \cdot (-1)^3 + A_2 \cdot 1^3 + A_3 \cdot 3 \cdot (-\tfrac{1}{2})^2 + A_4 \cdot 3 \cdot (\tfrac{1}{2})^2 =$$
$$= -A_1 + A_2 + 3/4\, A_3 + 3/4\, A_4$$

SISTEM:

$$\begin{cases} A_1 + A_2 = 2 \\ -A_1 + A_2 + A_3 + A_4 = 0 \\ A_1 + A_2 - A_3 + A_4 = 2/3 \\ -A_1 + A_2 + 3/4\, A_3 + 3/4\, A_4 = 0 \end{cases}$$

$$\begin{cases} 2A_2 + A_3 + A_4 = 2 \\ 2A_2 + 2A_4 = 2/3 \\ 2A_2 - 1/4\, A_3 + 7/4\, A_4 = 2/3 \end{cases}$$

$$\begin{cases} A_3 - A_4 = 4/3 \Rightarrow 2A_3 = \frac{4}{3} \Rightarrow \boxed{A_3 = \frac{2}{3}} \\ 1/4\, A_3 + 1/4\, A_4 = 0 \Rightarrow A_3 + A_4 = 0 \Rightarrow A_3 = -A_4 \end{cases}$$

$$\Rightarrow \boxed{A_4 = -\frac{2}{3}}$$

$$A_2 = \frac{1}{2}(2 - A_3 - A_4) = \frac{1}{2}(2 - \frac{2}{3} + \frac{2}{3}) = 1$$

$$A_1 = 2 - A_2 = 2 - 1 = 1$$

$$\Rightarrow \begin{cases} A_1 = 1 \\ A_2 = 1 \\ A_3 = 2/3 \\ A_4 = -2/3 \end{cases} \Rightarrow \boxed{\int_{-1}^{1} f(x)dx = f(-1) + f(1) + \frac{2}{3} f'(-\tfrac{1}{2}) - \frac{2}{3} f'(\tfrac{1}{2}) + R(f)}$$

b) Nalazimo alg. st. tačnosti:

$j = 4$:

$$(x^4)' = 4x^3$$

$$\int_{-1}^{1} x^4 dx = \frac{x^5}{5}\Big|_{-1}^{1} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

$$\int_{-1}^{1} x^4 dx \overset{?}{=} (-1)^4 + 1^4 + \frac{2}{3}\cdot 4 \cdot \left(-\frac{1}{2}\right)^3 - \frac{2}{3}\cdot 4\cdot\left(\frac{1}{2}\right)^3 =$$

$$= 1 + 1 + \frac{8}{3}\cdot\left(-\frac{1}{8}\right) - \frac{8}{3}\cdot\frac{1}{8} =$$

$$= 2 - \frac{1}{3} - \frac{1}{3} = \frac{4}{3} \neq 0$$

$\Rightarrow$ aritm. st. tačnosti je 3.

c) $\int_0^3 e^{-x^2} dx = ?$

$[0,3] \longrightarrow [-1,1]$

$x = at + b$

$x = 0: \quad 0 = -a + b$

$x = 3: \quad 3 = a + b$ $\Big\} +$

$3 = 2b \Rightarrow b = \frac{3}{2} \Rightarrow a = \frac{3}{2}$

$\Rightarrow \underline{x = \frac{3}{2}(t+1)} \Rightarrow \underline{dx = \frac{3}{2} dt}$

$$\int_0^3 e^{-x^2} dx = \int_{-1}^{1} \frac{3}{2}\cdot e^{-\left(\frac{3}{2}(t+1)\right)^2} dt = \frac{3}{2}\int_{-1}^{1} e^{-\frac{9}{4}(t+1)^2} dt$$

$\Rightarrow f(t) = e^{-\frac{9}{4}(t+1)^2}$

$$f'(t) = -\frac{9}{4} e^{-\frac{9}{4}(t+1)^2}\cdot 2\cdot(t+1) = -\frac{9}{2}(t+1)\cdot e^{-\frac{9}{4}(t+1)^2}$$

$$\int_0^3 e^{-x^2}\,dx \approx e^{-\frac{9}{4}(\cancelto{0}{-1+1})^2} + e^{-\frac{9}{\cancel{4}}(\cancelto{2}{1+1})^2} + \frac{2}{3}\cdot\left(-\frac{9}{2}\cdot\left(\cancelto{\frac{1}{2}}{-\frac{1}{2}+1}\right)\cdot e^{-\frac{9}{4}(\cancelto{\frac{1}{4}}{-\frac{1}{2}+1})^2}\right) -$$

$$-\frac{2}{3}\left(-\frac{9}{2}\left(\cancelto{\frac{3}{2}}{\frac{1}{2}+1}\right)\cdot e^{-\frac{9}{4}(\cancelto{\frac{9}{4}}{\frac{1}{2}+1})^2}\right) =$$

$$= 1 + e^{-9} + \frac{\cancel{2}}{\cancel{3}}\cdot\left(-\frac{\cancelto{3}{9}}{\cancelto{2}{4}}\cdot e^{-\frac{9}{16}}\right) - \frac{\cancel{2}}{\cancel{3}}\cdot\left(-\frac{\cancelto{9}{27}}{\cancelto{2}{4}}\cdot e^{-\frac{81}{16}}\right) =$$

$$= 1 + 0.00012 - 0.854 + 0.028 = \underline{\underline{0.17412}}$$