- NAĆI INVERZ MATRICE

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}.$$

-REŠENJE:

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 1 & 0 \\ 4 & -3 & 8 & 0 & 0 & 1 \end{array}\right] \underset{\simeq}{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 4 & -3 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{\simeq}{R_3 \to R_3 - 4R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & -4 & 0 & -4 & 1 \end{array}\right] \underset{\simeq}{R_3 \to R_3 + 3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 3 & -4 & 1 \end{array}\right]$$

$$\underset{\simeq}{R_3 \to \tfrac{1}{2} R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3/2 & -2 & 1/2 \end{array}\right] \underset{R_1 \to R_1 - 3R_3}{\overset{R_2 \to R_2 - 2R_3}{\simeq}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -9/2 & 7 & -3/2 \\ 0 & 1 & 0 & -2 & 4 & -1 \\ 0 & 0 & 1 & 3/2 & -2 & 1/2 \end{array}\right]$$

$$= [I \mid A^{-1}]$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -9/2 & 7 & -3/2 \\ -2 & 4 & -1 \\ 3/2 & -2 & 1/2 \end{bmatrix}$$

- DATA JE MATRICA

$$A = \begin{bmatrix} 4 & -2 & 0 & 0 \\ 2 & 4 & -2 & 0 \\ 0 & 2 & 4 & -2 \\ 0 & 0 & 2 & -4 \end{bmatrix}.$$

PRIMENOM GAUSOVOG ALGORITMA BEZ IZBORA GLAVNOG ELEMENTA NAĆI FAKTORIZACIJU $A=LR$, GDE JE L DONJE-TROUGAONA, A R GORNJE-TROUGAONA MATRICA, A ZATIM KORIŠĆENJEM OVE FAKTORIZACIJE REŠITI SISTEM JEDNAČINA

$$A\vec{x} = [0 \;\; 1 \;\; 0 \;\; 0]^T.$$

- REŠENJE:

MNOŽIMO PRVU VRSTU SA $m_{12} = \frac{2}{4} = \frac{1}{2}$,

ODUZIMAMO OD DRUGE

~~MNOŽIMO PRVU VRSTU SA~~ $\cancel{m_{13}}$ $m_{13} = m_{14} = 0$

$$A = \begin{bmatrix} 4 & -2 & 0 & 0 \\ 2 & 4 & -2 & 0 \\ 0 & 2 & 4 & -2 \\ 0 & 0 & 2 & -4 \end{bmatrix} \simeq \begin{bmatrix} 4 & -2 & 0 & 0 \\ 0 & 5 & -2 & 0 \\ 0 & 2 & 4 & -2 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

MNOŽIMO ~~PRVU~~ DRUGU VRSTU SA $m_{23} = \frac{2}{5}$,

ODUZIMAMO OD TREĆE

$\cancel{m_{24}}$ $m_{24} = 0$

$$A \simeq \begin{bmatrix} 4 & -2 & 0 & 0 \\ 0 & 5 & -2 & 0 \\ 0 & 0 & \frac{24}{5} & -2 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

MNOŽIMO TREĆU VRSTU SA $m_{34} = \dfrac{2}{24/5} = \dfrac{5}{12}$,

ODUZIMAMO OD ČETVRTE

$$A \simeq \begin{bmatrix} 4 & -2 & 0 & 0 \\ 0 & 5 & -2 & 0 \\ 0 & 0 & 24/5 & -2 \\ 0 & 0 & 0 & -19/6 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 2/5 & 1 & 0 \\ 0 & 0 & 5/12 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 4 & -2 & 0 & 0 \\ 0 & 5 & -2 & 0 \\ 0 & 0 & 24/5 & -2 \\ 0 & 0 & 0 & -19/6 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 4 & -2 & 0 & 0 & 0 \\ 2 & 4 & -2 & 0 & 1 \\ 0 & 2 & 4 & -2 & 0 \\ 0 & 0 & 2 & 4 & 0 \end{array}\right] \simeq \left[\begin{array}{cccc|c} 4 & -2 & 0 & 0 & 0 \\ 0 & 5 & -2 & 0 & 1 \\ 0 & 2 & 4 & -2 & 0 \\ 0 & 0 & 2 & -4 & 0 \end{array}\right]$$

$$\simeq \left[\begin{array}{cccc|c} 4 & -2 & 0 & 0 & 0 \\ 0 & 5 & -2 & 0 & 1 \\ 0 & 0 & 24/5 & -2 & -2/5 \\ 0 & 0 & 2 & -4 & 0 \end{array}\right] \simeq \left[\begin{array}{cccc|c} 4 & -2 & 0 & 0 & 0 \\ 0 & 5 & -2 & 0 & 1 \\ 0 & 0 & 24/5 & -2 & -2/5 \\ 0 & 0 & 0 & -19/6 & 1/6 \end{array}\right]$$

REŠAVAMO SISTEM:

$$\begin{aligned} 4x_1 - 2x_2 &= 0 \\ 5x_2 - 2x_3 &= 1 \\ \frac{24}{5}x_3 - 2x_4 &= -\frac{2}{5} \\ -\frac{19}{6}x_4 &= \frac{1}{6} \end{aligned}$$

$$\Rightarrow \quad x_4 = -\frac{1}{19}, \quad x_3 = -\frac{2}{19}, \quad x_2 = \frac{3}{19}, \quad x_1 = \frac{3}{38}$$

MATEMATIČKI METODI

Zadaci sa računskih vežbi

Rešeni zadaci

Numerički metodi u linearnoj algebri

1. zadatak

Gausovim metodom bez izbora glavnog elementa rešiti sistem linearnih jednačina *Ax*=*b*, gde je:

a) $A=\begin{pmatrix}1&0&2\\0&3&-4\\2&-4&10\end{pmatrix}$, $b=\begin{pmatrix}7\\-6\\24\end{pmatrix}$; b) $A=\begin{pmatrix}1&0&2&0\\0&3&-4&3\\2&-4&10&0\\1&2&3&100\end{pmatrix}$, $b=\begin{pmatrix}20\\-15\\74\\239\end{pmatrix}$.

Rešenje

b) Formiramo trougaoni oblik matrice *A* :

množimo prvu vrstu redom sa : $m_{12}=\dfrac{0}{1}=0$ i oduzimamo od druge;

$m_{13}=\dfrac{2}{1}=2$ i oduzimamo od treće;

$m_{14}=\dfrac{1}{1}=1$ i oduzimamo od četvrte;

```
-2 {1, 0, 2, 0, 20} + {2, -4, 10, 0, 74}
-1 {1, 0, 2, 0, 20} + {1, 2, 3, 100, 239}
```

```
{0, -4, 6, 0, 34}
```

```
{0, 2, 1, 100, 219}
```

$$A=\left(\begin{array}{cccc|c}1&0&2&0&20\\0&3&-4&3&-15\\2&-4&10&0&74\\1&2&3&100&239\end{array}\right)\simeq\left(\begin{array}{cccc|c}1&0&2&0&20\\0&3&-4&3&-15\\0&-4&6&0&34\\0&2&1&100&219\end{array}\right)$$

množimo drugu vrstu redom sa : $m_{23} = \frac{-4}{3}$ i oduzimamo od treće;

$m_{24} = \frac{2}{3}$ i oduzimamo od četvrte;

```
-(-4/3) {0, 3, -4, 3, -15} + {0, -4, 6, 0, 34}
-(2/3) {0, 3, -4, 3, -15} + {0, 2, 1, 100, 219}
```

$\{0, 0, \frac{2}{3}, 4, 14\}$

$\{0, 0, \frac{11}{3}, 98, 229\}$

$$A = \begin{pmatrix} 1 & 0 & 2 & 0 & | & 20 \\ 0 & 3 & -4 & 3 & | & -15 \\ 2 & -4 & 10 & 0 & | & 74 \\ 1 & 2 & 3 & 100 & | & 239 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & 2 & 0 & | & 20 \\ 0 & 3 & -4 & 3 & | & -15 \\ 0 & -4 & 6 & 0 & | & 34 \\ 0 & 2 & 1 & 100 & | & 219 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & 2 & 0 & | & 20 \\ 0 & 3 & -4 & 3 & | & 9 \\ 0 & 0 & 2/3 & 4 & | & 14 \\ 0 & 0 & 11/3 & 98 & | & 229 \end{pmatrix}$$

množimo treću vrstu sa : $m_{34} = \frac{11/3}{2/3} = \frac{11}{2}$ i oduzimamo od četvrte;

```
-((11/3)/(2/3)) {0, 0, 2/3, 4, 14} + {0, 0, 11/3, 98, 229}
```

$\{0, 0, 0, 76, 152\}$

$$A = \begin{pmatrix} 1 & 0 & 2 & 0 & | & 20 \\ 0 & 3 & -4 & 3 & | & -15 \\ 2 & -4 & 10 & 0 & | & 74 \\ 1 & 2 & 3 & 100 & | & 239 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & 2 & 0 & | & 20 \\ 0 & 3 & -4 & 3 & | & -15 \\ 0 & -4 & 6 & 0 & | & 34 \\ 0 & 2 & 1 & 100 & | & 219 \end{pmatrix} \simeq$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 & | & 20 \\ 0 & 3 & -4 & 3 & | & 9 \\ 0 & 0 & 2/3 & 4 & | & 14 \\ 0 & 0 & 11/3 & 98 & | & 229 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & 2 & 0 & | & 20 \\ 0 & 3 & -4 & 3 & | & 9 \\ 0 & 0 & 2/3 & 4 & | & 14 \\ 0 & 0 & 0 & 76 & | & 152 \end{pmatrix}$$

Resavamo sistem :

$$\begin{aligned} x_1 \quad + 2\,x_3 \quad &= 20 \\ 3\,x_2 - 4\,x_3 + 3\,x_4 &= 9 \\ \tfrac{2}{3}\,x_3 + 4\,x_4 &= 14 \\ 76\,x_4 &= 152 \end{aligned}$$

$$\begin{aligned} x_4 &= 2 \\ x_3 &= \tfrac{3}{2}\,(14 - 4\,x_4) = \tfrac{3}{2}\,(14 - 8) = 9 \\ x_2 &= \tfrac{1}{3}\left(9 + \tfrac{4}{3}\,x_3 - 3\,x_4\right) = \tfrac{1}{3}\left(9 + \tfrac{4}{3}\cdot 9 - 3\cdot 2\right) = 5 \\ x_1 &= 20 - 2\,x_3 = 20 - 2\cdot 9 = 2 \end{aligned}$$

```
Solve[{x1 + 2 x3 == 20, 3 x2 - 4 x3 + 3 x4 == -15,
  2 x1 - 4 x2 + 10 x3 == 74, x1 + 2 x2 + 3 x3 + 100 x4 == 239}, {x1, x2, x3, x4}]
```

```
{{x1 → 2, x2 → 5, x3 → 9, x4 → 2}}
```

2. zadatak

Odrediti *LR* fktorizaciju matrice:

a) $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & -4 \\ 2 & -4 & 10 \end{pmatrix}$, b) $A = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & -4 & 3 \\ 2 & -4 & 10 & 0 \\ 1 & 2 & 3 & 100 \end{pmatrix}$.

Rešenje

b) $A = LR$,

R je gornjetrougaona matrica (nule is pod g l a vne dijagonale)
koja se dobija primenom Gausovog algoritma;

L je donjetrougaona matrica (nule iznad g l a vne dijagonale) čiji su elementi 1 na glavnoj dijagonali,
a eliminacioni faktori koji se koriste u Gausovom algoritmu ispod glavne dijagonale.

$$R = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & -4 & 3 \\ 0 & 0 & 2/3 & 4 \\ 0 & 0 & 0 & 76 \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ m_{12} & 1 & 0 & 0 \\ m_{13} & m_{23} & 1 & 0 \\ m_{14} & m_{24} & m_{34} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & -4/3 & 1 & 0 \\ 1 & 2/3 & 11/2 & 1 \end{pmatrix}.$$

prva kolona : $m_{12} = \frac{0}{1} = 0, \quad m_{13} = \frac{2}{1} = 2, \quad m_{14} = \frac{1}{1} = 1.$

druga kolona : $m_{23} = \frac{-4}{3}, \quad m_{24} = \frac{2}{3}.$

treća kolona : $m_{34} = \frac{11}{2}.$

```
L = {{1, 0, 0, 0}, {0, 1, 0, 0}, {2, -4/3, 1, 0}, {1, 2/3, 11/2, 1}};
R = {{1, 0, 2, 0}, {0, 3, -4, 3}, {0, 0, 2/3, 4}, {0, 0, 0, 76}};
L.R
Print["L=", MatrixForm[L], ",    R=", MatrixForm[R], ",    L.R=", MatrixForm[L.R], "=A"];
```

{{1, 0, 2, 0}, {0, 3, -4, 3}, {2, -4, 10, 0}, {1, 2, 3, 100}}

$L=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & -\frac{4}{3} & 1 & 0 \\ 1 & \frac{2}{3} & \frac{11}{2} & 1 \end{pmatrix}, \quad R=\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & -4 & 3 \\ 0 & 0 & \frac{2}{3} & 4 \\ 0 & 0 & 0 & 76 \end{pmatrix}, \quad L.R=\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & -4 & 3 \\ 2 & -4 & 10 & 0 \\ 1 & 2 & 3 & 100 \end{pmatrix}=A$

3. zadatak

Primenom Gaussovog algoritma sa izborom glavnog elementa odrediti matrice
P, L i R u faktorizaciji $PA = LR$, gde je

$$A = \begin{pmatrix} 2 & -6 & 4 & -2 \\ 1 & -3 & 4 & 3 \\ 4 & 3 & -2 & 3 \\ 1 & -4 & 3 & 3 \end{pmatrix},$$

a zatim korišćenjem te faktorizacije rešiti sistem $Ax=b$, gde je $b = (8 \times 6 \times 3 \times 9)^T$.

Rešenje

Zbog izbora glavnog elementa vrši se zamena mesta vrstama matrice A,
tj. dobija se faktorizacija matrice $PA = LR$, gde je P permutaciona matrica. (Permutaciona matrica dobija se permutovanjem vrsta jedinične matrice.)

U mesto polaznog sistema rešava se transformisani sistem :

$$Ax = b,$$
$$PAx = Pb, \quad Pb = b'$$
$$LRx = b'$$

Označavanjem : $Rx = y$, dobijaju se dva trougaona sistema $Ly = b'$,
$Rx = y$, koji se rešavaju sukcesivno.

1. Faktorizacija :

Izbor glavnog elementa u prvom koraku zahteva zamenu mesta prve i treće vrste, što je označeno rednim brojevima vrsta polazne matrice A koji su napisani pored vrsta transformisane matrice $A1$.

$$A = \begin{array}{c} \begin{pmatrix} 2 & -6 & 4 & -2 \\ 1 & -3 & 4 & 3 \\ 4 & 3 & -2 & 3 \\ 1 & -4 & 3 & 3 \end{pmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \longrightarrow A1 = \begin{pmatrix} 4 & 3 & -2 & 3 \\ 1 & -3 & 4 & 3 \\ 2 & -6 & 4 & -2 \\ 1 & -4 & 3 & 3 \end{pmatrix} \begin{array}{c} 3 \\ 2 \\ 1 \\ 4 \end{array}$$

To znači da je prva permutaciona matrica koja transformiše $A \longrightarrow A1$ sledeća :

$$P1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \text{ jer :}$$

$$P1.A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -6 & 4 & -2 \\ 1 & -3 & 4 & 3 \\ 4 & 3 & -2 & 3 \\ 1 & -4 & 3 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 3 & -2 & 3 \\ 1 & -3 & 4 & 3 \\ 2 & -6 & 4 & -2 \\ 1 & -4 & 3 & 3 \end{pmatrix} = A1.$$

Nadalje neće biti pisane permutacione matrice,
već će zamena mesta vrstama biti označena brojevima pored matrice.

Primenjuje se Gausov algoritam,
ali se u istoj matrici upisuju elementi gornjetrougaone matrice R po vrstama i elementi donjetrougaone matrice L po kolonama. U matrici L elementi na glavnoj dijagonali su jednaki 1.

množimo prvu vrstu redom sa :

$m_{12} = \frac{1}{4}$ i oduzimamo od druge;

$m_{13} = \frac{2}{4} = \frac{1}{2}$ i oduzimamo od treće;

$m_{14} = \frac{1}{4}$ i oduzimamo od četvrte;

```
-1/4 {4, 3, -2, 3} + {1, -3, 4, 3}
-1/2 {4, 3, -2, 3} + {2, -6, 4, -2}
-1/4 {4, 3, -2, 3} + {1, -4, 3, 3}
```

$$\left\{0, -\frac{15}{4}, \frac{9}{2}, \frac{9}{4}\right\}$$

$$\left\{0, -\frac{15}{2}, 5, -\frac{7}{2}\right\}$$

$$\left\{0, -\frac{19}{4}, \frac{7}{2}, \frac{9}{4}\right\}$$

$$A = \begin{pmatrix} 2 & -6 & 4 & -2 \\ 1 & -3 & 4 & 3 \\ 4 & 3 & -2 & 3 \\ 1 & -4 & 3 & 3 \end{pmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \longrightarrow \begin{pmatrix} 4 & 3 & -2 & 3 \\ 1 & -3 & 4 & 3 \\ 2 & -6 & 4 & -2 \\ 1 & -4 & 3 & 3 \end{pmatrix} \begin{array}{c} 3 \\ 2 \\ 1 \\ 4 \end{array} \longrightarrow \left(\begin{array}{ccccc} 4 & & 3 & -2 & 3 \\ 1/4 & \vert & -15/4 & 9/2 & 9/4 \\ 1/2 & \vert & -15/2 & 5 & -7/2 \\ 1/4 & \vert & -19/4 & 7/2 & 9/4 \end{array}\right) \begin{array}{c} 3 \\ 2 \\ 1 \\ 4 \end{array}$$

$$\longrightarrow \left(\begin{array}{ccccc} 4 & & 3 & -2 & 3 \\ 1/2 & \vert & -15/2 & 5 & -7/2 \\ 1/4 & \vert & -15/4 & 9/2 & 9/4 \\ 1/4 & \vert & -19/4 & 7/2 & 9/4 \end{array}\right) \begin{array}{c} 3 \\ 1 \\ 2 \\ 4 \end{array}$$

množimo drugu vrstu redom sa :

$$m_{23} = \frac{-15/4}{-15/2} = \frac{1}{2}$$ i oduzimamo od treće;

$$m_{24} = \frac{-19/4}{-15/2} = \frac{19}{30}$$ i oduzimamo od četvrte;

```
-(1/2) {0, -15/2, 5, -7/2} + {0, -15/4, 9/2, 9/4}
-(19/30) {0, -15/2, 5, -7/2} + {0, -19/4, 7/2, 9/4}
```

$$\{0, 0, 2, 4\}$$

$$\left\{0, 0, \frac{1}{3}, \frac{67}{15}\right\}$$

$$A = \begin{pmatrix} 2 & -6 & 4 & -2 \\ 1 & -3 & 4 & 3 \\ 4 & 3 & -2 & 3 \\ 1 & -4 & 3 & 3 \end{pmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \longrightarrow \begin{pmatrix} 4 & 3 & -2 & 3 \\ 1 & -3 & 4 & 3 \\ 2 & -6 & 4 & -2 \\ 1 & -4 & 3 & 3 \end{pmatrix} \begin{array}{c} 3 \\ 2 \\ 1 \\ 4 \end{array} \longrightarrow \left(\begin{array}{ccccc} 4 & & 3 & -2 & 3 \\ 1/4 & \vert & -15/4 & 9/2 & 9/4 \\ 1/2 & \vert & -15/2 & 5 & -7/2 \\ 1/4 & \vert & -19/4 & 7/2 & 9/4 \end{array}\right) \begin{array}{c} 3 \\ 2 \\ 1 \\ 4 \end{array}$$

$$\longrightarrow \left(\begin{array}{ccccc} 4 & & 3 & -2 & 3 \\ 1/2 & \vert & -15/2 & 5 & -7/2 \\ 1/4 & \vert & -15/4 & 9/2 & 9/4 \\ 1/4 & \vert & -19/4 & 7/2 & 9/4 \end{array}\right) \begin{array}{c} 3 \\ 1 \\ 2 \\ 4 \end{array} \longrightarrow \left(\begin{array}{cccccc} 4 & & 3 & & -2 & 3 \\ 1/2 & \vert & -15/2 & & 5 & -7/2 \\ 1/4 & \vert & 1/2 & \vert & 2 & 4 \\ 1/4 & \vert & 19/30 & \vert & 1/3 & 67/15 \end{array}\right) \begin{array}{c} 3 \\ 1 \\ 2 \\ 4 \end{array}$$

množimo treću vrstu sa : $m_{34} = \dfrac{1/3}{2} = \dfrac{1}{6}$ i oduzimamo od cetvrte;

```
-1/6 {0, 0, 2, 4} + {0, 0, 1/3, 67/15}
```

$\left\{0, 0, 0, \frac{19}{5}\right\}$

$$A=\left(\begin{array}{cccc}2&-6&4&-2\\1&-3&4&3\\4&3&-2&3\\1&-4&3&3\end{array}\right)\begin{array}{c}1\\2\\3\\4\end{array}\longrightarrow\left(\begin{array}{cccc}4&3&-2&3\\1&-3&4&3\\2&-6&4&-2\\1&-4&3&3\end{array}\right)\begin{array}{c}3\\2\\1\\4\end{array}\longrightarrow\left(\begin{array}{cccc}4&3&-2&3\\1/4&|\,-15/4&9/2&9/4\\1/2&|\,-15/2&5&-7/2\\1/4&|\,-19/4&7/2&9/4\end{array}\right)\begin{array}{c}3\\2\\1\\4\end{array}$$

$$\longrightarrow\left(\begin{array}{cccc}4&3&-2&3\\1/2&|\,-15/2&5&-7/2\\1/4&|\,-15/4&9/2&9/4\\1/4&|\,-19/4&7/2&9/4\end{array}\right)\begin{array}{c}3\\1\\2\\4\end{array}\longrightarrow\left(\begin{array}{cccc}4&3&-2&3\\1/2&|\,-15/2&5&-7/2\\1/4&|\,1/2&|\,2&4\\1/4&|\,19/30&|\,1/3&67/15\end{array}\right)\begin{array}{c}3\\1\\2\\4\end{array}$$

$$\longrightarrow\left(\begin{array}{cccc}4&3&-2&3\\1/2&|\,-15/2&5&-7/2\\1/4&|\,1/2&|\,2&4\\1/4&|\,19/30&|\,1/6&|\,19/5\end{array}\right)\begin{array}{c}3\\1\\2\\4\end{array}$$

$$L=\begin{pmatrix}1&0&0&0\\1/2&1&0&0\\1/4&1/2&1&0\\1/4&19/30&1/6&1\end{pmatrix},$$

$$R=\begin{pmatrix}4&3&-2&3\\0&-15/2&5&-7/2\\0&0&2&4\\0&0&0&19/5\end{pmatrix},\quad P=\begin{pmatrix}0&0&1&0\\1&0&0&0\\0&1&0&0\\0&0&0&1\end{pmatrix}.$$

Provera :

```
L = {{1, 0, 0, 0}, {1/2, 1, 0, 0}, {1/4, 1/2, 1, 0}, {1/4, 19/30, 1/6, 1}};
R = {{4, 3, -2, 3}, {0, -15/2, 5, -7/2}, {0, 0, 2, 4}, {0, 0, 0, 19/5}};
L.R // MatrixForm
```

$$\begin{pmatrix}4&3&-2&3\\2&-6&4&-2\\1&-3&4&3\\1&-4&3&3\end{pmatrix}$$

2. Rešavanje sistema

$$PA=L.R=\begin{pmatrix}4&3&-2&3\\2&-6&4&-2\\1&-3&4&3\\1&-4&3&3\end{pmatrix},\qquad b'=Pb=\begin{pmatrix}0&0&1&0\\1&0&0&0\\0&1&0&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}8\\6\\3\\9\end{pmatrix}=\begin{pmatrix}3\\8\\6\\9\end{pmatrix}$$

$LRx = b'$; $\quad Ly = b'$, $y = Rx$

$Ly = b'$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 1/4 & 1/2 & 1 & 0 \\ 1/4 & 19/30 & 1/6 & 1 \end{pmatrix} \begin{pmatrix} y1 \\ y2 \\ y3 \\ y4 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 6 \\ 9 \end{pmatrix}$$

$$y1 = 3$$
$$\frac{1}{2} y1 + y2 = 8$$
$$\frac{1}{4} y1 + \frac{1}{2} y2 + y3 = 6$$
$$\frac{1}{4} y1 + \frac{19}{30} y2 + \frac{1}{6} y3 + y4 = 9$$

$$y1 = 3$$
$$y2 = 8 - \tfrac{1}{2} y1 = \tfrac{13}{2}$$
$$y3 = 6 - \tfrac{1}{4} y1 - \tfrac{1}{2} y2 = 6 - \tfrac{3}{4} - \tfrac{13}{4} = 2$$
$$y4 = 9 - \tfrac{1}{4} y1 - \tfrac{19}{30} y2 - \tfrac{1}{6} y3 = 9 - \frac{3}{4} - \frac{19}{30} * \frac{13}{2} - \frac{1}{3} = \frac{19}{5}$$

$$y = \left[3 \quad \tfrac{13}{2} \quad 2 \quad \tfrac{19}{5}\right]^T$$

$Rx = y$

$$\begin{pmatrix} 4 & 3 & -2 & 3 \\ 0 & -15/2 & 5 & -7/2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 19/5 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \begin{pmatrix} 3 \\ 13/2 \\ 2 \\ 19/5 \end{pmatrix}$$

$$\begin{aligned} 4\,x1 + 3\,x2 - 2\,x3 + 3\,x4 &= 3 \\ -\tfrac{15}{2}\,x2 + 5\,x3 - \tfrac{7}{2}\,x4 &= \tfrac{13}{2} \\ 2\,x3 + 4\,x4 &= 2 \\ \tfrac{19}{5}\,x4 &= \tfrac{19}{5} \end{aligned}$$

$$x4 = 1$$
$$x3 = \tfrac{1}{2}\,(2 - 4\,x4) = \tfrac{1}{2}\,(2 - 4) = -1$$
$$x2 = -\tfrac{2}{15}\left(\tfrac{13}{2} - 5\,x3 + \tfrac{7}{2}\,x4\right) = -\tfrac{2}{15}\left(\tfrac{13}{2} + 5 + \tfrac{7}{2}\right) = -2$$
$$x1 = \tfrac{1}{4}\,(3 - 3\,x2 + 2\,x3 - 3\,x4) = \tfrac{1}{4}\,(3 + 6 - 2 - 3) = 1$$

$$x = [1 \quad -2 \quad -1 \quad 1]^T.$$

Provera :

```
(*  Ry=b'  *)
```
$$\text{Solve}\left[\left\{y1 == 3,\ \frac{1}{2}y1 + y2 == 8,\ \frac{1}{4}y1 + \frac{1}{2}y2 + y3 == 6,\ \frac{1}{4}y1 + \frac{19}{30}y2 + \frac{1}{6}y3 + y4 == 9\right\},\ \{y1, y2, y3, y4\}\right]$$
```
(*  Lx=y  *)
```
$$\text{Solve}\left[\left\{4\,x1 + 3\,x2 - 2\,x3 + 3\,x4 == 3,\ -\frac{15}{2}x2 + 5\,x3 - \frac{7}{2}x4 == \frac{13}{2},\ 2\,x3 + 4\,x4 == 2,\ \frac{19}{5}x4 == \frac{19}{5}\right\},\ \{x1, x2, x3, x4\}\right]$$
```
(*  Ax=b  *)
Solve[{2 x1 - 6 x2 + 4 x3 - 2 x4 == 8, x1 - 3 x2 + 4 x3 + 3 x4 == 6,
  4 x1 + 3 x2 - 2 x3 + 3 x4 == 3, x1 - 4 x2 + 3 x3 + 3 x4 == 9}, {x1, x2, x3, x4}]
```
$$\left\{\left\{y1 \to 3,\ y2 \to \frac{13}{2},\ y3 \to 2,\ y4 \to \frac{19}{5}\right\}\right\}$$
```
{{x1 → 1, x2 → -2, x3 → -1, x4 → 1}}

{{x1 → 1, x2 → -2, x3 → -1, x4 → 1}}
```

4. zadatak

Data je matrica A

a. $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & -4 \\ 2 & -4 & 10 \end{pmatrix}$, b. $A = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & -4 & 0 \\ 2 & -4 & 10 & 0 \\ 0 & 0 & 0 & 100 \end{pmatrix}$.

Primenom metoda Čoleskog odrediti LR faktorizaciju matrice A.

Rešenje

Kako je matrica simetrična i pozitivno definitna, tražena faktorizacija je oblika $A=R^T R$.

```
Clear["Global`*"];
A = {{1., 0., 2.}, {0., 3., -4.}, {2., -4., 10.}};
A = {{1., 0., 2., 0}, {0., 3., -4., 0.}, {2., -4., 10., 0.}, {0., 0., 0., 100.}};
Print["A=", MatrixForm[A]];
n = Length[A];
R = Table[0, {i, 1, n}, {j, 1, n}];
For[i = 1, i ≤ n, i++,
```

$$R[[i, i]] = \sqrt{A[[i, i]] - \sum_{k=1}^{i-1} R[[k, i]]^2};$$

```
  For[j = i + 1, j ≤ n, j++,
```

$$R[[i, j]] = \frac{1}{R[[i, i]]}\left(A[[i, j]] - \sum_{k=1}^{i-1} R[[k, i]] * R[[k, j]]\right)$$

```
  ]
 ];
Print["R=", MatrixForm[R], ",    L=R^T=", MatrixForm[Transpose[R]]];
Print["L.R=", Transpose[R].R // MatrixForm]
```

$$A=\begin{pmatrix} 1. & 0. & 2. & 0 \\ 0. & 3. & -4. & 0. \\ 2. & -4. & 10. & 0. \\ 0. & 0. & 0. & 100. \end{pmatrix}$$

$$R=\begin{pmatrix} 1. & 0. & 2. & 0. \\ 0 & 1.73205 & -2.3094 & 0. \\ 0 & 0 & 0.816497 & 0. \\ 0 & 0 & 0 & 10. \end{pmatrix}, \quad L=R^T=\begin{pmatrix} 1. & 0 & 0 & 0 \\ 0. & 1.73205 & 0 & 0 \\ 2. & -2.3094 & 0.816497 & 0 \\ 0. & 0. & 0. & 10. \end{pmatrix}$$

$$L.R=\begin{pmatrix} 1. & 0. & 2. & 0. \\ 0. & 3. & -4. & 0. \\ 2. & -4. & 10. & 0. \\ 0. & 0. & 0. & 100. \end{pmatrix}$$